École doctorale SESAM

Laboratoire EQUIPPE

Thèse en Sciences Économiques soutenue le 15 octobre 2013

David Desmarchelier

Pollution externalities: a source of endogenous business cycles

Membres du jury :

Alain Ayong Le Kama (Rapporteur)

Professeur à l'université de Paris Ouest-Nanterre la Défense

Stefano Bosi

Professeur à l'université d' Evry Val d'Essonne, Directeur de thèse

Hubert Jayet

Professeur à l'université de Lille 1, Co-Directeur de thèse

Cuong Le Van (Rapporteur)

Directeur de recherche au C.N.R.S, Professeur à l'université de Paris 1

Lionel Ragot

Professeur à l'université de Paris Ouest-Nanterre la Défense

Thomas Seegmuller (Président de Jury)

Directeur de recherche au C.N.R.S, université d'Aix-Marseille

© 2014 Tous droits réservés.

Acknowledgments

I would like to express my deep gratitude to Professors Stefano Bosi and Hubert Jayet, my PhD supervisors, for their help and for all the fruitful discussions we had. As a teacher, Professor Stefano Bosi gave me his passion for time and for its representation which arguably constitute the essential elements of the present dissertation. As a laboratory director, Hubert Jayet gave me the possibility to present results of this dissertation to conferences in Europe (France, Germany and Portugal) and outside Europe (Russia). My intellectual debt to them is enormous.

I thank Professors Alain Ayong Le Kama and Cuong Le Van , who honor me by accepting to evaluate this work and I also thank Professor Lionel Ragot and Doctor Thomas Seegmuller who honor me by accepting to be members of my Jury.

I thank the laboratory EQUIPPE for hosting me during these three years. I would like to thank in particular my colleagues and friends: Kassoum Ayouba, Hamza Bennani, Radmila Datsenko, Marion Drut, Joseph Faye, Amaury Goguel, Aurélie Mahieux, Léa Marchal, Clément Nedoncelle, Romain Plassard, Paul Eliot Rabesandratana, Philippe Senave, Luo Yingchan and Linja Zhang. I wish them the best for their PhD.

© 2014 Tous droits réservés.

My twin brother, Benoît, takes a particular place in those acknowledgments. He has already completed his PhD in economics and He is now assistant professor in the Xi'an Jiaotong-Liverpool University. Our discussions since high school made me grow up. I wish him the best to realize his dreams.

I thank Charles Vandenbussche, my lifelong friend, for all those legendary night drives and to Thibault Corrion for his outstanding friendship and support.

Finally and most importantly, I dedicate this Thesis to my parents and to my partner, Véronique Drecq, for their unshakable support and for their encouragements during these long working years.

Lille, June 2013.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Il faut un certain courage pour pratiquer la pensée à long terme et prendre des décisions hardies, courageuses, anticipatrices dès que les problèmes apparaissent et avant qu'ils ne prennent des dimensions critiques. Il va à l'encontre de la prise de décision réactive à court terme qui caractérise trop souvent les élus politiques.

Jared Diamond, *Effondrement* (2005, page 789).

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Résumé

Depuis l'article de Zhang (1999), un nombre croissant de contributions académiques s'atèlent à explorer les canaux par lesquels la pollution peut être la source de cycles économiques endogènes. Nous sommes convaincus que cette ligne de recherche est d'une grande importance pour le décideur public car elle réconcilie ses impératifs de court terme avec le long terme qu'impose la préservation de l'environnement. C'est pourquoi, cette thèse se propose d'explorer de nouveaux canaux par lesquels la pollution peut induire l'apparition de cycles économiques endogènes.

Les chapitres 1,2 et 3 se basent sur des résultats empiriques récents arguant que la pollution agit negativement sur la productivité du travail et sur l'offre de travail. Au travers de ces chapitres, nous montrons que de tels effets de la pollution peuvent conduire à l'apparition de cycles économiques, tant déterministes que stochastiques, au voisinage de l'état stationnaire.

Le chapitre 4 se concentre sur l'étude du système de taxe verte existant dans laplupart des pays de l'OCDE. Nous montrons en particulier que sa régressivité par rapport aux revenus des ménages peut conduire à l'apparition d'équilibres à tâches solaires.

Mots-clés : Pollution, modèles OLG et à la Ramsey, bifurcations locales, indetermination

© 2014 Tous droits réservés.

http://doc.univ-lille1.fr

Abstract

Since Zhang (1999), a rising number of contributions explore channels by which pollution can induce endogenous business cycles. We believe that this research line is of great interest because it reconciles the short run imperative of policy leaders and the long run imperative of environmental preservation. Consequently, the present dissertation aims to contribute to this strand of literature by pointing out new channels by which pollution can induce endogenous business cycles.

Chapters 1, 2 and 3 depart from some new empirical findings who stress non-marginal negative effect of pollution on labor productivity and on labor supply. Within those chapters, we show that such pollution effects can lead to deterministic cycles as well as stochastic fluctuations around the steady state.

The chapter 4 is devoted to the study of the already existing green fiscal policies in most of OECD countries. We show in particular that their well-known regressivity, with respect to households' incomes, may promote sunspot equilibria.

key words: Pollution, Ramsey and OLG models, local bifurcations, indeterminacy

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Contents

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

General Introduction

Reading of the Diamond's *Collapse* (2005), it appears that all past and modern Human societies have been concerned by their ecological impacts. A good example from the past is to be found in the ancient Easter Island society, as its collapse seems to result from an extreme deforestation which destabilized the island ecosystem. Modern societies are also facing important environmental difficulties. Indeed, scientists are warning us about the global warming phenomenon: Earth's average surface temperature is rising since the beginning of the industrial revolution, and this rise will have severe consequences for Human activities. A recent report from the World Bank (2012) evaluates the dramatical consequences of a 4°C warmer world (than it was before the industrial revolution). Such a configuration will imply :

"the inundation of coastal cities; increasing risks for food production potentially leading to higher malnutrition rates; many dry regions becoming dryer, wet regions wetter; unprecedented heat waves in many regions, especially in the tropics; substantially exacerbated water scarcity in many regions; increased frequency of high-intensity tropical cyclones; and irreversible loss of biodiversity, including coral reef systems." (WB 2012).

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

There is an emerging consensus within the scientific community for imputing this global warming to human activities such as deforestation or burning of fossil fuels (WB 2012). Consequently, the World Bank advocates for stricter environmental policies. Since the seminal work of Denison (1979), who shows that stricter environmental regulations slow down the economic growth, it is a common belief that there exists a trade-off between growth and environmental protection. In our point of view, such a belief neglects the feedback from this pollution to human capital, which is the very engine of growth (Lucas 1988). This feedback on human capital is well documented, indeed :

1. Pollution increases children mortality (Chay and Greenstone 2003, Currie and Neidell 2005)
2. Pollution increases school absenteeism due to illness (Currie et al 2009, Frank et al 2001).
3. Pollution reduces life expectancy (Pautrel 2009, Mariani et al 2010).
4. Pollution reduces the labor supply (Hanna and Oliva 2011, Graff Zivin and Neidell 2010).
5. Pollution reduces labor productivity (Schlenker and Walker 2011 and Graff Zivin and Neidell 2012).

Those evidences show that pollution has a non marginal negative effect on human health and in turn on human capital. To this respect, environmental policies should

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

rather be seen as an investment in human capital. Despite the common belief, such policies could sustain the economic growth over the long run (Van Ewijk and Van Wijnbergen 1995 ; Bovenberg and Mooij 1997).

Despite these long run potential benefits, environmental policies are not on top of political leader's priorities. We belief that it is because they are above all concerned by short run problematics (Nordhaus 1975). Fortunately, over the last decade, some scholars have explored the possible short run economic effects of pollution. This strand of literature, introduced by Zhang (1999), tries to explore the mechanisms by which pollution can be a very source of business cycles. In what follows, we review this literature.

0.1 Pollution and endogenous fluctuations

Since Ramsey (1928), it is usual to represent the economy in the form of a dynamical system. Such a system gives many informations about how the economy approaches a steady state. In most cases, the trajectory followed by the economy is unique and monotonous. For example, the basic Ramsey framework (1928) is characterized by a unique monotonous trajectory leading to a unique saddle-path stable steady state[1].

However, under certain conditions, the trajectory followed by the economy can loose its uniqueness or monotonicity. Such a change in the behavior of a dynamical system is called a *bifurcation*, it defines a sudden change in the stability properties of the system following an arbitrarily small perturbation of a fundamental parameter

[1] See Koopmans (1965) among the others.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

(Bosi and Ragot 2011).

The literature that we are concerned exploits the possible occurrence of bifurcation due to pollution in socio-economic systems. Regarding the literature, we have found four channels by which pollution can induces endogenous business cycles through a bifurcation :

1. The existence of persistent habits of environmental quality (Chen and Li 2011 or Schumacher and Zou 2008).
2. The rate of pollution emission (Seegmuller and Verchère 2004-2007 ; Cao, Wang and Wang 2011 ; Zhang 1999 ; Antoci and Sodini 2009).
3. Pollution in utility function jointly with public policy. (Fernandez, Pérez and Ruiz 2012 ; Itaya 2008 ; Pérez and Ruiz 2007)
4. The effect of pollution on the household's discount factor (Yanase 2011).

In what follows, we will discuss each of those channels.

0.1.1 The existence of persistent habits of environmental quality

The frameworks used by Schumacher and Zou (2008) and by Chen and Li (2011) are very close. They develop an OLG model, in the spirit of John and Pecchenino (1994,1995), in which pollution dynamics is seen through the evolution of an environmental asset assumed to be a stock variable. As in most Environmental-growth

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

OLG models, the representative household lives for two periods, can invest in depollution but derives utility only in his old age. The main difference between those two contributions rests on the fact that Schumacher and Zou (2008) consider the pollution accumulation (P) while Chen and Li (2011) take into account the environmental quality (E). These scholars show that persistence of habits of environmental quality does matter for the occurrence of business cycles. Indeed, following Chen and Li (2011), the utility function (U) of an agent born at time t is defined by :

$$U = \ln(c_{t+1}) + \eta \ln(E_{t+1} - \phi E_t)$$

They show that a high ϕ implies local stability while a low ϕ induces chaos through a flip bifurcation if the pollution emission rate is high enough. Schumacher and Zou (2008) show also that there exists a value of ϕ for which a limit cycle can emerge through a Hopf bifurcation.

In addition, Chen and Li (2011) analyze the short run effects of a green consumption tax. They find that such a tax reduces the likelihood of endogenous business cycles.

0.1.2 The rate of pollution emission

In a framework very close to Chen and Li (2011) but without persistent environmental habits, Antoci and Sodini (2009), Cao, Wang and Wang (2011), Seegmuller and Verchère (2004)(2007), and Zhang (1999) have shown that the rate of pollution emission can destabilize the economy.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Following Seegmuller and Verchère (2004), pollution accumulates according to the following equation :

$$P_{t+1} = (1-m)\,P_t + ak_t - bd_t$$

where m, a and d denote respectively the natural rate of pollution absorption, the emission rate of pollution by unit of capital (k) and the coefficient which capture the abatement (d) efficacity. In this context, Seegmuller and Verchère (2004), Cao, Wang and Wang (2011) and Zhang (1999) show that if the emission rate of pollution (a) is sufficiently high regarding the coefficient of abatement (d), then two-period cycles emerge through the occurrence of a flip bifurcation. In addition, Seegmuller and Verchère (2007) show that when pollution is seen as a flow ($m = 0$), then a low emission rate of pollution (a) induces local indeterminacy. Zhang (1999) insists on the possibility of chaos while Cao, Wang and Wang (2011) apply the delayed feedback control method to stabilize the fluctuation due to the flip bifurcation.

The framework developed by Antoci and Sodini (2009) differs from the previous mentioned contributions in the sense that it assumes a non-linear linkage between the environment and the production level, indeed, for those authors :

$$E_{t+1} = \overline{E} - \eta\,[Y_t]^{\beta}$$

where $\overline{E}$ denotes the environmental quality without production activities (Y). Antoci and Sodini (2009) use this non linearity (captured by β) to show that local indeterminacy occurs through a saddle-node bifurcation.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

0.1.3 Pollution in utility function jointly with public policy

Fernandez, Pérez and Ruiz (2012), Itaya (2008) and Pérez and Ruiz (2007) study the dynamic properties of an endogenous growth model in which pollution is seen as a flow. Using a Romer's (1986) learning-by-doing model, with endogenous labor supply and in which pollution is seen as a by-product of firms' activities, Itaya (2008) shows that the effect of pollution on marginal utility of consumption makes indeterminacy more likely to occur. Indeed, since Pelloni and Waldman (2000), it is well-known that such a model, without environmental externalities, exhibits indeterminacy only for an intertemporal elasticity of substitution in consumption greater than unity. With a pollution externality, Itaya (2008) shows that indeterminacy may well arise, even if this condition is not satisfied. A very similar result is pointed out by Fernandez, Pérez and Ruiz (2012).

In addition, Itaya (2008) assumes that a Government levies a lump-sum tax on firm's activities and shows that, if the tax revenue is used to finance public abatement expenditures, it is more difficult for indeterminacy to emerge.

Pérez and Ruiz (2007) use a model *à la* Barro (1990) in which the Government finance, on one hand depollution expenditures and on the other hand, spendings that increases private labor productivity. In such a framework, these authors show that indeterminacy comes to the interaction between the two externalities, namely the negative pollution externality (through its effect on the utility function) and the positive one due to public productive services.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

0.1.4 The effect of pollution on the household's discount factor

Yanase (2011) develops a Ramsey model in which pollution affects the utility function and the individual discount factor, as in Ayong Le Kama and Schubert(2007). The main difference between these two contributions rests on the fact that Ayong Le Kama and Schubert(2007) analyze the social planner' solution while Yanase (2011) focuses on a decentralized equilibrium. In other words, environmental effects on the discount factor are not internalized by the representative household in Yanase (2011). In this framework, the Author shows that a positive linkage between pollution and the discount factor does matter for equilibrium indeterminacy.

0.2 Objective and realizations

The previous mentioned results are of a great interest, because they reconcile the short run thinking of policy leaders and the long run imperative of environment preservation. This is especially true if stricter environmental policies can fight the volatility induced by pollution (Itaya 2008).

The present dissertation aims to contribute to this new literature by exploring new mechanisms by which pollution can promote endogenous business cycles. It is organized in four chapters.

Within the first chapter, we note that the literature, who analyzes the occurrence

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

of endogenous business cycles induced by pollution in OLG economies, remains focused on the effects of pollution on the household' satisfaction. This observation is quite surprising because :

1. The literature on endogenous business cycles has stressed the role of technology in promoting deterministic as well as stochastic fluctuations in OLG economies (Azariadis 1981, Grandmont et al 1998, Cazzavillan et al 1998).
2. As we have already underlined, there is strong empirical evidences showing the negative effect of pollution on the labor productivity.

Starting from this observation, we develop a monetary OLG economy *à la* Samuelson (1958) in which pollution has a negative effect on labor productivity. In this very simple framework, we find that under dominant income effects, a lower pollution elasticity of labor productivity may promote the emergence of sunspot equilibria[2] through a Hopf bifurcation. This new result shows that the Graff Zivin and Neidell (2012)(among the others) empirical finding works as a destabilizing force for the economy.

In the second and third chapters, we note a certain disconnection between theoretical and empirical literatures. At first, number of theoretical papers analyze how pollution affect consumption behavior, while to the best of our knowledge, there is

[2] In this thesis, expectation-driven fluctuations, sunspot equiliria and local indeterminacy are used interchangeably.

© 2014 Tous droits réservés.
http://doc.univ-lille1.fr

no empirical evidence on such a phenomenon. Secondly, there is a rising number of empirical papers studying how pollution affects the labor supply, while such an effect is largely ignored in theoretical contributions.

From an empirical ground, the magnitude of the pollution effect on the labor supply is strong. For example, Hanna and Oliva (2011) point out that a one percent increase in air pollution results in a 0.61 percent decrease in worked hours. Departing from this result, we investigate its short and long run macroeconomic incidences in chapter 2 and 3.

In order to keep things simple, we choose to analyze this effect through a Ramsey model with separable preferences between consumption and labor supply.

In chapter 2, we assume non separable preferences between pollution and labor supply (separability rules out any direct effect of pollution on labor supply). In this very simple framework, we find that a sufficiently large (negative) effect of pollution on labor supply may promotes deterministic cycles (near the steady state) through a flip bifurcation.

In chapter 3, we assume non separable preferences between consumption and pollution and between labor supply and pollution. Through this framework, we find that the join pollution effects on consumption and on labor supply allows for a wide range of dynamics and in particular, local indeterminacy through saddle-node and Hopf bifurcations.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

These new results indicate that the observed pollution effects on labor supply promotes macroeconomic volatility through deterministic as well as stochastic fluctuations.

In chapter 4, we explore the short run incidence of the existing green fiscal policies. A lot of scholars have stressed the regressive nature of those policies with respect to households' incomes (Grainger and Kolstad (2009), Hasset, Mathur and Metcalf (2007), West and Williams (2004) among the others). Throughout this chapter, we develop a discrete time Ramsey economy that sustain such empirical evidences and we found in particular that sunspot equilibria occur if and only if:

1. Pollution has a sufficiently strong positive effect on marginal utility of consumption.

2. The ecotax regressivity is sufficiently strong.

3. The elasticity of capital-labor substitution is not too low.

It is well-known that the regressive feature of green fiscal policies implies negative distributional effects (Grainger and Kolstad (2009)), our result add another argument against such policies in a sense this regressivity may promote expectation-driven fluctuations.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Chapter 1

Demography and the effect of pollution on labor productivity

1.1 Introduction

In this chapter[1], we consider the dynamic interplay between demography and pollution. On the one side, pollution affects the productivity of workers because of its impact on health (Schlenker and Walker 2011). On the other side, demography changes the supply of labor, its impact on growth and pollution in turn. We consider short and long run effects, but the novelty mainly rests on the analysis of equilibrium multiplicity.

The dynamic aspects of pollution have been considered either in economies à la Ramsey or in OLG models. This chapter contributes to the OLG literature, pioneered by John and Pecchenino (1994) where the issue of sustainability of economic

[1] This chapter refers to a joint work with Stefano BOSI.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

growth was addressed.

In OLG framework à la Diamond (1965) (with capital accumulation), Seegmuller and Verchère (2004) and Schumaker and Zou (2008) prove that, when the consumer chooses between consumption and environmental quality, endogenous business cycles may appear in a neighborhood of the steady state. Seegmuller and Verchere (2007) study an OLG economy where households arbitrate between leisure, consumption and environmental quality. Considering pollution as a flow, these authors find that a low pollution emission rate promotes the emergence of sunspot equilibria. Those contributions insist on the role of the household's concern toward environmental quality for explaining endogenous business cycles. But, one can raise the question whether technology or preferences matter more in the occurrence of such endogenous fluctuations. Even if the debate is not ended, Azariadis (1981) stresses the role of technology in promoting equilibrium multiplicity. In this respect, Grandmont et al. (1998) focuses on the degree of capital-labor substitution and Cazzavillan and al. (1998) on the increasing returns to scale to demonstrate the existence of sunspot equilibria. Conversely, the level of gross substitutability between consumption and labor seems to play a little role to explain the equilibrium indeterminacy in OLG economies without pollution.

To rule out any misleading interference from capital accumulation, we focus on a simple monetary model à la Samuelson (1958) where money represents household's savings. In addition, we consider labor supply in terms of endogenous fertility as in Galor and Weil (1996) with an environmental dimension. In contrast to Seegmuller

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

and Verchere (2007), pollution does not enter household utility, but simply lowers labor productivity (Schlenker and Walker 2011). Moreover, we consider pollution as a stock instead of a flow, that is as a predetermined variable, and, in this sense, we remain close to John and Pecchenino (1994). In this context, we study the long run effects of pollution and demography on consumption. In the short run, we find that, under dominant income effects, a lower pollution elasticity of labor productivity may promote the emergence of sunspot equilibria through a Hopf bifurcation. In this sense, the joint effect of technology (through the externalities of pollution) and preferences (through the elasticity of intertemporal substitution) seems to play a role for the occurrence of endogenous fluctuations.

The chapter is organized as follow. In the next section, we present the model. The market clearing conditions are given in Section 3. In Sections 4 and 5, we study the steady state and the stability properties of equilibrium. In addition, we provide an interpretation for the effects of pollution in the long run and the occurrence of fluctuations in the short run. Section 6 concludes.

1.2 The model

The economy consists of an infinite sequence of overlapping generations living three periods: childhood, adulthood and the old age. Time is discrete and is indexed by $t = 0, 1, \ldots$ Agents self-replicate and divide their income between consumption and the number of children desired. They derive no satisfaction in childhood, they have children in adulthood and they consume in the old age. In this economy, a

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

single consumption good is produced using a technology with labor as single input. Without capital market, the agents transfer income from adulthood to the old age gaining currency. In addition, production generates pollution that reduces labor productivity.

1.2.1 Producers

There are q firms with no market power that transform a unique input l_{jt} (labor demand) in a unique output y_{jt} with $j = 1, \ldots, q$. Technology is represented by a linear function:

$$y_{jt} = A_t l_{jt}$$

The firm j maximizes the profit defined by :

$$\pi_{jt} \equiv p_t y_{jt} - w_t l_{jt} = p_t A_t l_{jt} - w_t l_{jt}$$

Let ω_t denote the real wage: $\omega_t \equiv w_t/p_t$. Profit maximization implies, at equilibrium, that the real wage is equal to the productivity of labor:

$$\omega_t = A_t$$

The aggregate production Y_t is given by

$$Y_t \equiv \sum_{j=1}^{q} y_{jt} = A_t \sum_{j=1}^{q} l_{jt} \equiv A_t L_t \tag{1.1}$$

and depends linearly on the aggregate labor demand L_t.

1.2.2 Consumers

The economy is populated by individuals who live for three periods. Consider an individual who is born at time $t-1$.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

During childhood (period $t-1$), he neither works nor consumes. The introduction of childhood does not matter in the model, but allows us to justify that an individual makes children only in his adult period.

In the working age (period t), he supplies labor at a nominal wage rate w_t, makes n_t children and saves through nominal balances m_{t+1} without consuming. For simplicity, we assume that an individual can generate n_t children alone. Rearing children takes time: z is the constant leisure-time needed per child. So, the opportunity cost of rearing children is given by $w_t z n_t$. We normalize the endowment of leisure time in the adult period to one. The individual labor supply turns out to be endogenous as a result of the endogenous fertility: $l_t = 1 - z n_t$.

At the end of his life-cycle (period $t+1$), the individual consumes the quantity c_{t+1} using the monetary savings.

Money is the numeraire and the budget constraints of second and third period become:

$$m_{t+1} \leq w_t \left(1 - z n_t\right) \tag{1.2}$$

$$p_{t+1} c_{t+1} \leq m_{t+1} \tag{1.3}$$

In addition, we require $n_t \leq 1/z$.

Rearing children is costly, but children represent a utility for parents. For simplicity, preferences are separable:

$$u\left(n_t\right) + v\left(c_{t+1}\right) \tag{1.4}$$

and satisfy the standard neoclassical assumptions.

Assumption 1 : *The utility functions $u, v : R_+ \rightarrow R$ are C^2, strictly increasing and concave: $u'\left(n_t\right), v'\left(c_{t+1}\right) > 0$, $u''\left(n_t\right), v''\left(c_{t+1}\right) \leq 0$ for $n_t, c_{t+1} > 0$. Additional*

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

boundary conditions hold: $u'(0), v'(0) = +\infty$ *and* $u'(+\infty), v'(+\infty) = 0$.

Since the utility functions are strictly increasing, the budget constraints are binding. (1.2) and (1.3) become

$$n_t = \frac{1}{z}\left(1 - \frac{m_{t+1}}{w_t}\right) \tag{1.5}$$

$$c_{t+1} = \frac{m_{t+1}}{p_{t+1}} \tag{1.6}$$

Budget constraint (1.6) states that real balances are equal to consumption:

$$c_t = \frac{m_t}{p_t} \tag{1.7}$$

Replacing (1.5) and (1.6) in (1.4), we obtain an equivalent program:

$$\max_{m_{t+1}} \left[u\left(\frac{1}{z}\left(1 - \frac{m_{t+1}}{w_t}\right)\right) + v\left(\frac{m_{t+1}}{p_{t+1}}\right)\right]$$

where saving (money demand) is the unique choice variable. Maximization gives the parity-consumption arbitrage (the lower the parity, the higher the labor income and the consumption):

$$u'(n_t) = \frac{z\omega_t}{\pi_{t+1}} v'(c_{t+1}) \tag{1.8}$$

where $\pi_{t+1} \equiv p_{t+1}/p_t$ is now the inflation factor.

1.2.3 Pollution

P_t denotes the aggregate stock of pollution at time t which is a pure externality. The pollution mechanism is two-sided: (1) production affects the pollution stock and (2) pollution in turn affects the production level.

Past pollution persists and technology is dirty. More precisely, we assume that the current stock of pollution depends on past pollution and the past level of eco-

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

nomic activity according to a linear process:

$$P_t = (1-\alpha) P_{t-1} + \gamma Y_{t-1} \tag{1.9}$$

where $\alpha \in (0,1]$ is the rate of natural absorption and γ captures the environmental impact of economic activity (see John and Pecchenino (1994) among the others).

The aggregate externality of pollution

$$A_t = A(P_t) \tag{1.10}$$

lowers the labor productivity as follows.

Assumption 2 : *The productivity function $A : R_+ \rightarrow R_+$ is C^1 and strictly decreasing: $A'(P_t) < 0$ for every $P_t \geq 0$. The following boundary conditions hold: $A(0) > 0$ and $A(+\infty) = 0$.*

A pollution elasticity of production is introduced:

$$\sigma(P_t) \equiv -\frac{P_t A'(P_t)}{A(P_t)} \in (0, +\infty)$$

For simplicity, we do not assume a direct effect of pollution on consumers' preferences.

1.3 Equilibrium

Let N_{t-1} the size of the generation born at time $t-1$. In this economy, there are three markets: the money market, the labor market and the goods market. In the following, we will investigate conditions for which each market is at the equilibrium.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

1.3.1 Money market

For simplicity, money supply is constant over time and equal to M. The equilibrium in the money market requires

$$M = m_{t+1} N_{t-1} \tag{1.11}$$

where $m_{t+1} N_{t-1}$ represents the aggregate demand for nominal balances.

1.3.2 Labor market

Individual labor supply at time t is given by $1 - z n_t$. The size of the working class at time t is N_{t-1}. Then, the equilibrium in the labor market is given by

$$N_{t-1} \left(1 - z n_t\right) = L_t \tag{1.12}$$

We observe that the working class growth factor is endogenous and given by

$$n_t = \frac{N_t}{N_{t-1}}$$

at time t.[2]

1.3.3 Goods market

The aggregate supply at period t is Y_t and is consumed by N_{t-2} old individuals (indeed, the old of period t are born at time $t-2$). Individual consumption at time

[2] Don't confuse this factor with the demographic growth factor

$$\delta_{t+1} \equiv \frac{N_{t-1} + N_t + N_{t+1}}{N_{t-2} + N_{t-1} + N_t} = \frac{1 + n_t + n_t n_{t+1}}{1 + n_{t-1} + n_{t-1} n_t} n_{t-1}$$

where $N_{t-2} + N_{t-1} + N_t$ is the size of population at the end of period t after the birth of children and before the death of the old.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

t is given by c_t. Equilibrium in the goods market requires

$$Y_t = N_{t-2} c_t \tag{1.13}$$

1.3.4 Intertemporal equilibrium

Focus on the money market. (1.11) gives $M = m_t N_{t-2} = m_{t+1} N_{t-1}$and by considering (1.7) it follow that :

$$\frac{c_t}{c_{t+1}} = \pi_{t+1} n_{t-1}$$

Focus on the real market. Budget constraints (1.5) and (1.6) give

$$n_t = \frac{1}{z}\left(1 - c_{t+1}\frac{\pi_{t+1}}{\omega_t}\right) \tag{1.14}$$

while the children-consumption arbitrage (1.8) writes

$$\frac{\pi_{t+1}}{\omega_t} = z\frac{v'(c_{t+1})}{u'(n_t)} \tag{1.15}$$

Replacing (1.15) in (1.14), we obtain

$$\frac{v'(c_{t+1})}{u'(n_t)} = \frac{1 - zn_t}{zc_{t+1}} \tag{1.16}$$

by applying the Implicit Function Theorem on (1.16), we obtain that the number of children made by the household is a function of the future consumption:

$$n_t = n(c_{t+1}) \tag{1.17}$$

Totally differentiating (1.16), we get

$$n'(c_{t+1}) = -\frac{zv'(c_{t+1}) + zc_{t+1}v''(c_{t+1})}{zu'(n_t) - (1 - zn_t)u''(n_t)} \tag{1.18}$$

We introduce the elasticities of intertemporal substitution:

$$\eta(n_t) \equiv -\frac{u'(n_t)}{n_t u''(n_t)} \geq 0$$
$$\theta(c_t) \equiv -\frac{v'(c_t)}{c_t v''(c_t)} \geq 0$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

We obtain

$$n'(c_{t+1}) = \frac{1-\theta(c_{t+1})}{\theta(c_{t+1})} \frac{zn_t\eta(n_t)}{1-zn_t+zn_t\eta(n_t)} \frac{v'(c_{t+1})}{u'(n_t)} \tag{1.19}$$

Since $zn_t < 1$, we have that $n'(c_{t+1}) > 0$ iff $\theta(c_{t+1}) < 1$ (dominant income effects).

We can introduce the equilibrium elasticity of natality:

$$\varepsilon(c_t) \equiv \frac{c_t n'(c_t)}{n(c_t)}$$

Replacing (1.16) in (1.19), we obtain

$$\varepsilon(c_{t+1}) = \frac{1-\theta(c_{t+1})}{\theta(c_{t+1})} \frac{(1-zn_t)\eta(n_t)}{1-zn_t+zn_t\eta(n_t)} \tag{1.20}$$

As above, $\varepsilon(c_{t+1}) > 0$ iff $\theta(c_{t+1}) < 1$ (dominant income effect).

From (1.1), (1.13), (1.10) and (1.12), the equilibrium in the goods market writes

$$N_{t-2}c_t = Y_t = A_t L_t = A(P_t) N_{t-1}(1-zn_t)$$

that is

$$c_t = A(P_t) n(c_t) [1 - zn(c_{t+1})] \tag{1.21}$$

Focus on pollution.

From (1.9), we have

$$\frac{Y_{t+1}}{Y_t} = \frac{P_{t+2} - (1-\alpha) P_{t+1}}{P_{t+1} - (1-\alpha) P_t} \tag{1.22}$$

Replacing (1.13) and (1.17) in (1.22), we obtain

$$n(c_t) \frac{c_{t+1}}{c_t} = \frac{P_{t+2} - (1-\alpha) P_{t+1}}{P_{t+1} - (1-\alpha) P_t}$$

Let us introduce the new variable

$$Q_t \equiv P_{t+1} \tag{1.23}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

to write

$$n(c_t)\frac{c_{t+1}}{c_t}=\frac{Q_{t+1}-(1-\alpha)P_{t+1}}{Q_t-(1-\alpha)P_t} \tag{1.24}$$

The main proposition follows:

Proposition 1 *An intertemporal equilibrium with perfect foresight is a non-negative sequence* $(c_t, P_t, Q_t)_{t=0}^{\infty}$ *satisfying equations*

$$n(c_t)\frac{c_{t+1}}{c_t}=\frac{Q_{t+1}-(1-\alpha)P_{t+1}}{Q_t-(1-\alpha)P_t} \tag{1.25}$$

$$c_t = A(P_t)\,n(c_t)\left[1-zn(c_{t+1})\right] \tag{1.26}$$

$$Q_t \equiv P_{t+1} \tag{1.27}$$

We observe that P_t is a predetermined variable at time t, while Q_t is non-predetermined. In addition, $c_t = m_t/p_t$ is non-predetermined at time t because p_t is non-predetermined.

1.4 Steady state

At the steady state, all variables are constant, thus, $c_{t+1} = c_t$ and $Q_{t+1} = Q_t = P_{t+1} = P_t$.Noting c and P the consumption and the the pollution at the steady state. By regarding (1.25), it follow that :

$$n(c) = 1$$

$$c = (1-z)\,A(P)$$

We notice also that the elasticity of natality becomes

$$\varepsilon(c)=\frac{1-\theta(c)}{\theta(c)}\frac{(1-z)\,\eta(1)}{1-z+z\eta(1)} \tag{1.28}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Under Assumptions 1 and 2, functions $n_t = n(c_{t+1})$ and $A_t = A(P_t)$ are invertible, so $c = n^{-1}(1)$. By substitution we get the steady-state value for the pollution stock:

$$P = A^{-1}\left(\frac{n^{-1}(1)}{1-z}\right)$$

therefore, there is a unique non-trivial steady state for this economy. Noting that at the steady-state, the demographic growth rate is 0. It is not a surprising conclusion in an exogenous economic growth framework.[3]

1.4.1 Example

We consider more explicit fundamentals. Let the elasticities $\eta(n_t)$ and $\theta(c_t)$ to be constant, with $\eta(n_t) = 1$ (logarithmic preferences for children) and $\theta(c_t) = \theta \in (0, +\infty)$. More precisely, consider the following utility function:

$$u(n_t) + v(c_{t+1}) \equiv \ln n_t + \beta \frac{c_{t+1}^{1-1/\theta}}{1-1/\theta} \tag{1.29}$$

Equation (1.16) gives

$$n_t = n(c_{t+1}) = \frac{1}{z\left(1+\beta c_{t+1}^{\frac{\theta-1}{\theta}}\right)} \tag{1.30}$$

At the steady state, we have $n(c) = 1$ and, so,

$$c = \left(\frac{\beta z}{1-z}\right)^{\frac{\theta}{1-\theta}} \tag{1.31}$$

In addition, we know that $c = (1-z) A(P)$. Setting

$$A(P_t) \equiv B P_t^{-\sigma} \tag{1.32}$$

[3] The same conclusion holds in Eckstein, Stern and Wolpin (1988).

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

we get

$$P=\left(B\frac{1-z}{c}\right)^{\frac{1}{\sigma}}=\left[B\left(1-z\right)\left(\frac{1-z}{\beta z}\right)^{\frac{\theta}{1-\theta}}\right]^{\frac{1}{\sigma}} \tag{1.33}$$

We get always meaningful interior solutions with $0<z<1$, $\sigma>0$ and $\theta\neq 1$.

Focus on the impact of parameters on the steady state. Consider the impact of parameters β, z and σ on the stationary levels of consumption and pollution c and P.

Proposition 2 *Under Assumptions 1 and 2 and specifications (1.29) and (1.32), we obtain $\partial P/\partial\sigma<0$ iff $P>1$, and $\partial c/\partial\sigma=0$. In addition,*

$$\frac{\partial c}{\partial\beta}>0,\ \frac{\partial c}{\partial z}>0,\ \frac{\partial P}{\partial\beta}<0,\ \frac{\partial P}{\partial z}<0 \textit{ iff } \theta<1$$

Proof. Differentiating (1.31) and (1.33), we get

$$\begin{aligned}
&\frac{\beta}{c}\frac{\partial c}{\partial\beta}=\frac{\theta}{1-\theta},\ \frac{z}{c}\frac{\partial c}{\partial z}=\frac{\theta}{1-\theta}\frac{1}{1-z},\ \frac{\sigma}{c}\frac{\partial c}{\partial\sigma}=0\\
&\frac{\beta}{P}\frac{\partial P}{\partial\beta}=-\frac{1}{\sigma}\frac{\theta}{1-\theta},\ \frac{z}{P}\frac{\partial P}{\partial z}=-\frac{1}{\sigma}\left(\frac{\theta}{1-\theta}+z\right)\frac{1}{1-z},\ \frac{\sigma}{P}\frac{\partial P}{\partial\sigma}=-\ln P
\end{aligned}$$

Proposition 2 follows. ■

Let us provide some intuitions for comparative statics.

Labor productivity $A(P)\equiv BP^{-\sigma}$ decreases with σ iff $P>1$. Thus, production and pollution decrease as well.

σ has no effects on the consumption because the natality function n in (1.30) does not depend on σ and $n=1$ at the steady state.

When β or z increases, individuals like less the parentage (expression (1.29)). As a general equilibrium effect, the relative price of children with respect to future consumption lowers. In the case of dominant income effect ($\theta<1$), this entails an increase in future consumption, while, in the case of dominant substitution effects

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

($\theta > 1$), a reduction in future consumption. Focus now on future variables: a higher (lower) consumption level requires a higher (lower) productivity of labor and, eventually, a lower (higher) pollution level.

1.5 Local dynamic

Consider now the general case. In order to study the local stability, we linearize system (1.25) to (1.27) around the steady state:

$$\begin{aligned}
\frac{1-\alpha}{\alpha}\frac{dP_{t+1}}{P_{t+1}} - \frac{1}{\alpha}\frac{dQ_{t+1}}{Q_{t+1}} + \frac{dc_{t+1}}{c_{t+1}} &= \frac{1-\alpha}{\alpha}\frac{dP_t}{P_t} - \frac{1}{\alpha}\frac{dQ_t}{Q_t} + (1-\varepsilon)\frac{dc_t}{c_t} \\
z\varepsilon\frac{dc_{t+1}}{c_{t+1}} &= -\sigma(1-z)\frac{dP_t}{P_t} + (\varepsilon-1)(1-z)\frac{dc_t}{c_t} \\
\frac{dP_{t+1}}{P_{t+1}} &= \frac{dQ_t}{Q_t}
\end{aligned}$$

This system is represented by the following Jacobian matrix:

$$J = \begin{bmatrix} 0 & 1 & 0 \\ \alpha - 1 - \alpha\frac{\sigma}{\varepsilon}\frac{1-z}{z} & 2-\alpha & \alpha\frac{\varepsilon-1}{\varepsilon}\left(\varepsilon + \frac{1-z}{z}\right) \\ -\frac{\sigma}{\varepsilon}\frac{1-z}{z} & 0 & \frac{\varepsilon-1}{\varepsilon}\frac{1-z}{z} \end{bmatrix} \tag{1.34}$$

What does it happens when the elasticity of natality ε is sufficiently close to one? According to (1.28), this case corresponds to the following elasticity of intertemporal substitution in consumption:

$$\theta^* = \frac{(1-z)\eta}{1-z+\eta} \in (0,1)$$

that is a case where, concerning the consumption choice, the income effect dominates the substitution effect.

The case of dominant income effects is relevant on the empirical ground. The existing literature does not provide a definitive estimate for the elasticity of intertemporal substitution in consumption θ. Although many standard RBC models

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

(Hansen (1985), King et al. (1988) among the others) consider relatively high values (around unity), more recent empirical works adopt values around 0.5 (Campbell (1999) suggest $(0.2, 0.6)$ as plausible interval).[4]

When $\theta = \theta^*$, the Jacobian matrix becomes

$$J^* = \begin{bmatrix} 0 & 1 & 0 \\ \alpha - 1 - \alpha\sigma\frac{1-z}{z} & 2-\alpha & 0 \\ -\sigma\frac{1-z}{z} & 0 & 0 \end{bmatrix}$$

with eigenvalues

$$\lambda_0^* \equiv 0$$

$$\lambda_1^* \equiv 1 - \frac{\alpha}{2} - \frac{1}{2}\sqrt{\alpha^2 - 4\alpha\sigma\frac{1-z}{z}} \tag{1.35}$$

$$\lambda_2^* \equiv 1 - \frac{\alpha}{2} + \frac{1}{2}\sqrt{\alpha^2 - 4\alpha\sigma\frac{1-z}{z}} \tag{1.36}$$

It is worthy to focus on values of θ in a neighborhood of $\theta^* \in (0,1)$ (region of dominant income effects). Under the assumptions of the Großman-Hartman Theorem (see Guckenheimer and Holmes (1983) among others)), the linearized dynamics are topologically equivalent to the nonlinear ones in a neighborhood of the steady state. In particular, this theorem requires $\det J \neq 0$, that is $\theta \neq \theta^*$. When θ is close to θ^* but different from θ^*, by continuity, the eigenvalues λ_0, λ_1, λ_2 of matrix (1.34) lies in a neighborhood of the eigenvalues λ_0^*, λ_1^*, λ_2^*.

Let us introduce two critical values for the pollution elasticity of production.

$$\sigma^* \equiv \frac{\alpha}{4}\frac{z}{1-z} < \frac{z}{1-z} \equiv \sigma_H$$

[4] If, for instance, $\eta = 1$ and $z = 1/2$

$$\theta = \frac{1-z}{2-z} = \frac{1}{3} \in (0.2, 0.6)$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Proposition 3 *Let θ be close to θ^* (dominant income effects).*

If $0 < \sigma < \sigma^*$, then λ_0, λ_1 and λ_2 are real with $\lambda_0 \approx 0 < \lambda_1 < \lambda_2 < 1$. Thus, equilibrium indeterminacy.

If $\sigma^* < \sigma < \sigma_H$, then λ_0 is real while λ_1 and λ_2 are nonreal with $\lambda_0 \approx 0$, $|\lambda_1| < 1$, $|\lambda_2| < 1$. Thus, equilibrium indeterminacy.

If $\sigma > \sigma_H$, then λ_0 is real while λ_1 and λ_2 are nonreal with $\lambda_0 \approx 0$, $|\lambda_1| > 1$, $|\lambda_2| > 1$. Thus, equilibrium determinacy.

When $\sigma = \sigma_H$, the economic system generically undergoes a Hopf bifurcation and a limit cycle arises around the steady state.

Proof. Simply, consider expressions (1.35) et (1.36). ∎

Woodford (1986) shows that equilibrium indeterminacy is not only a necessary but also a sufficient condition for sunspot equilibria.

To provide an economic intuition for the emergence of self-fulfilling prophecies in our model, focus on the case where the income effects dominate the substitution effects. Suppose that, at time t, the economy is at the steady state and suppose that the representative household rationally anticipates an exogenous increase in the pollution stock at time $t+1$. From (1.10) and (1.1), we have that, if P_{t+1} increases, then Y_{t+1} decreases. Market clearing condition (1.13) implies a decrease in the future consumption level c_{t+1}. Since the income effect is supposed to be stronger than the substitution one, we obtain a decrease in the parentage at time t and so an increase in the time devoted to work. This higher labor supply raises the output Y_t produced at time t and, eventually, the pollution (stock) at time $t+1$, making the prophecy self-fulfilling.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

1.6 Conclusion

We have considered and OLG economy with elastic labor supply and pollution externalities. As in John and Pecchenino (1994), but differently from Seegmuller and Verchere (2007), we have treated pollution as a stock and we have characterized the long run equilibrium and shown how sunspot equilibria can appear, under dominant income effects, for sufficiently low values of pollution elasticity of labor productivity (Schlenker and Walker 2011). Our argument takes in account technological aspects that the existing literature on pollution externalities has failed to consider.[5]

[5] See Schumacher and Zou (2008) and Seegmuller and Verchère (2007) among others.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Chapter 2

Pollution effects on labor supply and growth

2.1 Introduction

In the last chapter[1], we have shown that the negative effect of pollution on labor productivity, empirically stressed by Schlenker and Walker (2011) or Graff Zivin and Neidell (2012), does matter for the occurrence of endogenous business cycles. In the following chapter, we will investigate the pollution effects on labor supply. Indeed, recent empirical studies have documented nonmarginal impacts of pollution on labor supply (Graff Zivin and Neidell (2010), Carson, Koundouri and Nauges (2011), Hanna and Oliva (2011)). For example, using a recent data set for Mexico City, Hanna and Oliva (2011) find that a one percent increase in air pollution results in a 0.61 percent decrease in the worked hours. The magnitude of this phenomenon is quite surprising and leads us to address the question of its macroeconomic incidence

[1] This chapter refers to a joint work with Stefano BOSI and Lionel RAGOT.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

in the short and the long run.

The literature on the interplay between pollution and growth seems to neglect the possible influence of pollution on the consumption-leisure arbitrage and thereby on the labor supply. Pollution may affect labor demand through negative external effects on Total Factor Productivity (TFP), but the impact on labor supply is largely ignored, at least from a theoretical point of view.

In this respect, the theoretical effects of pollution on labor supply seem to be ambiguous. On the one hand, pollution may worsen working conditions (for instance, the negative impact of global warming rests on a positive correlation between heat and work painfulness) and give an incentive to substitute leisure to working time. On the other hand, households like to enjoy leisure in a healthy and pleasant environment (for example, air pollution may dissuade people from going outdoor and encourage them to work more).

Theoretical literature has pointed out the same ambiguity in the role of pollution on consumption. Keeler, Spence and Zeckhauser (1972) pioneered the class of Ramsey models with pollution accumulation. Pollution lowers the level of welfare as negative externality. Focusing on nonseparable preferences, they assume that consumption and environmental quality are normal goods in order to ensure the uniqueness of the steady state. Van der Ploeg and Withagen (1991) generalized the Ramsey model with pollution as a stock, by assuming additively separable preferences or a negative cross derivative (a marginal utility of consumption decreasing in

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

the pollution level). These conditions are sufficient for uniqueness and saddle-point stability of the steady state. Tahvonen and Kuuluvainen (1993) removed any restriction on the sign of the cross derivative in a Ramsey model. The most complete characterization of the interplay between consumption and pollution in a Ramsey model was given by Ryder and Heal (1973)[2]. Satiation is possible under assumptions on the first-order derivatives of the utility function and may promote the multiplicity of steady states. Assumptions on second-order derivatives and intertemporally dependent preferences may promote the occurrence of cycles through a Hopf bifurcation in the case of adjacent complementarity.

The effects of pollution on growth through the consumption channel were also studied by Michel and Rotillon (1996) in an endogenous growth model (AK). They find that a *distaste effect* of pollution on consumption (negative cross derivative) or separable preferences are incompatible with optimal endogenous growth. Conversely, sustained growth is optimal when the utility function exhibits a *compensation effect* (positive cross derivative). Endogenous growth occurs in the competitive equilibrium regardless of the effects of pollution on consumption.

Surprisingly, many theoretical works have considered the pollution effects on consumption behavior and the growth path, while, to the best of our knowledge, there are no empirical studies on that. Conversely, a rising number of empirical works study the pollution effects on labor supply, whereas these effects are largely ignored

[2] In Ryder and Heal (1973) pollution comes from consumption and it is interpreted as a habit effect. Heal (1982) considers explicitly the same variable as a pollution stock.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

in theoretical papers.

Our work contributes to shed light on the interplay between pollution and labor supply in the particular case of a productive economy with capital and pollution accumulation. An ideal framework to carry out this task is a Ramsey model with nonseparable preferences between pollution and labor supply (separability rules out any direct effect of pollution on labor disutility and supply). Thereby, the novelty of our analysis rests on the interchange of nonseparability assumptions: between pollution and leisure instead of between pollution and consumption.

Our oversimplified context allows us to prove that, under sufficiently large (negative) effects of pollution on labor supply, the economy may experience flip and period-doubling bifurcations, and deterministic fluctuations around the steady state. Thus, the pollution effect on labor supply, empirically stressed by Hanna and Oliva (2011), seems to promote macroeconomic volatility and destabilize the economic dynamics.

The rest of the chapter is articulated in three sections: (1) presentation of a general setting, (2) application to the separable case (separability between consumption and labor), (3) conclusion.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

2.2 The model

In the following, we consider a discrete-time Ramsey economy with pollution and capital accumulation. A representative household faces a consumption-leisure arbitrage by supplying a labor force to a sector of perfectly competitive firms. These firms produce a single commodity working either as capital or a consumption good. Because of the constant returns to scale, firms can be represented by a single aggregate firm. Pollution is a by-product of industrial activities and affects the individual welfare by distorting the consumption-leisure arbitrage.

2.2.1 Firms

At each date $t = 0, 1, \ldots$, a representative firm produces a single output Y_t. Technology is represented by a constant returns to scale production function: $Y_t = F(K_t, L_t)$, where K_t and L_t are the demands for capital and labor respectively.

Assumption 3 *The production function $F : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ is C^1, homogeneous of degree one, strictly increasing and concave. Standard Inada conditions hold.*

The firm chooses the amount of capital and labor to maximize the profit taking as given the real interest rate r_t and the real wage w_t. The program is correctly defined under Assumption 3: $\max_{K_t, L_t} [F(K_t, L_t) - r_t K_t - w_t L_t]$, and the first-order conditions write:

$$r_t = f'(k_t) \equiv r(k_t)$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t)$$

where $k_t \equiv K_t / L_t$ denotes the capital intensity. We introduce the capital share in

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

total income α and the elasticity of capital-labor substitution σ:

$$\begin{aligned}\alpha(k_t) &\equiv \frac{k_t f'(k_t)}{f(k_t)} \\ \sigma(k_t) &= \alpha(k_t)\frac{w(k_t)}{k_t w'(k_t)}\end{aligned} \tag{2.1}$$

In addition,

$$\frac{k_t r'(k_t)}{r(k_t)} = -\frac{1-\alpha(k_t)}{\sigma(k_t)} \tag{2.2}$$

$$\frac{k_t w'(k_t)}{w(k_t)} = \frac{\alpha(k_t)}{\sigma(k_t)} \tag{2.3}$$

2.2.2 Preferences

At each date $t = 0, 1, \ldots$, the household earns a capital income $r_t h_t$ and a labor income $w_t l_t$ where h_t and l_t denote the individual wealth and labor supply respectively. Income is consumed and saved/invested according to the budget constraint:

$$c_t + h_{t+1} - (1-\delta) h_t \leq r_t h_t + w_t l_t \tag{2.4}$$

The gross investment includes the capital depreciation at the rate δ.

For simplicity, the population of consumers-workers is constant over time: $N = 1$. Such normalization implies $L_t = Nl_t = l_t$, $K_t = Nh_t = h_t$ and $h_t = K_t/N = k_t l_t$.

The representative agent takes a utility from the consumption c_t and a disutility from the labor supply l_t and the amount of pollution P_t, that is an aggregate externality. The utility function $u_t = u(c_t, l_t, P_t)$ satisfies the following assumption.

Assumption 4 *The utility function $u : \mathbb{R}^3_+ \to \mathbb{R}$ is C^2, strictly increasing in c_t and strictly decreasing in l_t and P_t, and concave with respect to (c_t, l_t).*

If consumption and leisure are both normal goods, we have $u_{cc} - u_{lc}u_c/u_l < 0$ and $u_{ll} - u_{cl}u_l/u_c < 0$. These inequalities hold for instance if $u_{lc} \leq 0$ that is a sufficient condition.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

According to Michel and Rotillon (1996), pollution has a *distaste effect* on consumption if $u_{cP} < 0$: an increase in pollution reduces the marginal utility of consumption and thereby households' propensity to consume. These authors call the opposite effect ($u_{cP} > 0$) the *compensation effect*. An increase in pollution raises the propensity to consume.

This terminology can be extended to the effects of pollution on labor supply. Focusing on leisure, we will call *leisure effect* the positive effect of pollution on labor disutility ($u_{lP} < 0$) which decreases labor supply and increases in turn leisure demand. Conversely, we will call *disenchantment effect* the negative effect of pollution on labor disutility ($u_{lP} > 0$). In this case, leisure time decreases with pollution. As seen in the introduction, disenchantment for leisure comes from a more polluted and unpleasant environment.

The agent maximizes the intertemporal utility function $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, P_t)$ under the budget constraint (2.4) where $\beta \in (0,1)$ is a constant discount factor. This program is correctly defined under Assumption 4. The first-order conditions result in a static consumption-leisure arbitrage

$$u_l(c_t, l_t, P_t) = -u_c(c_t, l_t, P_t)\, w_t$$

and a dynamic Euler equation

$$\frac{u_c(c_t, l_t, P_t)}{u_c(c_{t+1}, l_{t+1}, P_{t+1})} = \beta(1 - \delta + r_{t+1})$$

jointly with the budget constraint (2.4) now binding.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

2.2.3 Pollution

The aggregate stock of pollution P_t is a pure externality. Technology is dirty and pollution persists. More explicitly, we assume that the stock of pollution tomorrow will depend on pollution and production today according to a simple linear process:

$$P_{t+1} = aP_t + bY_t \tag{2.5}$$

where $1 - a \in (0, 1]$ captures the natural rate of pollution absorption and $b > 0$ the environmental impact of production. Under Assumption 3, the process of pollution accumulation (2.5) writes:

$$P_{t+1} = aP_t + bL_t f(k_t) = aP_t + bl_t f(k_t)$$

2.2.4 Equilibrium

Good and labor markets clear. Noticing that $h_t = k_t l_t$, we find

$$c_t + k_{t+1}l_{t+1} = [1 - \delta + r(k_t)] k_t l_t + w(k_t) l_t \tag{2.6}$$

$$\frac{u_c(c_t, l_t, P_t)}{u_c(c_{t+1}, l_{t+1}, P_{t+1})} = \beta [1 - \delta + r(k_{t+1})] \tag{2.7}$$

$$P_{t+1} = aP_t + bl_t f(k_t) \tag{2.8}$$

$$u_l(c_t, l_t, P_t) = -u_c(c_t, l_t, P_t) w(k_t) \tag{2.9}$$

Applying the Implicit Function Theorem to the static arbitrage (2.9), we are able to compute the derivatives of the labor supply function $l_t = l(c_t, k_t, P_t)$. Indeed, differentiating $u_l(c_t, l_t, P_t) + u_c(c_t, l_t, P_t) w(k_t) = 0$ and keeping in mind that $w_t = -u_l/u_c$, we get

$$\frac{dl}{dc_t} = -\frac{\frac{u_{cl}}{u_l} - \frac{u_{cc}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}, \ \frac{dl}{dP_t} = -\frac{\frac{u_{lP}}{u_l} - \frac{u_{cP}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}, \ \frac{dl}{dk_t} = \frac{\frac{w'(k_t)}{w_t}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

These derivatives allow us to compute the second-order elasticities of the utility function:

$$E \equiv \begin{bmatrix} \varepsilon_{cc} & \varepsilon_{cl} & \varepsilon_{cP} \\ \varepsilon_{lc} & \varepsilon_{ll} & \varepsilon_{lP} \\ \varepsilon_{Pc} & \varepsilon_{Pl} & \varepsilon_{PP} \end{bmatrix} \equiv \begin{bmatrix} \frac{c_t u_{cc}}{u_c} & \frac{c_t u_{cl}}{u_l} & \frac{c_t u_{cP}}{u_P} \\ \frac{l_t u_{lc}}{u_c} & \frac{l_t u_{ll}}{u_l} & \frac{l_t u_{lP}}{u_P} \\ \frac{P_t u_{Pc}}{u_c} & \frac{P_t u_{Pl}}{u_l} & \frac{P_t u_{PP}}{uP} \end{bmatrix}$$

Using (2.1), we find the elasticities of labor supply:

$$\lambda_c \equiv \frac{c_t}{l_t}\frac{dl}{dc_t} = -\frac{\varepsilon_{cl} - \varepsilon_{cc}}{\varepsilon_{ll} - \varepsilon_{lc}}$$
$$\lambda_P \equiv \frac{P_t}{l_t}\frac{dl}{dP_t} = -\frac{\varepsilon_{Pl} - \varepsilon_{Pc}}{\varepsilon_{ll} - \varepsilon_{lc}}$$
$$\lambda_k \equiv \frac{k_t}{l_t}\frac{dl}{dk_t} = \frac{\alpha}{\sigma}\frac{1}{\varepsilon_{ll} - \varepsilon_{lc}}$$

All our economic analysis will rest on these crucial elasticities.

Replacing the labor supply $l(c_t, k_t, P_t)$ in (2.6), (2.7) and (2.8), we obtain a three-dimensional dynamic system.

Proposition 4 *An intertemporal equilibrium with perfect foresight is a nonnegative sequence* $(k_t, c_t, P_t)_{t=0}^{\infty}$ *satisfying the dynamic system*

$$c_t + k_{t+1} l(c_{t+1}, k_{t+1}, P_{t+1}) = ([1-\delta + r(k_t)]k_t + w(k_t))\, l(c_t, k_t, P_t) \tag{2.10}$$

$$\frac{u_c(c_t, l(c_t, k_t, P_t), P_t)}{u_c(c_{t+1}, l(c_{t+1}, k_{t+1}, P_{t+1}), P_{t+1})} = \beta[1-\delta + r(k_{t+1})] \tag{2.11}$$

$$P_{t+1} = aP_t + bf(k_t)\, l(c_t, k_t, P_t) \tag{2.12}$$

We observe that this system is three-dimensional with two predetermined variables (k_t, P_t) and one non-predetermined (c_t).

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

2.2.5 Steady state

Variables are constant over time: $(k_t, c_t, P_t) = (k, c, P)$ for every t. At the steady state, the dynamic system (2.10)-(2.12) writes:

$$r(k) = \frac{1}{\beta} - 1 + \delta \tag{2.13}$$

$$c = \left[\frac{1-\beta}{\beta}k + w(k)\right] l(c, k, P) \tag{2.14}$$

$$P = \frac{b}{1-a} f(k)\, l(c, k, P) \tag{2.15}$$

We obtain the stationary capital k from the first equation. Replacing k in (2.14) and (2.15) and solving system (2.14)-(2.15), we obtain also (c, P).

Given k, solution of (2.13), let

$$\mu(c) \equiv l\left(c, k, \frac{b}{1-a}\frac{f(k)}{f(k) - \delta k}c\right)$$

Proposition 5 *Let Assumptions 3 and 4 hold. If* $\lim_{c\to 0^+} \mu(c) > 0$ *and* $\mu'(c) < 0$ *for every* $c > 0$*, then there exists a unique steady state.*

Proof. Under Assumption 3, k is uniquely determined by (2.13). Replacing k in (2.14) and (2.15) we obtain a two-dimensional system in (c, P). We observe from (2.14) that

$$\frac{c}{l(c, k, P)} = \frac{1-\beta}{\beta}k + w(k) = f(k) - \delta k > 0 \tag{2.16}$$

Dividing equations (2.14) and (2.15) side by side and using (2.16), we get

$$P = \frac{b}{1-a}\frac{f(k)}{f(k) - \delta k}c \tag{2.17}$$

Replacing (2.17) in (2.14), we find

$$g(c) \equiv c - [f(k) - \delta k]\, \mu(c) = 0 \tag{2.18}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

a single equation in c. Under Assumption 4, we find $\lim_{c\to 0^+} g(c) < 0$ and $\lim_{c\to +\infty} g(c) = +\infty$. Under Assumption 3, g is a continuous function. Thus, a solution of equation (2.18) exists.

In addition, this solution is unique because, under Assumption 4, $g'(c) > 0$ for any $c > 0$. ■

We will see that, in the case of separable isoelastic preferences, the assumptions of Proposition 5 hold and a unique steady state exists.

Corollary 6 *Let Assumptions 3 and 4 hold. If consumption and leisure are normal goods, a distaste effect ($u_{cP} < 0$) jointly with a leisure effect ($u_{lP} < 0$) hold and* $\lim_{c\to 0^+} \mu(c) > 0$*, then there exists a unique steady state.*

Proof. We observe that

$$\mu'(c) = l_c + l_P \frac{b}{1-a} \frac{f(k)}{f(k) - \delta k} = l_c + l_P \frac{P}{c}$$
$$= -\frac{\frac{u_{cl}}{u_l} - \frac{u_{cc}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}} - \frac{\frac{u_{lP}}{u_l} - \frac{u_{cP}}{u_c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}} \frac{P}{c} = \frac{\frac{u_{cc}}{u_c} - \frac{u_{cl}}{u_l} + \left(\frac{u_{cP}}{u_c} - \frac{u_{lP}}{u_l}\right)\frac{P}{c}}{\frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c}}$$

Since consumption and leisure are both normal goods, we have

$$\frac{u_{cc}}{u_c} - \frac{u_{lc}}{u_l} < 0 \text{ and } \frac{u_{ll}}{u_l} - \frac{u_{cl}}{u_c} > 0$$

Thus, $\mu'(c) < 0$ iff

$$\frac{u_{cc}}{u_c} - \frac{u_{cl}}{u_l} + \left(\frac{u_{cP}}{u_c} - \frac{u_{lP}}{u_l}\right)\frac{P}{c} < 0 \tag{2.19}$$

If a distaste effect ($u_{cP} < 0$) jointly with a leisure effect ($u_{lP} < 0$) hold, we get (2.19), that is $\mu'(c) < 0$ for any $c > 0$. Eventually, Proposition 5 applies. ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

2.2.6 Long run

Focus on the comparative statics. In this general section, we have not specified technology and preferences. The only parameters we consider are a, b, β and δ. We compute their impact on c, k, P. In the isoelastic case (see below), we will consider also the impact of technology and preferences on the endogenous variables.

Differentiating (2.13) and using (2.2), we obtain the usual elasticities of Modified Golden Rule:

$$\frac{\beta}{k}\frac{\partial k}{\partial \beta} = \frac{\sigma(k)}{1-\alpha(k)}\frac{1}{\beta r(k)} > 0 \tag{2.20}$$

$$\frac{\delta}{k}\frac{\partial k}{\partial \delta} = -\frac{\sigma(k)}{1-\alpha(k)}\frac{\delta}{r(k)} < 0 \tag{2.21}$$

where $r(k)$ is given by (2.13).

Differentiating system (2.14)-(2.15) and using (2.20) and (2.21), we obtain:

$$\rho(1-\lambda_c)\frac{dc}{c} - \rho\lambda_P\frac{dP}{P} = \frac{\sigma_2}{\beta}\frac{d\beta}{\beta} - \delta(1+\sigma_2)\frac{d\delta}{\delta}$$
$$-\lambda_c\frac{dc}{c} + (1-\lambda_P)\frac{dP}{P} = -\frac{dz}{z} + \frac{db}{b} + \sigma_1\frac{d\beta}{\beta} - \beta\delta\sigma_1\frac{d\delta}{\delta}$$

where $z \equiv 1-a$ and

$$\rho \equiv \frac{c}{kl} = \frac{1-\beta}{\beta} + \frac{1-\alpha}{\alpha}r$$
$$\sigma_1 \equiv \frac{\sigma}{1-\alpha}\frac{\alpha+\lambda_k}{\beta r}$$
$$\sigma_2 \equiv \lambda_k\frac{\sigma}{\alpha} + (1+\lambda_k)\frac{\sigma}{1-\alpha}\frac{1-\beta}{\beta r}$$

that is

$$\begin{bmatrix} \frac{dc}{c} \\ \frac{dP}{P} \end{bmatrix} = \frac{M}{1-\lambda_c-\lambda_P}\begin{bmatrix} \frac{dz}{z} \\ \frac{db}{b} \\ \frac{d\beta}{\beta} \\ \frac{d\delta}{\delta} \end{bmatrix}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

with

$$M \equiv \begin{bmatrix} -\lambda_P & \lambda_P & \lambda_P\sigma_1 + (1-\lambda_P)\frac{\sigma_2}{\beta\rho} & -\lambda_P\beta\delta\sigma_1 - (1-\lambda_P)\frac{\delta(1+\sigma_2)}{\rho} \\ \lambda_c - 1 & 1-\lambda_c & \lambda_c\frac{\sigma_2}{\beta\rho} + (1-\lambda_c)\sigma_1 & -\lambda_c\frac{\delta(1+\sigma_2)}{\rho} - (1-\lambda_c)\beta\delta\sigma_1 \end{bmatrix} \tag{2.22}$$

We find the following elasticity of comparative statics:

$$\begin{bmatrix} \frac{z}{c}\frac{\partial c}{\partial z} & \frac{b}{c}\frac{\partial c}{\partial b} & \frac{\beta}{c}\frac{\partial c}{\partial \beta} & \frac{\delta}{c}\frac{\partial c}{\partial \delta} \\ \frac{z}{P}\frac{\partial P}{\partial z} & \frac{b}{P}\frac{\partial P}{\partial b} & \frac{\beta}{P}\frac{\partial P}{\partial \beta} & \frac{\delta}{P}\frac{\partial P}{\partial \delta} \end{bmatrix} = \frac{M}{1-\lambda_c-\lambda_P} \tag{2.23}$$

2.2.7 Short run

Focus on local dynamics. We linearize the dynamic system (2.10)-(2.12) around the steady state:

$$\begin{bmatrix} 1+\lambda_k & \lambda_c & \lambda_P \\ \varepsilon_{lc}\lambda_k - \frac{1-\alpha}{\sigma}\beta r & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P + \varepsilon_{Pc} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{dk_{t+1}}{k} \\ \frac{dc_{t+1}}{c} \\ \frac{dP_{t+1}}{P} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{\beta} + (1+\rho)\lambda_k & (1+\rho)\lambda_c - \rho & (1+\rho)\lambda_P \\ \varepsilon_{lc}\lambda_k & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P + \varepsilon_{Pc} \\ (1-a)(\alpha+\lambda_k) & (1-a)\lambda_c & a + (1-a)\lambda_P \end{bmatrix} \begin{bmatrix} \frac{dk_t}{k} \\ \frac{dc_t}{c} \\ \frac{dP_t}{P} \end{bmatrix}$$

In order to study the local stability of system (2.10)-(2.12), that is the shape of the characteristic polynomial, we assume preferences to be separable.

2.3 The separable model

In the case of separable utility: $u(c_t, l_t, P_t) = \tilde{u}(c_t) - \omega v(l_t, P_t)$, the elasticity matrix E becomes

$$E \equiv \begin{bmatrix} \varepsilon_{cc} & 0 & 0 \\ 0 & \varepsilon_{ll} & \varepsilon_{lP} \\ 0 & \varepsilon_{Pl} & \varepsilon_{PP} \end{bmatrix} \tag{2.24}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

and the elasticities of labor supply $l_t = l(c_t, k_t, P_t)$ write:

$$\lambda_c = \frac{\varepsilon_{cc}}{\varepsilon_{ll}}$$
$$\lambda_P = -\frac{\varepsilon_{Pl}}{\varepsilon_{ll}}$$
$$\lambda_k = \frac{\alpha}{\sigma}\frac{1}{\varepsilon_{ll}} \tag{2.25}$$

When pollution does not affect the labor supply $\varepsilon_{Pl} = 0$. Thus, the very difference with respect to the standard consumption-labor arbitrage is the elasticity ε_{Pl}. In the following, we will show that σ and ε_{Pl} play a role in the occurrence of endogenous business cycles.

From a theoretic point of view, the effect of pollution on labor supply is ambiguous: $\lambda_P \lesseqgtr 0$. However, according to the recent empirical studies and, in particular, to Hanna and Oliva (2011) work on Mexico City, a rise in pollution seems to have a negative effect on labor supply: $\lambda_P < 0$. In the following, we will consider this case.

2.3.1 Isoelastic form

In the isoelastic case, the elasticities are constant and notation simplifies: $\varepsilon \equiv -\varepsilon_{cc}$ and $\varphi \equiv \varepsilon_{ll}$. We consider explicit isoelastic separable preferences:

$$\tilde{u}(c_t) \equiv \frac{c_t^{1-\varepsilon}}{1-\varepsilon} \text{ and } v(l_t, P_t) \equiv \frac{(l_t P_t^{\gamma})^{1+\varphi}}{1+\varphi} \tag{2.26}$$

where $1/\varepsilon \geq 0$ is the consumption elasticity of intertemporal substitution and $1/\varphi \geq 0$ is the Frisch elasticity of intertemporal substitution.

In addition, this form allows us to express the key elasticity ε_{Pl} in terms of the structural parameters: $\varepsilon_{Pl} = \gamma(1+\varphi)$. Since

$$\lambda_P = -\frac{\varepsilon_{Pl}}{\varepsilon_{ll}} = -\gamma\frac{1+\varphi}{\varphi} \tag{2.27}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

and, according to Assumption 4, the utility function decreases with P_t, we obtain $\gamma > 0$ and, thus, $\lambda_P < 0$ or, equivalently, $dl/dP_t < 0$ (the labor supply decreases with pollution in turn).

From (2.27), labor disutility writes also

$$v\left(l_t, P_t\right) \equiv \frac{l_t^{1+\varphi} P_t^{-\varphi\lambda_P}}{1+\varphi}$$

In this case (2.26), with an intensive production function $f\left(k_t\right) = Ak_t^{\alpha}$, the labor supply function explicitly becomes

$$l_t = l\left(c_t, k_t, P_t\right) = m\left(c_t, k_t\right) P_t^{\lambda_P} \text{ with } m\left(c_t, k_t\right) \equiv \left(\frac{1-\alpha}{\omega}\frac{Ak_t^{\alpha}}{c_t^{\varepsilon}}\right)^{\frac{1}{\varphi}}$$

2.3.2 Long run (continued)

Preferences rationalized by functions (2.26) ensure the uniqueness of the steady state. Indeed, consumption and leisure are normal goods and a distaste effect jointly with a leisure effect hold. More explicitly, we find

$$\mu\left(c\right) = \left(\frac{1-\alpha}{\omega}Ak^{\alpha}\right)^{\frac{1}{\varphi}}\left(\frac{b}{1-a}\frac{Ak^{\alpha}}{Ak^{\alpha}-\delta k}\right)^{\lambda_P} c^{\lambda_P - \frac{\varepsilon}{\varphi}}$$

with $\lambda_P < 0$. Thus, $\lim_{c\to 0^+} \mu\left(c\right) = +\infty > 0$ and $\mu'\left(c\right) < 0$, and the assumptions of Proposition 5 hold.

Let

$$\lambda_P^* \equiv -\frac{1}{\varphi\sigma}\left(1 + \frac{r-\delta}{\delta}\frac{1+\varphi\sigma}{1-\alpha}\right) < 0$$

Proposition 7 *The the long-run effects of the fundamental parameters on the pollution stock are given by*

$$\frac{\partial P}{\partial a} > 0,\ \frac{\partial P}{\partial b} > 0,\ \frac{\partial P}{\partial \beta} > 0$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

and

$$\frac{\partial P}{\partial \delta} < 0 \text{ iff } \sigma > \frac{\frac{r-\alpha r}{r-\alpha\delta}\varepsilon - 1}{\frac{\delta-\alpha\delta}{r-\alpha\delta}\varepsilon + \varphi}$$

The the long-run effects of these parameters on the consumption level depend on the pollution elasticity of labor supply λ_P*,*

$$\begin{aligned}
&\frac{\partial c}{\partial a} < 0 \\
&\frac{\partial c}{\partial b} < 0 \\
&\frac{\partial c}{\partial \beta} > 0 \text{ iff } \lambda_P > \lambda_P^* \\
&\frac{\partial c}{\partial \delta} < 0 \text{ iff } \sigma < \frac{r}{\delta} \text{ or } \left(\sigma > \frac{r}{\delta} \text{ and } \lambda_P > \frac{r-\sigma\delta\lambda_P^*}{r-\sigma\delta}\,(<0)\right)
\end{aligned}$$

Proof. Reconsider (2.27) and the impact matrix (2.23). Notice that $z \equiv 1-a$ and

$$(\lambda_c, \lambda_k) = \left(-\frac{\varepsilon}{\varphi}, \frac{1}{\varphi}\frac{\alpha}{\sigma}\right)$$

The denominator $1-\lambda_c-\lambda_P$ is positive. (2.22) becomes

$$M \equiv \begin{bmatrix} -\lambda_P & \lambda_P & \frac{\alpha\delta\sigma}{\beta r}\frac{\lambda_P-\lambda_P^*}{r-\alpha\delta} & \frac{\alpha\delta}{r}\frac{r-\sigma\delta}{r-\alpha\delta}\left(\lambda_P - \frac{r-\sigma\delta\lambda_P^*}{r-\sigma\delta}\right) \\ -\frac{\varepsilon+\varphi}{\varphi} & \frac{\varepsilon+\varphi}{\varphi} & \frac{1}{\varphi\beta r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right) & \frac{\delta}{\varphi}\left[\frac{\varepsilon}{\rho} - \frac{1}{r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right)\right] \end{bmatrix}$$

We observe that $r > \alpha\delta$ and

$$\begin{aligned}
\frac{1}{\varphi\beta r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right) &> 0 \\
\frac{\delta}{\varphi}\left[\frac{\varepsilon}{\rho} - \frac{1}{r}\frac{\alpha}{1-\alpha}\left(1+\varphi\sigma+\varepsilon\sigma\frac{\delta-\alpha\delta}{r-\alpha\delta}\right)\right] &< 0 \text{ iff } \sigma > \frac{\frac{r-\alpha r}{r-\alpha\delta}\varepsilon - 1}{\frac{\delta-\alpha\delta}{r-\alpha\delta}\varepsilon + \varphi}
\end{aligned}$$

The proposition follows immediately. ■

Corollary 8 $\partial P/\partial\delta < 0$ *if*

$$\varepsilon < \frac{r-\alpha\delta}{r-\alpha r}\,(>1)$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Corollary 9 *If $\lambda_P = 0$ (that is $\gamma = 0$), we have*

$$\frac{\partial c}{\partial a} = \frac{\partial c}{\partial b} = 0,\ \frac{\partial c}{\partial \beta} > 0,\ \frac{\partial c}{\partial \delta} < 0$$

We recover in this case the usual conclusions of the Ramsey model.

Let us provide an interpretation of Proposition 7.

Focus on equations (2.15), (2.20) and (2.25). The higher the β, the larger the capital stock k and, in turn, the higher the labor supply l and the stock of pollution P. The effect of β on c in the long run depends on λ_P. Since $\lambda_P < 0$, the households substitutes leisure to working time. However, if this effect does not compensate the positive effect of k on labor supply, a higher β entails a higher consumption in the long run.

Focus now on equations (2.15), (2.21) and (2.25). The higher the depreciation rate δ, the lower the capital stock k and, in turn, the lower the labor supply l and and the pollution stock P. According to equation (2.21), the impact of δ on k and, in turn, on l, depends on the elasticity of capital-labor substitution σ. A stronger σ induces a larger negative effect of δ on k. Since the negative effect of δ on P depends crucially on its impact on k, it follows that the negative impact of δ on P is also magnified under a large elasticity σ. Notice that $\lambda_P < 0$. We know also that a higher δ implies a lower P. Let the pollution elasticity of labor supply be not too negative. In this case, under a sufficiently large σ, the effect of k on l dominates the effect of P on l. Thus, a higher δ leads the household to substitute leisure to working time, that is to consume less in the long run.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

2.3.3 Short run (continued)

Specification (2.26) gives the following Jacobian:

$$J = \begin{bmatrix} 1+\lambda_k & \lambda_c & \lambda_P \\ \varepsilon_{lc}\lambda_k - \frac{1-\alpha}{\sigma}\beta r & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P+\varepsilon_{Pc} \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\beta}+(1+\rho)\lambda_k & (1+\rho)\lambda_c-\rho & (1+\rho)\lambda_P \\ \varepsilon_{lc}\lambda_k & \varepsilon_{cc}+\varepsilon_{lc}\lambda_c & \varepsilon_{lc}\lambda_P+\varepsilon_{Pc} \\ (1-a)(\alpha+\lambda_k) & (1-a)\lambda_c & a+(1-a)\lambda_P \end{bmatrix}$$

with an elasticity matrix (2.24):

$$E = \begin{bmatrix} -\varepsilon & 0 & 0 \\ 0 & \varphi & \varepsilon_{lP} \\ 0 & \gamma(1+\varphi) & \varepsilon_{PP} \end{bmatrix}$$

Therefore,

$$J = \begin{bmatrix} 1+\lambda_k & \lambda_c & \lambda_P \\ -\frac{1-\alpha}{\sigma}\beta r & -\varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\beta}+(1+\rho)\lambda_k & (1+\rho)\lambda_c-\rho & (1+\rho)\lambda_P \\ 0 & -\varepsilon & 0 \\ (1-a)(\alpha+\lambda_k) & (1-a)\lambda_c & a+(1-a)\lambda_P \end{bmatrix} \tag{2.28}$$

with:

$$\rho = \frac{1-\beta}{\beta} + \frac{1-\alpha}{\alpha} r$$

and

$$\lambda_c = -\frac{\varepsilon}{\varphi},\ \lambda_P = -\gamma\frac{1+\varphi}{\varphi},\ \lambda_k = \frac{\alpha}{\sigma}\frac{1}{\varphi}$$

To know the location of the eigenvalues of the Jacobian matrix w.r.t. the unit circle, we study the sign of the characteristic polynomial $P(x)$ in $x = -1, 0, 1$.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Tedious computations give the following values:

$$P(0) = (1-a)\frac{(1-\alpha)(1-\delta)\varepsilon\sigma}{(1-\alpha)\beta r\lambda_c - \varepsilon\sigma(1+\lambda_k)}(\lambda_P - \lambda_1) > 0 \text{ iff } \lambda_P < \lambda_1 \tag{2.29}$$

$$P(1) = \frac{\beta r\rho(1-\alpha)(1-a)(1-\lambda_c-\lambda_P)}{(1-\alpha)\beta r\lambda_c - \varepsilon\sigma(1+\lambda_k)} < 0 \tag{2.30}$$

$$P(-1) = (1-a)\frac{(1-\alpha)[\beta r\rho + 2\varepsilon\sigma(2-\delta)]}{(1-\alpha)\beta r\lambda_c - \varepsilon\sigma(1+\lambda_k)}(\lambda_P - \lambda_2) > 0 \text{ iff } \lambda_P < \lambda_2 \tag{2.31}$$

where

$$\lambda_1 \equiv -\frac{a}{1-a}\frac{1+(1+\rho)\beta\lambda_k}{\beta(1-\delta)(1-\alpha)} < 0$$

$$\lambda_2 \equiv -\frac{1+a}{1-a}\frac{(1-\alpha)\beta r[\rho-(2+\rho)\lambda_c] + 2\varepsilon\sigma[1+1/\beta+(2+\rho)\lambda_k]}{(1-\alpha)[\beta r\rho + 2\varepsilon\sigma(2-\delta)]} < 0$$

Assumption 5 *$a < \beta$.*

We notice that, under Assumption 5,

$$\lambda_P < 0 < \frac{\beta-a}{1-a} < \lambda_3 \equiv \frac{\beta-a}{1-a}\frac{1+\varphi\sigma-\beta(1-\alpha)(1-\delta)}{\varphi\sigma\beta(1-\alpha)(1-\delta)}$$

Let D and T be the determinant and the trace of J respectively.

Lemma 10 *Under Assumption 5, $D < 1$.*

Proof. $D < 1$ is equivalent to

$$D = -P(0) = (1-a)\frac{(1-\alpha)(1-\delta)\varepsilon\sigma}{\varepsilon\sigma(1+\lambda_k) - (1-\alpha)\beta r\lambda_c}(\lambda_P - \lambda_1) < 1$$

that is to $\lambda_P < \lambda_3$. ■

Focus now on the issue of equilibrium uniqueness.

Two variables are predetermined (k_t and P_t), one is nonpredetermined (c_t). $P(1) < 0$ implies that one eigenvalue is real and greater than one. Thus, equilibrium determinacy (locally). The question now is whether there are zero, one or two eigenvalues inside the unit circle.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

There are two possible cases:

(1) $\lambda_1 < \lambda_2$,

(2) $\lambda_2 < \lambda_1$.

We observe that $\lambda_1 < \lambda_2$ iff

$$\frac{1+a}{a} < \frac{\left[1+(1+\rho)\,\beta\lambda_k\right]\left[\beta r\rho+2\varepsilon\sigma\,(2-\delta)\right]}{\beta\,(1-\delta)\left((1-\alpha)\,\beta r\left[\rho-(2+\rho)\,\lambda_c\right]+2\varepsilon\sigma\left[1+1/\beta+(2+\rho)\,\lambda_k\right]\right)}$$

Notice that the RHS does not depend on a.

Focus on the second case (for instance, if a is sufficiently small or $a=0$). In this case, $\lambda_2 < \lambda_1$.

Proposition 11 *(equilibrium uniqueness) Let a be null or sufficiently small and Assumption 5 hold. There are three cases.*

(1) $\lambda_P < \lambda_2 < \lambda_1 < 0$. The eigenvalues x_i are such that $x_1 < -1 < 0 < x_2 < 1 < x_3$: local overdeterminacy.

(2) $\lambda_2 < \lambda_P < \lambda_1 < 0$. The eigenvalues x_i are such that $-1 < x_1 < 0 < x_2 < 1 < x_3$: local determinacy.

(3) $\lambda_2 < \lambda_1 < \lambda_P < 0$. Under Assumption 5, there are two eigenvalues inside the unit circle and one outside: $|x_1|, |x_2| < 1 < x_3$. Thus, local determinacy.

When $\lambda_P = \lambda_2$ the system generically undergoes a flip bifurcation.

Proof. Consider the three eigenvalues: x_1, x_2 and x_3. We know that $x_3 > 1$ because $P(1) < 0$.

Points (1) and (2) are immediate: simply notice that $\lambda_2 < \lambda_1$ and consider the signs of expressions (2.29) and (2.31) in the cases $\lambda_P < \lambda_2 < \lambda_1 < 0$ and $\lambda_2 < \lambda_P < \lambda_1 < 0$ respectively.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Focus now on point (3). In this case, we get: $P(-1) < 0$, $P(0) < 0$ and $P(1) < 0$.

Under Assumption 5, Lemma 10 applies and $D = x_1 x_2 x_3 < 1$. Since $P(1) < 0$, we have $x_3 > 1$ and, so $x_1 x_2 < 1$. There are two cases: these eigenvalues are (3.1) real or (3.2) nonreal.

In the subcase (3.1), $D < 1$ implies that at least one of the two eigenvalues x_1 and $x_2 > x_1$ is inside the unit circle. Let, without loss of generality, x_2 be in the unit circle that is $-1 < x_2 < 1$. If $0 < x_2 < 1$, since $P(1) < 0$, there exists $\bar{x} \in (0, x_2)$ such that $P(\bar{x}) > 0$. Since $P(0) < 0$, we have also $0 < x_1 < \bar{x} < 1$. Thus, $0 < x_1 < x_2 < 1$. Similarly, one can show that $-1 < x_2 < 0$ implies $-1 < x_1 < x_2 < 0$ because $P(-1) < 0$.

In the subcase (3.2), x_1 and x_2 are nonreal and conjugated. Thus, $|x_1| |x_2| = |x_1 x_2| < 1$ and, since they have the same modulus, $|x_1| = |x_2| < 1$. ■

Corollary 12 *Under Assumption 5, there is no room for Hopf bifurcations.*

Proof. We know that $P(1) < 0$, that is $x_3 > 1$. Under Assumption 3, Lemma 10 applies and, therefore, $D = x_1 x_2 x_3 < 1$. Thus, $x_1 x_2 < 1$. The Hopf bifurcation generically arises when x_1 and x_2 are nonreal and $x_1 x_2 = 1$. Then Assumption 5 is incompatible with the occurrence of a Hopf bifurcation. ■

The occurrence of deterministic fluctuations deserves an interpretation. Focus on the case of a sufficiently small a, that is $\lambda_2 < \lambda_1 < 0$, and a sufficiently negative impact of pollution on labor supply (λ_P close to λ_2).

In this case, an increase in pollution lowers enough the labor supply. The penury of labor input decreases considerably the production and pollution in turn. Thus, a

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

rise in pollution is followed by a drop in pollution at the very end: a cycle of period two arises.

Notice that a is a measure of pollution persistence. The occurrence of cycles is magnified when a is close to zero because this inertia fails, pollution becomes more volatile and the comparative effect of production on the pollution process becomes maximal.

2.4 Conclusion

We have considered an economy à la Ramsey where production pollutes and the negative externality distorts the household's consumption-leisure choice. In this framework, we have proved that a sufficiently large (negative) effects of pollution on labor supply may promotes macroeconomic volatility (deterministic cycles near the steady state) through a flip bifurcation. It seems that, in the empirical case considered by Hanna and Oliva (2011), pollution works as a destabilizing force. In this sense, our work provides a theoretical argument in favor of environmental friendly fiscal policies.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Chapter 3

Households' preferences, pollution and competitive growth

3.1 Introduction

Externalities[1] of pollution may affect the economy through their effects on technology or preferences, that is on productivity of factors or utility of goods. In the case of preferences, their influence can be disentangled in (1) the effects on consumption demand and (2) those on labor supply.

(1) Michel and Rotillon (1996) have pointed out the ambiguous effect of pollution on consumption demand. Pollution can decrease the consumption level through a distate effect and increase it through a compensation effect. In the last three decades, scholars have payed attention to the dynamic complexity arising from the interplay between pollution and consumption. A seminal contribution is Heal (1982). Ryder

[1] This chapter refers to a joint work with Stefano BOSI and Lionel RAGOT.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

and Heal (1973) study the growth path in a Ramsey model where the marginal utility of consumption is affected by a weighted average of past consumption demands. Heal (1982) revisits the model by replacing the past consumption by a pollution stock. Under the assumption of adjacent complementarity, Heal finds that a limit cycle may occur through a Hopf bifurcation. Within an endogenous growth model with pollution, Michel and Rotillon (1996) recover the same dynamics under the assumption of a strong compensation effect.

More recently, some authors have considered the occurrence of local indeterminacy under the effect of pollution on consumption demand. Itaya (2008) shows that this effect may promote indeterminacy and Fernandez, Pérez and Ruiz (2012) stress the role of nonseparability between consumption and pollution in the utility function in an endogenous growth model with elastic labor supply.

All these contributions highlight potential benefits from adopting environmental policies to reduce the macroeconomic volatility.

Even if, to the best of our knowledge, there is no empirical evidence showing a direct effect of pollution on consumption behavior, there is a rising numbers of empirical studies pointing out a negative effect of pollution on labor supply (Graff Zivin and Neidell (2010), Carson, Koundouri and Nauges (2011), Hanna and Oliva (2011)). In accordance with this evidence, we have prove in the last chapter[2] that the consumption effect of pollution promotes the emergence of persistent cycles through

[2]See also Bosi, Desmarchelier and Ragot (2013).

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

a flip bifurcation and, hence, macroeconomic volatility.

(2) Pollution may affect leisure demand or, equivalently, labor supply. As pointed out in the last chapter, pollution may decrease or increase labor supply through a leisure or a disenchantment effect respectively[3]. However, in our previous model, pollution has no effect on consumption but only on labor supply. More precisely introducing separable preferences in consumption and pollution, rule out any direct affect of pollution on the marginal utility of consumption and, in turn, on consumption demand. By definition, the distate and the compensation effects highlighted by Michel and Rotillon (1996) no longer hold.

The present chapter aims at generalizing both Michel and Rotillon (1996) and Bosi, Desmarchelier and Ragot (2013), by setting up a model where pollution jointly affects the consumption demand and the labor supply. To carry out this task, we consider a continuous-time Ramsey model with separable preferences in consumption and labor but nonseparable either in consumption and pollution or in labor and pollution.

We study the economic dynamics in the long run (steady state and other attractors) and in the short run (stability properties).

On the one hand, we prove the uniqueness of the steady state under a distate effect or a weak compensation effect jointly with a leisure effect or a weak disen-

[3]See Bosi, Desmarchelier and Ragot (2013) for more details.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

chantment effect.

On the other hand, we show that a high compensation effect jointly with a large leisure effect (empirically found by Hanna and Oliva (2011) among the others) may promote the local indeterminacy of equilibria through a Hopf bifurcation.

The rest of the chapter is articulated in three parts: 1) presentation of the general model , 2) the analysis of case with separable preferences and constant elasticities and 3) conclusion.

3.2 Model

We consider a continuous-time Ramsey economy with pollution and capital accumulation. A representative household faces a consumption-leisure arbitrage by supplying a labor force to a sector of perfectly competitive firms. These firms produce a single commodity which plays the role of capital or consumption good. Because of the constant returns to scale, firms can be represented by a single aggregate firm. Pollution is a by-product of industrial activities and affects the individual welfare by distorting the consumption-leisure arbitrage.

3.2.1 Technology

At time t representative firm produces a single output $Y(t)$. Technology is represented by a constant returns to scale production function: $Y(t) = F(K(t), L(t))$, where $K(t)$ and $L(t)$ are the demands for capital and labor at time t.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Assumption 6 *The production function $F : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ is C^1, homogeneous of degree one, strictly increasing and concave. Standard Inada conditions hold.*

The firm chooses the amount of capital and labor to maximize the profit taking as given the real interest rate $r(t)$ and the real wage $w(t)$. In the following, for notational simplicity, we will omit the time argument t.

The program $\max_{K,L}[F(K,L) - rK - wL]$ is correctly defined under Assumption 6 and the first-order conditions write:

$$r = f'(k) \equiv r(k)$$
$$w = f(k) - kf'(k) \equiv w(k)$$

where $f(k) \equiv F(k,1)$ is the average productivity and $k = k(t) \equiv K(t)/L(t)$ denotes the capital intensity at time t. We introduce the capital share in total income α and the elasticity of capital-labor substitution σ:

$$\alpha(k) \equiv \frac{kf'(k)}{f(k)}$$
$$\sigma(k) = \alpha(k)\frac{w(k)}{kw'(k)} \tag{3.1}$$

In addition, we determine the elasticities of factor prices:

$$\frac{kr'(k)}{r(k)} = -\frac{1-\alpha(k)}{\sigma(k)} \tag{3.2}$$
$$\frac{kw'(k)}{w(k)} = \frac{\alpha(k)}{\sigma(k)} \tag{3.3}$$

3.2.2 Preferences

The household earns a capital income rh and a labor income wl where $h = h(t)$ and $l = l(t)$ denote the individual wealth and labor supply at time t. Income is

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

consumed and saved/invested according to the budget constraint:

$$\dot{h} \leq (r-\delta)\, h + wl - c \tag{3.4}$$

The gross investment includes the capital depreciation at the rate δ.

For simplicity, the population of consumers-workers is constant over time: $N = 1$. Such normalization implies $L = Nl = l$, $K = Nh = h$ and $h = K/N = kl$.

Assumption 7 *Preferences are separable in consumption and labor:*

$$U(c, l, P) \equiv u(c, P) - v(l, P) \tag{3.5}$$

with $u_c > 0$*,* $u_P < 0$*,* $v_l > 0$*,* $v_P > 0$ *as first-order restrictions,* $u_{cc} < 0$*,* $v_{ll} > 0$ *as second-order restrictions, and* $\lim_{c\to 0^+} u_c = \infty$*,* $\lim_{l\to 0^+} v_l = 0$ *as a limit conditions.*

We do not impose any restriction on the sign of the cross-derivatives u_{cP} and v_{lP}. Even if preferences are separable in consumption and labor supply, pollution affects the marginal utilities of both of them and, hence, the consumption-labor arbitrage through a general equilibrium effect.

The agent maximizes the intertemporal utility function $\int_0^\infty e^{-\rho t} U(c, l, P)\, dt$ under the budget constraint (3.4) where $\rho > 0$ is the rate of time preference. This program is correctly defined under Assumption 7.

Proposition 13 *The first-order conditions result in a static consumption-leisure arbitrage*

$$U_c = \lambda = -U_l/w \tag{3.6}$$

a dynamic Euler equation $\dot{\lambda} = \lambda(\rho + \delta - r)$ *and the budget constraint (3.4) now binding* $\dot{h} = (r-\delta)\, h + wl - c$ *jointly with the transversality condition* $\lim_{t\to\infty} e^{-\rho t}\lambda(t)\, h(t) = 0$.

Proof. See the Appendix. ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

3.2.3 Pollution

The aggregate stock of pollution P is a pure externality. Technology is dirty and pollution persists. We assume a simple linear process:

$$\dot{P} = -aP + bY \tag{3.7}$$

where $a \geq 0$ captures the natural rate of pollution absorption and $b \geq 0$ the environmental impact of production. Since, under Assumption 6, $Y = Lf(k) = lf(k)$, the process of pollution accumulation (3.7) writes:

$$\dot{P} = -aP + blf(k)$$

3.2.4 Equilibrium

At equilibrium, good and labor markets clear. Applying the Implicit Function Theorem to the consumption-labor arbitrage (3.6), we obtain (c, l) as a function of (λ, k, P), that is $c = c(\lambda, k, P)$ and $l = l(\lambda, k, P)$. Let us introduce the following second-order elasticities of the utility function:

$$E \equiv \begin{bmatrix} \varepsilon_{cc} & \varepsilon_{cl} & \varepsilon_{cP} \\ \varepsilon_{lc} & \varepsilon_{ll} & \varepsilon_{lP} \\ \varepsilon_{Pc} & \varepsilon_{Pl} & \varepsilon_{PP} \end{bmatrix} \equiv \begin{bmatrix} \frac{cU_{cc}}{U_c} & \frac{cU_{cl}}{U_l} & \frac{cU_{cP}}{U_P} \\ \frac{lU_{lc}}{U_c} & \frac{lU_{ll}}{U_l} & \frac{lU_{lP}}{U_P} \\ \frac{PU_{Pc}}{U_c} & \frac{PU_{Pl}}{U_l} & \frac{PU_{PP}}{UP} \end{bmatrix}$$

Proposition 14 *The matrix of partial elasticities is given by*

$$\begin{bmatrix} \frac{\lambda}{c}\frac{\partial c}{\partial \lambda} & \frac{k}{c}\frac{\partial c}{\partial k} & \frac{P}{c}\frac{\partial c}{\partial P} \\ \frac{\lambda}{l}\frac{\partial l}{\partial \lambda} & \frac{k}{l}\frac{\partial l}{\partial k} & \frac{P}{l}\frac{\partial l}{\partial P} \end{bmatrix} = \frac{M}{\varepsilon_{cc}\varepsilon_{ll} - \varepsilon_{lc}\varepsilon_{cl}} \tag{3.8}$$

where

$$M \equiv \begin{bmatrix} \varepsilon_{ll} - \varepsilon_{lc} & -\frac{\alpha}{\sigma}\varepsilon_{lc} & \varepsilon_{lc}\varepsilon_{Pl} - \varepsilon_{ll}\varepsilon_{Pc} \\ \varepsilon_{cc} - \varepsilon_{cl} & \frac{\alpha}{\sigma}\varepsilon_{cc} & \varepsilon_{cl}\varepsilon_{Pc} - \varepsilon_{cc}\varepsilon_{Pl} \end{bmatrix} \tag{3.9}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Proof. See the Appendix. ■

In the separable case, the elasticities matrix simplifies

$$E \equiv \begin{bmatrix} \varepsilon_{cc} & 0 & \varepsilon_{cP} \\ 0 & \varepsilon_{ll} & \varepsilon_{lP} \\ \varepsilon_{Pc} & \varepsilon_{Pl} & \varepsilon_{PP} \end{bmatrix} \tag{3.10}$$

and we get

$$\begin{bmatrix} \frac{\lambda}{c}\frac{\partial c}{\partial \lambda} & \frac{k}{c}\frac{\partial c}{\partial k} & \frac{P}{c}\frac{\partial c}{\partial P} \\ \frac{\lambda}{l}\frac{\partial l}{\partial \lambda} & \frac{k}{l}\frac{\partial l}{\partial k} & \frac{P}{l}\frac{\partial l}{\partial P} \end{bmatrix} = \frac{M}{\varepsilon_{cc}\varepsilon_{ll}} = \begin{bmatrix} \frac{1}{\varepsilon_{cc}} & 0 & -\frac{\varepsilon_{Pc}}{\varepsilon_{cc}} \\ \frac{1}{\varepsilon_{ll}} & \frac{\alpha}{\sigma}\frac{1}{\varepsilon_{ll}} & -\frac{\varepsilon_{Pl}}{\varepsilon_{ll}} \end{bmatrix} \tag{3.11}$$

Proposition 15 *The equilibrium transition is represented by the following dynamic system:*

$$\begin{aligned} \frac{\dot{\lambda}}{\lambda} &= \rho + \delta - r(k) \\ \frac{\dot{k}}{k} &= \frac{r(k) - \delta + \frac{w(k)}{k} - \frac{c(\lambda,k,P)}{kl(\lambda,k,P)} - \frac{\lambda}{l}\frac{\partial l}{\partial \lambda}\left[\rho + \delta - r(k)\right] - \frac{P}{l}\frac{\partial l}{\partial P}\left[b\frac{l(\lambda,k,P)f(k)}{P} - a\right]}{1 + \frac{k}{l}\frac{\partial l}{\partial k}} \\ \frac{\dot{P}}{P} &= b\frac{l(\lambda,k,P)f(k)}{P} - a \end{aligned} \tag{3.12}$$

Proof. See the Appendix. ■

3.2.5 Steady state

At the steady state, $\dot{\lambda} = \dot{k} = \dot{P} = 0$ and system (3.12) becomes

$$r(k) = \rho + \delta \tag{3.13}$$

$$c(\lambda, k, P) = [\rho k + w(k)]\, l(\lambda, k, P) = \frac{a}{b}P - \delta k l(\lambda, k, P) \tag{3.14}$$

$$l(\lambda, k, P) f(k) = \frac{a}{b}P \tag{3.15}$$

because $f(k) = kr(k) + w(k)$.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

According to Michel and Rotillon (1996), pollution has a *distaste effect* on consumption if $U_{cP} < 0$, that is $\varepsilon_{Pc} < 0$; while pollution has a *compensation effect* on consumption if $U_{cP} > 0$, that is $\varepsilon_{Pc} > 0$. According to Bosi, Desmarchelier and Ragot (2013), pollution has a *leisure effect* in the case of positive effect of pollution on labor disutility ($U_{lP} < 0$), that is $\varepsilon_{Pl} > 0$ (because, according to Assumption 7, $U_l < 0$); while pollution has a *disenchantment effect* in the case of negative effect of pollution on labor disutility ($U_{lP} > 0$), that is $\varepsilon_{Pl} < 0$ (because, from Assumption 7, $U_l < 0$).

Consider the system

$$\frac{c(\lambda, k, P)}{l(\lambda, k, P)} = \rho k + w(k) \tag{3.16}$$

$$l(\lambda, k, P) f(k) = \frac{a}{b} P \tag{3.17}$$

Let

$$\varsigma(\lambda) \equiv \frac{c(\lambda, k, P(\lambda))}{l(\lambda, k, P(\lambda))} > 0 \text{ and } \varepsilon_{\varsigma}(\lambda) \equiv \frac{\lambda \varsigma'(\lambda)}{\varsigma(\lambda)}$$

where $P(\lambda)$ is implicitly defined by (3.17).

Proposition 16 *(existence and uniqueness of the steady state) Let Assumptions 6 and 7 hold. A steady state exists. In addition, the steady state is unique if* $\varepsilon_{Pc} < -\varepsilon_{cc}$ *(distate effect (*$\varepsilon_{Pc} < 0$*) or weak compensation effect (*$0 < \varepsilon_{Pc} < -\varepsilon_{cc}$*)) jointly with* $\varepsilon_{Pl} > -\varepsilon_{ll}$ *(leisure effect (*$\varepsilon_{Pl} > 0$*) or weak disenchantment effect (*$-\varepsilon_{ll} < \varepsilon_{Pl} < 0$*)).*

Proof. See the Appendix. ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

3.2.6 Local dynamics

In order to study the local dynamics, we linearize the three-dimensional dynamic system (3.12) :

$$\dot{\lambda} = f_1(\lambda, k, P)$$

$$\dot{k} = f_2(\lambda, k, P)$$

$$\dot{P} = f_3(\lambda, k, P)$$

around the steady state and we obtain a Jacobian matrix :

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \lambda} & \frac{\partial f_1}{\partial k} & \frac{\partial f_1}{\partial P} \\ \frac{\partial f_2}{\partial \lambda} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} \\ \frac{\partial f_3}{\partial \lambda} & \frac{\partial f_3}{\partial k} & \frac{\partial f_3}{\partial P} \end{bmatrix} \tag{3.18}$$

Local bifurcations and local indeterminacy

In continuous time, a local bifurcation generically arises when the real part of an eigenvalue $\lambda(p)$ of the Jacobian matrix crosses zero in response to a change of parameter p. Denoting by p^* the critical parameter value of bifurcation, we get generically two cases: (1) saddle-node bifurcation when a real eigenvalue crosses zero: $\lambda(p^*) = 0$, (2) Hopf bifurcation when the real part of two complex and conjugate eigenvalues $\lambda(p) = a(p) \pm ib(p)$ crosses zero. More precisely, we require $a(p^*) = 0$ and $b(p) \neq 0$ in a neighborhood of p^* (see Bosi and Ragot (2011, p. 76)).

System (3.12) is three-dimensional with two predetermined variables (k and P) and one jump variable (λ). Thus, multiple equilibria (local indeterminacy) arise when the three eigenvalues of the Jacobian matrix (3.18) evaluated at the steady state have negative real parts: either $\lambda_1, \lambda_2, \lambda_3 < 0$ or $\operatorname{Re}\lambda_1, \operatorname{Re}\lambda_2 < 0$ and $\lambda_3 < 0$.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Saddle-node bifurcation

A saddle-node bifurcation generically occurs when a real eigenvalue crosses zero: $\lambda_3 = 0$.

Focus on the Jacobian matrix J and consider the determinant, the sum of minors of order two and the trace:

$$D = \lambda_1\lambda_2\lambda_3$$

$$S = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3$$

$$T = \lambda_1 + \lambda_2 + \lambda_3$$

Proposition 17 *(saddle-node characterization) A saddle-node bifurcation generically arises if and only if $D = 0$. In the saddle-node bifurcation value $p^* = p_S$, we have*

$$\lambda_1 (p_S) = \frac{T (p_S)}{2} - \sqrt{\left[\frac{T (p_S)}{2}\right]^2 - S (p_S)} \tag{3.19}$$

$$\lambda_2 (p_S) = \frac{T (p_S)}{2} + \sqrt{\left[\frac{T (p_S)}{2}\right]^2 - S (p_S)} \tag{3.20}$$

These eigenvalues are nonreal if and only if $T (p_S)^2 < 4S (p_S)$.

Proof. See the Appendix. ■

Hopf bifurcation

A Hopf bifurcation occurs when the real part of two complex and conjugate eigenvalues $\lambda (p) = a (p) \pm ib (p)$ crosses zero. More precisely, we require $a (0) = 0$ and $b (p) \neq 0$ in a neighborhood of $p = 0$, where $p = 0$ is the normalized bifurcation value of parameter (see Bosi and Ragot (2011)).

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Proposition 18 *(Hopf characterization) In the case of a three-dimensional system, a Hopf bifurcation generically arises if and only if* $D = ST$ *and* $S > 0$.

Proof. See the Appendix. ■

Local indeterminacy

In our economy, there are two predetermined variables (k and P) and a jump variable (λ). As seen above, indeterminacy requires the three eigenvalues with negative real parts: either $\lambda_1, \lambda_2, \lambda_3 < 0$ or $\mathrm{Re}\,\lambda_1, \mathrm{Re}\,\lambda_2 < 0$ and $\lambda_3 < 0$.

Proposition 19 *(local indeterminacy) In the case of system (3.22)-(3.24), if all the eigenvalues are real, the equilibrium is locally indeterminate if and only if* $D, T < 0$ *and* $S > 0$.

Proof. See the Appendix. ■

Focus on Proposition 17 and notice that $\lambda_1(p_S)$ and $\lambda_2(p_S)$ can be real or nonreal. If they are real $\mathrm{Re}\,\lambda_1(p_S) = \lambda_1(p_S)$ and $\mathrm{Re}\,\lambda_2(p_S) = \lambda_2(p_S)$.

Proposition 20 *(local indeterminacy through a saddle-node bifurcation) Let* p_S *be the saddle-node bifurcation value of a parameter* p *such that* $D(p_S) = 0$. *The equilibrium is generically locally indeterminate in a (left or right) neighborhood of* p_S *if and only if* $\mathrm{Re}\,\lambda_1(p_S), \mathrm{Re}\,\lambda_2(p_S) < 0$, *where* $\lambda_1(p_S)$ *and* $\lambda_2(p_S)$ *are given by (3.19) and (3.20).*

Proof. See the Appendix. ■

Corollary 21 *Local indeterminacy generically occurs through a saddle-node bifurcation at* $p = p_S$ *if and only if* $D(p_S) = 0$, $S(p_S) > 0$ *and* $T(p_S) < 0$.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Proof. See the Appendix. ■

Focus now on the possibility of local indeterminacy through a Hopf bifurcation.

Notice that, unfortunately, Proposition 19 is of little use because, it is difficult to know whether the eigenvalues are real. In the nonreal case, the necessary condition of Proposition 19 still holds. Indeed, indeterminacy ($\operatorname{Re}\lambda_1 = \operatorname{Re}\lambda_2 < 0$ and $\lambda_3 < 0$) implies

$$D = \lambda_1\lambda_2\lambda_3 = \left[(\operatorname{Re}\lambda_1)^2 + (\operatorname{Im}\lambda_1)^2\right]\lambda_3 < 0$$

$$S = \lambda_1\lambda_2 + (\lambda_1 + \lambda_2)\lambda_3 = (\operatorname{Re}\lambda_1)^2 + (\operatorname{Im}\lambda_1)^2 + 2\operatorname{Re}\lambda_1\lambda_3 > 0$$

$$T = \lambda_1 + \lambda_2 + \lambda_3 = 2\operatorname{Re}\lambda_1 + \lambda_3 < 0$$

However, the sufficient condition fails: even if

$$D = \lambda_1\lambda_2\lambda_3 = \left[(\operatorname{Re}\lambda_1)^2 + (\operatorname{Im}\lambda_1)^2\right]\lambda_3 < 0$$

still implies $\lambda_3 < 0$, conditions $D, T < 0$ and $S > 0$ do not rule out the unpleasant case $\operatorname{Re}\lambda_1 = \operatorname{Re}\lambda_2 > 0$.

We provide instead another sufficient condition for local indeterminacy, that is more restrictive.

Proposition 22 *(local indeterminacy through a Hopf bifurcation) Let p_H the Hopf bifurcation value of a parameter p such that $D(p_H) = S(p_H)T(p_H)$ and $S(p_H) > 0$. If $D(p_H) < 0$, the equilibrium is locally indeterminate for some value of p around p_H.*

Proof. See the Appendix. ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

3.3 Separable isoelastic case

The above conditions for local bifurcations and local indeterminacy are general and it is difficult to provide an economic interpretation. However, introducing more explicit form with exogenous parameters that have an easy clear economic meaning allows us to provide an economic intuition for local bifurcations and local indeterminacy.

The separable case (Assumption 7) is suitable for our local analysis because of the lack of direct cross effects between the marginal utility of consumption and labor. However, we need to introduce more structure for the purpose of economic analysis. In the isoelastic case, the elasticities of matrix (3.11) are constant and have an easy economic interpretation. Thus, we consider isoelastic separable preferences:

$$u(c,P) \equiv \frac{\left(cP^{-\eta}\right)^{1-\varepsilon}}{1-\varepsilon} \text{ and } v(l,P) \equiv \omega\frac{\left(lP^{\psi}\right)^{1+\varphi}}{1+\varphi}$$

where $1/\varepsilon \geq 0$ is the consumption elasticity of intertemporal substitution, $1/\varphi \geq 0$ is the Frisch elasticity of intertemporal substitution and $\omega > 0$ is the weight of disutility of labor in total utility. The elasticities on the RHS of matrix (3.11) appear only in the first two columns of the elasticities matrix E:

$$\tilde{E} \equiv \begin{bmatrix} \varepsilon_{cc} & \varepsilon_{cl} \\ \varepsilon_{lc} & \varepsilon_{ll} \\ \varepsilon_{Pc} & \varepsilon_{Pl} \end{bmatrix} = \begin{bmatrix} \frac{cu_{cc}}{u_c} & 0 \\ 0 & \frac{lv_{ll}}{v_l} \\ \frac{Pu_{Pc}}{u_c} & \frac{Pv_{Pl}}{v_l} \end{bmatrix} = \begin{bmatrix} -\varepsilon & 0 \\ 0 & \varphi \\ (\varepsilon-1)\,\eta & (1+\varphi)\,\psi \end{bmatrix}$$

The elasticities in the third column of E (see (3.10)) are more complicated: they are not merely parametric and involve all the variables: λ, k, P. Fortunately, we no longer need them in the following. Hence, matrix (3.11) simplifies:

$$\begin{bmatrix} \frac{\lambda}{c}\frac{\partial c}{\partial \lambda} & \frac{k}{c}\frac{\partial c}{\partial k} & \frac{P}{c}\frac{\partial c}{\partial P} \\ \frac{\lambda}{l}\frac{\partial l}{\partial \lambda} & \frac{k}{l}\frac{\partial l}{\partial k} & \frac{P}{l}\frac{\partial l}{\partial P} \end{bmatrix} = \begin{bmatrix} \frac{\lambda c_\lambda}{c} & \frac{kc_k}{c} & \frac{Pc_P}{c} \\ \frac{\lambda l_\lambda}{l} & \frac{kl_k}{l} & \frac{Pl_P}{l} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\varepsilon} & 0 & -\eta\frac{1-\varepsilon}{\varepsilon} \\ \frac{1}{\varphi} & \frac{\alpha}{\sigma}\frac{1}{\varphi} & -\psi\frac{1+\varphi}{\varphi} \end{bmatrix} \quad (3.21)$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

and the dynamic system writes:

$$\frac{\dot{\lambda}}{\lambda} = \rho + \delta - r(k) \tag{3.22}$$

$$\frac{\dot{k}}{k} = \frac{\rho + \frac{w(k)}{k} - \frac{c(\lambda,k,P)}{kl(\lambda,k,P)} - \frac{1+\varphi}{\varphi}\left(\rho + \delta - r(k) + \psi\left[a - b\frac{l(\lambda,k,P)f(k)}{P}\right]\right)}{1 + \frac{\alpha}{\sigma}\frac{1}{\varphi}} \tag{3.23}$$

$$\frac{\dot{P}}{P} = b\frac{l(\lambda,k,P)f(k)}{P} - a \tag{3.24}$$

3.3.1 Steady state

Proposition 23 *In the isoelastic case, a unique steady state exists if*

$$\frac{1}{\varepsilon} + \frac{1}{\varphi}\frac{1+\eta\frac{1-\varepsilon}{\varepsilon}}{1+\psi\frac{1+\varphi}{\varphi}} > 0 \tag{3.25}$$

Proof. See the Appendix. ■

According to (3.25), a unique stationary solution only when

$$\eta\frac{\varepsilon-1}{\varepsilon} < 1 + \frac{\varphi}{\varepsilon}\left(1+\psi\frac{1+\varphi}{\varphi}\right) \tag{3.26}$$

We observe that, according to (3.11),

$$\begin{aligned}\frac{P}{c}\frac{\partial c}{\partial P} &= -\frac{\varepsilon_{Pc}}{\varepsilon_{cc}} = \eta\frac{\varepsilon-1}{\varepsilon}\\ \frac{P}{l}\frac{\partial l}{\partial P} &= -\frac{\varepsilon_{Pl}}{\varepsilon_{ll}} = -\psi\frac{1+\varphi}{\varphi}\end{aligned}$$

Thus, inequality (3.26) becomes

$$\frac{P}{c}\frac{\partial c}{\partial P} < 1 + \frac{\varphi}{\varepsilon}\left(1 - \frac{P}{l}\frac{\partial l}{\partial P}\right) \tag{3.27}$$

and holds if the elasticity $(P/c)\,\partial c/\partial P$ is negative or positive but sufficiently small. The RHS of (3.27) becomes larger as soon as the elasticity $(P/l)\,\partial l/\partial P$ becomes sufficiently negative. Thus, the steady state is unique when pollution has a negative

© 2014 Tous droits réservés.
http://doc.univ-lille1.fr

impact on consumption or a moderate positive effect, and a large (negative) impact on labor supply.

3.3.2 Local dynamics

System (3.22)-(3.24) writes:

$$\begin{aligned}
&\dot{\lambda} = f_1(\lambda, k, P) \equiv \lambda\left[\rho + \delta - r(k)\right] \\
&\dot{k} = f_2(\lambda, k, P) \equiv \left(1 + \frac{\alpha(k)}{\sigma(k)}\frac{1}{\varphi}\right)^{-1} \\
&\quad\left[\rho k + w(k) - \frac{c(\lambda, k, P)}{l(\lambda, k, P)} - k\frac{1+\varphi}{\varphi}\left(\rho + \delta - r(k) + \psi\left[a - b\frac{l(\lambda, k, P) f(k)}{P}\right]\right)\right] \\
&\dot{P} = f_3(\lambda, k, P) \equiv bl(\lambda, k, P) f(k) - aP
\end{aligned}$$

We linearize it around the steady state.

In the following, let

$$\begin{aligned}
\theta(k) &\equiv \frac{\alpha(k) + \varphi\sigma(k)\alpha(k)}{\varphi\sigma(k)} \text{ and } \tau(k) \equiv \frac{\alpha(k) + \varphi\sigma(k)}{\varphi\sigma(k)} > \theta(k) \\
\mu &\equiv \psi\frac{1+\varphi}{\varphi},\ \gamma \equiv \frac{r}{\alpha} - \delta,\ s \equiv r\frac{1-\alpha}{\sigma} \\
n &\equiv \mu\frac{a}{\varphi} + \gamma\left(\frac{1}{\varepsilon} + \frac{1}{\varphi}\right) \text{ and } \xi \equiv \gamma\left(\mu + \eta\frac{\varepsilon - 1}{\varepsilon}\right)
\end{aligned}$$

Lemma 24 *Let D, S and T be the determinant, the sum of diagonal minors of order two and the trace. Thus,*

$$\begin{aligned}
D &= \lambda_1\lambda_2\lambda_3 = \frac{as}{\tau}\left[(1+\mu)\left(n - \frac{a\mu}{\varphi}\right) - \frac{\xi}{\varphi}\right] \\
S &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 = \sum_{i=1}^{3}\det M_{ii} = \frac{a\theta\xi - ns}{\tau} - a\rho(1+\mu) \\
T &= \lambda_1 + \lambda_2 + \lambda_3 = \rho - a + a\mu\frac{\theta - \tau}{\tau}
\end{aligned}$$

Proof. See the Appendix. ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Surprisingly and fortunately, D, S and T no longer depends on the steady state values (λ, k, P). However, α and σ depends on k. We will focus on functional forms such that α and σ are constant. For instance, $f(k_t) = Ak_t^{\alpha}$ is characterized by $\sigma = 1$ and α constant.

3.3.3 Local bifurcations and local indeterminacy

Saddle-node bifurcation

Proposition 25 *A saddle-node bifurcation generically occurs if and only if*

$$\xi = \xi_S \equiv (1+\mu)(\varphi n - a\mu)$$

or, equivalently, if and only if

$$\eta_S = \frac{\varepsilon + \varphi + \varphi\mu}{\varepsilon - 1} \tag{3.28}$$

Proof. See the Appendix. ■

We observe that (3.28) is equivalent to

$$\frac{P}{c}\frac{\partial c}{\partial P} = \eta\frac{\varepsilon - 1}{\varepsilon} = 1 + \frac{\varphi}{\varepsilon}(1+\mu)$$

In this case, a saddle-node bifurcation is not surprising. Indeed, a saddle-node is associated to a multiplicity of steady states. We have seen before that uniqueness requires $\varepsilon_{\varsigma}(\lambda) < 0$. But

$$\frac{P}{c}\frac{\partial c}{\partial P} = 1 + \frac{\varphi}{\varepsilon}(1+\mu) = 1 + \frac{\varphi}{\varepsilon}\left(1 - \frac{P}{l}\frac{\partial l}{\partial P}\right)$$

is equivalent to $\varepsilon_{\varsigma}(\lambda) = 0$ entailing a change in the number of steady states.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Hopf bifurcation

Let

$$\eta_H \equiv \frac{\varepsilon}{\varepsilon - 1}\left(\frac{\xi_H}{\gamma} - \mu\right)$$

with

$$\xi_H \equiv \frac{s\left(1+\mu\right)\left(n - \frac{a\mu}{\varphi}\right) + \left(\rho\tau\left(1+\mu\right) + \frac{ns}{a}\right)\left(\rho - a - a\mu\frac{\tau-\theta}{\tau}\right)}{\frac{s}{\varphi} + \theta\left(\rho - a - a\mu\frac{\tau-\theta}{\tau}\right)}$$

Proposition 26 *(limit cycles) There exists a parameter region such that, when η goes through η_H, the system undergoes a Hopf bifurcation.*

Proof. See the Appendix. ■

It is interesting to see that $\lim_{\alpha\to 1} \eta_H = \varepsilon/\left(\varepsilon - 1\right)$. Then, $\eta_H > 0$ iif $\varepsilon > 1$. Recall that

$$\frac{P}{c}\frac{\partial c}{\partial P} = -\eta\frac{1-\varepsilon}{\varepsilon}$$

that is, in this limit case, a Hopf bifurcation occurs only under a compensation effect ($\partial c/\partial P > 0$ or $\varepsilon_{Pc} > 0$) according to Michel and Rotillon (1996).

Assume a rise of P near the steady state. Since $\partial c/\partial P > 0$ and $\partial l/\partial P < 0$ (matrix (3.21)), this entails an increase of c jointly with a decrease of k and a decrease of l. These two effects imply a fall in the production level and, in turn, a decrease of pollution. By this channel, deterministic endogenous fluctuations occur near the steady state.

a represents the pollution inertia, that is, when ma is less than ρ, say $a \approx 0$, P becomes more volatile and the occurrence of cycle more likely.

It is interesting to notice that the curvature of the intensive production function (Cobb-Douglas) can moderate or exacerbate the environmental effects of a variation in the saving/investment level. Indeed, if, as before, as above $a \approx 0$, the pollution

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

accumulation process writes $\dot{P} = blAk^{\alpha}$. Thus, when α becomes closer to zero, a variation of k has a little effect on P, while when α tends to one, this effect is maximal and the occurrence of cycle more likely.

Local indeterminacy

Proposition 27 *(local indeterminacy through a Hopf bifurcation) If*

$$\frac{as}{\tau}\left[(1+\mu)\left(n-\frac{a\mu}{\varphi}\right)-\frac{\xi_H}{\varphi}\right]<0$$

then there exists a parameter region where indeterminacy occurs.

Proof. See the Appendix. ■

Proposition 28 *(local indeterminacy through a saddle-node bifurcation) Let p_S be the saddle-node bifurcation value of a parameter p such that $D(p_S)=0$. If*

$$\mathrm{Re}\left(\frac{T(p_S)}{2}-\sqrt{\left[\frac{T(p_S)}{2}\right]^2-S(p_S)}\right)<0$$
$$\mathrm{Re}\left(\frac{T(p_S)}{2}+\sqrt{\left[\frac{T(p_S)}{2}\right]^2-S(p_S)}\right)<0$$

then, generically, there exists a parameter region where indeterminacy occurs.

Proof. Apply Proposition 20. ■

The possibility of self-fulfilling prophecies rests on equilibrium indeterminacy. Let us provide an intuition for them in our economy.

Let the economy be at the steady state and assume that any consumer expect today an increase in the pollution level tomorrow. Since $\partial c/\partial P>0$ and $\partial l/\partial P<0$, she wants a higher consumption demand tomorrow jointly with a lower labor supply. She needs to save more today to finance a larger consumption tomorrow under a

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

lower labor income. The increase in capital intensity will enhance the production level and promote an increase in the pollution stock. Hence, the expectation of higher pollution tomorrow turns out to be self-fulfilling.

3.4 Conclusion

We have considered an economy à la Ramsey where production pollutes and the negative externality affects consumption demand and labor supply.

Within this simple framework, we have found that a strong compensation effect jointly with a strong leisure effect generates local indeterminacy through a Hopf bifurcation. Despite the lack of empirical evidences on the pollution effects on consumption demand, the leisure effect, pointed out in recent empirical contributions, legitimates our theoretical result.

© 2014 Tous droits réservés.
http://doc.univ-lille1.fr

3.5 Appendix

Proof of Proposition 13

The Hamiltonian writes $\tilde{H} = e^{-\rho t} U(c, l, P) + \tilde{\lambda}[(r-\delta)h + wl - c]$ and the first-order conditions

$$\partial \tilde{H}/\partial \tilde{\lambda} = (r-\delta)h + wl - c = \dot{h}$$

$$\partial \tilde{H}/\partial h = \tilde{\lambda}(r-\delta) = -\tilde{\lambda}'$$

$$\partial \tilde{H}/\partial c = e^{-\rho t} U_c - \tilde{\lambda} = 0$$

$$\partial \tilde{H}/\partial l = e^{-\rho t} U_l + \tilde{\lambda} w = 0$$

jointly with the transversality condition $\lim_{t\to\infty} \tilde{\lambda}(t) h(t) = 0$. Setting $\lambda \equiv e^{\rho t}\tilde{\lambda}$, we find $\dot{\lambda} - \rho\lambda = e^{\rho t}\tilde{\lambda}'$ and equations in Proposition 13. The discounted Hamiltonian $H \equiv e^{\rho t}\tilde{H}$ becomes $H = U(c, l, P) + \lambda[(r-\delta)h + wl - c]$. ■

Proof of Proposition 14

Differentiating the system

$$\lambda - U_c(c, l, P) = 0$$

$$\lambda w(k) + U_l(c, l, P) = 0$$

we get

$$\begin{aligned} \varepsilon_{cc}\frac{dc}{c} + \varepsilon_{lc}\frac{dl}{l} &= \frac{d\lambda}{\lambda} - \varepsilon_{Pc}\frac{dP}{P} \\ \varepsilon_{cl}\frac{dc}{c} + \varepsilon_{ll}\frac{dl}{l} &= \frac{d\lambda}{\lambda} + \frac{\alpha}{\sigma}\frac{dk}{k} - \varepsilon_{Pl}\frac{dP}{P} \end{aligned}$$

that is

$$\begin{bmatrix} \frac{dc}{c} \\ \frac{dl}{l} \end{bmatrix} = \frac{M}{\varepsilon_{cc}\varepsilon_{ll} - \varepsilon_{lc}\varepsilon_{cl}} \begin{bmatrix} \frac{d\lambda}{\lambda} \\ \frac{dk}{k} \\ \frac{dP}{P} \end{bmatrix}$$

© 2014 Tous droits réservés.

http://doc.univ-lille1.fr

where M is given by (3.9). Thus, we obtain the following matrix of partial elasticities (3.8). ■

Proof of Proposition 15

Let us reconsider the dynamic system:

$$\dot{\lambda} = \lambda \left[\rho + \delta - r(k)\right]$$

$$\dot{h} = (r - \delta) h + wl - c$$

$$\dot{P} = -aP + blf(k)$$

We observe that $h = kl$ and, thus,

$$\frac{\dot{h}}{h} = \frac{\dot{k}}{k} + \frac{\dot{l}}{l}$$

In addition, $l = l(\lambda, k, P)$ and, thus,

$$\frac{\dot{l}}{l} = \frac{\lambda}{l}\frac{\partial l}{\partial \lambda}\frac{\dot{\lambda}}{\lambda} + \frac{k}{l}\frac{\partial l}{\partial k}\frac{\dot{k}}{k} + \frac{P}{l}\frac{\partial l}{\partial P}\frac{\dot{P}}{P}$$

where the elasticities

$$\frac{\lambda}{l}\frac{\partial l}{\partial \lambda}, \ \frac{k}{l}\frac{\partial l}{\partial k}, \ \frac{P}{l}\frac{\partial l}{\partial P}$$

are given by (3.8).

We obtain the following three-dimensional dynamic system

$$\begin{aligned}
\frac{\dot{\lambda}}{\lambda} &= \rho + \delta - r(k) \\
\frac{\dot{k}}{k} &= r(k) - \delta + \frac{w(k)}{k} - \frac{c(\lambda, k, P)}{kl(\lambda, k, P)} - \frac{\dot{l}}{l} \\
&= r(k) - \delta + \frac{w(k)}{k} - \frac{c(\lambda, k, P)}{kl(\lambda, k, P)} - \frac{\lambda}{l}\frac{\partial l}{\partial \lambda}\frac{\dot{\lambda}}{\lambda} - \frac{k}{l}\frac{\partial l}{\partial k}\frac{\dot{k}}{k} - \frac{P}{l}\frac{\partial l}{\partial P}\frac{\dot{P}}{P} \\
\frac{\dot{P}}{P} &= b\frac{l(\lambda, k, P) f(k)}{P} - a
\end{aligned}$$

that is system (3.12). ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Proof of Proposition 16

Focus first on existence.

Assumption 6 ensures that a stationary level of capital k exists according to equation (3.13). The concavity of f ensures also that there is a unique stationary level of capital. The difficulty consists in proving that a pair (λ, P) satisfying system (3.14)-(3.15) exists and is unique, given k.

We apply first the Implicit Function Theorem to equation (3.17) to obtain a function $P(\lambda)$ with

$$P'(\lambda) = \frac{\frac{\lambda l_\lambda}{l}\frac{l}{\lambda}}{\frac{a}{bf(k)} - \frac{Pl_P}{l}\frac{l}{P}}$$

Noticing that, at the steady state, $l/P = a/(bf)$, we get the multiplier elasticity of pollution

$$\zeta \equiv \frac{\lambda P'(\lambda)}{P(\lambda)} = \frac{\frac{\lambda l_\lambda}{l}}{1 - \frac{Pl_P}{l}}$$

Replacing $P = P(\lambda)$ into equation (3.16), we find

$$\varsigma(\lambda) \equiv \frac{c(\lambda, k, P(\lambda))}{l(\lambda, k, P(\lambda))} = \rho k + w(k) > 0$$

with

$$\begin{aligned} \varepsilon_\varsigma(\lambda) \equiv \frac{\lambda \varsigma'(\lambda)}{\varsigma(\lambda)} &= \frac{\lambda c_\lambda}{c} - \frac{\lambda l_\lambda}{l} + \zeta\left(\frac{Pc_P}{c} - \frac{Pl_P}{l}\right) \\ &= \frac{\lambda c_\lambda}{c} - \frac{\lambda l_\lambda}{l}\frac{1 - \frac{Pc_P}{c}}{1 - \frac{Pl_P}{l}} \end{aligned} \tag{3.29}$$

Let us prove that $\lim_{\lambda\to 0}\varsigma(\lambda) > \rho k + w(k) > \lim_{\lambda\to\infty}\varsigma(\lambda)$. These boundary conditions are sufficient conditions for equilibrium existence.

From the static consumption-leisure arbitrage, we know that $\lambda = U_c$ and $\lambda w(k) = -U_l$. It follows that $\lambda \to 0$ implies $U_c \to 0$ and $U_l \to 0$ and in turn $c \to +\infty$ and $l \to 0$. In the same way, $\lambda \to +\infty$ implies that $U_c \to +\infty$ and $U_l \to -\infty$, in turn

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

$c \to 0$ and $l \to +\infty$. Therefore, $\lim_{\lambda \to 0} \varsigma(\lambda) = +\infty$ and $\lim_{\lambda \to \infty} \varsigma(\lambda) = 0$. Then, there exists a stationary solution.

Focus now on uniqueness.

Monotonicity of $\varsigma(\lambda)$ implies the steady state uniqueness. More precisely, since $\lim_{\lambda \to 0} \varsigma(0) > \rho k + w(k) > \lim_{\lambda \to \infty} \varsigma(\lambda)$, the negativity of ε_ς implies the steady state uniqueness.

Under Assumption 7 (separability), expression (3.29) writes

$$\varepsilon_\varsigma(\lambda) = \frac{\lambda c_\lambda}{c} - \frac{\lambda l_\lambda}{l} \frac{1 - \frac{P c_P}{c}}{1 - \frac{P l_P}{l}} = \frac{1}{\varepsilon_{cc}} - \frac{1}{\varepsilon_{ll}} \frac{1 + \frac{\varepsilon_{Pc}}{\varepsilon_{cc}}}{1 + \frac{\varepsilon_{Pl}}{\varepsilon_{ll}}} \tag{3.30}$$

(see elasticities (3.11)) with $\varepsilon_{cc} < 0$ and $\varepsilon_{ll} > 0$. Thus $\varepsilon_\varsigma(\lambda) < 0$ if and only if

$$\frac{1 + \frac{\varepsilon_{Pc}}{\varepsilon_{cc}}}{1 + \frac{\varepsilon_{Pl}}{\varepsilon_{ll}}} > \frac{\varepsilon_{ll}}{\varepsilon_{cc}}$$

Thus, a sufficient condition for $\varepsilon_\varsigma(\lambda) < 0$ is $\varepsilon_{Pc} < -\varepsilon_{cc}$ (distate effect ($\varepsilon_{Pc} < 0$) or weak compensation effect ($0 < \varepsilon_{Pc} < -\varepsilon_{cc}$)) jointly with $\varepsilon_{Pl} > -\varepsilon_{ll}$ (leisure effect ($\varepsilon_{Pl} > 0$) or weak disenchantment effect ($-\varepsilon_{ll} < \varepsilon_{Pl} < 0$)), we have $\varepsilon_\varsigma(\lambda) < 0$. In the other cases, multiple steady state may occur. ■

Proof of Proposition 17

Generically, $\lambda_3 = 0$ if and only if $D = 0$. In this case, $S = \lambda_1 \lambda_2$ and $T = \lambda_1 + \lambda_2$. Solving this system of two equations for λ_1 and λ_2, we get (3.19) and (3.20). ■

Proof of Proposition 18

Necessity In a three-dimensional dynamical system, we require at the bifurcation value: $\lambda_1 = ib = -\lambda_2$ with no generic restriction on λ_3 (see Bosi and Ragot (2011) or Kuznetsov (2004) among others). The characteristic polynomial of J is given by: $P(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) = \lambda^3 - T\lambda^2 + S\lambda - D$. Using $\lambda_1 = ib = -\lambda_2$, we find $D = b^2 \lambda_3$, $S = b^2$, $T = \lambda_3$. Thus, $D = ST$ and $S > 0$.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Sufficiency In the case of a three-dimensional system, one eigenvalue is always real, the others two are either real or nonreal and conjugated. Let us show that, if $D = ST$ and $S > 0$, these eigenvalues are nonreal with zero real part and, hence, a Hopf bifurcation generically occurs.

We observe that $D = ST$ implies

$$\lambda_1\lambda_2\lambda_3 = (\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)(\lambda_1 + \lambda_2 + \lambda_3)$$

or, equivalently,

$$(\lambda_1 + \lambda_2)\left[\lambda_3^2 + (\lambda_1 + \lambda_2)\lambda_3 + \lambda_1\lambda_2\right] = 0 \tag{3.31}$$

This equation holds if and only if $\lambda_1+\lambda_2 = 0$ or $\lambda_3^2+(\lambda_1 + \lambda_2)\lambda_3+\lambda_1\lambda_2 = 0$. Solving this second-degree equation for λ_3, we find $\lambda_3 = -\lambda_1$ or $-\lambda_2$. Thus, (3.31) holds if and only if $\lambda_1 + \lambda_2 = 0$ or $\lambda_1 + \lambda_3 = 0$ or $\lambda_2 + \lambda_3 = 0$. Without loss of generality, let $\lambda_1 + \lambda_2 = 0$ with, generically, $\lambda_3 \neq 0$ a real eigenvalue. Since $S > 0$, we have also $\lambda_1 = -\lambda_2 \neq 0$. We obtain $T = \lambda_3 \neq 0$ and $S = D/T = \lambda_1\lambda_2 = -\lambda_1^2 > 0$. This is possible only if λ_1 is nonreal. If λ_1 is nonreal, λ_2 is conjugated, and, since $\lambda_1 = -\lambda_2$, they have a zero real part. ■

Proof of Proposition 19

Necessity In the real case, we obtain $D = \lambda_1\lambda_2\lambda_3 < 0$, $S = \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3 > 0$ and $T = \lambda_1 + \lambda_2 + \lambda_3 < 0$.

Sufficiency We want to prove that, if $D, T < 0$ and $S > 0$, then $\lambda_1, \lambda_2, \lambda_3 < 0$. Notice that $D < 0$ implies $\lambda_1, \lambda_2, \lambda_3 \neq 0$.

$D < 0$ implies that at least one eigenvalue is negative. Let, without loss of generality, $\lambda_3 < 0$. Since $\lambda_3 < 0$ and $D = \lambda_1\lambda_2\lambda_3 < 0$, we have $\lambda_1\lambda_2 > 0$. Thus, there are two subcases: (1) $\lambda_1, \lambda_2 < 0$, (2) $\lambda_1, \lambda_2 > 0$. If $\lambda_1, \lambda_2 > 0$, $T < 0$ implies

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

$\lambda_3 < -(\lambda_1 + \lambda_2)$ and, hence,

$$S = \lambda_1\lambda_2 + (\lambda_1 + \lambda_2)\lambda_3 < \lambda_1\lambda_2 - (\lambda_1 + \lambda_2)^2 = -\lambda_1^2 - \lambda_2^2 - \lambda_1\lambda_2 < 0$$

a contradiction. Then, $\lambda_1, \lambda_2 < 0$. ■

Proof of Proposition 20

$D(p_S) = 0$ if and only if $\lambda_3(p_S) = 0$ without loss of generality.

Necessity If the equilibrium is locally indeterminate in a (left or right) neighborhood of p_S, then there exists $\varepsilon > 0$ such that $\operatorname{Re}\lambda_1(p), \operatorname{Re}\lambda_2(p), \lambda_3(p_S) < 0$ for any $p \in (p_S - \varepsilon, p_S)$ or for any $p \in (p_S, p_S + \varepsilon)$, and, hence, generically, $\operatorname{Re}\lambda_1(p_S), \operatorname{Re}\lambda_2(p_S) < 0$ and $\lambda_3(p_S) = 0$.

Sufficiency If $\operatorname{Re}\lambda_1(p_S), \operatorname{Re}\lambda_2(p_S) < 0$ and $\lambda_3(p_S) = 0$, then there exists $\varepsilon > 0$ such that $\operatorname{Re}\lambda_1(p), \operatorname{Re}\lambda_2(p), \lambda_3(p_S) < 0$ (local indeterminacy) for any $p \in (p_S - \varepsilon, p_S)$ or for any $p \in (p_S, p_S + \varepsilon)$. ■

Proof of Corollary 21

Necessity If local indeterminacy occurs through a saddle-node bifurcation at $p = p_S$, that is $\operatorname{Re}\lambda_1(p_S), \operatorname{Re}\lambda_2(p_S) < 0$ and $\lambda_3(p_S) = 0$ (Proposition 20), then, in the real case, $D(p_S) = \lambda_1(p_S)\lambda_2(p_S)\lambda_3(p_S) = 0$, $S(p_S) = \lambda_1(p_S)\lambda_2(p_S) > 0$ and $T(p_S) = \lambda_1(p_S) + \lambda_2(p_S) < 0$, and, in the nonreal case, $D(p_S) = \lambda_1(p_S)\lambda_2(p_S)\lambda_3(p_S) = 0$, $S(p_S) = \lambda_1(p_S)\lambda_2(p_S) = [\operatorname{Re}\lambda_1(p_S)]^2 + [\operatorname{Im}\lambda_1(p_S)]^2 > 0$ and $T(p_S) = \lambda_1(p_S) + \lambda_2(p_S) = 2\operatorname{Re}\lambda_1(p_S) < 0$.

Sufficiency Conversely, if $D(p_S) = 0$, $S(p_S) > 0$ and $T(p_S) < 0$, then $D(p_S) = \lambda_1(p_S)\lambda_2(p_S)\lambda_3(p_S) = 0$ implies without loss of generality $\lambda_3(p_S) = 0$, $S(p_S) = \lambda_1(p_S)\lambda_2(p_S)$ and $T(p_S) = \lambda_1(p_S) + \lambda_2(p_S)$. If $\lambda_1(p_S)$ and $\lambda_2(p_S)$ are real, $S(p_S) > 0$ and $T(p_S) < 0$ implies $\lambda_1(p_S), \lambda_2(p_S) < 0$, while, if $\lambda_1(p_S)$ and $\lambda_2(p_S)$ are nonreal $T(p_S) = 2\operatorname{Re}\lambda_1(p_S) < 0$, so that $\operatorname{Re}\lambda_1(p_S) = \operatorname{Re}\lambda_2(p_S) < 0$. Thus, in both the

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

cases, $\operatorname{Re}\lambda_1(p_S), \operatorname{Re}\lambda_2(p_S) < 0$ and $\lambda_3(p_S) = 0$, and Proposition 20 implies local indeterminacy through a saddle-node bifurcation at $p = p_S$.

Proof of Proposition 22

By Proposition 18, we have $\operatorname{Re}\lambda_1(p_H) = \operatorname{Re}\lambda_2(p_H) = 0$. $D(p_H) = [\operatorname{Im}\lambda_1(p_H)]^2 \lambda_3(p_H) < 0$ implies $\lambda_3(p_H) < 0$. Thus, there exists $\varepsilon > 0$ such that, generically, we have $\operatorname{Re}\lambda_1(p), \operatorname{Re}\lambda_2(p), \lambda_3(p) < 0$ (local indeterminacy) for any $p \in (p_H - \varepsilon, p_H)$ or, alternatively, for any $p \in (p_H, p_H + \varepsilon)$. ■

Proof of Proposition 23

$\varepsilon_\varsigma(\lambda) \equiv \lambda\varsigma'(\lambda)/\varsigma(\lambda)$ and $\varsigma(\lambda) > 0$. Expression (3.30) writes:

$$\varepsilon_\varsigma(\lambda) = \frac{\lambda c_\lambda}{c} - \frac{\lambda l_\lambda}{l}\frac{1-\frac{Pc_P}{c}}{1-\frac{Pl_P}{l}} = -\frac{1}{\varepsilon} - \frac{1}{\varphi}\frac{1+\eta\frac{1-\varepsilon}{\varepsilon}}{1+\psi\frac{1+\varphi}{\varphi}}$$

We know that $+\infty = \lim_{\lambda\to 0}\varsigma(\lambda) > \rho k + w(k) > \lim_{\lambda\to\infty}\varsigma(\lambda) = 0$. Thus, there exists a unique stationary solution only when $\varepsilon_\varsigma(\lambda) < 0$. ■

Proof of Lemma 24

The Jacobian matrix (3.18) becomes:

$$J = \begin{bmatrix} 0 & s\frac{\lambda}{k} & 0 \\ \frac{\partial f_2}{\partial \lambda} & \frac{\partial f_2}{\partial k} & \frac{\partial f_2}{\partial P} \\ a\frac{\lambda l_\lambda}{l}\frac{P}{\lambda} & a\left(\alpha + \frac{kl_k}{l}\right)\frac{P}{k} & a\left(\frac{Pl_P}{l} - 1\right) \end{bmatrix}$$

with

$$\begin{aligned}
\frac{\partial f_2}{\partial \lambda} &= \frac{1}{\tau}\frac{k}{\lambda}\left[a\mu\frac{\lambda l_\lambda}{l} + \gamma\left(\frac{\lambda l_\lambda}{l} - \frac{\lambda c_\lambda}{c}\right)\right] \\
\frac{\partial f_2}{\partial k} &= \frac{1}{\tau}\left[a\mu\left(\alpha + \frac{kl_k}{l}\right) + \gamma\left(\frac{kl_k}{l} - \frac{kc_k}{c}\right) + \rho - \frac{s}{\varphi}\right] \\
\frac{\partial f_2}{\partial P} &= \frac{1}{\tau}\frac{k}{P}\left[a\mu\left(\frac{Pl_P}{l} - 1\right) + \gamma\left(\frac{Pl_P}{l} - \frac{Pc_P}{c}\right)\right]
\end{aligned}$$

because, at the steady state,

$$\frac{c}{kl} = \gamma > 0,\ \frac{w}{k} = r\frac{1-\alpha}{\alpha} \text{ and } b\frac{lf(k)}{P} = a$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Using (3.2), (3.3) and (3.21), we find

$$J = (m_{ij}) = \begin{bmatrix} 0 & s\frac{\lambda}{k} & 0 \\ \frac{n}{\tau}\frac{k}{\lambda} & \rho + a\mu\frac{\theta}{\tau} & -\frac{\xi + a\mu(1+\mu)}{\tau}\frac{k}{P} \\ \frac{a}{\varphi}\frac{P}{\lambda} & a\theta\frac{P}{k} & -a(1+\mu) \end{bmatrix}$$

■

Proof of Proposition 25

A saddle-node bifurcation generically occurs if and only if

$$D = \frac{as}{\tau}\left[(1+\mu)\left(n - \frac{a\mu}{\varphi}\right) - \frac{\xi}{\varphi}\right] = 0$$

(Proposition 17). ■

Proof of Proposition 26

Focus on Proposition 18. We know that a Hopf bifurcation arises if and only if $D = ST$ and $S > 0$. We know that

$$\begin{aligned} D &= \frac{as}{\tau}\left[(1+\mu)\left(n - \frac{a\mu}{\varphi}\right) - \frac{\xi}{\varphi}\right] \\ S &= \frac{a\theta\xi - ns}{\tau} - a\rho(1+\mu) \\ T &= \rho - a + a\mu\frac{\theta - \tau}{\tau} \end{aligned}$$

Thus, a Hopf bifurcation arises if and only if

$$\begin{aligned} \frac{as}{\tau}\left[(1+\mu)\left(n - \frac{a\mu}{\varphi}\right) - \frac{\xi}{\varphi}\right] &= \left(\frac{a\theta\xi - ns}{\tau} - a\rho(1+\mu)\right)\left(\rho - a + a\mu\frac{\theta-\tau}{\tau}\right) \\ \frac{a\theta\xi - ns}{\tau} - a\rho(1+\mu) &> 0 \end{aligned}$$

that is if and only if

$$\begin{aligned} \xi_H &\equiv \frac{s(1+\mu)\left(n - \frac{a\mu}{\varphi}\right) + \left(\rho\tau(1+\mu) + \frac{ns}{a}\right)\left(\rho - a - a\mu\frac{\tau-\theta}{\tau}\right)}{\frac{s}{\varphi} + \theta\left(\rho - a - a\mu\frac{\tau-\theta}{\tau}\right)} \\ \xi_H &> \frac{ns + a\rho\tau(1+\mu)}{a\theta}\ (>0) \end{aligned}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

A Hopf bifurcation generically occurs if the following restriction is satisfied:

$$\frac{s\left(1+\mu\right)\left(n-\frac{a\mu}{\varphi}\right)+\left(\rho\tau\left(1+\mu\right)+\frac{ns}{a}\right)\left(\rho-a-a\mu\frac{\tau-\theta}{\tau}\right)}{\frac{s}{\varphi}+\theta\left(\rho-a-a\mu\frac{\tau-\theta}{\tau}\right)}>\frac{ns+a\rho\tau\left(1+\mu\right)}{a\theta} \tag{3.32}$$

If

$$\frac{s}{\varphi}+\theta\left(\rho-a-a\mu\frac{\tau-\theta}{\tau}\right)>0 \tag{3.33}$$

(3.32) becomes equivalent to

$$a\left(1+\mu\right)\left(\varphi n\theta-\tau\rho-a\mu\theta\right)-ns>0 \tag{3.34}$$

Let us show that inequalities (3.33) and (3.34) are satisfied for some parametric values. Consider the case $a<\rho$ and $\alpha\approx 1$. Inequalities (3.33) and (3.34) become

$$\lim_{\alpha\rightarrow 1}\left[\frac{s}{\varphi}+\theta\left(\rho-a-a\mu\frac{\tau-\theta}{\tau}\right)\right]=\theta\left(\rho-a\right)>0$$

and

$$\lim_{\alpha\rightarrow 1}\left[a\left(1+\mu\right)\left(\varphi n\theta-\tau\rho-a\mu\theta\right)-ns\right]=a\rho\tau\left(1+\mu\right)\frac{\varphi}{\varepsilon}>0$$

because

$$\lim_{\alpha\rightarrow 1}n\equiv\mu\frac{a}{\varphi}+\rho\left(\frac{1}{\varepsilon}+\frac{1}{\varphi}\right)$$

■

Proof of Proposition 27

Notice that

$$D\left(p_H\right)=\frac{as}{\tau}\left[\left(1+\mu\right)\left(n-\frac{a\mu}{\varphi}\right)-\frac{\xi_H}{\varphi}\right]<0$$

and apply Proposition 22. ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Chapter 4

Regressive environmental taxation and local indeterminacy

4.1 Introduction

The previous chapters have pointed out the dynamical complexity arising from the interplay between growth and pollution externalities. In the present chapter, we analyze the existing evironmental friendly fiscal policies.

A lot of scholars have pointed out the regressive nature of environmental taxation. Grainger and Kolstad (2009), Hasset, Mathur and Metcalf (2007) or West and Williams (2004) show evidences of carbon tax regressivity, with respect to household's incomes, for the United States and Wier et al (2005) for the Denmark. More generally, Speck (1999) shows that carbon tax are regressive in most of OECD countries while the EEA (2011) insists on this regressivity in most of European countries. For example, "in the United Kingdom, the sum of environmental taxes and charges clearly has a regressive impact on households, with the proportion of income paid

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

decreasing consistently as income levels rise"(EEA 2011).

Such a regressivity implies negative distributional effects (Grainger and Kolstad 2009) but this is possibly not the only disadvantage of such a tax scheme. Indeed, the literature point out the ambiguous effects of non-linear income taxes on the macroeconomic stability, especially when the Government faces a balanced budget constraint. On one hand, Schmitt-Grohé and Uribe (1997) and Guo and Lansing (1998) stress respectively the destabilizing effect of a regressive income tax and the stabilizing impact of a progressive tax scheme within a Ramsey model. On the other hand, Guo and Harrison (2001) analyze a two sectors economy and they find that a regressive tax policy can prevent the economy from expectations driven fluctuations when the investment externalities are strong enough. In addition, Bosi and Seegmuller (2010) show that progressive income tax can promote local indeterminacy in a Ramsey model when the households are heterogeneous as in Becker (1980). Thus, it appears that the (de)-stabilizing effect of regressive income taxes seems to depend crucially upon the framework of analysis. Departing from this observation, one can rise the question of the (de)-stabilizing effect of regressive income tax when the tax revenues serve to finance depollution expenditures.

To answer this important question, we develop a Ramsey economy where a pollution externality comes from the capital accumulation and reduces the household's satisfaction. The Government levies a regressive capital income tax to finance public depollution expenditures. Within this simple framework, we find that such a regressivity may promote expectation-driven fluctuations. Such a conclusion stress

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

another possible adverse effect of environmental taxes used in most of OECD countries.

In addition, our result contribute also to the literature on the interplay between pollution externalities and local indeterminacy. Indeed, some scholars have already explored the link between environmental externalities and local indeterminacy within a Ramsey framework. In particular, Itaya (2008) have shown that a pollution externality enhances the parameter region for which local indeterminacy occurs in an endogenous growth model very close to Benhabib and Farmer (1994). And more recently, Fernandez, Perez and Ruiz (2012) have stressed the role of non separability between consumption and pollution in the utility function for the occurrence of such dynamical phenomena. Our result complete this literature in three points: 1) we show that local indeterminacy can occur when pollution is seen as a stock variable while this literature remain focused on a flow pollution externality, 2) we stress the role of the capital-labor substitution while the literature assume in most cases a Cobb-Douglas production function and 3) We stress the role of environmental taxation to explain endogenous business cycles.

The rest of the chapter is organized as follows: 1) we present the model, 2) we analyze the local dynamics, 3) we give some economic interpretations and 4) we conclude the chapter.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

4.2 The model

We analyze a discrete time Ramsey economy where a pollution externality comes from the capital accumulation and reduces the household's utility. The production sector produces a single commodity which can be consumed/invested. A Government levies a regressive capital income tax to finance public depollution expenditures according to a balanced budget-rule.

4.2.1 The households

At each time t, the household earns a wage w_t from his inelastic labor supply and receives capital income $r_t k_t$ from his past saving. w_t and r_t are two competitive market prices. We assume that the Government levies a capital income tax to finance depollution expenditures, namely $\tau(r_t k_t)$. As in most European countries, such a green tax is regressive, namely $\tau''(r_t k_t) < 0$. The after tax capital income is represented by $\varphi(r_t k_t)$, it follows that $\varphi(r_t k_t) = r_t k_t [1 - \tau(r_t k_t)]$.

Assumption 8 [1]*$\varphi : \mathbb{R}^+ \to \mathbb{R}^+$, is C^2 and $\varphi(0) = 0$. Moreover $\varphi(x) \leq x$, $0 < \varphi'(x) \leq 1$, $\lim_{x \to 0} \varphi'(x) = 0$ and $\varphi''(x) \geq 0$.*

We define by μ and η the first and second order elasticities of φ, namely :

$$(\mu, \eta) \equiv \left(\frac{x\varphi'(x)}{\varphi(x)}, \frac{x\varphi''(x)}{\varphi'(x)} \right)$$

μ captures the rate of the ecotax while η captures its regressivity.

[1] We denote rk by x for a visibility convenience.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

The representative household uses his incomes to consume c_t and save $k_{t+1} - (1-\delta)\,k_t$, where $\delta \in (0,1)$ is the capital depreciation rate. The household's budget constraint is then :

$$c_t + k_{t+1} - (1-\delta)\,k_t = w_t + \varphi\,(r_t k_t) \tag{4.1}$$

The household's preferences are described by $u\,(c,P)$, where P is the aggregate pollution level.

Assumption 9 $u_1 > 0$, $u_2 < 0$, $u_{11} < 0$, $u_{12} \geq 0$ *and* $\lim\limits_{c\to 0} u_1\,(c_t, P_t) = +\infty$, $\lim\limits_{c\to+\infty} u_1\,(c_t, P_t) = 0$

We set :

$$(\varepsilon, \rho) \equiv \left(-\frac{cu_{11}\,(c,P)}{u_1\,(c,P)}, -\frac{Pu_{12}\,(c,P)}{u_1\,(c,P)}\right)$$

ε is the inverse of the intertemporal elasticity of substitution in consumption and ρ captures the impact of pollution on marginal utility of consumption. The sign of the cross derivative u_{12}, and in turn of ρ, is ambiguous. Following Michel and Rotillon (1996), there is a *distate effect* if $u_{12} < 0(\rho > 0)$ and a *compensation effect* if $u_{12} > 0(\rho < 0)$. Nevertheless, to the best of our knowledge, there is no empirical evidences which can drive our investigation. We will analyze two different cases through this chapter. First, we will investigate the case where the utility function is separable with respect to pollution and consumption, that is $u_{12} = 0$ and secondly, we will assume a *compensation effect*, namely $u_{12} > 0$ $(\rho < 0)$.

The household maximizes his intertemporal utility function $\sum_{t=0}^{+\infty} \beta^t u\,(c_t, P_t)$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

with respect to his budget constraint (4.1), where $\beta \in (0,1)$ is a discount factor. The first order conditions give the following dynamical Euler equation :

$$\beta u_1 (c_{t+1}, P_{t+1}) [r_{t+1}\varphi' (r_{t+1}k_{t+1}) + \gamma] = u_1 (c_t, P_t) \tag{4.2}$$

with $\gamma = (1-\delta)$, jointly with the transversality condition $\lim_{t\to+\infty} \beta^t u_1 (c_t, P_t) k_{t+1} = 0$.

4.2.2 The representative firm

At time t the representative firm produces a single output Y_t. Technology is represented by a constant returns to scale production function: $Y_t = AF(K_t, L_t)$, where K_t and L_t are the demands for capital and labor at time t and $A > 0$ is a scale parameter.

Assumption 10 *The production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is C^1, is homogeneous of degree one, strictly increasing and concave. Standard Inada conditions hold.*

The firm chooses the amount of capital and labor to maximize its profit taking as given the real interest rate r_t and the real wage w_t. The program :

$$\max_{K_t, L_t} [AF(K_t, L_t) - r_t K_t - w_t L_t]$$

is correctly defined under assumption 10 and the first-order conditions write:

$$r_t = Af'(k_t) \equiv r(k_t) \tag{4.3}$$

$$w_t = A[f(k_t) - k_t f'(k_t)] \equiv w(k_t) \tag{4.4}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

where $f(k) \equiv F(k,1)$ is the average productivity and $k_t \equiv K_t/L_t$ denotes the capital intensity at time t. For simplicity, we consider that there is no population growth and we normalize the population size (N) to the unity, it follows that $N = L = 1$ and then $k_t = K_t$. We set by $\alpha \in (0,1)$ and $\sigma > 0$ the capital share in total income and the elasticity of capital-labor substitution, namely :

$$\alpha \equiv \frac{k_t f'(k_t)}{f(k_t)} \tag{4.5}$$

$$\sigma \equiv -f'(k_t)\frac{[f(k_t) - k_t f'(k_t)]}{f''(k_t) f(k_t) k_t} \tag{4.6}$$

4.2.3 The pollution

As in Forster (1973), pollution (P) is an externality coming from the capital accumulation (K) according to this simple linear process :

$$P_{t+1} = (1-m) P_t + aK_t - bB_t \tag{4.7}$$

pollution persist and its inertia is captured by $m \in (0,1)$, the natural rate of pollution absorption. In addition, $a \in (0,1)$ and $b \in (0,1)$ capture respectively the impact of capital accumulation on the environment and the efficacity of depollution expenditures (B).

4.2.4 The Government

The Government uses the tax revenues to finance depollution expenditures according to a balanced budget rule, namely :

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

$$B_t = r_t k_t - \varphi(r_t k_t) \tag{4.8}$$

4.3 The dynamical system

4.3.1 Intertemporal general equilibrium

The economy is composed by three markets : 1) the goods market, 2) the labor market and 3) the capital market. Since the representative household supplies inelastically his labor and since the representative firm maximizes its profit, the labor market is always at the equilibrium. From the Walras law, if the goods market is at the equilibrium, then all markets in the economy are at the equilibrium.

The goods market clearing condition is given by :

$$Y_t = c_t + k_{t+1} - (1-\delta) k_t + B_t \tag{4.9}$$

from (4.3), (4.4) and (4.8), we can rewrite (4.9) as follows :

$$k_{t+1} = w(k_t) + \gamma k_t + \varphi(r_t k_t) - c_t$$

Proposition 29 *An intertemporal general equilibrium for this economy is a non-negative sequence $\{c_t, k_t, P_t\}_{t=0}^{+\infty}$ such that the following system is verified :*

$$u_1(c_t, P_t) = \beta u_1(c_{t+1}, P_{t+1}) \left[r(k_{t+1}) \varphi'(k_{t+1} r(k_{t+1})) + \gamma \right] \tag{4.10}$$

$$k_{t+1} = w(k_t) + \gamma k_t + \varphi(k_t r(k_t)) - c_t \tag{4.11}$$

$$P_{t+1} = (1-m) P_t + a k_t - b \left[k_t r(k_t) - \varphi(k_t r(k_t)) \right] \tag{4.12}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

The system formed by equations (4.10), (4.11) and (4.12) has two predetermined variables, namely k and P, and one jump variable, namely c.

4.3.2 Steady state

Our first task is to ensure the existence of a steady state for the system defined by equations (4.10)-(4.11)-(4.12), that is, the existence of a triplet $(k, c, P) \in \mathbb{R}_+^3$ such that $k_{t+1} = k_t = k$, $c_{t+1} = c_t = c$ and $P_{t+1} = P_t = P$.

At the steady state, equation (4.10) gives :

$$Af'(k)\,\varphi'(Af'(k)\,k) = \frac{1}{\beta} - \gamma \tag{4.13}$$

At the steady state, equations (4.11) and (4.12) imply that for every k satisfying (4.13), there exists a unique c and a unique P. Then, the existence, the uniqueness or the multiplicity of a steady state depends crucially upon the number of k satisfying (4.13).

Proposition 30 *Assume that there exists at least one k satisfying equation (4.13). If $\eta = 0$, k is unique. If $\eta > 0$ jointly with $\sigma < 1 - \alpha$, k is also unique. If $\eta > 0$ jointly with $\sigma > 1 - \alpha$, multiple k may satisfy equation (4.13).*

Proof. We have to question the monotonicity of the LHS of 4.13 w.r.t k :

$$\frac{\partial LHS}{\partial k} = \frac{1}{k}\left(\frac{1}{\beta} - \gamma\right)\left\{\eta\left[1 - \left(\frac{1-\alpha}{\sigma}\right)\right] - \frac{1-\alpha}{\sigma}\right\}$$

If $\eta = 0$, $\frac{\partial LHS}{\partial k} < 0$. If $\eta > 0$, the sign of $\frac{\partial LHS}{\partial k}$, depends crucially upon σ, if $\sigma < 1 - \alpha$, then $\frac{\partial LHS}{\partial k} < 0$ and if $\sigma > 1 - \alpha$, then $\frac{\partial LHS}{\partial k} \lesseqgtr 0$. Proposition 30 follows.

■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Empirical evidences suggest that $\sigma > 1-\alpha$ (Duffy and Papageorgiou 2000) and in turn that the economy possesses multiple steady state. To facilitate the exploration of local dynamics, we choose to normalize the steady state (Cazzavillan, Lloyd-Braga and Pintus 1998, Magris 2012).

Proposition 31 *(Existence of a normalized steady state (NSS)) There exists a unique A such that $k = 1$.*

Proof. Let us rewrite (4.13) as follows :

$$Af'(1) = \left(\frac{1}{\beta} - \gamma\right) \frac{1}{\varphi'(Af'(1))} \tag{4.14}$$

From assumption 8 :

$$\lim_{A\to 0} Af'(1) < \lim_{A\to 0} \left(\frac{1}{\beta} - \gamma\right) \frac{1}{\varphi'(Af'(1))} \tag{4.15}$$

$$\lim_{A\to +\infty} Af'(1) > \lim_{A\to +\infty} \left(\frac{1}{\beta} - \gamma\right) \frac{1}{\varphi'(Af'(1))} \tag{4.16}$$

In addition :

$$\frac{\partial}{\partial A}\left[Af'(1)\right] > 0 \tag{4.17}$$

$$\frac{\partial}{\partial A}\left[\left(\frac{1}{\beta} - \gamma\right) \frac{1}{\varphi'(Af'(1)H)}\right] < 0 \tag{4.18}$$

From the intermediate value theorem, relations (4.15), (4.16), (4.17) and (4.18) imply that there exist a unique A such that equation (4.14) is verified. ■

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

4.4 Local dynamics

Our aim is to analyze the stability properties of the system (4.10)-(4.11)-(4.12) near the normalized steady state (NSS). We compute the Jacobian matrix J evaluated at the NSS and we set λ an eigenvalue of J. Our task now is to define how λ moves, with respect to the unit circle, when η moves from 0 to $+\infty$, that is to say when the regressivity degree of the ecotax moves from 0 to $+\infty$. In other words, η is our bifurcation parameter.

When λ enters or leaves the unit circle, there is a sudden qualitative change of the behavior of the dynamical system. In such a situation, a local bifurcation occurs[2]. More precisely, when one eigenvalue is just equal to 1 a saddle-node or a transcritical or a pitchfork bifurcation occurs depending upon the number of steady states of the economy. We have seen in proposition 30 that the economy may experience multiple steady states but we do not know how many. For simplicity, we will say that a "saddle-node type" bifurcation occurs when $\lambda = 1$. When $\lambda = -1$ the economic system undergoes two-period cycles through a flip bifurcation. Finally, When two complex conjugate eigenvalues cross the unit-cycle, a limit cycle appears near the steady state through a Hopf bifurcation.

We set $\Phi(\lambda)$ the characteristic polynomial of J :

$$\Phi(\lambda) = \lambda^3 - T\lambda^2 + S\lambda - D \tag{4.19}$$

with, $T = \lambda_1 + \lambda_2 + \lambda_3$, $D = \lambda_1\lambda_2\lambda_3$ and $S = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3$, where λ_1, λ_2 and λ_3 are the three eigenvalues of J, namely the three roots of (4.19). Posing

[2] The reader can refer to Kuznetsov (2004) or Bosi and Ragot (2011) for a simple presentation of one-parameter bifurcations of fixed points in discrete-time dynamical systems.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

$\Phi(1) = 0$, $\Phi(-1) = 0$ and $\Phi(D) = 0$ give all the relations between T,S and D for which, respectively, a "saddle-node type", a flip and a Hopf bifurcation occurs. As it was noticed by Barinci and Drugeon (2004)(2008), for a fixed value of D, each of these three relations define a straight line in the (S, T)-plane. Figure 1 gives a representation of this two-dimensional plane and the typology[3] (namely, numbers in brackets) of the eigenvalues with respect to all possible values of D.

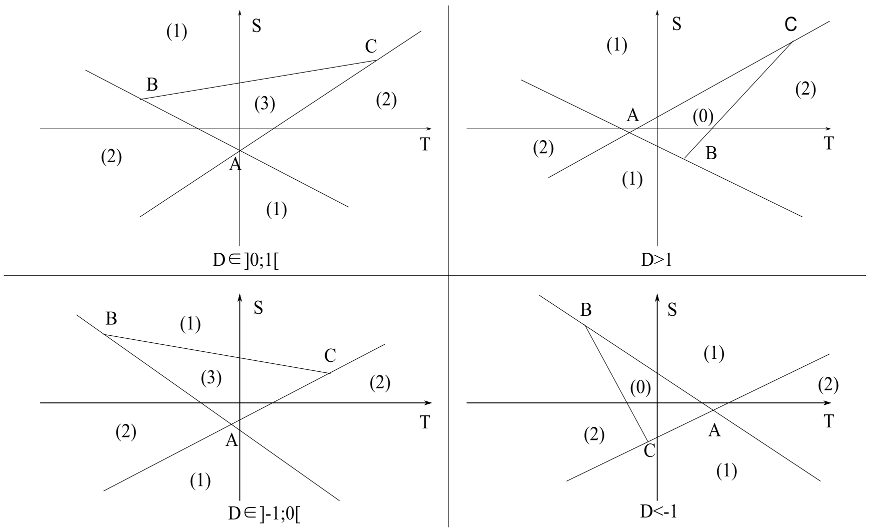

Figure 1

When (S, T) goes through (AC), $\lambda = 1$ and a "saddle-node type" bifurcation occurs, when (S, T) goes through (AB), $\lambda = -1$ and a flip bifurcation emerges and when (S, T) goes through $[BC]$, $\Phi(D) = 0$ and J possesses two complex conjugate eigenvalues with unit modulus, then a Hopf bifurcation occurs.

A variation of η defines a straight line (Δ) in the (S, T)-plane. Its position with respect to the ABC triangle tells us about the stability of the economic system near the NSS.

[3] The reader can refer to Barinci and Drugeon (1999, 2004) for more details.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Assumption 11 To simplify the exposition, as in Sorger (2002), we assume that there is no capital depreciation, namely $\gamma = 1$.

The linear operator J takes the following form :

$$J = \begin{bmatrix} \beta\left(\left(\frac{1}{\beta}-1\right)\left(\eta\left(1-\omega\right)-\omega\right)\right) & -\varepsilon & -\rho \\ 1 & 0 & 0 \\ 0 & 0 & \frac{B}{m} \end{bmatrix}^{-1} *$$

$$\begin{bmatrix} 0 & -\varepsilon & -\rho \\ 1+\left(\frac{1}{\beta}-1\right)\left(1-\omega\right)+r\omega & -r\left(\frac{1-\alpha}{\alpha}\right)-\varphi & 0 \\ a-b\left(1-\omega\right)\left(r-\left(\frac{1}{\beta}-1\right)\right) & 0 & \left(\frac{1-m}{m}\right)B \end{bmatrix}$$

With $\omega = \frac{1-\alpha}{\sigma}$, $B = a + b\left(\varphi - r\right)$ and, at the NSS $\varphi = r\left(1-\tau\right)$.

Remember that the dynamical system has two predetermined variables and one jump variable, then indeterminacy can occur if and only if J possesses three eigenvalues lying inside the unit circle (See Grandmont 2008). At this step of the reasoning, we can show that indeterminacy is ruled-out if the ecotax takes the form of a flat tax, it is the meaning of the following proposition :

Proposition 32 *Indeterminacy is ruled out if $\eta = 0$.*

Proof. Assume that the ecotax is a flat tax ($\eta = 0$), the characteristic polynomial evaluated for $\lambda = 1$ is defined by :

$$\Phi\left(1\right) = -\frac{m}{\alpha\sigma\varepsilon}\left(1-\beta\right)\left(r\left(1-\alpha\right)+\alpha\varphi\right)\left(1-\alpha\right) < 0 \tag{4.20}$$

notice that there is only two pre-determined variables, since $\lim_{\lambda\to+\infty}\Phi\left(\lambda\right) = +\infty$, relation (4.20) implies that J possesses always a real eigenvalue greater than 1. Thus,

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

indeterminacy is ruled-out because the dimension of the stable manifold can never exceed the number of pre-determined variables. ■

4.4.1 The separable model

Through this subsection, we investigate the case where the utility function is separable between consumption and pollution, that is $u_{12} = 0$ ($\rho = 0$). The determinant of J is given by :

$$D = \frac{(1-m)\left[(1-\alpha)\left(\beta\left(1+r\right)-1\right)+\sigma\right]}{\sigma\beta} > 0$$

We can make two major comments on this expression. First of all, D does not depend on η, then a variation of η does not modify the ABC triangle. Secondly, a high level of natural pollution absorption implies that $D \in (0,1)$.

Proposition 33 *Assume that $\sigma > 1-\alpha$ and $m \approx 1$:*

If $\eta \in (0, \eta^s)$, J possesses two eigenvalues[4] *inside and one outside the unit circle. Thus equilibrium determinacy and the steady state is locally stable.*

If $\eta \in \left(\eta^s, \eta^f\right)$, J possesses two eigenvalues[5] *outside and one inside the unit circle. Thus equilibrium determinacy and the steady state is locally unstable.*

If $\eta > \eta^f$, J possesses two eigenvalues inside and one outside the unit circle. Thus equilibrium determinacy and the steady state is locally stable.

When $\eta = \eta^s$, J possesses one eigenvalue just equal to 1 and a saddle-node type bifurcation occur and when $\eta = \eta^f$, J possesses one eigenvalue just equal to -1, then a two-periods cycle appears near the NSS through a flip bifurcation.

[4] With $\eta^s = \frac{1-\alpha}{\sigma-(1-\alpha)}$

[5] With $\eta^f = \frac{\beta(1-\alpha)(r((1-\alpha)(1-\beta)+2\alpha\varepsilon)+\alpha\varphi(1-\beta))+2\alpha\varepsilon(\sigma(1+\beta)-(1-\alpha)(1-\beta))}{\beta(1-\beta)(\sigma-(1-\alpha))(r(1-\alpha)+\alpha\varphi)}$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Proof. See Appendix. ■

If $\sigma < 1-\alpha$ then $\eta^s = \frac{1-\alpha}{\sigma-(1-\alpha)} < 0$ but a regressive feature requires that $\eta > 0$. In other words, the ecotax regressivity can induce a loss of stability only when $\sigma > 1-\alpha$.

Figure 2 gives an illustration of proposition 33:

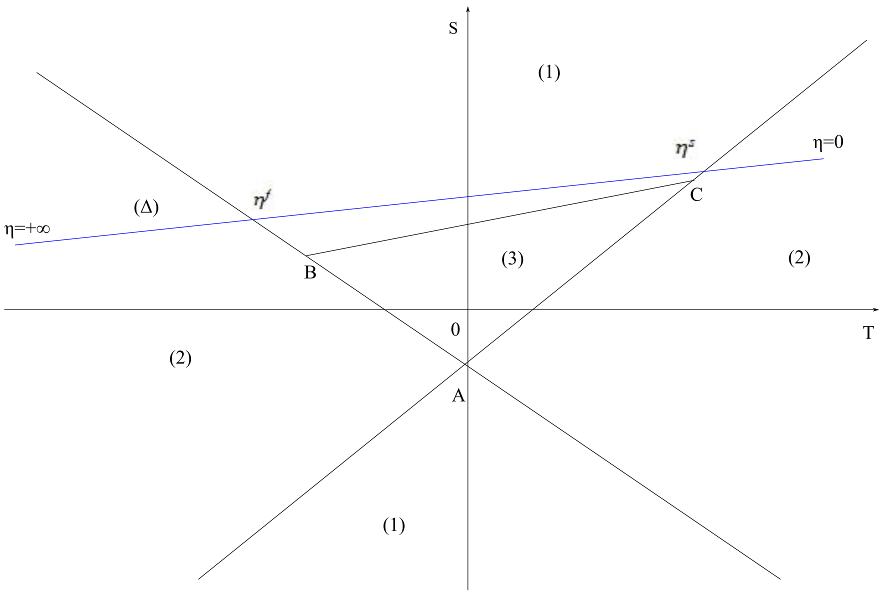

Figure 2

In order to show the relevance of the last proposition, let us analyze a simple numerical example. We fix $a = b = 0.5$, $m = 0.9$, $r = 0.02$, $\tau = 0.1$, $\beta = 0.99$, $\sigma = 1$, $\alpha = 0.33$, $\varepsilon = 2$. In such a situation, $\eta^s = \frac{1-\alpha}{\sigma-(1-\alpha)} = 2.0303$, we obtain the following results :

	λ_1	λ_2	λ_3
$\eta = 0$	0.1	0.992 09	1.024 8
$\eta = 1$	0.1	0.995 36	1.021 4
$\eta = 2.0303$	0.1	1.0	1.016 7
$\eta = 4$	0.1	$1.0083 - 1.1062 \times 10^{-2}i$	$1.0083 + 1.1062 \times 10^{-2}i$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

This illustrates that the economic equilibrium loose its local stability when the ecotax regressivity is high enough ($\eta > \eta^s$).

4.4.2 The non-separable model

Assumption 12 To simplify the discussion, we assume that $a = b$.

Through this subsection, we analyze a non-separable utility function: in what follows, we assume a *compensation effect*, namely $u_{12} > 0$ ($\rho < 0$). Such a configuration is more difficult to handle because $D'(\rho) \neq 0$, thus the ABC triangle depends crucially upon ρ. Surprisingly, D is a linear function of ρ :

$$\begin{aligned} D = &-\rho \frac{mb\left(r\left(1-\alpha\right)+\alpha\varphi\right)\left(\sigma\beta-\left(\sigma-\left(1-\alpha\right)\right)\left(\beta\left(1+r\right)-1\right)\right)}{\alpha\sigma\beta\varepsilon b\left(1+\varphi-r\right)} \\ &+\left(1-m\right)\frac{\left(\left(\sigma-\left(1-\alpha\right)\left(1-\beta\right)\right)+r\beta\left(1-\alpha\right)\right)}{\sigma\beta} \end{aligned} \tag{4.21}$$

It appears that $D'(\rho) < 0$ and fortunately, D does not depend upon the regressivity degree of the tax function. Then, for a fixed value of ρ, a variation of η does not modify the ABC triangle.

We set :

$$\rho^* = -\alpha \frac{\varepsilon\left(\left(1-r\right)+\varphi\right)}{r\left(1-\alpha\right)+\alpha\varphi} \frac{\left(1-\beta\right)\left(\sigma-\left(1-\alpha\right)\right)+r\beta\left(1-\alpha\right)}{\left(1-\alpha\right)\left(\beta\left(1+r\right)-1\right)+\sigma\left(1-r\beta\right)}$$

$$\begin{aligned} \rho^{**} = &-\frac{\alpha\varepsilon}{m}\frac{\left(1-r\right)+\varphi}{r\left(1-\alpha\right)+\alpha\varphi} \\ &* \frac{\left(m\left(\sigma-\left(1-\alpha\right)\left(1-\beta\right)\right)-\left(1-\beta\right)\left(\sigma-\left(1-\alpha\right)\right)\right)-r\beta\left(1-\alpha\right)\left(1-m\right)}{\sigma-\left(1-\alpha\right)\left(1-\beta\right)-r\beta\left(\sigma-\left(1-\alpha\right)\right)} \end{aligned}$$

When $\beta \approx 1$, it follows that $\rho^* < 0$, $\rho^{**} < 0$ with $\rho^* > \rho^{**}$.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Proposition 34 *$D \in (0,1)$ if and only if $\rho \in (0, \rho^{**})$.*

Proof. Simply remark that $D'(\rho) < 0$ with $D(0) \in (0,1)$ and $D(\rho^{**}) = 1$. ■

Proposition 35 *Assume that $\sigma > 1 - \alpha$ and $\rho \in (\rho^{**}, \rho^{*})$:*

If $\eta \in (0, \eta^{s})$, J possesses two eigenvalues inside and one outside the unit circle. Thus equilibrium determinacy and the steady state is locally stable.

If $\eta \in \left(\eta^{s}, \eta^{H}\right)$, J possesses three eigenvalues inside the unit circle. There exists a continuum of equilibrium near the steady state.

If $\eta \in \left(\eta^{H}, \eta^{f}\right)$, J possesses two eigenvalues outside and one inside the unit circle. The steady state is locally unstable.

If $\eta > \eta^{f}$, J possesses two eigenvalues inside and one outside the unit circle. Thus equilibrium determinacy and the steady state is locally stable.

When $\eta = \eta^{s}$ (with $\eta^{s} = \frac{1-\alpha}{\sigma-(1-\alpha)}$), J possesses one eigenvalue just equal to 1 and a saddle-node type bifurcation occur, when $\eta = \eta^{H}$, J possesses two complex conjugate eigenvalues with unit modulus, then a limit cycle occurs near the NSS through a Hopf bifurcation. Finally, when $\eta = \eta^{f}$, J possesses one eigenvalue just equal to -1, then a two-periods cycle appears near the NSS through a flip bifurcation.

Proof. See Appendix. ■

As before, it appears that $\eta^{s} > 0$ if and only if $\sigma > 1 - \alpha$, then sunspot equilibria can occur only for this parametric region.

Figure 3 gives an illustration of the last proposition :

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

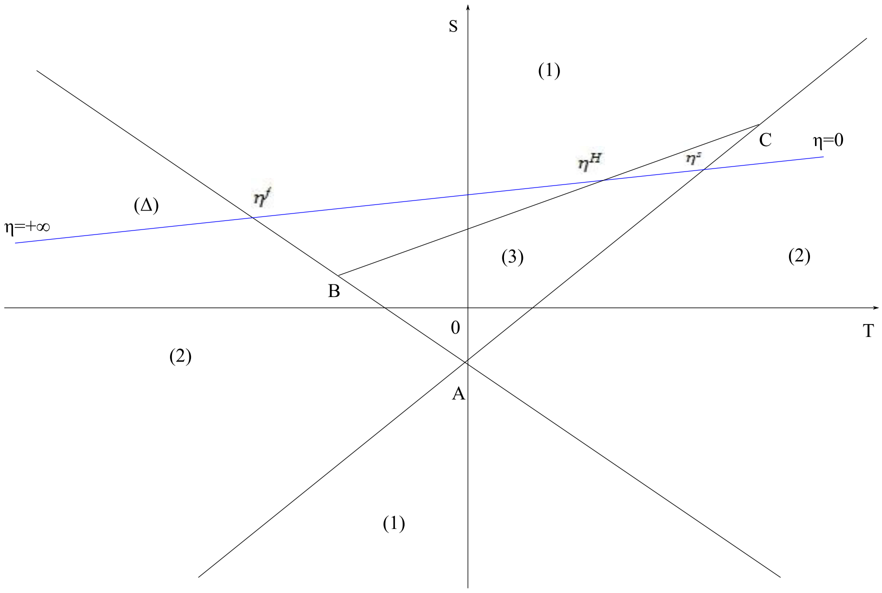

Figure 3

Remark 36 *We don't give the explicit expression of* η^H *due to its very complicated expression.*

If we consider the previous example, it follows that $\rho^* = -0.571\,77$ and $\rho^{**} = -34.106$. First, assume that $\rho > \rho^*$, for example $\rho = -0.2$, we obtain:

	λ_1	λ_2	λ_3
$\eta = 0$	$1.020\,7$	$0.990\,45$	$0.105\,78$
$\eta = 1$	$1.017\,0$	$0.994\,09$	$0.105\,78$
$\eta = 2.0303$	$1.011\,0$	1.0	$0.105\,78$
$\eta = 4$	$1.005\,4 + 1.275\,8 \times 10^{-2}i$	$1.005\,4 - 1.275\,8 \times 10^{-2}i$	$0.105\,78$

in this case, the stability properties of our dynamical system are the same as the separable model.

Now, assume that $\rho \in (\rho^{**}, \rho^*)$, for example, we set $\rho = -2$. In this case :

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

	λ_1	λ_2	λ_3
$\eta = 0$	1.0043	0.95102	0.1616
$\eta = 1$	1.0023	0.95296	0.1616
$\eta = 2.0303$	1.0	0.95513	0.1616
$\eta = 4$	0.99488	0.96006	0.1616

We can see that such a value of ρ implies local indeterminacy when $\eta > \eta^s$.

It is interesting to notice that for every ρ, we have always $\eta^s = \frac{1-\alpha}{\sigma-(1-\alpha)}$, it implies that a high elasticity of capital-labor substitution induces a low value of η^s. Duffy and Papageorgiou (2000) have found empirical evidences for $\sigma \in [1.24, 3.24]$, this induces that $\eta^s \in [0.2607, 1.1754]$ for $\alpha = 0.33$. It follows that sunspot fluctuations are more likely when the elasticity of capital-labor substitution is high.

4.5 Economic interpretations

Within the previous section, we have found that local indeterminacy occurs if and only if :

1. The compensation effect is high enough ($\rho \in (\rho^{**}, \rho^{*})$).

2. The ecotax regressivity is strong enough ($\eta > \eta^s$).

3. The elasticity of capital-labor substitution is not too low ($\sigma > 1 - \alpha$).

Since Azariadis (1981), it is well-known that an economic interpretation of local indeterminacy is the occurrence of self-fulfilling expectations.

Assume that the economy is at the NSS at time t and assume also that the patient household expects rationally an increase of P_{t+1}. Since $\rho < 0$, he knows that

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

he will increase c_{t+1}. In order to do so, he has to increase his present saving, namely k_t. At this step of the reasoning, it is interesting to have in mind that :

$$\frac{\partial\left[f'(k_t)Hk_t\right]}{\partial k_t}=Hf'(k_t)\left(1-\left(\frac{1-\alpha}{\sigma}\right)\right)$$

Since $\sigma>1-\alpha$, it follows that $\frac{\partial[f'(k_t)Hk_t]}{\partial k_t}>0$. That is, an increase of k_t produces an increase of $f'(k_t)Hk_t$. Remember now that :

$$P_{t+1}=(1-m)P_t+\left[aHk_t+b\varphi\left(f'(k_t)Hk_t\right)\right]-bf'(k_t)Hk_t \tag{4.22}$$

An increase of k_t induces an increase of $\left[aHk_t+b\varphi\left(f'(k_t)Hk_t\right)\right]$ and an increase of $bf'(k_t)Hk_t$. Recall that η measures the convexity of φ, from equation (4.22), it is obvious that there exists a convexity degree, namely a regressivity degree of the ecotax, for which the increase of $\left[aHk_t+b\varphi\left(f'(k_t)Hk_t\right)\right]$ exceeds the increase of $bf'(k_t)Hk_t$, renders the expectations on the pollution's level self-fulfilling.

It is interesting to remark that without the ecotax, such a mechanism cannot appear. In other work, the Government put in place an ecotax to reduce the pollution stock, but its regressivity induces a counterintuitive effect by which the pollution level increases. Such a relation looks like a *green paradox* (Sinn 2008, Smulders, Tsur and Zemel 2012).

4.6 Conclusion

Throughout this chapter, we have analyzed an environmental-Ramsey model where a Government uses a regressive capital income tax to finance depollution expenditures. By analyzing the dynamics around the normalized steady state, we have found

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

that the regressive feature of the ecotax is a necessary condition for which local indeterminacy occurs.

The literature have stressed the fact that a regressive ecotax, with respect to household's incomes, has negative distributional effects. The present result shows that, in addition, such ecotaxes may promote macroeconomic volatility.

© 2014 Tous droits réservés.

http://doc.univ-lille1.fr

4.7 Appendix

Proof of proposition 33 :

The characteristic polynomial evaluated at $\lambda = 1$ is given by :

$$\Phi(1) = \frac{m}{\alpha\sigma\varepsilon}(1-\beta)(r(1-\alpha)+\alpha\varphi)(\alpha-1+\eta(\alpha-1+\sigma))$$

when $\sigma > 1-\alpha$, $\Phi(1) < 0$ if $\eta < \frac{1-\alpha}{\sigma-(1-\alpha)}$, $\Phi(1) > 0$ if $\eta > \frac{1-\alpha}{\sigma-(1-\alpha)}$ and $\Phi(1) = 0$ if $\eta = \frac{1-\alpha}{\sigma-(1-\alpha)}$.

Since T and S are two linear function of η, it follows that a variation of η from 0 to $+\infty$ define an half-line (Δ) in the (S,T)-plane with origin at $(T(0), S(0))$ and slope $(\Delta)' = \frac{S'(\eta)}{T'(\eta)}$. Simple computation gives :

$$\begin{aligned}
S'(\eta) &= -\frac{\beta(1-\beta)(1-m)}{\alpha\sigma\beta\varepsilon}(\sigma-(1-\alpha))(r(1-\alpha)+\alpha\varphi) < 0 \\
T'(\eta) &= -\frac{(1-\beta)}{\alpha\sigma\varepsilon}(\sigma-(1-\alpha))(r(1-\alpha)+\alpha\varphi) < 0 \\
(\Delta)' &= 1-m > 0
\end{aligned}$$

since a saddle-node type bifurcation occur for $\eta = \frac{1-\alpha}{\sigma-(1-\alpha)} > 0$, it follows that $(T(0), S(0))$ is located under (A, C). By definition $[B, C]$ cuts (AC) when $T = D+2$. We define by T_s the value of T when $\eta = \frac{1-\alpha}{\sigma-(1-\alpha)}$. Simple computation allow us to say that :

$$T_s - (D+2) = \frac{m}{\sigma\beta}((1-\beta)(\sigma-(1-\alpha)) + r\beta(1-\alpha)) > 0$$

it follows that (Δ) cuts (AC) at a higher point than C.

Is it possible to have a Hopf bifurcation ? For answering this important question, we compare the slope of $[BC]$ and the one of (Δ). By definition, the slope of $[BC]$ is given by D, and it appears that :

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

$$D-(1-m)=\frac{1}{\sigma\beta}(1-m)((1-\beta)(\sigma-(1-\alpha))+r\beta(1-\alpha))>0$$

It follows that the slope of $[BC]$ is higher than the slope of (Δ) and since (Δ) cuts (AC) at a higher point than C, it follows that a Hopf bifurcation is impossible, thus indeterminacy is ruled-out. Proposition 33 follows. ■

Proof of proposition 35 :

Simple computation allow us to see that ρ has no impact on η_s and has no impact $(\Delta)'$, that is :

$$\eta_s=\frac{1-\alpha}{\sigma-(1-\alpha)}$$
$$(\Delta)'=1-m$$

interestingly, a variation of ρ from 0 to $-\infty$ induce an upward translation of (Δ) in the (S,T)-plane, indeed :

$$(\Delta):S=(1-m)T+D+\Omega$$

with $\Omega=-\frac{(m((1-\alpha)(1-\beta)-\sigma(\beta(2-m)+1))+\sigma\beta)-(1-\alpha)mr\beta}{\sigma\beta}$ and :

$$D=-\rho\frac{mb(r(1-\alpha)+\alpha\varphi)(\sigma\beta-(\sigma-(1-\alpha))(\beta(1+r)-1))}{\alpha\sigma\beta\varepsilon b(1+\varphi-r)}+(1-m)\frac{((\sigma-(1-\alpha)(1-\beta))+r\beta(1-\alpha))}{\sigma\beta}$$

In addition, D is a linear function of ρ such that $D'(\rho)<0$ (recall that $\rho<0$). Such a variation of D induces some transformation of the ABC triangle and in particular, it induces an increase of the slope of $[BC]$ and consequently an upward

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

motion of the point $C\left(T_C, S_C\right)$, indeed, T_C is such that $(AC) = [BC]$, in other words :

$$-1 + T_C + D = 1 + \left(T_C - D\right) D$$

thus :

$$T_C = D + 2$$

it follows that T_C increase linearly when ρ move from 0 to $-\infty$. Is there exist D for which $T_C > T_S$?

We begin by defining T_S, the value of T for which $(\Delta) = (AC)$:

$$\begin{aligned}(1 - m)\, T_S + D + \Omega &= -1 + T_S + D \\ T_S &= \frac{1 + \Omega}{m}\end{aligned}$$

and $T_C > T_S$ if and only if :

$$\begin{aligned}D + 2 &> \frac{1 + \Omega}{m} \\ D &> \frac{1 + \Omega}{m} - 2\end{aligned}$$

This appears only when $\rho < -\alpha \frac{\varepsilon((1-r)+\varphi)}{r(1-\alpha)+\alpha\varphi} \frac{(1-\beta)(\sigma-(1-\alpha))+r\beta(1-\alpha)}{(1-\alpha)(\beta(1+r)-1)+\sigma(1-r\beta)} = \rho^*$. Thus when $\rho \in (\rho^*, 0)$, the situation is close to the case when $\rho = 0$. A necessary condition for local indeterminacy is that :

$$\begin{cases} T_C > T_S \\ D \in (0, 1) \end{cases}$$

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Following proposition 34, $D \in (0,1)$ if and only if $\rho \in (0, \rho^{**})$. Since $\rho^{**} < \rho^{*}$, a necessary condition for local indeterminacy is that $\rho \in (\rho^{**}, \rho^{*})$. Thus, when η is slightly higher than η_s with $\rho \in (\rho^{**}, \rho^{*})$, the three eigenvalues of J are inside the unit circle and then local indeterminacy occurs. Proposition 35 follows. ∎

© 2014 Tous droits réservés.

http://doc.univ-lille1.fr

General conclusion

Through this dissertation, our objective was to bring out new mechanisms under which pollution may promotes endogenous business cycles. Within the chapter 1, we have considered a monetary OLG economy *à la* Samuelson (1958) where pollution reduces labor productivity. In this very simple context, we found that a strong negative effect of pollution on labor productivity may promote sunspot equilibria through a Hopf bifurcation. This result contributes to the literature in two points :

1. It shows that pollution's feedback on technology may promote endogenous business cycles while the existing literature insists on the pollution's feedback on the household's preferences.
2. It shows that the Graff Zivin and Neidell (2012) empirical finding works as a destabilizing force for the economy.

Within the chapters 2 and 3, we have explored the macroeconomic incidences of the pollution effects on labor supply empirically stressed by Hanna and Oliva (2011) (among the others). In chapter 2, we have developed a Ramsey economy in order to capture those effects. This allowed us to introduce two new concepts (see also Bosi, Desmarchelier and Ragot 2013) :

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

1. The *leisure effect* (a negative pollution effect on labor supply).

2. The *disenchantment effect* (a positive pollution effect on labor supply).

In this simple framework, we have found that a strong *leisure effect*, empirically stressed by Hanna and Oliva (2011) (among the others), induces two-period cycles near the steady state through a flip bifurcation.

In chapter 3, we have build a Ramsey model for capturing simultaneously the pollution effects on labor supply and on consumption behavior. This allow us to recover, in extreme cases, Bosi, Desmarchelier and Ragot (2013) and Michel and Rotillon (1996). Within this framework, we have found, in particular, that a strong *leisure effect* jointly with a strong *compensation effect* imply equilibrium indeterminacy through a Hopf bifurcation.

The new results found in chapter 1, 2 and 3 could give a strong incentive to policy makers for adopting stricter environmental policies. Indeed, by reducing pollution levels, ecological policies inhibit a very source of business cycles. In this sense, those new results contribute to reconcile the short run thinking of policy leaders (Nordhaus 1975) and the long run imperative of the environment preservation (WB 2012). Some scholars have stressed the fact that there are long-run benefits to adopt green policies (Van Ewijk and Van Wijnbergen 1995 ; Bovenberg and Mooij 1997), our results show that there are possibly also short-run benefits to them.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

In chapter 4, we have explored effects of the existing green taxes and we have shown that their regressive nature, with respect to household's incomes, may promote sunspot equilibria and then self-fulfilling expectations on the pollution's level. Through this result, we have stressed the fact that equilibrium indeterminacy can be seen as a channel for a perpetual rise of the pollution stock. It follows naturally from this finding that not all green taxes are good for the environment.

A natural extension to this dissertation is the short-run exploration of other green policies such as consumption taxes, especially in frameworks which takes into account pollution effects on labor supply.

Models developed through this dissertation are highly stylized and then can not describe the real world in its deep complexity. In addition, we have taken into account only temporal effects, our future research line aims to capture also some geographical aspects in order to analyze how pollution can affect, for example, the location choice of a representative household throughout his life-cycle.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

Bibliography

[1] Antoci A. and M. Sodini. (2009). Indeterminacy, bifurcations and chaos in an overlapping generations model with negative environmental externalities. *Chaos, Solitons and Fractals* **42**,1439–1450.

[2] Ayong Le Kama A. and K. Schubert. (2007). A note on the consequences of an endogenous discounting depending on the environmental quality. *Macroeconomic Dynamics* **11**, 272-289.

[3] Azariadis C. (1981). Self-fulfilling prophecies. *Journal of Economic Theory* **25**, 380-396.

[4] Barinci J-P. and J-P. Drugeon. (1999). Local Indeterminacy & Local Bifurcations In Discrete Time Three-Dimensional Dynamical Systems. *Cahier de la MSE-EUREQua* **1999-40**.

[5] Barinci J-P. and J-P. Drugeon. (2004). On Outward-Looking Comparison Utility, Heterogeneous Preferences & the Third Dimension: A Geometric Perspective. *SSNR Working Paper Series* and *Document de recherche EPEE* **05-07**.

[6] Barro R. (1990). Government spending in a simple model of endogeneous growth. *The Journal of Political Economy* **98**, 103-125.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[7] Becker R. (1980). On the Long-Run Steady State in a Simple Dynamic Model of Equilibrium with Heterogenous Households. *The Quarterly Journal of Economics* **95,** 375-382.

[8] Benhabib J. and R. Farmer. (1994). Indeterminacy and Increasing Returns. *Journal of Economic Theory* **63**, 19-41.

[9] Bosi S., D. Desmarchelier and L. Ragot. (2013). Pollution effects on labor supply and growth. *Mimeo.*

[10] Bosi S. and L. Ragot. (2011). *Introduction to discrete-time dynamics.* CLUEB, Bologna.

[11] Bosi S. and Seegmuller T. (2010). On the Role of Progressive Taxation in a Ramsey Model With Heterogeneous Households. *Journal of Mathematical Economics* **46**, 977-996.

[12] Bovenberg L. and R. Mooij. (1994). Environmental levies and distortionary taxation. *American Economic Review* **84**, 1085-1089.

[13] Bovenberg L. and R. Mooij. (1997). Environmental Levies and Distortionary Taxation: Reply. *American Economic Review* **87**, 252-253.

[14] Campbell J. (1999). Asset prices, consumption and the business cycle. In J.B. Taylor and M. Woodford eds., *Handbook of Macroeconomics*, North-Holland, Amsterdam, 1231-1303.

[15] Cao D., L. Wang. and Y. Wang. (2011). Endogenous fluctuations induced by nonlinear pollution accumulation in an OLG economy and the bifurcation control. *Economic Modelling* **28**, 2528–2531.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[16] Carson R.T., P. Koundouri. and C. Nauges. (2011). Arsenic mitigation in Bangladesh: A household labor market approach. *American Journal of Agricultural Economics* **93**, 407-414.

[17] Cazzavillan G., T. Lloyd-Braga and P. Pintus (1998). Multiple steady states and endogenous fuctuations with increasing returns to scale in production. *Journal of Economic Theory* **80** 60-107.

[18] Chay K. and M. Greenstone. (2003). The Impact of Air Pollution on Infant Mortality: Evidence from Geographic Variation in Pollution Shocks Induced by a Recession. *The Quarterly Journal of Economics* **118**, 1121-1167.

[19] Chen H-J. and M-C. Li. (2011). Environmental tax policy, habit formation and nonlinear dynamics. *Nonlinear Analysis: Real World Applications* **12**, 246–253.

[20] Currie J. and M. Neidell. (2005). Air pollution and infant health: What can we learn from california's recent experience?. *The Quarterly Journal of Economics* **120**, 1002-1030.

[21] Currie J., E. Hanushek., M. Kahn, M. Neidell. and S. Rivkin. (2009). Does pollution increase school absences?. *The Review of Economics and Statistics* **91**, 682-694.

[22] Denison E.F. (1979). Pollution abatement programs: Estimates of their effect upon output per unit of input, 1975-1978. *Survey of current business* **59**, 58-59.

[23] Diamond P.A. (1965). National debt in a neoclassical growth model. *American Economic Review* **55** 1126-1150.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[24] Diamond J. (2005). *Collapse : How societies choose to fail or succeed.* Viking Press.

[25] Duffy J. and C. Papageorgiou. (2000). A cross-country empirical investigation of the aggregate production function specification. *Journalof Economic Growth* **5**, 87-120.

[26] Eckstein Z., S. Stern and K. Wolpin (1988). Fertillity choice, land and the Malthusian hypothesis. *International Economic Review* **29** 353-361.

[27] EEA Technical report (2011). *Environmental tax reform in Europe: Implications For Income Distribution.*

[28] Fernandez E., R. Pérez. and J. Ruiz. (2012). The environmental Kuznets curve and equilibrium indeterminacy. *Journal of Economic Dynamics and Control* **36**, 1700-1717.

[29] Forster B. (1973). Optimal capital accumulation in a polluted environment. *Southern Economic Journal* **39**, 544-547.

[30] Frank G,. K. Berhane., E. Rappaport, D. Thomas., E. Avol,. J. Gauderman. and S. London. (2001). The Effects of Ambient Air Pollution on School Absenteeism Due to Respiratory Illness. *Epidemiology* **12**, 43-54.

[31] Galor O. and D. Weil. (1996). The gender gap, fertility, and growth. *American Economic Review* **86** 374-87.

[32] Graff Zivin J. and M. Neidell (2010). Temperature and the allocation of time: Implications for climate change. *NBER Working Paper* **15717.**

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[33] Graff Zivin J. and M. Neidell (2012). The impact of pollution on worker productivity. *American Economic Review* **102**, 3652-3673.

[34] Grainger A. and C. Kolstad. (2009). Who pays a price on carbon? *NBER Working Paper No* **15239**.

[35] Grandmont J.-M., P. Pintus and R. de Vilder (1998). Capital-labor substitution and competitive nonlinear endogenous business cycles. *Journal of Economic Theory* **80** 14-59.

[36] Grandmont J-M. (2008). Nonlinear difference equations, bifurcations and chaos: An introduction. *Research in Economics* **62**, 122–177.

[37] Guckenheimer J. and P. Holmes (1983). *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. Springer, New York.

[38] Guo J-T. and K. Lansing. (1998). Indeterminacy and Stabilization Policy. *Journal of Economic Theory* **82**, 481 490.

[39] Guo J-T. and S. Harrison. (2001). Tax Policy and Stability in a Model with Sector-Specific Externalities. *Review of Economic Dynamics* **4**, 75–89.

[40] Hanna R. and P. Oliva (2011). The effect of pollution on labor supply: Evidence from a natural experiment in mexico city. *NBER Working Paper* **17302**.

[41] Hansen G. (1985). Indivisible labor and the business cycle. *Journal of Monetary Economics* **16**, 309-327.

[42] Hassett K., A. Mathur. and G. Metcalf. (2007). The Incidence of a U.S. Carbon Tax: A Lifetime and Regional Analysis. *NBER Working Paper No* **13554.**

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[43] Heal G. (1982). The use of common property resources. In *Explorations in Natural Resource Economics*, The Johns Hopkins University Press for Resources for the Future, Baltimore.

[44] Itaya J-I. (2008). Can environmental taxation stimulate growth? The role of indeterminacy in endogenous growth models with environmental externalities. *Journal of Economic Dynamics and Control* **32**, 1156–1180.

[45] John A. and R. Pecchenino. (1994). An overlapping generations model of growth and the environment. *Economic Journal* **104** 1393-1410.

[46] John, A., R. Pecchenino., D. Schimmelpfennig. and S. Schreft. (1995). Short-lived agents and the long-lived environment. *Journal of Public Economics* **58**, 127-141.

[47] Keeler E., M. Spence. and R. Zeckhauser. (1971). The optimal control of pollution. *Journal of Economic Theory* **4**, 19-34.

[48] King R., C. Plosser. and S. Rebelo. (1988). Production, growth and business cycles, II. New directions. *Journal of Monetary Economics* **21**, 309-341.

[49] Koopmans T. (1965). On the Concept of Optimal Economic Growth. *Cowles Foundation Discussion Papers* **163**.

[50] Kuznetsov Y. (2004). *Elements of Applied Bifurcation Theory.* Springer.

[51] Lucas R. (1988). On the mechanics of economic development. *Journal of Monetary Economics* **22**, 3-42.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[52] Magris F. (2012). Indeterminacy and multiple steady states with sector-specific externalities. *Economic Modelling* **29**, 2664–2672.

[53] Mariani F., A. Pérez-Barahona and N. Raffin (2010). Life expectancy and the environment. *Journal of Economic Dynamics and Control* **34**, 798-815.

[54] Michel P. and G. Rotillon. (1996). Disutility of pollution and endogenous growth. *Environmental and Resource Economics* **6**, 279-300.

[55] Nordhaus W. (1975). The Political Business Cycle. *Review of Economic Studies* **42**, 169-190.

[56] Pautrel X. (2009). Pollution and life expectancy: How environmental policy can promote growth. *Ecological Economics* **68**, 1040-1051.

[57] Pelloni A. and R. Waldmann. (2000). Can waste improves welfare?. *Journal of Public Economics* **77**, 45–79.

[58] Pérez R. and J. Ruiz. (2007). Global and local indeterminacy and optimal environmental public policies in an economy with public abatement activities. *Economic Modelling* **24**, 431–452

[59] Ramsey F. (1928). A Mathematical Theory of Saving. *Economic Journal* **38**, 543-559.

[60] Romer P. (1986). Increasing Returns and Long-Run Growth. *The Journal of Political Economy* **94**, 1002-1037.

[61] Ryder H.E and G. Heal (1973). Optimal growth with intertemporally dependent preferences. *Review of Economic Studies* **40**, 1-33.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[62] Samuelson P.A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy* **66** 467-482.

[63] Schlenker W. and W. Walker (2011). Airports, air pollution, and contemporaneous health. *NBER Working Paper* **17684**.

[64] Schmitt-Grohé S. and M. Uribe. (1997). Balanced-Budget Rules, Distortionary Taxes, and Aggregate Instability. *Journal of Political Economy* **105**, 976-1000.

[65] Schumacher I. and B. Zou (2008). Pollution perception: A challenge for intergenerational equity. *Journal of Environmental Economics and Management* **55** 296-309.

[66] Seegmuller T. and A. Verchère (2004). Pollution as a source of endogenous fluctuations and periodic welfare inequality in OLG economies. *Economics Letters* **84** 363-369.

[67] Seegmuller T. and A. Verchère (2007). A note on indeterminacy in overlapping generations economies with environment and endogenous labor supply. *Macroeconomic Dynamics* **11** 423-429.

[68] Sinn H-W. (2008). Public policies against global warming: a supply side approach. *International Tax and Public Finance* **15**, 360–394.

[69] Smulders S., Y. Tsur. and A. Zemel. (2012). Announcing climate policy: Can a green paradox arise without scarcity?. *Journal of Environmental Economics and Management* **64**, 364–376.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[70] Sorger G. (2002). On the Long-Run Distribution of Capital in the Ramsey Model. *Journal of Economic Theory* **105**, 226-243.

[71] Speck S. (1999). Energy and carbon taxes and their distributional implications. *Energy Policy* **27**, 659-667.

[72] Tahvonen,O.and J. Kuuluvainen (1993). Economic growth, pollution, and renewable resources. *Journal of Environmental Economics and Management* **24**, 101-118.

[73] Van Der Ploeg R. and C. Withagen. (1991). Pollution control and the Ramsey problem. *Environmental and Resource Economics* **1**, 215-236.

[74] Van Ewijk C. and S. Van Wijnbergen (1995). Can abatement overcome the conflict between environment and economic growth?. *De Economist* **143**, 197-216.

[75] West S. and R Williams. (2004). Estimates from a consumer demand system: implications for the incidence of environmental taxes. *Journal of Environmental Economics and Management* **47**, 535–558.

[76] Wier M., K. Birr-Pedersen., H. Jacobsen. and J. Klok. (2005). Are CO2 taxes regressive? Evidence from the Danish experience. *Ecological Economics* **52,** 239– 251.

[77] Woodford M. (1986). Stationary sunspot equilibria: The case of small fluctuations around a deterministic steady state. *University of Chicago Working Papers.*

© 2014 Tous droits réservés. http://doc.univ-lille1.fr

[78] World Bank Report. (2012). *Turn down the heat: Why a 4°C warmer world must be avoided.*

[79] Yanase A. (2011). Impatience, pollution, and indeterminacy. *Journal of Economic Dynamics and Control* **35**, 1789–1799.

[80] Zhang J. (1999). Environmental sustainability, nonlinear dynamics and chaos. *Economic Theory* **14**, 489-500.

© 2014 Tous droits réservés. http://doc.univ-lille1.fr