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Essays on Public Policy: Public Budgets and Implicit Transfers with Productivity Differences, Public Input and FDI

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Abstract

This dissertation includes three essays on public policy, the aim of which is to better understand the economic mechanism behind the public policies. Chapter 2 works on public budget, implicit transfer and productivity differences. Chapter 3 extends the work of chapter 2 toward a fiscal competition model. Chapter 4 focuses on the relationship between public inputs and foreign direct investments.

In chapter 2, we set up a model with two asymmetric regions, differing in productivity, population being imperfectly mobile across regions while capital is perfectly mobile. A single utilitarian central authority uses taxes for providing public goods in each region. Looking at the optimal policies, we show that, when differences in productivity levels are exogenous, the first best policy generates an implicit transfer from the richer region to the poorer one as soon as capital is taxed. However, when differences in productivity levels are generated by an agglomeration externality, households are charged a higher tax in the poorer region, with the consequence that, if capital is not taxed, the first best policy generates an implicit transfer from the poorer to the richer region. Capital taxation narrows this implicit transfer.

In chapter 3 we extend the model to a decentralized setting with fiscal competition. There are two jurisdictions, heterogeneous in private productivity, inter-jurisdictional migration exists and is costly while capital is perfectly mobile. This part focusses on the relationship between public revenue collection and public goods supply in the case of fiscal competition. We keep the assumption that the jurisdictions are different in private productivity and each jurisdiction has a planner maximizing the local welfare of natives. In our model both tax on capital and labor are considered as the resource of public revenue to finance public services, public transfer is cut off between

jurisdictions. Then we work also on the model when the productivity difference between jurisdictions is generated by the externality of labor.

Chapter 4 works on the relationship between public inputs and foreign direct investment (FDI). Using ‘public inputs’, we look into details of how could the change of public inputs affect FDI flows. When we talk about public input, it means that the public goods can enter into the production process and influence the production function. We look into the decomposition of how do fiscal policies influence FDI flow through public inputs and what are the important economy characters that can influence the effect of public inputs.

Chapter 1 Introduction

This dissertation includes three essays related to fiscal policy studies. More precisely, we look at governments using revenue collection (tax) and expenditures (public goods) for influencing the economy (O'Sullivan and Sheffrin, 2003). In our three essays, the governments are benevolent. They maximize the social welfare, taking account of the fact that their fiscal choices influence the relocation of resources.

The choice of taxes and expenditures on public good may be analyzed from a normative point of view, focusing on optimal tax policy. Important early contributors are Ramsey (1927), who looked at the optimal repartition of taxes between various assets; Samuelson (1954) about the optimal level of public good provision; Tiebout (1956) about the decentralized provision of local public goods; Pigou who examined the link between taxation and externalities (the polluter-payer principle).

It may also be analyzed from a positive point of view: which are the incentives to tax and how do governments make their tax choice. From this point of view, an important question is the existence of tax interactions and their consequences. In most countries, the power to tax and to provide public goods is not restricted to the national level. Local authorities usually provide local public goods for the agents located in their constituency and have the power to tax, even when taxes are only a part of their resources.

There may be interactions in these fiscal choices, each jurisdiction being influenced by the other jurisdictions. The literature identifies several sources of interactions. The most well-known is the mobility of fiscal assets, which reacts do differences in taxes or local provision of public goods, flying out of jurisdictions where taxes are high or publics are poorly provided and moving toward jurisdictions where taxes are low or public goods are well provided. The economic analysis of these interactions has been largely developing since the seminal work of Zodrow and Miezkowski (1986), Wilson (1986) and Wildasin (1988). Another well-known one is yardstick competition (Besley and Case, 1995): knowing that an imperfectly informed elector tries to infer the performance of local politicians comparing them to each other, politicians who want to be re-elected mimick their neighbours.

Initially analyzed for jurisdictions within a country, fiscal interactions may also be at work at the international level, between countries. The ongoing globalization of the world economy and the economic integration in regions like Europe makes the mobility of agents easier (above all capital) and countries more comparable, leading to rising interactions among national public policies (Devereux and Griffith, 1998 and 2008).

Starting with theoretical models, the literature on fiscal interactions moved later on to empirical analyses, the main aim being to prove the existence of these interactions and to discriminate among competing theories explaining them. Two questions may be asked: First, are there mobiles assets sensitive to differences in tax levels? More precisely, capital being much more mobile than labor, this question leads to analyze the sensitivity of firms to tax differentials. The second question is: are the tax chosen by jurisdictions sensitive to the taxes charged by their direct competitors, usually their neighbors?

Empirical studies trying to answer the first question are carried out at the international level. Bolmstrom and Lipsey (1993) use US multinational data to study the link between the size of the firms and the size of the foreign activities of these firms. Head (1995) analyzed the decision of Japanese firms to invest in US and found out the important agglomeration effects. Brainard (1997) investigates the choices between exporting and producing locally using industry level data. Devereux and Griffith (1998) used the panel data of US multinationals and analyzed the factors that influence the location decisions of multinational firms. Using data for Norwegian municipalities, Carlsen (2005) found out that their mobility is systematically negatively related to tax level among municipalities in Norway. Using panel data on 18 OECD countries and measuring the extent of social welfare policies by Social over GDP ratio, Görg, Molana and Montagna (2009) tried to study the role of social expenditure and the interaction with corporate taxation in determining the destination of FDI flows. They found that redistributive social welfare state policies are valued by multinationals.

As for the second question, it has attracted a lot of empirical studies. At the infra-national level, we can cite Heyndels and Vuchelen, a pioneer paper in this field, focusing on the Belgian municipalities; Bordignon, Derniglia and Revelli (2003) who test the hypothesis of yardstick competition on Italian municipalities; Leprince, Madiès and Paty (2007), which is one of the earliest empirical studies on the French case ; Reulier and Rocaboy (2009), who examine this question at the level of French regions; Gerard, Jayet and Paty (2010) who look at the impact of the regional division of Belgium on tax interactions across Belgian municipalities; Cassette and alii (2012), who look at interactions between French and German municipalities along the Rhine valley. At the international level, among others, we can find Cassette and Paty (2008) who look at tax interactions among Eastern and Wester European countries; Cassette and alii (2013) who examine interactions generated by discretionary fiscal policies. The report by the French Concil of Economic Analysis (Saint-Etienne and Le Cacheux, 2005) provides a good synthesis.

Most of the models of fiscal interactions make two important simplifying assumptions. First, they make a symmetry assumption: at least ex ante, all the jurisdictions share the same characteristics: same size, same production technologies, and same preferences of the local population. Second, they assume capital to be perfectly mobile and population to be perfectly immobile.

In their recent review paper, Keen and Konrad (2012) note about the symmetry assumption: “It is worth stressing how unrealistic it is. The implication, for instance, is that there is no capital movement in equilibrium, and no gain from allowing capital to move; indeed there is a loss, given the inefficient tax-setting (if border were closed, each country would recognize the inelasticity of the tax base, and achieve the first-best) from allowing capital to move at all. While the asymmetric case is thus inherently more interesting, it is also much more complex.”

The assumption that population is perfectly immobile is also a useful assumption, leading to a theoretical framework contrasting a perfectly mobile asset (capital) and a perfectly immobile one (population). However, it does not help us answer an important motive for fiscal policies aiming at attracting capital: providing jobs to a local population which, without these jobs, would be led to migrate to other regions. This motive cannot be neglected: In France and several other countries, it led governmental authorities to make capital attraction as a component of their regional policies. But, behind this motive, there is the fact that population is imperfectly mobile (it migrates, at some cost, if there are not enough local jobs) and that this mobility interacts with capital mobility (an inflow of capital provides new local jobs, lowering the incentive to out migrate).

Some papers have relaxed the symmetry assumption. The earliest and probably the most well-known is Bucovetsky (1991), who looks at jurisdiction differing in the size of their local population and finds that the smallest jurisdiction attracts and disproportionately large share of capital. Wilson (1991) and Peralta and Van Ypersele (2005, 2006) examine the impact of differences in capital endowment and show that capital rich countries choose a less aggressive fiscal policy. Keen and Konrad (2012) also briefly look at the impact of differences in the valuation of public goods.

Surprisingly, the consequences of differences in the productivity of jurisdictions have never been analyzed. These differences are widespread and may be large: a worker in Ile de France earn a wage 20% higher than the average French wage. If we make the reasonable assumption that differences in wages reflect differences in productivity, the relative productivity differential between Ile de France and the less productive French region is probably around 25%. Moreover, these productivity differentials may be a good reason for engaging in aggressive fiscal policies trying to compensate the low productivity of a region by a lower level of taxation.

Then, it is natural to look at the links between productivity differentials, capital attraction, the imperfect mobility of population, and fiscal policies. This perspective is at the core of this dissertation.

Chapters 2 and 3 look at this question starting from a standard fiscal competition model, where we introduce imperfect mobility of workers and productivity differentials. Both chapter rest upon a similar framework. First, there is a productivity differential, introduced as a shift in the global factors productivity, similar to Hicks neutral technical product. This productivity may be purely exogenous and constant; or it may be endogenously generated by agglomeration externalities. Second, we stick to the standard assumption of fiscal competition model that the local public good is provided by benevolent governments from the private good, with a constant rate of transformation normalized to unity. However, because jurisdictions are not of fixed size, we need to make an assumption on divisibility of the local public good. We make the assumption of perfect divisibility. The reason behind this choice is that we do not want introduce in our model the consequences of scale economies which appear implicitly when the public good is imperfectly divisible.

Third, residents are mobile, but have a personal preference in their location choice. Similarly to Mayer (1993), we define this preference as a potential migrant cost or a possible welfare lost. There could be several reasons for this preference, for example, they enjoy the specific amenities provided by their preferred jurisdiction; or they have developed strong relations shared with relatives and friends, as well as their social network. Then, we assume that each worker has a willingness to pay for leaving in one jurisdiction relative to the other. This willingness to pay would be compared to the welfare difference resulting from local consumption of both the private and the public good. If the welfare gap between jurisdictions is larger than the willingness to pay, the worker chooses to migrate.

Chapter 2 adopts a normative point of view: which are the properties of an optimal repartition of capital and population, how this outcome can be implemented using taxes, which are the consequences of limits in tax setting?

In this chapter, the asymmetry and the fact that both fiscal assets are mobile (at different degrees, however), leads us to look at a question usually neglected because it does not make sense in a symmetric model: to which extent an optimal policy generates implicit fiscal transfers between jurisdictions? By implicit transfer, we mean the fact that, despite the governmental budget is balanced a the global level of the whole economy, it may not be balanced at the level of the jurisdictions: some jurisdictions generate more fiscal resources than the amount they need for covering the cost of providing their local public good, and this excess is implicitly transferred to jurisdiction that are not able to generate enough fiscal resources for providing their local public good.

These implicit transfers come from the fact that the taxes implementing an optimal outcome must provide the right incentives to mobile agents for the repartition of these agents across jurisdictions to be the optimal ones; while the optimal amount of local public good provided in each jurisdiction follows the standard Samuelson rule. There is no reason for the two mechanisms to lead to the same repartition of tax receipts and expenditures, leading to implicit transfers.

We show that, when productivity differentials are purely exogenous, implementation of a first best outcome implies implicit transfers from the most productive (and richest) region to the less productive (and poorer one) as soon as capital is taxed. However, this conclusion is not robust to the introduction of agglomeration externalities.

When the global factors productivity increases with the population, we prove that the Pigovian principle leads to tax inhabitants of the poorer region more than inhabitants of the reacher region, because migrants from the poorer to the richer regions generate a negative externality in the region they leave and a positive externality in the region they enter. If capital is not taxed, we are led to the undesirable conclusion that the households in the region must pay higher taxes than agents in the riche one, generating an implicit transfer from the poor region to the rich one. Taxation of capital may be needed for decreasing this implicit transfer.

Chapter 3 adopts a positive point of view, looking at the outcome of fiscal competition between decentralized benevolent planners, each planner bewaring of the welfare of the natives of its jurisdictions (who have a positive willingness to pay for staying in the jurisdiction managed by the local planner).

The analysis of fiscal competition in this asymmetric setting leads to new and interesting insights. The main one is that, starting from a first best outcome where capital is not taxed, when a complete set of taxes is available, jurisdictions have different incentives to tax capital. The richest and most productive region has an incentive to set a positive tax on capital, and then to benefit from a productivity rent analogous to the Baldwin's and Krugman's agglomeration rent (Baldwin and Krugman, 2004). Conversely, the poorest and less productive region has an incentive to subsidize capital, so as to compensate its disadvantage for being still able to attract capital; the cost of these subsidies bears upon inhabitants.

This result provides an interesting explanation of the reason why poor regions engage in capital attraction policies through the offer of subsidies. Note that it does not imply that, at the Nash equilibrium, the poorest region subsidizes capital. If the richest region taxes capital at a high enough level, the poorest region may only need to charge a low enough but still positive tax on capital for attracting it, resulting in a Nash equilibrium with positive tax in both regions. The type of equilibrium depends upon the productivity differential and population mobility. The larger the productivity differential and the lower the population mobility, the stronger are the incentives to tax capital in the most productive region and to subsidize it in the least productive one, the more likely is a Nash equilibrium where capital is subsidized in the poorest region and taxed in the richest one.

Chapter 4 analyzes the link between capital mobility, provision of public goods and taxation from a different point of view. We focus on a single small country and we consider inflows of capital as foreign direct investment (FDI). FDI is generally considered to be a useful resource that can bring in not only capital but also working opportunities and technology, which is extensively competed by public authorities at different levels. The very nice example is Asia. In the 1950s, South Korea, Hong Kong and Singapore had actively attracted foreign investors, mainly from the U.S. More recently, China, India, Vietnam and almost all the other Asian countries have been engaging in the competition for investments from South Korea, Japan and all the other investor countries. In China, the central government sets up special economic regions and offers tax incentives or subsidies to attract more foreign companies. The competition exists not only between countries, but also between different provinces within China. In order to get not only capital investments, but also more working opportunities and technology spillovers, the countries, even the regions within one country, fight for FDI.

However, does a single tax policy works well for attracting FDI? Research on FDI attraction widely considers tax policy and public good provision as double competition factors (Bénasy-Quéré and al. 2005). On the one hand, an increase in tax levels can expel foreign capital out of the region; on the other hand, if the collected tax revenues are used to finance the public goods in the host region, the increase in the supply of public goods can attract capital. This process is widely accepted by the scholars and is used directly in the empirical studies. However, a systematic theoretical analysis of the mechanism behind this double competition is still absent.

The main aim of chapter 4 is to contribute to this analysis. We focus on understanding the mechanism which is behind the influences of public input provision on FDI inflow. There is no doubt that, in the traditional tax competition model, taxing the mobile factor (capital) generates factor outflows and is a source of welfare distortion. However, the effects of public goods can be analyzed differently. Public goods can either be considered as a direct contributor to the welfare of residents, or as a factor directly reducing the cost of production for multinationals (Kellenberg, 2003). We take the assumption of Feehan (1998) that public goods can be considered as public input, which enters the production function of all the sectors in the economy, which implies that productivity is enhanced by the public input.

Following the double competition theory, Chapter 4 focusses on uncovering the detailed mechanisms of the influence of public inputs on FDI flows, and tries to find the important economic characteristics determining the effect of public inputs. Contrary to the previous two chapters, the public goods do not enter directly into the welfare function of consumers, but into the production process of each sector in the economy.

A general equilibrium model with three sectors is used to analyze the influence of public inputs on FDI flow: a sector providing public inputs from labor, financed by public budgets; and two competitive sectors providing traded private goods, the first one (agriculture) using labor and immobile land while the second one (manufacturing) uses labor and perfectly mobile capital provided by foreign investors. This model is used for analyzing the impact of an increase in public input provision under two main assumptions about the taxes used for financing this public input: lump sum taxation, taxation of capital income.

With this model, we can closely look at all the effects of policies using taxation for providing a public input. Looking first at public input provision financed out of lump sum taxation, we identify several mechanisms determining the effect of an increased provision of public input on FDI. There is first an effect coming from the increase in the marginal productivity of capital, which attracts new FDI, generates an increase in wages and substitution of capital to labor. Second, there is an effect through inter-sectoral transfers of labor. Third, there is an effect induced by the differential effect of public input provision on productivity of labor in each sector.

Combining these mechanisms, we prove several conditions for an increase in provision of the public input to generate an inflow of capital: when the public input is small, when the elasticity of total factor productivity to the quantity of public input is high enough in the agricultural sector, when the cost share of labor is high enough in the manufacturing sector, if the marginal rate of substitution is high enough in the manufacturing sector.

Moving to the case where the public input is financed by a tax on capital income, we have several additional effects. The most obvious one is that an increase in the tax implies that the pre-tax return to capital must increase of the post-tax return to be still at the international level. However, there are additional effects coming from the fact that this change in the pre-tax return to capital changes the relative price of factors. Combining these effects, we prove that, contrary to a well-establish belief, a decrease in the tax rate charged on capital income may lead to a FDI outflow instead of an inflow.

The structure of the dissertation is the following: Chapter 1 is the general introduction. Chapters 2 and 3 share the same basic model: where a local public good used by households is provided by two asymmetric jurisdictions differing in their productivity and inhabited by an imperfectly mobile population. Chapter 2 adopts a normative point of view and characterizes the first and second best policies under various assumptions, determining the implicit transfers between regions generated by these policies. Chapter 3 uses the same basic model as chapter 2, but adopts a positive point of view, examining fiscal competition between decentralized jurisdictions. In chapter 4, we move to the question of the provision of a public input on capital flows.

Chapter 2 Public Budgets, Productivity Differences and Implicit Transfers

2.1 Introduction

Governments use fiscal policies for getting resources they normally use for providing resources to households and firms. In a multi-regional context, the fiscal policy choices result in a spatial repartition of resources generated by tax collection; and choices for the provision of public goods generate a spatial repartition of public expenditures. These two geographical repartitions have no reason to coincide with each other, so that public policies generate transfers of resources across regions. Most of these transfers are implicit, as they do not result from an explicit decision to transfer public resources from one region to another.

The existence of these implicit transfers has been widely acknowledged by scholars. Surprisingly, if research has often been questioning the logic and the consequences of explicit transfers, e.g. systems of grants implemented by the central government, very little attention has been paid to implicit transfers. Their logic, the factors driving them, their consequences are ignored most of the time.

The source of these transfers is often attributed to the fact that central governments cannot differentiate tax payers across regions, charging the same taxes wherever taxpayers are located. However, even if central governments are allowed to differentiate tax levels across regions, their choices may be constrained by the economic consequences of this differentiation, which notably result from the interregional mobility of assets. And interregional implicit transfers may be an unwarranted consequence of these restrictions.

This neglect of implicit transfers may be linked to the fact that most theoretical models of tax policy work in a symmetric environment, all the regions being similar to each other. It is only when we work in an asymmetric environment, some regions being richer and other regions being poorer, that questions of transfers matter. In such an asymmetric setting, two questions appear. First what is the relationship between the spatial allotments of fiscal resources and public expenditures and which factors influence the existence and the amount of implicit public transfers across regions? Second, which is the impact of constraints making these public transfers impossible?

We notice that those two questions exist only in the asymmetric situation where the regions are different in their private productivity. In the symmetric situation, it is obvious that there will be identical treatment, identical distribution of resources and no transfers between jurisdictions. Here in this paper, we will focus on an analysis considering the asymmetric regions with different productivities.

In this chapter we are trying to answer the questions using a rather simple framework where a central planner provides local public goods out of tax resources, in a multiregional economy with two regions. Tax resources come from a personal tax on workers and a linear tax on capital. The two regions differ in their productivity, production using perfectly mobile capital and imperfectly mobile labor. Imperfect mobility of workers is generated by preferences, which generate mobility costs when a worker locates in a region that is not his best choice.

We first look at the central planner's first best policy when productivity differences between regions are purely exogenous. We show that, at the first best, as soon as capital is taxed, the planner gets tax revenue higher (lower) than local expenditures in the more (less) productive region; then, the public budget generates an implicit transfer from the more productive region to the less productive one, and this transfer is not motivated by equity considerations. If the planner is not allowed to make implicit transfers (local expenditures must be funded out of local tax revenues), the only solution for reaching a first best outcome is not to tax capital.

Then we look at the central planner's first best policy when productivity differences come from agglomeration economics, which make production more efficient in larger regions. The results are strikingly different: the application of the Pigovian principle to the externalities generated by migrants lead to higher taxation of households in the small (and less productive region) than in the large one, and then, if capital is not taxed, the public budget generates a transfer from the less productive region to the most productive one. For mitigating this transfer, one must tax capital.

Despite the different results, the same mechanism is at work. The mobility of capital and households, and externalities, has constraints on the fiscal choices of the central planner. In the first case, both capital and household tax levels are equalized across regions. In the second case, the application of the Pigovian taxation leads to a fixed interregional differential in household taxes. These constraints, jointly with the consequences of the asymmetry in productivity on the repartition of capital, generate differences in tax revenues that must be offset by transfers.

The structure of the chapter is organized as follows. Section 2 will be relative literature reviews considering about previous research on optimal tax policy related to tax choices and public goods two aspects, as well as the literature on fiscal equilibrium. In section 3, a basic theoretical model is presented based on a series of economic assumptions. Section 4 will have a purely exogenous asymmetry between regions for single planner case with the single budget and the correctness of assumptions can also be tested. After finding the initial equilibrium, an implementation research will be done. The case with single planner with un-transferable public service will follow. Section 5 examines the case of an asymmetry generated by agglomeration economics. Section 6 concludes.

2.2 Literature review

The two important aspects of fiscal policy theory are taxation and expenditure. On the one hand, the standard theory of optimal taxation works on the tax system which would maximize a social welfare function subject to a set of constraints (Mankiw, Weinzierl and Yagan, 2009). They pointed out that in order to simplify the problem, theoretical optimal taxation studies often assumed that all the residents has the same preferences over consumption and leisure. Ever since 1920s, researches on optimal tax policy are made by Ramsey (1927). Using a purely competitive system with no foreign trade and neglecting the questions of distribution and the marginal utility difference, his paper work on the problem of: whether given revenue should be financed by uniform taxes or different rate of taxes. The conclusion shows that taxes should be imposed in inverse proportion to the consumer's elasticity of the good demand. Another important fundamental work is contributed by James Mirrlees (1971). His work launched the optimal tax models which can formalize the planner's problem of dealing with unobserved heterogeneity among taxpayers (the difference of ability and effort of the taxpayers which will influence the income).

Plenty of researches on optimal taxation are development based on this two benchmark researches. The relationship between optimal taxation and resources is also studied by the previous scholars. For example one of most prominent result in dynamic optimal taxation model is that the capital income should not be taxed (Chamley, 1986 and Judd, 1985). However, there are also other scholars argued for the justification of capital taxation (Conesa, Kitao, and Krueger, 2009). Besides, taxation and capital follow are taken by the scholars (Jorgenson and Yun 1986) in a very practical way.

Generally speaking, the optimal taxation researches are very practical that even data of company level are used to support the theoretical researches (Brunori, 1997). Our research will be taken in a more general way which will employ a simple general equilibrium model. Our target is not to give the most appropriate policy suggestion but to better understand the mechanism of the fiscal budget and fiscal choices.

As we said in the beginning, the other important branch of fiscal policy study is public expenditure. The development this area of researches are started by Musgrave (1939) and Samuelson (1954). They pointed out that for the question of the provision of public goods, comparing to private sector, it's hard to find the optimal suggestion for the authorities. Tiebout (1956) worked on a model where every consumer has their specific preference corresponding to the level of a single public goods provide by the jurisdictions. His work is a great contribution to fiscal policy research and becomes the fundamental benchmark for further fiscal competition researches. There are series of researchers following, for example Pauly (1976) generate the question of whether there is equilibrium and of actual empirical implications. Then the relationship between taxation, fiscal expenditures and property values are generated after (Oates, 1969, Gronberg, 1979). Goldstein and Pauly (1980) developed the model in the situation that Tiebout-type migration is taken into account and showed that the estimated effects of personal characteristics would generally be biased.

Instead of simply offering fiscal policy suggestions for the authorities, our work will focus on analyzing the equilibrium when productivity asymmetry exists between jurisdictions. In the multi-jurisdiction economy, productivity difference could force the single central planner to make asymmetric policy choices in each jurisdiction. These policies could possibly generate explicit fiscal transfer between jurisdictions. Fiscal transfer is nothing new in the previous researches. The fiscal transfer studies have non-doubtable relationship with fiscal decentralization. One strand of the literature argues that fiscal decentralization could generate a common-pool problem that the governments' expenditure and revenue responsibilities are not adequately balanced (Ehdaie, 1994; Rodden, 2003). The imbalance is the main reason for the authorities to implement fiscal transfer. Scholars studied not only the fiscal transfers between jurisdiction s in the same level, but also the fiscal transfer between central government and local authorities. Most of the fiscal transfer researches are empirical and are taken in certain cases concerning specific countries.

For example the case of Germany, Ring (2002) notes that ecological functions are incorporated into the intergovernmental fiscal relations at the local level through conditional grants. Buettner (2009) uses a large panel of German municipalities to investigate the dynamic fiscal policy adjustment and the role of fiscal equalization in maintaining fiscal balance. The paper also compares the German case with the result of US case (Buettner and Wildasin, 2006). For the case of Canada, Courchene and Melvin (1980) indicate that the federal government is still committed to approximate fiscal equality. Dean (1986) works on the financial arrangement between the Government of the Northwest Territories and the Canadian government. Rangarajan and Srivastava (2004) examine the relevance and applicability of inter-governmental transfers in Canada and introduced this into another widely researched country: India. Govinda Rao (2003) looks at the incentivizing fiscal transfers in India and finds out that the incentive-linked transfers are too small to the fiscal performance, as well as uncover the design problem in the fiscal system of India federation. Rangarajan and Srivastava (2008) dedicate to reform India’s fiscal transfer system concerning both vertical and horizontal imbalances.

However, we notice that firstly the fiscal transfer researches often focus on empirical research using large amount of data. Furthermore, the fiscal transfer studies are usually closely related to the theory of fiscal equalization. The aim of fiscal equalization is to eliminating differences in net fiscal benefits and reducing the degree of inequality (Blochliger and Charbit 2008). The efficiency of fiscal equalization has long been questioned (Oakland 1994). Different from the fiscal equalization target, our model will focus on the single purpose of maximizing the social welfare and then look at whether the fiscal transfer will be generated by this simple requirement under the assumption of asymmetric private productivities.

2.3 Model

In this paper, we consider an economy with two regions, inhabited by a continuum of L inhabitants providing labor and consuming a private good and a publicly provided good. All the inhabitants share the same preferences for both goods. However, they differ from each other with respect to their preferences across regions. Each inhabitant has a specific willingness to pay for residing in region 2 instead of region 1. This willingness to pay is randomly distributed across the whole population.

In each region, the private good is produced from labor and capital using a constant return to scale technology. Labor is supplied by the individuals living in the region. There is a global fixed stock of perfectly mobile capital K, choosing to locate in the region where its return is highest. The property of capital is evenly shared over the whole population.

A planner produces the publicly provided good from the private good, at a constant unit rate of transformation. Production is financed out of taxes. The planner is able to use both a per capita tax on inhabitants and a proportional tax on capital invested in each region. Tax levels may differ across regions.

2.3.1 Workers

There is a continuum of $L = 1$ workers. L_1 workers choose to stay in region 1, L_2 workers choose to stay in region 2, with $L_1 + L_2 = L = 1$. Every worker supplies one labor unit and holds a fixed amount of capital, *k*. Therefore, the net income of a worker staying in region *i* is $w_i + \rho k - \tau_i$, where w_i is the wage in region *i*, ρ is the after tax rate of return to capital and τ_i is the income tax in region *i*.

This net income is used for consumption of a private monetary good so that, in region *i*, private consumption is

$$c_i = w_i + \rho k - \tau_i$$

Moreover, a worker staying in region *i* benefits from consumption of a publicly provided good z_i. Workers are endowed with preferences represented by the following utility function (for worker *l*):

$$V_{i,l} = U(c_i, z_i) + m_{i,l}$$

$$U(c_i, z_i) = c_\mathrm{i} + u(z_\mathrm{i})$$

where

$$m_{1,l} = \mu_l$$

$$m_{2,l} = 0$$

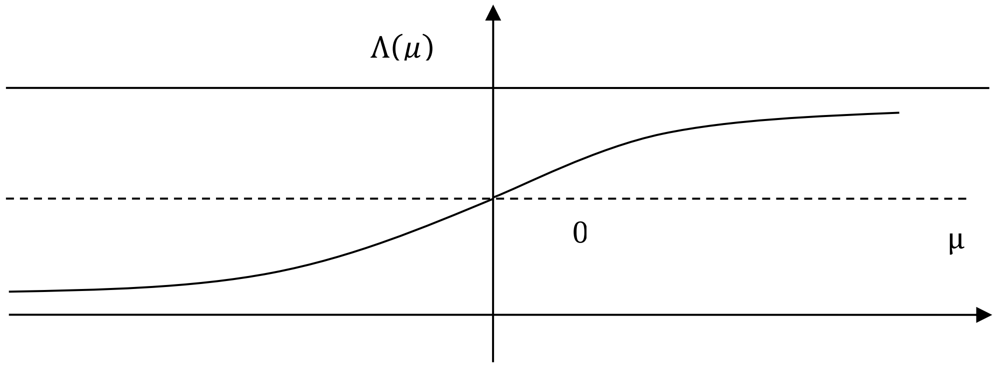
μ_l is the willingness to pay of worker *l* for staying in region 1 instead of region 2.

Workers differ from each other with respect to their preferences across regions, and then that μ varies across workers. The cumulative distribution function of μ is

$$\Lambda(\mu): \mathbb{R} \to]0,1[$$

with $0 < d\Lambda/d\mu < \infty$ and $\Lambda(0) = 0.5$. Therefore, for every $\mu \in \mathbb{R}$, $\Lambda(\mu)$ is the number of workers whose willingness to pay for staying in region 1 instead of region 2 is below μ. The shape of the function $\Lambda(\mu)$ is represented in Figure 1.

Figure 1: cumulative distribution function of willingness to pay



The inverse function of $\Lambda(\mu)$ is $M(l)$: for every $l \in]0,1[$, $M(l)$ is the maximal value of μ for the group of *l* workers whose willingness to pay for staying in region 1 instead of region 2 is lowest.

Workers are mobile across regions and choose the location where their utility is highest. Then, the choice of an individual whose willingness to pay μ is:

$$i = 1 \quad iff \;\; U(c_1, z_1) + \mu > U(c_2, z_2) \Leftrightarrow \mu > \Delta U$$

$$i = 2 \quad iff \;\; U(c_1, z_1) + \mu < U(c_2, z_2) \Leftrightarrow \mu < \Delta U$$

where $\Delta U = U(c_2, z_2) - U(c_1, z_1)$. Therefore, $L_2 = \Lambda(\Delta U)$ workers choose region 2 while $L_1 = 1 - \Lambda(\Delta U)$ choose region 1. Equivalently, for L_2 workers to choose region 2, the utility differential must be $\Delta U = M(L_2)$

2.3.2 Production of the private good

The private good is produced by private firms combining labor and capital using a constant return to scale technology. In region i, the production function is

$$F_{\mathrm{i}}(K_i, L_i) = \theta_i \, F(K_i, L_i) = \theta_i \, L_i f(k_i)$$

where K_i is the capital input, L_i is the labor input, $k_i = K_i/L_i$, $F(K_i, L_i)$ is an homogenous production function while $f(k_i)$ is a concave increasing function meeting the Inada conditions; θ_i is an efficiency parameter, which differs across regions. Without loss of generality, we assume that $\theta_1 = 1$ and that production is more efficient in region 2, so that $\theta_2 = \theta > 1$.

Sometimes, we will need the following property:

Single crossing property: Whatever $\theta > 1$ and $\delta > 0$, the curves $y = f'(k)$ and $y = \theta f'(k + \delta)$ cross once.

There is a fixed capital stock of capital available for production in both regions, *K*, so that

$$K_1 + K_2 = L_1 k_1 + L_2 k_2 = K$$

Capital is perfectly mobile across region and then the post-tax return to capital, ρ, is the same in both regions:

$$\rho = f'(k_1) - t_1 = \theta f'(k_2) - t_2$$

Where t_i is the tax rate on capital in region *i*.

2.3.3 Public good provision

We assume the publicly provided good to be divisible[1] and that there are no spillovers: for every consumer in region i to consume the quantity z_i of publicly provided goods, the government must provide the global quantity $Z_i = L_i z_i$ in the region. The publicly provided good is produced from the private good with a one-to-one rate of transformation.

A single planner provides the good in both regions, using both the tax on households and the tax on capital. The total amount of taxes collected in region i is

$$L_i \tau_i + K_i t_i = L_i(\tau_i + k_i t_i)$$

where τ_i is the amount of the tax on workers and t_i is the rate of the tax on capital. The central planner balances his whole budget:

$$L_1 z_1 + L_2 z_2 = L_1(\tau_1 + k_1 t_1) + L_2(\tau_2 + k_2 t_2)$$

2.3.4 Welfare

The central planner maximizes the total welfare for all the residents in both regions. Then he maximizes the standard utilitarian welfare function:

$$W = \int_0^1 \left[c_{i(l)} + u\left(z_{i(l)}\right) + m_{i(l),l}\right] dl$$

where $i(l)$ is the location of agent l.

[1] This assumption has been made because, with an indivisible public good, the largest region has an advantage in the provision of public good. We do not want our results to be driven by this advantage.

Straightforwardly, as workers are homogenous with respect to their contribution to production and the utility they derive from consumption, all the workers staying in region 1 must have a higher willingness to pay for staying in that region than the workers staying in region 2. Then, if there are L_1 workers in region 1 and L_2 workers in region 2 (with $L_1 + L_2 = 1$), for all the workers staying in region 1 (resp. region 2) we have $\mu > M(L_2)$ (resp. $\mu < M(L_2)$). Then, knowing that $m_{1,l} = \mu_l$ and $m_{2,l} = 0$, we can rewrite W as:

$$W = L_1[c_1 + u(z_1)] + L_2[c_2 + u(z_2)] + \int_{L_2}^{1} M(l)dl$$

which, up to the constant $\int_{0,5}^{1} M(l)dl$, may also be written as:

$$W = L_1[c_1 + u(z_1)] + L_2[c_2 + u(z_2)] - \Omega(L_2) \tag{1.1}$$

Where $\Omega(L_2) = \int_{0,5}^{L_2} M(l)dl$

Note $\Omega(L_2)$ may be interpreted as an aggregated migration cost. Let us assume that, initially, all the workers are located in the region they prefer: the 0.5 migrants with $\mu > 0$ are in region 1 while the 0.5 workers with $\mu < 0$ are in region 2. Then, taking account of the utility differential generated by consumption, ΔU, workers relocate. For a worker with willingness to pay μ, the migration cost is $|\mu|$. And, aggregating over all the migrating workers, we get the aggregated migration cost $\Omega(L_2)$.

2.4 Central planner taxation choice

2.4.1 First best optimum

At a first best optimum, the central planner determines the repartitions of population and capital across regions and the levels of public and private consumption, maximizing W under the following constraints:

$$L_1 f(k_1) + L_2 \theta f(k_2) = L_1(c_1 + z_1) + L_2(c_2 + z_2) \quad (1.2)$$

$$L_1 k_1 + L_2 k_2 = k \quad (1.3)$$

$$L_2 = L - L_1 \quad (1.4)$$

The first constraint is the budget constraint: the global level of production equals the global level of consumption. The second and third constraints describe the spatial repartitions of the global fixed stocks or capital and population.

In appendix 2.1, we prove the following proposition:

Proposition 2.1: *at the optimal outcome the following equalities hold:*

$$\theta f'(k_2) = f'(k_1)$$

$$u'(z_1) = u'(z_2) = 1$$

$$M(L_2) = \theta[f(k_2) - k_2 f'(k_2)] - [f(k_1) + k_1 f'(k_1)]$$

The first equality tells us that, at the first best optimum, the marginal productivity of capital is equalized across regions.

The second equality is the standard Samuelson rule: in both regions, the marginal rate of substitution of the private good to the publicly provided good equals the marginal rate of transformation, which equals unity. Together with the assumption that $\mathrm{u}(\mathrm{z_i})$ meets the Inada conditions, it implies that both regions should provide the same amount of public good, $z_1 = z_2 = z$, with $u'(z) = 1$.

As for the third equation, we know that $M(L_2)$ is the willingness to pay of the marginal migrant. $\theta[f(k_2) - k_2 f'(k_2)]$ is the marginal productivity of labor in region 2, while $f(k_1) - k_1 f'(k_1)$ is the marginal productivity of labor in region 1. Therefore, the third condition tells us that the marginal migrant (who can be located in either region) has a willingness to pay for staying in region 1 equal to the productivity differential between region 2 and region 1: the preference for region 1 exactly compensates the loss in productivity.

2.4.2 Equilibrium and implementation

In this section, we look at the possibility to implement the first best optimum defined above as equilibrium in an economy where the central planner uses the per capita tax on labor and the proportional tax on capital as policy instruments. The central planner starts, choosing his tax policy. Then, the workers freely choose their location and capital freely splits across regions, taking account of the taxes.

For region i, the central planer levies taxes on capital, t_i, and on households, τ_i. Capital and workers choose their own location based on the taxes. At equilibrium, the following conditions are met:

$$\rho = f'(k_1) - t_1 = \theta f'(k_2) - t_2 \tag{1.5}$$

$$Z_1 + Z_2 = L_1 z_1 + L_2 z_2 = L_1(\tau_1 + k_1 t_1) + L_2(\tau_2 + k_2 t_2) \tag{1.6}$$

$$M(L_2) = c_2 + u(z_2) - c_1 - u(z_1) \tag{1.7}$$

The first equation tells us that when capital is perfectly mobile, the after tax returns to capital must be the same across regions.

The second equation is the global public budget constraint faced by the planner.

The third equation tells us that, when migrants are perfectly mobile, the marginal migrant is indifferent living in either region, so that his marginal willingness to pay, $M(L_2)$, equals his utility differential: $c_2 + u(z_2) - c_1 - u(z_1)$.

Furthermore, consumption is determined by the private budget constraints:

$$c_1 = f(k_1) - k_1 f'(k_1) + \rho k - \tau_1$$

$$c_2 = \theta f(k_2) - k_2 \theta f'(k_2) + \rho k - \tau_2$$

Let us now compare the conditions for equilibrium and the conditions for a first best optimum. Propositions 2.2, 2.3, 2.4 are proved in appendix 2.2:

Proposition 2.2: *For equalizing marginal productivities, the planner has to tax capital at the same rate in both regions:* $t_1 = t_2 = t$

At the first best optimum, the marginal productivity of capital must be same in both regions. On the other hand, at equilibrium with perfectly mobile capital, the post-tax returns to capital must be the same in both regions. Therefore, for equilibrium to coincide with the first best optimum, the central planner has to tax capital at the same rate in both regions.

Proposition 2.3: *The choice made by the marginal migrant implies that he must be charged the same head tax in both regions:* $\tau_1 = \tau_2 = \tau$

At equilibrium, the marginal migrant equalizes his willingness to pay to the utility differential between regions. In order for this utility differential not to be modified by taxation, head taxes must be the same in both regions.

It's easy to see the central planner can implement the first best outcome as an equilibrium for every tax package equalizing both taxes in both regions ($\tau_1 = \tau_2 = \tau$ and $t_1 = t_2 = t$) and meeting the public budget constraint, which in that case becomes

$$z = \tau + kt$$

where z is the optimal quantity of the publicly provided good, which is the same in both regions ($z_1 = z_2 = z$, with $u'(z) = 1$). Then, the planner can choose any level of one of the taxes, say $t_1 = t_2 = t$ (resp. $\tau_1 = \tau_2 = \tau$), and choose the other tax, say $\tau_1 = \tau_2 = \tau$ (resp. $t_1 = t_2 = t$), so as to meet his budget constraint. Comparing cost of providing the public goods in both regions and the revenue of taxation, we get the following proposition (see the proof in appendix 2.3):

Proposition 2.4: *If equilibrium implements the first best optimum and capital is positively taxed, more taxes per capita will be collected in the more productive region than in the less productive region, so that providing the same level of public good in* *both regions implies a transfer of resources from the more productive to the less productive one.*

2.4.3 Single planner case with no public transfer

Since proposition 2.4 implies that there may be an implicit transfer between regions, what happens when this transfer is not allowed? Let us first note that if $t_1 = t_2 = 0$ then $z_1 = z_2 = z = \tau_1 = \tau_2$, so that the head tax collected in each region exactly covers the cost of the publicly provided good and there is no transfer between regions.

Then an equilibrium where the publicly provided good is funded out of the head tax only does not need any transfer. Moreover, this equilibrium is the only one implementing the first best outcome without transfers, for a simple reason: we proved in Proposition 2.4 that, for every strictly positive level of capital taxation, at the first best outcome, there is an implicit transfer. Then, not allowing transfers generates a binding constraint which prevents implementing the first best outcome as soon as capital is taxed.

Let us note that this necessity of not taxing capital when we do not allow for public interregional transfers differs from the standard result that decentralized jurisdictions must not tax perfectly mobile capital. In our model, capital is perfectly mobile across jurisdictions, obliging the central planner to tax capital at the same rate in both regions for capital taxation not to be distortive. But the global capital stock is fixed and then, at an efficient outcome, the common tax rate is undetermined. It is explicitly for not generating transfers that capital must not be taxed, because capital is concentrated in the most productive region and, jointly with the obligation to tax capital at the same rate in both concentration, as soon as capital is effectively taxed, this concentration of the tax based generates a concentration of the tax revenue that must be compensated by an implicit transfer.

2.4.4 Limits on the tax on capital

What happens if the planner is obliged to tax capital? If the planner is allowed to make implicit transfers between regions, the obligation to tax capital does not prevent the planner to implement a first best outcome. However, as noted above, implementing the first best outcome becomes impossible without transfers if the tax on capital is strictly positive. Looking for a second best optimum, we find the following proposition, proved in Appendix 2.4:

Proposition 2.5: *If the central planner is obliged to set a strictly positive tax on capital,* $t_1 = t_2 = t > 0$, *he still provides the optimal amount of public good in both regions* ($z_1 = z_2 = z$, with $u'(z) = 1$), *which leads him to levy a higher labor tax in region 1 than in region 2. There are more inhabitants in the more productive region than at the first best outcome and the unevenness between regions is larger the higher the capital tax rate.*

Then, being obliged to tax capital, the planner chooses a second best policy which exacerbates the inequalities between regions.

2.4.5 Limits on the head tax

Then we consider the symmetric case: what happens if the planner faces an upward limit on household taxation, implying that he cannot finance the whole cost of the public goods out of household taxes? We still look at the case where the planner is not allowed to make transfers between regions. The constraint on household taxation is:

$\tau_1 \leq \bar{\tau}$

$\tau_2 \leq \bar{\tau}$

with $\bar{\tau} < z$, where z is the first best level of public good $(u'(z) = 1)$. In Appendix 2.5, we prove the following proposition:

Proposition 2.6: *If the planner faces an upward limit on household taxation, $\bar{\tau}$, implying that he cannot finance the whole cost of the public goods out of household taxes, in both regions he underprovides the public good with respect to the first best. Moreover, a tighter limitation ($d\bar{\tau} < 0$) implies:*

- *Higher capital tax rates in both regions ($dt_1 > 0,\ dt_2 > 0$)*
- *A higher increase of the capital tax rate in the less productive region ($dt_1 - dt_2 > 0$).*
- *A lower endowment in capital per worker in the less productive region ($dk_1 < 0$)*
- *A lower level of public good provision in the less productive region ($dz_1 < 0$)*

We are unable to sign the impact on capital per worker and public good provision in the most productive region.

2.5 Agglomeration externalities

In this chapter, we no longer consider that the total factor productivity in each region is fixed. We look at the case where there is an agglomeration externality, the total factor productivity being an increasing function of the population of the region.

2.5.1 The agglomeration externality

We introduce an agglomeration externality in a very simple way: in each region, the efficiency parameter, θ_i, is an increasing function of the size of the labor force:

$$\theta_i = \theta(L_i)$$

where θ is a convex increasing function: $\theta'(L_i) > 0$ and $\theta"(L_i) > 0$. Note that, now, there is no longer any ex-ante asymmetry between the two regions: if $L_1 = L_2 = 1/2$, then $\theta_1 = \theta_2 = \theta(1/2)$. However, if agglomeration externalities are strong enough, mobile workers may tend to agglomerate in one region, leading to an ex-post asymmetric outcome.

2.5.2 Welfare analysis with an agglomeration externality

The maximization problem determining the first best outcome is the same as in section 2.4.1. The central planner chooses the repartitions of population and capital across regions and the levels of public and private consumption, maximizing W under the constraints (1.1), (1.2) and (1.3). The only difference is that now, we have to take account of the fact that, in each region, total factor productivity depends upon population, $\theta_i = \theta(L_i)$.

The characteristics of the optimal outcome are derived in Appendix 2.6. We again find the equality of marginal productivities of capital:

$$\theta_1 f'(k_1) = \theta_2 f'(k_2)$$

and the Samuelson rule:

$$u'(z_1) = u'(z_2) = 1 \Rightarrow \; z_1 = z_2$$

However, the condition for an optimal repartition of the population changes. It is now:

$$M(L_2) = \theta_2[f(k_2) - k_2 f'(k_2)] - \theta_1[f(k_1) - k_1 f'(k_1)] + [L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)]$$

Compared to the expression of Proposition 2.1, we now have an additional term, $L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)$. What happens is that, compared to the case of fixed total factor productivities, the introduction of agglomeration externalities generates an additional effect of migration. A migrant moving from region 2 to region 1 generates a decrease in the production of workers located in region 2, measured by $L_2\theta'_2 f(k_2)$, and in increase in region 1, measured by $L_1\theta'_1 f(k_1)$, hence a productivity differential $L_1\theta'_1 f(k_1) - L_2\theta'_2 f(k_2)$. Therefore, the willingness to pay of the marginal migrant $M(L_2)$, has now to compensate for both the differential in marginal productivity of the migrant and his impact on the production of the economy.

Let us note that the symmetric outcome ($L_1 = L_2 = 0.5$ and $k_1 = k_2 = k$) meets the first order conditions. However, it may not be an optimum, because of the non-concavity induced by the externality. Then, beyond the first order conditions, we also have to look at second order conditions. Maximizing W with respect to k_1, k_2, z_1 and z_2 for given L_2, one finds the welfare function $W(L_2)$. In appendix 1.6, we prove that, for $L_1 = L_2 = 0.5$:

$$\frac{\partial^2 W(L_2)}{\partial {L_2}^2} = \frac{2\theta' f(k)}{A}\left[A^2 + 2(1-\eta)A + \frac{\eta^2}{4k^2}\right]$$

where $A = L_i \frac{\theta''}{\theta'} = 0.5\frac{\theta''}{\theta'}$ is the elasticity with respect to population of the marginal increase in the agglomeration externality and $\eta = \frac{kf'(k)}{f(k)}$ is the elasticity of production function with respect to capital. The ratio $\frac{2\theta' f(k)}{A}$ being positive, $\frac{\partial^2 W(L_2)}{\partial {L_2}^2}$

has the same sign has the bracketed term, which is a quadratic expression. The determinant of this quadratic expression is

$$\Delta^2 = 4(1-\eta)^2 - \frac{\eta^2}{k^2} = \left[\left(2+\frac{1}{k}\right)\eta - 2\right]\left[\left(2-\frac{1}{k}\right)\eta - 2\right]$$

Two cases must be distinguished:

- When $\Delta^2 < 0 \Leftrightarrow \eta \in [\frac{2k}{2k+1}, \frac{2k}{2k-1}]$, $\frac{\partial^2 W}{\partial {L_2}^2} > 0$ for whatever A.
- When $\Delta^2 > 0 \Leftrightarrow \eta \notin [\frac{2k}{2k+1}, \frac{2k}{2k-1}]$, $\frac{\partial^2 W}{\partial {L_2}^2} \leq 0$ for $A \in [A_1, A_2]$ and $\frac{\partial^2 W}{\partial {L_2}^2} > 0$ for $A \notin [A_1, A_2]$, with $A_1 = 2(\eta - 1) - \Delta$ and $A_2 = 2(\eta - 1) + \Delta$

Then, as soon as the elasticity with respect to population of the marginal increase in the agglomeration externality, A, is high enough ($A > A_2$), we are sure that $\frac{\partial^2 W}{\partial {L_2}^2} > 0$ whatever η, and then the symmetric outcome is a local minimum. The first best outcome is necessarily asymmetric. From now on, we will focus on asymmetric outcomes.

2.5.3 Equilibrium and implementation

As in section 2.4.3, we look at the possibility to implement the first best optimum defined above as equilibrium in an economy where the central planner uses the per capita tax on labor and the proportional tax on capital as policy instruments. At equilibrium, the conditions (5), (6) and (7) defined in section 4.3 still hold.

In Appendix 2.7, we prove the following proposition:

Proposition 2.7: *A policy implementing the first best optimum is characterized by:*

$z_1 = z_2 = z \quad with \;\; u'(z) = 1$

$t_1 = t_2 = t$

$\tau_1 - \tau_2 = \Delta\tau \equiv L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)$

$z = L_1\tau_1 + L_2\tau_2 + kt$

Comparing to the case of fixed total factor productivities (Propositions 2.2 and 2.3), there is an important difference: the planner no longer equalizes the taxes paid by households across regions. When total factor productivities were fixed, taxes were equalized so as not to distort the location choice of the marginal migrant. As noted above, when there are agglomeration externalities, migration generates externalities, as it decreases the productivity of all the workers located in the origin region and increases the productivity of all the workers located in the destination region. For a move from region 1 to region 2, the net effect of these externalities is $L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)$. Following the Pigovian rule, the differential in household taxes must compensate for these externalities, hence the equality $\tau_1 - \tau_2 = L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)$.

When the first best outcome to be implemented is asymmetric, this application of the Pigovian principle has an important, and maybe undesirable, feature. Without loss of generality, let us assume that the largest region is region 2, so that $L_2 > 0.5 > L_1$, which implies $\theta_2 > \theta_1$ and $\theta'_2 > \theta'_1$; and, using the equality $\theta_1 f'(k_1) = \theta_2 f'(k_2)$, $k_2 > k > k_1$, leading to $f(k_2) > f(k_1)$. Then,

$$\tau_1 - \tau_2 = \Delta\tau > 0$$

The implication of this inequality is that households leaving in the smaller and poorer region (Region 1) pay more taxes than households leaving in the larger and richer region (Region 2). This result is a direct consequence of the application of the

Pigovian principle: a migrant moving from the largest and richest region to the smallest and poorest one generates a larger decrease in the production of the large region than the increase generated in the smaller region. Then, his global effect is negative and, for compensating this negative effect, he has to pay higher taxes in the smaller and poorer region. However, from an equity point of view, it may be undesirable, a point we will be looking at more closely in the nest section.

Let us also note that the planner has one degree of freedom: he can choose t $(= t_1 = t_2)$ arbitrarily and, once t has been chosen, τ_1 and τ_2 are determined by the last two equations of Proposition 2.7, hence:

$$\tau_1 = z - kt + (L_2)^2\theta'_2 f(k_2) - L_1 L_2 \theta'_1 f(k_1)$$

$$\tau_2 = z - kt - L_1 L_2 \theta'_2 f(k_2) + (L_1)^2\theta'_1 f(k_1)$$

2.5.4 Transfers

The per capita tax revenue in each region is

$$R_1 = \tau_1 + k_1 t = z - (k - k_1)t + L_2 \Delta\tau$$

$$R_2 = \tau_2 + k_2 t = z - (k - k_2)t - L_1 \Delta\tau$$

with $\Delta\tau = L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)$

As noted above, when the first best outcome to be implemented is asymmetric, region 2 being the largest one, we have:

$$\tau_1 - \tau_2 = \Delta\tau > 0$$

so that households leaving in the smaller and poorer region (Region 1) pay more taxes than households leaving in the larger and richer region (Region 2).

Then, if the planner chooses not to tax capital ($t = 0$), $R_1 = \tau_1 > \tau_2 = R_2$ and per capita tax revenue is higher in the smaller and poorer region 1 than in the larger and richer region 2, generating an implicit transfer from the poorer region to the richer one. However:

$$\frac{dR_1}{dt} = k_1 - k < 0$$

$$\frac{dR_2}{dt} = k_2 - k > 0$$

so that introducing taxation of capital decreases the per capita tax revenue in the poorer region and increases it in the richer one, reducing the level of implicit transfers from the poorer to the richer region: the central planner uses the fact that the capital is disproportionately located in region 2.

The transfers will be reversed if $R_1 \leq R_2$, which implies

$$t \geq \frac{\Delta\tau}{k_2 - k_1}$$

2.6 Conclusion

The central message of our analysis is a fairly simple one: in an economy where there are several regions, even if he is able to differentiate tax levels or tax rates between regions, efficiency considerations may limit the freedom of the central planner to use this differentiated taxation. These limits may come from interregional mobility of tax bases, as taxation must not distort the location choice of data base and/or may need to internalize externalities generated by the mobility of tax bases.

Then, for the central planner, the mobility of tax bases is constraining in two ways: it determines the interregional repartition of tax bases and it constraints the choice of tax levels or tax rates charged on these tax bases. The result of this double constraint is that the spatial repartition of tax revenues may not coincide with the spatial repartition of spatial expenditures, generating implicit transfers in the welfare system. The nature and the direction of these transfers depend upon the type of constraints imposed on the planner. In some cases, these implicit transfers may have undesirable characteristics, for example when a poor regions pay for a rich one.

However, despite the constraint he faces, the planner may still have some degrees of freedom in the choice of the tax menu. For example, in the model of this paper, the planner is still able to choose the repartition of tax revenues between capital taxation and household taxations. The planner may be using this menu for manipulating implicit transfers. For example, in the examples of this paper, the concentration of capital in the most productive region and the constraint that taxes on capital must be equalized across regions generate concentrated tax revenues in the richest region. Then, lower taxation of capital lowers the impact of the concentration of capital on interregional transfers. In the first situation described in this paper (exogenous productivity), this leads the planner not to tax capital if he is unable to make transfers. On the contrary, in the second situation, the planner may want to use capital taxation for compensating the distortion resulting from the fiscal internalization of agglomeration externalities generated by migration.

Appendix Chapter 2

Appendix 2.1

The central planner solves the following problem:

$$Max\ L_1[c_1 + u(z_1)] + L_2[c_2 + u(z_2)] - \Omega(L_2)$$

$s.t L_1 f(k_1) + L_2\theta f(k_2) = L_1(c_1 + z_1) + L_2(c_2 + z_2)$ λ

$L_1 k_1 + L_2 k_2 = k$ ξ

$L_1 + L_2 = 1$

Hence the Lagrangian:

$$\Upsilon_1 = (1 - L_2)[c_1 + u(z_1)] + L_2[c_2 + u(z_2)] - \Omega(L_2) + \lambda[(1 - L_2)(f(k_1) - c_1 - z_1) + L_2(\theta f(k_2) - c_2 - z_2)] - \xi[(1 - L_2)k_1 + L_2 k_2 - k]$$

Differentiating the Lagrangian with respect to $z_1,\ z_2,\ c_1,\ c_2,\ k_1,\ k_2$ and L_2, and rearranging, we get the following first order conditions:

$$u'(z_1) = u'(z_2) = 1$$

$$f'(k_1) = \theta f'(k_2)$$

$$M(L_2) = \theta f(k_2) - f(k_1) - \xi(k_2 - k_1)$$

$$= \theta[f(k_2) - k_2 f'(k_2)] - [f(k_1) - k_1 f'(k_1)]$$

Appendix 2.2

Looking at firms, at equilibrium with perfectly mobile capital, post tax returns to capital are equalized across regions:

$$\rho = f'(k_1) - t_1 = \theta f'(k_2) - t_2$$

We know that, at the first best optimum, marginal productivity is the same in both regions.

$$\theta f'(k_2) = f'(k_1)$$

For both equalities to hold simultaneously, we need to have $t_1 = t_2$

Now, looking at households, at equilibrium, the willingness to pay of the marginal households equals the utility differential:

$$M(L_2) = c_2 + u(z_2) - c_1 - u(z_1)$$

Knowing that at the first best optimum, $\mathrm{z}_1 = \mathrm{z}_2$, and using the expressions for consumption, we get:

$$M(L_2) = c_2 - c_1 = \theta[f(k_2) - k_2 f'(k_2)] - [f(k_1) - k_1 f'(k_1)] + \tau_1 - \tau_2$$

Furthermore, from Proposition 2.1, we know that

$$M(L_2) = \theta[f(k_2) - k_2 f'(k_2)] - [f(k_1) - k_1 f'(k_1)]$$

For an equilibrium to implement the first best optimum, both equalities must hold simultaneously, hence $\tau_1 = \tau_2$. The planner charges the same taxes on labor in both regions.

Appendix 2.3

Knowing that $\theta > 1$, $f'(k_1) = \theta f'(k_2)$ implies $f'(k_1) > f'(k_2)$ and then, knowing that $f''(k_i) < 0$, $k_1 < k_2$. Moreover, $k_1 < k_2$ and the equalities $k = L_1 k_1 + L_2 k_2$ and $L_1 + L_2 = 1$ imply:

$$k_1 < k < k_2$$

Tax revenue per capita is $R_1 = \tau_1 + k_1 t_1$ in region 1 and $R_2 = \tau_2 + k_2 t_2$. The central planner charging the same tax rate at capital $t_1 = t_2 = t$ and the same head tax $\tau_1 = \tau_2 = \tau$, we get:

$$t > 0 \Rightarrow R_1 = \tau + k_1 t < \tau + k_2 t = R_2$$

Moreover, knowing that $z_1 = z_2 = z$, the global public budget constraint implies:

$$z = \tau + kt$$

Which, knowing that $k_1 < k < k_2$, implies:

$$t > 0 \Rightarrow R_1 < z < R_2$$

Then, as soon as the central planner taxes capital, per capita tax revenue in region 1 is not high enough for covering per capital public expenditures, the difference being $R_1 - z < 0$; at the same time, per capita tax revenue in region 2 is higher than capital public expenditures, the difference being $R_2 - z > 0$. Then, the planner implicitly transfers tax revenue from region 2 to region 1.

Appendix 2.4

Now we consider the case where the central planner cannot tax capital at a lower rate than *t*, this constraint being binding in both regions, so that $t_1 = t_2 = t$.

When transfers between regions are not allowed, the planner's second best program maximizes W under the following constraints:

$$c_1 + z_1 = f(k_1) + (k - k_1)(f'(k_1) - t)$$

$$c_2 + z_2 = \theta f(k_2) + (k - k_2)(\theta f'(k_2) - t)$$

$$f'(k_1) - t = \theta f'(k_2) - t \implies f'(k_1) = \theta f'(k_2)$$

$$M(L_2) = c_2 + u(z_2) - c_1 - u(z_1)$$

$$L_1 k_1 + L_2 k_2 = k$$

The Lagrangian of this program is:

$$\Upsilon_4 = L_1[c_1 + u(z_1)] + L_2[c_2 + u(z_2)] - \Omega(L_2) - \lambda_1[c_1 + z_1 - f(k_1) - (k - k_1)(f'(k_1) - t)] - \lambda_2[c_2 + z_2 - \theta f(k_2) - (k - k_2)(\theta f'(k_2) - t)] - \mu[L_1 k_1 + L_2 k_2 - k] - \zeta[f'(k_1) - \theta f'(k_2)] - \xi[c_2 + u(z_2) - c_1 - u(z_1) - M(L_2)]$$

Differentiating the Lagrangian with respect to z_1, z_2, c_1, c_2, k_1, k_2 and L_2, and rearranging, we get the following first order conditions:

$$\xi = L_2 - \lambda_2 = \lambda_1 - L_1 \Rightarrow \lambda_1 = \xi + L_1 \quad \lambda_2 = L_2 - \xi$$

$$u'(z_1) = u'(z_2) = 1 \Rightarrow z_1 = z_2 = z \text{ with } u'(z) = 1$$

$$\mu L_1 = \lambda_1[t + (k - k_1) f"(k_1)] - \zeta f"(k_1)$$

$$\mu L_2 = \lambda_2[t + (k - k_2)\theta f''(k_2)] + \zeta\theta f''(k_2)$$

$$M(L_2) - \xi M'(L_2) = c_2 - c_1 - \mu(k_2 - k_1)$$

Note that, when $z_1 = z_2 = z$, the migration constraint becomes $M(L_2) = c_2 - c_1$ and then the last condition becomes

$$\xi M'(L_2) = \mu(k_2 - k_1)$$

The planner is still following the Samuelson rule, providing the same quantity of public good in both regions, $z_1 = z_2 = z$ with $u'(z) = 1$. Then,

$$z = \tau_1 + k_1 t = \tau_2 + k_2 t \Rightarrow \tau_1 - \tau_2 = (k_2 - k_1)t > 0$$

the later inequality being the consequence of the fact that capital is charge the same tax rate in both regions, which implies $f'(k_1) = \theta f'(k_2)$ and then, θ being above unity, $k_1 < k_2$.

The first order conditions imply that, for any given t, k_1, k_2 and L_2 solve the following system of equations:

$$M(L_2) = \theta f(k_2) - f(k_1) - (k_2 - k_1)(f'(k_1) - t)$$

$$\theta f'(k_2) = f'(k_1)$$

$$(1 - L_2)k_1 + L_2 k_2 = k$$

Differentiating this system or equations with respect to t, k_1, k_2 and L_2, after some straightforward calculations, we get:

$$\frac{dL_2}{dt} = \frac{k_2 - k_1}{D}$$

with

$$D = M'(L_2) - \frac{(k_1 - k_2)[f''(k_1) - \theta f''(k_2)]}{L_1 \theta f''(k_2) + L_2 f''(k_1)} t - \frac{(k_2 - k_1)^2 f''(k_1) \theta f''(k_2)}{L_1 \theta f''(k_2) + L_2 f''(k_1)}$$

Under the single crossing property, we have $f''(k_1) - \theta f''(k_2) < 0$, implying $D > 0$ and then

$$\frac{dL_2}{dt} > 0$$

The higher the tax rate the central planner has to charge on capital, the higher the population of region 2, the larger the disequilibrium between regions.

Appendix 2.5

Let us now consider the case of an upward limit on the labor tax, $\tau_1 \leq \bar{\tau} < z$ and $\tau_2 \leq \bar{\tau} < z$, with $u'(z)$, so that the planner is no longer able to rest upon households for financing the production of the public good. Now, the planner is maximizing W under the following constraints:

$$c_1 + z_1 = f(k_1) + (k - k_1)(f'(k_1) - t_1)$$

$$c_2 + z_2 = \theta f(k_2) + (k - k_2)(\theta f'(k_2) - t_2)$$

$$f'(k_1) - t_1 = \theta f'(k_2) - t_2$$

$$M(L_2) = c_2 + u(z_2) - c_1 - u(z_1)$$

$$L_1 k_1 + L_2 k_2 = k$$

$$z_1 \leq \bar{\tau} + k_1 t_1$$

$z_2 \leq \bar{\tau} + k_2 t_2$

hence the Lagrangian:

$\Upsilon_5 = L_1[c_1 + u(z_1)] + L_2[c_2 + u(z_2)] - \Omega(L_2) - \lambda_1[c_1 + z_1 - f(k_1) - (k - k_1)(f'(k_1) - t_1)] - \lambda_2[c_2 + z_2 - \theta f(k_2) - (k - k_2)(\theta f'(k_2) - t_2)] - \mu[L_1 k_1 + L_2 k_2 - k] - \zeta[f'(k_1) - t_1 - \theta f'(k_2) + t_2] - \xi[c_2 + u(z_2) - c_1 - u(z_1) - M(L_2)] - \eta_1(z_1 - \bar{\tau} - k_1 t_1) - \eta_2(z_2 - \bar{\tau} - k_2 t_2)$

Differentiating the Lagrangian with respect to z_1, z_2, c_1, c_2, k_1, k_2, t_1, t_2 and L_2, and rearranging, we get the following first order conditions for a second best outcome:

$\xi = L_2 - \lambda_2$

$\xi = \lambda_1 - L_1$

$(u'(z_1) - 1)\lambda_1 = \eta_1$

$(u'(z_2) - 1)\lambda_2 = \eta_2$

$\zeta = \lambda_1 k - (\lambda_1 + \eta_1)k_1$

$\zeta = (\eta_2 + \lambda_2)k_2 - \lambda_2 k$

$\mu L_1 = \lambda_1 u'(z_1)t_1 + \eta_1 k_1 f''(k_1)$

$\mu L_2 = \lambda_2 u'(z_2)t_2 + \eta_2 k_2 \theta f''(k_2)$

$M(L_2) = [c_2 + u(z_2)] - [c_1 + u(z_1)]$

$\xi M'(L_2) = \mu(k_2 - k_1)$

$c_1 + z_1 = f(k_1) + (k - k_1)(f'(k_1) - t_1)$

$c_2 + z_2 = \theta f(k_2) + (k - k_2)(\theta f'(k_2) - t_2)$

$f'(k_1) - t_1 = \theta f'(k_2) - t_2$

$z_1 = \bar{\tau} + k_1 t_1$

$z_2 = \bar{\tau} + k_2 t_2$

Note that, now,

$$u'(z_1) = 1 + \frac{\eta_1}{\lambda_1} > 1$$

$$u'(z_2) = 1 + \frac{\eta_2}{\lambda_2} > 1$$

and then the planner is no longer following the standard Samuelson rule; compared to the first best, he is underproviding the public good in both regions.

Differentiating this system with respect to all the unknowns and after some tedious calculations, we get the following derivatives

$$\frac{dt_1}{d\bar{\tau}} = \frac{F_1}{F} < 0$$

$$\frac{dt_2}{d\bar{\tau}} = \frac{F_2}{F} < 0$$

$$\frac{dt_1}{d\bar{\tau}} - \frac{dt_2}{d\bar{\tau}} = \frac{F_1 - F_2}{F} < 0$$

$$\frac{dk_1}{d\bar{\tau}} = \frac{E_1}{D_1 F} > 0$$

$$\frac{dk_2}{d\bar{\tau}} = \frac{E_2}{D_2 F}$$

$$\frac{dz_1}{d\bar{\tau}} = \frac{F + k_1 F_1}{F} > 0$$

$$\frac{dz_2}{d\bar{\tau}} = \frac{F + k_2 F_2}{F}$$

$$\frac{dL_2}{d\bar{\tau}} = -\frac{1}{k_2 - k_1}\left(\frac{L_1 E_1}{D_1 F} + \frac{L_2 E_2}{D_2 F}\right)$$

With :

$$A_1 = M'(L_2) k_1 L_1 u''(z_1) < 0$$

$$A_2 = M'(L_2) k_2 L_2 u''(z_2) < 0$$

$$B_1 = (k_2 - k_1)^2 k_1 f''(k_1) u''(z_1) > 0$$

$$B_2 = (k_2 - k_1)^2 k_2 \theta f''(k_2) u''(z_2) > 0$$

$$D_1 = -(k_2 - k_1)^2 f''(k_1) \theta f''(k_2) + L_2 M'(L_2) f''(k_1) + L_1 M'(L_2) \theta f''(k_2) < 0$$

$$D_2 = -(k_2 - k_1)^2 f''(k_1) \theta f''(k_2) + L_2 M'(L_2) f''(k_1) + L_1 M'(L_2) \theta f''(k_2) < 0$$

$$F = \left[k_1 k_2 (B_1 B_2 - A_1 B_2 - A_2 B_1) - \big(k_1 (A_1 - B_1) + k_2 (A_2 - B_2)\big)(k_2 - k_1)^2 + (k_2 - k_1)^4\right] > 0$$

$$F_1 = k_2 (A_1 B_2 + A_2 B_1 - B_1 B_2) - (k_2 - k_1)^2 (B_1 - A_1 - A_2) < 0$$

$$F_2 = k_1 (A_1 B_2 + A_2 B_1 - B_1 B_2) - (k_2 - k_1)^2 (B_2 - A_1 - A_2) < 0$$

$$F_1 - F_2 < 0$$

$$E_1 = L_2 M'(L_2)(F_1 - F_2) - (k_2 - k_1)^2 \theta f''(k_2) F_1 < 0$$

$E_2 = L_1 M'(L_2)(F_1 - F_2) - (k_2 - k_1)^2 f''(k_1) F_2$

Appendix 2.6

We will work in two stages. First, we look at the optimal choice of z_1, z_2, c_1, c_2, k_1, k_2 for L_2 given, hence population-dependent welfare function $W(L_2)$. Then, we look at the first and second order conditions for the maximization of $W(L_2)$.

The problem is similar to Appendix 2.1. L_2 being given, the planner maximizes W with respect to z_1, z_2, c_1, c_2, k_1, k_2, under the following constraints:

$L_1\theta_1 f(k_1) + L_2\theta_2 f(k_2) = L_1(c_1 + z_1) + L_2(c_2 + z_2)$

$L_1 k_1 + L_2 k_2 = k$

hence the Lagrangian:

$\Upsilon_8 = L_1\big(c_1 + u(z_1)\big) + L_2\big(c_2 + u(z_2)\big) - \Omega(L_2) + \lambda[L_1\theta_1 f(k_1) + L_2\theta_2 f(k_2) - L_1(c_1 + z_1) - L_2(c_2 + z_2)] - \xi[L_1 k_1 + L_2 k_2 - k]$

Differentiating the Lagrangian with respect to z_1, z_2, c_1, c_2, k_1, k_2, and rearranging, we get the following first order conditions:

$\theta_1 f'(k_1) = \theta_2 f'(k_2)$

$u'(z_1) = u'(z_2) = 1$

We again find the equality of marginal productivities and the Samuelson rule, the later implying $z_2 = z_1 = z$ with $u'(z) = 1$.

The solution to this problem leads to a solution correspondence, $W(L_2)$. Then, we maximize $W(L_2)$. Knowing that $\theta_1 = \theta(L_1) = \theta(1 - L_2)$ and $\theta_2 = \theta(L_2)$ and using the envelope theorem, we get the first order condition:

$$\frac{\partial W(L_2)}{\partial L_2} = \frac{\partial Y_8}{\partial L_2} = c_2 + u(z_2) - c_1 - u(z_1) + \lambda[L_2\theta'(L_2)f(k_2) + \theta(L_2)f(k_2) - c_2 - z_2 - L_1 f(k_1)\theta'(L_1) - \theta(L_1)f(k_1) + c_1 + z_1] - \xi(k_2 - k_1) - M(L_2) = 0$$

which simplifies to:

$$M(L_2) = \theta_2[f(k_2) - k_2 f'(k_2)] - \theta_1[f(k_1) - k_1 f'(k_1)] + [L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)]$$

Where $\theta'_1 = \theta'(L_1)$ and $\theta'_2 = \theta'(L_2)$

Let us note that the symmetric outcome, $L_1 = L_2 = 0.5$, which implies $\theta_1 = \theta_2 = \theta(0.5)$, $\theta'_1 = \theta'_2 = \theta'(0.5)$ and $k_1 = k_2 = k$, obviously meets this first order condition.

Because the function $\theta(L)$ is not concave, we have to beware of the second order condition. Differentiating $\frac{\partial W(L_2)}{\partial L_2}$ with respect to L_2, we get:

$$\frac{\partial^2 W}{\partial {L_2}^2} = \frac{\partial^2 Y_8}{\partial {L_2}^2} = \theta'_2[f(k_2) - k_2 f'(k_2)] + \theta_2[f'(k_2) - f'(k_2) - k_2 f''(k_2)]\frac{dk_2}{dL_2} + \theta'_1[f(k_1) - k_1 f'(k_1)] - \theta_1[f'(k_1) - f'(k_1) - k_1 f''(k_1)]\frac{dk_1}{dL_2} + \theta'_2 f(k_2) + L_2\theta''_2 f(k_2) + L_2\theta'_2 f'(k_2)\frac{dk_2}{dL_2} + \theta'_1 f(k_1) + L_1\theta''_1 f(k_1) - L_1\theta'_1 f'(k_1)\frac{dk_1}{dL_2} - M'(L_2)$$

And then, using the solution to the first order conditions:

$\frac{\partial^2 W}{\partial {L_2}^2} = 2\theta'_2 f(k_2) - k_2\theta'_2 f'(k_2) + L_2\theta''_2 f(k_2) + 2\theta'_1 f(k_1) - k_1\theta'_1 f'(k_1) +$

$L_1\theta''_1 f(k_1) - M'(L_2) +$

$[L_2\theta'_2 f'(k_2) - \theta_2 k_2 f''(k_2)]\frac{\theta_2 f''(k_2)(k_1-k_2)+L_2[\theta'_2 f'(k_2)+\theta'_1 f'(k_1)]}{L_2\theta_1 f''(k_1)+L_1\theta_2 f''(k_2)}$

$+[\theta_1 k_1 f''(k_1) - L_1\theta'_1 f'(k_1)]\frac{\theta_1 f''(k_1)(k_1-k_2)-L_1[\theta'_2 f'(k_2)+\theta'_1 f'(k_1)]}{L_2\theta_1 f''(k_1)+L_1\theta_2 f''(k_2)}$

At the symmetric outcome, $L_1 = L_2 = 0.5$, $\theta_1 = \theta_2 = \theta(0.5)$, $\theta'_1 = \theta'_2 = \theta'(0.5)$ and $k_1 = k_2 = k$, and assuming that $M'(0.5) = 0$, this derivative simplifies to:

$$\frac{\partial^2 W}{\partial {L_2}^2} = 4\theta' f(k) + \theta'' f(k) \ + \left(\frac{\theta' f'(k)}{\theta'' f(k)} - 4k\right)\theta' f'(k)$$

Where $\theta = \theta(0.5)$, $\theta' = \theta'(0.5)$ and $\theta" = \theta"(0.5)$. This expression may also be written as

$$\frac{\partial^2 W}{\partial {L_2}^2} = \theta' f(k)\left[4 + \frac{\theta''}{\theta'} + \left(\frac{\theta' f'(k)}{\theta'' f(k)} - 4k\right)\frac{f'(k)}{f(k)}\right]$$

$$= \theta' f(k)\left[4 + 2A + \frac{\eta^2}{2Ak^2} - 4\eta\right] = \frac{2\theta' f(k)}{A}\left[A^2 + 2(1-\eta)A + \frac{\eta^2}{4k^2}\right]$$

where $A = L_i\frac{\theta''}{\theta'} = 0.5\frac{\theta''}{\theta'}$ is the elasticity with respect to population of the marginal increase in the agglomeration externality and $\eta = \frac{kf'(k)}{f(k)}$ is the elasticity of production function with respect to capital.

The ratio $\frac{2\theta' f(k)}{A}$ being positive, $\frac{\partial^2 W}{\partial {L_2}^2}$ has the same sign as the quadratic polynomial $A^2 + 2(1-\eta)A + \frac{\eta^2}{4k^2}$. The discriminant of this polynomial is

$$\Delta^2 = 4(\eta-1)^2 - \frac{\eta^2}{k^2} = \left[\left(2+\frac{1}{k}\right)\eta - 2\right]\left[\left(2-\frac{1}{k}\right)\eta - 2\right]$$

Two cases must be distinguished:

- When $\Delta^2 < 0 \Leftrightarrow \eta \in [\frac{2k}{2k+1}, \frac{2k}{2k-1}]$, $\frac{\partial^2 W}{\partial {L_2}^2} > 0$ whatever A.
- When $\Delta^2 > 0 \Leftrightarrow \eta \notin [\frac{2k}{2k+1}, \frac{2k}{2k-1}]$, $\frac{\partial^2 W}{\partial {L_2}^2} \leq 0$ for $A \in [A_1, A_2]$ and $\frac{\partial^2 W}{\partial {L_2}^2} > 0$ for $A \notin [A_1, A_2]$, with $A_1 = 2(\eta - 1) - \Delta$ and $A_2 = 2(\eta - 1) + \Delta$

Then, as soon as the elasticity with respect to population of the marginal increase in the agglomeration externality, A, is high enough ($A > A_2$), we are sure that $\frac{\partial^2 W}{\partial {L_2}^2} > 0$ whatever η, and then the symmetric outcome is a local minimum. The first best outcome is necessarily asymmetric.

Appendix 2.7

We look at the case of an asymmetric first best outcome; without loss of generality, we can assume that the largest (and most productive) region is region 2: $L_2 > 0.5$.

We know from Appendix 2.6 that, at the first best outcome, the following first order conditions must be met:

$$\theta_1 f'(k_1) = \theta_2 f'(k_2) \qquad (A7.1)$$

$$u'(z_1) = u'(z_2) = 1 \Longrightarrow z_1 = z_2 = z \quad with \;\; u'(z) = 1$$

$$M(L_2) = \theta_2[f(k_2) - k_2 f'(k_2)] - \theta_1[f(k_1) - k_1 f'(k_1)] + [L_2 \theta'_2 f(k_2) - L_1 \theta'_1 f(k_1)] \qquad (A7.2)$$

When the planner charges taxes t_1, t_2, τ_1 and τ_2, as in Appendix 2.2, at equilibrium, the following conditions must be met:

$$\rho = f'(k_1) - t_1 = \theta f'(k_2) - t_2 \qquad (A7.3)$$

$$M(L_2) = c_2 + u(z_2) - c_1 - u(z_1) \qquad (A7.4)$$

For the equilibrium to implement the first best optimum, (A7.1), (A7.2), (A7.3) and (A7.4) must hold simultaneously. (A7.1) and (A7.3) jointly imply $t_1 = t_2$. Knowing that at the first best optimum, $\mathrm{z}_1 = \mathrm{z}_2 = \mathrm{z}$, and using the expressions for consumption, (A7.4) becomes:

$$M(L_2) = c_2 - c_1 = \theta_2[f(k_2) - k_2 f'(k_2)] - \theta_1[f(k_1) - k_1 f'(k_1)] + \tau_1 - \tau_2$$

And then, using (A7.2):

$$\tau_1 - \tau_2 = L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)$$

Then, a policy implementing the first best is characterized by:

$$z_1 = z_2 = z \quad with \;\; u'(z) = 1$$

$$t_1 = t_2 = t$$

$$\tau_1 - \tau_2 = L_2\theta'_2 f(k_2) - L_1\theta'_1 f(k_1)$$

$$z = L_1\tau_1 + L_2\tau_2 + kt$$

Chapter 3 Fiscal competition with imperfect migration and asymmetric productivity

3.1 Introduction

In chapter 2, we looked at the single central planner case and found that, as soon as the planner taxes capital, at the first best optimum there is an implicit public transfer from the most productive jurisdiction to the less productive one. In this chapter, we look at what happens when governance is decentralized, each region being governed by a local planner, so that there is fiscal competition: each local government makes its choice taking account of the choices made by the other government and the mobility of fiscal assets.

Standard fiscal competition models have been widely studied ever since 1980s. Most of them share two bass characteristics. First, capital is a useful resource freely moving across countries while labor is immobile. Second, all the jurisdictions share the same basic characteristics, and particularly the same technologies.

In this chapter, we relax both these assumptions: as in Chapter 2, workers are imperfectly mobile and the production of the private good is more efficient in one of the jurisdictions compared to the other. The difference with Chapter 2 is that, instead of a central planner managing both jurisdictions, we have two independent authorities, one for each jurisdiction, instead of the single central planner. Then, the objective of this chapter is twofold. First, it is an extension of Chapter 2, where we look at the consequences of fiscal decentralization in an economy where population is mobile between two jurisdictions differing in their productivity level; these consequences are examined both when the jurisdictions can tax both capital and workers and when they are restricted. Second, it is an extension of the standard fiscal competition, where we relax the assumptions of households' immobility and identical technologies.

The structure of the paper is organized as follows. Section 2 is a brief review of the literature on fiscal and tax competition. Section 3 presents the model. The main results are presented in section 4. Then, in section 5, we look at an extension, where differences in productivity levels are endogenous generated by an agglomeration externality. Section 6 concludes.

3.2 Literature review

Since the seminal work of Tiebout (1956), the literature on the decentralized provision of local public goods has been strongly developing. Early contributions were neglecting fiscal interactions between decentralized jurisdictions. The analysis of these interactions started with Zodrow and Mieszkowski (1986) and Wilson (1986), further developed by Wildasin (1988) and many other scholars.

The standard model developed by the literature has a set of decentralized jurisdictions with immobile households, perfectly mobile capital, a private public good produced under constant returns combining capital and labor, and benevolent local planners who use capital taxation for providing a local public good. The perfect mobility of capital generates fiscal interactions across jurisdictions, as capital reacts to a tax increase in a jurisdiction flying out to other jurisdictions. These interactions are not internalized by the local planner, leading to the popular result that fiscal competition leads to under provision of local public goods.

The standard fiscal competition model is a symmetric one, with all the jurisdictions sharing the same underlying characteristics: same size, same technology for producing both the private and the public good, same preferences of the inhabitants.

There are very few papers dealing with asymmetries. The most well-known one is Wilson (1991), who considers differences in size and finds that the smallest jurisdictions charge a lower capital tax rate than the largest one, and then attracts more capital per worker. We do not know of any paper looking at the consequences of differences in productivity.

There is a much larger literature looking at fiscal interactions when population is mobile: Boadway (1982), Brown and Oates (1987), Wildasin (1991), Arthur, Richard and Emilson (2000), Bucovetsky (2010). An important point examined in this literature is the impact of population mobility on decentralized redistribution: when the population is mobile, rich households tend to move to locations where there is few redistribution whole poor households tends to move to locations where redistribution is high. The consequence is that decentralized redistribution becomes difficult or even impossible because jurisdictions hosting the poor population are not able to get the fiscal resources needed for redistribution. However, this literature neglects the mobility of capital and then is not looking at the simultaneous impact of capital and population mobility.

Wildasin emphasized in 2011 that difference in the degree of mobility of different types of factors of production matter. It is we are looking at in this paper, jointly with differences in productivity levels.

3.3 Basic Model

Our model has the same basic structure as in chapter 2. There are two regions, inhabited by a continuum of $L = 1$ inhabitants providing labor and consuming a private good and a publicly provided good. Private goods are produced by two primary factors: capital and labor. The production function has constant return to scale. Capital is perfectly mobile while labor is imperfectly mobile. Inhabitants in different jurisdictions have the same preferences for goods but different preferences for locations. Each inhabitant has a specific willingness to pay for residing in region 2 instead of region 1, which is randomly distributed in $[-\infty, +\infty]$.

Each worker inelastically supplies one unit of labor, hence a total quantity of labor equaling $L = 1$. There is a fixed stock of perfectly mobile capital K, choosing to locate in the region where its return is higher. The property of capital is evenly shared over the whole population. Public goods are financed by the collection of tax on both capital and inhabitants settled in the local region.

There are L_i workers staying in jurisdiction i, with $L_1 + L_2 = L = 1$. The net income of a worker staying in jurisdiction i is $w_i + \rho k - \tau_i$, where w_i is the local wage, ρ the after tax rate of return to capital, k is the fixed amount of capital and τ_i the tax paid by households in jurisdiction i. This income is used for consumption of a private monetary good in jurisdiction i.

Workers staying in jurisdiction i benefit from consumption of a publicly provided good. This good is perfectly divisible and the quantity per worker is z_i. Workers are endowed with preferences represented by the following utility function (for worker *l*): $V_i = U(c_i, z_i) + m_{i,l}$,where $m_{1,l} = \mu_l$, $m_{2,l} = 0$ and $U(c_i, z_i) = c_i + u(z_i)$. ($\mu_l$ is the willingness to pay of worker l for staying in jurisdiction 1 instead of jurisdiction 2.

Workers are mobile across jurisdictions and choose the location where their utility is highest. Then, the choice of an individual whose willingness to pay μ is:

$$i = 1 \text{ iff } U(c_1, z_1) + \mu > U(c_2, z_2) \Leftrightarrow \mu > \Delta U$$

$$i = 2 \text{ iff } U(c_1, z_1) + \mu < U(c_2, z_2) \Leftrightarrow \mu < \Delta U$$

where $\Delta U = U(c_2, z_2) - U(c_1, z_1)$. Therefore, $L_2 = \Lambda(\Delta U)$ workers choose jurisdiction 2 while $L_1 = 1 - \Lambda(\Delta U)$ workers choose jurisdiction 1. The function Λ is increasing from $\mathbb{R}$ to $[0,1]$, with $\Lambda(-\infty) = 0$, $\Lambda(0) = 0.5$ and $\Lambda(+\infty) = 1$. The assumption $\Lambda(0) = 0.5$ implies that half the workers prefer to leave in jurisdiction 1 in the sense that their willingness to pay for staying in jurisdiction 1 instead of jurisdiction 2 is positive; and half the workers prefer to leave in jurisdiction 2 in the sense that their willingness to pay for staying in jurisdiction 1 instead of jurisdiction 2 is negative

The inverse function of Λ is $\Delta U = M(L_2)$, where $M(L_2)$ may be interpreted as the migration cost of the marginal migrant (who is indifferent between the two jurisdictions) when there are L_2 on jurisdiction 2. The assumptions we make on Λ imply that M is an increasing function from $[0,1]$ to $\mathbb{R}$, with $M(0) = -\infty$, $M(0.5) = 0$ and $M(1) = \infty$.

The private good is produced by firms combining labor and capital using a constant returns to scale technology. In jurisdiction i, the production function is

$$F_{\mathrm{i}}(K_i, L_i) = \theta_i\, F(K_i, L_i) = \theta_i\, L_i f(k_i)$$

where K_i is the capital input, L_i is the labor input, $k_i = K_i/L_i$, $F(K_i, L_i)$ is an homogenous production function while $f(k_i)$ is a concave increasing function meeting the Inada conditions; θ_i is an efficiency parameter, which may differ across regions. Without loss of generality, we assume that $\theta_1 = 1$ and that production is more efficient in jurisdiction 2, so that $\theta_2 = \theta > \theta_1 = 1$.

There is a fixed capital stock of capital available for production in both jurisdictions, K, so that

$$K_1 + K_2 = L_1 k_1 + L_2 k_2 = K$$

Capital is perfectly mobile across jurisdictions and then the post-tax return to capital, ρ, is the same in both jurisdictions:

$$\rho = f'(k_1) - t_1 = \theta f'(k_2) - t_2$$

where t_i is the tax rate on capital in jurisdiction *i*.

In each jurisdiction, the local public good is provided by a local government out of taxes on both capital and households. The public budget constraint of government *i* is

$$z_i = \tau_i + k_i t_i$$

where τ_i is the tax paid by each household and t_i the tax rate charged on capital invested in the jurisdiction.

Each local government only takes account of the welfare of its "natives". Natives are households who started leaving in the jurisdiction, this starting period leading them to prefer leaving there. Then, natives from region 1 are households who have a negative willingness to pay for staying in region 2 instead of region 1, while natives from region 2 are households who have a positive willingness to pay for staying in region 2 instead of region 1. Assumptions made earlier on about the distribution of willingness to pay imply that each jurisdiction has 0.5 natives.

In the standard situation we will be working on, production is more efficient in region 2 than in region 1 ($\theta > 1$) and then attracts more workers ($L_2 > 0.5$), so that there are $L_2 - 0.5$ migrants, *id est* natives from region 1 who choose to work in region 2. Then, taking account of migration costs, the aggregate welfare of the natives from region 1 is

$$W_1 = L_1[c_1 + u(z_1)] + (0.5 - L_1)[c_2 + u(z_2)] - \Omega(L_2)$$

where[2]

$$\Omega(L_2) = \int_{0,5}^{L_2} M(l)dl$$

is the aggregate migration cost of the migrants from region 1 to region 1.

Conversely, all the natives from region 2 work in region 2, so that they do not bear any migration cost and their aggregate welfare is

$$W_2 = 0.5[c_2 + u(z_2)]$$

Note that, by construction, the aggregate welfare of the whole economy is the sum of the aggregate welfares of both categories of natives, $W = W_1 + W_2$, where W is the global welfare function maximized by the central planner in Chapter 2.

3.4 Equilibrium

This section is devoted to the analysis of the Nash equilibria of a standard fiscal competition between the two jurisdictions. We will compare these equilibria with the optimal outcomes presented in the previous chapters.

3.4.1 Equilibrium when all the tax instruments are available

In each jurisdiction, the local government has to choose three quantities: the per-capita tax on workers, the tax rate on capital, and the quantity of local public good to be provided. These three quantities are linked to each other by the public budget constraint, and then only two of them can be used as strategic instruments. Here, we

[2] See Chapter 2, section 2.3.4 for the derivation of this expression.

focus on the case where the strategic instruments are the quantity of local public good, z_i, provided by the jurisdiction and the tax on capital, t_i, the level of the tax paid by households being determined by the local public budget constraint. Then, the strategy of government *i* is characterized by the package (t_i, z_i)

The equilibrium with perfect mobility of capital and imperfect mobility of households is determined by the following systems of equations:

$$c_1 = f(k_1) - k_1 f'(k_1) + \rho k - \tau_1 \tag{3.1}$$

$$c_2 = \theta f(k_2) - k_2 \theta f'(k_2) + \rho k - \tau_2 \tag{3.2}$$

$$M(L_2) = c_2 + u(z_2) - c_1 - u(z_1) \tag{3.3}$$

$$\rho = f'(k_1) - t_1 \tag{3.4}$$

$$\rho = \theta f'(k_2) - t_2 \tag{3.5}$$

$$z_1 = \tau_1 + k_1 t_1 \tag{3.6}$$

$$z_2 = \tau_2 + k_2 t_2 \tag{3.7}$$

$$(1 - L_2)k_1 + L_2 k_2 = k \tag{3.8}$$

Equations (3.1) and (3.2) are the private budget constraints. Equation (3.3) tells that the marginal migrant is indifferent between residing in either region. Equations (3.4) and (3.5) result from equalization of post-tax capital returns across jurisdictions. Equations (3.6) and (3.7) are the public budget constraints. Equation (3.8) is the constraint imposed by the fixed stock of capital.

This system has eight equations and 12 unknowns: Once all the four tax levels (t_1, t_2, τ_1 and τ_2) are known, the other equilibrium quantities are determined by the following system of 8 equations with 8 unknowns (c_1, k_1, z_1, t_1, τ_1, c_2, k_2, z_2, t_2, τ_2, L_2, and ρ). Then, once both capital tax rates (t_1 and t_2) and quantities of local public goods (z_1 and z_2) are known, the system determines all the other equilibrium quantities. Then, for determining the best reply of jurisdiction 1 (resp jurisdiction 2) to the package (t_2, z_2) (resp (t_1, z_1)) choosen by jurisdiction 2 (resp jurisdiction 1), we solve (2.1) to (2.8) for t_2 and z_2 (resp t_1 and z_1) given, adding two equations resulting from the optimization of its welfare function by jurisdiction 1 (resp jurisdiction 2).

The best reply of jurisdiction 1 to the package (t_2, z_2) chosen by jurisdiction 2 is the package (t_1, z_1) maximizing the jurisdiction's welfare function:

$$W_1 = L_1[c_1 + u(z_1)] + (0.5 - L_1)[c_2 + u(z_2)] - \Omega(L_2)$$

under the constraints (3.1) to (3.8).

Similarly, the best reply of jurisdiction 2 to the package (t_1, z_1) chosen by jurisdiction 1 is the package (t_2, z_2) maximizing the jurisdiction's welfare function:

$$W_2 = 0.5[c_2 + u(z_2)]$$

under the same constraints, (3.1) to (3.8).

Let us first look at the best reply by jurisdiction 1. We write z_{opt} for the first best quantity of public good, ($u'(z_{opt}) = 1$). In Appendix 3.1, we prove the following proposition:

Proposition 3.1: *At the best reply by jurisdiction 1, the values of* c_1, k_1, τ_1, t_1, c_2, k_2, τ_2, L_2, *and* ρ *solve the equations* (3.1) *to* (3.8) *together with the equality*

$$z_1 = z_{opt}$$

and the equation:

$$t_1 = \frac{L_1 M'(L_2)[L_1(k_2 - k_1)\theta f''(k_2) + (L_2 - L_1)u'(z_2)t_2]}{[u'(z_2)t_2 - L_1(k_2 - k_1)\theta f''(k_2)](k_2 - k_1) + 2L_1 L_2 M'(L_2)}$$

Let us now look at the best reply by jurisdiction 2. In Appendix 3.2, we prove the following proposition:

Proposition 3.2: *At the best reply by jurisdiction 2, the values of* $c_1,\ k_1,\ z_1,\ c_2,\ k_2,\ t_2,\ \tau_2,\ L_2$, and ρ *solve the equations* (3.1) *to* (3.8) *together with the equality*

$$z_2 = z_{opt}$$

and the equation:

$$t_2 = \frac{L_1 L_2 M'(L_2)(k_2 - k_1)f''(k_1)}{(k_2 - k_1)[L_2(k_2 - k_1)f''(k_1) + u'(z_1)t_1] - L_1 M'(L_2)}$$

A direct consequence of Propositions 3.1 and 3.2 is that, at Nash equilibrium, both jurisdictions choose to provide the first best quantity of local public good. This is a direct consequence of the fact that they are unrestricted in their ability to tax households, who are the only agents who benefit from the local public good. Then, the jurisdictions are able to compare the marginal utility of an extra unit of local public good and its cost using households' taxation, leading them to meet the Samuelson rule.

What about taxation at the Nash equilibrium? It is impossible to have direct evidence combining Propositions 3.1 and 3.2. However, we can have indirect evidence looking at the incentives to change the capital tax rate when the economy is at its first best outcome. Let us remind that, from Chapter 2, we know that the first best outcome can be implemented taxing the households only for providing the local public good:

$u'(z_1) = u'(z_2) = 1$, $t_1 = t_2 = 0$, and then $\tau_1 = \tau_2 = z_{opt}$. Using the reduced welfare functions[3] $\bar{W}_1(t_1)$ and $\bar{W}_2(t_2)$, these incentives can be determined looking at the signs of the derivatives $d\bar{W}_1/dt_1$ and $d\bar{W}_2/dt_2$. In Appendix 3.3, we prove the following proposition:

Proposition 3.3: *At the first best outcome* ($u'(z_1) = u'(z_2) = 1$, $t_1 = t_2 = 0$, *and then* $\tau_1 = \tau_2 = z_{opt}$), *jurisdiction 1 has an incentive to subsidize capital* ($d\bar{W}_1/dt_1 < 0$) *while jurisdiction 2 has an incentive to tax capital* ($d\bar{W}_2/dt_2 > 0$). *Following these incentives leads to a decrease in the difference in capital per worker ratios,* $k_2 - k_1$. *However, this incentive cannot no lead to a lower capital per worker ratio in jurisdiction 2 compared to jurisdiction 1.*

Then, jurisdiction 1 (the less productive region) has an incentive to subsidize capital while jurisdiction 2 (the most productive region) has an incentive to tax capital. Where this difference in behavior does come from?

Let us remind that, whatever happens, both jurisdictions choose to provide the same quantity of local public good, the first best one. Then, expenditures on the local public good are fixed and changing the level of the tax on capital implies changing the repartition of the budget between tax receipts (or expenditures) from the capital tax and tax receipts from the households. In this context, increasing the capital tax rate has two effects. The direct and obvious one is that the local post-tax rate of return to capital decreases, leading capital to flow out from the jurisdiction until returns are equalized

[3] $\bar{W}_1(t_1)$ is the welfare level of jurisdiction 1 when it provides the optimal quantity of local public good ($z_1 = z_{opt}$) and taxes capital at the rate t_1. Similarly, $\bar{W}_2(t_2)$ is the welfare level of jurisdiction 1 when it provides the optimal quantity of local public good ($z_2 = z_{opt}$) and taxes capital at the rate t_2.

across regions. In the expression of the derivative of the welfare function with respect to the tax rate[4], $d\overline{W}_i/dt_i = \lambda_{i,i}k_i - \mu_{i,i}$, this effect is measured by the second term, $-\mu_{i,i}$, where $\mu_{i,i}$ is the Lagrange multiplier of the constraint on post-tax returns to capital.

There is however an indirect and less obvious effect. If the increase in the tax rate does not generate too large a capital flow out of the jurisdiction, it allows the local government to decrease the tax on households. Then, the private consumption of local households evolves under the influence of two forces: the decrease generated by the capital outflow depressing wages and the increase generated by the lower tax charged by the local government. The net effect is measured by the first term of $d\overline{W}_i/dt_i$, $\lambda_{i,i}k_i$, where $\lambda_{i,i}$ is the multiplier of the private budget constraint; it is positive. Note that both effects are influenced by the mobility of households, who react to the changes in welfare by migrating, leading to a reduction of the tax base in the jurisdiction they leave and an enlargement of the same tax base in the jurisdiction they enter.

The difference of behavior between jurisdictions depends upon the respective impacts of both effects. In the less productive jurisdiction (jurisdiction 1), the detrimental effect of increased capital taxation dominates its beneficial effect. Or, the other way round, subsidizing capital attracts enough capital for the wages of local workers to increase more than the extra amount of capital they pay for covering these subsidies, so that subsidizing capital is attractive. In the most productive jurisdiction, it is the opposite. The detrimental effect of increasing the tax on capital is dominated by its beneficial effect and then taxation is attractive.

[4] See appendices 1 and 2.

Property 3 does not imply that, at Nash equilibrium, jurisdiction 1 subsidizes capital while jurisdiction 2 taxes capital. The reason is simple: jurisdiction 1 subsidizes capital for attracting more capital. However, what mainly matters for attracting capital is the tax differential between jurisdictions 1 and 2. If jurisdiction 2 follows its intensive, increasing its capital tax rate, the tax differential increases and then, with a larger tax differential, jurisdiction 1 has a lower incentive to decrease its tax rate. The final result may be positive tax rates in both jurisdictions, with a lower tax in jurisdiction 1 compared to jurisdiction 2.

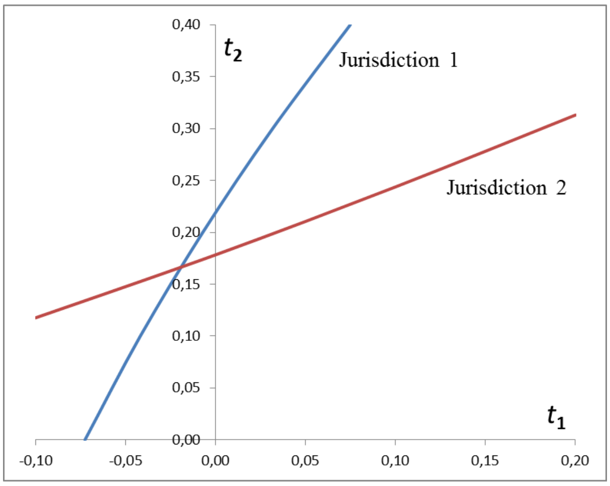
We can illustrate this point with some simulations. We choose the following simple specifications:

- For the utility of the local public good: $u(z) = z^{\alpha}$
- For the production function: $f(k) = k^{\beta}$
- For the migration function: $\gamma \ln \frac{L_2}{1-L_2}$

Figure 2 presents the best reply function of each jurisdiction for a standard configuration of the parameters[5]. The blue line is the best reply function of jurisdiction 1, while the red line is the best reply function of jurisdiction 2. As expected from Property 3, jurisdiction 1 choosing a negative tax rate when jurisdiction 2 does not tax capital, the best reply of jurisdiction 1 intersects the horizontal axis on the left hand side; and jurisdiction 2 choosing a positive tax rate when jurisdiction 1 does not tax capital, the best reply of jurisdiction 2 intersects the vertical axis above the origin.

[5] For this simulation, the values of the parameters are $\alpha = 0.5$, $\beta = 1/3$, $\gamma = 2$. The total population is $L = 1$ and the global capital stock is $k = 1$.

Figure 2: Response functions, base scenario

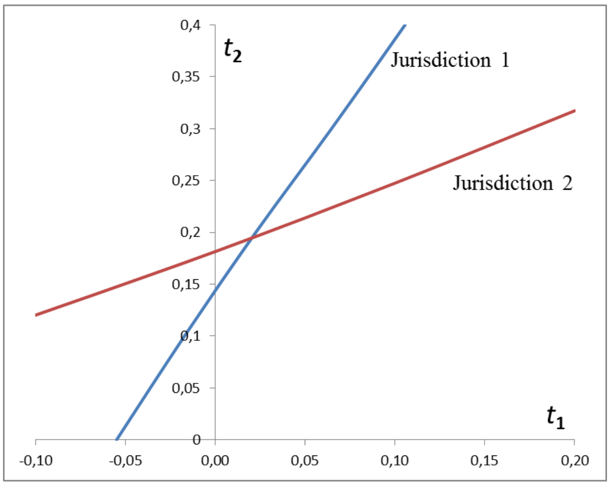


As expected (despite a complete formal proof is not available), both best reply functions are upward sloping: when one of the jurisdictions increases its capital tax rate, capital flows toward the other jurisdiction, allowing it to increase its tax rate. In Figure 2, the Nash equilibrium corresponds to a negative tax rate by jurisdiction 1 and a positive tax rate by jurisdiction 2. Then, the incentives for subsidizing capital in the less productive jurisdiction and taxing it the most productive jurisdiction are still at work.

Figure 3 displays the response functions when the population is more mobile[6]. There is almost no change in the best reply by jurisdiction 2 (in fact a very small downward shift), while the best reply by jurisdiction 1 has shifted to the right.

[6] The only change with respect to Figure 2 is the value of γ, which changes from $\gamma = 2$ to $\gamma = 1$. A lower value of γ corresponds to lower migration costs and then to a higher mobility of population. In the limit, when $\gamma = 0$, there are no migration costs and the population is perfectly mobile. And, when $\gamma = \infty$, migration costs are so high that the population is perfectly immobile.

Figure 3: Response functions, higher mobility



This shift is easy to understand. If jurisdiction 1 decreases its tax rate for attracting capital, as the quantity of local public good stays at its optimal level, it has to increase the tax paid by household and then to depress there private consumption. If households are more mobile, more households migrate to region 2, generating a higher welfare loss due to aggregate migration costs. Then, the welfare gain from a capital subsidy (or a low tax rate on capital) is lower and the jurisdiction chooses a higher tax rate on capital than when the population is perfectly mobile.

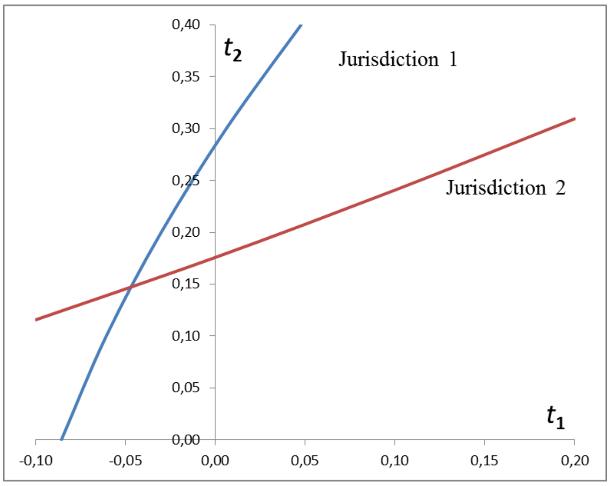
The straightforward consequence of this shift is that the Nash equilibrium moves to the right and, at Nash equilibrium, both taxes may be positive, as it is the case in Figure 3.

Conversely, a decrease in households' mobility increases the incentives of jurisdiction 1 to decrease its capital tax rate, as a less mobile population generates lower welfare losses due to migration. Figure 4 illustrates this situation[7]. The lower mobility of households generates a leftward shift of the best reply by jurisdiction 1 with the

[7] The only change with respect to Figures 2 and 3 is again the value of γ, which is now $\gamma = 5$.

consequence that the Nash equilibrium is more asymmetric than is the base scenario of figure 2, with a higher subsidy rate by jurisdiction 1 and a lower subsidy rate by jurisdiction 2.

Figure 4: Response functions, lower mobility



This impact of workers mobility clearly appears when we compare the two extreme situations, perfect mobility and perfect immobility. Workers are perfectly mobile when there are no migration costs: $M(L_2) = M'(L_2) = 0$, whatever L_2. Conversely, whorkers are perfectly immobile when migration costs are infinite. Therefore, as soon as L_2 differs from 0.5, $M(L_2)$ is infinity. More precisely, $M(L_2) = -\infty$ for $L_2 < 0.5$, $M(L_2) = +\infty$ for $L_2 > 0.5$, and $M'(L_2) = +\infty$ for $L_2 = 0.5$.

Straightforwardly, when $M(L_2) = M'(L_2) = 0$, Properties 1 and 2 imply $t_1 = t_2 = 0$. With perfectly mobile workers, the incentives to tax or subsidize disappear and the first best outcome is a Nash equilibrium. It is interesting to note that we are in a situation where both fiscal assets are perfectly mobile, but one of them only is taxed.

Conversely, when workers are perfectly immobile, $L_2 = 0.5$ whatever the difference in consumption level, $c_2 - c_1$ and $M'(0.5) = +\infty$. At $t_1 = t_2 = 0$, the first order derivatives of the reduced welfare functions are:

$$\frac{d\bar{W}_1}{dt_1} = -\frac{1}{4}\frac{(k_2 - k_1)\theta f''(k_2)}{[\theta f''(k_2) + f''(k_1)]} < 0$$

$$\frac{d\bar{W}_2}{dt_2} = \frac{1}{4}\frac{(k_2 - k_1)f''(k_1)}{[\theta f''(k_2) + f''(k_1)]} > 0$$

So that, as in the general case, jurisdiction 1 has an incentive to subsidize capital and jurisdiction 2 has an incentive to tax it. These incentives are confirmed by the expressions of taxes at Nash equilibrium. Using Properties 1 and 2, we find that, when $L_2 = 0.5$ and, $M'(0.5) = +\infty$:

$$t_1 = \frac{(k_2 - k_1)\theta f''(k_2)}{2} < 0$$

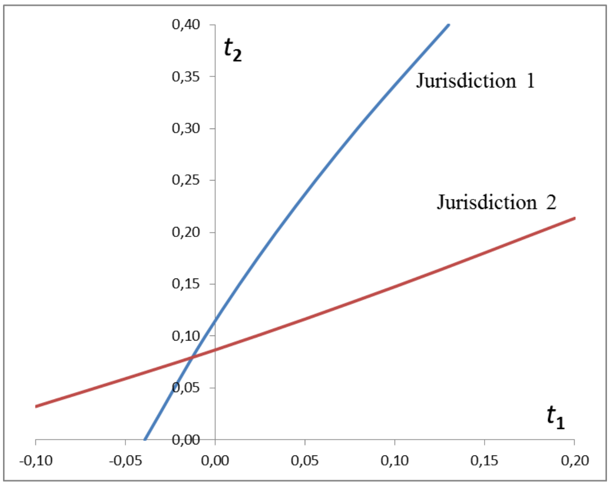
$$t_2 = -\frac{(k_2 - k_1)f''(k_1)}{2} > 0$$

Then, the incentives to subsidize capital in the less productive jurisdiction and to tax it in the most productive jurisdiction look to be highest when workers are perfectly mobile. However, some asymmetry between jurisdictions must exist.

This observation leads us to examine the role of productive asymmetries between jurisdictions. Clearly, some asymmetry is needed for jurisdictions to have incentives for taxing or subsidizing capital. More precisely, in the limiting case where there is no difference in productivity levels, $\theta = 1$, both regions are perfectly similar, and we are led to look at a symmetric equilibrium, with $t_1 = t_2$. But, when, $\theta = 1$, $t_1 = t_2$ implies $f'(k_1) = f'(k_2)$ and then $k_1 = k_2$. Now, looking at the first best outcome ($t_1 = t_2 = 0$), $k_1 = k_2$ implies that the derivatives of the reduced welfare functions are zero: $d\bar{W}_1/dt_1 = d\bar{W}_2/dt_2 = 0$. Then, when there is no difference in productivity, the incentives to tax or subsidize capital disappear and the first best is still an optimum.

Then, we can expect smaller differences in productivity levels to lead to tax levels closer to zero in both jurisdictions, and closer to each other. Figure 5 illustrates this point.

Figure 5: Response functions, small difference in productivity levels



In the scenario used for building Figure 5, the productivity differential is much lower[8] than in the base scenario of Figure 2. The consequence is a rightward shift of the best reply curve of jurisdiction 1, a backward shift of the best reply curve of jurisdiction 2, and Nash equilibrium where both tax rates are much closer to zero.

Conversely, for large productivity differentials, we expect much higher capital tax rates in the most productive region. The productive advantage of region 2 results in an agglomeration rent of the Baldwin and Krugman type (see Baldwin and Krugman, 2004). When this productive advantage is very large, capital is reluctant to

[8] More precisely, θ has decreased from $\theta = 2$ to $\theta = 1.5$, all the other parameters being unchanged. Let us remind that there is no productivity differential when $\theta = 1$.

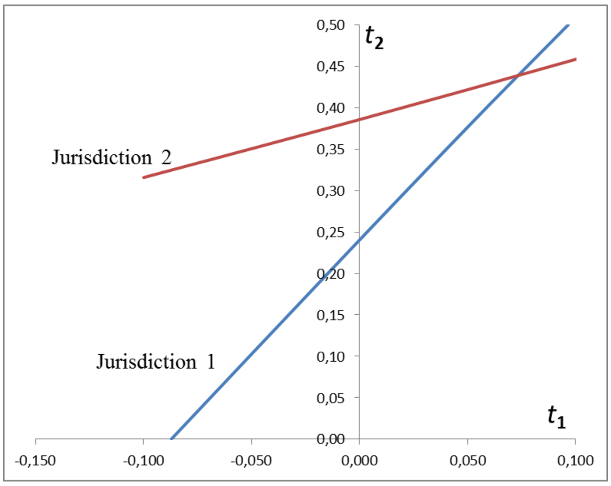
fly out of the jurisdiction and then less mobile. Then, it accepts higher tax levels, which allows the jurisdiction to set higher capital taxes and results in an upward shift of its best reply curve.

The consequences of a larger productivity differential are more ambiguous for the less productive jurisdiction. Jurisdiction needs to compensate its productive disadvantage with a high tax differential: the levels of its capital tax must be much lower than the level chosen by jurisdiction 2 for jurisdiction 1 to be able to attract capital. The consequence is a leftward shift of its best reply function. But this leftward shift does not necessarily lead jurisdiction 1 to subsidize capital, for a simple reason: when the tax level is very high in jurisdiction 2, even a positive tax in jurisdiction (but close enough to 0) may be enough for the tax differential to be large.

A formal proof of this conjecture is not available because the interactions between the variables lead to highly complex expressions of the derivatives of the best reply functions with respect to the parameters, which are not interpretable. However, they can be illustrated easily. In the scenario used for building Figure 6, the productivity differential is higher[9] than in the base scenario of Figure 2.

[9] More precisely, θ has increased from $\theta = 2$ to $\theta = 3$, all the other parameters being unchanged.

Figure 6: Response functions, large difference in productivity levels



As expected, the consequence is a leftward shift of the best reply curve of jurisdiction 1, and an upward shift of the best reply curve of jurisdiction 2. At Nash equilibrium, compared to Figure 2, the tax rate is much higher in jurisdiction 2 and much less negative in jurisdiction 1.

In the scenario used for building Figure 6, the productivity differential is much higher[10] than in the scenario of Figure 5.

3.4.2 Equilibrium when there are constraints on capital taxation

In this section, we briefly look at what happens when the choice of the tax rate on capital is constrained. It is natural to look at a constraint applying similarly to both jurisdictions ($t_1 = t_2 = \bar{t}$). Moreover, knowing that a central planner implements the first best with a zero tax rate on capital, a natural question is to examine how the decentralized jurisdictions react when they are not allowed to tax capital: $\bar{t} = 0$, and then $t_1 = t_2 = 0$.

[10] The value of θ is now $\theta = 4$, all the other parameters being unchanged.

Let us note that, in Appendices 1 and 2, the fact that at its best reply each jurisdiction provides the first best quantity of local public good ($z_i = z_{opt}$) has been proved for every choice of the capital tax rate and then this property still holds when the jurisdiction is constrained. This result has a strong implication: when jurisdictions are not allowed to tax capital, as soon as both jurisdictions are constrained, the equilibrium is determined by the system of equations (3.1) to (3.8) jointly with the equalities $t_1 = t_2 = \bar{t}$ and $z_1 = z_2 = z_{opt}$ (or, equivalently, $u'(z_1) = u'(z_2) = 1$). But, when $\bar{t} = 0$, the solution to this system of equations is a first best optimum. Then, it is possible to implement the first best by not allowing jurisdictions to tax capital.

Note that, for this implementation of the first best by a constraint on business taxation, the constraint must be an equality constraint. An equality constraint will not work. If jurisdictions are not allowed to subsidize capital ($t_1 \geq \bar{t} = 0$ and $t_2 \geq \bar{t} = 0$), jurisdiction 2 will never be constrained as, at Nash equilibrium, it always chooses a positive tax rate. And jurisdiction 1 will be constrained only when, at Nash equilibrium, it subsidizes capital. If jurisdictions are not allowed to tax capital ($t_1 \leq \bar{t} = 0$ and $t_2 \leq \bar{t} = 0$), the constraint can never be binding on both jurisdictions. This assertion is straightforward when, at unconstrained Nash equilibrium, jurisdiction 1 subsidizes capital, implying that it is unconstrained. However, even when at the unconstrained Nash equilibrium both jurisdiction tax capital, at the constrained Nash equilibrium, jurisdiction 1 subsidizes capital. This is a direct consequence of Proposition 3.3: when $t_1 = t_2 = 0$, jurisdiction 1 is better off subsidizing capital so that, at a constrained Nash equilibrium where the constraint $t_2 \leq 0$ is binding for jurisdiction 2, jurisdiction 1 chooses a negative tax rate.

3.4.3 Equilibrium when there are constraints on workers' taxation

Let us now examine what happens when jurisdictions are limited in their capacity to tax households. More precisely, we assume that there is an upward limit in the level of the household tax. As before, it is natural to look at a constraint applying similarly to both jurisdictions : $\tau_1 \leq \bar{\tau}$ and $\tau_2 \leq \bar{\tau}$.

A first consequence of the constraint is that a constrained jurisdiction only has one strategic instrument. When the constraint is binding, the tax on households is fixed by the constraint. Then, the tax on capital and the quantity of local public good are linked to each other by the budget constraint. We adopt the standard framework of fiscal competition models, choosing the tax on capital as the strategic instrument, the quantity of local public good resulting from the budget constraint.

Let us first look at the best reply by jurisdiction 1. In Appendix 3.4, we prove the following proposition:

Proposition 3.4: *When the constraint $\tau_1 \leq \bar{\tau}$ is binding ($\tau_1 = \bar{\tau}$), at its best reply, jurisdiction 1 sets $u'(z_1) > 1$ and then under provides the local public good. The values of c_1, k_1, z_1, t_1, c_2, k_2, z_2, L_2, and ρ solve the equations* (3.1) *to* (3.8) *together with the equation:*

$$t_1 = \frac{L_1 M'(L_2)}{u'(z_1)} \frac{(k_1+k_2-2u'(z_1)k_1)L_1\theta f"(k_2)+(L_2-L_1)u'(z_2)t_2}{2L_1L_2M'(L_2)-(k_2-k_1)[(k_2-k)\theta f"(k_2)-u'(z_2)t_2]} - \left(1-\frac{1}{u'(z_1)}\right) k_1 f"(k_1)$$

Let us now look at the best reply by jurisdiction 2. In Appendix 3.5, we prove the following proposition:

Proposition 3.5: *When the constraint $\tau_2 \leq \bar{\tau}$ is binding ($\tau_2 = \bar{\tau}$), at its best reply, jurisdiction* 2 *sets $u'(z_2) > 1$ and then under provides the local public good. The* *values of* c_1, k_1, z_1, c_2, k_2, z_2, t_2, L_2, *and* ρ *solve the equations* (3.1) *to* (3.8) *together with the equation:*

$$t_2 = \frac{L_2 M'(L_2)}{u'(z_2)} \frac{(u'(z_2)k_2 - k)f''(k_1)}{(k_2-k_1)[u'(z_1)t_1 + (k-k_1)f''(k_1)] - L_1 M'(L_2)} - \left(1 - \frac{1}{u'(z_2)}\right) k_2 \theta f''(k_2)$$

A straightforward consequence of Propositions 3.4 and 3.5 is that, as soon as they are constrained on the level of household's taxation, both jurisdictions underprovide the local public good. We are back to the standard outcome of fiscal competition. When jurisdictions are constrained on households' taxation, increasing the quantity of local public good provided by the jurisdiction is costly because it implies a higher tax level on capital and generates a capital fly toward the competing jurisdiction. Then, jurisdictions arbitrate between provision of the public good and taxation of capital.

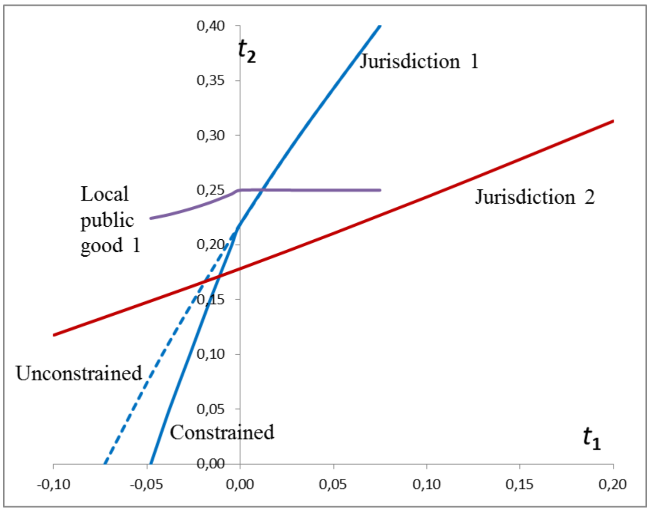
The consequences of the constraint depend upon its intensity. Let us note the capital tax rates at the unconstrained Nash equilibrium as t_1^U for jurisdiction 1 and t_2^U for jurisdiction 2. From the analysis of section 3.4.3, we know that $t_1^U < t_2^U$, where t_2^U is always positive and t_1^U may be negative. Moreover, we know that the quantity of capital per worker is higher in jurisdiction 2: $k_1 < k_2$. Then, $k_1 t_1^U < k_2 t_2^U$, so that $\tau_1^U = z_{\text{opt}} - k_1 t_1^U > z_{\text{opt}} - k_2 t_2^U = \tau_2^U$: at the unconstrained Nash equilibrium, households are charged higher taxes in jurisdiction 1 than in jurisdiction 2. The consequence is that, when $\bar{\tau} < \tau_1^U$, but not too low, only jurisdiction 1 is constrained. When $\bar{\tau}$ is low enough, both jurisdictions are constrained.

Let us look at the situation where only jurisdiction 1 is constrained. Then, for jurisdiction 2, the best reply is still given by Proposition 3.2, and then the best response function is unchanged. As for jurisdiction 1, its best reply is given by Proposition 3.4. A soon as the constraint is binding for jurisdiction 1, the local planner has to arbitrate between the quantity of local public good it provides and the tax rate charged on capital.

Compared to the first best outcome, it is likely to decrease the quantity of local public good and to increase the rate of the tax on capital.

Figure 7 illustrates this situation. We start from the base scenario of Figure 2, and we introduce the constraint $\tau_i \leq \bar{\tau} = z_{\text{opt}}$, $i = 1{,}2$. Let us remind that, in the base scenario, at the Nash equilibrium, jurisdiction 1 subsidizes capital while jurisdiction 2 taxes it. Because jurisdiction 2 always taxes capital, it never needs to tax households more than $\bar{\tau} = z_{\text{opt}}$, and then it is never constrained. Its response function (the red line on Figure 7) is unchanged.

Figure 7: Response functions, constraint on the tax on households



Conversely, when it subsidizes capital, jurisdiction 1 taxes households at a higher level than $\bar{\tau} = z_{\text{opt}}$, so that the constraint will be binding. Then, as soon as the unconstrained best reply is $t_1 < 0$ (the dotted blue line), the constraint generates a shift of this best reply to the right (the plain blue line), together with a decrease of the provision of local public good (the pink line). Jurisdiction 1 does not stop immediately subsidizing capital: it shares the missing resources between lower subsidies to firms and a lower level of local public good.

The consequence is that the Nash equilibrium moves along the unconstrained best reply curve of jurisdiction 2, with higher tax rates on capital (or, equivalently, lower subsidies) for both jurisdictions.

3.5 Equilibrium with agglomeration economies

We now move to the analysis of fiscal competition when differences in productivity levels are generated by an agglomeration externality. More precisely, as in Chapter 2, we assume that, in each region $i = 1{,}2$, the production function is $Y_i = \theta(L_i)F(K_i, L_i)$ where $F(K_i, L_i)$ is a constant returns to scale production function function and the efficiency parameter $\theta(L_i)$ is an increasing function of the size of the region, measured by its population. Using per capita quantities, this production function may be written as $y_i = \theta(L_i)f(k_i)$.

The system of equilibrium equations becomes:

$$c_1 = \theta_1 f(k_1) - k_1\theta_1 f'(k_1) + \rho k - \tau_1 \tag{3.1'}$$

$$c_2 = \theta_2 f(k_2) - k_2\theta_2 f'(k_2) + \rho k - \tau_2 \tag{3.2'}$$

$$M(L_2) = c_2 + u(z_2) - c_1 - u(z_1) \tag{3.3'}$$

$$\rho = \theta_1 f'(k_1) - t_1 \tag{3.4'}$$

$$\rho = \theta_2 f'(k_2) - t_2 \tag{3.5'}$$

$$z_1 = \tau_1 + k_1 t_1 \tag{3.6'}$$

$$z_2 = \tau_2 + k_2 t_2 \tag{3.7'}$$

$$(1 - L_2)k_1 + L_2k_2 = k \tag{3.8'}$$

Where, for simplicity, $\theta_i = \theta(L_i)$ and $\theta'_i = \theta'(L_i)$

We make the same assumption as in section 3.4 about the strategic instruments: the strategic instruments are the quantity of local public good, z_i, provided by the jurisdiction and the tax on capital, t_i, the level of the tax paid by households being determined by the local public budget constraint. Then, the strategy of government *i* is characterized by the package (t_i, z_i)

Our model is now completely symmetric: *a priori*, both regions share the same characteristics. Of course, this symmetry does not exclude dissymmetric outcomes as increasing returns to scale may generate a concentration of capital and population in one region. Without loss of generality we assume that, at a dissymmetric outcome, region 1 is the smallest and least productive one while region 2 is the largest and most productive one.

The best reply of jurisdiction 1 to the package (t_2, z_2) chosen by jurisdiction 2 is the package (t_1, z_1) maximizing the jurisdiction's welfare function:

$$W_1 = L_1[c_1 + u(z_1)] + (0.5 - L_1)[c_2 + u(z_2)] - \Omega(L_2)$$

under the constraints (3.1') to (3.8').

Similarly, the best reply of jurisdiction 2 to the package (t_1, z_1) chosen by jurisdiction 1 is the tax package (t_2, z_2) maximizing the jurisdiction's welfare function:

$$W_2 = 0.5[c_2 + u(z_2)]$$

under the same constraints, (3.1’) to (3.8’).

Let us first look at the best reply by jurisdiction 1 (the smallest one). In Appendix 3.6, we prove the following proposition:

Proposition 3.6: *At the best reply by jurisdiction 1, the values of* c_1, k_1, τ_1, t_1, c_2, k_2, τ_2, L_2, *and* ρ *solve the equations* (3.1’) *to* (3.8’) *together with the equality*

$$z_1 = z_{opt}$$

and the equation:

$$t_1 = \frac{L_1 M'(L_2)\left[(1-2L_1)u'(z_2)t_2+(k_2-k)\theta_2 f''(k_2)\right]+\left[f(k_2)-(k_2-k)f'(k_2)\right]\left[u'(z_2)t_2-(k_2-k_1)\theta_2 f''(k_2)\right]L_1\theta'_2}{2L_1L_2M'(L_2)+[u'(z_2)t_2-(k_2-k)\theta_2 f''(k_2)](k_2-k_1)-[f(k_2)-(k_2-k)f'(k_2)]L_2\theta'_2}$$

Let us now look at the best reply by jurisdiction 2. In Appendix 3.7, we prove the following proposition:

Proposition 3.7: *At the best reply by jurisdiction 2, the values of* c_1, k_1, z_1, c_2, k_2, t_2, τ_2, L_2, and ρ *solve the equations* (3.1’) *to* (3.8’) *together with the equality*

$$z_2 = z_{opt}$$

and the equation:

$$t_2 = \frac{\left[(k_2-k)\theta'_1 f'(k_1)-\theta'_2 f(k_2)\right]L_2 u'(z_1)t_1-\left[L_2\theta'_2 f(k_2)+L_1(\theta'_1 f(k_1)+M'(L_2))\right](k-k_1)\theta_1 f''(k_1)}{L_1M'(L_2)-[u'(z_1)t_1+(k-k_1)\theta_1 f''(k_1)](k_2-k_1)+[f(k_1)+(k-k_1)f'(k_1)]L_1\theta'_1}$$

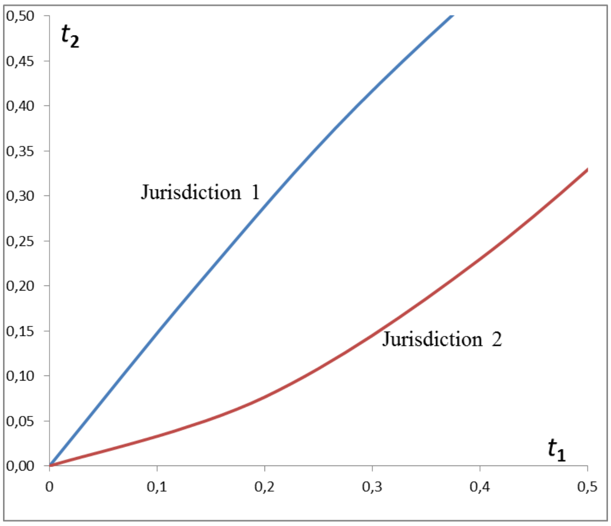
As in the case without agglomeration externalities, both jurisdictions choose to provide the first best quantity of local public good.

What about Nash equilibrium? Taking account of the symmetry of the problem, we can expect the existence of a symmetric Nash equilibrium, where both jurisdictions charge the same tax on capital ($t_1 = t_2$), leading to the same capital endowment ($k_1 = k_2 = k$) and the same population ($L_1 = L_2 = 0.5$). Propositions 3.6 and 3.7 imply that this Nash equilibrium exists, with zero taxation of capital: $t_1 = t_2 = 0$. The question is now: is this equilibrium stable? Are there other Nash equilibria?

As for the first question, the complexity of the response functions prevents proposing a general answer. Simulations always lead to the conclusion that the symmetric Nash equilibrium is stable. Figure 7 illustrates this situation. The basic functions and parameters are the same we used for the base situation illustrated in Figure 2. The only difference is the efficiency function, which takes the form $\theta(L_i) = 1 + \zeta L_i$. When $\zeta = 0$, there are no agglomeration externalities and, the higher ζ, the stronger are agglomeration externalities. For figure 7, we have $\zeta = 3$.

Clearly, the symmetric Nash equilibrium is stable. Each jurisdiction reacts to a deviation by its competitor choosing a lower tax rate than the other jurisdiction. Hence, a race back toward the equilibrium zero tax rate.

Figure 7: Response functions with agglomeration externalities



We suspect this result to be fairly general. Indeed, as noted in the previous section, the best response of each jurisdiction is an increasing function of the tax rate chosen by its competitor: a higher tax rate in jurisdiction j relaxes the constraint $\theta_i f'(k_i) - t_i \geq \theta_j f'(k_j) - t_j$, faced by jurisdiction i, allowing it to charge a higher tax rate. Moreover, we expect this increase to be lower than its competitor, so that, along the reaction function of jurisdiction i, $dt_i/dt_j < 1$, which leads to the situation described in Figure 7 and then to the unicity and the stability of the Nash equilibrium.

3.6 Conclusion

This chapter complements chapter 2 looking at the consequence of the decentralization of public decision, each jurisdiction being managed by a local government choosing the taxes to charge and the quantity of local public good to provide; local governments are utilitarian and take care of the welfare of their natives.

Our most important result is that the interaction between households' mobility and productivity differentials generate divergent incentives between jurisdictions. Starting from a situation where capital is not taxed by any jurisdiction, the less productive jurisdiction has an incentive to subsidize capital while the most productive jurisdiction has an incentive to tax capital. These incentives are stronger the less mobile are the workers and the larger the productivity differential.

However, these incentives do not imply that, at Nash equilibrium, capital is subsidized in the less productive jurisdiction and taxed in the most productive one. The interaction between jurisdictions modifies incentives and, facing a competitor taxing capital, the less productive jurisdiction does not necessarily subsidize capital: a low but positive tax rate may be enough for attracting capital. Then, depending upon the productivity differential and the mobility of households, at Nash equilibrium the less productive jurisdiction may be taxing or subsidizing households.

Another important result is that, if jurisdictions provide the first best efficient quantity of public good as long as they are freely able to choose the tax they charge on households, this is no longer the case when they are constrained. In that case, we are back to the results from the standard fiscal competition models. A binding constraint on households' taxation leads to under provision of the local public good. The constrained jurisdiction chooses a compromise between charging a higher tax on capital for compensating the loss in revenues generated by the constraint and a providing a lower quantity of local public good. When only the less productive jurisdiction is constrained, the constraint leads to an increase in taxes on capital charged by both jurisdictions.

A third important result is that, if the productivity differential is endogenously generated by an agglomeration externality, there is a symmetric Nash equilibrium without taxation of capital and this equilibrium looks stable. In Chapter 2, we found that there may be dissymmetric optimal outcomes: if the agglomeration externality is strong enough, global welfare is maximized when a jurisdiction is smaller (and less productive), the other jurisdiction being larger (and more productive). The stability of the Nash equilibrium implies that it is impossible to generate such a dissymmetric outcome with decentralized jurisdictions.

There is still some work to carry out. It would be useful to provide a more complete characterization of the levels of mobility and productivity differentials leading to subsidies to capital in the less productive jurisdiction. The case of productivity differences generated by an agglomeration externality is still to be developed, with more complete analyses and the introduction on restrictions to taxes.

Appendix chapter 3

Appendix 3.1

The Lagrangian of the maximization problem determining the best reply function of jurisdiction 1 is:

$$\Upsilon_1 = L_1[c_1 + u(z_1)] + (L_2 - 0.5)[c_2 + u(z_2)] - \Omega(L_2) - \lambda_{11}[c_1 - f(k_1) + k_1 f'(k_1) - \rho k + \tau_1] - \lambda_{12}[c_2 - \theta f(k_2) + k_2 \theta f'(k_2) - \rho k + \tau_2] - \mu_{11}[\rho - f'(k_1) + t_1] - \mu_{12}[\rho - \theta f'(k_2) + t_2] - \nu_{11}[z_1 - \tau_1 - k_1 t_1] - \nu_{12}[z_2 - \tau_2 - k_2 t_2] - \xi_1[c_2 + u(z_2) - c_1 - u(z_1) - M(L_2)] - \zeta_1[L_1 k_1 + L_2 k_2 - k]$$

Hence the first order conditions:

$$\frac{d\Upsilon_1}{dc_1} = L_1 - \lambda_{11} + \xi_1 = 0$$

$$\frac{d\Upsilon_1}{dc_2} = L_2 - 0.5 - \lambda_{12} - \xi_1 = 0$$

$$\frac{d\Upsilon_1}{d\tau_1} = -\lambda_{11} + \nu_{11} = 0$$

$$\frac{d\Upsilon_1}{dz_1} = (L_1 + \xi_1)u'(z_1) - \nu_{11} = 0$$

$$\frac{d\Upsilon_1}{dz_2} = (L_2 - 0.5 - \xi_1)u'(z_2) - \nu_{12} = 0$$

$$\frac{d\Upsilon_1}{dk_1} = -\lambda_{11} k_1 f''(k_1) + \mu_{11} f''(k_1) + \nu_{11} t_1 - \zeta_1 L_1 = 0$$

$$\frac{d\Upsilon_1}{dk_2} = -\lambda_{12} k_2 \theta f''(k_2) + \mu_{12} \theta f''(k_2) + \nu_{12} t_2 - \zeta_1 L_2 = 0$$

$$\frac{d\Upsilon_1}{d\rho} = (\lambda_{11} + \lambda_{12})k - \mu_{11} - \mu_{12} = 0$$

$\frac{dY_1}{dt_1} = \nu_{11}k_1 - \mu_{11} = 0$

$\frac{dY_1}{dL_2} = c_2 + u(z_2) - c_1 - u(z_1) - M(L_2) + \xi_1 M'(L_2) - \zeta_1(k_2 - k_1) = 0$

Instead of looking immediately at the solution of all these first order conditions, we will be working in two stages. In a first stage, we consider t_1 as a parameter and then we ignore the first order condition $dY_1/dt_1 = \nu_{11}k_1 - \mu_{11} = 0$. The value function of this first stage is a reduced welfare function, $\bar{W}_1(t_1)$. At its best reply, jurisdiction 1 maximizes $\bar{W}_1(t_1)$ and then, using the envelope theorem.

$$\frac{d\bar{W}_1}{dt_1} = \frac{dY_1}{dt_1} = \nu_{11}k_1 - \mu_{11} = 0$$

In the first stage, for t_1 given, the first order conditions lead to

$\nu_{11} = \lambda_{11}$

$\xi_1 = \lambda_{11} - L_1$

$\lambda_{12} = 0.5 - \lambda_{11}$

$\nu_{12} = (0.5 - \lambda_{11})u'(z_2)$

$\mu_{12} = 0.5k - \mu_{11}$

$u'(z_1) = 1$

$\lambda_{11}(t_1 - k_1 f''(k_1)) + \mu_{11} f''(k_1) - \zeta_1 L_1 = 0$

$(0.5 - \lambda_{11})(u'(z_2)t_2 - k_2\theta f''(k_2)) + (0.5k - \mu_{11})\,\theta f''(k_2) - \zeta_1 L_2 = 0$

$(\lambda_{11} - L_1)M'(L_2) = \zeta_1(k_2 - k_1)$

The condition $u'(z_1) = 1$ implies that, whatever the value of t_1, at its best reply, jurisdiction 1 chooses to provide the first best quantity of local public good, $z_1 = z_{opt}$.

Then, solving the last three equations, we get

$$\lambda_{11} = \frac{1}{2}\frac{2L_1M'(L_2)A + (k_2-k_1)\left[u'(z_2)t_2 - L_1(k_2-k_1)\theta f"(k_2)\right]f"(k_1)}{(k_2-k_1)[u'(z_2)t_2f"(k_1) - t_1\theta f"(k_2) - (k_2-k_1)\theta f"(k_1)f"(k_2)] + M'(L_2)A} \qquad \text{(A3.1.1)}$$

$with\ \ A = L_2 f"(k_1) + L_1\theta f"(k_2)$

Moving to the second stage, we start from the equation

$$\lambda_{11}(t_1 - k_1 f"(k_1)) + \mu_{11}\, f"(k_1) - \zeta_1 L_1 = 0$$

may be written as

$$[(k_2 - k_1)(k_1 f"(k_1) - t_1) + L_1 M'(L_2)]\lambda_{11} - \ (k_2 - k_1)f"(k_1)\mu_{11} = {L_1}^2 M'(L_2)$$

and then as

$$(k_2 - k_1)f"(k_1)(k_1\lambda_{11} - \mu_{11}) = {L_1}^2 M'(L_2) - [L_1 M'(L_2) - (k_2 - k_1)t_1]\lambda_{11}$$

Knowing that $\frac{d\overline{W}_1}{dt_1} = \frac{dY_1}{dt_1} = \nu_{11}k_1 - \mu_{11} = \lambda_{11}k_1 - \mu_{11}$, we get:

$$\frac{d\overline{W}_1}{dt_1} = \frac{{L_1}^2 M'(L_2) + [(k_2 - k_1)t_1 - L_1 M'(L_2)]\lambda_{11}}{(k_2 - k_1)f"(k_1)}$$

And, using (A1.1), after standard calculations:

$$\frac{d\overline{W}_1}{dt_1} = \frac{1}{2}\frac{\left[u'(z_2)t_2 - L_1(k_2-k_1)\theta f"(k_2)\right](k_2-k_1)t_1 - L_1M'(L_2)\left[L_1(k_2-k_1)\theta f"(k_2) + (L_2-L_1)u'(z_2)t_2 - 2t_1L_2\right]}{(k_2-k_1)[u'(z_2)t_2f"(k_1) - t_1\theta f"(k_2) - (k_2-k_1)\theta f"(k_1)f"(k_2)] + M'(L_2)A}$$

(A3.1.2)

Then, at the best response of jurisdiction 1, this derivative being zero, the following equality is met:

$$[u'(z_2)t_2 - L_1(k_2 - k_1)\theta f''(k_2)](k_2 - k_1)t_1 = L_1 M'(L_2)[L_1(k_2 - k_1)\theta f''(k_1) + (L_2 - L_1)u'(z_2)t_2 - 2t_1 L_2]$$

which may also be written as:

$$t_1 = \frac{L_1 M'(L_2)[L_1(k_2-k_1)\theta f''(k_1)+(L_2-L_1)u'(z_2)t_2]}{[u'(z_2)t_2-L_1(k_2-k_1)\theta f''(k_2)](k_2-k_1)+2L_1L_2M'(L_2)} \qquad \text{(A3.1.3)}$$

Reminding that $z_1 = z_{opt}$, adding (A3.1.3), to the equations (3.1) to (3.8), we get a system of 9 equations determining the values of the 9 unknowns t_1, c_1, k_1, τ_1, c_2, k_2, z_2, L_2, and ρ at a best response of jurisdiction 1 to the tax package (t_2, τ_2) chosen by jurisdiction 2.

Appendix 3.2

The Lagrangian of the maximization problem determining the best reply function of jurisdiction 2 is:

$$\Upsilon_2 = 0.5[c_2 + u(z_2)] - \lambda_{21}[c_1 - f(k_1) + k_1 f'(k_1) - \rho k + \tau_1] - \lambda_{22}[c_2 - \theta f(k_2) + k_2\theta f'(k_2) - \rho k + \tau_2] - \mu_{21}[\rho - f'(k_1) + t_1] - \mu_{22}[\rho - \theta f'(k_2) + t_2] - \nu_{21}[z_1 - \tau_1 - k_1 t_1] - \nu_{22}[z_2 - \tau_2 - k_2 t_2] - \xi_2[c_2 + u(z_2) - c_1 - u(z_1) - M(L_2)] - \zeta_2[L_1 k_1 + L_2 k_2 - k]$$

Hence the first order conditions:

$$\frac{d\Upsilon_2}{dc_1} = -\lambda_{21} + \xi_2 = 0$$

$$\frac{d\Upsilon_2}{dc_2} = 0.5 - \lambda_{22} - \xi_2 = 0$$

$\frac{dY_2}{d\tau_2} = -\lambda_{22} + \nu_{22} = 0$

$\frac{dY_2}{dz_1} = \xi_2 u'(z_1) - \nu_{21} = 0$

$\frac{dY_2}{dz_2} = (0.5 - \xi_2)u'(z_2) - \nu_{22} = 0$

$\frac{dY_2}{dk_1} = -\lambda_{21}k_1 f''(k_1) + \mu_{21}\, f''(k_1) + \nu_{21}t_1 - \zeta_2 L_1 = 0$

$\frac{dY_2}{dk_2} = -\lambda_{22}k_2\, \theta f''(k_2) + \mu_{22}\, \theta f''(k_2) + \nu_{22}t_2 - \zeta_2 L_2 = 0$

$\frac{dY_2}{d\rho} = (\lambda_{21} + \lambda_{22})k - \mu_{21} - \mu_{22} = 0$

$\frac{dY_2}{dt_2} = \nu_{22}k_2 - \mu_{22} = 0$

$\frac{dY_2}{dL_2} = \xi_2 M'(L_2) - \zeta_2(k_2 - k_1) = 0$

We will be using the same two stages method as in Appendix 3.1. In a first stage, we consider t_2 as a parameter and then we ignore the first order condition $dY_2/dt_2 = \nu_{22}k_2 - \mu_{22} = 0$. The value function of this first stage is a reduced welfare function, $\overline{W}_2(t_2)$. At its best reply, jurisdiction 2 maximizes $\overline{W}_2(t_2)$ and then, using the envelope theorem,

$$\frac{d\overline{W}_2}{dt_2} = \frac{dY_2}{dt_2} = \nu_{22}k_2 - \mu_{22} = 0$$

In the first stage, for t_2 given, the first order conditions lead to

$\lambda_{21} = \xi_2 = 0.5 - \lambda_{22}$

$\nu_{22} = \lambda_{22}$

$\nu_{21} = (0.5 - \lambda_{22})u'(z_1)$

$\mu_{21} = 0.5k - \mu_{22}$

$u'(z_2) = 1$

$(0.5k - \mu_{22})f"(k_1) + (0.5 - \lambda_{22})(u'(z_1)t_1 - k_1 f"(k_1)) = \zeta_2 L_1$

$(\mu_{22} - \lambda_{22}k_2)\theta f"(k_2) + \lambda_{22}t_2 = \zeta_2 L_2$

$\zeta_2(k_2 - k_1) = (0.5 - \lambda_{22})M'(L_2)$

The equality $u'(z_2) = 1$ implies that jurisdiction 2 provides the first best quantity of local public good, $z_2 = z_{opt}$.

Then, solving the last three equations, we get

$$\lambda_{22} = \frac{[L_2(k_2-k_1)f"(k_1)+u'(z_1)t_1](k_2-k_1)\theta f"(k_2)-M'(L_2)A}{2\{(k_2-k_1)[(k_2-k_1)f"(k_1)\theta f"(k_2)+u'(z_1)t_1\theta f"(k_2)-t_2 f"(k_1)]-M'(L_2)A\}} \qquad \text{(A3.2.1)}$$

Where, as in Appendix 3.1, $A = L_2 f"(k_1) + L_1\theta f"(k_2)$

Moving to the second stage, we start from the equation

$$(\mu_{22} - \lambda_{22}k_2)\theta f"(k_2) + \lambda_{22}t_2 = \zeta_2 L_2$$

may be written as

$$2(k_2 - k_1)\theta f"(k_2)(\lambda_{22}k_2 - \mu_{22}) = 2[(k_2 - k_1)t_2 + L_2 M'(L_2)]\lambda_{22} - L_2 M'(L_2)$$

Knowing that $\frac{d\bar{W}_2}{dt_2} = \frac{dY_2}{dt_2} = \nu_{22}k_2 - \mu_{22} = \lambda_{22}k_2 - \mu_{22}$, we get:

$$\frac{d\bar{W}_2}{dt_2} = \frac{2[(k_2 - k_1)t_2 + L_2 M'(L_2)]\lambda_{22} - L_2 M'(L_2)}{2(k_2 - k_1)\theta f"(k_2)}$$

And, using (A2.1), after standard calculations:

$$\frac{d\bar{W}_2}{dt_2} = \frac{1}{2}\frac{\{(k_2-k_1)[L_2(k_2-k_1)f"(k_1)+u'(z_1)t_1]-L_1M'(L_2)\}t_2-L_1L_2M'(L_2)(k_2-k_1)f"(k_1)}{(k_2-k_1)[(k_2-k_1)f"(k_1)\theta f"(k_2)+u'(z_1)t_1\theta f"(k_2)-t_2f"(k_1)]-M'(L_2)A} \qquad \text{(A3.2.2)}$$

Then, at the best response of jurisdiction 2, this derivative being zero, the following equality is met:

$$\{(k_2-k_1)[L_2(k_2-k_1)f"(k_1)+u'(z_1)t_1]-L_1M'(L_2)\}t_2 = L_1L_2M'(L_2)(k_2-k_1)f"(k_1)$$

which may also be written as:

$$t_2 = \frac{L_1L_2M'(L_2)(k_2-k_1)f"(k_1)}{(k_2-k_1)[L_2(k_2-k_1)f"(k_1)+u'(z_1)t_1]-L_1M'(L_2)} \qquad \text{(A3.2.3)}$$

Reminding that $z_2 = z_{opt}$, adding (A3.2.2) to the equations (3.1) to (3.8), we get a system of 9 equations determining the values of the 9 unknowns c_1, k_1, z_1, c_2, k_2, t_2, τ_2, L_2, and ρ at a best response of jurisdiction 2 to the tax package (t_1, τ_1) chosen by jurisdiction 1.

Appendix 3.3

Let us remind that, in Chapter 2, we proved that, even when in each region the public good must be provided using local tax collection, the central planner can implement the first best outcome using the taxation of households only for providing the public good: $u'(z_1) = u'(z_2) = 1$, $t_1 = t_2 = 0$, and then $\tau_1 = \tau_2 = z_{opt}$.

Combining these equalities with (A3.1.3), and knowing that $t_1 = t_2$ implies $k_2 - k_1 > 0$, we get:

$$\frac{d\bar{W}_1}{dt_1} = -\frac{1}{2}\frac{{L_1}^2M'(L_2)(k_2-k_1)\theta f"(k_2)}{M'(L_2)[L_1\theta f"(k_2)+L_2f"(k_1)]-(k_2-k_1)^2\theta f"(k_1)f"(k_2)} < 0$$

$$\frac{d\bar{W}_2}{dt_2} = \frac{1}{2}\frac{L_1 L_2 M'(L_2)(k_2 - k_1) f"(k_1)}{M'(L_2)[L_1 \theta f"(k_2) + L_2 f"(k_1)] - (k_2 - k_1)^2 f"(k_1)\theta f"(k_2)} > 0$$

Then, starting from the first best, jurisdiction 1 has an incentive to subsidize capital, while jurisdiction 2 has an incentive to tax capital.

However, when jurisdiction 1 subsidizes capital ($t_1 < 0$) and jurisdiction 2 taxes capital ($t_2 > 0$), equality of post-tax returns to capital implies

$$\theta f'(k_2) - f'(k_1) = t_2 - t_1 > 0$$

When $t_1 = t_2$, the fact that region 2 is more productive ($\theta > 1$) implies that its capital per worker ratio is higher: $k_2 - k_1 > 0$. The fact that now $\theta f'(k_2) - f'(k_1)$ becomes positive implies that the difference $k_2 - k_1$ decreases. Is it possible to have an inversion of the difference in ratios, $k_2 - k_1 < 0$?

For answering this question, let us examine the derivatives of the reduced welfare function when $k_1 = k_2$. Knowing that, at their best reply, both jurisdictions choose to provide the optimal quantity of local public good, (A3.1.2) and (A3.2.2) become

$$\frac{d\bar{W}_1}{dt_1} = \frac{L_1}{2}\frac{2t_1 L_2 - (L_2 - L_1)t_2}{L_1 \theta f"(k_2) + L_2 f"(k_1)}$$

$$\frac{d\bar{W}_2}{dt_2} = \frac{1}{2}\frac{L_1 t_2}{L_1 \theta f"(k_2) + L_2 f"(k_1)}$$

The equalities $k_1 = k_2$ and $\theta f'(k_2) - f'(k_1) = t_2 - t_1$ imply $t_2 - t_1 > 0$. Therefore, both jurisdictions providing the same optimal quantity of local public good ($z_1 = z_2 = z_{opt}$), the lower taxation of capital by jurisdiction 1 implies a higher taxation of households ($\tau_1 = z_{opt} - k_1 t_1 > z_{opt} - k_2 t_2 = \tau_2$). But $k_1 = k_2$ also implies the equality of pre-tax incomes across jurisdictions and then, with a higher households tax and the same level of local public goods, utility derived from consumption is lower an jurisdiction 1, implying that some workers move to jurisdiction 2, so that $L_1 < L_2$. Then, the derivative $d\overline{W}_1/dt_1$ is ambiguous in sign, as its numerator is the difference between two positive terms.

However, the derivative $d\overline{W}_2/dt_2$ is unambiguously negative. Then, jurisdiction 2, which has an incentive to increase its tax rate on capital when $t_1 = t_2 = 0$, has an incentive to decrease it when $t_2 > 0$ and the difference $t_2 - t_1$ is high enough for capital per worker ratios to be equalized across jurisdictions. This change in sign implies that, for jurisdiction 2 to be at its best reply, $k_2 - k_1$ must be still positive.

Appendix 3.4

The Lagrangian is the same as in Appendix 3.1, but now, when the constraint is binding, $\tau_1 = \bar{\tau}$ and the first order condition $dY_1/d\tau_1 = \nu_{11} - \lambda_{11} = 0$ disappears. Then, the system of first order conditions is

$$L_1 - \lambda_{11} + \xi_1 = 0$$

$$L_2 - 0.5 - \lambda_{12} - \xi_1 = 0$$

$$(L_1 + \xi_1)u'(z_1) - \nu_{11} = 0$$

$$(L_2 - 0.5 - \xi_1)u'(z_2) - \nu_{12} = 0$$

$$-\lambda_{11}k_1 f''(k_1) + \mu_{11} f''(k_1) + \nu_{11}t_1 - \zeta_1 L_1 = 0$$

$$-\lambda_{12}k_2\theta f''(k_2) + \mu_{12}\, \theta f''(k_2) + \nu_{12}t_2 - \zeta_1 L_2 = 0$$

$$(\lambda_{11} + \lambda_{12})k - \mu_{11} - \mu_{12} = 0$$

$$-\mu_{11} + \nu_{11}k_1 = 0$$

$\xi_1 M'(L_2) - \zeta_1(k_2 - k_1) = 0$

After straightforward calculations, these first order conditions lead to :

$\xi_1 = \lambda_{11} - L_1$

$\lambda_{12} = 0.5 - \lambda_{11}$

$\nu_{12} = (0.5 - \lambda_{11})u'(z_2)$

$\mu_{11} = \nu_{11}k_1 = \lambda_{11}u'(z_1)k_1$

$\mu_{12} = 0.5k - \mu_{11} = 0.5k - \nu_{11}k_1 = 0.5k - \lambda_{11}u'(z_1)k_1$

$\nu_{11} = \lambda_{11}u'(z_1)$

$\zeta_1(k_2 - k_1) = \xi_1 M'(L_2) = (\lambda_{11} - L_1)M'(L_2)$

$\{[(u'(z_1) - 1)k_1 f''(k_1) + u'(z_1)t_1](k_2 - k_1) - L_1 M'(L_2)\}\lambda_{11} = -L_1 L_1 M'(L_2)$

$\{[(k_2 - u'(z_1)k_1)\theta f''(k_2) - u'(z_2)t_2](k_2 - k_1) - L_2 M'(L_2)\}\lambda_{11} = 0.5(k_2 - k_1)[(k_2 - k)\theta f''(k_2) - u'(z_2)t_2] - L_1 L_2 M'(L_2)$

When the constraint $\tau_1 \leq \bar{\tau}$ is binding, the first order derivative $dY_1/d\tau_1 = \nu_{11} - \lambda_{11}$, is positive at $\tau_1 = \bar{\tau}$ (the jurisdiction would be better off with a higher value of $\tau_1 = \bar{\tau}$) and then $\nu_{11} > \lambda_{11}$, with the consequence that $u'(z_1) = \nu_{11}/\lambda_{11} > 1$, so that $z_1 < z_{opt}$: jurisdiction 1 underprovides the local public good.

Then, dividing the last two equations term by term, we get

$$\frac{[(k_2 - u'(z_1)k_1)\theta f''(k_2) - u'(z_2)t_2](k_2 - k_1) - L_2 M'(L_2)}{[(u'(z_1) - 1)k_1 f''(k_1) + u'(z_1)t_1](k_2 - k_1) - L_1 M'(L_2)} = \frac{0.5(k_2 - k_1)[(k_2 - k)\theta f''(k_2) - u'(z_2)t_2] - L_1 L_2 M'(L_2)}{-L_1{}^2 M'(L_2)}$$

which simplifies to:

$$\frac{(k_1+k_2-2u'(z_1)k_1)L_1\theta f''(k_2)+(L_2-L_1)u'(z_2)t_2}{(u'(z_1)-1)k_1 f''(k_1)+u'(z_1)t_1} = \frac{2L_1L_2M'(L_2)-(k_2-k_1)\left[(k_2-k)\theta f''(k_2)-u'(z_2)t_2\right]}{L_1M'(L_2)}$$

and may be written as

$$t_1 = \frac{L_1M'(L_2)}{u'(z_1)}\frac{(k_1+k_2-2u'(z_1)k_1)L_1\theta f''(k_2)+(L_2-L_1)u'(z_2)t_2}{2L_1L_2M'(L_2)-(k_2-k_1)[(k_2-k)\theta f''(k_2)-u'(z_2)t_2]} - \left(1-\frac{1}{u'(z_1)}\right)k_1 f''(k_1)$$

Appendix 3.5

The Lagrangian is the same as in Appendix 3.2, but now, when the constraint is binding, $\tau_2 = \bar{\tau}$ and the first order condition $dY_2/d\tau_2 = \nu_{22} - \lambda_{22} = 0$ disappears. Then, the system of first order conditions is

$$-\lambda_{21} + \xi_2 = 0$$

$$0.5 - \lambda_{22} - \xi_2 = 0$$

$$\xi_2 u'(z_1) - \nu_{21} = 0$$

$$(0.5 - \xi_2)u'(z_2) - \nu_{22} = 0$$

$$-\lambda_{21}k_1 f''(k_1) + \mu_{21}\, f''(k_1) + \nu_{21}t_1 - \zeta_2 L_1 = 0$$

$$-\lambda_{22}k_2\, \theta f''(k_2) + \mu_{22}\, \theta f''(k_2) + \nu_{22}t_2 - \zeta_2 L_2 = 0$$

$$(\lambda_{21} + \lambda_{22})k - \mu_{21} - \mu_{22} = 0$$

$$-\mu_{22} + \nu_{22}k_2 = 0$$

$$\xi_2 M'(L_2) - \zeta_2(k_2 - k_1) = 0$$

After straightforward calculations, these first order conditions lead to:

$\xi_2 = \lambda_{21} = 0.5 - \lambda_{22}$

$\nu_{21} = \xi_2 u'(z_1) = (0.5 - \lambda_{22})u'(z_1)$

$\nu_{22} = \lambda_{22}u'(z_2)$

$\mu_{22} = \nu_{22}k_2 = \lambda_{22}u'(z_2)k_2$

$\mu_{21} = 0.5k - \mu_{22} = 0.5k - \lambda_{22}u'(z_2)k_2$

$\zeta_2(k_2 - k_1) = \xi_2 M'(L_2) = (0.5 - \lambda_{22})M'(L_2)$

$2\{(k_2 - k_1)[(u'(z_2)k_2 - k_1)f"(k_1) + u'(z_1)t_1] - L_1 M'(L_2)\}\lambda_{22} = (k_2 - k_1)[u'(z_1)t_1 + (k - k_1)f"(k_1)] - L_1 M'(L_2)$

$2\{(k_2 - k_1)[(u'(z_2) - 1)k_2\theta f"(k_2) + u'(z_2)t_2] + L_2 M'(L_2)\}\lambda_{22} = L_2 M'(L_2)$

When the constraint $\tau_2 \leq \bar{\tau}$ is binding, we must have $dY_2/d\tau_2 = \nu_{22} - \lambda_{22} > 0$ at $\tau_2 = \bar{\tau}$ (the jurisdiction would be better off with a higher value of $\tau_2 = \bar{\tau}$) so that $\nu_{22} > \lambda_{22}$ and then $u'(z_2) = \nu_{22}/\lambda_{22} > 1$. This equality implies $z_2 < z_{opt}$: jurisdiction 2 underprovides the local public good.

Then, dividing the last two equations term by term, we get

$$\frac{(k_2-k_1)[(u'(z_2)k_2-k_1)f"(k_1)+u'(z_1)t_1]-L_1M'(L_2)}{(k_2-k_1)[(u'(z_2)-1)k_2\theta f"(k_2)+u'(z_2)t_2]+L_2M'(L_2)} = \frac{(k_2-k_1)[u'(z_1)t_1+(k-k_1)f"(k_1)]-L_1M'(L_2)}{L_2M'(L_2)}$$

which simplifies to:

$$\frac{(u'(z_2)k_2-k)f"(k_1)}{(u'(z_2)-1)k_2\theta f"(k_2)+u'(z_2)t_2} = \frac{(k_2-k_1)[u'(z_1)t_1+(k-k_1)f"(k_1)]}{L_2M'(L_2)} - \frac{L_1}{L_2}$$

and may be written as

$$t_2 = \frac{L_2 M'(L_2)}{u'(z_2)} \frac{(u\prime(z_2)k_2 - k)f"(k_1)}{(k_2 - k_1)[u'(z_1)t_1 + (k - k_1)f"(k_1)] - L_1 M'(L_2)} - \left(1 - \frac{1}{u'(z_2)}\right) k_2 \theta f"(k_2)$$

Appendix 3.6

The Lagrangian of the maximization problem determining the best reply function of jurisdiction 1 is:

$$\Upsilon_1 = L_1[c_1 + u(z_1)] + (L_2 - 0.5)[c_2 + u(z_2)] - \Omega(L_2) - \lambda_{11}[c_1 - \theta_1 f(k_1) + k_1\theta_1 f'(k_1) - \rho k + \tau_1\,] - \lambda_{12}[c_2 - \theta_2 f(k_2) + k_2\theta_2 f'(k_2) - \rho k + \tau_2] - \mu_{11}[\rho - \theta_1 f'(k_1) + t_1] - \mu_{12}[\rho - \theta_2 f'(k_2) + t_2] - \nu_{11}[z_1 - \tau_1 - k_1 t_1] - \nu_{12}[z_2 - \tau_2 - k_2 t_2] - \xi_1[c_2 + u(z_2) - c_1 - u(z_1) - M(L_2)] - \zeta_1[L_1 k_1 + L_2 k_2 - k]$$

Hence the first order conditions:

$$\frac{d\Upsilon_1}{dc_1} = L_1 - \lambda_{11} + \xi_1 = 0$$

$$\frac{d\Upsilon_1}{dc_2} = L_2 - 0.5 - \lambda_{12} - \xi_1 = 0$$

$$\frac{d\Upsilon_1}{d\tau_1} = -\lambda_{11} + \nu_{11} = 0$$

$$\frac{d\Upsilon_1}{dz_1} = (L_1 + \xi_1)u'(z_1) - \nu_{11} = 0$$

$$\frac{d\Upsilon_1}{dz_2} = (L_2 - 0.5 - \xi_1)u'(z_2) - \nu_{12} = 0$$

$$\frac{d\Upsilon_1}{dk_1} = -\lambda_{11}k_1\theta_1 f''(k_1) + \mu_{11}\theta_1 f''(k_1) + \nu_{11}t_1 - \zeta_1 L_1 = 0$$

$$\frac{d\Upsilon_1}{dk_2} = -\lambda_{12}k_2\theta_2 f''(k_2) + \mu_{12}\,\theta_2 f''(k_2) + \nu_{12}t_2 - \zeta_1 L_2 = 0$$

$$\frac{d\Upsilon_1}{d\rho} = (\lambda_{11} + \lambda_{12})k - \mu_{11} - \mu_{12} = 0$$

$\frac{dY_1}{dt_1} = \nu_{11}k_1 - \mu_{11} = 0$

$\frac{dY_1}{dL_2} = c_2 + u(z_2) - c_1 - u(z_1) - M(L_2) + \lambda_{11}[f(k_1) - k_1 f'(k_1)]\theta'_1 + \lambda_{12}[f(k_2) - k_2 f'(k_2)]\theta'_2 + \mu_{11} f'(k_1)\theta'_1 + \mu_{12} f'(k_2)\theta'_2 + \xi_1 M'(L_2) - \zeta_1(k_2 - k_1) = 0$

These first order conditions lead to

$\nu_{11} = \lambda_{11}$

$\xi_1 = \lambda_{11} - L_1$

$\lambda_{12} = 0.5 - \lambda_{11}$

$\nu_{12} = (0.5 - \lambda_{11})u'(z_2)$

$\mu_{12} = 0.5k - \mu_{11}$

$u'(z_1) = 1$

$\mu_{11} = \nu_{11}k_1 = \lambda_{11}k_1$

$\lambda_{11}t_1 = \zeta_1 L_1$

$0.5[u'(z_2)t_2 - (k_2 - k)\theta_2 f"(k_2)] - [u'(z_2)t_2 - (k_2 - k_1)\theta_2 f"(k_2)]\lambda_{11} = \zeta_1 L_2$

$\zeta_1(k_2 - k_1) = -L_1 M'(L_2) + \lambda_{11}\{M'(L_2) + f(k_1)\theta'_1 - [f(k_2) - (k_2 - k_1)f'(k_2)]\theta'_2\} + 0.5[f(k_2) - (k_2 - k)f'(k_2)]\theta'_2$

The condition $u'(z_1) = 1$ implies that, whatever the value of t_1, at its best reply, jurisdiction 1 chooses to provide the first best quantity of local public good, $z_1 = z_{opt}$.

The last two equations may be written as:

$[t_1 L_2 + u'(z_2) L_1 t_2 - L_1(k_2 - k_1)\theta_2 f"(k_2)]\lambda_{11} = 0.5 L_1[u'(z_2)t_2 - (k_2 - k)\theta_2 f"(k_2)]$

$\{L_1 M'(L_2) - t_1(k_2 - k_1) + L_1 f(k_1)\theta'_1 - L_1[f(k_2) - (k_2 - k_1)f'(k_2)]\theta'_2\}\lambda_{11} = L_1 L_1 M'(L_2) - 0.5 L_1[f(k_2) - (k_2 - k)f'(k_2)]\theta'_2$

leading to:

$$\frac{1}{\lambda_{11}} = \frac{t_1 L_2 + u'(z_2) L_1 t_2 - L_1(k_2 - k_1)\theta_2 f"(k_2)}{0.5 L_1[u'(z_2)t_2 - (k_2 - k)\theta_2 f"(k_2)]} = \frac{L_1 M'(L_2) - t_1(k_2 - k_1) + L_1 f(k_1)\theta'_1 - L_1[f(k_2) - (k_2 - k_1)f'(k_2)]\theta'_2}{L_1 L_1 M'(L_2) - 0.5 L_1[f(k_2) - (k_2 - k)f'(k_2)]\theta'_2}$$

Then, reorganizing terms and after straightforward calculations, we get

$$t_1 = \frac{L_1 M'(L_2)[(1 - 2L_1)u'(z_2)t_2 + (k_2 - k)\theta_2 f"(k_2)] + [f(k_2) - (k_2 - k)f'(k_2)][u'(z_2)t_2 - (k_2 - k_1)\theta_2 f"(k_2)]L_1\theta'_2}{2 L_1 L_2 M'(L_2) + [u'(z_2)t_2 - (k_2 - k)\theta_2 f"(k_2)](k_2 - k_1) - [f(k_2) - (k_2 - k)f'(k_2)]L_2\theta'_2}$$

Appendix 3.7

The Lagrangian of the maximization problem determining the best reply function of jurisdiction 2 is:

$\Upsilon_2 = 0.5[c_2 + u(z_2)] - \lambda_{21}[c_1 - \theta_1 f(k_1) + k_1\theta_1 f'(k_1) - \rho k + \tau_1] - \lambda_{22}[c_2 - \theta_2 f(k_2) + k_2\theta_2 f'(k_2) - \rho k + \tau_2] - \mu_{21}[\rho - \theta_1 f'(k_1) + t_1] - \mu_{22}[\rho - \theta_2 f'(k_2) + t_2] - \nu_{21}[z_1 - \tau_1 - k_1 t_1] - \nu_{22}[z_2 - \tau_2 - k_2 t_2] - \xi_2[c_2 + u(z_2) - c_1 - u(z_1) - M(L_2)] - \zeta_2[L_1 k_1 + L_2 k_2 - k]$

Hence the first order conditions:

$$\frac{dY_2}{dc_1} = -\lambda_{21} + \xi_2 = 0$$

$$\frac{dY_2}{dc_2} = 0.5 - \lambda_{22} - \xi_2 = 0$$

$$\frac{dY_2}{d\tau_2} = -\lambda_{22} + \nu_{22} = 0$$

$$\frac{dY_2}{dz_1} = \xi_2 u'(z_1) - \nu_{21} = 0$$

$$\frac{dY_2}{dz_2} = (0.5 - \xi_2)u'(z_2) - \nu_{22} = 0$$

$$\frac{dY_2}{dk_1} = -\lambda_{21}k_1\theta_1 f''(k_1) + \mu_{21}\theta_1 f''(k_1) + \nu_{21}t_1 - \zeta_2 L_1 = 0$$

$$\frac{dY_2}{dk_2} = -\lambda_{22}k_2\,\theta_2 f''(k_2) + \mu_{22}\theta_2 f''(k_2) + \nu_{22}t_2 - \zeta_2 L_2 = 0$$

$$\frac{dY_2}{d\rho} = (\lambda_{21} + \lambda_{22})k - \mu_{21} - \mu_{22} = 0$$

$$\frac{dY_2}{dt_2} = \nu_{22}k_2 - \mu_{22} = 0$$

$$\frac{dY_2}{dL_2} = \lambda_{21}[f(k_1) - k_1 f'(k_1)]\theta'_1 + \lambda_{22}[f(k_2) - k_2 f'(k_2)]\theta'_2 + \mu_{21}f'(k_1)\theta'_1 +$$

$$\mu_{22}f'(k_2)\theta'_2 + \xi_2 M'(L_2) - \zeta_2(k_2 - k_1) = 0$$

These first order conditions lead to

$$\lambda_{21} = \xi_2 = 0.5 - \lambda_{22}$$

$$\nu_{22} = \lambda_{22}$$

$$\nu_{21} = (0.5 - \lambda_{22})u'(z_1)$$

$$\mu_{21} = 0.5k - \mu_{22} = 0.5k - k_2\lambda_{22}$$

$$u'(z_2) = 1$$

$$\mu_{22} = \nu_{22} k_2 = \lambda_{22} k_2$$

$$\lambda_{22} t_2 = \zeta_2 L_2$$

$$\lambda_{22}[u'(z_1)t_1 + (k_2 - k_1)\theta_1 f''(k_1)] - 0.5[u'(z_1)t_1 + (k - k_1)\theta_1 f''(k_1)] = -\zeta_2 L_1$$

$$2\{f(k_2)\theta'_2 - [f(k_1) + (k_2 - k_1)f'(k_1)]\theta'_1 - M'(L_2)\}\lambda_{22} + [f(k_1) + (k - k_1)f'(k_1)]\theta'_1 + M'(L_2) = 2\zeta_2(k_2 - k_1)$$

The condition $u'(z_2) = 1$ implies that, whatever the value of t_1, at its best reply, jurisdiction 2 chooses to provide the first best quantity of local public good, $z_2 = z_{opt}$.

The last two equations may be written as:

$$\{L_2[u'(z_1)t_1 + (k_2 - k_1)\theta_1 f''(k_1)] + t_2 L_1\} 2\lambda_{22} = L_2[u'(z_1)t_1 + (k - k_1)\theta_1 f''(k_1)]$$

$$\{L_2 M'(L_2) + t_2(k_2 - k_1) + [f(k_1) + (k_2 - k_1)f'(k_1)]L_2\theta'_1 - L_2 f(k_2)\theta'_2\} 2\lambda_{22} = [f(k_1) + (k - k_1)f'(k_1)]L_2\theta'_1 + L_2 M'(L_2)$$

so that:

$$\frac{1}{2\lambda_{22}} = \frac{L_2[u'(z_1)t_1 + (k_2 - k_1)\theta_1 f''(k_1)] + t_2 L_1}{L_2[u'(z_1)t_1 + (k - k_1)\theta_1 f''(k_1)]} = \frac{L_2 M'(L_2) + t_2(k_2 - k_1) + [f(k_1) + (k_2 - k_1)f'(k_1)]L_2\theta'_1 - L_2 f(k_2)\theta'_2}{[f(k_1) + (k - k_1)f'(k_1)]L_2\theta'_1 + L_2 M'(L_2)}$$

Then, reorganizing terms and after straightforward calculations, we get

$$t_2 = \frac{[(k_2 - k)\theta'_1 f'(k_1) - \theta'_2 f(k_2)]L_2 u'(z_1)t_1 - [L_2\theta'_2 f(k_2) + L_1(\theta'_1 f(k_1) + M'(L_2))](k - k_1)\theta_1 f''(k_1)}{L_1 M'(L_2) - [u'(z_1)t_1 + (k - k_1)\theta_1 f''(k_1)](k_2 - k_1) + [f(k_1) + (k - k_1)f'(k_1)]L_1\theta'_1}$$

Chapter 4 Public inputs and foreign direct investments

4.1 Introduction

Foreign direct investment is considered to play an undoubtable significant role in the development of countries. Back to 1950s, the successful development of several Asian countries (for example Singapore) benefited from the presence of multinational corporations. FDI is considered to bring not only capital investment, but also working opportunities and technology transfers to the host countries. Competition between regions in their effort to attract FDI has been intensive since then. Research over FDI attraction factors has become a hot topic too. A government can take different fiscal policies to attract foreign direct investment.

There are different kinds of policies used to attract FDI such as tax rate changes, direct subsidies or public goods. Subsidies can be considered as negative taxes. Tax competition is generally accepted as a misleading way which would generate an under provision of public goods (D. Wildasin, 1988). Similarly, subsidies are a common policy which could also raise the risk of a 'race to the bottom' (J. Bruckner, 2002). The combination of tax competition and public goods provision is known as "double competition" (Bénasy-Quéré et al. 2005).

The standard research on how to attract FDI started with tax competition: the authorities use taxes as policy instrument to compete for capital. Zodrow and Mieszkowski (1986) introduced the canonical model of tax competition. Based on the assumption that an optimal combination of public and private goods is reached in a closed economy, the model illustrated that capital mobility leads to a sub-optimal provision of public goods. An important question is generated from the conclusion here: is one-sided research over tax competition enough? Obviously, the answer is no. Public goods such as transportation infrastructures, educations might influence the FDI flow. How? If the public goods act as an input in the private good production, the government could use the tax collected to provide such goods, and public choices will be definitely changed. In the previous literature, the positive impact of public goods on FDI attraction is widely accepted. However, the systematical theoretical analysis of how public goods affect FDI flow is absent in most of the previous papers.

There are several theoretical studies over the relationship between public goods and FDI which are incomplete. On the one hand, lower taxation gives higher capital revenue which will attract FDI inflow. On the other hand, authorities use the collected tax to provide public goods which may have positive effect on productivities, which will also be attractive for FDI. However, systematically research on 'how do public goods affect FDI flows' is absent and worth to be worked with. The aim of our paper is to contribute to fill this gap. Several questions are tackled:

How do public goods affect FDI flow? Are there specific conditions for public goods to attract foreign capital? Could public goods be detrimental for FDI? What are the mechanisms behind?

In order to answer these questions, we use a simple general equilibrium model of a small open economy with three sectors and a public input. The basic structure is similar to Kellenberg's (2003) model and Haaland and Wooton's (1999) general equilibrium framework, which has been developed from Markusen and Venables' partial equilibrium model (1999). We take the assumption of a small open economy with several sectors and assume that public goods are produced in one of the sectors and used in other ones. However, when introducing the public goods into the model, we take the method of Ishizawa (1991) and Altenburg (1992), but not Kellenberg's way of considering public goods as a direct reduce on the cost of the foreign firms. Instead of taking "public provided private goods" or a constant resource to reduce production costs, we consider public goods as imparting increasing returns to scale to the economy, which is called "factor augmenting public input". This assumption can represent the positive effect of public goods over productivities and is similar to a Hicksian technical change (the ratio between primary factors does not change). We set up our general equilibrium model following the structure of Jones (1965).

Assuming that the public goods are financed by fiscal revenue collected by taxes, it is worth to look at different cases. We first consider the non-distortionary case: the provision of the public inputs is financed by lump-sum taxation. Then we look at the case when public inputs are financed by a tax on foreign capital revenue. This assumption will connect the tax policy directly to the mobile factor of our model (capital) and could generate distortions. This assumption is more related to the double competition research and is worth to be closely looked at.

One of the main assumptions of this kind of research is the mobility of factors. As discussed before, in the traditional fiscal competition model, capital is considered to be the single mobile factor while other factors (land or labor) are fixed. Other papers made the assumption that both capital and labor are mobile (Burbidge and Myers, 1994; Wilson, 1997). Different from the previous two chapters, we focus here on the relationship between public inputs and capital flows. Therefore we follow the usual assumption of fiscal competition model where only one factor is internationally mobile: capital.

The paper will include the following parts: section 2 is a brief literature review. Section 3 describes the basic model. Section 4 analyses the case with a lump-sum tax.

In this part, the effects of public inputs on FDI are disentangled, and the optimality rule for provision of the Hicksian public input in the present of FDI is derived. Section 5 studies the case with a tax on capital income. Finally section 6 is the conclusion.

4.2 Literature review

As part of an open and effective international economic system, foreign direct investment (FDI) is often regarded as an important factor that drives economic growth both for the countries of origin and countries of destination. Its contributions, however, do not appear so automatic and are not distributed evenly across countries, sectors and local communities. The importance of FDI for economic development and its unbalanced distribution have led to substantial recent interest by the international economics literature to theoretically and empirically investigate the determinants of FDI (Faeth, 2009).

Various theories have been developed since the 1960s to explain FDI. These theories proclaim a number of determinants that could explain foreign direct investment flows, such as ownership advantages of the company, cost reduction, barriers to entry, availabilities of resources, etc. The main theoretical framework includes two important branches: the partial equilibrium analysis of FDI decision and location (Hartman, 1984 and 1985; Slemrod, 1990; Froot and Stein, 1991; Swenson, 1994; Desai, Foley and Hines, 2004, etc.) and the General equilibrium analysis of FDI decision and location (Carr, Markusen and Maskus, 2001; Eckholm, Forslid, and Markusen, 2003 ; Bergstrand and Egger, 2007, etc.). Meantime, the empirical applications mostly use econometric models to asses the relevance of « presupposed determinants » in attracting FDI in various countries (Deichmann *et al*., 2003; Asiedu, 2006; Dunning and Lundan, 2008; Cheung and Qian, 2009; Mohamed and Sidiropoulos, 2010, etc.).

Public goods and FDI

In the literature, a lot of works have been done to study how exogenous macroeconomic factors affect the FDI decisions of the firms using partial equilibrium firm-level or country-level framework. This is because that it is difficult for a general equilibrium model to be tied back to microeconomic decision. The concern with evidence from partial equilibrium models is that they ignore important long-run general-equilibrium factors that affect FDI decisions and locations. This can then lead to omitted variable bias in the empirical specification.

Furthermore, researchers have paid a lot of attentions on the influence of environmental variables such as, institutions, taxes, etc. Public goods are often ignored in the analysis. However, if a country has good quality infrastructure attracts more FDI, (Vijayakumar et al., 2010), it may be expected that there is a strong relationship between this determinant and FDI. But the conclusions are not unanimous, since significant positive relations have been found, while others do not find any statistical evidence that infrastructure attracts FDI (Cleeve, 2008; Mohamed and Sidiropoulos, 2010). The latter finding may be due to the fact that the authors were working with a small scale sample made up of countries with fairly similar structures, and the causality problem between tax, infrastructures, and FDI are not quit well treated in these applications.

Cleeve (2008) used an indicator of the education and skills level of the population in the study, but he did not obtain conclusive results for this indicator either, maybe because of the small variability in the illiteracy rates of the countries in the sample.

As a matter of fact, the relations among tax, infrastructures and FDI are quite complicated in economics. An obvious hypothesis is that higher taxes discourage FDI, but the literature on tax competition underlines the possible compensation of high taxation by the provision of public goods. Therefore, some of the more well-placed articles in the literature have highlighted why such a number may be quite misleading. As these papers point out, the effects of taxes on FDI can vary substantially by type of taxes, measurement of FDI activity, and tax treatment in the host and parent countries.

And in the literature, compare to the richly developed empirical researches, the theoretical research over the relationship between public goods and FDI is incomplete. For example, Kellenberg's discussion paper in 2003 worked on this topic which analyzed the direct and indirect effects of public inputs on the cost of multinationals. However, there are several unclear parts in his research: The research focus on multinational choice instead of government policy. The equilibriums are defined by graphics directly while the mechanisms behind are not clear. Bénassy-Quéré (2005) presented a working paper on 'Tax Competition and Public Input' which tried to review the double function of tax and public service. Using the data of investments from US to 18 EU countries, the paper proved empirically that for some countries like France and Germany, high tax-high public service mode works. However, the theoretical model based on the change of marginal substitution rate between public goods and private goods. The specific positive effects of public goods are not clearly illustrated.

We already know that the double competition is widely accepted theoretically and empirically. However, the more details of a systematically research on 'how do public goods affect FDI flow' is absent and worth to be worked with. The decomposition of the effect process needs to be closely observed. The important economic characteristics which can influence the effect of public inputs on capital flows are worth to be defined. Under what circumstances are the positive effects dominant? The aim of this chapter is to contribute to fill these gaps.

4.3 Basic model

Let us assume a small economy open to international trade and capital flows. There are three sectors which produce three goods:

- $\mathrm{S_A}$ is the agricultural sector; its output is called $\mathrm{X_A}$.
- $\mathrm{S_M}$ is the manufacturing sector; its output is called $\mathrm{X_M}$.
- $\mathrm{S_G}$ is the public service sector; its output is called G.

Both agricultural and manufacturing products are internationally traded. G is the amount of the public input which is produced in the public service sector.

There are several ways to model the production of public goods. The inputs in public sector could be labor, land or capital, or an intermediate input. In our basic model, we follow the standard practice by assuming that public goods are produced with primary factors under a technology characterized by constant returns to scale. For sake of simplicity, we assume that only labor is used (as public good mainly consists of services):

$$G = F_G(L_G) \tag{4.1}$$

The public input is produced by the government and is made freely available to the private-good industries. We start with the assumption of a lump-sum taxation to pay for the production of the public good. This assumption will be extended to the case when the public good is funded by a capital tax, which will be used as a policy instrument.

The public good enters the production function of other sectors. For example, transportation infrastructure, information provision infrastructure or education will influence the private-sector production functions. Kaizuka (1965) introduces a production function with constant returns in all factors, including public input:

$$X_i = F_i(L_i, K_i, G)$$

However this expression cannot represent the factor-augmenting effect of public input over other factors. McMillan (1979) suggests a more realistic specification of the production function as $X_i = F_i(h_{iL}(G)L_i, h_{iK}(G)K_i)$.

Our model follows the assumption that the impact of additional public inputs fits the Hicks-neutral technological change. We take Feehan's (1998) way to model the effect of the public good into the production functions of other sectors. In his model, the impact of additional G is akin to Hicks-neural technological change, which can be represented as a multiplying effect on production functions in both agriculture and manufacturing sectors, as following:

$$X_A = h(G)F_A(L_A, T,) \tag{4.2}$$

$$X_M = g(G)F_M(L_M, K_M^*) \tag{4.3}$$

where L_j is the labor used in sector j (L is the only input which is used in all three sectors), T is the land resource within the economy which is used only in the agricultural sector A; K_M^* is the stock of foreign direct investments, only used in manufacturing sector M.

F_A and F_M are constant returns to scale production functions.

$h(G)$ and $g(G)$ denote the Hicksian impact of G on the production functions of the agricultural and the manufacturing sectors. The function $h(G)$ and $g(G)$ are supposed to be positive, strictly increasing and concave. Moreover, we assume that they are continuously twice differentiable:

$$\frac{dh(G)}{dG} > 0,\ \frac{d^2h(G)}{dG^2} < 0\ and\ \frac{dg(G)}{dG} > 0,\ \frac{d^2g(G)}{dG^2} < 0$$

Labor and land are nationally and internationally immobile resources (with a fixed endowment), while capital is internationally mobile (but nationally immobile).

Let us assume that there is perfect competition in both agriculture and manufacturing sectors. With the small open economy assumption, the good prices in each sector are exogenous and fixed on international markets. Considering the agriculture products as the numéraire, we have:

$$P_A = \overline{P_A} = 1 \tag{4.4}$$

$$P_M = \overline{P_M} \tag{4.5}$$

The country is considered as small on the international market of capital as well. The capital income equals its international level:

$$r = r^* \tag{4.6}$$

Let us define the elasticity of function h (g) with respect to G. This elasticity describes the intensity of the effect of the public good G on the production in sector A (M):

$$E_A = \frac{dh(G)}{dG}\frac{G}{h(G)} \tag{4.7}$$

$$E_M = \frac{dg(G)}{dG}\frac{G}{g(G)} \tag{4.8}$$

The input-output coefficients (from profit maximization) are:

$$a_{iA} = a_{iA}(w, t, G) \quad i = L, T \tag{4.9}$$

$$a_{iM} = a_{iM}(w, r^*, G) \quad i = L, K \tag{4.10}$$

Under perfect competition, producers have zero profit and the cost equals the price. Furthermore, with the small and open economy assumption, the price of tradable products equals its international level. Therefore, we get:

$$a_{LA}w + a_{TA}t = \overline{P_A} = 1 \tag{4.11}$$

$$a_{LM}w + a_{KM}r^* = \overline{P_M} \tag{4.12}$$

$$a_{LG}w = P_G \tag{4.13}$$

Since labor is the unique input in the constant returns to scale public service sector, we can assume that $a_{LG} = 1$ without any loss of generality. Therefore, we have:

$$w = P_G \tag{4.14}$$

The full employment assumption ensures that:

$$a_{LA}x_A + a_{LM}x_M + a_{LG}G = L \tag{4.15}$$

$$a_{TA}x_A = T \tag{4.16}$$

$$a_{KM}x_M = K_M^* \tag{4.17}$$

In order to answer the questions raised in the introduction, we will use the variation form of the model.

With Hicks-Allen elasticity of substitution, the changes of the input-output ratios can be expressed as following:

$$\hat{a}_{ij} = \sum_{h=1}^{n} \theta_{hj} \cdot \sigma_{ih}^{j} \cdot \widehat{w_h} - E_j \cdot \hat{G}$$

The hat form represents the change rate of the variables ($\hat{x} = \frac{dx}{x}$).

i and h denote the primary factors $(L, K \; or \; T)$, and j denotes the sector $(A \; or \; M)$.

n is the total number of private factors.

w_h is the income of factor h.

θ_{ij} denotes the factor cost share of factor i in sector j ($i = L, K, T$ and $j = A, M$).

σ_{ih}^{j} is the Hicks-Allen elasticity between factor i and h in sector j.

The homogeneity condition is:

$$\sum_{h=1}^{n} \theta_{hj} \cdot \sigma_{ih}^{j} = 0$$

The full variation form of zero profit and full employment conditions is given in Appendix 4.1.

In spite of its simplicity, this basic model is enough to analyze the effects of a change in the public good amount on the whole economy, especially on foreign direct investments. We will start with a lump-sum tax, and then we will move to the case of a public good which is financed by a tax on capital income.

4.4 The case of a lump-sum tax

4.4.1 Decomposition of the effect of public input over FDI

When the public service is financed by a lump-sum tax, the government budget constraint equation is:

$$P_G.G = TR \tag{4.18}$$

with TR being the lump-sum transfer.

The amount of public good decided by the government, G, is the policy instrument. Let us look at how the wage rate, the land rent and the capital flows react to a change in the amount of the public good G.

First, we can get the relationship between the change rate of the wage rate and the public good (see Appendix 4.2):

$$\widehat{w} = \frac{E_M}{\theta_{LM}}\widehat{G} \tag{4.19}$$

As $E_M > 0$ and $\theta_{LM} > 0$, we always have $\frac{E_M}{\theta_{LM}} > 0$.

This means that any additional amount of the public service ($\widehat{G} > 0$) always increases the wage rate in the economy ($\widehat{w} > 0$). This is because it raises the domestic marginal product of labor and capital in sector M. The domestic capital revenue goes up: $r > r^*$. This extra revenue attracts foreign capital into the economy, which also raises the marginal product of labor. The wage rate increases further while the domestic marginal productivity of capital goes back to its international level r^*. Therefore, labor reaps all the additional product of M due to the additional amount of G.

Then, let us look at how the land rent reacts to a public good change. The Appendix 4.3 gives the relationship between the change rates of land rent and public goods:

$$\hat{t} = \left(-\frac{\theta_{LA}}{\theta_{TA}}\frac{E_M}{\theta_{LM}} + \frac{E_A}{\theta_{TA}}\right)\widehat{G} = \left(\frac{E_A}{\theta_{LA}} - \frac{E_M}{\theta_{LM}}\right)\frac{\theta_{LA}}{\theta_{TA}}\widehat{G} \tag{4.20}$$

The impact of a public good change on the land income depends on the sign of $(\frac{E_A}{\theta_{\mathrm{LA}}} - \frac{E_M}{\theta_{LM}})$. There are two effects at the same time. First, any increase of G acts as an Hicksian technical progress, which raises the land income t (in proportion to $\frac{E_A}{\theta_{\mathrm{LA}}}$). At the same time, it increases the wage rate w, which comes into conflict with the land income (in proportion to $\frac{E_M}{\theta_{LM}}$). The final effect on t will depend on the strength of these two forces.

If the public good has a stronger effect in sector A than in sector M ($E_A > E_M$) and sector M is more labour-intensive than sector A ($\theta_{\mathrm{LA}} < \theta_{LM}$), then land income will rise as a final result of the increase in G.

Let us now focus on the effect of a public good variation on foreign capital flows, $\widehat{\mathrm{K}_{\mathrm{M}}^{*}}$. Under the lump-sum tax assumption, we get the following equation (see Appendix 4.5):

$$\widehat{K_M^*} = \hat{G}.\left[\underbrace{\frac{\sigma_{LK}^{M}}{\theta_{LM}}.E_M}_{a} - \underbrace{\frac{\lambda_{LG}}{\lambda_{LM}}}_{b} + \underbrace{\left(\frac{E_M}{\theta_{LM}} - E_A\right)\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^{A}}{\theta_{TA}}}_{c}\right] \tag{4.21}$$

λ_{Lj} denotes the ratio of total labor used in sector j $(\frac{L_j}{L})$, and θ_{ij} is the factor cost share of factor i in sector j ($i = L, K, M$ and $j = A, T, G$).

There are three effects working on foreign direct investment flows when the amount of public service increases.

<u>First effect</u>: a (always positive).

This is an indirect effect of the increase in the marginal productivity of capital. It works through a change in factor prices. We have already seen that any additional amount of G raises the domestic marginal product of labor and capital in sector M. The wage rate increases while foreign capital flows into the country. This FDI inflow brings the domestic marginal productivity of capital back to its international level r^* and raises further the marginal product of labor. (This is why labor reaps all the additional product of M due to the additional amount of G, as expressed by the equation: $\widehat{w} = \frac{E_M}{\theta_{LM}} \widehat{G}$.) What does the additional amount of foreign capital become? As the ratio w/r increases (the price of labor goes up relative to the price of capital), firms want to substitute capital for labor. More units of capital are used per unit of labor, which relaxes the constraint on output due to the fixed amount of labor in M and lets more M output to be produced. More foreign direct investment will be attracted into the host country.

<u>Second effect</u>: b (always negative).

Any expansion of G requires more labor. This labor is taken from sector M only. This is because sector M has a variable and internationally mobile specific factor, whereas sector A has a fixed and internationally immobile specific factor[11]. The decline of labor in sector M decreases the marginal product of capital, which makes capital flow out of the country. (These outflows of both labor and capital induce a decline of sector M's output.)

<u>Third effect</u>: c (ambiguous: can be positive or negative).

More public input G raises the marginal product of labor in both sectors, A and M, but not in the same proportion. It increases by E_A in sector A and by $\frac{E_M}{\theta_{LM}}$ in sector M.

If $E_A < \frac{E_M}{\theta_{LM}}$, then labor moves from sector A towards sector M. This increases the marginal product of capital in sector M and foreign capital flows into the country. (Sector's M output increases.)

If $E_A > \frac{E_M}{\theta_{LM}}$, labor moves from sector M towards sector A. The marginal product of capital decreases, making capital flow out of the country. (Sector's M output decreases.)

[11] Let us assume that sector G takes labor from both sectors, M and A. In both sectors, the marginal product of labor increases, while the marginal product of the specific factor decreases. As capital is internationally mobile, it flows out, which reduces the marginal product of labor in sector M back to its initial level. This does not happen in sector A, as land is internationally immobile. Therefore, labor moves from sector M to sector A until the marginal products of labor and land come back to their initial level in A. Finally, all the labor needed by sector G comes from sector M and sector A is not affected.

With regard to the effect of an increase in G on the output of the manufacturing sector, Appendix 4.4 gives us the following expression:

$$\widehat{X_M}=\widehat{G}.\left[\underbrace{\theta_{KM}.\frac{\sigma_{LK}^{M}}{\theta_{LM}}.E_M}_{a}-\underbrace{\frac{\lambda_{LG}}{\lambda_{LM}}}_{b}+\underbrace{\left(\frac{E_M}{\theta_{LM}}-E_A\right)\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^{A}}{\theta_{TA}}}_{c}+\underbrace{E_M}_{d}\right] \tag{4.22}$$

The first three effects (a, b and c) are closely linked to the three above described effects[12]. The fourth effect (d) is the (positive) Hicksian productivity effect of the public input G on M's output[13].

4.4.2 Some propositions

Several propositions can be drawn from these results.

Proposition 4.1: *If the size of the government is small enough, then any increase in the amount of public good will attract foreign direct investments into the country.*

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[12] With regard to effect **a**, recall that more public input increases the ratio w/r. Firms substitute capital for labour and more units of capital are used per unit of labour, which relaxes the constraint due to the fixed amount of labour in M and increases M's output. However, because the increase of public input also reduces the capital requirement per unit of output, this positive effect on output is partly offset. Therefore, the variation of output in sector M is less than the variation of capital.

[13] While having a direct effect on M's output, the variations of G have no direct effect on M's capital. This is because any increase in G reduces the capital requirement for each unit of M's output ($\hat{a}_{KM}$<0).

The size of the government can be captured by the ratio of total labor used in sector G: λ_{LG} $(= \frac{L_G}{L})$. Equation (4.21) tells us that:

$$\frac{\widehat{K_M^*}}{\hat{G}} > 0 \quad if \ \lambda_{LG} < \left(\frac{E_M}{\theta_{LM}} - E_A\right)\frac{\lambda_{LA}\sigma_{LT}^A}{\theta_{TA}} + \frac{E_M \lambda_{LM}}{\theta_{LM}} . \sigma_{LK}^M$$

Let us call Γ_λ the right-hand side of the inequality.

If the size of the government is small enough (less than the threshold Γ), any increase of the public good will attract FDI inflow. However, if the size of the government, in terms of employment, exceeds the threshold Γ_λ, then any additional of public good will make FDI flow out of the country:

$$\frac{\widehat{K_M^*}}{\hat{G}} < 0 \quad if \ \ \lambda_{LG} > \Gamma_\lambda = \left(\frac{E_M}{\theta_{LM}} - E_A\right)\frac{\lambda_{LA}\sigma_{LT}^A}{\theta_{TA}} + \frac{E_M \lambda_{LM}}{\theta_{LM}} . \sigma_{LK}^M$$

This is explained by the effect b in equation (4.21).

The policy suggestion deduced from proposition 4.1 is that the government should be aware of its size when considering a policy of public input provision to attract FDI. A very large public sector could be detrimental for attracting foreign capital.

Proposition 4.2: *When the elasticity of the Hicksian impact function* g *with respect to the public input in the multinational sector (E_M) is large enough, any increase of public services will attract foreign capital.*

According to equation (4.21), the effect of the public input over FDI also depends on the elasticity E_M. (Recall that $E_M = \frac{dg(G)}{dG}\frac{G}{g(G)}$.) This elasticity describes the intensity of the effect of the public good G on the production in sector M.

This elasticity must be high enough in order for the public input to have an attractive effect on FDI. It is only if the output of M is enough sensitive to the public input G that any increase in G will attract FDI:

$$\frac{\widehat{K_M^*}}{\hat{G}} > 0 \quad if \ E_M > \frac{E_A \theta_{LM} \frac{\lambda_{LA}\sigma_{LT}^A}{\theta_{TA}} + \lambda_{LG}\theta_{LM}}{\sigma_{LK}^M \lambda_{LM} + \frac{\lambda_{LA}\sigma_{LT}^A}{\theta_{TA}}} \quad (= \Gamma_{EM})$$

If the production function of the manufacturing sector does not react enough to G ($E_M < \Gamma_{EM}$), then any increase in G will drive the international capital out of the domestic country ($\frac{\widehat{K_M^*}}{\hat{G}} < 0$).

In a similar way, the sensitivity of sector A to the public input G, i.e E_A, also matters.

Proposition 4.3: *If the elasticity of the Hicksian impact function* h *with respect to the public input in the agricultural sector (E_A) is too large, then any increase in G will be detrimental to FDI.*

From equation (4.21), we get the following condition:

$$\frac{\widehat{K_M^*}}{\hat{G}} < 0 \quad if \ E_A > \frac{E_M}{\theta_{LM}} . \left(1 + \frac{\lambda_{LM}}{\lambda_{LA}} \frac{\theta_{TA}}{\sigma_{LT}^A} \sigma_{LK}^M\right) - \frac{\lambda_{LG}}{\lambda_{LM}} \frac{\lambda_{LM}}{\lambda_{LA}} \frac{\theta_{TA}}{\sigma_{LT}^A}$$

Let us call Γ_{EA} the right-hand side of the inequality. FDI will decline as a result of an additional amount of G if the production function of the agricultural sector A reacts too much to G, i.e. if its elasticity E_A exceeds the threshold Γ_{EA}. This is explained by effect c in equation (4.21). We have seen that more public input G raises the marginal product of labor in both sectors, A and M, but not in the same proportion. If it increases more in sector A than in sector M ($E_A > \Gamma_{EA}$), then labor moves from sector M towards sector A. The marginal product of capital decreases, making capital flow out of the country.

Therefore, when the government makes a policy choice to attract FDI, it should closely look at the characteristics of the private sectors, especially at their sensitivity to the public input.

Besides the sensitivity of the private sectors to the public input, there are other characteristics that influence the effect of G on FDI. One of them is the unit cost of labor in the multinational sector θ_{LM} $(=\frac{w.a_{LM}}{P_M.X_M})$, which also represents the labor intensity in sector M.

Proposition 4.4: *The more labor intensive the manufacturing sector, the more likely the benefits of an increase in G on FDI.*

$$\frac{\widehat{\mathrm{K}_{\mathrm{M}}^{*}}}{\hat{G}} > 0 \text{ if } \theta_{LM} > \frac{\sigma_{LK}^{M}\lambda_{\mathrm{LM}}\theta_{\mathrm{TA}}+\lambda_{\mathrm{LA}}\sigma_{LT}^{A}}{\lambda_{\mathrm{LG}}\theta_{\mathrm{TA}}+E_A\lambda_{\mathrm{LA}}\sigma_{LT}^{A}}E_M \ (=\Gamma_{LM})$$

The policy suggestion deduced from proposition 4.4 is that the government should take into account the labor intensity of the multinational sector when considering a policy of public goods to attract FDI. In an economy with a very capital intensive manufacturing sector (small θ_{LM}, at least smaller than Γ_{LM}), any policy supplying more public input will be detrimental for attracting foreign capital.

4.4.3 First best optimum with lump-sum taxation

This sector provides an analysis of the optimality rule for provision of the Hicksian public input in the presence of FDI and a lump-sum tax.

Recall that under the assumption of a small open economy, the prices of the products are exogenous. The demand for agricultural and manufactured goods are D_A and D_M. The income (Y)-expenditure equation is:

$$D_A + P_M.D_M = Y = X_A + P_M.X_M + P_G.G - r^*.K - TR \tag{4.23}$$

where TR is the lump-sum taxation.

The government's public budget constraint is:

$$P_G.G = TR \tag{4.24}$$

Since the prices are fixed, the variation of the real income (dy) equals to the variation of the national income (dY):

$$dy = dD_A + P_M.dD_M = dY = (dX_A + P_M.dX_M + P_G.dG) + G.dP_G - r^*.dK - dTR \tag{4.25}$$

Because of the small country assumption, the rental rate of capital, r^*, is exogenous ($dr^* = 0$). The variation form of the public budget constraints is:

$$P_G.dG + G.dP_G = dTR \tag{4.26}$$

In variation form, the full employment of labor is:

$$dL_A + dL_M + dL_G = d\bar{L} = 0 \tag{4.27}$$

We know that the production functions are:

$$X_A = h(G)F_A(L_A, T,)$$

$$X_M = g(G)F_M(L_M, K_M^*)$$

$$G = F_G(L_G)$$

In variation form, we get:

$$dX_A = F_A(L_A, T).h'(G).dG + h(G).\frac{\partial F_A(L_A,T)}{\partial L_A}.dL_A + h(G).\frac{\partial F_A(L_A,T)}{\partial T}.dT$$

$$dX_M = F_M(L_M, K).g'(G).dG + g(G).\frac{\partial F_M(L_M,K)}{\partial L_M}.dL_M + g(G).\frac{\partial F_M(L_M,K)}{\partial K}.dK$$

$$dG = dL_G \tag{4.28}$$

After extracting dL_A and dL_M from these equations and substituting them into (4.27), assuming that land is immobile and that its endowment is fixed (d$T = 0$), using (4.25), (4.26) and the perfect competition conditions, we get the variation of the real income, dy.

As d$y = 0$ at the optimum, we finally obtain the following first-best optimality condition:

$$E_A.\left(\frac{X_A}{G}\right) + E_M.\left(\frac{P_M.X_M}{G}\right) = P_G \tag{4.29}$$

Or, using the definitions of the E_j:

$$\left(\frac{\partial X_A}{\partial G}\right) + P_M.\left(\frac{\partial X_M}{\partial G}\right) = P_G \tag{4.29'}$$

This expression is consistent with the Kaizuka (1965) first-best optimality condition for public good provision and offers a clear insight as to the marginal benefit and cost of an extra public input. The left-hand side of (4.29) is the marginal social benefit of the public input G. It is the sum of the value of the marginal products of G across sectors: any extra public input adds to the real income by increasing outputs (in value) across sectors in proportion to their elasticity with respect to G.

P_G measures the opportunity cost of G (i.e the change in value of the private goods production resulting from a withdrawal of labor to make one unit of G).

The condition (4.29) tells us that the public input G should be used up to the point where its marginal social benefit equals to its marginal social cost (or opportunity cost, P_G). This optimal rule for provision of public input is the same as in Feehan (1998) with no FDI. The presence of FDI does not change the rule to achieve the optimal G in the lump-sum tax case. However, this will change when the public good provision is financed by a tax on capital.

4.5 Financing the public input by a tax on capital income

4.5.1 Decomposition of the effect of public input over FDI

Let now assume that the public input is financed through a tax levied on capital income $r \cdot K_M^*$. The government's budget constraint is:

$$P_G \cdot G = \tau \cdot r \cdot K_M^* \tag{4.30}$$

Where τ is the tax rate ($0 < \tau < 1$). This tax rate creates a wedge between the national and the international rates of return on capital. The arbitrage condition ensures that

$$r \cdot (1 - \tau) = r^* \tag{4.31}$$

At the equilibrium, the domestic **net** rate of return to capital is equal to the international rate of return to capital. The domestic **gross** rate of return is given by

$$r = \frac{r^*}{(1-\tau)} \tag{4.32}$$

Given the assumption of a small open economy, r^* can be considered as given. The variation form of (4.32) is

$$\hat{r} = \frac{\tau}{(1-\tau)}\hat{\tau} \tag{4.33}$$

Using (4.14), (4.30) and (4.32), the government's budget constraint in variation form can be expressed as

$$\widehat{w} + \widehat{G} = \hat{\tau} + \frac{\tau}{(1-\tau)}\hat{\tau} + \widehat{K_M^*} \tag{4.34}$$

The tax on capital income τ is now the policy instrument. The government controls the tax rate to influence the amount of public input, which serves as an indirect instrument. The question is now: how does a variation of the tax rate influence FDI?

Equation (4.34) can be rewritten as

$$\widehat{G} = \frac{1}{(1-\tau)}\hat{\tau} + \widehat{K_M^*} - \widehat{w} \tag{4.35}$$

Contrary to the lump-sum tax case, a variation of the amount of public input is not due to a tax modification only, but also on the induced changes of foreign capital and the wage rate. The details will be explained below.

In Appendix 4.6 we calculate the expression of $\widehat{\mathrm{K}_\mathrm{M}^*}$:

$$\widehat{K_M^*} = \left(\frac{1}{1-\Omega}\right).\left(\frac{\theta_{LM}}{(1-\tau).(\theta_{LM}+E_M)}\right).\left\{\frac{\sigma_{LK}^M}{\theta_{LM}}.\left[\underbrace{E_M}_{a} - \underbrace{\tau.(1+E_M)}_{a'}\right] - \frac{\lambda_{LG}}{\lambda_{LM}}.\left(\underbrace{1}_{b} + \underbrace{\tau.\frac{\theta_{KM}}{\theta_{LM}}}_{b'}\right) + \left(\underbrace{\frac{E_M}{\theta_{LM}} - E_A}_{c} - \underbrace{\tau.\frac{\theta_{KM}}{\theta_{LM}}.[1+E_A]}_{c'}\right).\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^A}{\theta_{TA}}\right\}.\hat{\tau} \qquad (4.36)$$

with

$$\Omega = \frac{\theta_{LM}}{(\theta_{LM}+E_M)}.\left\{\frac{\sigma_{LK}^M}{\theta_{LM}}.E_M - \frac{\lambda_{LG}}{\lambda_{LM}} + \left(\frac{E_M}{\theta_{LM}} - E_A\right).\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^A}{\theta_{TA}}\right\}$$

Any increase of τ provides more tax, and more public input can be financed and produced. This triggers the same three mechanisms as in the case when the public input G is financed by a lump-sum tax (terms a, b and c in equation (4.36)).

However, an increase of τ has also many additional effects as it changes the relative price of factors.

1) In sector M, it lowers the domestic net return to capital, which becomes less than the international rate: $(1-\tau).r < r^*$. This makes capital go out of the country ($\widehat{K_M^*} < 0$). It is expressed by term (a') in equation (4.36). The outflow of FDI raises the marginal product of capital (therefore the domestic rental cost of capital r), and decreases the marginal product of labor in sector M (therefore the wage rate): $r\uparrow$, $w\downarrow$.

This fall of the wage rate w induced by the increase of τ gives rise to other capital flow movements through its effects on both sectors A and G.

Before examining these effects in A and G, let us first determine the net result of an increase in τ in sector M. This net result is the combination of effects (a) and (a'). Let us recall that effect (a) is caused by the additional amount of G (financed by the increased tax). By raising the domestic marginal product of capital and labor, it attracts capital into the country ($\widehat{K_M^*} > 0$). This FDI inflow makes the domestic marginal product of capital decline and raises further the marginal product of labor. The wage rate goes up (w↑), whereas the rental cost of capital goes back to its initial level.

If effect (a) is larger than effect (a') [$\tau < E_M/1 + E_M$ in equation (36)], then more capital is coming into the country than going out of it. As the wage rate rises relative to the rate of return to capital (w/r↑), firms in sector M substitute the incoming capital for labor.

If effect (a') is larger than effect (a) [$\tau > E_M/1 + E_M$ in equation (4.36)], then more capital is going out of the country than coming into it. As the wage rate declines relative to the rate of return to capital (*w/r*↓), firms in sector M substitute labor to the lost capital.

2) In sector G, the fall of the wage rate w induced by the increase of τ relaxes the government's budget constraint [see equation (4.35): $\hat{G} = \frac{1}{(1-\tau)}\hat{\tau} + \widehat{K_M^*} - \hat{w}$]. This enables the government to provide more public input by taking labor from sector M. This withdrawal of labor from sector M reduces the marginal product of capital and drives more capital away (term b' in equation (4.36)).

3) In sector A, the same fall of the wage rate w induced by the increase of τ puts up the rental price of land (t↑). This increase of the rental price t is accentuated by the rise of the marginal product of land since more public input has been made available by the falling wage rate (relaxed budget constraint). This raises sector A's profitability and therefore its demand for labor. As sector A takes labor from sector M, the marginal product of capital declines and more capital flows out of the country (term c' in equation (4.36)).

Moreover, with a public input financed by a tax levied on capital income, there is also a multiplier effect, as revealed by the term $\frac{1}{1-\Omega}$ in equation (4.36). Any movement of capital changes the tax base, the government's budget constraint and thus the amount of public input available for firms [equation (4.35): $\hat{G} = \frac{1}{(1-\tau)}\hat{\tau} + \widehat{K_M^*} - \hat{w}$]. Let us take the example of a capital inflow. A capital inflow widens the tax base and augments the available amount of public input G. This extra public input triggers the same three mechanisms (a), (b) and (c) as in the case of a lump-sum tax (as revealed by Ω), except that the variations are now smaller[14]. If the net effect of (a), (b) and (c) is positive, then more capital flows into the country, widening again the tax base… The final result is an infinite geometric series whose common ratio is Ω.

Of course, the absolute value of Ω has to be less than one in order for the series to converge to a finite sum (this ensures the existence of a multiplier effect: $\frac{1}{1-\Omega} > 1$ and the stability of the model).

Let us make this assumption from now on: $-1 < \Omega < 1$.

Several results can be drawn from equation (4.36).

Proposition 4.5: a decrease of a capital income tax does not always attract FDI inflow.

[14] The induced increase of the wage rate tightens the government's budget constraint and leads to a reduced final amount of public input G.

A popular view, especially for developing countries, is that tax reductions can be used as a fiscal policy to attract FDI. The rationale is the following: if the government reduces tax rates on foreign enterprises, they earn more and will prefer to invest in the host country. However, our model tells us that this proposition may not be true when a public input is financed by capital taxation.

This is because the sign of $\frac{\widehat{K_M^*}}{\hat{\tau}}$ is ambiguous in (4.36). Especially, this sign can be positive. Since $-1 < \Omega < 1$ (convergence of the geometric series) and $\frac{\theta_{LM}}{(1-\tau).(\theta_{LM}+E_M)} > 0$, a necessary and sufficient condition is:

$$\frac{\sigma_{LK}^M}{\theta_{LM}}.[E_M - \tau.(1+E_M)] - \frac{\lambda_{LG}}{\lambda_{LM}}.\left(1+\tau.\frac{\theta_{KM}}{\theta_{LM}}\right) + \left(\frac{E_M}{\theta_{LM}} - E_A - \tau.\frac{\theta_{KM}}{\theta_{LM}}.[1+E_A]\right).\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^A}{\theta_{TA}} \boldsymbol{>0}$$

This condition can be fulfilled for different values of the parameters.

Let us for example consider the size of the government, as captured by the share of total labor used in sector G: λ_{LG}. The size of the government has to be less than a given threshold in order for an income tax reduction to be detrimental for FDI ($\hat{\tau} < 0 \longrightarrow \widehat{K_M^*} < 0$):

$$\frac{\widehat{K_M^*}}{\hat{\tau}} > 0 \text{ if } \lambda_{LG} < \Gamma_{\lambda\tau}\text{, with}$$

$$\Gamma_{\lambda\tau} = \frac{\lambda_{LM}}{\left(1+\tau.\frac{\theta_{KM}}{\theta_{LM}}\right)}.\left\{\frac{\sigma_{LK}^M}{\theta_{LM}}.[E_M - \tau.(1+E_M)] + \left(\frac{E_M}{\theta_{LM}} - E_A - \tau.\frac{\theta_{KM}}{\theta_{LM}}.[1+E_A]\right).\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^A}{\theta_{TA}}\right\}$$

It can be seen that, the larger the *initial* income tax rate τ, the smaller the threshold $\Gamma_{\lambda\tau}$, and thus the smaller the size of the government has to be for the unconventional result to happen.

A similar remark holds for the elasticity E_A (elasticity of the Hicksian impact function h with respect to the public input in the agricultural sector): the larger the elasticity E_A (sensitivity of the production in sector A to the public input), the smaller the threshold $\Gamma_{\lambda\tau}$, and thus the smaller the size of the government has to be. This is explained by mechanisms (c) and (c').

Note that the elasticity E_M has an opposite effect on the critical size of the government: the larger it is, the larger $\Gamma_{\lambda\tau}$, and thus the larger the size of the government can be.

To sum up, in opposition to the usual view, a capital income tax reduction can be detrimental for FDI provided that:

- the size of the government is small
- the production of the agricultural sector is not very sensitive to the public input
- the production of the manufacturing sector is very sensitive to the public input
- the manufacturing sector M is labor intensive (high θ_{LM} and low θ_{KM}).

4.6 Conclusion

This chapter works specifically on the relationship between public inputs and FDI flow. We follow the assumption that local public goods have multiply effect on production function. In our basic model concerning only the public inputs, the products of public goods sector affect the production function in the other two sectors (agriculture sector and manufacturing sector). We start with the case when public goods are financed by lump-sum tax.

Our main conclusion is that we find the decompositions of the effect of public input over capital flow. The results shows that if labor is the only shared factor among all three sectors, the effect of public inputs over capital flow works throw three sub-effects: the augmentation effect of public inputs, the labor outflow effect generated by the expansion of public sector, and the substitute of labor between A and M sector. The combination of these three effects decide whether the increase of public inputs have positive or negative effect on capital inflow. Then we point out several important economy factors which can influence the effect of public inputs over capital inflow. To make sure a positive effect of public goods, the size of the public sector should be small enough; the sensitivity of production function to public goods in M sector should be large while that of A sector should be small; the labor intensity in M sector should be large too.

Then we try to define the first best optimum and find that the public input G should be used up to the point where its marginal social benefit equals to its marginal social cost.

In the final part we work on a model when the public inputs are financed by a tax on capital income. Now the fiscal instrument is taxation instead of public goods itself. We get the similar conclusion and point out that the decrease of taxation on capital won't always help to attract FDI. These propositions are helpful for the authorities to understand the mechanism of a fiscal policy and to make appropriate fiscal choices.

Appendix chapter 4

Appendix 4.1

The Hicks-Allen elasticity of substitution $\hat{a}_{ij} = \sum_{h=1}^{n} \theta_{hj} \cdot \sigma_{ih}^{j} \cdot \widehat{w_h} - E_j \cdot \hat{G}$ gives us the following ratio variation equations:

$$\hat{a}_{LA} = \theta_{LA}\sigma_{LL}^{A}\widehat{w} + \theta_{TA}\sigma_{LT}^{A}\hat{t} - E_A\hat{G} \quad (A4.1.1)$$

$$\hat{a}_{TA} = \theta_{LA}\sigma_{LT}^{A}\widehat{w} + \theta_{TA}\sigma_{TT}^{A}\hat{t} - E_A\hat{G} \quad (A4.1.2)$$

$$\hat{a}_{LM} = \theta_{LM}\sigma_{LL}^{M}\widehat{w} + \theta_{KM}\sigma_{LK}^{M}\hat{r} - E_M\hat{G} = \theta_{LM}\sigma_{LL}^{M}\widehat{w} - E_M\hat{G} \quad (A4.1.3)$$

$$\hat{a}_{KM} = \theta_{LM}\sigma_{KL}^{M}\widehat{w} + \theta_{KM}\sigma_{KK}^{M}\hat{r} - E_M\hat{G} = \theta_{LM}\sigma_{KL}^{M}\widehat{w} - E_M\hat{G} \quad (A4.1.4)$$

With the small and open economic assumption, at the equilibrium, capital revenue equals the international one, which can be considered to be fixed. There is always $\hat{r} = 0$.

The homogeneity conditions $\sum_{h=1}^{n} \theta_{hj} \cdot \sigma_{ih}^{j} = 0$ are:

$$\theta_{LA}\sigma_{LL}^{A} + \theta_{TA}\sigma_{LT}^{A} = 0 \quad (A4.1.5)$$

$$\theta_{LA}\sigma_{LT}^{A} + \theta_{TA}\sigma_{TT}^{A} = 0 \quad (A4.1.6)$$

$$\theta_{LM}\sigma_{LL}^{M} + \theta_{KM}\sigma_{LK}^{M} = 0 \quad (A4.1.7)$$

$$\theta_{LM}\sigma_{KL}^{M} + \theta_{KM}\sigma_{KK}^{M} = 0 \quad (A4.1.8)$$

The ratio variation forms of zero profit condition (equations 4.11-4.13) are:

$$\frac{a_{LA}w}{P_A}(\hat{a}_{LA}+\widehat{w})+\frac{a_{TA}t}{P_A}(\hat{a}_{TA}+\hat{t})=0$$

$$\frac{a_{LM}w}{P_M}(\hat{a}_{LM}+\widehat{w})+\frac{a_{KM}r}{P_M}(\hat{a}_{KM}+\hat{r})=0$$

θ_{ij} denotes the ratio cost of factor i for product in sector j

$$\theta_{LA}=\frac{a_{LA}w}{P_A}$$

$$\theta_{TA}=\frac{a_{TA}t}{P_A}$$

$$\theta_{LM}=\frac{a_{LM}w}{P_M}$$

$$\theta_{KM}=\frac{a_{KM}r}{P_M}$$

The equations can be expressed as:

$$\theta_{LA}\widehat{w}+\theta_{TA}\hat{t}=-\theta_{LA}\hat{a}_{LA}-\theta_{TA}\hat{a}_{TA} \qquad (A4.1.9)$$

$$\widehat{w}\theta_{LM}=-\theta_{LM}\hat{a}_{LM}-\theta_{KM}\hat{a}_{KM} \qquad (A4.1.10)$$

Similarly, denote λ_{Lj} as the ratio of labour used in sector j, the ratio variation forms to express the full employment condition (equations 4.14 to 4.16) are as followed:

$$\hat{a}_{LA}\lambda_{LA}+\hat{x}_A\lambda_{LA}+\hat{a}_{LM}\lambda_{LM}+\hat{x}_M\lambda_{LM}+\hat{a}_{LG}\lambda_{LG}+\hat{x}_G\lambda_{LG}=\hat{L}=0 \qquad (A4.1.11)$$

$$\hat{a}_{TA}+\hat{x}_A=\hat{T}=0 \qquad (A4.1.12)$$

$$\hat{a}_{KM}+\hat{x}_M=\widehat{K^*}_M \qquad (A4.1.13)$$

With $\lambda_{LA} = \frac{a_{LA}x_A}{L}$, $\lambda_{LM} = \frac{a_{LM}x_M}{L}$, $\lambda_{LG} = \frac{a_{LG}G}{L}$ denote the labor share which represents the share of total labor endowment $\bar{L}$ used by sector *j*.

The labor and land resource are constant in host country, so the change rate of them are zero ($\hat{L} = 0$ and $\hat{T} = 0$).

Appendix 4.2

With the ratio form of our model, firstly we try to determine the reaction of wage rate change over public goods changes.

Substitute $\hat{a}_{LM}$ and $\hat{a}_{KM}$ in equation (A4.1.10) with (A4.1.3) and (A4.1.4):

$$\hat{w}\theta_{LM} = -\theta_{LM}\hat{a}_{LM} - \theta_{KM}\hat{a}_{KM}$$

$$= -\theta_{LM}(\theta_{LM}\sigma_{LL}^{M}\hat{w} - E_M\hat{G}) - \theta_{KM}(\theta_{LM}\sigma_{KL}^{M}\hat{w} - E_M\hat{G})$$

So that:

$$(1 + \theta_{LM}\sigma_{LL}^{M} + \theta_{KM}\sigma_{KL}^{M})\hat{w} = (1 + \frac{\theta_{KM}}{\theta_{LM}})E_M\hat{G}$$

With (A1.7), there is $\theta_{LM}\sigma_{LL}^{M} = -\theta_{KM}\sigma_{KL}^{M}$

$$\hat{w} = \left(1 + \frac{\theta_{KM}}{\theta_{LM}}\right)E_M\hat{G} \qquad (A4.2.1)$$

With equation (4.8), there is $\hat{w} = (1 + \frac{\theta_{KM}}{\theta_{LM}})\frac{dh(G)}{dG}\frac{G}{h(G)}\hat{G}$

With the definition of θ_{ij} and the zero profit assumption, there is:

$$\theta_{LM} + \theta_{KM} = 1$$

Then (A2.1) can be changed into:

$$\widehat{w} = \frac{E_M}{\theta_{LM}}\widehat{G} \qquad (A4.2.2)$$

$$E_M > 0, \theta_{LM} > 0 \Rightarrow \frac{E_M}{\theta_{LM}} > 0$$

$$If\ \widehat{G} > 0,\ \widehat{w} > 0$$

Appendix 4.3

Substitute (A4.1.1), (A4.1.2) in to the ratio variation forms of zero profit condition (A4.1.9), we get:

$$\theta_{LA}\widehat{w} + \theta_{TA}\hat{t} = -\theta_{LA}(\theta_{LA}\sigma_{LL}^{A}\widehat{w} + \theta_{TA}\sigma_{LT}^{A}\hat{t} - E_A\widehat{G}) - \theta_{TA}(\theta_{LA}\sigma_{LT}^{A}\widehat{w} + \theta_{TA}\sigma_{TT}^{A}\hat{t} - E_A\widehat{G})$$

$$\theta_{TA}\hat{t}(1 + \theta_{LA}\sigma_{LT}^{A} + \theta_{TA}\sigma_{TT}^{A}) = -\theta_{LA}\widehat{w}(1 + \theta_{LA}\sigma_{LL}^{A} + \theta_{TA}\sigma_{LT}^{A}) + (\theta_{LA} + \theta_{TA})E_A\widehat{G}$$

$\hat{t}$ can be presented by the following equation:

$$\hat{t} = -\frac{\theta_{LA}}{\theta_{TA}}\widehat{w} + \frac{(\theta_{LA}+\theta_{TA})}{(1+\theta_{LA}\sigma_{LT}^{A}+\theta_{TA}\sigma_{TT}^{A})\theta_{TA}}E_A\widehat{G}$$

With $(A4.1.6)$, there is $\theta_{LA}\sigma_{LT}^{A} = -\theta_{TA}\sigma_{TT}^{A}$. The equation can be changed as:

$$\hat{t} = -\frac{\theta_{LA}}{\theta_{TA}}\widehat{w} + \frac{E_A}{\theta_{TA}}\widehat{G}$$

Substitute $\widehat{w}$ with (A4.2.2), there is:

$$\hat{t} = \left(-\frac{\theta_{LA}}{\theta_{TA}}\frac{E_M}{\theta_{LM}} + \frac{E_A}{\theta_{TA}}\right)\widehat{G} = \left(\frac{E_A}{\theta_{LA}} - \frac{E_M}{\theta_{LM}}\right)\frac{\theta_{LA}}{\theta_{TA}}\widehat{G} \qquad (A4.3.1)$$

Appendix 4.4

Equation (A1.12) can be changed as the expression of the change rate of outputs in sector A ($\hat{x}_A$):

$$\hat{x}_A = -\hat{a}_{TA} = -\theta_{LA}\sigma_{LT}^{A}\hat{w} - \theta_{TA}\sigma_{TT}^{A}\hat{t} + E_A\hat{G} = \theta_{LA}\sigma_{LT}^{A}(\hat{t} - \hat{w}) + \frac{dg(G)}{dG}\frac{G}{g(G)}\hat{G}$$

Substitute with $\hat{w}$ and $\hat{t}$ with equation with (4.18), (4.19), there is:

$$\hat{x}_A = \theta_{LA}\sigma_{LT}^{A}\left[\left(\frac{E_A}{\theta_{LA}} - \frac{E_M}{\theta_{LM}}\right)\frac{\theta_{LA}}{\theta_{TA}}\hat{G} - \frac{E_M}{\theta_{LM}}\hat{G}\right] + E_A\hat{G} = \{[\frac{\theta_{LA}\sigma_{LT}^{A}}{\theta_{LM}\theta_{TA}}[(\theta_{LM}E_A - \theta_{LA}E_M) - E_M] + E_A\}\hat{G}$$

$$= \left\{\frac{\theta_{LA}\sigma_{LT}^{A}}{\theta_{LM}\theta_{TA}}[(\theta_{LM}E_A - \theta_{LA}E_M) - \theta_{TA}E_M] + E_A\right\}\hat{G}$$

$$= \left\{\frac{\theta_{LA}\sigma_{LT}^{A}}{\theta_{LM}\theta_{TA}}(\theta_{LM}E_A - E_M) + E_A\right\}\hat{G} \qquad (A\ 4.4.1)$$

$$= [\frac{\theta_{LA}\sigma_{LT}^{A}}{\theta_{TA}}E_A - \frac{\theta_{LA}\sigma_{LT}^{A}}{\theta_{LM}\theta_{TA}}E_M + E_A]\hat{G}$$

Equation (A4.1.11) can be changed as:

$$\hat{x}_A\lambda_{LA} + \hat{x}_M\lambda_{LM} + \hat{G}\lambda_{LG} = -\hat{a}_{LA}\lambda_{LA} - \hat{a}_{LM}\lambda_{LM} - \hat{a}_{LG}\lambda_{LG}$$

$$\hat{x}_M\lambda_{LM} = -\hat{x}_A\lambda_{LA} - \hat{G}\lambda_{LG} - \hat{a}_{LA}\lambda_{LA} - \hat{a}_{LM}\lambda_{LM} - \hat{a}_{LG}\lambda_{LG} \qquad (A4.4.2)$$

Since we have assumed $a_{LG} = 1$, there is always $\hat{a}_{LG} = 0$

Substitute $\hat{x}_A$ with (A4.4.1) into equation (A4.4.2):

$$\hat{x}_M\lambda_{LM} = \hat{a}_{TA}\lambda_{LA} - \hat{G}\lambda_{LG} - \hat{a}_{LA}\lambda_{LA} - \hat{a}_{LM}\lambda_{LM}$$

$$= [\hat{a}_{TA} - \hat{a}_{LA}]\lambda_{LA} - \hat{G}\lambda_{LG} - \hat{a}_{LM}\lambda_{LM}$$

$$\hat{x}_M = -\hat{a}_{LM} + [\hat{a}_{TA} - \hat{a}_{LA}]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}} = E_M\hat{G} - \theta_{LM}\sigma_{LL}^M\hat{w} + \big[\theta_{LA}\sigma_{LL}^A\hat{w} +$$

$$\theta_{TA}\sigma_{LT}^A\hat{t} - E_A\hat{G} - \theta_{LA}\sigma_{LT}^A\hat{w} - \theta_{TA}\sigma_{TT}^A\hat{t} + E_A\hat{G}\big]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}}$$

$$= E_M\hat{G} - \theta_{LM}\sigma_{LL}^M\frac{E_M}{\theta_{LM}}\hat{G} + [\theta_{LA}\sigma_{LL}^A\hat{w} + \theta_{TA}\sigma_{LT}^A\hat{t} - \theta_{LA}\sigma_{LT}^A\hat{w} - \theta_{TA}\sigma_{TT}^A\hat{t}]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}}$$

$$= E_M\hat{G} - \sigma_{LL}^M E_M\hat{G} + [(\theta_{LA}\sigma_{LL}^A - \theta_{LA}\sigma_{LT}^A)\hat{w} + (\theta_{TA}\sigma_{LT}^A - \theta_{TA}\sigma_{TT}^A)\hat{t}]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}} \qquad (A4.4.3)$$

(A1.6) can be changed as $\theta_{LA}\sigma_{LL}^A = -\theta_{TA}\sigma_{LT}^A$

In equation $(A4.3)$:

$$\theta_{LA}\sigma_{LL}^A - \theta_{LA}\sigma_{LT}^A = -\theta_{TA}\sigma_{LT}^A - \theta_{LA}\sigma_{LT}^A = -(\theta_{TA} + \theta_{LA})\sigma_{LT}^A = -\sigma_{LT}^A$$

$$-\theta_{TA}\sigma_{TT}^A = \theta_{LA}\sigma_{LT}^A$$

$$\theta_{TA}\sigma_{LT}^A - \theta_{TA}\sigma_{TT}^A = \theta_{TA}\sigma_{LT}^A + \theta_{LA}\sigma_{LT}^A = (\theta_{TA} + \theta_{LA})\sigma_{LT}^A = \sigma_{LT}^A$$

So $(A4.3)$ can be changed as following:

$$\hat{x}_M = E_M\hat{G} - \sigma_{LL}^M E_M\hat{G} + [-\sigma_{LT}^A\hat{w} + \sigma_{LT}^A\hat{t}]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}}$$

$$= E_M\hat{G} - \sigma_{LL}^M E_M\hat{G} + [-\sigma_{LT}^A\hat{w} + \sigma_{LT}^A(-\frac{\theta_{LA}}{\theta_{TA}}\hat{w} + \frac{E_A}{\theta_{TA}}\hat{G})]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}}$$

$$= E_M\hat{G} - \sigma_{LL}^M E_M\hat{G} + [-\sigma_{LT}^A\hat{w} - \frac{\theta_{LA}\sigma_{LT}^A}{\theta_{TA}}\hat{w} + \frac{E_A}{\theta_{TA}}\sigma_{LT}^A\hat{G})]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}}$$

$$= E_M\hat{G} - \sigma_{LL}^M E_M\hat{G} + [-\frac{(\theta_{LA}+\theta_{TA})\sigma_{LT}^A}{\theta_{TA}}\hat{w} + \frac{E_A}{\theta_{TA}}\sigma_{LT}^A\hat{G})]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}}$$

$$=E_M\hat{G} - \sigma_{LL}^{M}E_M\hat{G} + [-\frac{\sigma_{LT}^{A}}{\theta_{TA}}\hat{w} + \frac{E_A}{\theta_{TA}}\sigma_{LT}^{A}\hat{G})]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}} \qquad (A4.4.4)$$

With equation (4.20) $\hat{w} = \frac{E_M}{\theta_{LM}}\hat{G}$

$$\hat{x}_M = E_M\hat{G} - \sigma_{LL}^{M}E_M\hat{G} + [-\frac{1}{\theta_{TA}}\frac{E_M}{\theta_{LM}}\sigma_{LT}^{A}\hat{G} + \frac{E_A}{\theta_{TA}}\sigma_{LT}^{A}\hat{G})]\frac{\lambda_{LA}}{\lambda_{LM}} - \hat{G}\frac{\lambda_{LG}}{\lambda_{LM}}$$

$$= \hat{G}\left[E_M + \frac{\theta_{KM}E_M}{\theta_{LM}}.\sigma_{LK}^{M} + \left(\frac{E_M}{\theta_{LM}} - E_A\right)\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^{A}}{\theta_{TA}} - \frac{\lambda_{LG}}{\lambda_{LM}}\right] \qquad (A4.4.5)$$

Appendix 4.5

Then we can define $\widehat{K^*}_M$ by substitute (A4.4.5) into (A4.1.13):

$$\widehat{K^*}_M = \hat{a}_{KM} + \hat{x}_M$$

$$= \theta_{LM}\sigma_{KL}^{M}\hat{w} - E_M\hat{G} + \hat{G}[E_M + (\frac{E_M}{\theta_{LM}} - E_A)\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^{A}}{\theta_{TA}} - \frac{\lambda_{LG}}{\lambda_{LM}} + \frac{\theta_{KM}E_M}{\theta_{LM}}.\sigma_{LK}^{M}]$$

$$= \theta_{LM}\sigma_{KL}^{M}\frac{E_M}{\theta_{LM}}\hat{G} + \hat{G}[(\frac{E_M}{\theta_{LM}} - E_A)\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^{A}}{\theta_{TA}} - \frac{\lambda_{LG}}{\lambda_{LM}} + \frac{\theta_{KM}E_M}{\theta_{LM}}.\sigma_{LK}^{M}]$$

$$= \hat{G}[(\frac{E_M}{\theta_{LM}} - E_A)\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^{A}}{\theta_{TA}} - \frac{\lambda_{LG}}{\lambda_{LM}} + \frac{(\theta_{KM}+\theta_{LM})E_M}{\theta_{LM}}.\sigma_{LK}^{M}]$$

$$= \hat{G}\left[\frac{E_M}{\theta_{LM}}.\sigma_{LK}^{M} - \frac{\lambda_{LG}}{\lambda_{LM}} + \left(\frac{E_M}{\theta_{LM}} - E_A\right)\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^{A}}{\theta_{TA}}\right] \qquad (A4.5.1)$$

$\widehat{K^*}_M$ represents the relative variations of FDI. G has no direct effect on FDI. Factor productivity increase has direct effect on output but since $\hat{a}_{KM}$ act sensible to G, capital requirement for each unit of products reduced according to the increase of G, the direct effect of G over FDI relative variation disappears.

Appendix 4.6

Now let us look at the case when public goods are bought by the government with taxes on capital revenue. Similar with the lump-sum tax case, we will look at the ratio variation form in order to define the relationship between the change rates of variables. We have equation (4.36), (4.39):

$$\hat{r} = \frac{\tau}{(1-\tau)}\hat{t}$$

$$\hat{G} = \hat{t} + \frac{\tau}{(1-\tau)}\hat{t} + \widehat{K_M^*} - \widehat{w} = \frac{1}{(1-\tau)}\hat{t} + \widehat{K_M^*} - \widehat{w}$$

The Hicks-Allen elasticity of substitution, zero profit constrains always exit:

For the agriculture section, the ratio variation equations stays the same as equation (A4.1.1) and (A4.1.2)

$$\hat{a}_{LA} = \theta_{LA}\sigma_{LL}^{A}\widehat{w} + \theta_{TA}\sigma_{LT}^{A}\hat{t} - E_A\hat{G} = \theta_{TA}\sigma_{LT}^{A}(\hat{t} - \widehat{w}) - E_A\hat{G} \qquad (A4.6.1)$$

$$\hat{a}_{TA} = \theta_{LA}\sigma_{LT}^{A}\widehat{w} + \theta_{TA}\sigma_{TT}^{A}\hat{t} - E_A\hat{G} = \theta_{LA}\sigma_{LT}^{A}(\widehat{w} - \hat{t}) - E_A\hat{G} \qquad (A4.6.2)$$

$$\theta_{LA}\widehat{w} + \theta_{TA}\hat{t} = -\theta_{LA}\hat{a}_{LA} - \theta_{TA}\hat{a}_{TA} \qquad (A4.6.3)$$

For the manufacturing sector, since now $\hat{r}$ is represented by equation (4.25), the ratio variation forms are changed as:

$$\hat{a}_{LM} = \theta_{LM}\sigma_{LL}^{M}\widehat{w} + \theta_{KM}\sigma_{LK}^{M}\hat{r} - E_M\hat{G} = \theta_{KM}\sigma_{LK}^{M}(\hat{r} - \widehat{w}) - E_M\hat{G} \qquad (A4.6.4)$$

$$\hat{a}_{KM} = \theta_{LM}\sigma_{KL}^{M}\widehat{w} + \theta_{KM}\sigma_{KK}^{M}\hat{r} - E_M\hat{G} = \theta_{LM}\sigma_{KL}^{M}(\widehat{w} - \hat{r}) - E_M\hat{G} \qquad (A4.6.5)$$

$$\theta_{LM}\widehat{w} = -\theta_{LM}\hat{a}_{LM} - \theta_{KM}\hat{a}_{KM} - \theta_{KM}\hat{r} \qquad (A4.6.6)$$

Substitute (A1.1), (A1.2) into (A1.9), there is:

$$\theta_{LA}\widehat{w}+\theta_{TA}\hat{t}=-\theta_{LA}\hat{a}_{LA}-\theta_{TA}\hat{a}_{TA}=E_A\widehat{G} \qquad (A4.6.7)$$

Similarly, there is:

$$\theta_{LM}\widehat{w}+\theta_{KM}\hat{r}=E_M\widehat{G} \qquad (A4.6.8)$$

$$\theta_{LA}\widehat{w}+\theta_{TA}\hat{t}=E_A(\frac{1}{(1-\tau)}\hat{\tau}+\widehat{K_M^*}-\widehat{w}) \qquad (A4.6.9)$$

$$\theta_{LM}\widehat{w}+\theta_{KM}\hat{r}=E_M(\frac{1}{(1-\tau)}\hat{\tau}+\widehat{K_M^*}-\widehat{w}) \qquad (A4.6.10)$$

Which shows that

$$(\theta_{LA}+E_A)\widehat{w}+\theta_{TA}\hat{t}=E_A\frac{1}{(1-\tau)}\hat{\tau}+E_A\widehat{K_M^*} \qquad (A4.6.11)$$

$$(\theta_{LM}+E_M)\widehat{w}=\left(E_M\frac{1}{(1-\tau)}-\theta_{KM}\frac{\tau}{(1-\tau)}\right)\hat{\tau}+E_M\widehat{K_M^*} \qquad (A4.6.12)$$

Full employment condition requires:

$$\hat{a}_{LA}\lambda_{LA}+\hat{x}_A\lambda_{LA}+\hat{a}_{LM}\lambda_{LM}+\hat{x}_M\lambda_{LM}+\hat{a}_{LG}\lambda_{LG}+\hat{x}_G\lambda_{LG}=\hat{L}=0 \qquad (\text{A4.1.11})$$

$$\hat{a}_{TA}+\hat{x}_A=\hat{T}=0 \qquad (\text{A4.1.12})$$

$$\hat{a}_{KM}+\hat{x}_M=\widehat{K^*}_M \qquad (\text{A4.1.13})$$

$(A4.1.13)$ can be rewritten as

$$\hat{x}_A=-\hat{a}_{TA}=\theta_{LA}\sigma_{LT}^A(\hat{t}-\widehat{w})+E_A\widehat{G} \qquad (A4.6.13)$$

$(A4.1.11)$ can be rewritten as

$$\hat{x}_A\lambda_{LA}+\hat{x}_M\lambda_{LM}+\hat{a}_{LG}\lambda_{LG}=-\hat{a}_{LA}\lambda_{LA}-\hat{a}_{LM}\lambda_{LM} \qquad (A4.6.14)$$

(A4.6.12) gives us:

$$\widehat{w} = \frac{E_M \widehat{K_M^*} + [E_M \frac{1}{1-\tau} - \theta_{KM} \frac{\tau}{1-\tau}]\hat{\tau}}{(\theta_{LM} + E_M)} \qquad (A4.6.15)$$

(A4.6.11) and (A4.6.12) shows that:

$$\hat{t} = \widehat{K_M^*} \frac{1}{\theta_{TA}} \left[E_A - E_M \frac{(\theta_{LA}+E_A)}{(\theta_{LM}+E_M)} \right]$$

$$+\hat{\tau} \frac{1}{\theta_{TA}} \left[E_A \frac{1}{1-\tau} - \frac{(\theta_{LA}+E_A)}{(\theta_{LM}+E_M)} \left(E_M \frac{1}{1-\tau} - \theta_{KM} \frac{\tau}{1-\tau} \right) \right] \qquad (A4.6.16)$$

Substitute $(4.36), (A4.6.1), (A4.6.4), (A4.6.13), (A4.6.15), (A4.6.16)$ into $(A4.6.14)$:

$$\lambda_{LM} \hat{x}_M = \widehat{K_M^*} \left\{ \left(\frac{\lambda_{LA}\sigma^A + \lambda_{LM}\sigma^M \theta_{KM} + \lambda_{LG} - \lambda_{LM} E_M}{\theta_{LM}+E_M} \right) E_M + (\lambda_{LM} E_M - \lambda_{LG}) - \frac{\lambda_{LA}\sigma^A}{\theta_{TA}} \left[E_A - E_M \frac{(\theta_{LA}+E_A)}{(\theta_{LM}+E_M)} \right] \right\} + \frac{\hat{\tau}}{1-\tau} \left\{ \left(\frac{\lambda_{LA}\sigma^A + \lambda_{LM}\sigma^M \theta_{KM} + \lambda_{LG} - \lambda_{LM} E_M}{\theta_{LM}+E_M} \right) \left(E_M - \theta_{KM}\tau + (\lambda_{LM} E_M - \lambda_{LG} - \lambda_{LM}\sigma^M \theta_{KM}\tau) - \frac{\lambda_{LA}\sigma^A}{\theta_{TA}} \left[E_A - \frac{(\theta_{LA}+E_A)}{(\theta_{LM}+E_M)} (E_M - \theta_{KM}\tau) \right] \right\} \qquad (A4.6.17)$$

With (A4.1.13) $\widehat{K_M^*} = \hat{a}_{KM} + \hat{x}_M$, substitute with $(4.36), (A4.6.5)$, (A4.6.15),

(A4.6.17), we get the expression of the change rate of capital:

$$\widehat{K_M^*} = \widehat{K_M^*} \left\{ \frac{\theta_{LM}\sigma^M}{\theta_{LM}+E_M} E_M - E_M + \frac{E_M}{\theta_{LM}+E_M} E_M + \frac{1}{\lambda_{LM}} \left[\left(\frac{\lambda_{LA}\sigma^A + \lambda_{LM}\sigma^M \theta_{KM} + \lambda_{LG} - \lambda_{LM} E_M}{\theta_{LM}+E_M} \right) E_M + (\lambda_{LM} E_M - \lambda_{LG}) - \frac{\lambda_{LA}\sigma^A}{\theta_{TA}} \left(E_A - E_M \frac{(\theta_{LA}+E_A)}{(\theta_{LM}+E_M)} \right) \right] \right\} + \frac{\hat{\tau}}{1-\tau} \{ \frac{\theta_{LM}\sigma^M}{\theta_{LM}+E_M} (E_M - \theta_{KM}\tau) - \theta_{LM}\sigma^M\tau - E_M - \frac{E_M}{\theta_{LM}+E_M} (E_M - \theta_{KM}\tau) + \frac{1}{\lambda_{LM}} \ [\ \left(\frac{\lambda_{LA}\sigma^A + \lambda_{LM}\sigma^M \theta_{KM} + \lambda_{LG} - \lambda_{LM} E_M}{\theta_{LM}+E_M} \right) (E_M - \theta_{KM}\tau) + (\lambda_{LM} E_M - \lambda_{LG} - \lambda_{LM}\sigma^M \theta_{KM}\tau) - \frac{\lambda_{LA}\sigma^A}{\theta_{TA}} (E_A - \frac{(\theta_{LA}+E_A)}{(\theta_{LM}+E_M)} (E_M - \theta_{KM}\tau))]\}$$

In a clearer form, there is:

$\widehat{K_M^*} = \hat{\tau}.\left(\frac{\theta_{LM}}{(1-\tau).(\theta_{LM}+E_M)}\right).\left(\frac{1}{1-\Omega}\right).$

$\left\{\left(\frac{E_M}{\theta_{LM}} - E_A - \tau.\frac{\theta_{KM}}{\theta_{LM}}.[1+E_A]\right).\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^A}{\theta_{TA}} \quad - \frac{\lambda_{LG}}{\lambda_{LM}}.\left(1+\tau.\frac{\theta_{KM}}{\theta_{LM}}\right) + \frac{\sigma_{LK}^M}{\theta_{LM}}.[E_M.(1-\tau) - \tau]\right\}$ $(A4.6.18)$

with

$$\Omega = \frac{\theta_{LM}}{(\theta_{LM}+E_M)}.\left\{\left(\frac{E_M}{\theta_{LM}} - E_A\right).\frac{\lambda_{LA}}{\lambda_{LM}}\frac{\sigma_{LT}^A}{\theta_{TA}} - \frac{\lambda_{LG}}{\lambda_{LM}} + \sigma_{LK}^M.\frac{E_M}{\theta_{LM}}\right\} \qquad (A4.6.19)$$

Substitute (A4.6.18) back into (A4.6.17):

$\hat{x}_M = \frac{\hat{\tau}}{1-\tau}\frac{1}{\theta_{LM}+E_M}\{\frac{1}{(\theta_{LM}+E_M)(1-\Omega)}\left[\frac{\lambda_{LA}\sigma^A}{\theta_{TA}\lambda_{LM}}(E_M - E_A\theta_{LM}) + \theta_{KM}E_M(\sigma^M+1) - \frac{\lambda_{LG}}{\lambda_{LM}}\theta_{LM}\right] + \left[\frac{\lambda_{LA}\sigma^A}{\theta_{TA}\lambda_{LM}}(E_M - \theta_{KM}\tau - E_M\theta_{LA} - E_A\theta_{KM}\tau) + \theta_{KM}E_M\tau(1-\sigma^M) + \theta_{LM}E_M - \frac{\lambda_{LG}}{\lambda_{LM}}(\theta_{KM}\tau + \theta_{LM})\right]\}$ $(A4.6.20)$

Substitute $(4.36), (A4.1.1), (A4.6.4), (A4.6.13), (A4.6.15), (A4.6.16), (A4.6.19)$ into $(A4.6.5)$:

$\hat{a}_{KM} = \theta_{LM}\sigma_{KL}^M(\widehat{w} - \hat{r}) - E_M\hat{G} = \theta_{LM}\sigma_{KL}^M(\widehat{w} - \hat{r}) - E_M\left(\frac{1}{(1-\tau)}\hat{\tau} + \widehat{K_M^*} - \widehat{w}\right) =$

$\theta_{LM}\sigma_{KL}^M\widehat{w} - \theta_{LM}\sigma_{KL}^M\hat{r} - \frac{E_M}{(1-\tau)}\hat{\tau} - E_M\widehat{K_M^*} + E_M\widehat{w} =$

$(\theta_{LM}\sigma_{KL}^M + E_M)\widehat{w} - \theta_{LM}\sigma_{KL}^M\frac{\tau}{(1-\tau)}\hat{\tau} - \frac{E_M}{(1-\tau)}\hat{\tau} - E_M\widehat{K_M^*} = (\theta_{LM}\sigma_{KL}^M + E_M)\left(\frac{E_M\widehat{K_M^*}}{(\theta_{LM}+E_M)} + \frac{E_M - \theta_{KM}\tau}{(\theta_{LM}+E_M)(1-\tau)}\hat{\tau}\right) - \frac{\theta_{LM}\sigma_{KL}^M\tau + E_M}{(1-\tau)}\hat{\tau} - E_M\widehat{K_M^*} =$

$\left(\frac{\theta_{LM}\sigma_{KL}^M + E_M}{\theta_{LM}+E_M} - 1\right)E_M\widehat{K_M^*} + \left(\frac{(\theta_{LM}\sigma_{KL}^M + E_M)(E_M - \theta_{KM}\tau)}{(\theta_{LM}+E_M)(1-\tau)} - \frac{\theta_{LM}\sigma_{KL}^M\tau + E_M}{(1-\tau)}\right)\hat{\tau} =$

$\frac{\sigma_{KL}^M - 1}{\theta_{LM}+E_M}\theta_{LM}E_M\widehat{K_M^*} + \frac{(\theta_{LM}\sigma_{KL}^M + E_M)(E_M - \theta_{KM}\tau) - (\theta_{LM}+E_M)(\theta_{LM}\sigma_{KL}^M\tau + E_M)}{(\theta_{LM}+E_M)(1-\tau)}\hat{\tau} =$

$\frac{\sigma_{KL}^M - 1}{\theta_{LM}+E_M}\theta_{LM}E_M\widehat{K_M^*} +$

$$\frac{\theta_{LM}\sigma_{KL}^{M}E_M+E_M{}^2-\theta_{LM}\sigma_{KL}^{M}\theta_{KM}\tau-\theta_{KM}\tau E_M-\theta_{LM}{}^2\sigma_{KL}^{M}\tau-\theta_{LM}\sigma_{KL}^{M}\tau E_M-\theta_{LM}E_M-E_M{}^2}{(\theta_{LM}+E_M)(1-\tau)}\hat{\tau}=$$

$$\frac{\sigma_{KL}^{M}-1}{\theta_{LM}+E_M}\theta_{LM}E_M\widehat{K_M^*}+\frac{\theta_{LM}E_M(\sigma_{KL}^{M}-1)-(\theta_{KM}+\theta_{LM}+E_M)\theta_{LM}\sigma_{KL}^{M}\tau-\theta_{KM}E_M\tau}{(\theta_{LM}+E_M)(1-\tau)}\hat{\tau} \qquad (A4.6.21)$$

Substitute (A4.6.18), (A4.6.21) into (A4.1.13) $\hat{x}_M=\widehat{K_M^*}-\hat{a}_{KM}$, we can get the expression of $\hat{x}_M$:

$$\hat{x}_M=\widehat{K_M^*}-\hat{a}_{KM}$$

$$=\hat{\tau}.\left(\frac{1}{1-\tau}\right).\left(\frac{\theta_{LM}}{\theta_{LM}+E_M}\right).\left(\frac{1}{1-\Omega}\right).\{\left[E_M-\left(E_A-\frac{E_M}{\theta_{LM}}\right]-\frac{\lambda_{LG}}{\lambda_{LM}}+\frac{E_M\sigma_M\theta_{KM}}{\theta_{LM}}\right]$$

$$+\tau[-\frac{\theta_{KM}}{\theta_{LM}}(1+E_A).A-\frac{\theta_{KM}}{\theta_{LM}}.E_M.A+E_A.\sigma_M.A-E_M.\sigma_M.A$$

$$+\frac{\lambda_{LG}}{\lambda_{LM}}\left(\sigma_M-\frac{\theta_{KM}}{\theta_{LM}}\right)$$

$$+\frac{E_M}{\theta_{LM}}\theta_{KM}-\frac{E_M}{\theta_{LM}}\sigma_M-\frac{E_M}{\theta_{LM}}\sigma_M\theta_{KM}-\frac{\sigma_M}{\theta_{LM}}+\sigma_M]\} \qquad (A4.6.22)$$

Conclusion

The analysis of fiscal policy has long been widely done by the scholars for different levels of government and under different combinations of assumptions. This research can help us to better understand the mechanism behind fiscal policies and then to make appropriate policy suggestions to the governments. Our three parts of the dissertation work separately on different topics in the fiscal policy field. Chapter 2 looks at the choice of a public budget in a multiregional setting with productivity differences and imperfectly mobile labor, and examines the implicit transfers generated by this budget. Chapter 3 works on a fiscal competition in the same type of model, and explicits the circumstances leading a decentralized jurisdiction to subsidize or tax capital. Chapter 4 analyzes the relationship between public inputs and FDI inflow, examining the various impacts on FDI of a change in the provision of public inputs.

In this conclusion, we will be starting from some of the main results of this dissertation and looking at the developments they call for.

A first interesting result is the nature of implicit transfers between regions generated by fiscal policies. In a context where productivity differentials are fully exogenous, we find that, as soon as capital is taxed, implementing the first best allocation implies an implicit transfer from the richest region to the poorest one. The reason behind this transfer is that efficiency implies taxing both firms and households at the same level across regions. Capital being over represented in the most productive region, the taxation of capital at the same rate across regions generates more tax resources in the most productive region, which must be compensated by a transfer.

Let us note that this transfer appears when the central planner is purely utilitarian and then is not motivated by any preference for redistribution. Then, the decision of taxing capital, which may be driven by motives outside the model, implies implicit redistribution for pure efficiency motives.

This result calls for two qualifications. First, it has been obtained in a context where the public good is a pure consumption good and perfectly divisible. What happens when the public good also contributes to production (it is also a public input, to some extent)? What happens when the public good is imperfectly divisible? The answer to these questions is left for further research. Second, it has been obtained in a context where the planner faces no constraint on taxation. In chapter 2, we start exploring the impact of a limitation on the power to tax households. We find that this limitation exacerbates the inequality between regions, the endowment in capital per worker and the level of public good provision being lower in the less productive region while this region taxes capital at a higher level than in the most productive region.

Third, this result has been obtained in a context where asymmetry between regions comes from a purely exogenous productivity differential. In Chapter 2, we start exploring what happens when the productivity differential is endogenously generated by an agglomeration externality, the total factors productivity being an increasing function of the population. We determine the conditions leading to an asymmetric first best allocation and we characterize fiscal policies implementing this first best optimum.

This policy internalizes the externality generated by migrants, who decrease the productivity of the region they leave and increase the productivity of the region they enter. This internalization reverses the result obtained when the productivity differential was purely exogenous: if capital tax rates are still equalized across regions, workers now pay higher taxes in the poor region that in the rich one. If the planner chooses not to tax capital, this higher taxation of households in the poor region generates an implicit transfer to the rich one. Capital being overrepresented in the rich region, the planner can use taxation of capital for decreasing the amount of this implicit transfer and reaching a less unequal outcome.

It would be interesting to look at what happens when asymmetries are endogenously generated by other mechanisms. Instead of an agglomeration externality generated by the population, we can look at an agglomeration externality generated by the local stock of capital. In that case, the application of the Pigovian principle would probably lead to differentials in capital tax rates, with results that may differ from the results obtained in Chapter. Another form of externality which is worth exploring is a negative externality generated by outmigration: natives who leave a jurisdiction decrease the welfare of stayers. This impact on the welfare of stayers may come from the disorganization of family groups and social networks and is often expressed as a source of concern by local policy makers. Looking at these other forms of externality is left for further research.

A second important result comes out from Chapter 3, where we found that, starting from a fist best allocation where capital is not taxed, the less productive jurisdiction has an incentive to subsidize capital for compensating its disadvantage while the most productive jurisdiction has an incentive to tax capital for benefitting from an agglomeration rent.

This result provides a rationale for explaining the differences in taxation policies between jurisdictions and the existence of attraction policies using capital subsidies. It would be worth confronting it with empirical data from a country with large productivity differences between regions and a mobile population. Taking the case of China as an example, population is mobile and productivity differences are really large between different provinces or between different regions: South versus North, East versus West. It is possible to look at the fiscal policies and fiscal transfers between provinces or between regions.

However, we cannot neglect the specific hypothesis leading to this result. What happens when the public good also contributes to production (it is also a public input, to some extent)? What happens when the public good is imperfectly divisible? What happens when the externality is generated by the stock of capital instead of the population? The answer to these questions is left for further research.

In chapter 4, we focus on the relationship between public inputs and FDI flow. We identify the effects of an increase in public inputs on capital flow: an indirect effect of the increase in the marginal productivity of capital through a change in factor prices; a labor outflow effect generated by the expansion of the sector producing public goods; and the contribution of public inputs to the efficiency of production. The final effect of public inputs on capital flows result from the combination of these three effects.

Then, we find a first interesting result: the public sector must be small enough for an increase in public inputs to attract FDI inflows. A very large public sector could be detrimental for attracting foreign capital. Furthermore, if the Hicksian elasticity of the impact function with respect to public inputs in the manufacturing sector needs to be large enough for the effect of public inputs on a capital inflow to be positive, the agriculture sector also matters. If the Hicksian elasticity in the agricultural sector is large enough, even with a small elasticity in the manufacturing sector, the increase in public inputs can still generate a capital inflow. The importance of the cost share of labor and the substitution rate between labor and capital in the manufacturing sector are also considered. Both of them should be large enough to assure the positive effect of public inputs on capital inflow.

Then the study is extended into the case when the public instrument is the taxation which finances the public inputs instead of the public input itself. A very interesting conclusion is that the decrease of tax would not always attract FDI inflow. This conclusion could be a contradiction for the tax competition policy taken by a lot of developing countries. Always taking the example of China, the government sets up a lot of special trade areas all around the country and provides tax holidays to attract FDI. This movement won't always work. The characteristics of the whole economy should be considered before making the decision.

The future improvement of the work could be that we extend the kind of taxation using to finance public inputs. The possible choices could be tax on agriculture products.

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