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A Cross-efficiency approach to portfolio selection

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Résumé

Le processus de sélection de portefeuille peut être effectué en deux étapes: la première consiste à évaluer les actifs financiers et la deuxième à déterminer la combinaison d'actifs qui permettrai d'‘allouer de façon optimale la richesse. La combinaison des actifs financiers retenue à la fin de ce processus se doit de répondre simultanément et de façon optimale aux différents objectifs de l'‘investisseur. Le problème de sélection de portefeuille peut être considéré comme un processus de décision multicritère. Dans cette thèse, plusieurs critères ont été analysés et on a tenté de répondre à la question de combien et où investir ?

On a proposé une nouvelle approche multicritère basée sur la méthode d'‘enveloppement des données (DEA) et l'‘approche de l'‘efficacité croisée pour sélectionner un portefeuille d'actifs financier. Afin d'évaluer la performance des actifs financiers, la méthode d'efficacité croisée basée sur DEA considère les critères à minimiser comme inputs tandis que ceux à maximiser comme outputs. La première méthodologie proposée consiste à incorporer la méthode d'efficacité croisée dans un espace Moyenne-Variance-Skewness-Kurtosis (MVSK). Ce modèle a l'avantage de considérer les moments d'ordre supérieur dans le processus de sélection de portefeuille. Le deuxième modèle combine la mesure de l'efficacité croisée par l'enveloppement de données vue comme solution d'un jeu avec la composante risque pour choisir un portefeuille. Finalement, on a proposé d'incorporer la mesure de l'efficacité croisée par l'enveloppement de données vue comme solution d'un jeu dans un modèle d'arbitrage entre profitabilité et efficacité afin de sélectionner un portefeuille. On a appliqué les approches proposées à un échantillon d'entreprises cotées sur la bourse de Paris et on a démontré que les portefeuilles obtenus sont plus performants du point de vue risque et rendement que les portefeuilles de référence, qui sont les indices de marché, et ce pour une période de 6 années s'étalant de 2010 à 2015. Globalement, ces méthodes ont permis la discrimination entre les actifs financiers et de leur donner un classement unique dans un premier temps, ensuite de sélectionner un portefeuille en prenant en considération les préférences du décideur.

Mots clés: sélection du portefeuille, Méthode d'enveloppement des données, efficacité croisée, moments d'ordre supérieur, bourse de Paris, décision multicritère, profitabilité

General Abstract

The process of portfolio selection could be divided into two stages: the first one is the evaluation of financial assets and the second is to choose the best ones to construct portfolio. It can be considered as Multi Criteria Decision Making (MCDM) process. It consists in selecting a combination of financial assets that can best meet the investors' objective. In this dissertation, different criteria are analyzed and the question of where and how much money to allocate to each of the financial asset is processed. We propose a new multi-criteria analysis approach to portfolio selection based on Data Envelopment Analysis (DEA) cross-efficiency model. To assess financial assets performance, the DEA cross-efficiency framework considers the attributes to minimize as inputs and those to maximize as outputs. The first methodology consists in nesting the DEA cross-efficiency model into the Mean-Variance-Skewness-Kurtosis (MVSK) space. We then cover the merit of considering higher order moments in portfolio selection process. The second model combines the DEA game cross-efficiency approach with risk component to select portfolio. Finally, we propose a model incorporating the DEA game cross-efficiency into Profitability-Efficiency framework. We apply the proposed approach to firms listed on the Paris stock Exchange, and demonstrate that the resulting portfolio yields higher risk-adjusted returns than other benchmark portfolios for a 6-year sample period from 2010 to 2015. Overall, these methodologies provide more discrimination for financial assets by providing unique ranks in a first step and permit to select portfolio by underlying preferences of the decision-maker in a second step.

Keywords: Portfolio selection, Data Envelopment Analysis, Cross-efficiency, higher order moments, Paris stock exchange, Multi-criteria-decision-making, profitability

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List of abbreviations

AHP: Analytic Hierarchy Process

AP: Arbitrary Portfolio

CAPM: Capital Asset Pricing Model

CE: Cross-Efficiency

CCR: Charnes Cooper & Rhodes

CRS: Constant Returns to scale

DDF: Directional Distance Function

DEA: Data Envelopment Analysis

DMU: Decision Making Units

EMH: Efficient Market Hypothesis

FDH: Free Disposal Hull

FPI: False Positive Index

GAM: Game

GCP: Game Cross Portfolio

LP: Linear Program

MAV: Maverick

MCDA: Multi Criteria Decision Analysis

MCDM: Multi criteria Decision Making

MM: Mean-Maverick

MILP: Mixed-Integer6linear-Programing

MV: Mean-Variance

MVSK: Mean-Variance-Skewness-Kurtosis

NSGA: Non-Dominated Sorting genetic Algorithm

OWA: Ordered Weighetd Averging

PE: Profitability-Efficiency

R&D: Research & Development

SCBB: Studentized Circular Block Bootstrap

SD: Standard Deviation

SK: Skewness-Kurtosis

SR: Sharpe Ratio

VNM: Von Neumann Morgenstern

General Introduction

There is no denying that stock markets are some of the most important parts of today’s global economy. One role of stock markets is to act as an intermediary for large and small investors seeking to make money outside the realm of standard banking institutions. Thus, a fundamental question in finance is how to invest? In other words, how to select portfolio?

Portfolio selection problem is an important issue in the theory and practice of finance. Having considered a number of risky assets, it may be defined as the decision whereby a basket of the best set of financial assets is selected from many different alternatives. Markowitz (1952) conceived the process of portfolio selection as a two-step procedure. The first one starts with observation and experience and ends with beliefs about the future performances of available financial assets, it consists in securities performance evaluation. The second stage starts with the relevant beliefs about future performances and ends with choosing the optimal and the most attractive investment with optimal wealth allocation given preferences of the investor. The decision of selecting portfolio is of greater importance, since choosing the right assets is a significant resource allocation decision that can lead to high profits and in the worst case to huge losses.

Portfolio performance has been evaluated for a long time based only on assets’ returns. Afterward, the assessment has been extended to the risk component. Then, the performance evaluation has been based on performance measures combining information on both return and risk.

According to the Mean-Variance (MV) approach introduced by Markowitz (1952), an investor is arbitrating between two basic conflicting objectives that are the maximization of the return and the minimization of the risk. Afterwards, several other models were developed based on the MV framework. Besides the two fundamental criteria of return and risk, a number of important criteria such as investor preferences, consistent returns, marketability, liquidity, capital growth, risk diversification, taxes planning and price earnings ratio have been considered through developing realistic multi-criteria models. Since the decision attributes may be conflicting, the portfolio selection can be considered as a Multi Criteria Decision Making (MCDM) process. It consists in selecting a combination of financial assets that can best meet the investors’ objective.

In this dissertation, different criteria are analyzed and the question of where and how much money to allocate to each of the financial assets is processed. Thus, this work mainly responds to a practical need for portfolio selection and management.

In recent years, the development of multi-criteria analysis gave rise to new ways of modeling portfolio management model. As an alternative MCDM tool, the application of Data Envelopment Analysis (DEA) has been considered as a promising tool for evaluation the discrete multi-criteria decision process since it does not require additional information than the values associated with each attribute. It has gained more attentions in the portfolio selection area. The methodological connection between DEA and MCDM analysis is established by considering the non-preferred criteria as inputs, whereas the preferred criteria are considered as outputs. The use of DEA as a benchmarking tool is relevant in the case of MCDM. It permits to consider several attributes together to present a single composite efficiency score, to assist in the decision making process of the investor.

In this study, we use a cross-efficiency approach based on DEA approach to evaluate and discriminate among securities. Developed by Charnes et al. (1978), DEA is used to measure the relative efficiencies of a set of Decision Making Units (DMUs) with multiple inputs and outputs. It determines the efficient DMUs and generates potential improvements for inefficient DMUs. Moreover, DEA is a useful tool in MCDM which evaluates the efficiency of DMUs without any information of the relative importance of inputs and outputs. This leads to confirm its adequacy to resolve portfolio selection problem, given that an investor is not able to express a preference toward attributes at the beginning of the portfolio selection process.

When we apply DEA in a MCDM context, a common problem arises, that is a multiplicity of 100% efficient DMUs. Thus ranking DMUs can be quite hard. Indeed, this further support the lack of discrimination power of this methodology and the unrealistic weighting scheme. Proposed by Sexton et al., (1986) and developed in Doyle and Green (1994a), the DEA cross-efficiency method provides a unique ordering of DMUs and eliminates unrealistic weights schemes. It uses DEA in a peer-evaluation instead of self-evaluation.

However the use of cross efficiency as such suffers from the non-uniqueness of the DEA optimal weights. In this dissertation we propose the DEA Game Cross efficiency

Framework which considers the factor of competition in ranking different assets and provides stable weights and Nash equilibrium efficiency scores to evaluate assets.

Overall, The DEA cross-efficiency would be a suitable multi-criteria method to solve portfolio selection problem as it allows explicit tradeoffs and interactions among criteria. Nonetheless, in the peer-evaluation, it is likely to choose assets whose performance is somewhat good on all measures and exclude those whose performance is good for only a subset of criteria. In fact a variation in one criterion could be compensated in a director or opposite way by other criteria. This leads to a portfolio made of too similar financial assets which causes a problem of lack of diversification.

To deal with this limitation we fit the cross-efficiency model into optimization model allowing us to select diversified portfolio depending on investor objectives. In this dissertation, we follow Markowitz portfolio selection process definition by evaluating financial assets considering criteria and beliefs in the first step and choosing among them in the second step.

In this work, the first stage of portfolio selection process is based on the DEA cross-efficiency approach. The second stage is based on optimization models that translate other investors' preferences.

In the first place, we propose a portfolio selection model which takes into account the investors preferences for higher return moments that are skewness and kurtosis.

Secondly, based on DEA game cross-efficiency approach, we propose that the maverick index (the deviation between the DMU self-appraised score and its peer-appraised score) of a DMU could be a consistent measure of risk degree with respect to changes in weights. We then select portfolio by making a tradeoff between efficiency and this novel risk indicator.

Finally, as the efficiency of a financial asset may result from a good mix of criteria, the profitability criterion matters a lot to an investor. Since efficiency does not mean profitability, we consider making gains as objective in the evaluation process of financial asset.

As illustrations to the developed approaches, we report case studies involving a 6-year sample period from 2010 to 2015 of firms listed in the Paris Stock exchange.

This dissertation is a compilation of articles. It is made by three articles and an introductive chapter that put the entire thesis into context and connecting the different papers.

In the first chapter of the thesis, we provide an overview of the cross-efficiency as an extension to DEA approach, we also present a survey on the related developed literature, and we show the usefulness of cross-efficiency evaluation in portfolio selection area. We also briefly recall some criteria that could be used in portfolio selection field. We finally cover the merit of considering higher order moments in portfolio selection process: We present the theory of decision making under risk due to the original contribution of Von Neumann and Morgenstern (1947). In particular, we describe the necessary and sufficient conditions for the presentation of portfolio problem. We also provide the relationship between the MV approach and the expected utility approach in details and show the importance of including higher order moments into portfolio selection decision process.

The second chapter consists in nesting the DEA cross-efficiency model into the Mean-Variance-Skewness-Kurtosis (MVSK) space. The DEA cross-efficiency evaluation suggests that each DMU is not only to be self-evaluated but also to be peer-evaluated. Therefore, the cross-efficiency evaluation can guarantee a unique ordering of the DMUs. The optimal choice is a portfolio that performs optimally with respect to many criteria, which are the first four moments of returns distribution. The developed framework permits to (i) take into account investor attitudes, (ii) determine endogenously two tradeoffs parameters, one between the mean and the variance of the return's distribution and a second between the skewness and the kurtosis of that same distribution and (iii) to allocate decision maker's wealth in an optimal way to select portfolio. To illustrate how our new methodology works, we applied it on a sample of financial assets and we test its robustness by comparing the obtained portfolio to benchmark portfolios in terms of risk-return performance.

In the third chapter, we combine the DEA game cross-efficiency approach with risk component to select portfolio. We evaluate performance of financial assets by providing Nash equilibrium efficiency scores. We suggest the maverick index as a consistent risk indicator. We then incorporate the game cross-efficiency into Mean-Maverick framework to select portfolios. The developed approach has multiple merits; it (i) gives Nash equilibrium efficiency scores and a unique rank to the financial assets, (ii) provides a relevant and novel measure of risk based on game cross-efficiency method in portfolio area, (iii) permits to select well-diversified portfolio. To demonstrate that the developed method could be a promising tool for portfolio selection we report a case study involving 500∼508 firms from the Paris Stock Exchange.

In the fourth chapter, we propose a model incorporating the DEA game cross-efficiency into Profitability-Efficiency (PE) model by considering profitability criterion on one hand and efficiency resulting from a good mix of attributes to portfolio selection on the other hand. The developed model permits to (i) obtain game cross-evaluation scores, which constitute a Nash equilibrium solution to financial assets. (ii) gives the scope to make decision about profits. (iii) reflects investor' preferences, by using higher order moments of returns distribution as inputs and outputs to compute efficiency scores of financial assets. A large sample of firms listed on Paris stock exchange during the period 2010-2015 served as illustration of this framework.

Chapter 1:

On the use of Cross-efficiency approach to portfolio performance evaluation with higher order moments

Chapter 1: On the use of Cross-efficiency approach to portfolio performance evaluation with higher order moments

Markowitz, the father of modern portfolio theory, introduced the notion of two moment efficiency analysis, where investors should consider the variance in addition to return levels in their decision making process. In fact, the tradeoff between mean and veriance involves minimizing risk for a given level of expected return, or equivalently, maximizing expected return for a given level of risk. This model assumes that all agents have similar expectations about market condition, which leads to impose that investors have a quadratic utility function and asset's returns are normally distributed. This does not seem too realistic.

Thus, many other critical factors and criteria which directly or indirectly influence the investor decision should be considered in the portfolio selection process. The portfolio selection is then a logical consequence of the investor's attitudes towards information concerning stocks. These factors and information may be considered together in a single composite Data Envelopment Analysis (DEA) efficiency score to evaluate financial asset.

The use of DEA as a Multi Criteria Decision Making Analysis (MCDA) tool is established by considering the non-preferred criteria as inputs, whereas the preferred criteria are considered as outputs. Moreover, DEA is useful in MCDM because it permits to evaluate assets without any information of the relative importance of the inputs and outputs, this leads to confirm its adequacy to resolve portfolio selection problem, given that an investor is not able to express a preference toward attributes at the beginning of the portfolio selection process.

However, the DEA provides each financial asset a good opportunity to self-evaluate its efficiency relative to other homogenous financial assets based on its favorable weights, this leads to a problem of unrealistic factors weights. Furthermore, the ranking of financial assets can be quite hard using efficiency scores because there is a multiplicity of the 100% efficient DMUs. These problems are dealt with cross-efficiency framework which provides a unique ordering of DMUs and eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from the decision maker.

In the first section, we briefly recall criteria that have been used in portfolio selection field and we cover the merit of considering higher order moments in portfolio selection process. In the second section, we provide an overview of the cross-efficiency as an extension to DEA approach, we also present a survey on the related developed literature, and we show the usefulness of cross-efficiency evaluation in portfolio selection area.

1. An investigation of the promising Criteria in portfolio selection decision making: Higher order moments

According to Cook et al. (2014), if the DEA issue is a general benchmarking problem where it is employed as Multi Criteria Decision Making (MCDM) tool, then *"the inputs are usually "less-the-better" type of performance measures and the outputs are usually the "more-the-better" type of performance measures".* In such context, inputs and outputs are two sets of performance attributes, where one set is non-preferred (inputs) to be minimized and the other is preferred (outputs) to be maximized.

In portfolio selection problem, the investor splits his wealth amongst the most desirable stocks based on a given characteristics and preferences. Since the pioneering work of Markowitz (1952), several portfolio selection methods have been investigated based on two criteria that are: the profitability (expected return) and the second order risk (variance). Since the investor desires to minimize the portfolio risk, the variance is the criterion to minimize. However, the expected return is to maximize because the investor prefer getting gains. Multi-criteria approaches build realistic models and processes by taking into consideration besides the two basic criteria of return and risk, a number of important desirable and undesirable attributes, such as solvency (Bouri et al. (2002), liquidity (see.eg., Steuer et al. (2007) and Jana et al. (2009)), marketability (see.eg. Zopounidis et al. (1998) and Aouni (2009)), growth of the dividends (Bower and Bower (1969)), profitability (Ballestero et al. (1996)), financial structure and others. (Aouni et al. (2018)) present a good work on the importance of multiple criteria decision aid methods for portfolio selection and a large survey on the criteria in use to portfolio selection. In DEA model, criteria are used as inputs and outputs. The DEA method offers investors the possibility to consider simultaneously a mix of attributes with direct control over the priority level paid to each criterion. It permits to take into account the specific preferences of investors.

Despite the significant development of portfolio selection based on MV framework, it has been shown that we should not neglect the higher order moments of returns. Recent techniques enlarge the assessment dimension to the skewness and kurtosis in order to take into consideration the non-normality of return distributions and the non-quadratic utility function of the investor.

The MV model is established on the assumptions that investors have a quadratic utility function and/or the returns of the financial assets are normally distributed. The simplifying assumption of return normality implies that the investor can lose more than his initial wealth. Moreover, quadratic utility function supposes that investors are equally averse to deviations above the mean as they are to deviations below the mean. This means that they are averse to upward movements in the same way as to downward movements and that they sometimes prefer less wealth to more wealth. Literature has proven that these assumptions are not literally true. In fact, empirical evidence suggests that asset returns of financial assets exhibit significant departures from the normality; they have heavier tails than implied by the normal assumption and are often not symmetric.

1.1 Basic utility theory for portfolio selection

While the approaches that tradeoff risk and return have always been of particular interest, the expected utility approach has proven a natural framework for the analysis of financial decision problems.

John von Neumann and Oskar Morgenstern developed the Von Neumann–Morgenstern utility function, an extension of the theory of consumer preferences that incorporates a theory of behaviour toward risk variance in theory of Games and Economic Behavior (Von Neumann and Morgenstern (1947)). Utility theory shows that if an agent is faced with a choice of outcomes subject to various levels of probability, the optimal decision will be the one that maximizes the expected value of the utility derived from the choice made. Expected value is defined as the sum of the products of the various utilities and their associated probabilities. The individual is expected to be able to rank the items or outcomes in terms of preference, but the expected value will be conditioned by their probability of occurrence. The expected utility hypothesis presents the most popular approach to decision making under uncertainty in finance.

Let an individual's initial wealth W_i to be invested in portfolio Ω. There are N risky assets with proportions to invest $P=(p_1\,p_2...p_N)$ and random returns $R=(r_1\,r_2...r_N)$. At time 0, the investor has to decide about the composition of Ω to be held until the period 1.

At the end of the period 1, the final wealth W_f of the investor is computed as follows:

$$W_f = W_i\left(1+\sum_{i=1}^{N} p_i r_i\right) = W_i\left(1+r_\Omega\right) \tag{1.1}$$

Where $r_\Omega = \sum_{i=1}^{N} p_i r_i$ is the portfolio return and assets' weights p_i are as $\sum_{i=1}^{N} p_i = 1$.

The final wealth allows the individual to consume goods which creates him pleasure and utility. Thus, the utility function $U(.)$ describes the relationship between wealth and the utility extracted from consuming this wealth. The utility function varies from an investor to another. To solve the portfolio problem, the expected utility hypothesis assumes that an investor selects the portfolio by maximizing his expected utility value as follows:

$$\begin{aligned} &\max_P\ E\left[U\left(W_1\right)\right] = E\left\{U\left[W_0\left(1+\sum_{i=1}^{N} p_i r_i\right)\right]\right\} = E\left\{U\left[W_0\left(1+r_\Omega\right)\right]\right\} \\ &s.t \quad \sum_{i=1}^{N} p_i = 1 \end{aligned} \tag{1.2}$$

This expected utility function is called the Von Neumann-Morgenstern utility function. An expected utility function is unique up to affine transformations. This property is described as follows:

$$U(.) = p_1 U(.) + p_2 \tag{1.3}$$

Where $U(.)$ is an individual utility function and $p_1, p_2 > 0$

Let $U(.)$ be an individual utility function, W_1 is the income if state of the world 1 occurs with probability p_1 and W_2 is the income if state of the world 2 occurs with probability p_2. To describe preferences over ex ante risky income bundles, the expected utility model is as follows:

$$U\left(W_1, W_2\right) = p_1 U\left(W_1\right) + p_2 U\left(W_2\right) \text{ where } p_1, p_2 > 0 \tag{1.4}$$

In fact, the investor chooses between risky bundles based on the expected utility to get from them. If he expects to extract more utility from one bundle than another he will choose this bundle rather than the other.

Moreover, the Von Neumann-Morgenstern utility function is characterized by a positive marginal utility. This property means that "*the more is always better*". It is the rate at which total utility increases as the level of income rises. That is the positivity of the first partial derivative of the utility function with respect to income W.

$$\frac{\partial U\left(W\right)}{\partial W} = U'\left(W\right) > 0 \tag{1.5}$$

1.1.1 Risk aversion and expected utility analysis

Let U be the strictly increasing utility function of an agent. Assuming that U is continuous and continuously differentiable, the risk aversion is equivalent to having a concave function i.e. $U'' < 0$. To derive this result, we use the utility premium tool suggested by Friedman and Savage (1948) which measure the degree of pain or harm implied in adding risk. While the usefulness of the utility premium concept in comparison between agents, it presents a promising tool for analyzing choices made by agents.

For a random risk ε, the utility premium is defined as follows:

$$V(W) = E\left[U(W+\varepsilon)\right] - U\left(W\right) \tag{1.6}$$

V presents the additional utility amount involved by adding the risk ε. If the agent is risk-averse, his utility decreases by adding risk, thus $V(W) < 0$.

Note that an individual is called risk-averse if at any wealth level; he dislikes every lottery with an expected payoff of zero. He prefers receiving the expected outcome of a lottery with certainty rather than the lottery itself i.e. $E\left[U\left(W+\varepsilon\right)\right] < U\left(W\right) \quad \forall\, W \textit{ and } \varepsilon$ where $Z = E\left(Z\right) + \varepsilon$ is a random outcome and ε is a zero-mean random variable.

A risk-averse individual is an individual who dislikes zero-mean risks. He may like risky lotteries if the expected payoffs that they yield are large enough.

In a general view, Rothschild and Stiglitz (1970) defined risk aversion as an aversion to mean-preserving spreads. It has been shown by Jensen's inequality that the necessary and sufficient condition for risk aversion is the concavity of the expected utility function, i.e. $U''(W) < 0 \; \forall \; W$. In the context of portfolio selection problem, the utility function must be decreasing and concave.

Following Eeckhoudt and Schlesinger (2006), we present risk aversion as preferences over lottery pairs. Let W be a positive initial wealth. We assume that an agent prefers more wealth to less wealth. Let k_1 and k_2 be two positive constants. In lottery $B_2 = [W - k_1, W - k_2]$, the agent has equally likely payoffs $W - k_1$ or $W - k_2$ payoffs. He has a 50-50 chance of either receiving a sure loss k_1 or the other sure loss k_2. However, in lottery $A_2 = [W, W - k_1 - k_2]$, the agent has a 50-50 chance of either receiving both losses ("harms") together $k_1 + k_2$ or receiving neither one. An agent is defined as risk-averse if he prefers B_2 to A_2, i.e. ($B_2 \succ A_2$) for every arbitrary positive parameter values W, k_1 *and* k_2. In fact, a risk-averse agent prefers to disaggregate sure losses k_1 *and* k_2 across states of the world. This risk aversion behavior describing disaggregating harms is defined by Eeckhoudt and Schlesinger (2006) as the concept of risk apportionment. This preference is equivalent to concavity of utility function $U'' < 0$. Moreover, within utility approach, the lottery B_2 is less risky than the lottery A_2 in the sense of Rothschild and Stiglitz (1970) since A_2 may be seen as a simple mean-preserving spread of B_2.

The certainty equivalent or the cash equivalent as introduced by Pratt (1964) is the certain level of wealth obtained in the 'good' state of nature ('no sickness') that yields the same level of satisfaction as the expected utility.

Since a risk-averse investor does not like zero-mean risks, we can measure the risk aversion by the amount which the investor is willing to pay to avoid a zero-mean risk ε. We call this amount as the risk premium π and presented as follows:

$$E\left[U(W+\varepsilon)\right] = U(W-\pi)$$

(1.7)

The risk premium is the amount that an individual would pay to achieve the same expected total utility when he replaces the lottery with its expected value (Eeckhoudt and Hammitt (2001)).

The risk premium is always a function of the utility function $U(.)$, the initial wealth W_0 and the distribution of zero-mean risk ε.

Considering a very small risk ε, we approximate the Equation (1.7) by a first order and a second order Taylor approximation of the left-hand and the right hand side as follows:

$$E\left[U(W+\varepsilon)\right] \simeq E\left[U(W) + \varepsilon U'(W) + \frac{\varepsilon^2}{2}U''(W)\right]$$
$$= U(W) + \frac{\sigma_\varepsilon^2}{2}U''(W)$$

and

$$U(W-\pi) \simeq U(W) - \pi U'(W) \tag{1.8}$$

Where $\sigma_\varepsilon^2 = E(\varepsilon^2)$ is the variance of ε.

Substituting into Equation (1.7), we find:

$$\pi \simeq \frac{\sigma_\varepsilon^2}{2}A(W) \tag{1.9}$$

Where $A(W) = -\frac{U''(W)}{U'(W)}$ is the Arrow-Pratt measure of absolute risk-aversion or a measure of the utility function concavity's degree. Moreover, dividing by the first derivative of the utility function $U'(W)$ shows that the Arrow-Pratt measure of absolute risk aversion is independent of affine transformations of the utility function and therefore the preference ordering would not been changed.

The absolute risk aversion and the risk premium should be decreasing functions of the initial wealth, i.e. $\frac{\partial \pi}{\partial W} < 0$ or equivalently $\frac{\partial A}{\partial W} < 0$, which mean $A'(W) < 0 \, \forall W$.

According to the global risk aversion theorem (Pratt (1964)), an investor A is more risk-averse than an investor 2 if for the same initial wealth amount, they have $\pi_1 > \pi_2 \ \forall \ \varepsilon \textit{ and } W$, or $A_1(W) > A_2(W) \forall W$ or equivalently $U_1(W) = G(U_2(W))$ where G is a strictly increasing concave function and U_1 and U_2 are the utility functions of investor 1 and 2 respectively. While $A(W) = -\frac{U''(W)}{U'(W)}$ measures the absolute risk aversion, $W \times A(W) = -W\frac{U''(W)}{U'(W)}$ is the relative risk aversion measure (Kimball (1990)).

Note that the first derivative of the absolute risk aversion is as follows:

$$A'(W) = -\frac{U'''(W)U'(W) - U''(W)^2}{U'(W)^2} \tag{1.10}$$

Since $A'(W) < 0$, the third partial derivative of the utility function $U'''(W)$ must be strictly positive.

1.1.2 Correspondences between Mean-Variance analysis and expected utility theory

1.1.2.1 Mean-variance preferences

According to MV analysis process, investors attempt to make more efficient investment choice by maximizing expected return for a given level of variance or minimizing variance for a given level of expected return. To determine the efficient set of portfolios, Markowitz framework is based on means and variances of returns and covariance between assets returns. It assumes that investor's preferences are described by preference function $\Phi(\mu,\sigma)$ over the mean μ and the standard deviation σ of the portfolio return. The first derivative of Φ with respect to the mean should be positive, i.e. $\Phi_\mu = \frac{\partial \Phi(\mu,\sigma)}{\partial \mu} > 0$; whereas with respect to the standard deviation, it should be negative, i.e. $\Phi_\sigma = \frac{\partial \Phi(\mu,\sigma)}{\partial \sigma} > 0$. Assuming that the variance is a risk measure, the above hypotheses are interpreted as risk investor conditions. That means the investor prefers realize gains and dislikes risk. Nonetheless, in general, the MV model is not necessarily equivalent to the expected utility approach. It has been proven by literature that in order to make MV evaluation reconcilable with the expected utility approach, we must make assumptions either about the quadratic utility function of the investor or about the assets returns distribution.

1.1.2.2 Restricting the utility function: Quadratic utility

In order to match the expected utility approach and the MV framework, we must assume that the investor's expected utility function is quadratic. Since the expected utility functions are unique only up to an affine transformation, the most general form of quadratic utility may be given by

$$U(W) = W - \frac{b}{2}W^2 \text{ , } b > 0 \tag{1.11}$$

Note that the marginal utility function $U'(W) = 1 - bW$ is positive for $W < \frac{1}{b}$. If not the marginal utility function becomes negative. In addition, the risk aversion is foolproof if the second derivative of utility function $U''(W) = -b$ is negative, that means for $b > 0$.

The expected utility function is given as follows

$$\begin{aligned} E\left[U(W)\right] &= E\left[W - \frac{b}{2}W^2\right] \\ &= E(W) - \frac{b}{2}\left[Var(W) + E^2(W)\right] \\ &= \mu - \frac{b}{2}\left(\sigma^2 + \mu^2\right) \\ &= \Phi(\mu, \sigma) \end{aligned} \tag{1.12}$$

Where $\Phi(\mu,\sigma)$ is the preference function mean μ and standard deviation σ. The first derivative of the preference function with respect with the mean $\Phi_\mu = \frac{\partial \Phi(\mu,\sigma)}{\partial \mu} = 1 - b\mu > 0$ for $\mu < \frac{1}{b}$. Moreover, the first derivative of the function with respect to the standard deviation $\Phi_\sigma = \frac{\partial \Phi(\mu,\sigma)}{\partial \sigma} = -b\sigma < 0$ for $b > 0$.

Finally, as shown by Pratt (1971) the quadratic utility assumption involves that wealthier agents invest less in risky investments, which is conflicting with intuition and the logic. In fact, the quadratic utility function requires globally increasing absolute risk aversion

$$A(W) = -\frac{U''(W)}{U'(W)} = \frac{b}{1 - bW} \tag{1.13}$$

The Arrow-Pratt risk aversion measure $A(W)$ is increasing in b $A'(W)=\left(\frac{b}{1-bW}\right)^2$, which is reasonable but contradicts the fact.

1.1.2.3 Restricting the return distribution: Normality

The normality of return's distribution implies the correspondence between the expected utility theory and the MV approach. A portfolio return (or wealth) is normally distributed if assets' returns have a normal distribution. Given that the wealth is normally distributed, the expected utility function is given as follows:

$$
\begin{aligned}
E\left[U(W)\right] &= \int_{-\infty}^{+\infty} U(W)\frac{1}{\sqrt{2\pi\sigma}}\exp\left\{-\frac{(W-\mu)^2}{2\sigma^2}\right\}dW \\
&= \int_{-\infty}^{+\infty} U(\sigma W+\mu)\phi(W)dW \\
&= \Phi(\mu,\sigma)
\end{aligned}
\tag{1.14}
$$

Where $\phi(W)$ is the normal density and $\Phi(\mu,\sigma)$ is the MV preference function.

The marginal utility is positive since the first derivative of Φ with respect to the mean is positive, $\Phi_\mu=\frac{\partial\Phi(\mu,\sigma)}{\partial\mu}=\int_{-\infty}^{+\infty}U'(\sigma W+\mu)\phi(W)dW \succ 0$ .Morevoer, as $\Phi(\mu,\sigma)$ is symmetric we have

$$
\Phi_\sigma=\frac{\partial\Phi(\mu,\sigma)}{\partial\sigma}=\int_{-\infty}^{+\infty}WU'(\sigma W+\mu)\phi(W)dW
$$

$$
\begin{aligned}
&= \int_{-\infty}^{0} WU'(\sigma W+\mu)\phi(W)dW+\int_{0}^{+\infty} WU'(\sigma W+\mu)\phi(W)dW \\
&= \int_{0}^{+\infty} W\left[U'(\sigma W+\mu)-U'(-\sigma W+\mu)\phi(W)\right]\phi(W)\,dW
\end{aligned}
\tag{1.15}
$$

If the investor is risk-averse ($U''(W)<0\ \forall W$), we find

$$U'(\sigma W+\mu)<U'(-\sigma W+\mu)\ \ for\ W>0$$

Thereby the first derivative of preference function with respect to the standard deviation is negative $\Phi_{\sigma} < 0$.

In total, individuals prefer maximizing expected returns and dislike return volatility (variance). Studies have shown that normality of returns simplifying assumption does not hold.

1.2 The parallel development of the expected utility theory and the modern portfolio theory

Since uncertainty governs the portfolio selection decision, the theory of choice under risk is important in financial investment process. While Bernoulli (1738/1954), Arrow (1965) and Pratt (1964) have proven the universal importance of risk aversion, Leland (1968), Sandmo (1970), Kimball (1990) and Kimball (1993) demonstrated that the higher-order risk attitudes prudence and temperance complement significantly risk aversion attitude. For example, in the saving behavior area, risk averse agent prefers smoothing consumption over time (Modigliani and Ando (1957)), prudence manages the change of saving behavior when future income becomes riskier that is defined by Kimball (1990) as the precautionary saving, and temperance define the sensitivity of saving behavior to changes in macroeconomic risks such as interest rate risk (Eeckhoudt and Schlesinger (2008)). Agents' prudence and temperance degrees have significant involvement on a wide range of financial and economic works such as healthcare field (see for example, Courbage and Rey (2006), Bui et al. (2005) and Eeckhoudt et al. (2007)), bargaining (White (2008)), sustainable development and climate change (Bramoullé and Treich (2009)), rent seeking (Treich (2010)), insurance holding (Eisenhauer and Halek (1999)), competitive firm underprice uncertainty (Wong (2004)), labor supply (Flodén (2006)) and portfolio choice (Briec and Kerstens (2010)) among other fields.

Literature shows that portfolio selection decisions depend crucially on higher order risk attitudes. Within the expected utility approach, prudence and temperance are properties of the third and fourth derivatives of the utility function. In fact, as termed by Kimball (1990), prudence is defined by a convex first derivative of the utility function (convex marginal utility) and has direct implications for saving behavior that is greater savings in response to an increase in background risk. Based on precautionary saving theory, prudence is positively correlated with saving motives, whereas temperance is defined by a concave second derivative of the utility function and is negatively correlated with the riskiness of portfolio choices.

In the following subsection, we define prudence and temperance concepts, we show how they are equivalent to signing derivatives of the utility function within an expected utility framework and we develop preferences over lotteries that correspond to prudence and temperance. We finally relate these higher order risk concepts to preferences in portfolio area.

1.2.1 On the direction of preference of higher order moments than the variance for portfolio selection

1.2.1.1 About the prudence

The notion of prudence in determining precautionary savings demand was firstly noted by Leland (1968) and Sandmo (1970). Then, Kimball (1990) has coined the prudence term as equivalent to a precautionary demand for savings. He defined prudence as "*the sensitivity of the optimal choice of a decision variable to risk*"; likewise it is "*meant to suggest the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to 'risk aversion', which is how much one dislikes uncertainty and would turn away from uncertainty if possible*", in consumption-saving decision, it is defined as "*the intensity of the precautionary saving motive*" which means uncertainty about future incomes implies the reduction of the current consumption in favor of the increase of the current saving.
Gollier (2001) defines *"an agent as prudent if adding an uninsurable zero-mean risk to his future wealth rises his optimal saving."*

Within the expected utility approach, the sign of every derivative of the Von Neumann Morgenstern utility function has some economic interpretation. In fact, classifying individuals as prudent and temperate may be based on the signs of the derivatives of their utility functions (Eeckhoudt and Schlesinger (2006)).

Let W be a risky investment, $\tilde{W} = E(W)$ is its expected value and U is the VNM utility function, the condition $E\left[U'(W)\right] > U'\left(\tilde{W}\right)$ implies the convexity of U', i.e. prudence attitude. An agent is called prudent if his marginal utility is convex.
Much earlier, Menezes et al. (1980) relate the sign of third derivative of expected utility function to aversion to downside risk which is equivalent to prudence. A pure rise in downside risk does not vary the mean or the variance of a risky wealth prospect, but it does decrease the skewness.

In fact, Eeckhoudt and Schlesinger (2006) used the utility premium $V(W)$ to show the relationship between the expected utility approach and the prudence concept. The derivative of the utility premium with respect to wealth is as follows:

$$V'(W) = E\left[U'(W+\varepsilon)\right] - U'\left(W\right) \tag{1.16}$$

Using Jensen'inequality, we find that $V'(W)$ is positive whenever U' is convex function i.e. $U''' > 0$. Since $V\left(W\right) < 0$, we interpret $V'\left(W\right) > 0$ as meaning that the size of the utility premium gets smaller as initial wealth W increases. A prudent agent prefers taking on an unavoidable risk in a relatively high income state of the nature.

Eeckhoudt and Schlesinger (2006) define prudence notion as a type of preference over lotteries. It is a type of preference for disaggregating two risks. A prudent agent is more willing to accept an extra risk when wealth is higher than when wealth is lower. Let W be a positive initial wealth. Let k be a positive constant and $\tilde{\varepsilon}$ be a zero-mean random variable.

Following Eeckhoudt and Schlesinger (2006), the third-order risk attitude of prudence (or downside risk aversion) is presented in lottery. Losing k and adding the random variable $\tilde{\varepsilon}$ present the pair of harms in this lottery case. A prudent agent prefers disaggregating these two harms across different states of the world. He prefers $B_3 = \left[W-k, W+\tilde{\varepsilon}\right]$ over $A_3 = \left[W, W+\tilde{\varepsilon}-k\right]$ for all wealth levels W, sure wealth contraction k and zero-mean risks $\tilde{\varepsilon}$. In other words, the individual is prudent if he prefers adding the zero-mean variable $\tilde{\varepsilon}$ to the state of the nature with the higher wealth W than with the state with lower wealth $W-k$. In the same way, a prudent agent prefers attaching the sure loss k to the state with no risk than to the state of the world with the random risk $\tilde{\varepsilon}$. In overall, an unavoidable risk preferred when wealth is higher. In terms of risk apportionment, as mentioned by Eeckhoudt and Schlesinger (2006) *"a prudent individual prefers to apportion the two harms by placing one in each state."*

The prudence premium φ is defined by Kimball (1990) and Bleichrodt and Eeckhoudt (2005) as the solution of the following equation:

$$E\left[U'\left(W+\varepsilon\right)\right] = U'\left(W-\varphi\right) \tag{1.17}$$

The prudence premium indicates how much of initial wealth has to be reduced by agent in order to maintain its expected marginal utility constant. It represents the degree of prudence. Higher prudence premium leads to higher saving.

The index of absolute prudence represents the strength of the precautionary saving motives according to Kimball (1990) and is presented as follows:

$$P\left(W,c\right) = P\left(s\right) = -\frac{U'''\left(s\right)}{U''\left(s\right)} \tag{1.18}$$

Where the amount of saving $s = W - c$ is the difference between the wealth W and the consumption c.

According to Kimball (1990), the index of relative prudence is presented as follows:

$$W \times P(W,c) = W \times P(s) = -W \times \frac{U'''(s)}{U''(s)} \tag{1.19}$$

1.2.1.2 Theoretical background about the temperance

Temperance behavior was first introduced by Kimball (1991) as the sense of moderation in accepting risks. Logically, an investor should hedge against a risk that is correlated with the risk that he is already exposed. In presence of an unavoidable risk, a temperate agent reduces exposure to another risk even if these two risks are statistically independent.

Temperance is also defined by Eeckhoudt and Schlesinger (2006) as a type of preference for disaggregation of two independent zero-mean risks. Temperance was explained in terms of utility as a negative fourth derivative of the utility function. Let W be a risky investment, $\tilde{W} = E(W)$ is its expected value and U is the VNM utility function, the condition $E\left[U''(W)\right] < U''\left(\tilde{W}\right)$ is equivalent to concavity of U'' and thus temperance behavior.

Indeed, Eeckhoudt and Schlesinger (2006) used the utility premium $V(W)$ to show the relationship between the expected utility approach and the temperance concept. The second derivative of the utility premium with respect to wealth is as follows:

$$V''(W) = E\left[U''(W+\varepsilon)\right] - U''(W) \tag{1.20}$$

Using Jensen'inequality, we find that $V''(W)$ is negative whenever U'' is concave function i.e. $U'''' < 0$. That concavity of the utility premium is equivalent to a preference for risks disaggregating. Since we have a decreasing utility premium, we interpret $V''(W) < 0$ as meaning that the rate of decrease in the utility premium lessens as wealth increases.

Moreover, Menezes and Wang (2005) show "*that temperance can be interpreted as aversion to outer risk. Temperate individuals dislike relocations of dispersion from the center of a distribution to its tails*". Thus, temperate agent dislikes kurtosis in the same way of risk-averse person dislikes higher variance.

Besides the equivalence between temperance behavior and negative fourth derivative of utility function, the temperance attitude was defined by Eeckhoudt and Schlesinger (2006) as a type of preference over lotteries. It is a type of preference for disaggregating two risks. Let W be a positive initial wealth. Let $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ be two independent and distinct zero-mean random variables. A temperate individual prefers $B_4 = \left[W + \tilde{\varepsilon}_1, W + \tilde{\varepsilon}_2\right]$ over $A_4 = \left[W, W + \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2\right]$. He prefers to apportion the two harms by placing one in each state of the nature.

The temperance premium τ is defined by Kimball (1990) and Bleichrodt and Eeckhoudt (2005) as the solution of the following equation:

$$E\left[U''(W+\varepsilon)\right] = U''(W-\tau) \tag{1.21}$$

Introduced by Eeckhoudt et al. (1996) , the index of absolute temperance is as follows :

$$T(W) = -\frac{U''''(W)}{U'''(W)}$$

As shown by Eeckhoudt and Schlesinger (2006) and Wang and Li (2010), the index of relative temperance is presented as follows:

$$W \times T(W) = -W\frac{U''''(W)}{U'''(W)} \tag{1.22}$$

As conclusion, Kimball (1991) has shown that a prudent individual responds to risk by accumulating more wealth however a temperate agent responds to an unavoidable risk by reducing exposure to other risk even when risks are statistically independent.

We summarize measurement details of risk aversion, prudence and temperance in Table 1.1.

Table 1.1: Summary Table on risk attitudes

Attitude	Utility function	Lottery preference	Absolute measure	Relative measure	Premium
Risk aversion	$U'' < 0$	$\left[W-k_1, W-k_2\right] \succ \left[W, W-k_1-k_2\right]$	$W \times A(W) = -W\dfrac{U''(W)}{U'(W)}$	$A(W) = -\dfrac{U''(W)}{U'(W)}$	$E\left[U(W+\varepsilon)\right] = U(W-\pi)$
Prudence	$U''' > 0$	$\left[W-k, W+\tilde{\varepsilon}\right] \succ \left[W, W+\tilde{\varepsilon}-k\right]$	$P(W,c) = P(s) = -\dfrac{U'''(s)}{U''(s)}$	$W \times P(W,c) = W \times P(s) = -W \times \dfrac{U'''(s)}{U''(s)}$	$E\left[U'(W+\varepsilon)\right] = U'(W-\varphi)$
Temperance	$U'''' < 0$	$\left[W+\tilde{\varepsilon}_1, W+\tilde{\varepsilon}_2\right] \succ \left[W, W+\tilde{\varepsilon}_1+\tilde{\varepsilon}_2\right]$	$T(W) = -\dfrac{U''''(W)}{U'''(W)}$	$W \times T(W) = -W\dfrac{U''''(W)}{U'''(W)}$	$E\left[U''(W+\varepsilon)\right] = U''(W-\tau)$

1.2.2 On prudence, temperance and portfolio optimization

As assumed by Eeckhoudt and Schlesinger (2006), an agent dislikes two states of the world: a certain diminution in wealth and adding a zero-mean independent noise random variable to the distribution of the wealth. Prudence is defined as a way of preference for disaggregation of these two untoward states of the world. Replacing the certain reduction in wealth with a second independent zero-mean risk, temperance is defined as a type of preference for disaggregation of these untoward events. In other words, prudent agent prefers adding an unavoidable zero-mean risk to a state in which income is high, rather than adding it to a state in which income is low. However, temperate agent prefers disaggregating two independent zero-mean risks across different states of the world, rather than facing them at the same time in a single state. As we have shown, if W is a risky investment, $\tilde{W} = E(W)$ is its expected value and U is the VNM utility function, then the condition $E\left[U(W)\right] < U\left(\tilde{W}\right)$ involves concavity of U, i.e. risk aversion behavior. The condition $E\left[U'(W)\right] > U'\left(\tilde{W}\right)$ implies the convexity of U', i.e. prudence attitude. The condition $E\left[U''(W)\right] < U''\left(\tilde{W}\right)$ is equivalent to concavity of U'' and thus temperance behavior.

As Levy (1989) pointed out, the MV and the Von-Neumann expected utility function models identify the same efficient sets of assets when the investor is risk-averse. In fact, investors choose exactly the same random prospects when their preferences are characterized by a quadratic utility function or when financial asset returns are random variables with fixed expected return and variance. A concave quadratic utility function is characterized by positive first order and negative second order derivative and null higher order derivatives. This assumption is a major source of criticisms of the MV framework. Indeed, it has been repeatedly shown that financial asset returns are not normally distributed, and we should not ignore empirical evidence that extreme value distribution characterize them. This strongly suggests that moments larger than the second do matter in financial problems. In fact, assuming investors to have quadratic utility function is equivalent to assume that the market is efficient. The Efficient Market Hypothesis (EMH) formulated by Fama (1970) suggests that "a Market in which prices always 'fully reflect' available information is called 'efficient'". Nonetheless, information included in the stock prices reflect other relevant information, the likes of political, social and economic events among others, in addition to financial information.

Ambiguous factors usually exist in stock prices and investors need to consider not only random conditions but also ambiguous and subjective conditions for portfolio selection problems (Urrutia (1995)). Therefore, recent studies enlarge the evaluation dimension to the third and fourth distribution moment.

The literature shows that portfolio selection decisions depend crucially on higher order risk attitudes. Within the expected utility approach, prudence and temperance are properties of the third and fourth derivatives of the utility function.

In an uncertain environment, an investor seeks usually to maximize his risk premium π. According to Eeckhoudt and Hammitt (2001), the risk premium presents the amount that an individual would pay to achieve the same expected total utility when he replaces the lottery with its expected value. It is presented as follows:

$$U\left(E(W)-\pi\right)=E\left[U(W)\right] \tag{1.23}$$

Where W is the end-of-period wealth and $E(.)$ is the expectation operator.

As developed by Arrow (1965) and Pratt (1964), the first order Taylor expansion of $U\left(E(W)-\pi\right)$ about W, is given by

$$U\left(E(W)-\pi\right)=U\left(E(W)\right)-\pi U^{(1)}\left(E(W)\right) \tag{1.24}$$

While the higher order Taylor expansion is as follows:

$$\begin{aligned}U(W)=&U\left(E(W)\right)+\left(W-E(W)\right)U^{(1)}\left(E(W)\right)+\frac{\left(W-E(W)\right)^2}{2!}U^{(2)}\left(E(W)\right)\\&+\frac{\left(W-E(W)\right)^3}{3!}U^{(3)}\left(E(W)\right)+\frac{\left(W-E(W)\right)^4}{4!}U^{(4)}\left(E(W)\right)+\sum_{k=5}^{\infty}\frac{\left(W-E(W)\right)^k}{k!}U^{(k)}\left(E(W)\right)\end{aligned} \tag{1.25}$$

Where $U^{(i)}$ corresponds to the i^{th} derivative of the utility function with respect to the final wealth.

Substituting Equation (1.23) in Equation (1.25) we obtain:

$$\begin{aligned}E\left[U(W)\right]=&U\left(E(W)\right)+E\left[W-E(W)\right]U^{(1)}\left(E(W)\right)+\frac{E\left[\left(W-E(W)\right)^2\right]}{2!}U^{(2)}\left(E(W)\right)\\&+\frac{E\left[\left(W-E(W)\right)^3\right]}{3!}+\frac{E\left[\left(W-E(W)\right)^4\right]}{4!}U^{(4)}\left(E(W)\right)+\sum_{k=5}^{\infty}\frac{E\left[\left(W-E(W)\right)^k\right]}{k!}U^{(k)}\left(E(W)\right)\end{aligned} \tag{1.26}$$

Where $\sum_{k=5}^{\infty}\frac{E\left[\left(W-E(W)\right)^k\right]}{k!}U^{(k)}\left(E(W)\right)$ is the remainder term of the Taylor expansion, that is of small order and negligible (Jondeau and Rockinger (2006) and Garlappi and Skoulakis (2011)). Moreover, it is suggested by Berenyi (2001), among others, that the fourth term could improve substantially the quality of the approximation of the expected utility function. Therefore, we may choose to truncate the Taylor series after the fourth term since the remainder is negligible and the fourth order term improves the approximation and allows us to introduce explicitly the skewness and kurtosis in the analysis of portfolio selection.

Eventually, the expected utility function may be expressed as follows:

$$E\left[U(W)\right]=U\left(E(W)\right)+\frac{m_2(W)}{2!}U^{(2)}\left(E(W)\right)+\frac{m_3(W)}{3!}U^{(3)}\left(E(W)\right)+\frac{m_4(W)}{4!}U^{(4)}\left(E(W)\right) \qquad (1.27)$$

Where $m_2(W)$, $m_3(W)$ and $m_4(W)$ are the second to the fourth order moment of the final wealth W.

This is equivalent to

$$U\left(E(W)\right)-\pi U'\left(E(W)\right)=U\left(E(W)\right)+\frac{m_2(W)}{2}U^{(2)}\left(E(W)\right)+\frac{m_3(W)}{3!}U^{(3)}\left(E(W)\right) + \frac{m_4(W)}{4!}U^{(4)}\left(E(W)\right) \qquad (1.28)$$

From which the expression of the premium π, that an investor wish to maximize, is presented as follows:

$$\pi=\left[-\frac{U^{(2)}\left(E(W)\right)}{U^{(1)}\left(E(W)\right)}\right]\frac{m_2(W)}{2}-\left[\frac{U^{(3)}\left(E(W)\right)}{U^{(1)}\left(E(W)\right)}\right]\frac{m_3(W)}{3!}+\left[-\frac{U^{(4)}\left(E(W)\right)}{U^{(1)}\left(E(W)\right)}\right]\frac{m_4(W)}{4!}$$
$$=\frac{1}{2}\lambda m_2(W)-\frac{1}{6}\psi m_3(W)+\frac{1}{24}\xi m_4(W) \qquad (1.29)$$

Where $\lambda=-\frac{U^{(2)}\left(E(W)\right)}{U^{(1)}\left(E(W)\right)}$ is the absolute risk aversion measure, $\psi=\frac{U^{(3)}\left(E(W)\right)}{U^{(1)}\left(E(W)\right)}$ is the coefficient of appetite toward symmetry (the third centered distribution moment) and $\zeta=-\frac{U^{(4)}\left(E(W)\right)}{U^{(1)}\left(E(W)\right)}$ is the coefficient of aversion to leptokurticity (distribution tail thickness).[1]

According to the previous development, an investor dislikes variance and kurtosis and likes skewness.

Note that the final wealth of the investor may be expressed as follows:

$$W=W_0\left(1+R\right) \qquad (1.30)$$

[1] For more detail, see (Courtois 2012).

Where W_0 is the initial wealth (equal to the unity by simplification[2]) and $R=(W-W_0)/W_0$ is the portfolio return.

Thereby the final wealth and its expectation are as follows:

$$W=1+R \tag{1.31}$$

$$E(W)=1+E(R) \tag{1.32}$$

Which give :

$$W-E(W)=R-E(R) \tag{1.33}$$

Substituting Equation (1.33) in Equation (1.26) truncated after the fourth term we find

$$\begin{aligned} E\left[U(R)\right] &= U\left(E(R)\right)+E\left[R-E(R)\right]U^{(1)}\left(E(R)\right)+\frac{E\left[\left(R-E(R)\right)^2\right]}{2!}U^{(2)}\left(E(R)\right) \\ &+\frac{E\left[\left(R-E(R)\right)^3\right]}{3!}+\frac{E\left[\left(R-E(R)\right)^4\right]}{4!}U^{(4)}\left(E(R)\right) \end{aligned} \tag{1.34}$$

Let r_h be a random variable representing the rate of return of the asset h and w_h be the weight of the asset h. The rate of portfolio return, $R=r(w)$, is a function of the portfolio weights $\boldsymbol{w}=(w_1,...,w_n)$ and is given by:

$$R=r(w)=\sum_{h=1}^{n} w_h r_h \tag{1.35}$$

The mean of the individual returns, $E(r_h)=\mu_h$, the vector of the means is given by $\boldsymbol{\mu}=[\mu_h]$

The mean, variance, skewness and kurtosis of the portfolio return $r(w)$, denoted $\mu(w), v(w)$, $s(w)$ and $\kappa(w)$ respectively, are defined as follow:

$$\begin{aligned} \mu(w) &= E\left[r(w)\right]=\sum_{h=1}^{n} w_h E(r_h)=\sum_{h=1}^{n} w_h\mu_h = w^T\boldsymbol{\mu}, \\ v(w) &= E\left[\left(r(w)-\mu(w)\right)^2\right]=\sum_{h=1}^{n}\sum_{j=1}^{n} w_h w_j\sigma_{hj} = \boldsymbol{w}^T M_2\boldsymbol{w}, \\ s(w) &= E\left[\left(r(w)-\mu(w)\right)^3\right]=\sum_{h=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} w_h w_j w_k s_{hjk} = \boldsymbol{w}^T M_3\left(\boldsymbol{w}\otimes\boldsymbol{w}\right), \\ \kappa(w) &= E\left[\left(r(w)-\mu(w)\right)^4\right]=\sum_{h=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} w_h w_j w_k w_l\kappa_{hjkl} = \boldsymbol{w}^T M_4\left(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}\right). \end{aligned} \tag{1.36}$$

Where

[2] Scaling the initial agent wealth to one is commonly used in the investor preference literature since it ensure the complete equivalence between the expected utility expressions both in terms of return and end-of-period wealth (see for more details Jondeau and Rockinger (2006) and Brockett and Golden (1987)).

$M_2 = E\left[(r_h-\mu)(r_j-\mu)'\right] = \left\{\sigma_{hj}\right\}$ is the (n,n) variance-covariance matrix, $M_3 = E\left[\left(r_h-\mu\right)\left(r_j-\mu\right)' \otimes \left(r_k-\mu\right)'\right] = \left\{s_{hjk}\right\}$ is the $\left(n,n^2\right)$ skewness co-skewness matrix, $M_4 = E\left[\left(r_h-\mu\right)\left(r_j-\mu\right)' \otimes \left(r_k-\mu\right)' \otimes \left(r_l-\mu\right)'\right] = \left\{\kappa_{hjkl}\right\}$ is the $\left(n,n^3\right)$ kurtosis co-kurtosis matrix, $h,j,k,l=1,...,n$ and the sign $\otimes$ standing for the symbol of the kronecker product.

The utility of an investor is a function of the rate of return, so we can write the investor's preferences as $U(r(w))$. The Taylor expansion of $U(r(w))$ around $\mu(w)$ up to the fourth moment is given by:

$$\begin{aligned} U(r(w)) &= U(\mu(w)) + U^{(1)}(\mu(w))(r(w)-\mu(w)) + \frac{U^{(2)}(\mu(w))}{2}(r(w)-\mu(w))^2 \\ &+ \frac{U^{(3)}(\mu(w))}{3!}(r(w)-\mu(w))^3 + \frac{U^{(4)}(\mu(w))}{4!}(r(w)-\mu(w))^4 \end{aligned} \tag{1.37}$$

This implies

$$\begin{aligned} E\left[U(r(w))\right] &= E\left[U(\mu(w))\right] + U^{(1)}(\mu(w))\,\mathrm{E}\left[(r(w)-\mu(w))\right] + \frac{U^{(2)}(\mu(w))}{2}\mathrm{E}\left[(r(w)-\mu(w))^2\right] \\ &+ \frac{U^{(3)}(\mu(w))}{3!}\mathrm{E}\left[(r(w)-\mu(w))^3\right] + \frac{U^{(4)}(\mu(w))}{4!}\mathrm{E}\left[(r(w)-\mu(w))^4\right] \end{aligned} \tag{1.38}$$

where $U^{(i)}$ is the i^{th} derivative of $U\left[r\left(w\right)\right]$.

Following Benishay (1992), Joro and Na (2006) and Jurczenko, Maillet and Merlin (2006), we focus on terms up to the fourth, so we approximate the expected utility by the following preference function

$$E\left[U(r(w))\right] = U^{(1)}(\mu(w)) + \frac{U^{(2)}(\mu(w))}{2}v(w) + \frac{U^{(3)}(\mu(w))}{3!}s(w) + \frac{U^{(4)}(\mu(w))}{4!}k(w) \tag{1.39}$$

In the case of two-asset portfolio, the $\left(2,2\right)$ variance-covariance matrix is illustrated as follows:

$$M_2 = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}$$

M_2 contains $\dfrac{n\left(n+1\right)}{2} = 3$ distinct elements in the above matrix as follows

σ_{11},
$\sigma_{12} = \sigma_{21}$,
σ_{22}.

The $\left(2,2^2\right)$ skewness co-skewness matrix is presented as follows:

$$M_3 = \begin{pmatrix} s_{111} & s_{112} & s_{211} & s_{212} \\ s_{111} & s_{122} & s_{221} & s_{222} \end{pmatrix}$$

Where only $\frac{n(n+1)(n+2)}{6} = 4$ distinct elements have to be computed as follows:

s_{111},
$s_{112} = s_{121} = s_{211}$,
$s_{122} = s_{212} = s_{221}$,
s_{222}.

The $(2, 2^3)$ kurtosis co-kurtosis matrix is as follows:

$$M_3 = \begin{pmatrix} k_{1111} & k_{1112} & k_{1121} & k_{1122} & k_{1211} & k_{1212} & k_{1221} & k_{1222} \\ k_{2111} & k_{2112} & k_{2121} & k_{2122} & k_{2211} & k_{2212} & k_{2221} & k_{2222} \end{pmatrix}$$

Where we have $\frac{n(n+1)(n+2)(n+3)}{24} = 5$ distinct elements:

k_{1111},
$k_{1112} = k_{1121} = k_{1211} = k_{2111}$,
$k_{1122} = k_{1212} = k_{1221} = k_{2112} = k_{2211}$,
$k_{1222} = k_{2122} = k_{2212} = k_{2221}$,
k_{2222}.

Under Scott and Horvath (1980) conditions, the expected utility depends positively on the expected return and skewness and negatively on the variance and kurtosis of the portfolio.

As a complex multi-criteria problem, portfolio selection has seen the investigation of various methods and procedures and an important number of criteria with additional to the mean, the variance and the higher moments of returns.

The process of portfolio selection could be divided into two stages: the first one is the evaluation of financial assets and the second one is to choose the best ones to construct portfolio. As it has been listed by Aouni et al. (2018) the Multi Criteria Decision Analysis (MCDA) techniques applied in security analysis/evaluation phase are the Analytic Hierarchy Process (AHP), TOPSIS, VICOR, ELECTRE, MACBETH, UTADIS based, Fuzzy multi-criteria Expert Systems. However, the MCDA techniques applied in portfolio construction or optimization stage are the Goal programming, Compromise programming, ε-Constraint method, PROMETHEE V, Reference Point Method, MAUT, weighting approach, Reservation level driven Tchebycheff procedure, Interactive methods, IPSSIS, ADELAIS and Fuzzy Mathematical Programming.

Yet, it is well proved that actual assessment criteria may be more complicated and differ extensively from theoretical formulations. Not only are there several criteria to take into account and each one being associated with a priority level, but also these criteria and their importance level (weight) are usually quite specific to each decision maker. The need to consider simultaneously multiple criteria while considering investors' own preferences is obvious since they have not always the same objective function, attitude toward the risk, wealth and utility function among other specifications. From such perspectives, the Data Envelopment Analysis approach (DEA) seems to be an impressive multi-criteria tool. In fact, it has the merit to consider many criteria at the same time, together with a direct control over the weights (importance level) paid to each criterion by means of an optimization program developed by Charnes et al. (1978). Given these merits in performance gauging, the DEA approach has been widely used in portfolio evaluation field (see for example, Basso and Funari (2001), Edirisinghe and Zhang (2007) and Chen et al. (2018)).

While DEA can provide a solution to the problem of aggregating multi-performance measures into one key indicator, DEA suffers from high flexibility thus allowing for a weak discrimination among Decision Making Units (DMUs). In fact, we may have more than 100% efficient DMU. As a result, ranking DMUs can be quite hard. Thereby, the cross-efficiency method, proposed by Sexton et al. (1986) and investigated by Doyle and Green (1994a) provides a unique ordering of DMUs and eliminates unrealistic weight schemes through peer evaluation.

2. Improving discrimination in DEA: From DEA cross-efficiency to DEA game cross-efficiency and Nash equilibrium

2.1 DEA Cross-efficiency: Derivations, meanings and uses

The DEA efficiency score is a weighted sum of a DMU outputs divided by a weighted sum of its inputs. Despite the effectiveness of DEA method in identifying the best practice frontiers, its flexibility in weighting inputs and outputs presents a major limitation. In fact, DEA technique permits to determine these weights as a linear program by allowing each DMU to appear in the best possible light (maximize its own measured efficiency relative to the other DMUs) given a minimal set of constraints on the weights. Introduced by Sexton et al. (1986) and investigated by Doyle and Green (1994a), the DEA cross-efficiency could present a solution to this weights flexibility problem. The main idea was to use the optimal factor (inputs and outputs) weights found for a particular DMU, using DEA, to compute the cross-efficiency of each of the other DMUs, as seen by the original DMU. Repeating this procedure for all DMUs we obtain a matrix of cross-efficiencies, row-by-row. The simple efficiencies (self-evaluation) scores for each DMU are found in the leading diagonal of this matrix. Whereas, the other values in the matrix are the peer-evaluation scores.

Given a set of n DMUs where a DMU_j $(j=1,...,n)$ chooses its own weights μ_{jy} (for j's y^{th} output O) and ω_{jx} (for j's x^{th} input I). Using the weights that j has chosen, the cross-efficiency of DMU m is $E_{jm}=\sum_y O_{my}\mu_{jy} \Big/ \sum_x I_{mx}\omega_{jx}$. Following Doyle and Green (1994a), we present the matrix of cross-efficiencies for n DMUs in Table 1.2 as follows:

Table 1.2 : Cross-efficiency matrix

Rating DMU	Rated DMU				Averaged appraisal of peers
	1	2	$\cdots$	n	
1	E_{11}	E_{12}	$\cdots$	E_{1n}	A_1
2	E_{21}	E_{22}	$\cdots$	E_{2n}	A_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	E_{n1}	E_{n2}	$\cdots$	E_{nn}	A_n
	e_1	e_2	$\cdots$	e_n	

The simple efficiency (self-evaluation) may be interpreted as a special case of cross-efficiency; it is when a DMU rates itself. The overall cross-efficiency scores e_j is obtained by averaging down column j. Instead of the average, we can use the median, the minimum, the maximum, the variance or even the range of ratings to obtain the overall cross-efficiency scores. There are two principal merits of the cross-efficiency approach: its discrimination power by providing a unique ordering among DMUs and the elimination of unrealistic weight schemes without requiring the elicitation of weight restrictions from the decision maker.

Given these advantages, cross-efficiency method has been used in various applications such as health care (Sexton et al. (1986), Lam (2010) and Lozano (2012)), R&D projects (Oral et al. (1991) and Shang and Sueyoshi (1995)), preference voting and project ranking (Green et al. (1996), Wu et al. (2009c), Liang et al. (2008a) and Chen and Zhu (2011)), Scheduling problem (Chai et al. (2013)), transportation (Ruiz (2013) and Sarkis (2000)), Energy and environement (Chen (2002), Lu and Lo, (2007), Rezaee et al. (2012), Liu et al. (2017b) and Chen et al. (2017a)), supply chains (Yu et al. (2010), Ho et al. (2010) , Wang and Li (2014), Ma et al. (2014b), Gregoriou et al. (2005a), Sun et al. (2016) and Falagario et al. (2012)), education field (Yang et al. (2013), Yang et al. (2012), Doyle and Green, (1994b), Wu et al. (2012) , Oral et al. (2015) and Liu et al. (2017a)), Banking (Zerafat et al. (2013), Li et al. (2018) and Ma et al. (2014a)), Manufacturing (Sun and Lu (2005), Wei and Wang (2017), Wang and Wang (2013), and Jahanshahloo et al. (2011b)), sport competition (Wu et al. (2009a), Oukil and Amin (2015), Roboredo et al. (2015), Gutiérrez and Ruiz (2013), Wu et al. (2009b) and Aizemberg et al. (2014)) and portfolio performance (Mashayekhi and Omrani (2016), Sanei and Banihashemi (2013), Sanei and Banihashemi (2014), Banihashemi and Sanei (2015) and Lim et al. (2014)).

However, DEA cross-efficiency suffers from the problem of non-uniqueness of the DEA optimal weights as noted by Doyle and Green (1994a). In fact, cross-efficiency scores depend on the generated optimal solutions by the DEA linear program in use. More specifically, it depends on the resolution of the used software (Despotis (2002)). Sexton et al. (1986) and Doyle and Green (1994a) propose the implement secondary goals to deal with the non-uniqueness problem. They present aggressive and benevolent model formulations. The idea of the benevolent model is to identify the optimal weights that not only maximize the efficiency of a particular DMU under evaluation, but at the same time, maximize the average efficiency of other DMUs. In the case of the aggressive model, one seeks weights that minimize the average efficiency of those other DMUs. Afterwards, a wide literature has been developed to improve the robustness of cross-efficiency approach.

Cross-efficiency approach is identified by Liu et al. (2016) as one of the four research fronts in DEA. Due to its democratic process and powerful discrimination ability, the method has seen several theoretical developments and was successfully applied to a wide real world problem. Table 1.3 presents most of these works.

Table 1.3 : Summary of literature on cross-efficiency method

Work	**Cross-efficiency extension**	**Topic**	**Research area**
Sexton et al. (1986)	DEA Cross-efficiency	Efficiency of nursing home evaluation	

Lam (2010)	Mixed-integer linear programming (MILP) to determine suitable weight sets for cross-evaluation	Efficiency of hospitals evaluation	Health care
Lozano (2012)	Cooperative DEA game cross-efficiency	Hospitals (illustrative example)	
Jahanshahloo et al. (2011a)	Aggressive and benevolent cross-efficiency	Nursing home (illustrative example)	
Oral et al. (1991)	DEA Cross-efficiency	R&D Project Evaluation	Preferences voting and R&D Projects Selection
Shang and Sueyoshi (1995)	Analytic Hierarchy Process (AHP)+ DEA cross-efficiency	selection of a Flexible Manufacturing System	
Green et al. (1996)	Benevolent cross-efficiency Aggressive cross-efficiency DEA cross-efficiency	Performance of R&D projects	
Liang et al. (2008a)	DEA game cross-efficiency	Preference Voting and R&D Projects Selection	
Chen and Zhu (2011)	Bootstrapped DEA game cross-efficiency	R&D Project Budgeting	
Chai et al. (2013)	Free disposal hull (FDH) cross-efficiency	The scheduling problem	
Ruiz (2013)	Cross-efficiency evaluation with Directional Distance Functions (DDF)	Performance of International airlines	Transportation
Sarkis (2000)	Aggressive cross-efficiency DEA cross-efficiency	Airports	
Chen (2002)	DEA cross-efficiency + cluster analysis	Electricity distribution sector	Energy
Lu and Lo (2007)	Cross-efficiency	economic-environmental performance	
Rezaee et al. (2012)	Bargaining DEA game efficiency + cross-efficiency	evaluation of thermal power plants	
Liu et al. (2017b)	DEA cross-efficiency evaluation considering undesirable output and ranking priority	eco-efficiency analysis of coal-fired power plants	

Chen et al. (2017b)	Game cross-efficiency	electric energy efficiency	
Oukil and Amin (2015)	Maximum appreciative cross-efficiency in DEA	baseball players	Games
Roboredo et al. (2015)	DEA game cross-efficiency	the Brazilian football championship	
Gutiérrez and Ruiz (2013)	DEA cross-efficiency	Performance of Players in the Spanish Handball League	
Aizemberg et al. (2014)	DEA game cross-efficiency	Basketball teams	
Yu et al. (2010)	DEA cross-efficiency	Supply chains	Supply chains
Wang and Li (2014)	Nash bargaining game model + cross-efficiency	Supply chain	
Ma et al. (2014b)	DEA Game Cross-efficiency	Supplier selection	
Gregoriou et al. (2005a)	Simple and cross-efficiency	commodity trading advisor	
Sun et al. (2016)	DEA game cross-efficiency	Public Infrastructure Investment	
Falagario et al. (2012)	DEA cross-efficiency	public procurement tenders (supplier selection)	
Jahanshahloo et al. (2011b)	Cross-efficiency with data are Intervals	Network problem	
Baker and Talluri (1997)	Cross-efficiency	Industrial robot selection	Manufacturing
Sun and Lu (2005)	Cross-efficiency profiling	evaluating robot performance	
Liu et al. (2017a)	An aggressive DEA game cross-efficiency	evaluating robot performance	
Wang and Wang (2013)	Approaches to determining the relative importance weights for cross-efficiency aggregation in data envelopment analysis	evaluating robot performance	
Talluri and Yoon	Cross-efficiency	justification of advanced	

(2000)		manufacturing technology	
Ertay and Ruan (2005)	Cross-efficiency	determination of the best labor assignment in a cellular manufacturing system	
Zerafat et al. (2013)	Cross-efficiency + cross ranking approaches	Bank branches	Banking
Ma et al. (2014a)	Game Cross-Efficiency for Systems with Two-Stage Structures	Commercial banks	
Li et al. (2018)	DEA game cross-efficiency	Bank branches	
Doyle and Green (1994b)	Cross-efficiency	Higher education	Education
Oral et al. (2015)	Cross-efficiency in DEA: A maximum resonated appreciative model	faculty	
Wang et al. (2011a)	Neutral DEA cross-efficiency	Departments of university	
Wang et al. (2011b)	Cross-efficiency evaluation based on ideal and anti-ideal decision making units	Numerical examples	
Wang and Chin (2011)	Using OWA operator weights for cross-efficiency aggregation		
Ramón et al. (2011)	A ‘‘peer-restricted’’ cross-efficiency evaluation to reduce differences between profiles of weights		
Ramón et al. (2010)	DEA cross-efficiency with slacks		
Alcaraz et al. (2013)	Cross-efficiency evaluation with ranking range		
Du et al. (2014)	Fixed cost and resource allocation based on DEA cross-efficiency		
Wang and Chin (2010a)	Alternative model for cross-efficiency (minimizing or maximizing the total deviation from the ideal point)		
Wu et al. (2011)	Determination of weights for ultimate cross-efficiency using Shannon entropy		

Wang et al. (2012)	DEA models for minimizing weight disparity in cross-efficiency evaluation	
Wang and Wang (2013)	Approaches to determining the relative importance weights for cross-efficiency aggregation in data envelopment analysis	
Bao et al. (2008)	Slack-Based Ranking Method: cross-efficiency	
Liang et al. (2008b)	Alternative secondary goals in cross-efficiency: Minimizing total deviation from the ideal point, Minimizing the maximum d-efficiency score, Minimizing the mean absolute deviation	

2.2 Portfolio selection: from DEA cross-efficiency approach to Nash equilibrium

Despite its effectiveness in multi-criteria analysis, the cross-efficiency approach has been rarely investigated in portfolio performance assessment. Sanei and Banihashemi (2013) work was the first to evaluate securities and to select portfolio using cross-efficiency approach considering negative data. They consider the variance of assets as input, whereas the expected return and skewness are considered as output. To illustrate and prove the robustness of the approach, they used a sample of Iranian stock companies. The second work was of Sanei and Banihashemi (2014), it prove that the cross-efficiency evaluation is an effective way of ranking and evaluating portfolios and asset selection. While the most widely used approach is to evaluate the efficiencies in each row or column in the cross-efficiency matrix with equal weights into an average cross-efficiency score for each DMU and consider it as the overall cross-efficiency measurement of the DMU, Sanei and Banihashemi (2014) propose the use of Ordered Weighted Averaging (OWA) operator weights for cross-efficiency evaluation. The OWA operator weights are generated by the minimax disparity approach and allow investor to select the best assets that are characterized by an orness degree. They used risk (variance) as input and the return as output and illustrated the approach using a sample of mutual funds. The third work was of Lim et al. (2014), it proposes to incorporate the DEA cross-efficiency evaluation in Mean-Variance (MV) space to portfolio selection. The approach permits to select well-diversified portfolios in terms of their performance on multiple evaluation criteria, and to alleviate the ‘‘ganging together’’ phenomenon of DEA cross-efficiency evaluation in portfolio selection. The proposed approach was illustrated to stock portfolio selection in the Korean stock market. 16 financial metrics indicating profitability, asset utilization, liquidity, leverage, and growth performance perspective are employed as input and output variables.

The fourth application to the portfolio selection area was Banihashemi and Sanei (2015) paper, which focuses on the evaluation of efficiency of assets using the cross-efficiency matrix with negative data and proposes the use of ordered weighted averaging (OWA) operator weights for cross-efficiency evaluation. An application to Iranian stock companies was performed using the variance as input and the expected return as output.

To the best of our knowledge, the last work is of Mashayekhi and Omrani (2016). This research propose a novel multi-objective model for portfolio selection by incorporating the DEA cross-efficiency into Markowitz mean-variance model taking into account the return, risk and efficiency of the portfolio. Also, in order to take uncertainty in the proposed model, the asset returns are considered as trapezoidal fuzzy numbers. Due to the computational complication of the proposed model, the second version of non-dominated sorting genetic algorithm (NSGA-II) is applied. An empirical illustration is performed using 52 firms listed in stock exchange market of Iran.

Despite the robustness of the cross-efficiency approach as a multi-criteria analysis method, its application in portfolio management remains very rare. This has mainly motivated our research work in this thesis. According to Liang et al. (2008a), the simple use of cross-efficiency method suffers from the issue of the instability and the unpredictability of efficiency scores and propose the game cross-efficiency as solution to provide a Nash equilibrium efficiency scores to evaluate DMUs.

As the portfolio selection process is the logical consequences of the investor's preferences towards information concerning stocks. Choosing between assets puts them in competition in the eyes of investors. To consider the factor of competition in ranking different assets, we propose to use DEA game cross-efficiency approach proposed by Liang et al. (2008a). Specifically, each DMU is viewed as a player that looks for maximizing its own efficiency, under the condition that the cross-efficiency of each of the other DMUs does not deteriorate. The average game cross-efficiency score is obtained when the DMU's own maximized efficiency scores are averaged. The DEA game cross-efficiency approach has been used to evaluate organizations in a competitive context such as banks (Ma et al. (2014a)), supply chain (Wang and Li (2014)), preference voting and R&D projects selection preference (Chen and Zhu (2011)) among others. To the best of our knowledge, the DEA game cross-efficiency approach has not been investigated to evaluate financial assets and to select portfolio.

While the DEA frontier can be interpreted as a production frontier, it may be seem as efficient frontier where the best practice or benchmarks are lying. In fact, the DEA method is considered as a tool for multiple-criteria evaluation issues where DMUs (securities) are alternatives and each DMU is represented by its performance in multiple criteria which are coined as DEA input and output variables. Multi-criteria DEA analysis may resolve the inherent multi-criteria nature of the portfolio selection problem.

Chapter 2:

Optimal Portfolio Selection Under Higher Moments: DEA Cross-Efficiency Approach

Chapter 2

Optimal Portfolio Selection Under Higher Moments: DEA Cross-Efficiency Approach

1. Introduction

The process of selecting a portfolio might be understood as the result of two basic stages. The first stage consists in financial assets performance gauging and the second chooses the optimal investment among them with optimal wealth allocation, given the investor's preferences. This paper deals with these two stages at the same time. Markowitz (1952) was the first to propose a quantitative approach to identify the optimal tradeoff between return and risk. In order to do this, he introduced the Mean-Variance (MV) model, which involves minimizing risk for a given level of expected return, or equivalently, maximizing expected return for a given level of risk. This amounts to define an efficient frontier concept as a Pareto-optimal subset of portfolios. That is, these are portfolios whose expected return cannot increase unless their variance increases. The Markowitz efficient frontier is defined as the MV efficient set of financial assets, given the knowledge of the true multivariate normal distribution of share returns. The major issue in portfolio performance evaluation is the appropriate benchmark to be used for comparison. It is difficult to identify benchmarks when multiple performance metrics exists. The portfolio frontier approach assesses the performance of a portfolio by measuring its distance to the efficient portfolio frontier. The major limitation of Markowitz model is that it maintains strong assumptions on the probability distributions and uses the Von Neumann-Morgenstern utility functions. In addition, another major problem at the time was the computational cost of solving quadratic programs. These limitations of the Markowitz model have triggered many developments, in particular equilibrium models, such as the capital asset pricing model (CAPM). This model assumes that all agents have similar expectations about the market conditions. This leads to impose that investors have quadratic utility functions, which does not seem too realistic. Afterwards, several empirical extensions have been developed using other parametric approaches to analyze portfolio performance (see, e.g., Yao, Lai, and Hao (2013), Chiu and

Wong (2014), Shen, Zhangand and Siu (2014), Bernard and Vanduffel (2014), Palczewski and Palczewski (2014) and Yao, Li and Chen (2014)).

In this paper, we extend the MV efficient frontier to include skewness and kurtosis in the analysis, leading to the Mean-Variance-Skewness-Kurtosis (MVSK) efficient frontier model. We then determine the best practice assets to include in the portfolio depending on the preferences of the investor. We construct an efficient frontier using nonparametric methods. Indeed, nonparametric approaches are a robust alternative to parametric methods in the portfolio management area, as they do not require a specific functional form for the portfolio efficiency frontier. Sengupta and Barbara (1989) were the first to propose a nonparametric approach for the specification and estimation of a portfolio efficiency frontier. Afterwards, Morey and Morey (1999) presented two basic quadratic programming approaches, risk contraction and mean-return augmentation, that are benchmarking efficiency frontier methods based on Data Envelopment Analysis (DEA) concepts. It has been shown that DEA could be a good tool to evaluate financial assets performance and help investors make their portfolio selection.[3] In fact, DEA provides each decision making unit (DMU) a good opportunity to self-evaluate its efficiency relative to other homogenous DMUs. Nevertheless, the DEA approach suffers from a major intrinsic drawback in its lack of discrimination power (Berger and Humphrey (1997)). That is, the self-evaluation lets each DMU rates its efficiency based on the most favorable weights computed by the DEA algorithm. That is, inputs and outputs favorable to a particular DMU are heavily weighted, while those not favorable to the DMU are assigned a small weight or simply ignored. Thus, the weights determined by the self-evaluation may sometimes be unrealistic. Furthermore, more than one DMU might be deemed efficient and as the method works it is not possible to discriminate between them.

One way to address these limitations is the DEA cross-efficiency approach, proposed by Sexton (1986) and examined by Liang et al. (2008a), Doyle and Green (1994a), Lim (2012) among others. This approach is an extension of the DEA method. DEA cross-efficiency evaluation suggests that each DMU is not only to be self-evaluated but also to be peer-evaluated. Therefore, the cross-efficiency evaluation can guarantee a unique ordering of the DMUs. This framework has several advantages. Firstly, the optimal choice is a DMU that performs better than the others with respect to many criteria (i.e. the first four moments of

[3] See, e.g., Murthi, Choi and Desai (1997), Basso and Funari (2001) and Gregoriou et al. (2005b).

returns distribution). Secondly, cross-efficiency evaluation eliminates unrealistic weight schemes without inputs and outputs weight restrictions.[4] Finally, our framework is developed based on DEA cross-efficiency approach, which allows us not only to peer-evaluate stocks but also to return a unique ordering of them.

While DEA cross-efficiency approach permits to discriminate further among DMU, it suffers from the non-uniqueness of DEA solutions. In fact, the cross-efficiency scores are computed through a weight structure that is not necessarily unique. That is, there are potentially many input and output optimal weights that would satisfy the conditions for a solution to the DEA problem. Furthermore, not all softwares treat the optimal solution the same way, so the results are sensitive to the algorithm used (Despotis (2002)).

As cross-efficiency evaluation is mainly based on the calculation of cross-efficiency matrix and since the cross-efficiency score is the arithmetic average of peer-evaluations and self-evaluation, weights of scores are used equally in cross-efficiency aggregation. The equal weights for cross-efficiency aggregation ($(1/n)$ if there are n DMUs to be evaluated) presents a limitation of the method. The problems are that it pays little attention to the aggregation of cross-efficiency scores and the self-evaluation score is of little importance in the final overall assessment and ranking (Wang and Chin (2011)). Consequently, the subjective preferences of decision maker on the best relative efficiencies in the final overall evaluation are ignored.

In spite of this limitation, but mainly due to its discrimination power between DMUs, the cross-efficiency evaluation has been widely used in the DEA literature. Nonetheless, only very few applications in portfolio selection used used this method, namely Pätäri et al (2012), Lim, Oh and Zhu (2014) and Mashayekhi and Omrani (2016). For portfolio selection, the performance measures are obtained using Multi Criteria Decision Making (MCDM) DEA approach where criteria (inputs and outputs) weights are determined exogenously and the effect of each criteria can not be considered alone and must always be seen as a tradeoff with respect to other criteria. These weights may not remain the same over time and may vary considerably depending on the changing environment. Any change in each input or output variables can change the decision priorities for other variables. This argument supports the use of cross-efficiency evaluation instead of the standard DEA model. In fact, the DEA

[4] The model can be restructured to allow for weight restrictions if necessary (Baker and Talluri (1997)).

cross-efficiency would be a suitable method for multi-criteria portfolio selection problem since it allows explicit tradeoffs and interactions among criteria. In the peer-evaluation, it is likely to choose DMUs whose performance is somewhat good on all measures and exclude those whose performance is good for only a subset of the criteria. Variation in one criterion could be compensated in a direct or opposite way by other criteria. Nonetheless, this leads to portfolio made of "too" similar DMUs. This is the "ganging together" problem (Tofallis (1996)) and the resulting lack of diversification of the portfolio (Lim et al (2014)). A diversified portfolio is made of weakly correlated assets and as a consequence is less risky than each individual component of the portfolio. In other words, adding a highly volatile asset does not necessarily increase the aggregate risk of the portfolio if the asset is weakly correlated with other components of the portfolio. Therefore, the more diversified a portfolio is, the smaller its risk is. A portfolio of risky assets can have a low overall risk, since it is made of weakly correlated assets. Hence, an investor generally looks for a diversified portfolio in MV space.[5] The use of DEA cross-efficiency as such is irrelevant to portfolio selection because it does not discriminate on the asset risk correlation dimension of the portfolio selection. To deal with this problem, specific to portfolio selection using DEA cross-efficiency method, Lim et al (2014) incorporate the DEA cross-efficiency into the MV formulation.

While the simultaneous maximization of return and minimization of risk of second-order (variance) seems quite straightforward, this risk is not the only risk that has been considered as "bad" in the literature. In fact, the variance does not convey the necessary information to determine if the deviations from the mean return are below or over the mean; it does not discriminate between the upward and downward shifts. In fact, using only the variance amount to assume implicitly that investor dislike any changes, including upward shits of the returns. This is not realistic, but the discrimination between these asymmetric shifts can be taken into account with higher moments of the distribution (in particular skewness and kurtosis). In fact, neglecting higher moments may present a major drawback in portfolio selection. To deal with this limitation, we fit cross-efficiency model into the MVSK space. The framework we introduce here allows us to select diversified portfolio, depending to investor's attitude with respect to risk aversion, prudence and temperance. The model captures investor attitude toward risk aversion, prudence and temperance. To do this, we use MV tradeoff and Skewness-Kurtosis (SK) tradeoff parameters. Using econometric tools, the

[5] "What exactly is diversification? It simply means: *do not put all your eggs in one basket*! "

tradeoff parameter between the mean and the variance may be estimated with mixing data sampling (MIDAS) estimator (Ghysels, Santa-Clara and Valkanov (2005)) or with overlapping data inference (Hedegaard and Hodrick (2016)) among other tools. It may be also defined arbitrarily (see for example Lim et al (2014)). Nonetheless, to the best of our knowledge, the SK tradeoff parameter has never been specified in previous works. Here MV and SK tradeoff parameters are determined endogenously using DEA cross-efficiency framework. Finally, our approach returns the stocks with the best performance but also the exact optimal proportions to invest in each individual stock. In order to build the optimal portfolio and the optimal weights we use the stocks first four moments and we use the odd moments as inputs and the even moments as outputs. To illustrate how this framework works, we apply it to 21 assets listed in Paris stock exchange during the period 2010-2015. We obtain well-diversified portfolios and we show the robustness of our framework using Ledoit and Wolf (2008) Sharpe test.

2. The utility analysis of choice in terms of higher order moments and portfolio selection problem

As Levy (1989) pointed out, the MV and the Von-Neumann expected utility function models identify the same efficient sets of assets when the investor is risk-averse. In fact, investors choose exactly the same random prospects when their preferences are characterized by a quadratic utility function or when financial asset returns are random variables with fixed expected return and variance. A concave quadratic utility function is characterized by positive first order and negative second order derivative and null higher order derivatives. This assumption is a major source of criticisms of the MV framework. Indeed, it has been repeatedly shown that financial asset returns are not normally distributed, and we should not ignore empirical evidence that extreme value distribution characterize them. This strongly suggests that moments larger than the second do matter in financial problems. In fact, assuming investors to have quadratic utility function is equivalent to assume that the market is efficient. The Efficient Market Hypothesis (EMH) formulated by Fama (1970) suggests that "a Market in which prices always 'fully reflect' available information is called 'efficient'". Nonetheless, the information included in the stock prices reflect other relevant information, the likes of political, social and economic events among others, in addition to financial information.

Ambiguous factors usually exist in stock prices and investors need to consider not only random conditions but also ambiguous and subjective conditions for portfolio selection problems (Urrutia (1995)). Therefore, recent studies enlarge the evaluation dimension to the third and fourth distribution moment in order to take into account the non-normality of return distributions (see, eg., Gregoriou et al. (2005b) and Pendaraki (2012)). It can be argued that investors have a positive preference for odd moments and a negative preference for even moments (see, eg., Scott and Horvath (1980), Brockett and Kahane (1992), Brockett and Garven (1998) and Jondeau and Rockinger (2006)). Yet, it is shown that investors' utility function with positive third derivative have a preference for distributions with a higher skewness, while a negative fourth derivative indicates a preference for distributions with a lower kurtosis. The signs of the third and fourth derivatives of the utility function have defined respectively prudence and temperance notions (Scott and Horvath (1980)). Notice that the third moment of the distribution is often related to 'downside risk', it is called the risk of extreme losses, and corresponds to values of the returns located on the "left" of the distribution. "Prudence" as defined by Kimball (1991) is a *"precautionary save motive" that can cause an agent to respond to a risk by accumulating more wealth"*.[6] This is contrasted to risk aversion that suggests how much one dislikes uncertainty and wants to avoid it. Kurtosis aversion is the consequence of a negative fourth derivative of the Von Neumann–Morgenstern utility function, this is known as "temperance". It is defined by Kimball (1991) as "*the desire to moderate total exposure to risk"*.[7] Temperance *"can cause an agent to respond to an unavoidable risk by reducing exposure to other risks even when the other risks are statistically independent of the first"*.[8] It is the sense of moderation in accepting risks.

The literature shows that portfolio selection decisions depend crucially on higher order risk attitudes. Within the expected utility approach, prudence and temperance are properties of the third and fourth derivatives of the utility function. Our model requires that we state and present some results from the expected utility theory and the modern portfolio theory.

The investor's preferences are over wealth, in general. However, we may express them over the rate of return on the wealth. Note that the final wealth of the investor may be expressed as $W = W_0(1+R)$ where W_0 is the initial wealth and $R = (W - W_0)/W_0$ is the portfolio return. Suppose that we normalize the wealth,[9] then $W = 1+R$, $E(W) = 1+E(R)$, and

[6]Kimball (1991). precautionary motives for holding assets. NBER Workin Papers Series NO 3586. Page 1
[7]Ibid
[8]Ibid
[9] For sake of simplicity we normalize $W_0 = 1$. Scaling the initial agent wealth to one is commonly used in the investor preference literature since it ensure the complete equivalence between the expected utility expressions both in terms of return and end-of-period wealth (see for more details Jondeau and Rockinger (2006) and Brockett and Golden (1987)).

$W-E(W)=R-E(R)$. Consequently, we assume that the investor has preferences on the wealth rate of return, *U(R)*. We wish to find a relationship between this utility function and the moments of the returns. In order to do so, consider the following Taylor expansion of *U(R)* about *E(R)*:

$$\begin{aligned} U(R) &= U(E(R)) + (R-E(R))U^{(1)}(E(R)) + \frac{(R-E(R))^2}{2!}U^{(2)}(E(R)) \\ &+ \frac{(R-E(R))^3}{3!}U^{(3)}(E(R)) + \frac{(R-E(R))^4}{4!}U^{(4)}(E(R)) + \sum_{k=5}^{\infty}\frac{(R-E(R))^k}{k!}U^{(k)}(E(R)) \end{aligned} \tag{2.1}$$

where $U^{(i)}$ corresponds to the i^{th} derivative of the utility function with respect to the final wealth. Taking expectation on both sides gives:

$$\begin{aligned} E[U(R)] &= U(E(R)) + \frac{E\left[(R-E(R))^2\right]}{2!}U^{(2)}(E(R)) + \frac{E\left[(R-E(R))^3\right]}{3!}U^{(3)}(E(R)) \\ &+ \frac{E\left[(R-E(R))^4\right]}{4!}U^{(4)}(E(R)) + \sum_{k=5}^{\infty}\frac{E\left[(R-E(R))^k\right]}{k!}U^{(k)}(E(R)) \end{aligned} \tag{2.2}$$

where $\sum_{k=5}^{\infty}U^{(k)}(E(R))E\left[(R-E(R))^k\right]\Big/k!$, is the remainder term of the Taylor expansion. Berenyi (2001), among others, claims that the fourth term plays a substantial role in the Taylor expansion of the expected utility function. Combining this with the fact that the remainder is of small order and negligible (Jondeau and Rockinger (2006) and (Garlappi and Skoulakis (2011)), we may choose to truncate the Taylor series after the fourth term. Doing so allows us to introduce explicitly the skewness and kurtosis in the analysis of portfolio selection. That is, the expected utility function may be expressed as follows:

$$E[U(R)] = U(E(R)) + \frac{m_2(R)}{2!}U^{(2)}(E(R)) + \frac{m_3(R)}{3!}U^{(3)}(E(R)) + \frac{m_4(R)}{4!}U^{(4)}(E(R)), \tag{2.3}$$

where $m_2(R)$, $m_3(R)$ and $m_4(R)$ are the second, third, and fourth order moment of the rate of return on wealth, *R*.

Now, to understand how these moments are related to preferences note that, as in Eeckhoudt and Hammitt (2001), we may define the risk premium as the amount that an individual is willing to pay on the expected return to bring her utility to the lottery's expected value, $U(E(R)-\pi)=E[U(R)]$. A first order Taylor expansion gives $U(E(R)-\pi)=U(E(R))-\pi U^{(1)}(E(R))$. Using these results we obtain:

$$E[U(R)] = U(E(R)) - \pi U^{(1)}(E(R)) \tag{2.4}$$

Using Equations (2.3) and (2.4) and rearranging leads to the following expression:

$$\begin{aligned}\pi &= \left[-\frac{U^{(2)}\left(E\left(R\right)\right)}{U^{(1)}\left(E\left(R\right)\right)}\right]\frac{m_2\left(R\right)}{2} - \left[\frac{U^{(3)}\left(E\left(R\right)\right)}{U^{(1)}\left(E\left(R\right)\right)}\right]\frac{m_3\left(R\right)}{3!} + \left[-\frac{U^{(4)}\left(E\left(R\right)\right)}{U^{(1)}\left(E\left(R\right)\right)}\right]\frac{m_4\left(R\right)}{4!} \\ &= \frac{1}{2}\lambda m_2\left(R\right) - \frac{1}{6}\psi m_3\left(R\right) + \frac{1}{24}\varphi m_4\left(R\right)\end{aligned} \tag{2.5}$$

Where $\lambda = -U^{(2)}\left(E\left(R\right)\right)/U^{(1)}\left(E\left(R\right)\right)$ is the absolute risk aversion measure, $\psi = -U^{(3)}\left(E\left(R\right)\right)/U^{(1)}\left(E\left(R\right)\right)$ is the coefficient of appetite toward symmetry (the third centered distribution moment) and $\varphi = -U^{(4)}\left(E\left(R\right)\right)/U^{(1)}\left(E\left(R\right)\right)$ is the coefficient of aversion to leptokurticity (distribution tail thickness).[10] The even number derivatives of the utility function are negative while the odd number derivatives are positive. It means that the risk premium would increase with variance of kurtosis while it would decrease with skewness. This confirms the standard theories that tell us that an investor dislikes variance and kurtosis and likes skewness.

The overall return R is for a portfolio. Let r_h be a random variable representing the rate of return of the asset h and w_h be the weight of the asset h. Given the individual rate of returns, r_h, the portfolio rate of return, $R = r(w)$, is a function of the weights $\boldsymbol{w} = (w_1,...,w_n)$, and is given by:

$$R = r(w) = \sum_{h=1}^{n} w_h r_h \tag{2.6}$$

Given the mean of the individual returns, $E\left(r_h\right) = \mu_h$, the vector of the means is given by $\boldsymbol{\mu} = [\mu_h]$. The mean, variance, skewness and kurtosis of the portfolio rate of return $r(w)$, are denoted $\mu(w), v(w)$, $s(w)$ and $\kappa(w)$ respectively, and are defined as follows:

$$\begin{aligned}\mu(w) &= E\left[r(w)\right] = \sum_{h=1}^{n} w_h E\left(r_h\right) = \sum_{h=1}^{n} w_h \mu_h = w^T \boldsymbol{\mu}, \\ v(w) &= E\left[\left(r(w) - \mu(w)\right)^2\right] = \sum_{h=1}^{n}\sum_{j=1}^{n} w_h w_j \sigma_{hj} = \boldsymbol{w}^T M_2 \boldsymbol{w}, \\ s(w) &= E\left[\left(r(w) - \mu(w)\right)^3\right] = \sum_{h=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} w_h w_j w_k s_{hjk} = \boldsymbol{w}^T M_3\left(\boldsymbol{w}\otimes\boldsymbol{w}\right), \\ \kappa(w) &= E\left[\left(r(w) - \mu(w)\right)^4\right] = \sum_{h=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} w_h w_j w_k w_l \kappa_{hjkl} = \boldsymbol{w}^T M_4\left(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}\right).\end{aligned} \tag{2.7}$$

[10] For more detail, see COURTOIS (2012).

where $M_2 = E\left[(r_h-\mu)(r_j-\mu)'\right] = \left\{\sigma_{hj}\right\}$ is the (n,n) variance-covariance matrix, $M_3 = E\left[\left(r_h-\mu\right)\left(r_j-\mu\right)' \otimes \left(r_k-\mu\right)'\right] = \left\{s_{hjk}\right\}$ is the $\left(n,n^2\right)$ skewness co-skewness matrix, $M_4 = E\left[\left(r_h-\mu\right)\left(r_j-\mu\right)' \otimes \left(r_k-\mu\right)' \otimes \left(r_l-\mu\right)'\right] = \left\{\kappa_{hjkl}\right\}$ is the $\left(n,n^3\right)$ kurtosis co-kurtosis matrix, $h,j,k,l=1,...,n$ and $\otimes$ denotes the kronecker product. Note that M_2 contains $\left[n\left(n+1\right)\right]/2$ distinct elements, while M_3 and M_4 contains respectively $\left[n\left(n+1\right)\left(n+2\right)\right]/6$ and $\left[n\left(n+1\right)\left(n+2\right)\left(n+3\right)\right]/24$ distinct elements.

Using Equation (2.3), the expected utility of an investor as a function of the assets rate of return, denoted $U(r(w))$, is given by:

$$\begin{aligned}
E\left[U\left(r\left(w\right)\right)\right] &= E\left[U\left(\mu\left(W\right)\right)\right] + \frac{U^{(2)}\left(\mu\left(W\right)\right)}{2} E\left[\left(r\left(w\right)-\mu\left(w\right)\right)^2\right] \\
&+ \frac{U^{(3)}\left(\mu\left(W\right)\right)}{3!} E\left[\left(r\left(w\right)-\mu\left(w\right)\right)^3\right] + \frac{U^{(4)}\left(\mu\left(W\right)\right)}{4!} E\left[\left(r\left(w\right)-\mu\left(w\right)\right)^4\right] \\
&= U\left(\mu\left(W\right)\right) + \frac{U^{(2)}\left(\mu\left(W\right)\right)}{2!} v\left(W\right) + \frac{U^{(3)}\left(\mu\left(W\right)\right)}{3!} s\left(W\right) + \frac{U^{(4)}\left(\mu\left(W\right)\right)}{4!} \kappa\left(W\right)
\end{aligned} \tag{2.8}$$

Under Scott and Horvath (1980) conditions, the expected utility depends positively on expected return and skewness and negatively on variance and kurtosis of the portfolio. This shows that the skewness and kurtosis are an integral part of the investors' decision-making process, and we just have to implement these concepts into our empirical decision model.

3. DEA cross-efficiency evaluation

The use of DEA for the evaluation of financial assets allows us to determine the set of all best practices (our benchmarks) observed on the market and the potentially feasible practices (assuming some conditions characterizing the technology). This set is what we call the efficient frontier. From a production perspective, a best practice is the one that convert the given inputs into the maximum levels of output (or the minimum levels of input for an observed output). From the benchmarking perspective, inputs are the attributes to minimize and/or outputs are the criteria to maximize (Stewart (1996)). The main idea of the cross-efficiency is to consider not only the self-evaluation inherent in conventional DEA analysis, but also to consider peer-evaluation. Self-evaluation means that a given DMU is allowed to choose the most favorable input-output weights freely to achieve its best possible relative efficiency, while peer-evaluation means that the DMU is evaluated using other DMU weights. A typical cross-efficiency analysis is implemented as a two-stage process: In the first stage, classical DEA method is performed in a self-evaluation and the optimal weights of inputs and outputs are computed for each DMU. Then in the second stage, we use the weights obtained in the first stage to calculate the peer-evaluation for each other DMUs.

In the first stage~~,~~ the standard CCR model of Charnes, Cooper and Rhodes (1978) is used to induce the best possible relative efficiency for a specific DMU. To be specific, we assume that DMU_j $\left(j=1,...,n\right)$ uses m inputs $x_{ij}(i=1,...,m)$ to produce s outputs $y_{rj}=\left(r=1,...,s\right)$. The efficiency score of a given DMU_h under evaluation is the optimal value of the following problem

$$
\begin{aligned}
& Max\ e_{hh}=\sum_{r=1}^{s}u_{rh}y_{rh} \\
& \quad s.t.\quad \sum_{i=1}^{m}v_{ih}x_{ij}-\sum_{r=1}^{s}u_{rh}y_{rj}\geq 0,\quad j=1,...,n \\
& \qquad\qquad \sum_{i}^{m}v_{ih}x_{ih}=1 \\
& \qquad\qquad v_{ih}\geq 0 \qquad i=1,...,m \\
& \qquad\qquad u_{rh}\geq 0 \qquad r=1,...,s
\end{aligned}
\tag{2.9}
$$

where v_{ih} and u_{rh} are the weights assigned to input *i* and output *r*, respectively, to be determined by optimizing the model.

In the second stage, we use the optimal solutions to problem (2.9) to compute the cross-efficiencies. Specifically, if $(v^{*}_{1h},...,v^{*}_{mh})$ and $(u^{*}_{1h},...,u^{*}_{sh})$ are an optimal solution to (2.9) for a given DMU_h, then the cross-efficiency of DMU_j, $j=1,...,n$ using the optimal weights of DMU_h, namely e_{hj}, can be computed as follows:

$$
e_{hj}=\frac{\sum_{r=1}^{s}u^{*}_{rh}y_{rj}}{\sum_{i=1}^{m}v^{*}_{ih}x_{ij}},\quad j=1,...,n \tag{2.10}
$$

where e_{hj} is the peer-evaluation of DMU_j by DMU_h.

Collecting all cross-efficiencies of all DMUs, a matrix of cross-efficiencies is constructed such that the element in the diagonal e_{hh} is the efficiency score for each DMU_h using problem (2.9). The other elements, e_{hj}, are the cross-efficiency of one DMU_j using the optimal weights of DMU_h. A cross-efficiency score for DMU_j is defined as the average of all e_{hj} $(h = 1,...,n)$ cross-efficiencies.

$$\bar{e}_j = \frac{1}{n}\sum_{h=1}^{n} e_{hj}, \quad j = 1,...,n \ . \tag{2.11}$$

It returns a peer-evaluation of DMU_j. This score is used to rank the DMUs.

4. Optimal Mean-Variance-Skewness-Kurtosis portfolio

A tradeoff is when you have to give up something to get something else. In the field of portfolio management, a tradeoff between risk and return is a situational decision that involves diminishing risk to increase the return on an asset. There are various ways to approach this tradeoff empirically. Ghysels, Santa-clara and Valkanov (2005) estimate a risk-return parameter by a quasi-maximum likelihood method. In behavioral models, the tradeoff between the mean and the variance is determined by models that account for individual differences (Libby and Fishburn (1977)). For portfolio selection, Lim et al (2014) propose a set of arbitrary values of return-risk tradeoff parameter to take into account the individual differences between investors. Because there exists a positive preference for skewness and a negative preference for kurtosis, it has been suggested that a skewness-kurtosis tradeoff may be required (Scott and Horvath (1980)). Based on the duality between the indirect utility function of investor and the shortage function, Briec and Kerstens (2010) have defined parameters representing the degree of absolute risk-aversion, prudence and temperance. To solve the problem of mean-variance-skewness-kurtosis conflicting objectives, Lai, Yu and Wang (2006) and Davies, Kat and Lu (2008) construct a polynomial goal programming model.

Our approach is different and consists in obtaining the optimal solution to the following models. Let Ω be a portfolio of assets and Ω^* be the optimal MVSK portfolio to be identified. In this section, we present a procedure for selecting the assets to put in portfolio Ω^* and to determine the optimal share of each asset. The basic idea of the procedure is to use cross-efficiency scores given the decision makers' preferences over its aversion to risk, prudence and temperance.

In a nutshell, the algorithm to find the optimal portfolio works as follows. First we compute the DEA cross-efficiency scores. Then, in Step 2, we use the scores computed in Step 1 to derive the first four statistical moments. Step 3 determines the optimal SK tradeoff parameter and determines efficiency of optimal SK portfolio. Finally, in Step 4, we determine an optimal MV tradeoff parameter to select the optimal MVSK portfolio. Formally, we have:

Step 1. For each $h=1,...,n$ solve model (2.9) and obtain an optimal weights for each DMU_h. Let $(v^*_{1h},...,v^*_{mh})$ and $(u^*_{1h},...,u^*_{sh})$ be the optimal weights of DMU_h. Use these weights to compute the cross-efficiency scores, e_{hj}, of DMU_j defined in (2.10), and average cross-efficiency scores, $\overline{e}_j$, defined in (2.11), for $j=1,...,n$.

Step 2. Let the decision variable w_j be the weight of DMU_j inside the portfolio Ω, and let $\boldsymbol{w}=(w_1,...,w_n)\in\Re^n_+$ be the weight vector. The portfolio's return, variance, skewness and kurtosis are defined as follow:

- **2.1** Use $\overline{e}_j, j=1,...,n$ obtained in *Step 1* to define the portfolio return as the weighted sum of the average cross-efficiency $E_\Omega=\sum_{j=1}^{n}\overline{e}_j w_j$.
- **2.2** Use M_2 to construct the variance of portfolio Ω, $V_\Omega=\boldsymbol{w}^T M_2\boldsymbol{w}=\sum_{h=1}^{n}w_h^2\sigma_h^2+\sum_{h=1}^{n}\sum_{j=1,h\neq j}^{n}w_h w_j\sigma_{hj}$. This is the weighted sum of the variances of each individual DMU's cross-efficiencies $\left(\sum_{h=1}^{n}w_h^2\sigma_h^2\right)$ and the covariance of each pair of DMU's cross-efficiencies $\left(\sum_{h=1}^{n}\sum_{j=1,h\neq j}^{n}w_h w_j\sigma_{hj}\right)$.
- **2.3** Use M_3 to construct the skewness of portfolio Ω, $S_\Omega=\boldsymbol{w}^T M_3(\boldsymbol{w}\otimes\boldsymbol{w})=\sum_{h=1}^{n}w_h^3 s_h^3+3\sum_{h=1}^{n}\left(\sum_{j=1,h\neq j}^{n}w_h^2 w_j s_{hhj}+\sum_{j=1,h\neq j}^{n}w_h w_j^2 s_{hjj}\right)$. This is the weighted sum of the skewness of each individual DMU's cross-efficiencies

$\left(\sum_{h=1}^{n} w_h^3 s_h^3\right)$ and the co-skewness of each pair of DMU's cross-efficiencies $\left(3\sum_{h=1}^{n}\left(\sum_{j=1,h\neq j}^{n} w_h^2 w_j s_{hhj} + \sum_{j=1,h\neq j}^{n} w_h w_j^2 s_{hjj}\right)\right)$.

- **2.4** Use M_4 to construct the kurtosis of portfolio Ω,

- $$K_\Omega = \sum_{h=1}^{n} w_h^4 k_h^4 + 4\sum_{h=1}^{n}\left(\sum_{j=1,h\neq j}^{n} w_h^3 w_j k_{hhhj} + \sum_{j=1,h\neq j}^{n} w_j^3 w_h k_{hhjj}\right) + 6\sum_{h=1}^{n}\sum_{j=1,h\neq j}^{n} w_h^2 w_j^2 k_{hhjj}$$
$$= \boldsymbol{w}^T M_4\left(\boldsymbol{w}\otimes\boldsymbol{w}\otimes\boldsymbol{w}\right)$$

This is the weighted sum of the kurtosis of each individual DMU's cross-efficiencies .

$\left(\sum_{h=1}^{n} w_h^4 k_h^4\right)$ and the co-kurtosis of each pair of DMU's cross-efficiencies $\left(4\sum_{h=1}^{n}\left(\sum_{j=1}^{n} w_h^3 w_j k_{hhhj} + \sum_{j=1}^{n} w_h w_j^3 k_{hjjj}\right) + 6\sum_{h=1}^{n}\sum_{j=1,h\neq j}^{n} w_h^2 w_j^2 k_{hhjj}\right)$.

Step 3. Let Ω be a portfolio with individual DMUs weighted using a weight vector, $\boldsymbol{w}\in\Re_+^n$, where $w_i \geq 0,\ i=1,\ldots,n$. It means that an asset cannot be short in the portfolio.[11] Let $\sum_{h=1}^{n} w_h = 1$ to preserve the budget constraint. Then, set the number of the SK parameter to Z, and for each SK tradeoff parameter value, $\delta\in\{\delta_1,\ldots,\delta_Z\}$, solve the following problem:

$$\begin{aligned} &\underset{\boldsymbol{w}}{Max}\ \ S_\Omega \\ &s.t\ \ \ K_\Omega \leq (1-\delta)K^* \\ &\qquad \mathrm{I}^T\boldsymbol{w} = 1 \\ &\qquad \boldsymbol{w}\geq 0 \end{aligned} \tag{2.12}$$

Where $K^* = Min\{k_h\}$ is the smallest kurtosis of the asset series, and I is a vector of ones. We find, for each value of δ, an assets weights matrix $\boldsymbol{w}^\delta = \left(\boldsymbol{w}_1^\delta\ \ \boldsymbol{w}_2^\delta \ldots\ \boldsymbol{w}_h^\delta\right)$ that allows us to compute the pseudo-outputs matrix $\boldsymbol{y}_h^\delta = (\boldsymbol{y}_{1h}^\delta\ \ \ \boldsymbol{y}_{2h}^\delta\ \ \ldots\ \ \boldsymbol{y}_{sh}^\delta),\ \ h=1,\ldots,n$ where $\boldsymbol{y}_{rh}^\delta = \boldsymbol{w}_h^\delta.\boldsymbol{y}_{rh}$ for *r=1,…,s*. These pseudo output can be combined with the initial inputs, to solve the

[11] Sometimes it is possible to sell an asset that we do not own. This is called short selling. It presents an usual regulated type of market transaction. It consists in selling assets that are borrowed in expectation of a fall in the assets' price. When and if the price declines, the investor buys an equivalent number of assets at the new lower price and returns to the lender the assets that were borrowed.

problem defined in equation (2.9) to compute the DEA efficiency scores of DMU$_h$, e_{hh}^{δ}. For each δ, we compute an average DEA scores, $\overline{e}_{\delta} = \frac{1}{n}\sum_{h=1}^{n} e_{hh}^{\delta}$. Then choose the optimal SK tradeoff parameter as the value of delta that maximizes the cross-efficiencies. That is $\delta^* = \arg\max_{\delta}\left\{\overline{e}_{\delta_1},...,\overline{e}_{\delta_z}\right\}$.

Using δ^* we solve the following problem for the optimal weights:

$$\begin{aligned} &\underset{w}{Max}\ \ S_{\Omega} \\ &s.t \quad K_{\Omega} \le \left(1-\delta^*\right)K^* \\ &\qquad \mathrm{I}^T \boldsymbol{w} = 1 \\ &\qquad \boldsymbol{w} \ge 0 \end{aligned} \tag{2.13}$$

The optimal assets weights $\boldsymbol{w}^{\delta^*} = \left(\boldsymbol{w_1}^{\delta^*} \quad \boldsymbol{w_2}^{\delta^*} \quad \dots \quad \boldsymbol{w_h}^{\delta^*}\right)$ returns the optimal SK-portfolio, denoted Ω_{δ^*}. The efficiency score of the optimal-SK portfolio Ω_{δ^*} is $E_{\Omega_{\delta^*}} = \sum_{h}^{N} \overline{e}_h . w_h^{\delta^*}$.

Step 4. To determine the optimal MV tradeoff parameter γ^*, use $E_{\Omega_{\delta^*}}$. Let F be the number of arbitrary MV tradeoff parameter, for each value of $\gamma \in \left\{\gamma_1,...,\gamma_F\right\}$ solve the following problem

$$\begin{aligned} &\underset{w}{Min}\ \ V_{\Omega} \\ &s.t \quad E_{\Omega} \ge \left(1-\gamma\right)E_{\Omega_{\delta^*}}^* \\ &\qquad \mathrm{I}^T \boldsymbol{w} = 1 \\ &\qquad \boldsymbol{w} \ge 0 \end{aligned} \tag{2.14}$$

For each γ, use the critical assets weights matrix $\boldsymbol{w}^{\gamma} = \left(\boldsymbol{w_1}^{\gamma_1} \quad \boldsymbol{w_2}^{\gamma_2} \quad \dots \quad \boldsymbol{w_h}^{\gamma_F}\right)$ to compute pseudo-outputs matrix $\boldsymbol{y_h}^{\gamma} = (\boldsymbol{y_{1h}}^{\gamma_1} \quad \boldsymbol{y_{2h}}^{\gamma_2} \quad \dots \quad \boldsymbol{y_{sh}}^{\gamma_F})$, $h = 1,...,n$ where $\boldsymbol{y_{rh}}^{\gamma_F} = \boldsymbol{w_h}^{\gamma_F} . \boldsymbol{y_{rh}}$ for *r=1,…,s*, and use (2.9) to compute DEA scores $e_{hh}^{\gamma_F}$. Use these results to compute the average cross-efficiency, $\overline{e}_{\gamma} = \frac{1}{n}\sum_{h=1}^{n} e_{hh}^{\gamma}$. The optimal MV tradeoff parameter is the one that maximizes the average cross-efficiency. That is $\gamma^* = \arg\max_{\gamma}\left\{\overline{e}_{\gamma_1},...,\overline{e}_{\gamma_F}\right\}$.

Given γ^* and $E_{\Omega_{\delta^*}}$, the optimal weights are obtained by solving the following problem:

$$\begin{aligned} &\underset{\boldsymbol{w}}{Min}\ V_{\Omega} \\ &s.t \quad E_{\Omega} \geq \left(1-\gamma^*\right) E^*_{\Omega_{\delta^*}} \\ &\qquad \mathrm{I}^T \boldsymbol{w} = 1 \\ &\qquad \boldsymbol{w} \geq 0 \end{aligned} \tag{2.15}$$

The solution to problem (2.15) returns the optimal assets weights $\boldsymbol{w}^* = \left(\boldsymbol{w}_1^* \quad \boldsymbol{w}_2^* \quad \ldots \quad \boldsymbol{w}_n^*\right)$ defining the MVSK-portfolio Ω^*.

5. Empirical illustration: application to portfolio selection in the Paris stock market

To illustrate how our new methodology works, we propose apply it on stock values to create a new portfolio based on the MVSK-method. To test its robustness we compare our portfolio to market portfolios in terms of return-risk performance. Our sample contains 21 assets from the French CAC40 index between January 2010 and December 2015. This sample contains observations on 72 monthly returns.[12] For all these assets the first four centered moments have been computed for each year in the sample year.

5.1 Variables and data

As discussed above, rational investor looks for mean and skewness expansion and kurtosis and variance contraction. Consequently the variance and the kurtosis work as inputs, and outputs are the mean and skewness. We calculate for each asset h the monthly return $R_{ht} = \left(P_{ht} - P_{ht-1}\right)/P_{ht-1}$, where P_{ht} is the current closing price of h in the last day in the month and P_{ht-1} is its current closing price in the first day in the month. We present computational details and interpretation of the input and output variables in Table 2.1.

Table 2.1: Inputs and outputs

Variables	Investor attitude	Formulas	Signification
Input1: Variance	Risk-aversion	$\sigma_h^2 = \frac{1}{T}\sum_{t=1}^{T}\left(R_{ht} - \bar{R}_h\right)^2$	The basic measure of variability is the standard deviation, also known as the volatility, or the variance. For a stock, the variance is used to measure the variability of daily returns presenting

[12] Data stock price are available using www.euronext.com website.

			the total risk of stock price.
Input 2: Kurtosis	Temperance	$\frac{1}{T}\sum_{t=1}^{T}\left(\frac{R_{ht}-\overline{R}_h}{\sigma_h}\right)^4-3$	Kurtosis is a measure of peakness degree (fourth central moment minus 3 is called excess kurtosis). When the data has more peakness than the normal distribution (long tails), kurtosis is greater than three (leptokurtosis) while in the case we have lower peak we have platy kurtosis (bounded distribution). The normal distribution has Kurtosis equal to three.
Output1: Mean	Return	$\overline{R}_h=\frac{1}{T}\sum_{t=1}^{T}R_{ht}$	The arithmetic mean is a basic measure of stock price return.
Output 2: Skewness	Prudence	$\frac{1}{T}\sum_{t=1}^{T}\left(\frac{R_{ht}-\overline{R}_h}{\sigma_h}\right)^3$	Skewness is a measure of the symmetry of distribution and refers to the "third moment" of frequency distribution. Normal distribution has zero skewness. When skewness is positive (skewed to the right) then the frequency distribution has a long "right tail", while when we have negative skewness (skewed to the left), then large negative returns are more common than large positive returns and the tail distribution is heavier on the left

West, Finch and Curran (1995), Glosten, Jagannanthan and Runkle (1993) and Bekaert, Erb, Harvey and Viskanta (1998) among others proposed the departure from normality as a definition of excess kurtosis (negative or positive). For practical reasons, excess kurtosis is used instead of kurtosis when examining historical returns of stocks or portfolios. In fact, the larger the excess kurtosis, the more likely it is that future returns will be either extremely large or extremely small. This is why rational investors prefer negative excess kurtosis.

The mean, skewness and kurtosis can assume negative values, and this is not compatible with DEA models, so we have to ensure the "positivity" of these variables.[13] A common method to manage the problem of negative values in DEA is the addition of a sufficiently large positive constant to the input or output that assumes negative values. It is shown that data rescaling or translation adjustment is neutral with respect to the DEA results. In fact, Pastor (1996) and Ali and Seiford (1990) have shown that data translation does not alter the efficient frontier for certain DEA formulations; this is the "*translation invariance property*". In our case, to tackle the negative values problem, we add to the variable one plus the absolute value of the smallest value it assumes. This transformation does not affect the input oriented analysis (Annaert, van den Broeck and Vander Vennet (2003) and Daraio and Simar (2006)). The descriptive statistics of the inputs and outputs are reported in Table 2.2. The average variance goes from 0.0032 in 2014 to 0.0076 in 2011. In addition, the minimum

[13] Charnes et al (1991) provides a model to relax this requirement, but we prefer not to use it in our case.

variance is about 0.0008 in 2013. The maximum variance is about 0.0225 in 2012. The mean ranges from -0.472 in 2010 to 0.412 in 2011 for the excess kurtosis. The minimum of the excess kurtosis is -1.695 in 2012 and its maximum is 4.41 in 2014. The mean of the returns has an average ranging between -0.01037 in 2011 and 0.012 in 2013, with minimum at -0.054 in 2015 and maximum at 0.056 in 2014. The average skewness goes from -0.097 in 2012 to 0.38 in 2015. The minimum of the skewness is -1.5 in 2012 and its maximum is 1.81 in 2014.

Table 2.2 : Descriptive statistics

Variable	Year	Mean	S.D.	Max	Min
Input 1	2010	0.00603709	0.004916135	0.02164537	0.000985535
	2011	0.00764214	0.004803425	0.015996088	0.001865065
	2012	0.00615885	0.004799163	0.022598138	0.001026176
	2013	0.00347891	0.00224609	0.008620316	0.000809397
	2014	0.00322455	0.001703077	0.006588672	0.001187213
	2015	0.00673249	0.004011608	0.017444435	0.002400661
Input 2	2010	-0.47208317	0.841237405	2.081882074	-1.655713539
	2011	0.41292556	1.077362214	2.652507873	-1.147394676
	2012	0.28945254	1.351959409	3.013296757	-1.695651783
	2013	-0.19044926	1.119469009	3.804748344	-1.353898004
	2014	0.1069616	1.366988813	4.412316814	-1.467494395
	2015	0.01442134	0.944876734	1.642490042	-1.40581694
Output 1	2010	-0.00108483	0.018095024	0.032987937	-0.026540006
	2011	-0.01037581	0.016655136	0.035365136	-0.041128174
	2012	0.00707787	0.017712834	0.032315892	-0.03227863
	2013	0.01201687	0.014324896	0.045403072	-0.012852251
	2014	0.01035713	0.014744475	0.056078123	-0.016688947
	2015	0.00927307	0.01780523	0.036006252	-0,054797973
Output 2	2010	0.26628261	0.467892968	1.213646859	-0.736964265
	2011	0.29567929	0.414843924	0.991517498	-0.425979966
	2012	-0.09733502	0.772366629	1.605113048	-1.505251044
	2013	0.14264504	0.626717795	1.185324356	-1.191236189
	2014	0.27881626	0.688277817	1.810356949	-1.020627594
	2015	0.3833061	0.438620753	1.278311448	-0.293172269

In our application, we use the smallest value of the excess kurtosis for each year, denoted $\min\{k_h\}$, to solve models (2.12) and (2.13) in Step 3 of the algorithm. These values are reported in the Table 2.3.

Table 2.3 : The smallest Kurtosis values

Year	$Min\{k_h\}$
2010	-1.655713539
2011	-1.147394676
2012	-1.695651783
2013	-1.353898004
2014	-1.467494395
2015	-1.40581694

5.2 Cross-efficiency as a complement or alternative to simple efficiency

The results for the DEA scores, the cross-efficiency scores and asset ranks are reported in Table 2.4. The table contains for every year and each stock, its DEA efficiency score (the self-evaluation score), the cross-efficiency score (the peer-evaluation score) and for each method the rank of the asset. The means of DEA efficiency scores are 62.14%, 57.23%, 49.52%, 71.95%, 69.24% and 69.47% for the years 2010 through 2015, respectively. Whereas those for the DEA cross-efficiency scores are in comparatively smaller. That is, the average scores are 53%, 43.66%, 36.42%, 56.38%, 51.95% and 54% for the years 2010 through 2015, respectively. A low dispersion of the efficiency scores (DEA and cross-efficiency) shows a typical behavior shared by all financial assets. The efficient stocks change over time. In 2010, DANONE, L'OREAL and ENGIE were efficient and ranked first while BNP PARIBAS ACT,A were the worst performer based on the DEA scores. Using Cross-efficiency scores we found that DANONE (with a score of 100%) is the best performer, ENGIE (83%) is second, L'OREAL (74%) is third while MICHELIN (30%) is the worst performer. In 2011, AIR LIQUIDE (100%) was the only efficient DMU and ARCELOR-MITTAL (23%) was the worst performer whatever the method we used. In 2012, DANONE and L'OREAL were the only two DEA efficient DMUs, and were both ranked first. Using cross-efficiency score, L'OREAL (100%) is the most efficient DMU, followed by DANONE (62%). In 2013, using DEA scores to rank the DMUs, AIRLIQUID and LEGRAND were the best performers, while ORANGE was the worst performer with a DEA score equal to 41%. However, using the peer-evaluation method, the best performer was LEGRAND (94%), the second DMU was AXA (88%) and the worst performer DMU was PUBLICS GROUPE SA with cross-efficiency score equal to 18%. In 2014, there were four DEA efficient firms, they were BNP PARIBAS ACT,A, DANONE, KERING and L'OREAL. Using Cross-efficiency evaluation, DANONE (65%) was ranked first, BNP PARIBAS ACT,A (92%) was second, ARCELOR-MITAL (75%) was third, and the fourth was KERING (58%). In 2015, BNP

PARIBAS ACT,A and BOUYGUES were the best performers using the CCR self-evaluation. However, with the peer-evaluation method, BNP PARIBAS ACT,A (98%) performed better than BOUYGUES (79%). These results show the discrimination power of cross-efficiency evaluation.

Table 2.4: Efficiency, cross-efficiency and stocks returns

YEAR	2010				2011				2012				2013				2014				2015			
DMU	DEA[14]	RANK	CE[15]	RANK	DEA	RANK	CE	RANK	DEA	RANK	CE	RANK	DEA	RANK	CE	RANK	DEA	RANK	CE	RANK	DEA	RANK	CE	RANK
ACCOR	0.53	13	0.49	10	0.28	20	0.24	20	0 .52	8	0.39	6	0.58	17	0.51	14	0.81	7	0.61	6	0.55	16	0.41	18
AIRBUS GROUP	0.43	20	0.39	12	0.46	14	0.43	11	0.31	19	0 .21	19	0.63	14	0.55	10	0.68	10	0.43	13	0.87	4	0.53	9
AIR LIQUIDE	0.50	17	0.47	19	1.00	1	1.00	1	0.82	3	0.37	7	1.00	1	0.86	3	0.67	11	0.38	14	0.70	12	0.56	8
ARCELORMITTAL	0.71	5	0.58	6	0.23	21	0.20	21	0.71	4	0.51	3	0.93	4	0 .58	9	0.98	5	0.75	3	0.40	19	0.26	21
AXA	0.72	4	0.59	5	0.52	12	0.39	14	0.42	11	0.31	13	0.96	3	0.88	2	0.89	6	0.68	5	0.86	6	0.68	5
BNP PARIBAS ACT,A	0.36	21	0.31	20	0.70	8	0.43	10	0.47	9	0.33	11	0.52	18	0.45	16	1.00	1	0.92	2	1.00	1	0.98	1
BOUYGUES	0.67	7	0.64	4	0.48	13	0.39	13	0.31	18	0.23	18	0.66	13	0.52	13	0.43	20	0.29	21	1.00	1	0.79	2
CAP GEMINI	0.58	10	0.43	17	0.73	6	0.56	3	0.43	10	0.33	10	0.81	9	0.70	5	0.53	15	0.47	11	0.86	5	0.50	13
CARREFOUR	0.48	18	0.45	15	0.67	9	0.52	6	0.34	16	0.27	16	0.58	16	0.40	17	0.56	14	0.47	10	0.75	8	0.70	4
CREDIT AGRICOLE	0.56	11	0.44	16	0.66	10	0.50	8	0.40	13	0.27	15	0.43	20	0.38	18	0.50	16	0.36	17	0.39	20	0.37	19
DANONE	1.00	1	1.00	1	0.78	4	0.51	7	1.00	1	0.62	2	0.81	8	0.60	8	1.00	1	0.96	1	0.61	15	0.45	14
ENGIE	1.00	1	0.83	2	0.74	5	0.54	4	0.29	20	0.20	20	0.69	11	0.54	11	0.72	9	0.51	9	0.73	9	0.51	12
KERING	0.51	16	0.47	11	0.34	17	0.31	16	0.38	14	0.35	8	0.69	10	0.52	12	1.00	1	0.74	4	0.49	18	0.42	17
KLEPIERRE	0.70	6	0.58	7	0.43	15	0.35	15	0 .57	7	0.25	17	0.60	15	0.49	15	0.80	8	0.58	7	0.71	11	0.51	11
LEGRAND	0.52	14	0.41	18	0.43	16	0.28	17	0.34	15	0.31	12	1.00	1	0.94	1	0.50	17	0.35	18	0.97	3	0.77	3
L'OREAL	1.00	1	0.74	3	0.80	3	0.47	9	1.00	1	1.00	1	0.82	7	0.65	7	1.00	1	0.57	8	0.54	17	0.42	16
LVMH	0.63	9	0.57	8	0.30	19	0.26	18	0.40	12	0.34	9	0.89	6	0.72	4	0.60	13	0.33	19	0.67	13	0.52	10,
Michelin	0.46	19	0.30	21	0.53	11	0.41	12	0.18	21	0.14	21	0.91	5	0.68	6	0.47	18	0.38	15	0.71	10	0.64	6
ORANGE	0.51	15	0.45	14	0.94	2	0.54	5	0.31	17	0.28	14	0.41	21	0.33	20	0.45	19	0.37	16	0.63	14	0.43	15
PUBLICIS GROUPE SA	0.53	12	0.52	9	0.70	7	0.60	2	0.59	6	0.50	4	0.52	19	0.18	21	0.36	21	0.32	20	0.80	7	0.62	7
RENAULT	0.65	8	0.47	13	0.30	18	0.24	19	0.61	5	0.44	5	0.67	12	0.36	19	0.63	12	0.44	12	0.35	21	0.27	20
Mean	0.62		0.53		0.57		0 .44		0.50		0.36		0.72		0.57		0.69		0.52		0.70		0.54	
S.D	0.18		0.17		0.22		0.18		0.23		0.18		0.19		0.19		0.22		0.19		0.19		0.18	
Min	0.36		0.30		0.23		0.20		0.18		0.14		0.41		0.18		0.36		0.29		0.35		0.26	

[14] DEA score
[15] Cross-efficiency score

Max	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.94	1.00	0.96	1.00	0.98

To measure the false positiveness of firms, we compute the maverick index, also called the False Positive Index (FPI) in the literature (Baker and Talluri (1997)). The maverick index is suggested by Doyle and Green (1994a) to measure the percentage increment of the simple efficiency score when it moves from its peer-evaluation to its self-evaluation. A firm presents a maverick case if it is considered efficient under self-evaluation but fail to be benchmark for inefficient firms. The false positive index is defined as follows:

$$FPI_j = \left(e_{jj} - \bar{e}_j\right) / \bar{e}_j \, , \tag{2.16}$$

where e_{jj} is the self-appraisal of DMU_j determined by optimizing the CCR model (2.9) and $\bar{e}_j$ is the average of all cross-efficiency scores e_{hj}, that is the peer-appraisal of DMU_j. A low maverick index means that the firm is all-round performer, however a firm with high maverick index does not perform well on most of the agreed upon best factors.

The maverick index estimations are presented in Table 2.5. MICHELIN, ORANGE, KLEPIERRE, PUBLICS GROUPE SA and CAP GEMINI are the DMUs with the largest false positive index values for the years 2010 through 2015 respectively, with respective maverick indexes equal to 53.33%, 74.07%, 128%, 188.89%, 76.32% and 72%. These firms make strong cases for maverick firms. We find these results and interpret them as maverick cases because the DEA model allows each DMU to be assessed as efficient by using the most favorable inputs/outputs weights, that is the optimal weights select a single or few inputs and/or outputs to appear efficient, while the cross-efficiency take into account all inputs and outputs. DANONE, AIR LIQUIDE, L'OREAL, BNP PARIBAS ACT.A have the smallest FPI indexes, 0%, 0%, 0%, 6.38% and 4.17% and 2.04% respectively, for the period 2010 through 2015. A low FPI for an asset indicates that it benefits the least when moving from peer-appraisal to self-appraisal and so these DMUs are good overall firms. Selection portfolio with cross-efficiency evaluation leads to selecting stocks robust with respect to the risk of change in inputs/outputs weights.

Table 2. 5 : Maverick index

	Maverick index (%)					
	2010	**2011**	**2012**	**2013**	**2014**	**2015**
ACCOR	8.16	16.67	33.33	13.73	32.79	34.15
AIRBUS GROUP	10.26	6.98	47.62	14.55	58.14	64.15
AIR LIQUIDE	6.38	0.00	121.62	16.28	**76.32**	25.00
ARCELORMITTAL	22.41	15.00	39.22	60.34	30.67	53.85
AXA	22.03	33.33	35.48	9.09	30.88	26.47
BNP PARIBAS ACT.A	16.13	62.79	42.42	15.56	8.70	2.04
BOUYGUES	4.69	23.08	34.78	26.92	48.28	26.58
CAP GEMINI	34.88	30.36	30.30	15.71	12.77	**72.00**
CARREFOUR	6.67	28.85	25.93	45.00	19.15	7.14
CREDIT AGRICOLE	27.27	32.00	48.15	13.16	38.89	5.41
DANONE	0	52.94	61.29	35.00	4.17	35.56
ENGIE	20.48	37.04	45.00	27.78	41.18	43.14
KERING	8.51	9.68	8.57	32.69	35.14	16.67
KLEPIERRE	20.69	22.86	**128.00**	22.45	37.93	39.22
LEGRAND	26.83	53.57	9.68	6.38	42.86	25.97
L'OREAL	35.14	70.21	0.00	26.15	75.44	28.57
LVMH	10.53	15.38	17.65	23.61	81.82	28.85
MICHELIN	**53.33**	29.27	28.57	33.82	23.68	10.94
ORANGE	13.33	**74.07**	10.71	24.24	21.62	46.51
PUBLICIS GROUPE SA	1.92	16.67	18.00	**188.89**	12.50	29.03
RENAULT	38.3	25.00	38.64	86.11	43.18	29.63

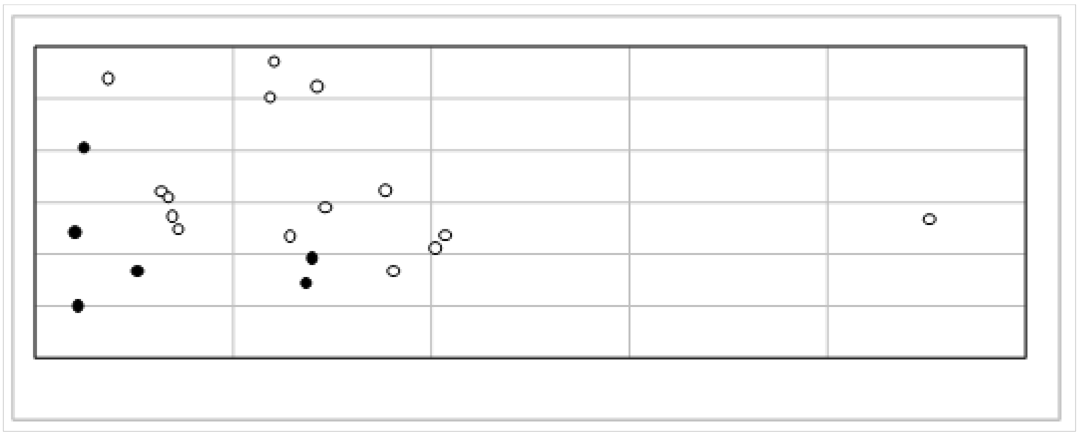
The simple use of cross-efficiency evaluation suffers from the ganging together phenomenon (Tofallis (1996)) and therefore induces a problem of insufficient diversification of the portfolio (Lim et al (2014)). In fact, under peer-evaluation, DMUs are highly ranked since their performance is at least moderately good on all measures or factors (inputs and outputs). Comparatively, DMUs that are lowly ranked perform well on only a subset of measures. The problem here is that DMUs with similar factors “vote” for each other. This leads to selection of a specialized portfolio made of similar DMUs and consequently with little diversification.

Table 2.6 : Inputs vs. Efficiency and cross-efficiency scores (year 2012)

Year 2012					
DMU	**Input 1**	**Input 2**	**DEA**	**Cross**	**Rank**
ACCOR	**0.00702495**	**1.92015577**	**0.5239**	**0.3926**	**6**
AIRBUS GROUP	0.00886475	3.23406096	0.3076	0.2106	19
AIR LIQUIDE	**0.00125788**	**4.05385983**	**0.8185**	**0.3725**	**7**
ARCELORMITTAL	**0.00686456**	**1.44946974**	**0.7101**	**0.5107**	**3**
AXA	0.01037369	2.36999844	0.4237	0.3114	13
BNP PARIBAS ACT, A	0.01012075	2.11883596	0.4739	0.3282	11
BOUYGUES	0.0059614	5.00052744	0.3084	0.2265	18
CAP GEMINI	0.0064478	2.34574152	0.4293	0.3298	10
CARREFOUR	0.00734413	2.91206268	0.3406	0.2694	16
CREDIT AGRICOLE	0.02259814	2.68218982	0.3995	0.2724	15
DANONE	**0.00102618**	**2.42775931**	**1.0000**	**0.6168**	**2**
ENGIE	0.00713514	5.22131654	0.2890	0.2042	20
KERING	0.00347499	2.72760608	0.3788	0.3530	8
KLEPIERRE	0.00185422	5.37093407	0.5677	0.2518	17
LEGRAND	0.00320158	3.2045444	0.3406	0.3125	12
L'OREAL	**0.00108997**	**1**	**1.0000**	**0.9961**	**1**
LVMH	0.00362291	2.4758752	0.4030	0.3429	9
MICHELIN	0.00605979	5.70894854	0.1829	0.1413	21
ORANGE	0.00337631	3.10506461	0.3091	0.2765	14
PUBLICIS GROUPE SA	**0.00258371**	**1.68150939**	**0.5935**	**0.5031**	**4**
RENAULT	**0.00905303**	**1.67673046**	**0.6077**	**0.4403**	**5**

This shortcoming is illustrated using the 2012 results presented in Table 2.6 and plotted in Figure 2.1. We have created a scatter plot to examine the diversification level on the input and output space of the cross-efficiency framework. We select the first seven shares (one third of the sample) having the highest scores in 2012 (L'OREAL, DANONE, ARCELORMITAL, PUBLICS GROUPS SA, ACCOR and AIR LIQUIDE) and we include them in a portfolio. These firms are identified as black circles on the figure, while the other DMUs are white circle. It is obvious that the selected stocks have relatively similar inputs and therefore are relatively similar. In fact, they are clustered around the central position. This makes the selected portfolio badly diversified in terms of its performance on the multiple input-output factors and thus vulnerable to weights change risk on these two inputs.

Figure 2.1 : Inputs versus portfolio selection with DEA cross-efficiency (year 2012)



5.3 Mean-Variance-Skewness-Kurtosis framework results

For every SK tradeoff parameter, δ, for which values are chosen in the unit interval $\delta \in [0.1, 0.2, ...0.9]$, we solve the SK tradeoff problem (12). We note a relative slight sensitivity of portfolio selection, resources allocation and thus portfolios efficiency to this parameter variation. To solve the MV tradeoff, we solve problem (2.14) for every γ chosen in the unit interval, $\gamma \in [0.1, 0.2, ..., 0.9]$. We find that the MV tradeoff parameter changes over time. Table 2.7 provides the optimal SK and MV tradeoff parameters values over the period of the study.

Table 2.7: Optimal SK and MV tradeoff parameters values

Year	δ^*	γ^*
2010	0.9	0.5
2011	0.9	0.5
2012	0.5	0.5
2013	0.5	0.2
2014	0.5	0.9
2015	0.5	0.3

By setting the tradeoff parameters that characterizes investor aversion to risk (prudence and temperance) optimally, our framework allows us to generate a better set of shares to invest in each stock than what was used to be done in the literature and it obviously returns the optimal wealth allocation. Table 2.8 presents the optimal composition of the portfolio using MVSK cross-approach for the whole period.

Table 2.8: Optimal resources allocation with MVSK approach (in percent)

DMU	2010	2011	2012	2013	2014	2015
ACCOR	1.65	5.771	3.21	-	-	-
AIRBUS GROUP	4.296	5.824	6.51	-	20.698	-
AIR LIQUIDE	4.652	5.603	2.337	-	-	-
ARCELORMITTAL	-	4.891	-	8.959	-	30.129
AXA	-	2.047	4.598	-	-	-
BNP PARIBAS ACT. A	5.533	6.854	4.978	-	-	-
BOUYGUES	-	4.499	11.073	-	19.066	-
CAP GEMINI	10.178	2.098	4.483	-	-	15.119
CARREFOUR	6.299	2.071	5.588	-	-	-
CREDIT AGRICOLE	-	2.122	4.326	-	23.091	-
DANONE	-	6.908	17.331	-	-	-
ENGIE	-	2.968	10.229	-	-	5.501
KERING	4.343	5.058	5.909	-	-	-
KLEPIERRE	-	4.394	-	-	-	-
LEGRAND	13.225	2.94	5.737	-	18.677	-
L'OREAL	14.2	5.16	-	38.054	18.466	-
LVMH	5.26	6.67	3.502	-	-	-
Michelin	19.113	3.041	4.248	-	-	-
ORANGE	0.979	8.386	3.393	-	-	-
PUBLICIS GROUPE SA	7.606	8.239	0.595	29.173	-	-
RENAULT	2.668	4.456	1.953	23.813	-	49.249
E_{Ω^*}	0.4602	0.4431	0.3379	0.4382	0.3994	0.315
V_{Ω^*}	0.0186	0.0214	0.0341	0.0209	0.0131	0.0128
γ^*	0.5	0.5	0.5	0.2	0.9	0.3
δ^*	0.9	0.9	0.5	0.5	0.5	0.5

To illustrate how the procedure works, let us use the year 2013 as an example. For the optimal value $\gamma^* = 0.2$ and $\delta^* = 0.5$, the optimal portfolio put weights 38.054%, 29.173%, 23.813% and 8.959% on L'OREAL, PUBLICS GROUP SA, RENAULT and ARECLOR-MITTAL stocks respectively. This portfolio achieves a cross-efficiency score equal to 0.4382 and has a variance equal to 0.0209.

To show that our approach allows us to select a well-diversified portfolio, we use an example based on the results for 2012, as plotted in Figure 2.2 and presented by Table 2.6. We select the first four shares with the highest proportions, DANONE (17.331%), BOUYGUES (11.073%), ENGIE (10.229%) and AIRBUS GROUP (6.51%)) to create a new portfolio. The selected assets corresponds to the black circles on Figure 2.2, the other stocks are represented with white circles. It is obvious that the selected stocks have relatively distinct inputs and outputs mixes and are not similar. They are relatively scattered and dispersed over the input space, and clearly not clustered around a central position. This makes the selected portfolio well diversified, more efficient, and less risky.

Figure 2.2: Inputs versus portfolio selection with MVSK cross-efficiency (year 2012)

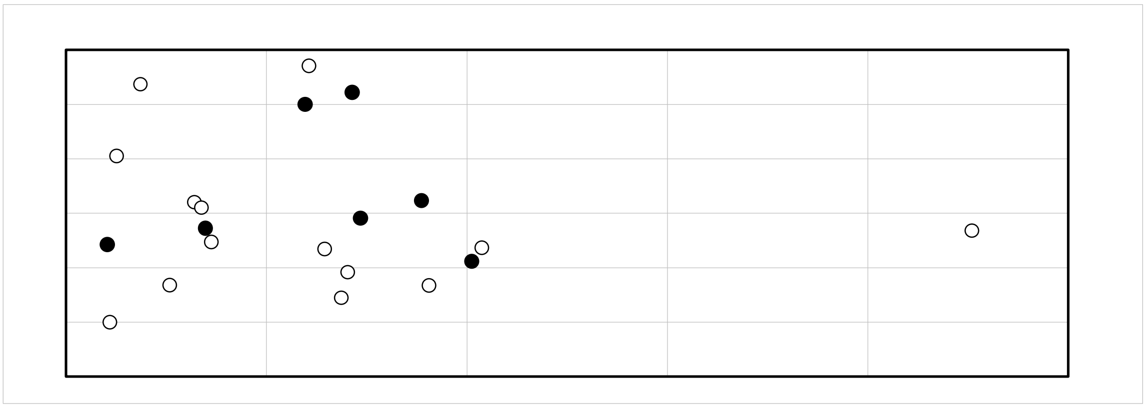
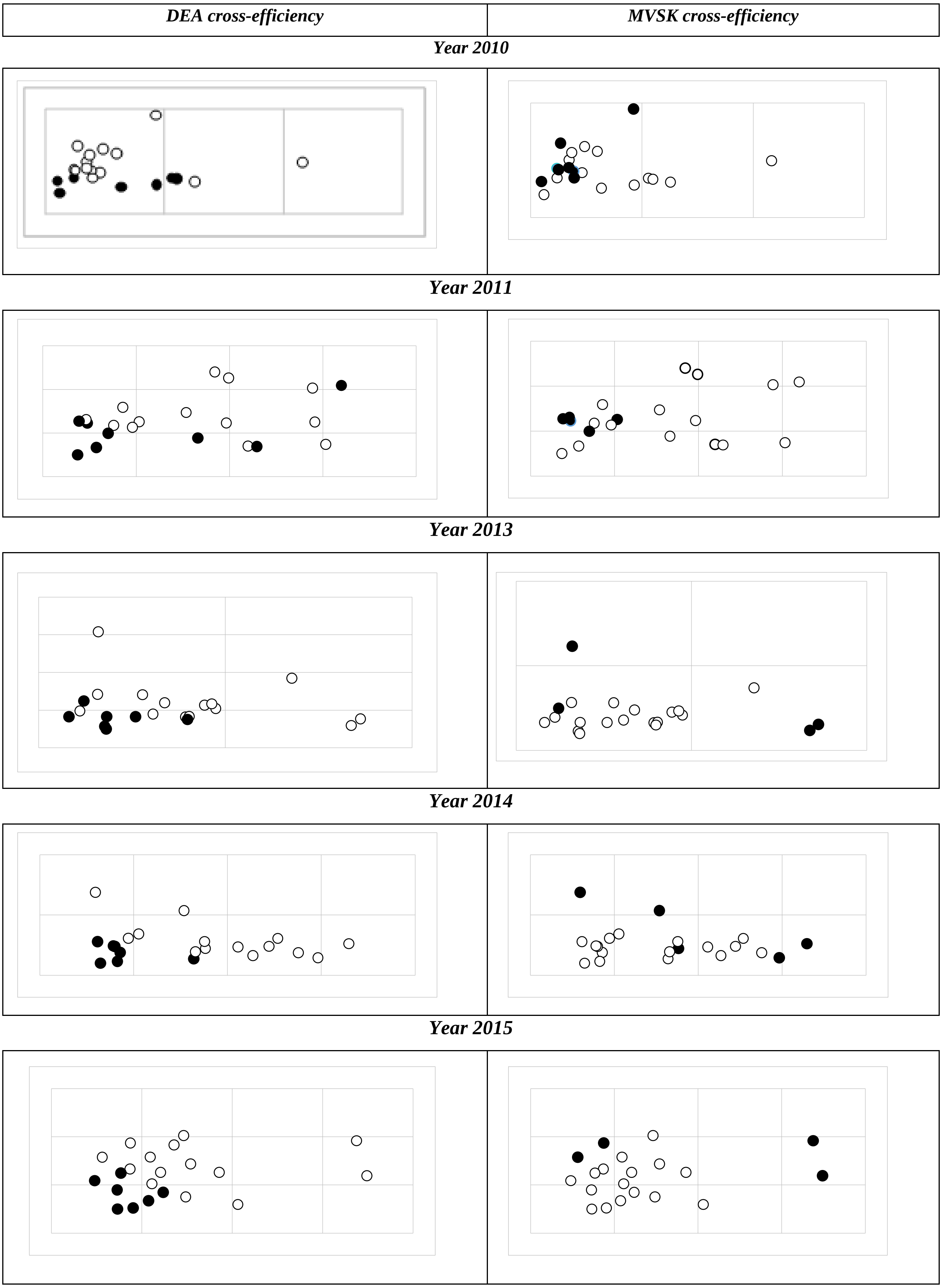


Figure 2.3: DEA cross-efficiency vs. MVSK cross-efficiency: Diversification in portfolio selection



On Figure 2.3, we plot the same type of results for the remaining years to show the effectiveness of MVSK cross framework to select diversified portfolios relatively to the simple use of DEA cross-efficiency framework. In 2013, the selected stocks with DEA cross-efficiency portfolio are LE GRAND, AXA, AIRLIQUIDE, LVMH, CAP GEMINI, MICHELIN and L'OREAL and are all clustered around the central position. These DMUs have relatively similar inputs and therefore are relatively similar. This is very different than the results obtained from the MVSK cross-efficiency, the portfolio would include ARECLORMITAL, L'OREAL, PUBLICS GROUP SA and RENAULT, which are scattered and dispersed in the input space. This is a good indication that portfolios selected when higher order moments are included in the DEA cross-efficiency method are more diversified so very likely less risky and more efficient than portfolio constructed using the simple DEA cross-efficiency method.

The MVSK DEA cross-efficiency method allows investors to allocate their wealth optimally and in precise proportions taking into account prudence, temperance and risk aversion

5.4 Portfolio performance: Robustness check

To check for the robustness of our method, we make the following experiment. We use the solution to the MVSK cross-efficiency model to select an optimal portfolio called the "MVSK cross". Suppose this portfolio is kept an investment horizon of 1 year. At the end of the year, we revise the portfolio stock composition with new stocks proportions. We repeat this procedure whenever we start a new investment period.[16] We compare this procedure with the standard portfolio selection using a simple DEA cross-efficiency method, and we call that portfolio "DEA cross". This portfolio is made of seven assets (the first seven shares). Then we test the performance of these portfolios with the Sharpe ratio.

[16] We ignore transaction costs in our analysis.

Table 2.9: Portfolios annual excess return (%) and performance comparison

Year	MVSK Cross	CAC40	AEX	BEL20	PSI20	DEA cross
2010	12.7	-8.56	0.01	-2.78	-15.14	-11.9
2011	-21.48	-22.15	-16.32	-23.99	-31.84	-21.05
2012	9.3	10.75	5.93	14.85	-1.47	16.84
2013	27.77	12.62	12.3	13.81	10.42	17.5
2014	3.22	0.24	5.33	13.22	-28.36	-3.01
2015	14.36	8.05	3.63	11.76	8.44	14.38
Geometric average annual excess return mean	6.5	-0.65	1.39	3.4	-11.24	0.97
Annualized volatiliy	16.41	13.44	9.74	15.41	18.27	16.52
Sharpe ratio	0.3959	-0.0483	0.1431	0.2207	-0.6153	0.0585

The Sharpe ratio has been one of the most referenced measures of risk to return $(risk/return)$ measures in the finance literature. It describes how much excess return received for the extra volatility that investor endure for holding a riskier asset. It is defined as the portfolio geometric average annual excess return over the risk-free return divided by the annualized excess return volatility. The common benchmark used to represent a risk-free return in France is the interest rate of 10-year treasury bond (OAT), which range from 0.995% to 3.35% per year over the sample period.[17]

We compare the risk-adjusted performance of “MVSK cross” portfolio with benchmark portfolios (AEX index, BEL20 index and PSI20 index), these indexes are chosen for comparison because they have dimension ‘closed’ to “MVSK cross” portfolio. We compare also the “MVSK cross” portfolio to the “DEA Cross” portfolio made of the seven cross-efficiency’ best-performer stocks. Results are presented in Table 2.9. We find that the “MVSK cross” portfolio reaches the highest geometric average annual excess return (6.5%) and the largest Sharpe ratio (0.3959), followed by BEL20 index then AEX index. “DEA cross” portfolio is performing better than the CAC40 index and the PSI20 index. Unfortunately, the standard deviation type procedures like the Sharpe ratio do not account for extreme returns. In fact, this performance measure is not valid when returns have tails heavier than the normal distribution, so we have to explore other robustness checks. Consequently, we use the studentized circular block bootstrap (SCBB) developed by Ledoit and Wolf (2008). This test takes into account the skewness, kurtosis and autocorrelation effects when

[17] We use the rate of 31 December day of each year.

comparing two Sharpe ratios for the differences between performances of derived portfolios with benchmark indexes are statistically significant.

The SCBB procedure is a two-sided hypothesis test formulated as follows:

$$\textit{Test } 1:\ H_0 : (\textit{MVSK vs. DEA cross}) : \textit{Sharpe ratio } (\textit{MVSK}) - \textit{Sharpe ratio } (\textit{DEA cross}) = 0$$
$$\textit{Test } 2:\ H_0 : (\textit{MVSK vs. AEX index}) : \textit{Sharpe ratio}(\textit{MVSK}) - \textit{Sharpe ratio } (\textit{AEX index}) = 0$$

Table 2.10 presents the test results.[18] $H_0 : (\textit{MVSK vs AEX index})$ is rejected at 6% (p-value = 0.06). This means that the Sharpe ratio of MVSK cross portfolio is significantly greater than AEX index Sharpe ratio (the difference is equal to 0.28). The test, $H_0 : (\textit{MVSK vs. DEA} - \textit{cross})$ is not rejected (p-value=49.6%) indicating that corresponding two Sharpe ratios are not significantly different. This result dims a little the performance of our procedure, but we have already shown that it performs better on the risk side of the problem.

Table 2.10 : Sharpe ratios using Ledoit-Wolf (2008) test

Null hypothesis	Difference	P-value
H0: Sharpe ratio (MVSK cross)=Sharpe ratio (DEA cross)	0.3372	0.496
H0: Sharpe ratio (MVSK cross)=Sharpe ratio (AEX index)	0.28	0.06

Overall, these results show that MVSK cross-efficiency approach is a promising tool for stock portfolio selection leading to better wealth allocation. We have demonstrated that DEA cross-efficiency with higher moments is more effective than using the simple DEA cross-efficiency.

6. Conclusion

The framework developed by Lim et al (2014) introduced a new way of using DEA cross-efficiency evaluation in portfolio selection in the Mean-Variance space. In this paper we have extended this method to allow investors to improve wealth allocation in the Mean-Variance-Skewness-Kurtosis space. Our procedure computes the optimal proportions of shares to invest in a portfolio. It also deals with the problem of non-uniqueness of cross-efficiencies, the poor diversification problem caused by the ganging-together phenomenon associated with

[18] We used the R package of Ledoit and Wolf (2008) using the default parameter settings.

the standard DEA cross-efficiency evaluation. Furthermore, it allows us to take into account investor risk aversion, prudence and temperance at the same time. The MV tradeoff parameter and SK tradeoff parameter are endogenously determined using a nonparametric framework. We have applied our approach to a sample of firms from the French CAC40 index for a period of 6 years. We use rational investor's preference for positive odd moments and low even moments of stocks returns as outputs and inputs respectively in a DEA frontier estimation framework. We find that portfolio selection is slightly sensitive to tradeoff parameters values. Our results confirm the intuition that higher order moments can significantly change the optimal portfolio selection. Our findings show the effectiveness of our approach to overcome the lack of diversification problem in portfolio selection associated to the simple use of DEA cross-efficiency approach in portfolio selection, thereby contributing to improve the performance of such DEA cross-efficiency approaches. Our results are robust when tested using the Ledoit and Wolf (2008) Sharpe test against a benchmark portfolio. There is a caveat however, because the developed MVSK model is quartic, its application on large sample could not be easily performed. It may be possible to resolve the model by employing a heuristic approach, however, but this is beyond the scope of this paper.

Chapter 3:

A Mean-Maverick Game Cross-Efficiency Approach to Portfolio Selection: An Application to Paris Stock Exchange[19]

[19] Essid Hédi, Ganouati Janet and Vigeant Stéphane (2018), 'A Mean-Maverick Game Cross-Efficiency Approach to Portfolio Selection: An Application to Paris Stock Exchange', Expert Systems With Applications 113, 161–85 https://doi.org/10.1016/j.eswa.2018.06.040 (Accepted for publication on June 16, 2018)

Chapter 3

A Mean-Maverick Game Cross-Efficiency Approach to Portfolio Selection: An Application to Paris Stock Exchange

1. Introduction

Portfolio selection is the decision whereby the best set of financial assets is selected from many different alternatives. Even though various tools and sophisticated techniques were developed to handle investment problems, uncertainties still govern the process. This is still problematic even for the most qualified experts. Indeed, evaluating and comparing the performance of financial assets present an important issue for managers and investors. Hence, there is a pressing need for a trustful assessment tool to evaluate and discriminate in a better way among different listed firms. Recently, many techniques dealing with problem of assets evaluation and portfolio selection have been proposed by the literature, such as heuristic algorithms to solve Markowitz model proposed by Soleimani et al. (2009), particle swarm optimization suggested by Deng et al. (2012) for solving the Cardinality Constraints Markowitz Portfolio Optimization problem, P-Spline clustering analysis proposed by Iorio et al. (2018) among others. Selecting portfolios is an important and recurring task for investors and managers managing clients' assets. It is the resulting process of how investors make investment decisions. A rational investor has certainly special insights and preferences, that must be translated into optimization model or/and algorithm to serve his investment decision making process. In this paper, we propose a novel method for portfolio selection based on Data Envelopment Analysis (DEA) game cross-efficiency approach using the maverick index, as a consistent risk measure.

The DEA, as non-parametric programming, measures the relative efficiency of peer many-input and many-output decision making units (DMUs). The efficiency of a DMU is a ratio of weighted sum of its output divided by a weighted sum of its inputs. According to Emrouznejad and Yang (2017) study, there is 9577 articles on DEA models during the period 1978-2016. Recently, DEA has seen an exponential growth in the number of publications related to theory (see for example, Wei and Wang (2017) and Wen et al.(2017)). The wide range of DEA applications enforces its rapid development. DEA has been applied to DMUs in various forms, such as environment field (see for example Deilmann et al.(2016), Chen et al. (2017a) and Deilmann et al. (2018)), financial institutions performance (Rahman et al. (2016), Kaffash and Marra (2017) and (Du et al. 2018), education area (Tüselmann et al. (2015) and Sagarra et al. (2017)), Agriculture area (Bojnec and Latruffe (2008) and Atici and Podinovski (2015)), health care (Khushalani and Ozcan (2017) and Asandului et al. (2014)), and so on. The applications of DEA to the evaluation of portfolio's performance have become more and more numerous in the last years. From a perspective of evaluation based on endogenous benchmarks, Zhao et al. (2011) propose two quadratic-constrained DEA models to evaluate mutual funds' performance and show that the ranking of mutual funds depends mostly on system risk control. Using DEA framework, Liu et al. (2015) work introduces the notion of like allocation efficiency and the scale efficiency among other into evaluation of portfolio. Sanei et al. (2016) estimate the Markowitz efficient frontier using DEA model and consider the variance and the value at risk as risk measure. Branda (2016) introduces a new diversification consistent DEA model based on directional distance measure and considers the value at risk as risk indicator to assess efficiency of investment opportunities available on financial markets. Tarnaud and Leleu (2017) define the financial production process as the generation of a distribution of returns by an initial investment, and consider the return's distribution as input/output variables to evaluate portfolio performance with DEA method. Zhou et al. (2017) extend DEA model to a general return-risk model to assess mutual funds' performance. They build portfolio rebalancing strategies which are able to show sustainability in future investment. Rezaee et al. (2018) propose an integrating dynamic fuzzy c-means, data envelopment analysis, and artificial neural network to clusters and evaluates listed companies. While DEA can provide a remedy to the issue of aggregating multi performance measures into a key indicator, DEA suffers from high flexibility thus allowing for weak discrimination among DMUs. In fact, we may have more than 100% efficient DMU. As a result, ranking DMUs can be quite hard. Thereby, the cross-efficiency method, proposed by Sexton, Silkman, and Hogan (1986) as an extension to DEA and investigated by (Doyle and Green 1994a), provides a unique ordering of DMUs and eliminates unrealistic weight schemes through peer evaluation.

Presenting a good alternative to DEA self-evaluation, the DEA cross-efficiency evaluation considers simultaneously self-appraisal and peer-appraisal, exhibiting an enhanced discriminative power. In fact, cross-efficiency is a democratic process with less of the arbitrariness of additional weight restrictions, as opposed to the DEA externally imposed weights and the self-evaluation process. Cross-efficiency evaluation approach identifies good overall performs and ranks DMUs. Liu et al. (2016) identify DEA cross-efficiency approach as one of the four research fronts in DEA. Due to the democratic process and powerful discrimination ability, the cross-efficiency evaluation has been applied in a wide variety of areas such as the application of Oral et al. (1991) to industrial R&D projects, Shang and Sueyoshi (1995), Sun (2002) and Song and Liu (2018) to manufacturing organization, Dotoli et al. (2015) to healthcare system, Liu et al. (2017b) to environment area and the work of Wu et al. (2017) to banking system evaluation. Even though cross-efficiency was successfully applied to multiple real world problems, it still has some issues as the non-uniqueness of the DEA optimal weights which may reduce its usefulness. Specifically, there is possibility of multiple optimal weights in the DEA model depending on the used optimization software (Despotis (2002)). Thereby, avoiding the non-uniqueness of DEA solutions on the one hand and unrealistic DEA weighting schemes on the other hand, several approaches have been developed. Various secondary goals (benevolent and aggressive formulations) have been developed by Sexton et al. (1986) and Doyle and Green (1994a) to choose the weights among optimal solutions. These formulations are extended by Liang et al. (2008b). They introduced a number of alternative secondary goals for the cross-efficiency approach. In a single input situation, Anderson et al. (2002) derived and demonstrated that cross-efficiency applies an implicit fixed weighting scheme to each and every DMU, which is a weighted-average of the weights used by all of the DMUs in the sample. Sun and Lu (2005) presented a cross-efficiency profiling model to improve discrimination power of DEA. Lim (2012) proposed aggressive and benevolent formulations of cross-efficiency in DEA, where a Minimax or a Maximin type secondary objective is incorporated. Liang et al. (2008a) generalized the original DEA cross-efficiency concept to game cross-efficiency and showed that the optimal game cross-efficiency scores constitute a Nash equilibrium point. In their work, cross-efficiency evaluation has been examined in the context of cooperative game. In fact, DMUs are viewed as players and the cross-efficiency scores as payoffs. Also, each DMU may choose to take a non-cooperative game stance to boost its efficiency scores given a weight selection strategy. Liang et al. (2008a) developed an algorithm converging to Nash equilibrium. Due to these advantages, DEA game cross-efficiency evaluation has been recently applied in performance evaluation of Olympic ranking (Roboredo et al. (2015)), supplier performance (Ma et al. (2014)), the infrastructure investment (Sun et al. (2016)), University' departments and international passenger airlines (Wang and Chin (2010)), R&D project selection and budgeting (Chen and Zhu (2011)), and so on. This work involves the game cross-efficiency approach in portfolio selection field. In fact, a direct competition may exist among financial assets and the decision to invest in one asset and not the other is crucial. As there are competitions existing among financial assets, it is rational to use the DEA game cross-efficiency model to evaluate the comprehensive efficiency of each stock and to discriminate among them in order to select portfolio.

Several models have been suggested as a remedy to the cross-efficiency issue in portfolio selection area (see,e.g., Lim et al. (2014), Mashayekhi and Omrani (2016) and Wu et al. (2016)). Indeed, a naive usage of DEA cross-efficiency in portfolio selection is the selection of the best performer stocks with the highest efficiency score. Even though this simple use yielded a better result than simple DEA, Lim et al. (2014) pointed out a major limitation of cross-efficiency application in portfolio selection, which is the lack of portfolio diversification. As a remedy to this issue, Lim et al. (2014) developed a mean-variance (MV) framework, using cross-efficiency scores and their variance. A similar method was developed by Chen and Zhu (2011) proposing the bootstrap game cross-efficiency distributions to gather information regarding efficiency variations and correlations, and then adopting the MV formulation to obtain a risk-minimizing resource allocation portfolio. Indeed, the variance was always considered as the most common metric to assess volatility and relative risk of potential investment. However, it is still irrelevant for asymmetrical return distributions for which MV models punish the upside potential in the same fashion as the downside risk (Grootveld and Hallerbach (1999)). In fact, it has been shown that the market portfolio is highly and significantly inefficient. Thus, asset returns cannot be described by the mean and the variance only. Thereby, MV criterion was replaced with a more general efficiency criterion that accounts for higher-order central moments, particularly skewness and kurtosis (Nalpas et al. (2017) and Neumann and Skiadopoulos (2013)), and lower partial moments such as the Value at Risk (Lwin et al. (2017) and Zhang and Gao (2017)), the expected shortfall (Broda et al. (2017)) and semi-variance metrics (Lobato et al. (2017)). Nevertheless these risk measures present several practical limitations. In fact, there are several approaches to measure the Value at Risk and the Expected Shortfall that can lead to different results with the same portfolio (Krause (2003)). Furthermore, the semi-variance measurement cannot be an objective measure of risk (Levy (1998)). Moreover, the main problem of considering higher order moments in portfolio optimization is rather the computational aspect which is not an easy task.

In this work, we try to solve the problem of the instability and the unpredictability of the cross-efficiency method. In fact, the Nash equilibrium efficiency scores provided by the game cross-efficiency serve to evaluate and rank financial assets performance. In addition, derived from the DEA game cross-efficiency method, we attempt to develop a novel risk indicator in the portfolio area. This method provides a relevant and novel measure of risk. Indeed, the maverick index, named also the False Positive Index (FPI) presents the sensitivity level to environmental changes; or further a good indicator of risk degree. It measures the deviation of (DMU) self-evaluation score from its Nash equilibrium score. This indicates the risk degree for change in performance of the different factors. Furthermore, by making a tradeoff between efficiency and risk we select a very well-diversified portfolio. As an illustration of our approach, we report a case study involving $500 \sim 508$ firms from the Paris Stock Exchange. We use actual financial data from 2010 to 2015. We demonstrate that our approach can represent a promising tool for financial assets portfolio selection by showing that the resulting portfolio yields higher risk-adjusted returns than other benchmark portfolios for a 6-year sample period. Furthermore, we show that the formed portfolio is well-diversified, superior to some portfolios based purely on game cross-efficiency and on simple cross-efficiency. This indicates the effectiveness of our approach as a reliable tool to portfolio selection.

The contribution is structured as follows. The next section reviews DEA, cross-efficiency and game cross-efficiency approaches. Section 3 provides a discussion on the maverick index and the risk degree issue. Section 4 develops the proposed approach. Section 5 presents an empirical application on a large sample of firms listed in Paris Stock exchange. Finally, section 6 summarizes the key results.

2. DEA game cross-efficiency evaluation and portfolio efficiency

Portfolio selection is the logical consequences of the investor's attitudes towards information concerning stocks. Choosing between stocks puts them in competition in the eyes of investors. To consider the factor of competition in ranking different assets , we use the DEA game cross-efficiency approach of Liang et al. (2008a). In fact, ranking different stocks in a more efficient manner may obviously enrich the decision aids.

Given a set of n DMUs where a $DMU_j\,(j = 1, 2, ..., n)$ that utilizes a set of m inputs $x_{ij}\,(i = 1, 2, ..., m)$ to produce s outputs $y_{rj}\,(r = 1, 2, ..., s)$ where $x_{ij}, y_{rj} \geq 0$, the standard input-oriented Charnes et al. (1978) model (CCR), for any given DMU_d under evaluation, in linear format, can be represented as follow:

$$
\begin{aligned}
&Max \;\; E_{dd} = \sum_{r}^{s} \mu_r y_{rd} \\
&s.t. \quad \sum_{r}^{s} \mu_r y_{rj} - \sum_{i}^{m} \omega_i x_{ij} \leq 0 \quad j = 1,\ldots,n \\
&\qquad\;\; \sum_{i}^{m} \omega_i x_{id} = 1 \\
&\qquad\;\; \omega_i \geq 0 \quad i = 1,\ldots,m \\
&\qquad\;\; \mu_r \geq 0 \quad r = 1,\ldots,s
\end{aligned}
\tag{3.1}
$$

Where μ_r and ω_i are the set of output and input weights, respectively to be determined through solving the above model. Upon solving this model, an efficiency score of DMU_d is obtained from which a cross-efficiency score E_{dj} for each of the other $(n-1)$ DMUs will be determined based on DMU_d ‘s optimal weights.

$$
E_{dj} = \frac{\sum_{r}^{s} \mu_r^d y_{rj}}{\sum_{i}^{m} \omega_i^d x_{ij}}, \quad j = 1, 2, \ldots, (n-1) \tag{3.2}
$$

Where d denote the optimal weights of DMU_d . Finally, each DMU’s cross-efficiency score $\bar{E}_j$ will be determined through averaging its peer ratings as follows:

$$
\bar{E}_j = \frac{1}{n} \sum_{d}^{n} E_{dj} \tag{3.3}
$$

DEA gives much flexibility to each DMU. Indeed, letting each DMU choose its own set of weights will actually lead to unrealistic weights scheme. In fact, the DMU under evaluation heavily weighs few favorable inputs/outputs and completely ignores the other to maximize its own performance score. Under a Multi Criteria Decision Making (MCDM) context and more specifically portfolio selection, this can be a serious problem. Furthermore, under an MCDM context the weights are better determined exogenously. Hence, each DMU is subjected to risk change in weights given the surrounding environment (Lim et al. (2014)). This consideration justifies the use of cross-efficiency evaluation. However, the latter method suffers from major shortcoming residing in the non-uniqueness of weights depending on the used software (Despotis (2002)). For this reason, the use of game cross-efficiency is justified as it provides more stable weights and a Nash equilibrium evaluation score. The game cross-efficiency model was developed by (Liang et al. (2008a)) through the addition of a second goal to the basic DEA model.

Given an agent DMU_d with an efficiency score α_d, the other player DMU_j tries to select a set of strategies (weights selection) to maximize its own efficiency while ensuring that α_d would not decrease. Formally we have,

$$\begin{aligned}
& Max \ \sum_{r}^{s} \mu_{rj}^{d} y_{rj} \\
& s.t. \quad \sum_{r}^{s} \mu_{rj}^{d} y_{rl} - \sum_{i}^{m} \omega_{ij}^{d} x_{il} \leq 0 \quad , l = 1, \ldots, n \\
& \quad \sum_{i}^{m} \omega_{ij}^{d} x_{ij} = 1 \\
& \quad \alpha_d \times \sum_{i}^{m} \omega_{ij}^{d} x_{id} - \sum_{r}^{s} \mu_{rj}^{d} y_{rd} \leq 0 \\
& \quad \mu_{rj}^{d} \geq 0 \quad r = 1, \ldots, s \\
& \quad \omega_{ij}^{d} \geq 0 \quad i = 1, \ldots, m
\end{aligned} \tag{3.4}$$

We note that $\alpha_d \leq 1$ initially takes the value $\bar{E}_d$ from (3.3). It is the average cross-efficiency of DMU_d, when the algorithm converges, this α_d becomes the game cross-efficiency. In addition, the constraint $\alpha_d \times \sum_{i}^{m} \omega_{ij}^{d} x_{id} - \sum_{r}^{s} \mu_{rj}^{d} y_{rd} \leq 0$ in model (3.4) is equivalent to $\sum_{r}^{s} \mu_{rj}^{d} y_{rd} / \sum_{i}^{m} \omega_{ij}^{d} x_{id} \geq \alpha_d$ which implies the restriction of DMU_d initial score to ensure that it would not deteriorate. The above model is solved once for each DMU_d and hence n times. In addition, the optimal value of model (4) will represent a game cross-efficiency with respect to DMU_d. In fact, the average game cross-efficiency score for DMU_j would be $\alpha_j = \frac{1}{n} \sum_{d}^{n} \sum_{r}^{s} \mu_{rj}^{d*}(\alpha_d) y_{rj}$ where $\mu_{rj}^{d*}(\alpha_d)$ is an optimal solution of model (3.4).

Liang et al. (2008b) present the steps to determine Nash equilibrium efficiency score. In the following algorithm, α_j^t represents the efficiency of DMU_j at iteration *t*.

Require: ε

Step 1. Set t=1. For each DMU_d. Calculate the average cross-efficiency $\bar{E}_d$ and set $\alpha_d^t = \bar{E}_d \quad \forall d \in \{1,...,n\}$.

Step 2. For each pair of DMUs d and j, solve model (4) and obtain $\bar{E}_{dj}$.

Step 3. Set $\alpha_d^{t+1} = \frac{1}{n}\sum_{d}^{n} E_{dj}$

Step 4. If for some d, $\left|\alpha_d^{t+1} - \alpha_d^t\right| > \varepsilon$, then return to step 2. Otherwise α_d^{t+1} is the optimum game cross-efficiency of DMU_d and the algorithm stops.

The game cross-efficiency determined by solving the proposed algorithm above is a Nash equilibrium point of the DEA game. Thus, it presents a stable solution. Therefore, the results and decisions based upon game cross-efficiency analysis are reliable.

3. The maverick index: a consistent measure of risk to financial assets

An effective way to measure the false positiveness of financial assets is by computing the maverick index. Developed by Doyle and Green (1994b), the index measures the deviation between the DMU self-appraised score and its peer-appraised score. The higher the value of the index, the more the financial asset is considered as maverick. The maverick index score can fit the benchmarking process, by which the stocks considered efficient under self-evaluation but fail to appear in the reference sets of inefficient stocks will mostly achieve a high maverick index value. The stocks achieving a low index are in general all-round performers and are frequently both self and peer efficient. Lim et al. (2014) show that because the variances of mavericks' cross-efficiency are very likely to be significantly large, mavericks units should not be selected in a portfolio. Under game cross-efficiency evaluation, all players (DMUs) are assigned their Nash equilibrium scores. While DMUs are highly ranked due to their good performance on all measures, they are not always the least maverick. Here comes the advantage of the maverick index to analyze performance of financial assets. Considering the game cross-efficiency framework, the maverick index measures the relative difference between DEA self-evaluation and the Nash equilibrium score. We define the maverick index score M_j of DMU_j as follows:

$$M_j = \frac{\left(CCR_j - \alpha_j\right)}{\alpha_j} \tag{3.5}$$

Where CCR_j is the self-appraisal score and α_j is the relevant optimum game cross-efficiency of DMU_j.

With the cross evaluation approach, we are in what might be a democratic vote, where a set of factors is voted to be of high importance by the majority of DMUs while the rest are of low importance. Indeed, a high maverick index M_j of DMU_j represents a higher deviation from the equilibrium efficiency score, which is an extreme case representing a highly performing DMU only on few of the agreed upon factors. In other words, the least mavericks are those DMUs who perform well on most of the agreed upon best factors; but what about those who perform extremely well on few of the agreed upon best factors?

In fact, some of those can reach top players with moderately high efficiency score due to that high level of focus on some of the agreed upon factors. These DMUs when self-evaluating themselves, they set some pretty high weights on few factors of the voted best factors (the population evaluation standards). A typical maverick that will reach the top players with a high score would then outweigh very few criteria, which make its ranking and score very sensitive to the changes in the environment. Such sensitivity would make DMUs with high maverick values strongly risk intense. In addition, we can conclude that the maverick index can measure the risk level and variation in the input and output space. For this reason, we can propose the maverick index of a DMU as a measure of its risk degree with respect to changes in weights. The higher the maverick value of a portfolio, the more risky is the portfolio.

We have noted that the round player with the highest game cross-efficiency (or simple cross- efficiency score) does not have the least risk level (maverick score). Indeed, this can be troublesome in the context of portfolio selection as it may result in a portfolio that is highly sensitive to the environment changes. This, motivates our development of a Mean-Maverick (MM) framework of portfolio selection based on DEA game cross-efficiency evaluation. In this context, we seek to minimize the overall risk degree. This will be detailed in the subsequent section.

4. A Mean-Maverick framework of portfolio selection based on game cross-efficiency evaluation

Any transaction with an element of uncertainty as to its future outcome carries an element of risk which is related to uncertainty, volatility and complexity. In the development framework, the measure for risk is defined in terms of sensitivity to the volatility or change in the environment. The portfolio is therefore selected to minimize the portfolio risk subject to a given level of efficiency.

As demonstrated in the previous section, while the simple use of DEA game cross-efficiency approach can result in moderately risk-consistent portfolio, especially for individual risks, it does not take into account the risk level of each DMU. To address this issue of tradeoff between efficiency and risk degree, we develop a MM framework of portfolio selection based on game cross-efficiency. To perform portfolio selection, we assume that there is no stock which is combination of other stocks, no taxes, no transaction costs, and no short sales. We also assume perfect liquidity and the assumption about imperfect correlation of stocks.

Given a stock j, the efficiency and risk degree characteristics are defined as its game cross-efficiency score α_j and its maverick index score M_j, respectively.

Let a portfolio Ω of n stocks being equally weighted. The vector $\boldsymbol{w}(w_1\ w_2 ... w_n)$ is the vector of all ones where $w_j = 1\ \ if\ \ j \in \Omega$, 0 otherwise (when the stock is not selected in the portfolio). Let K represents the size of the desired portfolio where $\sum_{j}^{n} w_j = K$. In our analysis, we choose $K = 30$ stocks. The efficiency is the weighted average efficiency of the assets that comprises the portfolio $E_\Omega = \frac{1}{K}\sum_{j=1}^{n} w_j \alpha_j$ and the risk degree characteristic of portfolio is defined as the weighted average risk of the shares forming the portfolio $I_\Omega = \frac{1}{K}\sum_{j=1}^{n} w_j M_j$.

An optimal portfolio Ω^* is determined by solving the following linear optimization model:

$$
\begin{aligned}
&\underset{w_j}{Min}\ \ I_\Omega \\
&s.t.\quad E_\Omega \geq (1-\gamma) E_m \\
&\qquad \sum_{j=1}^{n} w_j = K \\
&\qquad w_j \in \{0,1\} \quad j = 1,...,n
\end{aligned}
\tag{3.6}
$$

Where γ is the return-risk tradeoff parameter and E_m represents the maximum achievable efficiency, that is the highest game cross-efficiency value detected among stocks in the sample. The simple use of game cross-efficiency evaluation for portfolio selection can be effective in reducing the change risk. However, it fails to consider its intensity level. In contrast, our approach based on model (3.6) can reduce both parts, resulting in a consistent tradeoff between risk degree and efficiency. Model (3.6) minimizes the portfolio risk degree (the level of sensitivity to changes in the environment) while imposing a lower bound on the portfolio efficiency. To prove the empirical effectiveness of the above-described model, we report a case study involving firms from the Paris stock Exchange.

5. An application to stock portfolio selection in the Paris stock Exchange

5.1 Data and input/output variables

We have illustrated empirically the above models on data of the Paris stock exchange. We have considered the monthly returns of $500 \sim 508$ listed firms' stocks[20]. We have used actual financial data from 2010 to 2015. To form the sample, we have included only those firms without missing values in the sample. Data have been collected from the Euronext Paris website. We use the current closing price that are indicated at the first and the last day of each month to compute monthly returns. The return of the share i in the month t is calculated with the following formula:

$$R_{it} = \frac{P_{it} - P_{it-1}}{P_{it-1}} \tag{3.7}$$

Where P_{it} is the current closing price of the last day in the month and P_{it-1} is the current closing price of the first day in the month.

In the first part of the empirical analysis, we try to select portfolios using DEA, DEA cross-efficiency, DEA game cross-efficiency and Mean-Maverick game cross-efficiency frameworks. Then, we evaluate the performance of the obtained portfolios. Thirdly, we examine the diversification level of the Mean-Maverick portfolio. In the last part of the analysis, we prove the robustness of our developed approach from volatility-return perspective.

[20] The sample changes from year to year beacause the entry, exit and survival of firms listed on the Paris Stock exchange.

DEA has received much attention in portfolio area for modeling preferences of investor. To define the financial efficient frontier in portfolio performance evaluation, several inputs and outputs have been proposed in the literature. Using the efficiency ratio, each share will seek to maximize its desirability for investment through weighting different attributes (inputs and outputs). We cast the problem of portfolio selection in a financial production process (Tarnaud and Leleu (2017)) by taking the criteria to be minimized as inputs and those to be maximized as outputs. For this reason, we will model the different attributes as investor preference or aversion. We use the four first moments of the returns ‘distribution as inputs and outputs. Following Scott and Horvath (1980), Kimball (1990), Kimball (1991) and Briec et al. (2007), a rational decision maker prefers odd moments (mean and skewness) and dislike even ones. On the one hand, an investor prefers maximize gain, approximated by the arithmetic mean return (Markowitz (1952)). A positive skewness is preferred by the investor since it implies a low probability of obtaining large negative returns (Crainich and Eeckhoudt (2008)). On the other hand, he seeks to minimize the volatility approximated by the variance (Pratt (1964)). And as shown by (Menezes and Wang (2005)), the investor must be averse to kurtosis in terms of transfer of actuarially neutral noise from the center of a distribution to its tail. Indeed, when data has more peakdness than the normal distribution (long tails), kurtosis is greater than three. While in case we have lower peak we have platy kurtosis (bounded distribution). For practical reasons, the excess kurtosis is used in this work to examine historical returns of a stock (West et al. (1995)). In order to evaluate portfolio adopting DEA model, several studies adopted the use of higher order moments to define inputs and outputs (see for example Gregoriou et al. (2005b), Joro and Na (2006) and Nguyen-Thi-Thanh (2006) among others)).

Considering the monthly return R_{it} of a stock i in the month t, $(t = 1...T)$, we present computation details of inputs and outputs per year in Table 3.1.

Table 3.1: Inputs/outputs matrix

Inputs	**Outputs**
Input1: Variance = $\sigma_i^2 = \frac{1}{T}\sum_{t=1}^{T}\left(R_{it} - \bar{R}_i\right)^2$	Output1: The annual mean return = $\bar{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it}$
Input 2: Kurtosis = $\frac{1}{T}\sum_{t=1}^{T}\left(\frac{R_{it} - \bar{R}_i}{\sigma_i}\right)^4 - 3$	Output 2: Skewness= $\frac{1}{T}\sum_{t=1}^{T}\left(\frac{R_{it} - \bar{R}_i}{\sigma_i}\right)^3$

In Table 3.2, we report the descriptive statistics for the four input/output variables. The variance variable has an average between 0.01170 in 2012 and 0.02802 and in 2010, with a minimum of about 10^{-7} and maximum of about 8.09. The average kurtosis goes from 0.69328 in 2010 to 1.07229 in 2014. The minimum kurtosis is about -2.06950. The maximum kurtosis is about 11.95180. Regarding the outputs, the mean variable has an average between -0.00920 in 2011 and 0.02041 in 2013, with a minimum of about -0.21863 and a maximum of about 1.32518. The average skewness is between 0.17172 in 2011 and 0.48392 in 2015, with a minimum of about -3.43831and a maximum of about 3.45448.

Table 3.2: Descriptive Statistics

Years		2010	2011	2012	2013	2014	2015
Number of firms		505	508	500	500	502	501
Input 1: Variance	Mean	0.02802	0.01417	0.01170	0.01370	0.01353	0.01429
	SD	0.36103	0.03445	0.01645	0.04856	0.03348	0.03354
	Median	0.00605	0.00677	0.00632	0.00449	0.00549	0.00557
	Min	0.00007	0.00020	0.000001	0.00001	6.19261	0.00005
	Max	8.09060	0.48306	0.16850	0.78392	0.43569	0.34032
Input 2: Kurtosis	Mean	0.69328	0.82682	0.88967	0.83699	1.07229	0.83599
	SD	2.04623	2.03163	2.10029	2.12155	2.33403	2.16771
	Median	0.13780	0.28155	0.28374	0.17542	0.35323	0.17618
	Min	-1.86712	-1.94858	-1.73831	-1.91877	-1.83729	-2.06950
	Max	11.95180	11.58814	11.87129	10.34938	11.94657	10.20691
Output 1: Mean return	Mean	0.00990	-0.00920	0.00636	0.02041	0.01107	0.01409
	SD	0.05037	0.03753	0.03432	0.07296	0.03811	0.04000
	Median	0.00704	-0.01048	0.00709	0.01283	0.00935	0.01187
	Min	-0.19268	-0.20662	-0.16219	-0.21808	-0.18535	-0.21863
	Max	0.75259	0.28533	0.28464	1.32518	0.40305	0.24052
Output 2: Skewness	Mean	0.30745	0.17172	0.23793	0.33719	0.46271	0.48392
	SD.	0.87636	0.90296	0.93395	0.90963	0.92997	0.85872
	Median	0.26992	0.12013	0.15933	0.29090	0.40561	0.38977
	Min	-2.86149	-2.66648	-3.43831	-2.70227	-2.45524	-1.98872
	Max	3.45448	3.38537	3.17121	3.11380	3.45373	3.10527

To estimate scores efficiency we use the model (1), which is the CCR model. The usage of the distribution moments leads to the rise of negative values both in the inputs and outputs. Considering that DEA model cannot be used with negative data, it would be appropriate to use data transformation. Indeed, As developed by Charnes et al. (1978), the CCR model requires strict positivity of all input and output values, given that a CRS assumption imposes that any movement can be radially expanded or contracted to form other feasible movement and thereafter any proportion of an efficient unit must be also efficient (Portela et al. (2004). Applying data transformation when CCR model are used, would result in altering the efficiency values. However, in the context of our analysis we seek a benchmarking DEA approach among the stocks. Therefore, based on the translation invariance property in DEA (Ali and Seiford (1990)) and since the efficiency classification is persevered, portfolio construction would not be affected. In order to deal with negative data, we use the following formula to transform negative inputs and outputs to positive ones:

$$W^k = V^k + \left|\min V_j^k, j = 1,...,n\right| \qquad (3.8)$$

Where W^k is the transformed variable of input or output k, V^k is an input or output variable and $\min V_j^k$ is the smallest input/output value.

5.2 Mean-Maverick game cross-efficiency selection strategy and empirical results

In order to construct the different portfolios, we consider a buy-and-hold strategy. The constructed portfolio is held for an investment horizon of one year and revised (new stock selection based on model solutions) for each new investment horizon. For each investment horizon, we choose the top 30 stocks (about 6% of the sample) to form the portfolio the AP and GCP portfolios. Thus, the portfolio size is fixed at *K*=30 with equally weighted stocks. More specifically, at the beginning of each investment horizon the set of stocks will be selected through solving models already mentioned. The arbitrary portfolio (AP) is the portfolio selected using DEA cross-efficiency method (model 3.1 - 3.3). The game cross portfolio (GCP) is selected through solving the DEA game cross-efficiency model (model 3.4). The Mean-Maverick portfolio (MM) is the portfolio selected using the developed Mean-Maverick DEA game cross-efficiency model (model 3.6). To test the sensitivity of the results to investor' attitude, we examine the MM model for different return-risk tradeoff parameter values $\gamma \in \{5\%, 10\%, 15\%, 20\%, 25\%, 30\%\}$. Once a portfolio is selected, we assume that the same dollar amount will be invested in each of the stocks constituting the portfolio with no more transaction to be made until the end of the investment horizon. This strategy will imply that investment cost will only be incurred only at the end of each investment horizon.

Table 3.3 shows the AP portfolio composition during the whole period of study and the cross-efficiency scores of the best performers shares. Results highlight the discrimination power of cross-efficiency method. A unique order of firms is given. Firms are ranked in a descending order of performance. EUTELSAT COMMUNIC. (0.9736), SES (0.9563), L'OREAL (0.9575), EULER HERMES GROUP (0.979), MERCK AND CO INC (0.975) and ALTAREIT (0.9788) are the best performers for the period 2010 to 2015 respectively. The worst practices stocks are BAINS MER MONACO (0.4701), KORIAN (0.4911), DANONE (0.2447), INTERPARFUMS (0.4193), EULER HERMES GROUP (0.4779) and FONCIERE DE PARIS (0.3755) for the years 2010-2015 respectively. The average cross-efficiency score goes from 0.4043 in 2012 to 0.6840 in 2011.

However, using the DEA game cross-efficiency, the average efficiency score is between 0.4335 in 2012 and 0.7332 in 2011. The best performers firms are similar to those found with the AP except for the year 2011. MAROC TELECOM is the best practitioner with game cross-efficiency score equal to 0.9975, followed by SES (0.9966). FDL (0.5521), ANF IMMOBILIER (0.539), UNIBAIL-RODAMCO (0.261), IRDNORDPASDECALAIS (0.4781), ARCELORMITTAL (0.5487) and FIDUCIAL OFF.SOL. (0.4349) are the worst performers in the selected GCP portfolio during the period from 2011 to 2015 respectively. GCP composition and the cross-efficiency scores are presented in Table 3.4.

Tables 3.5 to 3.10 present the selected MM portfolios, the game cross-efficiency scores and maverick index scores when $\gamma \in \{5\%, 10\%, 15\%, 20\%, 25\%, 30\%\}$ respectively. Portfolios include the 30 best performers shares. The $MM\,(\gamma = 5\%)$ portfolio has average game cross-efficiency scores equal to 0.68, 0.7, 0.41, 0.58, 0.63 and 0.51 and average maverick scores equal to 0.22, 0.03, 0.02, 0.1, 0.12 and 0.2. At $\gamma = 10\%$ level, the highest average game cross-efficiency score (0.66) and the smallest maverick score (0.02) are recorded in 2011. The $MM\,(\gamma = 15\%)$ portfolio has the highest average game cross-efficiency score (0.62) and the smallest maverick score (0.01) in 2011. Similarly for the MM portfolio at 20%, 25% and 30% return-risk tradeoff parameter level, the greatest efficiency score and the smallest maverick score are obtained in 2011. Whatever the return-risk tradeoff parameter value, the smallest average efficiency score is recorded in 2012. However, the greatest average efficiency score is obtained in 2011.

Table 3.3: AP portfolio selection and DEA cross-efficiency scores

SHARES	2010	SHARES	2011	SHARES	2012	SHARES	2013	SHARES	2014	SHARES	2015
EUTELSAT COMMUNIC.	0.9736	SES	0.9563	L'OREAL	0.9575	EULER HERMES GROUP	0.979	MERCK AND CO INC	0.975	ALTAREIT	0.9788
DANONE	0.9024	MAROC TELECOM	0.9495	IVALIS	0.9557	MAROC TELECOM	0.7624	DANONE	0.8362	CFAO	0.7314
DASSAULT AVIATION	0.8661	TIPIAK	0.946	CNIM CONSTR.FRF 10	0.7365	TURENNE INV	0.7453	BNP PARIBAS ACT.A	0.7291	CRCAM NORM.SEINE	0.5922
ESSO	0.8525	FDL	0.8894	SABETON	0.7353	VIEL ET COMPAGNIE	0.6872	LE NOBLE AGE	0.7256	CHARGEURS	0.5718
L'OREAL	0.7493	ESSILOR INTL.	0.8763	VILMORIN	0.6828	HI-MEDIA	0.6616	SIGNAUX GIROD	0.6738	SELECTIRENTE	0.5378
ST DUPONT	0.7053	CA TOULOUSE 31 CCI	0.8584	SOFRAGI	0.6793	RUBIS	0.6099	SABETON	0.6561	CHAUF.URB.	0.525
FREY	0.7029	BONDUELLE	0.8575	ZODIAC AEROSPACE	0.4392	VETOQUINOL	0.6008	PAREF	0.6457	INSTALLUX	0.5138
RAMSAY GEN SANTE	0.6968	EUROSIC	0.854	TFF GROUP	0.3554	CIC	0.5784	EXEL INDUSTRIES	0.6305	PRECIA	0.4755
ESSILOR INTL.	0.6823	SODEXO	0.7551	BERNARD LOISEAU	0.3551	QUANTEL	0.57	CRCAM TOURAINE CCI	0.6292	CRCAM LANGUED CCI	0.4686
SELECTIRENTE	0.6576	BOLLORE	0.725	LEBON	0.353	PROCTER GAMBLE	0.563	NETBOOSTER	0.6081	LINEDATA SERVICES	0.4587
STEF	0.6364	EUTELSAT COMMUNIC.	0.716	MERCIALYS	0.3469	LEGRAND	0.5572	GROUPE EUROTUNNEL	0.6012	FLEURY MICHON	0.4584
MERCK AND CO INC	0.6342	BIC	0.7153	DASSAULT SYSTEMES	0.3422	UNIBAIL-RODAMCO	0.5434	SOFRAGI	0.5986	SAINT GOBAIN	0.4451
BIC	0.6321	HOPSCOTCH GROUPE	0.7134	SES	0.3406	EUROGERM	0.5397	ALES GROUPE	0.5859	COURTOIS	0.4326
FONCIERE EURIS	0.5942	ITS GROUP	0.7061	GAMELOFT SE	0.3308	SELECTIRENTE	0.5322	CFAO	0.585	EUROMEDIS GROUPE	0.4302
ROTHSCHILD	0.5675	PARFEX	0.6581	BRICORAMA	0.3245	AXA	0.5247	MONTEA C.V.A.	0.5771	SMTPC	0.4279
PLANT ADVANCED	0.5448	AGTA RECORD	0.649	INSTALLUX	0.3192	AIR LIQUIDE	0.5178	BIOMERIEUX	0.5771	EFESO CONSULTING	0.4238
CATERING INTL SCES	0.5419	AIR LIQUIDE	0.6256	SARTORIUS STED BIO	0.3161	ATOS	0.5109	ADL PARTNER	0.5731	VEOLIA ENVIRON.	0.4222
VIEL ET COMPAGNIE	0.5353	PLANT ADVANCED	0.619	PERRIER (GERARD)	0.3138	HOPSCOTCH GROUPE	0.5091	TFF GROUP	0.5596	TURENNE INV	0.4208
CA TOULOUSE 31 CCI	0.5342	SANOFI	0.6181	STEF	0.3103	SCOR SE	0.5037	BIC	0.5405	CRCAM SUD R.A.CCI	0.4145
IRDNORDPASDECALAIS	0.5238	MONTEA C.V.A.	0.5799	ADL PARTNER	0.3032	AFFINE R.E.	0.4997	DASSAULT SYSTEMES	0.5313	NEURONES	0.4115
FROMAGERIES BEL	0.5119	MR BRICOLAGE	0.5664	TRILOGIQ	0.2814	GROUPE PARTOUCHE	0.4993	BERNARD LOISEAU	0.5294	LAFUMA	0.4109
GRAND MARNIER	0.5066	SCBSM	0.5632	HOTELS DE PARIS	0.2809	CRCAM ATL.VEND.CCI	0.4708	SELECTIRENTE	0.5241	EURAZEO	0.406
SANOFI	0.5045	IGE XAO	0.5323	SIDETRADE	0.2798	BOURBON	0.4683	COURTOIS	0.5219	SCBSM	0.4042
KORIAN	0.4971	RADIALL	0.5304	RAMSAY GEN SANTE	0.2701	NATUREX	0.4616	HSBC HOLDINGS	0.5159	SALVEPAR	0.3902
FONCIERE INEA	0.4909	TFF GROUP	0.5275	ENVIRONNEMENT SA	0.2696	NETBOOSTER	0.4513	PROCTER GAMBLE	0.5138	ATOS	0.3839
SOMFY SA	0.4837	TURENNE INV	0.5253	ICADE	0.2623	VDI GROUP	0.4451	ESPERITE	0.5035	PATRIMOINE ET COMM	0.3824
TURENNE INV	0.478	O2I	0.5239	BIC	0.2509	FONCIERE LYONNAISE	0.4378	SIMO INTERNATIONAL	0.4911	GROUPE EUROTUNNEL	0.3814

TOUPARGEL GROUPE	0.4755	MERCIALYS	0.501	BONDUELLE	0.2492	GIORGIO FEDON	0.4342	SAFT	0.4889	TERREIS	0.3793
GROUPE FLO	0.4744	RUBIS	0.4925	PRODWARE	0.2453	ESSO	0.4259	CASINO GUICHARD	0.4788	THERMADOR GROUPE	0.376
BAINS MER MONACO	0.4701	KORIAN	0.4911	DANONE	0.2447	INTERPARFUMS	0.4193	EULER HERMES GROUP	0.4779	FONCIERE DE PARIS	0.3755
MEAN	**0.6141**	**MEAN**	**0.6840**	**MEAN**	**0.4043**	**MEAN**	**0.55032**	**MEAN**	**0.5961**	**MEAN**	**0.4676**

Table 3.4: GCP portfolio selection and DEA game cross-efficiency scores

SHARES	2010	SHARES	2011	SHARES	2012	SHARES	2013	SHARES	2014	SHARES	2015
EUTELSAT COMMUNIC.	0.999	MAROC TELECOM	0.9975	L'OREAL	0.993	EULER HERMES GROUP	1	MERCK AND CO INC	1	ALTAREIT	1
DASSAULT AVIATION	0.9689	SES	0.9966	IVALIS	0.9918	MAROC TELECOM	0.8447	DANONE	0.9069	CFAO	0.8052
DANONE	0.9372	TIPIAK	0.9823	CNIM CONSTR.FRF 10	0.7985	TURENNE INV	0.8299	BNP PARIBAS ACT.A	0.824	CRCAM NORM.SEINE	0.6544
ESSO	0.9122	FDL	0.9468	SABETON	0.7843	VIEL ET COMPAGNIE	0.8093	LE NOBLE AGE	0.8238	CHAUF.URB.	0.6528
L'OREAL	0.8475	ESSILOR INTL.	0.9422	SOFRAGI	0.7544	HI-MEDIA	0.7232	SIGNAUX GIROD	0.783	CHARGEURS	0.6453
FREY	0.8365	BONDUELLE	0.9138	VILMORIN	0.7304	RUBIS	0.6736	EXEL INDUSTRIES	0.7515	PRECIA	0.6102
RAMSAY GEN SANTE	0.8283	CA TOULOUSE 31 CCI	0.8948	ZODIAC AEROSPACE	0.4704	VETOQUINOL	0.6512	SABETON	0.7219	SELECTIRENTE	0.6009
ESSILOR INTL.	0.7836	EUROSIC	0.89	TFF GROUP	0.3853	QUANTEL	0.643	PAREF	0.7202	INSTALLUX	0.6008
ST DUPONT	0.7745	SODEXO	0.7923	BERNARD LOISEAU	0.3796	EUROGERM	0.641	CRCAM TOURAINE CCI	0.7132	CRCAM LANGUED CCI	0.5616
BIC	0.7707	HOPSCOTCH GROUPE	0.7909	DASSAULT SYSTEMES	0.3699	UNIBAIL-RODAMCO	0.6238	NETBOOSTER	0.696	EUROMEDIS GROUPE	0.5429
PLANT ADVANCED	0.7608	AGTA RECORD	0.7688	MERCIALYS	0.3694	CIC	0.6146	SOFRAGI	0.6766	COURTOIS	0.5219
SELECTIRENTE	0.7453	BOLLORE	0.7563	LEBON	0.3693	PROCTER GAMBLE	0.6016	ADL PARTNER	0.6615	FLEURY MICHON	0.5089
STEF	0.7405	BIC	0.7516	SES	0.3603	AIR LIQUIDE	0.5899	GROUPE EUROTUNNEL	0.6562	TURENNE INV	0.5008
MERCK AND CO INC	0.7147	PLANT ADVANCED	0.7474	STEF	0.3603	LEGRAND	0.5884	CFAO	0.6492	BRICORAMA	0.4976
IRDNORDPASDECALAIS	0.7126	EUTELSAT COMMUNIC.	0.7469	GAMELOFT SE	0.3505	GROUPE PARTOUCHE	0.5833	MONTEA C.V.A.	0.6434	VEOLIA ENVIRON.	0.4959
ROTHSCHILD	0.6679	ITS GROUP	0.7374	BRICORAMA	0.3496	NETBOOSTER	0.5736	ALES GROUPE	0.6387	MALTERIES FCO-BEL.	0.4897
BAINS MER MONACO	0.6638	PARFEX	0.7086	INSTALLUX	0.3392	SCOR SE	0.5694	BIOMERIEUX	0.6217	SCBSM	0.4872
FONCIERE EURIS	0.6633	AIR LIQUIDE	0.6542	SARTORIUS STED BIO	0.338	AXA	0.5556	TFF GROUP	0.6166	SMTPC	0.4828
FONCIERE INEA	0.6632	SANOFI	0.645	PERRIER (GERARD)	0.3335	ATOS	0.5547	BERNARD LOISEAU	0.6077	LINEDATA SERVICES	0.4787
TURENNE INV	0.6118	MONTEA C.V.A.	0.633	ADL PARTNER	0.3286	SELECTIRENTE	0.5537	SELECTIRENTE	0.5895	EFESO CONSULTING	0.478
KORIAN	0.6012	MR BRICOLAGE	0.6093	SIDETRADE	0.3113	HOPSCOTCH GROUPE	0.5537	COURTOIS	0.5886	SAINT GOBAIN	0.4734

CATERING INTL SCES	0.5989	TFF GROUP	0.5895	TRILOGIQ	0.3031	AFFINE R.E.	0.5469	DASSAULT SYSTEMES	0.5826	CRCAM SUD R.A.CCI	0.4719
VIEL ET COMPAGNIE	0.5946	SCBSM	0.5877	HOTELS DE PARIS	0.3008	BOURBON	0.5371	BIC	0.5798	LAFUMA	0.4688
FROMAGERIES BEL	0.5928	TURENNE INV	0.5823	RAMSAY GEN SANTE	0.2928	NATUREX	0.5215	ESPERITE	0.5788	NEURONES	0.4618
CA TOULOUSE 31 CCI	0.5914	IGE XAO	0.5811	ICADE	0.2891	VDI GROUP	0.5155	PROCTER GAMBLE	0.5774	EURAZEO	0.4617
IGE XAO	0.5899	RADIALL	0.5565	ENVIRONNEMENT SA	0.2846	FONCIERE LYONNAISE	0.5134	SAFT	0.5665	SALVEPAR	0.4615
GRAND MARNIER	0.5883	O2I	0.5549	BIC	0.2709	CRCAM ATL.VEND.CCI	0.5077	SIMO INTERNATIONAL	0.5661	PATRIMOINE ET COMM	0.4434
TOUPARGEL GROUPE	0.5557	MGI DIGITAL GRAPHI	0.5533	BONDUELLE	0.268	ESSO	0.4919	CASINO GUICHARD	0.5498	CAPELLI	0.4401
TOUAX	0.5543	MERCIALYS	0.5489	DANONE	0.2672	MEDIA 6	0.4799	HSBC HOLDINGS	0.5493	ATOS	0.4356
FDL	0.5521	ANF IMMOBILIER	0.539	UNIBAIL-RODAMCO	0.261	IRDNORDPASDECALAIS	0.4781	ARCELORMITTAL	0.5487	FIDUCIAL OFF.SOL.	0.4349
MEAN	**0.71405**	**MEAN**	**0.7332**	**MEAN**	**0.4335**	**MEAN**	**0.6123**	**MEAN**	**0.6663**	**MEAN**	**0.5389**

Table 3.5: $MM\ (\gamma = 5\%)$ **portfolio selection, game cross-efficiency and mavericks scores**

2010	GAM	MAV	2011	GAM	MAV	2012	GAM	MAV	2013	GAM	MAV	2014	GAM	MAV	2015	GAM	MAV
EUTELSAT COMMUNIC.	1,00	0,03	SES	1,00	0,00	L'OREAL	0,99	0,01	EULER HERMES GROUP	1,00	0,00	MERCK AND CO INC	1,00	0,00	ALTAREIT	1,00	0,00
DANONE	0,94	0,05	MAROC TELECOM	1,00	0,00	IVALIS	0,99	0,01	MAROC TELECOM	0,84	0,18	DANONE	0,91	0,06	CFAO	0,81	0,24
DASSAULT AVIATION	0,97	0,15	TIPIAK	0,98	0,02	CNIM CONSTR.FRF 10	0,80	0,03	TURENNE INV	0,83	0,20	BNP PARIBAS ACT.A	0,82	0,07	CRCAM NORM.SEINE	0,65	0,15
ESSO	0,91	0,09	FDL	0,95	0,06	SABETON	0,78	0,01	VIEL ET COMPAGNIE	0,81	0,12	LE NOBLE AGE	0,82	0,21	CHARGEURS	0,65	0,55
L'OREAL	0,85	0,19	ESSILOR INTL.	0,94	0,06	VILMORIN	0,73	0,02	HI-MEDIA	0,72	0,15	SIGNAUX GIROD	0,78	0,17	SELECTIRENTE	0,60	0,23
ST DUPONT	0,77	0,42	CA TOULOUSE 31 CCI	0,89	0,01	SOFRAGI	0,75	0,05	RUBIS	0,67	0,12	SABETON	0,72	0,24	CHAUF.URB.	0,65	0,53
FREY	0,84	0,34	BONDUELLE	0,91	0,09	ZODIAC AEROSPACE	0,47	0,05	VETOQUINOL	0,65	0,08	EXEL INDUSTRIES	0,75	0,19	INSTALLUX	0,60	0,33
RAMSAY GEN SANTE	0,83	0,36	EUROSIC	0,89	0,03	TFF GROUP	0,39	0,03	CIC	0,61	0,05	CRCAM TOURAINE CCI	0,71	0,10	PRECIA	0,61	0,48
ESSILOR INTL.	0,78	0,25	SODEXO	0,79	0,05	BERNARD LOISEAU	0,38	0,01	QUANTEL	0,64	0,26	NETBOOSTER	0,70	0,16	CRCAM LANGUED CCI	0,56	0,42
SELECTIRENTE	0,75	0,23	BOLLORE	0,76	0,01	LEBON	0,37	0,01	PROCTER GAMBLE	0,60	0,04	GROUPE EUROTUNNEL	0,66	0,20	LINEDATA SERVICES	0,48	0,01
STEF	0,74	0,28	EUTELSAT COMMUNIC.	0,75	0,01	MERCIALYS	0,37	0,01	LEGRAND	0,59	0,05	ALES GROUPE	0,64	0,05	FLEURY MICHON	0,51	0,19
MERCK AND CO INC	0,71	0,18	BIC	0,75	0,05	DASSAULT SYSTEMES	0,37	0,03	UNIBAIL-RODAMCO	0,62	0,31	CFAO	0,65	0,30	SAINT GOBAIN	0,47	0,07
BIC	0,77	0,42	HOPSCOTCH GROUPE	0,79	0,14	SES	0,36	0,01	SELECTIRENTE	0,55	0,02	BIOMERIEUX	0,62	0,16	SMTPC	0,48	0,25
FONCIERE EURIS	0,66	0,15	ITS GROUP	0,74	0,01	BRICORAMA	0,35	0,02	AXA	0,56	0,03	ADL PARTNER	0,66	0,20	EFESO CONSULTING	0,48	0,24

ROTHSCHILD	0,67	0,29	PARFEX	0,71	0,12	INSTALLUX	0,34	0,02	AIR LIQUIDE	0,59	0,17	BIC	0,58	0,01	VEOLIA ENVIRON.	0,50	0,35
CATERING INTL SCES	0,60	0,13	AIR LIQUIDE	0,65	0,01	SARTORIUS STED BIO	0,34	0,01	ATOS	0,55	0,09	DASSAULT SYSTEMES	0,58	0,10	CRCAM SUD R.A.CCI	0,47	0,21
CA TOULOUSE 31 CCI	0,59	0,16	SANOFI	0,65	0,01	PERRIER (GERARD)	0,33	0,03	HOPSCOTCH GROUPE	0,55	0,08	BERNARD LOISEAU	0,61	0,11	LAFUMA	0,47	0,18
FROMAGERIES BEL	0,59	0,29	MR BRICOLAGE	0,61	0,03	ADL PARTNER	0,33	0,03	SCOR SE	0,57	0,18	HSBC HOLDINGS	0,55	0,06	EURAZEO	0,46	0,14
GRAND MARNIER	0,59	0,28	SCBSM	0,59	0,01	TRILOGIQ	0,30	0,02	AFFINE R.E.	0,55	0,10	ESPERITE	0,58	0,12	ATOS	0,44	0,20
SANOFI	0,55	0,11	RADIALL	0,56	0,01	HOTELS DE PARIS	0,30	0,02	CRCAM ATL.VEND.CCI	0,51	0,09	CASINO GUICHARD	0,55	0,17	PATRIMOINE ET COMM	0,44	0,23
KORIAN	0,60	0,38	O2I	0,55	0,02	RAMSAY GEN SANTE	0,29	0,03	NATUREX	0,52	0,18	EULER HERMES GROUP	0,52	0,05	TERREIS	0,41	0,05
SOMFY SA	0,55	0,23	RUBIS	0,52	0,01	ENVIRONNEMENT SA	0,28	0,01	GIORGIO FEDON	0,46	0,02	ARCELORMITTAL	0,55	0,11	THERMADOR GROUPE	0,41	0,12
TOUPARGEL GROUPE	0,56	0,26	KORIAN	0,51	0,02	BONDUELLE	0,27	0,03	ESSO	0,49	0,13	VIRBAC	0,53	0,13	FONCIERE DE PARIS	0,42	0,19
GROUPE FLO	0,52	0,11	ARGAN	0,52	0,02	PRODWARE	0,26	0,01	INTERPARFUMS	0,45	0,07	JC DECAUX SA.	0,53	0,13	SYNERGIE	0,42	0,15
MEDICREA INTERNAT.	0,52	0,22	CRCAM LANGUED CCI	0,51	0,02	COFIDUR	0,25	0,02	EURO DISNEY	0,44	0,03	AKKA TECHNOLOGIES	0,50	0,06	ROBERTET	0,41	0,06
PHARMAGEST INTER.	0,50	0,11	MALTERIES FCO-BEL.	0,50	0,01	VALEO	0,22	0,01	IDI	0,45	0,07	CRCAM NORM.SEINE	0,51	0,06	MAKHEIA GROUP	0,41	0,12
PHILIP MORRIS INTL	0,53	0,29	ESPERITE	0,50	0,03	SI PARTICIPATIONS	0,20	0,01	CAP GEMINI	0,43	0,08	ORPEA	0,49	0,05	SES	0,40	0,04
HEURTEY PETROCHEM	0,51	0,24	IMPRIMERIE CHIRAT	0,48	0,01	OCTO TECHNOLOGY	0,20	0,01	FONCIERE EURIS	0,43	0,11	WENDEL	0,52	0,12	OBER	0,39	0,04
ALTAREA	0,49	0,15	EXPLOS.PROD.CHI.PF	0,47	0,01	RIBER	0,18	0,01	EIFFAGE	0,39	0,03	THALES	0,48	0,02	HARVEST	0,39	0,08
STORE ELECTRONICS	0,48	0,20	ROTHSCHILD	0,44	0,01	FREY	0,16	0,01	INFOTEL	0,36	0,03	ESI GROUP	0,46	0,04	GEVELOT	0,38	0,05
MEAN	**0,68**	**0,22**	**MEAN**	**0,70**	**0,03**	**MEAN**	**0,41**	**0,02**	**MEAN**	**0,58**	**0,10**	**MEAN**	**0,63**	**0,12**	**MEAN**	**0,51**	**0,20**

Table 3.6: $MM\ (\gamma = 10\%)$ **portfolio selection, game cross-efficiency and mavericks scores**

2010	GAM	MAV	2011	GAM	MAV	2012	GAM	MAV	2013	GAM	MAV	2014	GAM	MAV	2015	GAM	MAV
EUTELSAT COMMUNIC.	1,00	0,03	SES	1,00	0,00	L'OREAL	0,99	0,01	EULER HERMES GROUP	1,00	0,00	MERCK AND CO INC	1,00	0,00	ALTAREIT	1,00	0,00
DANONE	0,94	0,05	MAROC TELECOM	1,00	0,00	IVALIS	0,99	0,01	MAROC TELECOM	0,84	0,18	DANONE	0,91	0,06	CFAO	0,81	0,24
DASSAULT AVIATION	0,97	0,15	TIPIAK	0,98	0,02	CNIM CONSTR.FRF 10	0,80	0,03	TURENNE INV	0,83	0,20	BNP PARIBAS ACT.A	0,82	0,07	CRCAM NORM.SEINE	0,65	0,15
ESSO	0,91	0,09	FDL	0,95	0,06	SABETON	0,78	0,01	VIEL ET COMPAGNIE	0,81	0,12	LE NOBLE AGE	0,82	0,21	SELECTIRENTE	0,60	0,23
L'OREAL	0,85	0,19	ESSILOR INTL.	0,94	0,06	VILMORIN	0,73	0,02	HI-MEDIA	0,72	0,15	SIGNAUX GIROD	0,78	0,17	CHAUF.URB.	0,65	0,53
ST DUPONT	0,77	0,42	CA TOULOUSE 31 CCI	0,89	0,01	SOFRAGI	0,75	0,05	RUBIS	0,67	0,12	EXEL INDUSTRIES	0,75	0,19	INSTALLUX	0,60	0,33
FREY	0,84	0,34	EUROSIC	0,89	0,03	TFF GROUP	0,39	0,03	VETOQUINOL	0,65	0,08	CRCAM TOURAINE CCI	0,71	0,10	LINEDATA SERVICES	0,48	0,01
RAMSAY GEN SANTE	0,83	0,36	SODEXO	0,79	0,05	BERNARD LOISEAU	0,38	0,01	CIC	0,61	0,05	NETBOOSTER	0,70	0,16	FLEURY MICHON	0,51	0,19

ESSILOR INTL.	0,78	0,25	BOLLORE	0,76	0,01	LEBON	0,37	0,01	PROCTER GAMBLE	0,60	0,04	GROUPE EUROTUNNEL	0,66	0,20	SAINT GOBAIN	0,47	0,07
SELECTIRENTE	0,75	0,23	EUTELSAT COMMUNIC.	0,75	0,01	MERCIALYS	0,37	0,01	LEGRAND	0,59	0,05	ALES GROUPE	0,64	0,05	SMTPC	0,48	0,25
STEF	0,74	0,28	BIC	0,75	0,05	SES	0,36	0,01	SELECTIRENTE	0,55	0,02	BIOMERIEUX	0,62	0,16	EFESO CONSULTING	0,48	0,24
MERCK AND CO INC	0,71	0,18	ITS GROUP	0,74	0,01	BRICORAMA	0,35	0,02	AXA	0,56	0,03	ADL PARTNER	0,66	0,20	CRCAM SUD R.A.CCI	0,47	0,21
BIC	0,77	0,42	AIR LIQUIDE	0,65	0,01	INSTALLUX	0,34	0,02	AIR LIQUIDE	0,59	0,17	BIC	0,58	0,01	LAFUMA	0,47	0,18
FONCIERE EURIS	0,66	0,15	SANOFI	0,65	0,01	SARTORIUS STED BIO	0,34	0,01	ATOS	0,55	0,09	DASSAULT SYSTEMES	0,58	0,10	EURAZEO	0,46	0,14
ROTHSCHILD	0,67	0,29	MR BRICOLAGE	0,61	0,03	PERRIER (GERARD)	0,33	0,03	HOPSCOTCH GROUPE	0,55	0,08	BERNARD LOISEAU	0,61	0,11	ATOS	0,44	0,20
CATERING INTL SCES	0,60	0,13	SCBSM	0,59	0,01	ADL PARTNER	0,33	0,03	AFFINE R.E.	0,55	0,10	HSBC HOLDINGS	0,55	0,06	PATRIMOINE ET COMM	0,44	0,23
CA TOULOUSE 31 CCI	0,59	0,16	RADIALL	0,56	0,01	TRILOGIQ	0,30	0,02	CRCAM ATL.VEND.CCI	0,51	0,09	ESPERITE	0,58	0,12	TERREIS	0,41	0,05
GRAND MARNIER	0,59	0,28	O2I	0,55	0,02	HOTELS DE PARIS	0,30	0,02	GIORGIO FEDON	0,46	0,02	EULER HERMES GROUP	0,52	0,05	THERMADOR GROUPE	0,41	0,12
SANOFI	0,55	0,11	RUBIS	0,52	0,01	RAMSAY GEN SANTE	0,29	0,03	ESSO	0,49	0,13	ARCELORMITTAL	0,55	0,11	FONCIERE DE PARIS	0,42	0,19
SOMFY SA	0,55	0,23	KORIAN	0,51	0,02	ENVIRONNEMENT SA	0,28	0,01	INTERPARFUMS	0,45	0,07	AKKA TECHNOLOGIES	0,50	0,06	SYNERGIE	0,42	0,15
GROUPE FLO	0,52	0,11	ARGAN	0,52	0,02	PRODWARE	0,26	0,01	EURO DISNEY	0,44	0,03	CRCAM NORM.SEINE	0,51	0,06	ROBERTET	0,41	0,06
MEDICREA INTERNAT.	0,52	0,22	CRCAM LANGUED CCI	0,51	0,02	COFIDUR	0,25	0,02	IDI	0,45	0,07	ORPEA	0,49	0,05	MAKHEIA GROUP	0,41	0,12
PHARMAGEST INTER.	0,50	0,11	MALTERIES FCO-BEL.	0,50	0,01	VALEO	0,22	0,01	NEOPOST	0,44	0,11	WENDEL	0,52	0,12	SES	0,40	0,04
ALTAREA	0,49	0,15	IMPRIMERIE CHIRAT	0,48	0,01	SI PARTICIPATIONS	0,20	0,01	CAP GEMINI	0,43	0,08	THALES	0,48	0,02	OBER	0,39	0,04
EXPLOSIFS PROD.CHI	0,46	0,16	EXPLOS.PROD.CHI.PF	0,47	0,01	OCTO TECHNOLOGY	0,20	0,01	EIFFAGE	0,39	0,03	ESI GROUP	0,46	0,04	HARVEST	0,39	0,08
SOLVAY	0,41	0,13	ORCHESTRA PREMAMAN	0,47	0,02	RIBER	0,18	0,01	SWORD GROUP	0,37	0,03	NEURONES	0,43	0,06	GEVELOT	0,38	0,05
GROUPE OPEN	0,40	0,14	EUROGERM	0,46	0,02	FREY	0,16	0,01	VINCI	0,37	0,06	M.R.M	0,44	0,03	NEURONES	0,46	0,23
ESI GROUP	0,37	0,11	MANUTAN INTL	0,45	0,02	STREAMWIDE	0,16	0,01	INFOTEL	0,36	0,03	EUROFINS SCIENT.	0,41	0,04	ALTAMIR	0,36	0,06
MGI DIGITAL GRAPHI	0,31	0,07	ROTHSCHILD	0,44	0,01	KERING	0,15	0,01	S.E.B.	0,35	0,05	OENEO	0,36	0,00	SQLI	0,35	0,11
DALENYS	0,26	0,03	EXPLOSIFS PROD.CHI	0,42	0,01	CLASQUIN	0,15	0,01	LISI	0,34	0,03	COHERIS	0,35	0,01	KORIAN	0,32	0,02
MEAN	**0,64**	**0,19**	**MEAN**	**0,66**	**0,02**	**MEAN**	**0,39**	**0,02**	**MEAN**	**0,55**	**0,08**	**MEAN**	**0,60**	**0,09**	**MEAN**	**0,49**	**0,15**

Table 3.7: $MM\ (\gamma = 15\%)$ **portfolio selection, game cross-efficiency and mavericks scores**

2010	GAM	MAV	2011	GAM	MAV	2012	GAM	MAV	2013	GAM	MAV	2014	GAM	MAV	2015	GAM	MAV
EUTELSAT COMMUNIC.	1,00	0,03	SES	1,00	0,00	L'OREAL	0,99	0,01	EULER HERMES GROUP	1,00	0,00	MERCK AND CO INC	1,00	0,00	ALTAREIT	1,00	0,00
DANONE	0,94	0,05	MAROC TELECOM	1,00	0,00	IVALIS	0,99	0,01	MAROC TELECOM	0,84	0,18	DANONE	0,91	0,06	CFAO	0,81	0,24

DASSAULT AVIATION	0,97	0,15	TIPIAK	0,98	0,02	CNIM CONSTR.FRF 10	0,80	0,03	TURENNE INV	0,83	0,20	BNP PARIBAS ACT.A	0,82	0,07	CRCAM NORM.SEINE	0,65	0,15
ESSO	0,91	0,09	FDL	0,95	0,06	SABETON	0,78	0,01	VIEL ET COMPAGNIE	0,81	0,12	LE NOBLE AGE	0,82	0,21	SELECTIRENTE	0,60	0,23
L'OREAL	0,85	0,19	CA TOULOUSE 31 CCI	0,89	0,01	VILMORIN	0,73	0,02	HI-MEDIA	0,72	0,15	SIGNAUX GIROD	0,78	0,17	INSTALLUX	0,60	0,33
FREY	0,84	0,34	EUROSIC	0,89	0,03	SOFRAGI	0,75	0,05	RUBIS	0,67	0,12	EXEL INDUSTRIES	0,75	0,19	LINEDATA SERVICES	0,48	0,01
RAMSAY GEN SANTE	0,83	0,36	SODEXO	0,79	0,05	BERNARD LOISEAU	0,38	0,01	VETOQUINOL	0,65	0,08	CRCAM TOURAINE CCI	0,71	0,10	FLEURY MICHON	0,51	0,19
ESSILOR INTL.	0,78	0,25	BOLLORE	0,76	0,01	LEBON	0,37	0,01	CIC	0,61	0,05	NETBOOSTER	0,70	0,16	SAINT GOBAIN	0,47	0,07
SELECTIRENTE	0,75	0,23	EUTELSAT COMMUNIC.	0,75	0,01	MERCIALYS	0,37	0,01	PROCTER GAMBLE	0,60	0,04	ALES GROUPE	0,64	0,05	EFESO CONSULTING	0,48	0,24
STEF	0,74	0,28	ITS GROUP	0,74	0,01	SES	0,36	0,01	LEGRAND	0,59	0,05	BIC	0,58	0,01	CRCAM SUD R.A.CCI	0,47	0,21
MERCK AND CO INC	0,71	0,18	AIR LIQUIDE	0,65	0,01	BRICORAMA	0,35	0,02	SELECTIRENTE	0,55	0,02	DASSAULT SYSTEMES	0,58	0,10	LAFUMA	0,47	0,18
FONCIERE EURIS	0,66	0,15	SANOFI	0,65	0,01	INSTALLUX	0,34	0,02	AXA	0,56	0,03	BERNARD LOISEAU	0,61	0,11	EURAZEO	0,46	0,14
ROTHSCHILD	0,67	0,29	MR BRICOLAGE	0,61	0,03	SARTORIUS STED BIO	0,34	0,01	ATOS	0,55	0,09	HSBC HOLDINGS	0,55	0,06	ATOS	0,44	0,20
CATERING INTL SCES	0,60	0,13	SCBSM	0,59	0,01	TRILOGIQ	0,30	0,02	HOPSCOTCH GROUPE	0,55	0,08	ESPERITE	0,58	0,12	TERREIS	0,41	0,05
CA TOULOUSE 31 CCI	0,59	0,16	RADIALL	0,56	0,01	HOTELS DE PARIS	0,30	0,02	AFFINE R.E.	0,55	0,10	EULER HERMES GROUP	0,52	0,05	THERMADOR GROUPE	0,41	0,12
SANOFI	0,55	0,11	O2I	0,55	0,02	ENVIRONNEMENT SA	0,28	0,01	CRCAM ATL.VEND.CCI	0,51	0,09	ARCELORMITTAL	0,55	0,11	FONCIERE DE PARIS	0,42	0,19
SOMFY SA	0,55	0,23	RUBIS	0,52	0,01	PRODWARE	0,26	0,01	GIORGIO FEDON	0,46	0,02	AKKA TECHNOLOGIES	0,50	0,06	SYNERGIE	0,42	0,15
GROUPE FLO	0,52	0,11	KORIAN	0,51	0,02	COFIDUR	0,25	0,02	INTERPARFUMS	0,45	0,07	CRCAM NORM.SEINE	0,51	0,06	ROBERTET	0,41	0,06
MEDICREA INTERNAT.	0,52	0,22	ARGAN	0,52	0,02	VALEO	0,22	0,01	EURO DISNEY	0,44	0,03	ORPEA	0,49	0,05	MAKHEIA GROUP	0,41	0,12
PHARMAGEST INTER.	0,50	0,11	CRCAM LANGUED CCI	0,51	0,02	SAFRAN	0,20	0,02	IDI	0,45	0,07	THALES	0,48	0,02	SES	0,40	0,04
ALTAREA	0,49	0,15	MALTERIES FCO-BEL.	0,50	0,01	SI PARTICIPATIONS	0,20	0,01	EIFFAGE	0,39	0,03	ESI GROUP	0,46	0,04	OBER	0,39	0,04
EXPLOSIFS PROD.CHI	0,46	0,16	IMPRIMERIE CHIRAT	0,48	0,01	OCTO TECHNOLOGY	0,20	0,01	SWORD GROUP	0,37	0,03	SARTORIUS STED BIO	0,45	0,07	HARVEST	0,39	0,08
SOLVAY	0,41	0,13	EXPLOS.PROD.CHI.PF	0,47	0,01	LECTRA	0,18	0,02	VINCI	0,37	0,06	NEURONES	0,43	0,06	GEVELOT	0,38	0,05
COURTOIS	0,42	0,17	ROTHSCHILD	0,44	0,01	RIBER	0,18	0,01	INFOTEL	0,36	0,03	M.R.M	0,44	0,03	ALTAMIR	0,36	0,06
ILIAD	0,38	0,13	NEURONES	0,43	0,01	FREY	0,16	0,01	S.E.B.	0,35	0,05	EUROFINS SCIENT.	0,41	0,04	SOLUCOM	0,36	0,12
BIOMERIEUX	0,38	0,13	CHAUF.URB.	0,42	0,01	STREAMWIDE	0,16	0,01	LISI	0,34	0,03	SUEZ ENVIRONNEMENT	0,38	0,04	SARTORIUS STED BIO	0,34	0,08
ESI GROUP	0,37	0,11	EXPLOSIFS PROD.CHI	0,42	0,01	KERING	0,15	0,01	TESSI	0,33	0,05	OENEO	0,36	0,00	CRCAM NORD CCI	0,33	0,08
MGI DIGITAL GRAPHI	0,31	0,07	CRCAM NORM.SEINE	0,40	0,01	OENEO	0,15	0,01	DOM SECURITY	0,25	0,02	COHERIS	0,35	0,01	KORIAN	0,32	0,02
INTERPARFUMS	0,29	0,09	SOMFY SA	0,39	0,01	CLASQUIN	0,15	0,01	CRCAM ALP.PROV.CCI	0,23	0,02	CAP GEMINI	0,32	0,02	COHERIS	0,28	0,02
DALENYS	0,26	0,03	KINDY	0,35	0,01	VETOQUINOL	0,15	0,01	BIC	0,22	0,01	INSTALLUX	0,31	0,04	GASCOGNE	0,27	0,03
MEAN	**0,61**	**0,16**	**MEAN**	**0,62**	**0,01**	**MEAN**	**0,37**	**0,02**	**MEAN**	**0,52**	**0,06**	**MEAN**	**0,57**	**0,07**	**MEAN**	**0,46**	**0,12**

Table 3.8: $MM\ (\gamma = 20\%)$ **portfolio selection, game cross-efficiency and mavericks scores**

2010	GAM	MAV	2011	GAM	MAV	2012	GAM	MAV	2013	GAM	MAV	2014	GAM	MAV	2015	GAM	MAV
EUTELSAT COMMUNIC.	1,00	0,03	SES	1,00	0,00	L'OREAL	0,99	0,01	EULER HERMES GROUP	1,00	0,00	MERCK AND CO INC	1,00	0,00	ALTAREIT	1,00	0,00
DANONE	0,94	0,05	MAROC TELECOM	1,00	0,00	IVALIS	0,99	0,01	MAROC TELECOM	0,84	0,18	DANONE	0,91	0,06	CFAO	0,81	0,24
DASSAULT AVIATION	0,97	0,15	TIPIAK	0,98	0,02	CNIM CONSTR.FRF 10	0,80	0,03	VIEL ET COMPAGNIE	0,81	0,12	BNP PARIBAS ACT.A	0,82	0,07	CRCAM NORM.SEINE	0,65	0,15
ESSO	0,91	0,09	CA TOULOUSE 31 CCI	0,89	0,01	SABETON	0,78	0,01	RUBIS	0,67	0,12	SIGNAUX GIROD	0,78	0,17	SELECTIRENTE	0,60	0,23
L'OREAL	0,85	0,19	EUROSIC	0,89	0,03	VILMORIN	0,73	0,02	VETOQUINOL	0,65	0,08	CRCAM TOURAINE CCI	0,71	0,10	LINEDATA SERVICES	0,48	0,01
FREY	0,84	0,34	BOLLORE	0,76	0,01	BERNARD LOISEAU	0,38	0,01	CIC	0,61	0,05	NETBOOSTER	0,70	0,16	FLEURY MICHON	0,51	0,19
ESSILOR INTL.	0,78	0,25	EUTELSAT COMMUNIC.	0,75	0,01	LEBON	0,37	0,01	PROCTER GAMBLE	0,60	0,04	ALES GROUPE	0,64	0,05	SAINT GOBAIN	0,47	0,07
SELECTIRENTE	0,75	0,23	ITS GROUP	0,74	0,01	MERCIALYS	0,37	0,01	LEGRAND	0,59	0,05	BIC	0,58	0,01	CRCAM SUD R.A.CCI	0,47	0,21
STEF	0,74	0,28	AIR LIQUIDE	0,65	0,01	SES	0,36	0,01	SELECTIRENTE	0,55	0,02	DASSAULT SYSTEMES	0,58	0,10	LAFUMA	0,47	0,18
MERCK AND CO INC	0,71	0,18	SANOFI	0,65	0,01	BRICORAMA	0,35	0,02	AXA	0,56	0,03	BERNARD LOISEAU	0,61	0,11	EURAZEO	0,46	0,14
FONCIERE EURIS	0,66	0,15	MR BRICOLAGE	0,61	0,03	INSTALLUX	0,34	0,02	ATOS	0,55	0,09	HSBC HOLDINGS	0,55	0,06	TERREIS	0,41	0,05
ROTHSCHILD	0,67	0,29	SCBSM	0,59	0,01	SARTORIUS STED BIO	0,34	0,01	HOPSCOTCH GROUPE	0,55	0,08	EULER HERMES GROUP	0,52	0,05	THERMADOR GROUPE	0,41	0,12
CATERING INTL SCES	0,60	0,13	RADIALL	0,56	0,01	TRILOGIQ	0,30	0,02	AFFINE R.E.	0,55	0,10	ARCELORMITTAL	0,55	0,11	SYNERGIE	0,42	0,15
CA TOULOUSE 31 CCI	0,59	0,16	O2I	0,55	0,02	HOTELS DE PARIS	0,30	0,02	CRCAM ATL.VEND.CCI	0,51	0,09	AKKA TECHNOLOGIES	0,50	0,06	ROBERTET	0,41	0,06
SANOFI	0,55	0,11	RUBIS	0,52	0,01	ENVIRONNEMENT SA	0,28	0,01	GIORGIO FEDON	0,46	0,02	CRCAM NORM.SEINE	0,51	0,06	MAKHEIA GROUP	0,41	0,12
GROUPE FLO	0,52	0,11	ARGAN	0,52	0,02	PRODWARE	0,26	0,01	INTERPARFUMS	0,45	0,07	ORPEA	0,49	0,05	SES	0,40	0,04
PHARMAGEST INTER.	0,50	0,11	MALTERIES FCO-BEL.	0,50	0,01	COFIDUR	0,25	0,02	EURO DISNEY	0,44	0,03	THALES	0,48	0,02	OBER	0,39	0,04
ALTAREA	0,49	0,15	IMPRIMERIE CHIRAT	0,48	0,01	VALEO	0,22	0,01	IDI	0,45	0,07	ESI GROUP	0,46	0,04	HARVEST	0,39	0,08
EXPLOSIFS PROD.CHI	0,46	0,16	EXPLOS.PROD.CHI.PF	0,47	0,01	SAFRAN	0,20	0,02	EIFFAGE	0,39	0,03	ZODIAC AEROSPACE	0,46	0,09	GEVELOT	0,38	0,05
SOLVAY	0,41	0,13	EUROGERM	0,46	0,02	SI PARTICIPATIONS	0,20	0,01	SWORD GROUP	0,37	0,03	SARTORIUS STED BIO	0,45	0,07	ALTAMIR	0,36	0,06
GROUPE OPEN	0,40	0,14	MANUTAN INTL	0,45	0,02	OCTO TECHNOLOGY	0,20	0,01	VINCI	0,37	0,06	NEURONES	0,43	0,06	SQLI	0,35	0,11
COURTOIS	0,42	0,17	ROTHSCHILD	0,44	0,01	LECTRA	0,18	0,02	INFOTEL	0,36	0,03	M.R.M	0,44	0,03	SARTORIUS STED BIO	0,34	0,08
ILIAD	0,38	0,13	NEURONES	0,43	0,01	RIBER	0,18	0,01	S.E.B.	0,35	0,05	EUROFINS SCIENT.	0,41	0,04	CRCAM NORD CCI	0,33	0,08
BIOMERIEUX	0,38	0,13	CHAUF.URB.	0,42	0,01	FREY	0,16	0,01	OENEO	0,35	0,05	VRANKEN-POMMERY	0,38	0,05	KORIAN	0,32	0,02
ESI GROUP	0,37	0,11	PROCTER GAMBLE	0,42	0,02	STREAMWIDE	0,16	0,01	LISI	0,34	0,03	SUEZ ENVIRONNEMENT	0,38	0,04	SWORD GROUP	0,31	0,08
MGI DIGITAL GRAPHI	0,31	0,07	EXPLOSIFS PROD.CHI	0,42	0,01	KERING	0,15	0,01	TESSI	0,33	0,05	OENEO	0,36	0,00	BELIER	0,31	0,07
INTERPARFUMS	0,29	0,09	CRCAM NORM.SEINE	0,40	0,01	OENEO	0,15	0,01	DOM SECURITY	0,25	0,02	COHERIS	0,35	0,01	COHERIS	0,28	0,02

DALENYS	0,26	0,03	SOMFY SA	0,39	0,01	CLASQUIN	0,15	0,01	SIGNAUX GIROD	0,26	0,02	CARREFOUR	0,33	0,04	GASCOGNE	0,27	0,03
ENCRES DUBUIT	0,23	0,06	KINDY	0,35	0,01	ALES GROUPE	0,13	0,01	CNIM CONSTR.FRF 10	0,24	0,02	CAP GEMINI	0,32	0,02	LE NOBLE AGE	0,27	0,03
BUSINESS ET DECIS.	0,19	0,04	DELTA PLUS GROUP	0,33	0,01	IPSEN	0,12	0,01	CRCAM ALP.PROV.CCI	0,23	0,02	S.E.B.	0,29	0,03	GL EVENTS	0,26	0,03
MEAN	**0,57**	**0,14**	**MEAN**	**0,59**	**0,01**	**MEAN**	**0,35**	**0,01**	**MEAN**	**0,49**	**0,05**	**MEAN**	**0,53**	**0,06**	**MEAN**	**0,43**	**0,09**

Table 3.9: $MM\ (\gamma = 25\%)$ **portfolio selection, game cross-efficiency and mavericks scores**

2010	GAM	MAV	2011	GAM	MAV	2012	GAM	MAV	2013	GAM	MAV	2014	GAM	MAV	2015	GAM	MAV
EUTELSAT COMMUNIC.	1,00	0,03	SES	1,00	0,00	L'OREAL	0,99	0,01	EULER HERMES GROUP	1,00	0,00	MERCK AND CO INC	1,00	0,00	ALTAREIT	1,00	0,00
DANONE	0,94	0,05	MAROC TELECOM	1,00	0,00	IVALIS	0,99	0,01	VIEL ET COMPAGNIE	0,81	0,12	DANONE	0,91	0,06	CFAO	0,81	0,24
DASSAULT AVIATION	0,97	0,15	TIPIAK	0,98	0,02	CNIM CONSTR.FRF 10	0,80	0,03	RUBIS	0,67	0,12	BNP PARIBAS ACT.A	0,82	0,07	CRCAM NORM.SEINE	0,65	0,15
ESSO	0,91	0,09	CA TOULOUSE 31 CCI	0,89	0,01	SABETON	0,78	0,01	VETOQUINOL	0,65	0,08	CRCAM TOURAINE CCI	0,71	0,10	SELECTIRENTE	0,60	0,23
L'OREAL	0,85	0,19	EUROSIC	0,89	0,03	VILMORIN	0,73	0,02	CIC	0,61	0,05	ALES GROUPE	0,64	0,05	LINEDATA SERVICES	0,48	0,01
ESSILOR INTL.	0,78	0,25	BOLLORE	0,76	0,01	BERNARD LOISEAU	0,38	0,01	PROCTER GAMBLE	0,60	0,04	BIC	0,58	0,01	FLEURY MICHON	0,51	0,19
SELECTIRENTE	0,75	0,23	EUTELSAT COMMUNIC.	0,75	0,01	LEBON	0,37	0,01	LEGRAND	0,59	0,05	DASSAULT SYSTEMES	0,58	0,10	SAINT GOBAIN	0,47	0,07
STEF	0,74	0,28	ITS GROUP	0,74	0,01	MERCIALYS	0,37	0,01	SELECTIRENTE	0,55	0,02	BERNARD LOISEAU	0,61	0,11	EURAZEO	0,46	0,14
MERCK AND CO INC	0,71	0,18	AIR LIQUIDE	0,65	0,01	SES	0,36	0,01	AXA	0,56	0,03	HSBC HOLDINGS	0,55	0,06	TERREIS	0,41	0,05
FONCIERE EURIS	0,66	0,15	SANOFI	0,65	0,01	BRICORAMA	0,35	0,02	ATOS	0,55	0,09	EULER HERMES GROUP	0,52	0,05	THERMADOR GROUPE	0,41	0,12
CATERING INTL SCES	0,60	0,13	SCBSM	0,59	0,01	INSTALLUX	0,34	0,02	HOPSCOTCH GROUPE	0,55	0,08	ARCELORMITTAL	0,55	0,11	ROBERTET	0,41	0,06
CA TOULOUSE 31 CCI	0,59	0,16	RADIALL	0,56	0,01	SARTORIUS STED BIO	0,34	0,01	CRCAM ATL.VEND.CCI	0,51	0,09	AKKA TECHNOLOGIES	0,50	0,06	MAKHEIA GROUP	0,41	0,12
SANOFI	0,55	0,11	O2I	0,55	0,02	ENVIRONNEMENT SA	0,28	0,01	GIORGIO FEDON	0,46	0,02	CRCAM NORM.SEINE	0,51	0,06	SES	0,40	0,04
GROUPE FLO	0,52	0,11	RUBIS	0,52	0,01	PRODWARE	0,26	0,01	INTERPARFUMS	0,45	0,07	ORPEA	0,49	0,05	OBER	0,39	0,04
PHARMAGEST INTER.	0,50	0,11	MALTERIES FCO-BEL.	0,50	0,01	COFIDUR	0,25	0,02	EURO DISNEY	0,44	0,03	THALES	0,48	0,02	HARVEST	0,39	0,08
ALTAREA	0,49	0,15	IMPRIMERIE CHIRAT	0,48	0,01	VALEO	0,22	0,01	IDI	0,45	0,07	ESI GROUP	0,46	0,04	GEVELOT	0,38	0,05
EXPLOSIFS PROD.CHI	0,46	0,16	EXPLOS.PROD.CHI.PF	0,47	0,01	SI PARTICIPATIONS	0,20	0,01	EIFFAGE	0,39	0,03	SARTORIUS STED BIO	0,45	0,07	ALTAMIR	0,36	0,06
BOUYGUES	0,44	0,18	ROTHSCHILD	0,44	0,01	OCTO TECHNOLOGY	0,20	0,01	SWORD GROUP	0,37	0,03	NEURONES	0,43	0,06	SARTORIUS STED BIO	0,34	0,08
SOLVAY	0,41	0,13	NEURONES	0,43	0,01	LECTRA	0,18	0,02	VINCI	0,37	0,06	M.R.M	0,44	0,03	CRCAM NORD CCI	0,33	0,08
GROUPE OPEN	0,40	0,14	CHAUF.URB.	0,42	0,01	RIBER	0,18	0,01	INFOTEL	0,36	0,03	TOUAX	0,41	0,07	KORIAN	0,32	0,02
ILIAD	0,38	0,13	EXPLOSIFS PROD.CHI	0,42	0,01	FREY	0,16	0,01	S.E.B.	0,35	0,05	EUROFINS SCIENT.	0,41	0,04	SWORD GROUP	0,31	0,08

BIOMERIEUX	0,38	0,13	CRCAM NORM.SEINE	0,40	0,01	STREAMWIDE	0,16	0,01	OENEO	0,35	0,05	VRANKEN-POMMERY	0,38	0,05	BELIER	0,31	0,07
ESI GROUP	0,37	0,11	SOMFY SA	0,39	0,01	KERING	0,15	0,01	LISI	0,34	0,03	SUEZ ENVIRONNEMENT	0,38	0,04	COHERIS	0,28	0,02
MGI DIGITAL GRAPHI	0,31	0,07	KINDY	0,35	0,01	CLASQUIN	0,15	0,01	TESSI	0,33	0,05	OENEO	0,36	0,00	GASCOGNE	0,27	0,03
INTERPARFUMS	0,29	0,09	SI PARTICIPATIONS	0,33	0,01	ALES GROUPE	0,13	0,01	FIDUCIAL REAL EST.	0,27	0,03	COHERIS	0,35	0,01	LE NOBLE AGE	0,27	0,03
DALENYS	0,26	0,03	DELTA PLUS GROUP	0,33	0,01	AUGROS COSMETICS	0,12	0,01	DOM SECURITY	0,25	0,02	CARREFOUR	0,33	0,04	GL EVENTS	0,26	0,03
CHAUF.URB.	0,23	0,07	GROUPE JAJ	0,30	0,01	IPSEN	0,12	0,01	SIGNAUX GIROD	0,26	0,02	CAP GEMINI	0,32	0,02	AXA	0,24	0,04
ENCRES DUBUIT	0,23	0,06	ALTAREA	0,29	0,01	PSB INDUSTRIES	0,08	0,01	PERNOD RICARD	0,23	0,02	S.E.B.	0,29	0,03	M.R.M	0,23	0,02
BUSINESS ET DECIS.	0,19	0,04	PERRIER (GERARD)	0,24	0,01	BOURBON	0,06	0,01	CRCAM ALP.PROV.CCI	0,23	0,02	ARKEMA	0,28	0,04	PHARMAGEST INTER.	0,22	0,01
INNELEC MULTIMEDIA	0,19	0,06	INDLE FIN.ENTREPR.	0,21	0,01	COIL	0,05	0,01	BIC	0,22	0,01	ORANGE	0,24	0,01	EXEL INDUSTRIES	0,21	0,01
MEAN	**0,54**	**0,13**	**MEAN**	**0,55**	**0,01**	**MEAN**	**0,33**	**0,01**	**MEAN**	**0,46**	**0,05**	**MEAN**	**0,50**	**0,05**	**MEAN**	**0,40**	**0,07**

Table 3.10: $MM\ (\gamma = 30\%)$ **portfolio selection, game cross-efficiency and mavericks scores**

2010	GAM	MAV	2011	GAM	MAV	2012	GAM	MAV	2013	GAM	MAV	2014	GAM	MAV	2015	GAM	MAV
EUTELSAT COMMUNIC.	1,00	0,03	SES	1,00	0,00	L'OREAL	0,99	0,01	EULER HERMES GROUP	1,00	0,00	MERCK AND CO INC	1,00	0,00	ALTAREIT	1,00	0,00
DANONE	0,94	0,05	MAROC TELECOM	1,00	0,00	IVALIS	0,99	0,01	VIEL ET COMPAGNIE	0,81	0,12	DANONE	0,91	0,06	CFAO	0,81	0,24
DASSAULT AVIATION	0,97	0,15	TIPIAK	0,98	0,02	CNIM CONSTR.FRF 10	0,80	0,03	VETOQUINOL	0,65	0,08	BNP PARIBAS ACT.A	0,82	0,07	CRCAM NORM.SEINE	0,65	0,15
ESSO	0,91	0,09	CA TOULOUSE 31 CCI	0,89	0,01	SABETON	0,78	0,01	CIC	0,61	0,05	CRCAM TOURAINE CCI	0,71	0,10	CHARGEURS	0,65	0,55
L'OREAL	0,85	0,19	BOLLORE	0,76	0,01	VILMORIN	0,73	0,02	PROCTER GAMBLE	0,60	0,04	ALES GROUPE	0,64	0,05	SELECTIRENTE	0,60	0,23
ESSILOR INTL.	0,78	0,25	EUTELSAT COMMUNIC.	0,75	0,01	BERNARD LOISEAU	0,38	0,01	LEGRAND	0,59	0,05	BIC	0,58	0,01	CHAUF.URB.	0,65	0,53
SELECTIRENTE	0,75	0,23	ITS GROUP	0,74	0,01	LEBON	0,37	0,01	SELECTIRENTE	0,55	0,02	DASSAULT SYSTEMES	0,58	0,10	INSTALLUX	0,60	0,33
MERCK AND CO INC	0,71	0,18	AIR LIQUIDE	0,65	0,01	MERCIALYS	0,37	0,01	AXA	0,56	0,03	HSBC HOLDINGS	0,55	0,06	PRECIA	0,61	0,48
FONCIERE EURIS	0,66	0,15	SANOFI	0,65	0,01	SES	0,36	0,01	ATOS	0,55	0,09	EULER HERMES GROUP	0,52	0,05	CRCAM LANGUED CCI	0,56	0,42
CATERING INTL SCES	0,60	0,13	SCBSM	0,59	0,01	BRICORAMA	0,35	0,02	HOPSCOTCH GROUPE	0,55	0,08	AKKA TECHNOLOGIES	0,50	0,06	LINEDATA SERVICES	0,48	0,01
CA TOULOUSE 31 CCI	0,59	0,16	RADIALL	0,56	0,01	SARTORIUS STED BIO	0,34	0,01	GIORGIO FEDON	0,46	0,02	CRCAM NORM.SEINE	0,51	0,06	FLEURY MICHON	0,51	0,19
SANOFI	0,55	0,11	RUBIS	0,52	0,01	ENVIRONNEMENT SA	0,28	0,01	INTERPARFUMS	0,45	0,07	ORPEA	0,49	0,05	SAINT GOBAIN	0,47	0,07
GROUPE FLO	0,52	0,11	MALTERIES FCO-BEL.	0,50	0,01	PRODWARE	0,26	0,01	EURO DISNEY	0,44	0,03	THALES	0,48	0,02	SMTPC	0,48	0,25
PHARMAGEST INTER.	0,50	0,11	IMPRIMERIE CHIRAT	0,48	0,01	VALEO	0,22	0,01	IDI	0,45	0,07	ESI GROUP	0,46	0,04	EFESO CONSULTING	0,48	0,24
ALTAREA	0,49	0,15	ROTHSCHILD	0,44	0,01	SI PARTICIPATIONS	0,20	0,01	EIFFAGE	0,39	0,03	NEURONES	0,43	0,06	VEOLIA ENVIRON.	0,50	0,35

EXPLOSIFS PROD.CHI	0,46	0,16	NEURONES	0,43	0,01	OCTO TECHNOLOGY	0,20	0,01	SWORD GROUP	0,37	0,03	M.R.M	0,44	0,03	CRCAM SUD R.A.CCI	0,47	0,21
SOLVAY	0,41	0,13	CHAUF.URB.	0,42	0,01	RIBER	0,18	0,01	VINCI	0,37	0,06	EUROFINS SCIENT.	0,41	0,04	LAFUMA	0,47	0,18
ILIAD	0,38	0,13	EXPLOSIFS PROD.CHI	0,42	0,01	FREY	0,16	0,01	INFOTEL	0,36	0,03	VRANKEN-POMMERY	0,38	0,05	EURAZEO	0,46	0,14
BIOMERIEUX	0,38	0,13	CRCAM NORM.SEINE	0,40	0,01	STREAMWIDE	0,16	0,01	S.E.B.	0,35	0,05	SUEZ ENVIRONNEMENT	0,38	0,04	ATOS	0,44	0,20
ESI GROUP	0,37	0,11	SOMFY SA	0,39	0,01	KERING	0,15	0,01	LISI	0,34	0,03	OENEO	0,36	0,00	PATRIMOINE ET COMM	0,44	0,23
MGI DIGITAL GRAPHI	0,31	0,07	KINDY	0,35	0,01	CLASQUIN	0,15	0,01	TESSI	0,33	0,05	COHERIS	0,35	0,01	TERREIS	0,41	0,05
INTERPARFUMS	0,29	0,09	SI PARTICIPATIONS	0,33	0,01	ALES GROUPE	0,13	0,01	FIDUCIAL REAL EST.	0,27	0,03	CARREFOUR	0,33	0,04	THERMADOR GROUPE	0,41	0,12
DALENYS	0,26	0,03	DELTA PLUS GROUP	0,33	0,01	AUGROS COSMETICS	0,12	0,01	DOM SECURITY	0,25	0,02	CAP GEMINI	0,32	0,02	FONCIERE DE PARIS	0,42	0,19
CHAUF.URB.	0,23	0,07	PRECIA	0,31	0,01	IPSEN	0,12	0,01	SIGNAUX GIROD	0,26	0,02	INSTALLUX	0,31	0,04	SYNERGIE	0,42	0,15
ENCRES DUBUIT	0,23	0,06	GROUPE JAJ	0,30	0,01	PSB INDUSTRIES	0,08	0,01	CNIM CONSTR.FRF 10	0,24	0,02	S.E.B.	0,29	0,03	ROBERTET	0,41	0,06
BUSINESS ET DECIS.	0,19	0,04	ALTAREA	0,29	0,01	FERM.CAS.MUN.CANNE	0,08	0,01	PERNOD RICARD	0,23	0,02	TOTAL	0,27	0,03	MAKHEIA GROUP	0,41	0,12
INNELEC MULTIMEDIA	0,19	0,06	VINCI	0,29	0,01	BOURBON	0,06	0,01	CRCAM ALP.PROV.CCI	0,23	0,02	ARKEMA	0,28	0,04	SES	0,40	0,04
JC DECAUX SA.	0,18	0,06	PERRIER (GERARD)	0,24	0,01	COIL	0,05	0,01	BIC	0,22	0,01	ORANGE	0,24	0,01	OBER	0,39	0,04
LAURENT-PERRIER	0,18	0,07	AUSY	0,22	0,01	TRAQUEUR	0,02	0,01	BNP PARIBAS ACT.A	0,22	0,03	TRILOGIQ	0,24	0,02	HARVEST	0,39	0,08
TOUR EIFFEL	0,12	0,04	INDLE FIN.ENTREPR.	0,21	0,01	MEMSCAP REGPT	0,02	0,01	RIBER	0,14	0,01	O2I	0,21	0,02	GEVELOT	0,38	0,05
MEAN	**0,50**	**0,11**	**MEAN**	**0,51**	**0,01**	**MEAN**	**0,30**	**0,01**	**MEAN**	**0,43**	**0,04**	**MEAN**	**0,47**	**0,04**	**MEAN**	**0,51**	**0,20**

Table 3.11 presents cross-efficiency scores $E_{\Omega} = \frac{1}{K}\sum_{j}^{n} w_j E_{jj}$ and risk measurement $I_{\Omega} = \frac{1}{K}\sum_{j}^{n} w_j M_j$ for each portfolio over the whole period of study where E_{jj} is the cross-efficiency score of share j making part of the selected portfolio and M_j is the maverick index score of asset j. As assets are equally weighted in the selected portfolio, $w_j = 1$, $j = 1...n$. Hence, the efficiency of portfolio is the sum of individual stocks' efficiency and the risk of the portfolio is the sum of the individual shares' risk.

Table 3.11: Selected portfolios: Efficiency Vs. Risk

Selected portfolios	Efficiency (E_{Ω})	Risk (I_{Ω})
AP	0.71405	0.3875
GCP	0.71405	0.3875
MM ($\gamma = 5\%$)	0.67863	0.21929
MM ($\gamma = 10\%$)	0.6428	0.1855
MM ($\gamma = 15\%$)	0.60736	0.1607
MM ($\gamma = 20\%$)	0.5714	0.1422
MM ($\gamma = 25\%$)	0.5361	0.1259
MM ($\gamma = 30\%$)	0.49987	0.1115

The selected portfolios AP and GCP have a slightly higher efficiency score (0.714) than (MM) portfolio (from 0.49987 to 0.67863). However, MM portfolios are much less riskier than AP and GCP. In addition, we note that the higher the tradeoff parameter, the less risky is the MM portfolio and therefore the less efficient.

It is recognized that the most effective strategy to minimize risk is diversification. Note that diversification is the practice for mixing a wide variety of shares within a portfolio. To examine diversification level, we have created a scatter plot on the input and output space of the developed model at each return-risk tradeoff level. Figure 3.1 shows that the portfolio components of the GCP and AP form a highly dense arc in the lower left of the input space and a dense ellipse in the upper left of the output space. These populated areas employ similar weights which result in higher arbitrary cross and game cross-efficiencies leading to a poorly diversified portfolio. In contradiction to the MM results, Figures 3.2 through 3.4 demonstrate that MM model provides a more diversified portfolio whatever the tradeoff parameter value in terms of performance on multiple input and output factors are. Moreover, the figures show that the increase in the level of diversification is positively related to γ value. Our model provides a more diversified portfolio figures (3.2-3.4) show very well spread components on the input and output space; especially for $\gamma = 30\%$. This result is further confirmed by the decrease on the risk degree $I_\Omega = 11.62552$ of GCP to $I_\Omega = 3.345269$ for $\gamma = 30\%$.

Figure 3.1 : AP Vs GCP, Input & output space

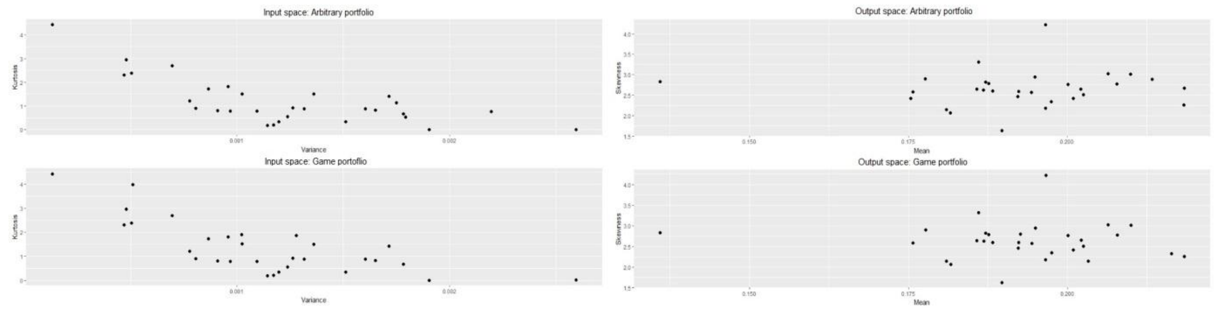


Figure 3.2 : $\gamma = 5\%$ $(E_\Omega = 0.678, I_\Omega = 0.21929)$ $\gamma = 10\%$ $(E_\Omega = 0.6428 \quad I_\Omega = 0.1855)$

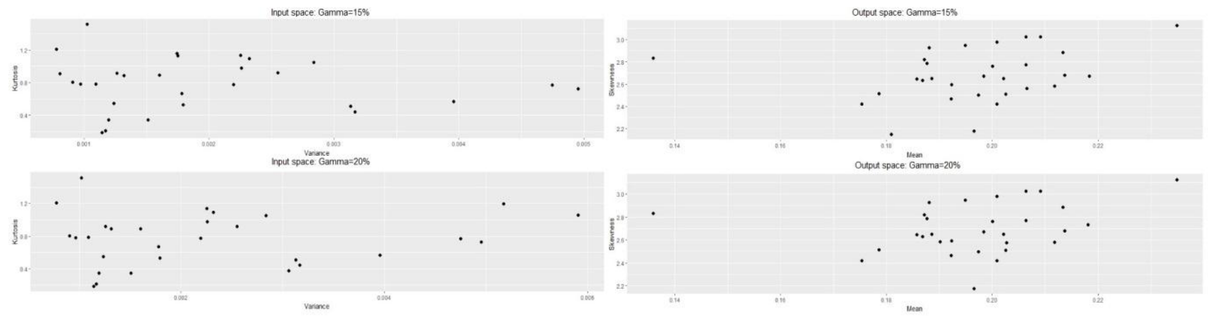


Figure 3.3 : $\gamma = 15\%$ $(E_\Omega = 0.60736 \quad I_\Omega = 0.1607)$ $\gamma = 20\%$ $(E_\Omega = 0.5714\ ,\ I_\Omega = 0.1422)$

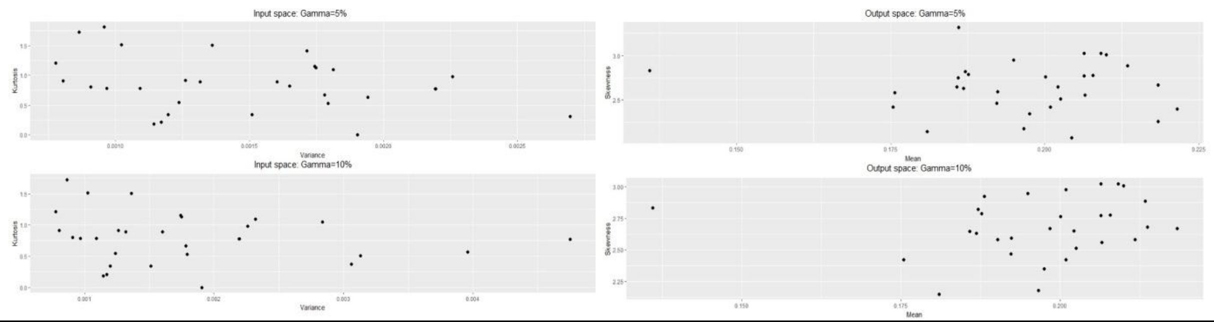
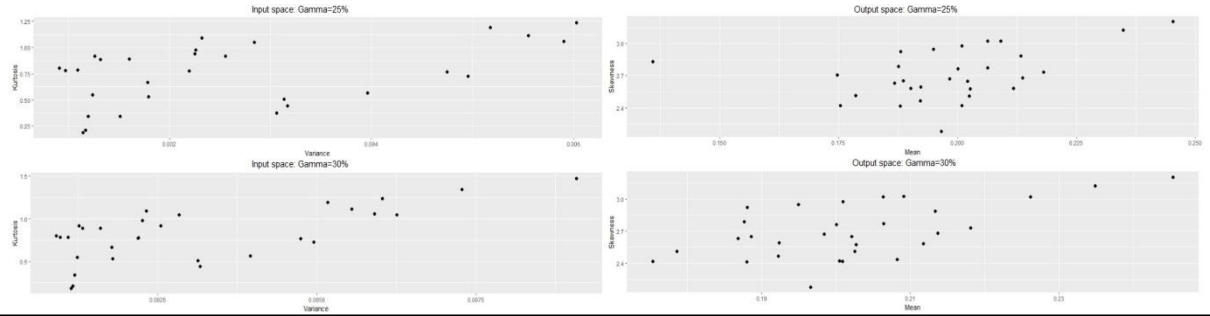


Figure 3.4 : $\gamma = 25\%$ $(E_\Omega = 0.5361\,,\ I_\Omega = 0.1259)$ $\gamma = 30\%$ $(E_\Omega = 0.49987\,,\ I_\Omega = 0.1115)$



Moreover, we examine the 6-year performance of the portfolio MM generated by the model (3.5) and compare it to those of the four market indexes, CAC40, AEX, BEL20 and PSI20 over the study period starting from 2010 to 2015. These benchmark indexes are the main national indices of the stock exchange group Euronext. The intent of this comparison is to show that the selected model outperforms the best Euronext indices and beat that of GCP and AP. Figures 3.5 through 3.10 present cumulative return curves of portfolios when $\gamma = 5\%, 10\%, 15\%, 20\%, 25\%$ and 30% compared to AP and GCP and benchmark portfolios. MM portfolios have the highest growth of hypothetical initial investment over the 6 years of study when the return-risk tradeoff parameter is higher than 20% (see Figures 3.8, 3.9 and 3.10). However, when $\gamma = 5\%, 10\%$ and 15%, the MM portfolios derived from the developed approach have higher cumulative returns than that of the index market (see Figures 3.5, 3.6 and 3.7). It follows that the bigger the tradeoff parameter, the greater the cumulative return of the portfolio (Figure 3.11). In fact, the black curve of MM portfolios grows exponentially and the cumulative return reaches more than 90% in December 2015 when $\gamma = 30\%$. However, other curves are less than 60% during the whole period (Figure 3.10). Market indices have the smallest cumulative returns during the period 2010-2015 as compared to other portfolios.

Figure 3.5 : Cumulative return curves ($\gamma = 5\%$)

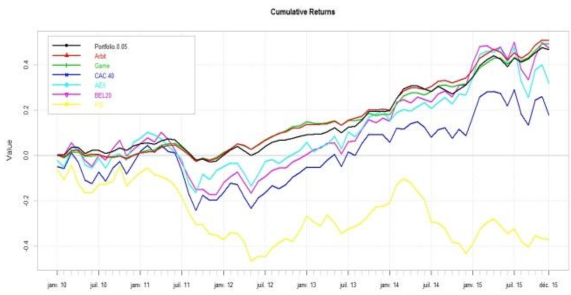


Figure 3.6 : Cumulative return curves ($\gamma = 10\%$)

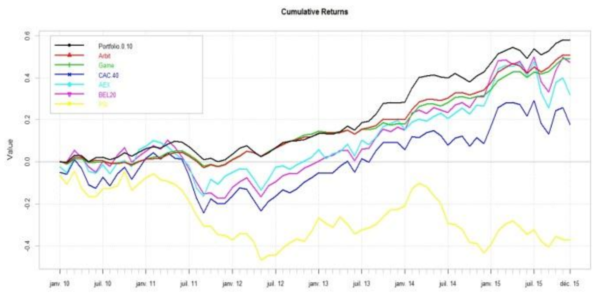


Figure 3.7 : Cumulative return curves ($\gamma = 15\%$)

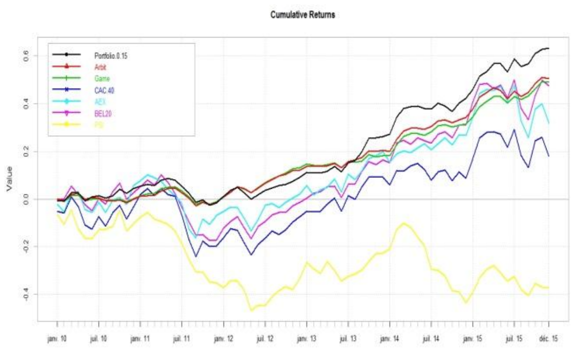


Figure 3.8 : Cumulative return curves ($\gamma = 20\%$)

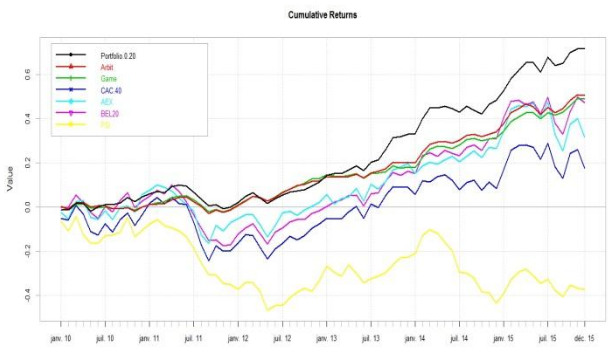


Figure 3.9 : Cumulative return curves ($\gamma = 25\%$)

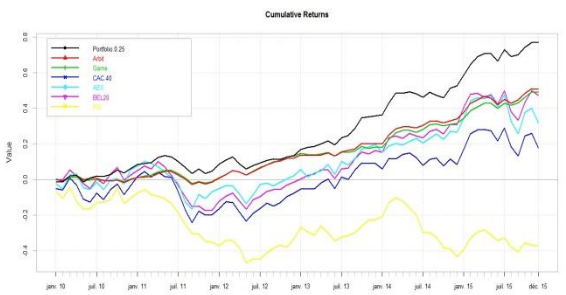
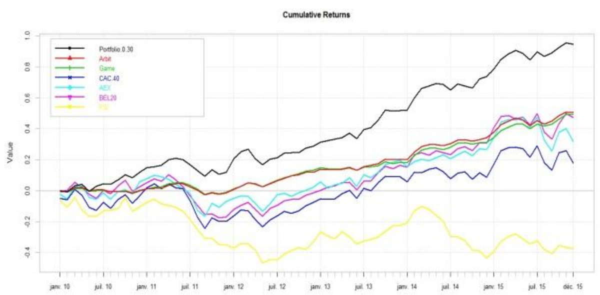


Figure 3.10 : Cumulative return curves ($\gamma = 30\%$)



According to losses analysis, Figures 3.12 to 3.17 present the drawdown curves of portfolios when the tradeoff parameter is equal to 5%, 10%, 15%, 20%, 25%, and 30% respectively. In all cases, MM portfolios have a minimum drawdown in comparison with market indices, indicating the minimum amount of loss during the period. Losses are then between zero and 10%. Moreover, MM portfolios, game portfolio and arbitrary portfolio have lower and more stable drawdown compared to market indices. Finally, conclusions can not be drawn about proportionality between portfolio losses and different levels of return-risk tradeoff parameter levels (see Figure 3.18).

Figure 3.11 : Cumulative return curves of MM portfolio

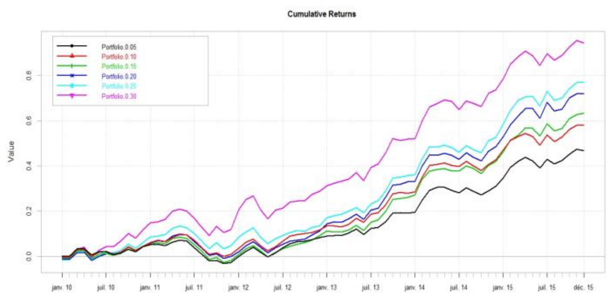


Figure 3.12 : Drawdown curves ($\gamma = 5\%$)

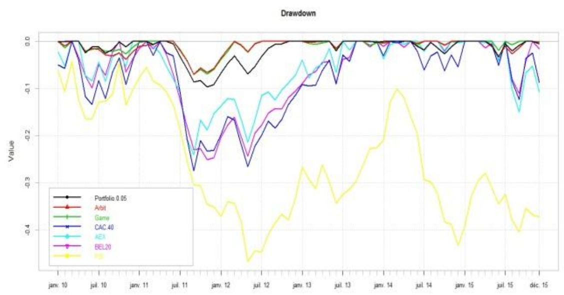


Figure 3.13 : Drawdown curves ($\gamma = 10\%$)

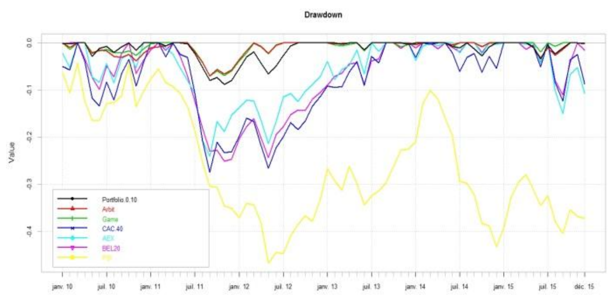


Figure 3.14 : Drawdown curves ($\gamma = 15\%$)

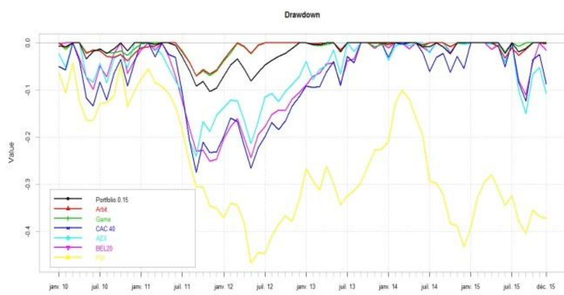


Figure 3.15 : Drawdown curves ($\gamma = 20\%$)

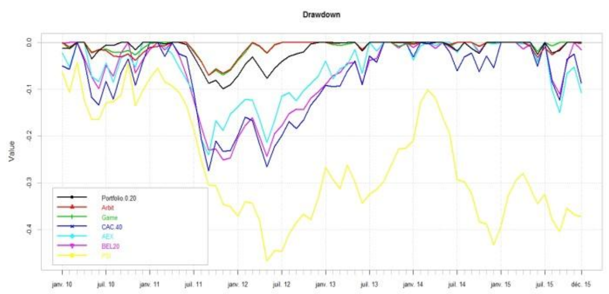


Figure 3.16 : Drawdown curves ($\gamma = 25\%$)

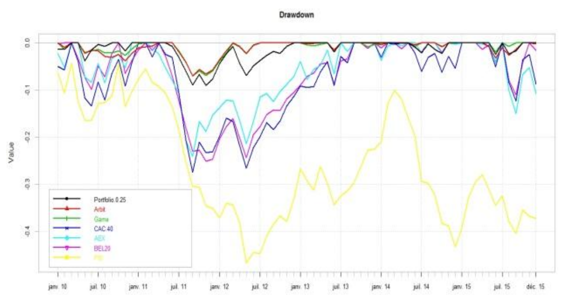


Figure 3.17 : Drawdown curves ($\gamma = 30\%$)

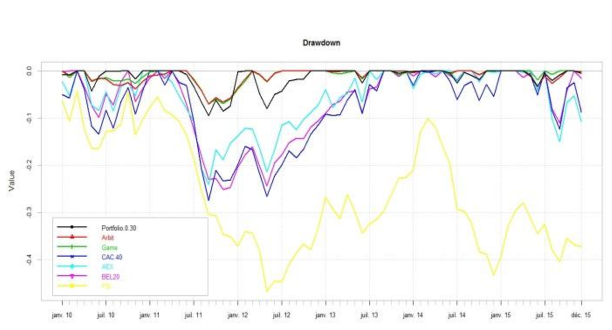
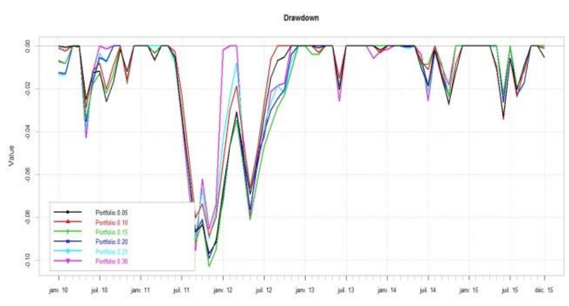


Figure 3.18 : Drawdown curves of MM portfolios



Since our contribution is to develop a novel risk-return model, it is important to compare the developed MM framework to MV portfolio model, which was first formulated by Markowitz (1952). The optimal MV portfolio is determined through resolving the following optimization program:

$$
\begin{aligned}
& Min \quad \sigma_{\Omega}^{2} = Var\left(\Omega\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \\
& s.t. \quad R_{\Omega} = \sum_{i=1}^{n} w_i R_i \geq \left(1-\delta\right) R^{*} \\
& \qquad \sum_{i=1}^{n} w_i = 1 \\
& \qquad w_i \geq 0 \;,\; i = 1 \ldots n
\end{aligned}
\tag{3.9}
$$

Where Ω is a portfolio with n different assets, R_i is the return of asset i , σ_{ij} is the covariance between R_i and R_j. w_i is the relative amount of the value of the portfolio invested in asset i, δ is a return-risk tradeoff parameter (fixed at 30% in this analysis) and R^* is the greatest return in the asset series. It consists of minimizing the portfolio variance for a given level of return. Table 3.12 provides MV model results. The MV portfolio consists of only 2, 8, 7, 4, 7 and 11 stocks during the study' years respectively. The MV portfolio is therefore badly-diversified. In fact, according to Evans and Archer (1968), a well-diversified portfolio must include 10 or more stocks.

Table 3.12: MV portfolio selection and shares' weights

2010		2011		2012		2013		2014		2015	
SHARES	w_j	**SHARES**	w_j	**SHARES**	w_j	**SHARES**	w_j	**SHARES**	w_j	**SHARES**	w_j
LEGRAND	0,662942	LECTRA	0,636974677	VDI GROUP	0,623093	EASYVISTA	0,672095	EASYVISTA	0,46741	STEF	0,487589
HOLOSFIND	0,337048	LEXIBOOK LINGUIST.	0,096901178	NICOX	0,132048	LIONAX	0,224088	GENFIT	0,295013	CELLECTIS	0,128088
		AVANQUEST	0,086051509	BD MULTI MEDIA	0,119943	REWORLD MEDIA	0,079049	GEVELOT	0,105842	PARROT	0,074382
		VDI GROUP	0,071133934	MEMSCAP REGPT	0,068593	CHINA SUPER POWER	0,024762	VDI GROUP	0,089707	GROUPIMO	0,074215
		EUROLAND CORPORATE	0,046701452	LE TANNEUR	0,04041			CELLECTIS	0,026875	DEXIA	0,058749
		GRAINES VOLTZ	0,031575546	TOUAXBSAR0316	0,010044			EUROLAND CORPORATE	0,007633	SODITECH ING	0,055152
		FONC. PARIS NORD	0,03016757	TRAQUEUR	0,005857			VERNEUIL PARTICIP	0,007509	VET AFFAIRES	0,045096
		ST DUPONT	0,000489075							OXIS INTL	0,034267
										RECYLEX S A	0,020376
										ARTEA	0,017002
										SPOREVER	0,005072

To take into account volatility and return together, we use the Sharpe ratio to examine the risk-adjusted performance of portfolios. We define the Sharpe ratio SR of portfolio Ω developed by Sharpe (1963) as the earned excess average return of the risk-free rate per unit of volatility The higher the ratio, the more reward an investment provides for the risk incurred. It is determined using the following formula:

$$SR_{\Omega} = \frac{\left(R_{\Omega} - R_f\right)}{\sigma_{\Omega}} \tag{3.10}$$

Where R_{Ω} is the geometric average return, R_f is the risk-free return approximated by the Interest Rates of Government Securities & Treasury Bills for France (INTGSTFRM193N) monthly rates and σ_{Ω} is the standard deviation of the portfolio return.

Table 3.13 shows the annual excess return $\left(R_{\Omega} - R_f\right)$ of each portfolio Ω over the 6-year study period. For $\gamma = 5\%, 10\%, 15\%, 20\%, 25\%$ and 30%, the MM portfolios attain the highest geometric mean excess return of 10.44%, 7.74%, 7.24%, 6.31% and 5.72% respectively, which are quite higher than those of the CAC40 (0.66%), AEX (2.56%), BEL20 (4.48%), PSI20 (-9.39%), AP (4.70%) and GCP (4.89%). Moreover, whatever the tradeoff parameter level is, MM portfolios keep a volatility level lower than that of each benchmark index. In addition, the MM portfolios have the highest Sharpe ratios over the period of study, which means that portfolios derived from the model MM are the most performing in terms of risk and profitability. In fact, the MM portfolios for $\gamma = 30\%, 25\%, 20\%$ and 15% attain respectively the highest Sharpe ratios of 1.895, 1.036, 1.0242, 0.9175 levels respectively succeeded by that of AP (0.9059), GCP (0.8961), MM portfolio when $\gamma = 10\%$ (0.8784), MM portfolio when $\gamma = 5\%$ (0.7196), BEL20 (0.3379), AEX (0.1674), CAC40 (0.0395) and PSI20 (-0.4884). For the same return-risk tradeoff parameter value (30%), the MM portfolio has a higher geometric average excess return (10.44) than that of the MV portfolio (4.027). Furthermore, while the volatility of the MV portfolio is smaller than that of the MM portfolio, we find that the Sharpe ratio of the MM portfolio is higher than that of the MV portfolio. In overall, the MM portfolio has a better risk-adjusted return than the MV portfolio.

Table 3.13 : Portfolios performance

	γ=0.05	γ=0.1	γ=0.15	γ=0.2	γ=0.25	γ=0.3	CAC40	AEX	BEL20	PSI20	AP	GCP	MV
Ret (%) 2010	1.91	1.90	1.89	1.82	3.86	**8.49**	-2.44	-2.33	-8.56	0.01	-2.78	-15.14	1.42
Ret (%) 2011	-9.31	-6.08	-9.04	-7.31	-4.57	**-3.55**	-4.10	-4.01	-22.15	-16.32	-23.99	-31.84	8.77
Ret (%) 2012	8.57	8.09	8.28	8.31	5.24	11.89	12.60	13.27	10.75	5.93	**14.85**	-1.47	10.94
Ret (%) 2013	8.55	13.01	13.89	**17.63**	17.41	15.73	5.87	2.72	12.62	12.30	13.81	10.42	1.11
Ret (%) 2014	9.44	11.69	12.50	11.09	12.49	**14.69**	10.99	10.78	0.24	5.33	13.22	-28.36	4.66
Ret (%) 2015	12.40	10.41	15.82	16.60	16.18	**19.60**	12.32	13.53	8.05	3.63	11.76	8.44	5.98
A. G Ret	4.43	5.72	6.31	7.24	7.74	10.44	0.66	2.56	4.48	-9.39	4.70	4.89	4.027
A. SD	6.15	6.51	6.88	7.07	7.47	8.78	16.61	15.28	13.27	19.23	5.19	5.45	3.928
Sharpe Ratio	**0.7196**	**0.8784**	**0.9175**	**1.0242**	**1.036**	**1.1895**	**0.0395**	**0.167**	**0.3379**	**-0.4884**	**0.9059**	**0.8961**	**1.025**

Ret: Excess return
A.G Ret: Annualized geometric excess return
A. SD: Annualized standard deviation

Since Sharpe ratio does not take into account how returns are distributed, this performance measure could be not valid when returns are not normally distributed. Thus, we examine the statistical significance of the difference between two Sharpe ratios (MM portfolios compared to each of the other market indices). We use the studentized circular block bootstrap (SCBB) developed by Ledoit and Wolf (2008), which takes into consideration the skewness, kurtosis and autocorrelation effects when comparing two Sharpe ratios. We focus on the MM portfolios at six levels of return-risk tradeoff parameter γ and we formulate 42 two-sided hypotheses. For each level of γ we test the following hypothesis:

$$\begin{aligned}
&H_0 : Sharpe\ \ ratio\ (MM) - Sharpe\ \ ratio\ (AP) = 0\\
&H_0 : Sharpe\ \ ratio\ (MM) - Sharpe\ \ ratio\ (\mathrm{G}\,P) = 0\\
&H_0 : Sharpe\ \ ratio\ (MM) - Sharpe\ \ ratio\ (\mathrm{CAC\,40}) = 0\\
&H_0 : Sharpe\ \ ratio\ (MM) - Sharpe\ \ ratio\ (\mathrm{AEX}) = 0\\
&H_0 : Sharpe\ \ ratio\ (MM) - Sharpe\ \ ratio\ (\mathrm{BEL\,20}) = 0\\
&H_0 : Sharpe\ \ ratio\ (MM) - Sharpe\ \ ratio\ (\mathrm{PSI\,20}) = 0\\
&H_0 : Sharpe\ \ ratio\ (MM) - Sharpe\ \ ratio\ (MV) = 0
\end{aligned} \tag{3.11}$$

We use the R implementation of Ledoit and Wolf (2008) to test the above hypotheses. We apply the test on pairs of monthly excess returns. Table 3.14 summarizes the test statistics resulting from the R code using the default parameter setting with B=10.

Table 3.14: Two-sided Sharpe difference test: The SCBB (B=10. M=4999)

		CAC 40	AEX	BEL20	PSI20	GCP	AP	MV
γ=5%	Difference	0.1772	0.1428	0.0978	0.3322	-0.051	-0.0484	-0.7326
	P value	(0.195)	(0.3838)	(0.327)	**(0.0446) ****	(0.5628)	(0.4512)	(0.492)
γ=10%	Difference	0.2212	0.1868	0.1419	0.3762	-0.007	-0.0044	-0.5059
	P value	(0.1376)	(0.2)	(0.1696)	**(0.0214) ****	(0.9478)	(0.9542)	0.662
γ=15%	Difference	0.2321	0.1977	0.1527	0.3871	0.0038	0.0064	-0.6269
	P value	**(0.0766) ***	(0.1798)	**(0.0988) ***	**(0.017) ****	(0.9644)	(0.9372)	**0.002*****
γ=20%	Difference	0.2611	0.2267	0.1818	0.4161	0.0329	0.0355	-0.5499
	P value	**(0.0668) ***	(0.1248)	**(0.0518) ***	**(0.0112) ****	(0.7262)	(0.6774)	0.728
γ=25%	Difference	0.2643	0.23	0.185	0.4194	0.0361	0.0387	-0.5499
	P value	**(0.0572) ***	**(0.0942) ***	**(0.03) ****	**(0.0054) *****	(0.731)	(0.6954)	0.728
γ=30%	Difference	0.3052	0.2708	0.2258	0.4602	0.077	0.0796	0.3988
	P value	**(0.0148) ****	**(0.0648) ***	**(0.0294) ****	**(0.003) *****	(0.5076)	(0.4924)	0.734
GCP	Difference	0.2282	0.1938	0.1489	0.3832	---------	0.0026	-0.0015
	P value	**(0.087) ***	(0.2244)	(0.1722)	**(0.0788) ****	---------	(0.9398)	0.232
AP	Difference	0.2256	0.1912	0.1463	0.3806	-0.0026	---------	-1.9252
	P value	**(0.0768) ***	(0.175)	(0.167)	**(0.0482) ****	(0.9392)	---------	**0.008*****

Significance level 1%***, 5%**, 10%*

Table 3.14 shows that for 10% significance level, the Sharpe ratio of the MM portfolio when $\gamma = 30\%$ and $\gamma = 25\%$ is significantly greater than all those of the market indices, while for $\gamma = 20\%$ and $\gamma = 15\%$, the Sharpe ratios exceed only those of CAC40, BEL20 & PSI20. Finally, for $\gamma = 10\%$ and $\gamma = 5\%$ the ratios exceed only that of PSI20. We fail to prove the significance of the Sharpe ratio differences of our developed model compared to that of AP and GCP. Moreover, we show that the MV portfolio has a significant better risk-adjusted return at 1% level (P-value=0.008) than the AP portfolio. However, the positive difference (0.3988) between the MM portfolio $\gamma = 30\%$ and the MV portfolio is not significant.

The results above demonstrate and support the effectiveness of our approach as promising tool for portfolio selection. Our results also show that the Mean-Maverick game cross-efficiency approach is more effective than the one based on the simple use of cross-efficiency and game cross-efficiency approaches; at least for this particular study.

6. Concluding Remarks

In this paper, the methodology DEA game cross-efficiency proposed by Liang et al. (2008a) is suggested to evaluate the performance of financial assets. The maverick index, which is the deviation from the Nash equilibrium score is considered as a risk indicator in this work. We incorporate the game cross-efficiency into Mean-Maverick framework to select portfolios. This development is motivated by the observation that the traditional simple use of cross-efficiency scores in portfolio selection per se does not incorporate the risk level of individual stocks. More specifically, it does not account for the sensitivity level to changes in the input/output environment. This problem is mainly due to the democratic vote characteristic of DEA game cross-efficiency evaluation known as the ganging up effect which also results in poorly diversified portfolio. We have found that this issue arises because the simple use of game cross-efficiency evaluation in portfolio selection fails to the sensitivity level of environmental changes of the constituting firms. Moreover, our approach permits to select well diversified portfolios. We use Ledoit and Wolf (2008) Sharpe test to prove the robustness of our methodology. The formed portfolio beats all Benchmark portfolios in the Paris Stock exchange. Finally, it would be of great interest to opt for our approach to select portfolios using another large data set and taking into consideration taxes and transaction costs.

Chapter 4:

On DEA game cross-efficiency approach to portfolio selection: Does profitability criterion help or hurt?

Chapter 4

On DEA game cross-efficiency approach to portfolio selection: Does profitability criterion help or hurt?

1. Introduction

A critical aspect of portfolio management is the decision whereby the best set of stocks, or financial assets, is selected from many different alternatives. In many cases, the stakes are high because selecting the right assets is a significant resource allocation decision that can lead to high profits and in the worst case to a huge loss. This work attempts to solve this question of portfolio selection by supporting profitability criterion on the one hand and efficiency of financial assets on the other hand. In fact, in early portfolio theory, performance of portfolio was commonly measured only in terms of returns. Afterwards, risk has been recognized as important criterion for investment decisions. The main problem in modern portfolio theory was finding the stock portfolio which may achieve the highest possible return for a given level of risk or a minimum possible risk for a given level of return (Markowitz (1952)). Besides the modern portfolio theory, a huge literature has started to evaluate portfolios in a Mean-Variance (MV) framework. Studies on portfolio performance have been developed based on only the first two moments of returns' distribution (see for example, Murthi et al. (1997), Morey and Morey (1999), Briec et al. (2004), Levy and Ritov (2011), Liu and Yong-jun (2014) and Shigeta (2017)). However, since Mandelbrott (1963), many works have shown that returns of financial assets are usually not normally distributed. Thus, using higher moments of returns allow taking into consideration the higher order utility function of investor and the non-normality of returns' distribution to assess performance of portfolios (Briec et al. (2007), Ghysels and Pereira (2008) and Glawischnig and Sommersguter-Reichmann (2010)). An investor's choice of a portfolio is then a function of various criteria. In fact, selecting assets to invest from many assets is a crucial decision for investors and managers, where efforts must be made to evaluate and choose optimal sets of assets to be undertaken. The most significant criteria that have to be considered in the portfolio selection process are investor's preferences, which may be multiple and often conflicting. To tackle this problem, various decision making approaches have been proposed in the literature such as mathematical programming (Tsionas (2018), analytic hierarchy process (Maghsoud et al. (2015)), machine learning techniques (Oh et al. (2005)) and the Data

Envelopment Analysis (DEA) (see, e.g., Mehlawat et al.(2018), Rezaee et al. (2018), Branda (2016), Tarnaud and Leleu (2017)). Various models based on DEA could deal with the problem of portfolio selection. Indeed, using DEA permits to analyze financial assets based on the production theory or the benchmarking theory. From a production perspective, DEA model evaluates efficiency of Decision Making Units (DMUs) which consume a fixed number of inputs in order to produce a fixed number of outputs. However, as mentioned by Cook et al. (2014), the use of DEA as a benchmarking tool is relevant in the case of Multi Criteria Decision Making (MCDM). It permits to consider several attributes together to presents a single composite efficiency score to assist in the decision making process of the investor. Developed by Charnes et al. (1978), DEA is used to measure the relative efficiencies of a set of DMUs with multiple inputs and outputs. In fact, a single score is simpler to interpret than interpreting various factors. DEA framework considers higher order moments in financial assets performance evaluation through taking the attributes to be minimized as inputs (even moments: variance and kurtosis) and that to be maximized as outputs (odd moments: mean and skewness). The derived DEA efficiency score is obtained as the maximum of a ratio of weighted outputs to weighted inputs. However, while the efficiency of a financial asset may result from a good mix of attributes (mean, variance, skewness and kurtosis of returns), the profitability criterion matters a lot to an investor. A risk averse and prudent investor seeks to minimize variance and maximize skewness of financial asset returns, respectively. A temperate investor looks to maximize the kurtosis. Thus obviously efficiency does not mean profitability. Thereby, considering profitability as objective in the evaluation process of financial asset may be very important for an investor looking for gains by acquiring efficient assets. This work introduces a new methodology for performance analysis by arbitrating between profitability and efficiency in DEA game cross-efficiency framework.

Applying DEA in a MCDM context, a common problem rose, that of multiplicity of 100% efficient DMUs. Thus ranking DMUs can be quite hard. Indeed, this further support the lack of discrimination power of this methodology (Berger and Humphrey (1997)) and the unrealistic weighting scheme. Proposed by Sexton et al. (1986) and developed in Doyle and Green (1994a), the DEA cross-efficiency method provides a unique ordering of DMUs and eliminates unrealistic weights schemes. It uses DEA in a peer-evaluation instead of self-evaluation. Liu et al. (2016) identified DEA cross-efficiency method as one of the four research fronts in DEA. In fact, this approach has been widely used to rank DMUs in many various areas. See, among others, the application in Wang and Chin (2011) to industrial robots, in Dotoli et al. (2015) to healthcare systems, in Liu et al.(2017b) to coal-fire power plants in energy field, in Wu et al. (2016) to R&D projects. Even though cross-efficiency was applied successfully to multiple real world problems, it still has some shortcomings as the non-uniqueness of the DEA optimal weights (Despotis 2002), which may reduce its usefulness. Several models have been suggested as a remedy to the cross-efficiency issue. The most known solution that has been suggested is to introduce a secondary goal to resolve the ambiguity of which alternative solution to the Linear program (LP) to use. The well-known models using multiple objective LP are the aggressive and benevolent formulations (Sexton et al. (1986), Doyle and Green (1994a) and Wu et al. (2016)). However, the various cross-efficiency scores proposed with secondary goals lead to different average cross-efficiencies and different rankings. To solve this issue, Liang et al. (2008a) adopted a non-cooperative game approach in which DMUs are competing among each other. In fact, DEA game-cross evaluation scores are considered as payoffs and each DMU may choose to take a non-cooperative game stance to the extent that it will attempt to maximize its worst possible payoff. The set of strategies played by each DMU would be weights selection. The obtained game cross-efficiency scores are unique and constitute Nash equilibrium point. DEA game cross-efficiency assessment has received much attention in the related literature (see, for example, Wu et al. (2009a), Wang and Chin (2010b), Chen and Zhu (2011), Roboredo et al.(2015), Wu and Liang (2012), Chen et al. (2017b) and Liu et al. (2017a)). Despite the effectiveness of DEA cross-efficiency evaluation, works involving this method are still rare especially in portfolio management. Lim et al. (2014) was the first to propose a MV DEA cross-efficiency to portfolio selection, succeeded by Mashayekhi and Omrani (2016) work which introduces a multi-objective Markowitz–DEA cross-efficiency model with fuzzy returns for portfolio selection. Combining DEA cross-efficiency technique and Analytic Hierarchy Process, Danesh et al. (2017) develop a method to portfolio selection. In portfolio area, the DEA cross-efficiency assessment is a democratic process with combining the self-assessment and the peer-assessment. However, the non-uniqueness of the average cross-efficiency scores reduces the robustness of the framework. In this work, we propose the use of DEA game cross-efficiency to portfolio performance assessment. We deal with the problem of non-uniqueness of optimal weights when using the simple cross-efficiency approach. The DEA game cross-efficiency evaluation discriminates in a better way among financial assets. Therefore, the stocks efficiency evaluation and rankings based upon DEA game cross-efficiency method are more reliable and will benefit the investor (Essid et al. (2018)). Our approach gives also the scope to make decision about profits. In fact, investor may make a tradeoff between efficiency which is a result of a set of attributes in the one hand and gains in the other hand. In order to reflect investor' preferences, we use higher order moments of returns distribution as inputs and outputs to compute efficiency scores of financial assets. Firms listed on Paris stock exchange serves as illustration of this framework.

The paper is organized as follows. Section 2 reviews the DEA game cross-efficiency model and the problems arising in the simple use of such method as an evaluation tool; section 3 describes the development of the proposed approach, followed by the case study in section 4. Section 5 concludes.

2. DEA Game cross-efficiency evaluation

DEA is commonly used to evaluate the relative efficiency of a number of financial assets. It allows every DMU to pick its own particular weights so as to amplify its own overall ratings subject to certain conditions. The problem of measuring efficiency is formulated as a LP which is solved for each DMU under evaluation. The analysis in this section requires the definition of Charnes et al. (1978) DEA model. Given a set of n DMUs where a $DMU_j\left(j=1,2,\dots,n\right)$ utilizes a set of m inputs $x_{ij}\left(i=1,\dots,m\right)$ to produce s outputs $y_{rj}\left(r=1,\dots,s\right)$ where $y_{rj}, x_{ij} \geq 0$. The standard input-oriented constant returns to scale (CRS) DEA model can be represented as follows:

$$
\begin{aligned}
& E_{dd} = Max \;\; Z = \sum_{r}^{s} \mu_r y_{rd} \\
& s.t. \quad \sum_{r}^{s} \mu_r y_{rj} - \sum_{i}^{m} \omega_i x_{ij} \leq 0 \quad j=1,\dots,n \\
& \qquad \sum_{i}^{m} \omega_i x_{id} = 1 \\
& \qquad \omega_i, \mu_r \geq 0
\end{aligned}
\tag{4.1}
$$

Where μ_r and ω_i are the set of output and input weights respectively to be determined through solving the above model. DEA framework permits to have more than 100% efficient DMU. Thus, discrimination among all DMUs is problematic and ranking DMUs can be quiet hard. Furthermore, the flexibility in weight selection provided by this method can lead to unrealistic weights scheme as the DMUs under evaluation heavily weights few favorable inputs/outputs to maximize its own performance score. In order to solve such issues, cross-efficiency was developed by Doyle and Green (1994a). This approach is based on the idea of peer-evaluation upon solving model (4.1), an efficiency score of DMU_d is obtained from which a cross-efficiency score E_{dj} for each of the other $(n-1)$ DMUs will be determined based on DMU_d ‘s optimal weights.

$$E_{dj} = \frac{\sum_{r}^{s} \mu_r^d y_{rj}}{\sum_{i}^{m} \omega_i^d x_{ij}} \quad j = 1, ..., (n-1) \tag{4.2}$$

Where d denote the optimal weights of DMU_d. Finally, each DMU’s cross-efficiency score $\bar{E}_j$ will be given through averaging its peer ratings as follows:

$$\bar{E}_j = \frac{1}{n} \sum_{d}^{n} E_{dj} \tag{4.3}$$

Even though this methodology provides more discrimination for DMU ranking, its main shortcoming is the non-uniqueness of weights obtained from solving the DEA model (4.1), resulting in arbitrary efficiency score depending on the optimal solution generated by the software in use (Despotis (2002)). For this reason, the use of game cross-efficiency is justified as it provides more stable weights and a Nash equilibrium evaluation score. The game cross-efficiency model was developed by Liang et al.(2008a) through the addition of a secondary goal to the basic DEA model. Furthermore, the authors developed an algorithm to find the Nash equilibrium.

Given an agent DMU_d with an efficiency score α_d, the other DMU_j tries to select a set of strategies (Weights selection) to maximize its own efficiency while ensuring that α_d won’t decrease.

$$
\begin{aligned}
& Max \;\; Z = \sum_{r}^{s} \mu_{rj}^{d} y_{rj} \\
& s.t. \;\; \sum_{r}^{s} \mu_{rj}^{d} y_{rl} - \sum_{i}^{m} \omega_{ij}^{d} x_{il} \le 0 \quad , l = 1,\dots,n \\
& \qquad \sum_{i}^{m} \omega_{ij}^{d} x_{ij} = 1 \\
& \qquad \alpha_d \sum_{i}^{m} \omega_{ij}^{d} x_{id} - \sum_{r}^{s} \mu_{rj}^{d} y_{rd} \le 0 \\
& \qquad \mu_{rj}^{d} \ge 0 \;\; \forall \;\; r = 1,\dots,s \\
& \qquad \omega_{ij}^{d} \ge 0 \;\; \forall \;\; i = 1,\dots,m \\
& \qquad \omega_i, \mu_r \ge 0
\end{aligned}
\tag{4.4}
$$

Note $\alpha_d \le 1$ which takes initially the value $\overline{E}_d$ from (4.3), is the average cross-efficiency of DMU_d, when the algorithm converges, this α_d becomes the game cross-efficiency. In addition, the constraint $\alpha_d \sum_{i}^{m} \omega_{ij}^{d} x_{id} - \sum_{r}^{s} \mu_{rj}^{d} y_{rd} \le 0$ in model (4.4) is equivalent to $\dfrac{\sum_{r}^{s} \mu_{rj}^{d} y_{rd}}{\sum_{i}^{m} \omega_{ij}^{d} x_{id}} \ge \alpha_d$ which implies the restriction of DMU_d initial score to ensure that it won't deteriorate. The above model is solved once for each DMU_d thus n times, in addition the optimal value to model (4.4) will represent a game cross-efficiency with respect to DMU_d. Indeed, the average game cross-efficiency score for DMU_j would be $\alpha_j = \dfrac{1}{n} \sum_{d}^{n} \sum_{r}^{s} \mu_{rj}^{d*}(\alpha_d) y_{rj}$ where $\mu_{rj}^{d*}(\alpha_d)$ is an optimal solution to model (4.4). Under game cross-efficiency evaluation, all players (DMUs) are assigned their Nash equilibrium scores.

3. Profitability Game cross-efficiency evaluation approach to portfolio selection

We consider the problem of an investor selecting a portfolio among a set of risky assets. While the simple use of game cross-efficiency can result in moderately consistent portfolio compared to the market indices it still lacks the control over the minimum achievable return. For this reason, we seek to develop a profitability game cross-efficiency model in which we seek to maximize the overall efficiency of the portfolio subject to given level of profit to be set by the investor.

In fact, under a cross-evaluation approach we are in what might be a democratic vote, where a set of factors are voted to be of high importance by the majority of DMUs while the rest are of low importance. Moreover, as we are in a multi-criteria context, we are in a tradeoff between the different attributes or criteria. Indeed, the round players in game cross-efficiency are not necessarily the one holding the highest return in that time horizon, given that the fact of having high return does not imply good performance on the other attributes. The developed model maximizes the portfolio overall efficiency subject to a minimum achievable return set by the investor. The annual return r_j of a stock j in the year t is determined as follows: $r_{jt} = Log(P_{jt}) - Log(P_{jt-1})$ where P_{jt} is the stock price in the year t and P_{jt-1} is the stock price in the year $t-1$.

Given the annual return r_j and the DEA game cross-efficiency score e_j of a stock j $(j = 1...n)$, for a portfolio Ω we define the portfolio return R_Ω and the portfolio efficiency E_Ω as follows: $R_\Omega = \frac{1}{K}\sum_{j=1}^{n} w_j r_j$ and $E_\Omega = \frac{1}{K}\sum_{j=1}^{n} w_j e_j$ where $\begin{cases} w_j = 1 \ \ if \ \ j \in \Omega \\ \quad = 0 \ \ otherwise \end{cases}$, $\sum_{j=1}^{n} w_j = K$ and $K \in \{1...n\}$ is the size of the portfolio, i.e. the number of stocks in a portfolio.

An optimal "PE portfolio" is determined by solving the following linear optimization model:

$$
\begin{aligned}
&\underset{w_j}{Max}\ E_\Omega \\
&s.t \quad R_\Omega \geq (1-\gamma)\,\mathrm{R}_m \\
&\qquad \sum_{j=1}^{n} w_j = K \\
&\qquad w_j \in \{0,1\},\ j = 1,...,n
\end{aligned}
\tag{4.5}
$$

where γ is the Profitability-Efficiency (PE) tradeoff parameter and R_m represent the minimum achievable return to be fixed by the investor.

In the empirical illustration of our approach (model (4.5)), among n stocks we set $K = 30$ for the "PE portfolio" that is about 6% of the sample size. We also select a "CCR portfolio" containing the 30 best performer stocks based on DEA approach (model (4.1)), "Game portfolio" which includes the 30 best performer stocks according to the DEA game cross-efficiency framework (model (4.4)) and the "Arbitrary cross portfolio" which holds the

30 best performer stocks according to the DEA cross-efficiency approach (model (4.3)). Furthermore, to consider various investor preferences, we will explore the results at three different arbitrary PE tradeoff level: 10%, 20%, and 30%.

4. An application to portfolio selection in the Paris stock exchange

4.1 Data and input/output matrix

Several criteria have been proposed in the literature regarding the choice of input and output and consequently the definition of financial efficient frontier in portfolio analysis. To assess financial assets efficiency, we use DEA under benchmarking approach (Tarnaud and Leleu (2017)). We choose criteria relating the inferred behavior of investor towards input or output variables from what is assumed to be their preference or aversion to these variables. Our approach is underlying preferences of the decision-makers only. In fact, we identify which variables are to be maximized and which variables are to be minimized instead of identifying which variables produce or produced by the others. Any variable “less-the-better” type according to the terminology used by Cook et al. (2014) or “small preferred” performance measures in Wilkens and Zhu (2001) ought to be considered as input. Whereas any variable of the “more-the-better” type or the “large-preferred” performance measures ought to be considered as output. Then, we use the four first moments of the distribution of returns in the analysis, considering variance and kurtosis as inputs and skewness and mean as outputs. Indeed, investors make their investment choices by considering the returns approximated by the mathematical mean return and the risk measured by the return dispersion represented by the variance (Basso and Funari (2001)). The variance is used to measure the variability of returns, presenting the total risk of financial asset. Because of non-normality of return distribution, recent studies enlarge the analysis dimension to the skewness or/and kurtosis. In fact, investors prefer positive skewness since it implies a low probability of obtaining a large negative return (Gregoriou et al. (2005b)). When distribution is skewed to the right, then the frequency distribution has a long right tail while when it is skewed to the left, then large negative returns are more common than large positive returns and the tail distribution is heavier on the left. As to kurtosis aversion, Menezes and Wang (2005) define outer risk in terms of transfer of actuarially neutral noise from the center of a distribution to its tail. In fact, when data has more peakdness than the normal distribution (long tails), kurtosis is greater than three. While in case we have lower peak we have platy kurtosis (bounded distribution).

As an illustration of our approach, we report a case study involving 500 ~ 508 firms[21] from the Paris Exchange. We have used actual financial data from 2010 to 2015. Our approach is applied as mean for stocks selection to portfolio. To form the sample, we check the completeness of the monthly stock return information and include only those firms without missing values. It contains 72 monthly return[22] observations in common for all assets on which the first fourth centered moments have been calculated by year. Table 4.1 presents computation details and descriptive statistics of input/output variables.

Table 4.1: Descriptive statistics

			2010	2011	2012	2013	2014	2015
Variables	**Number of firms**		**505**	**508**	**500**	**500**	**502**	**501**
Inputs	**Variance** $=\sigma^2=\frac{1}{T}\sum_{t=1}^{T}(r_{jt}-\bar{r}_j)^2$	Mean	0.02802	0.01417	0.01170	0.01370	0.01353	0.01429
		SD	0.36103	0.03445	0.01645	0.04856	0.03348	0.03354
		Median	0.00605	0.00677	0.00632	0.00449	0.00549	0.00557
		Min	0.00007	0.00020	0.00000	0.00001	6.19261	0.00005
		Max	8.09060	0.48306	0.16850	0.78392	0.43569	0.34032
	Kurtosis $=\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{jt}-\bar{r}_j}{\sigma_j}\right)^4\right)-3$	Mean	0.69328	0.82682	0.88967	0.83699	1.07229	0.83599
		SD	2.04623	2.03163	2.10029	2.12155	2.33403	2.16771
		Median	0.13780	0.28155	0.28374	0.17542	0.35323	0.17618
		Min	-1.86712	-1.94858	-1.73831	-1.91877	-1.83729	-2.06950
		Max	11.95180	11.58814	11.87129	10.34938	11.94657	10.20691
Outputs	**Mean** $=\bar{r}_j=\frac{1}{T}\sum_{t=1}^{T}r_{jt}$	Mean	0.00990	-0.00920	0.00636	0.02041	0.01107	0.01409
		SD	0.05037	0.03753	0.03432	0.07296	0.03811	0.04000
		Median	0.00704	-0.01048	0.00709	0.01283	0.00935	0.01187
		Min	-0.19268	-0.20662	-0.16219	-0.21808	-0.18535	-0.21863
		Max	0.75259	0.28533	0.28464	1.32518	0.40305	0.24052
	Skewness $=\frac{1}{T}\sum_{t=1}^{T}\left(\frac{r_{jt}-\bar{r}_j}{\sigma_j}\right)^3$	Mean	0.30745	0.17172	0.23793	0.33719	0.46271	0.48392
		SD.	0.87636	0.90296	0.93395	0.90963	0.92997	0.85872
		Median	0.26992	0.12013	0.15933	0.29090	0.40561	0.38977
		Min	-2.86149	-2.66648	-3.43831	-2.70227	-2.45524	-1.98872
		Max	3.45448	3.38537	3.17121	3.11380	3.45373	3.10527

We compute for each firm the monthly return and we select variance and kurtosis as inputs on the one hand and skewness and mean returns as outputs on the other hand. To eliminate the problem of negative values in DEA, we add to the variable one plus the absolute value of the smallest value it assumes.In fact, according to "the translation invariance property" for DEA formulations, the above transformation would conserve each variable scale

[21] The sample changes from year to year beacause the entry, exit and survival of firms listed on the Paris Stock exchange.
[22] Data stock price are available using www.euronext.com website.

without impacting efficiency classification (Ali and Seiford (1990)).

4.2 Results and discussion

In order to select portfolios, we consider a buy-and-hold strategy, where this year optimal portfolio is selected through solving CCR, cross-efficiency, game cross-efficiency or PE DEA game cross-efficiency approaches. Then, we compare the performance of these portfolios in terms of volatility and returns. Each portfolio is held for an investment horizon of one year and revised each new investment horizon. For each investment horizon, the portfolio sizes are fixed to a certain level (30 stocks) with equally weighted stocks. More specifically, at the beginning of each investment horizon the set of stocks are ranked based on each model's solution in a decreasing order of efficiency. Once a portfolio is selected, we assume that the same dollar amount is invested in each of the stocks belonging to the portfolio with no more transaction or taxes to be made until the end of the investment horizon. This strategy implies that investment cost will only be incurred only at the end of each investment horizon. The results of the analysis of CCR, arbitrary cross-efficiency and game cross-efficiency scores are summarized in Table 4.2.

Table 4.2: Statistical description of efficiency scores using CCR, DEA cross-efficiency, DEA Game cross-efficiency approaches

	2010			2011			2012			2013			2014			2015		
	CCR	**CROS**	**GAME**	**CCR**	**CROS**	**GAME**	**CCR**	**CROS**	**GAME**	**CCR**	**CROS**	**GAME**	**CCR**	**CROS**	**GAME**	**CCR**	**CROS**	**GAME**
Mean	0 .262	0.203	0.234	0.243	0.209	0.226	0.140	0.110	0.121	0.273	0.204	0.232	0.300	0 .219	0.251	0.255	0.179	0.207
Median	0.207	0.170	0.189	0.193	0.164	0.179	0.103	0.084	0.092	0.237	0.178	0.203	0.258	0.185	0.213	0.222	0.163	0.187
Min	0.001	0.000	0.000	0.006	0.006	0.006	0.009	0.007	0.008	0.029	0.010	0.016	0.024	0.012	0.014	0.023	0.012	0.015
Max	1	0.973	0.999	1	0.956	0.997	1	0.957	0.993	1	0.979	1	1	0.975	1	1	0.978	1

Analyzing the efficiencies of firms from a self-evaluation point of view (CCR model), only five of 505, nine of 508, five of 500, six of 500, six of 502 and six of 501 are evaluated as efficient for the year 2010, 2011, 2012, 2013, 2014 and 2015 respectively. The median analysis shows that more than half of the financial assets got efficiencies that are smaller than 0.207, 0.193, 0.103, 0.237, 0.258 and 0.222 over a 6 year period from 2010 to 2015. Moreover, the average DEA efficiency scores of firms are 0.262, 0.243, 0.140, 0.273, 0.300 and 0.255 for the study period which are higher than the respective median efficiency score values. These results prove that most of firms listed on Paris stock exchange did not perform well and some actions need to be taken to improve efficiency of these firms.

Table 4.3-4.7 contain results[23] about portfolios composition for the years 2010, 2012, 2014 and 2015. The 30 best-performer assets using self-evaluation method are presented in Table 4.3. A portfolio composed from these stocks have the average DEA scores (0.8322, 0.8156, 0.5702, 0.7486, 0.8482, 0.7466) over the study years respectively.

Table 4.3: Portfolio selection based on CCR framework

DMU 2010	CCR*	DMU 2012	CCR	DMU 2014	CCR	DMU 2015	CCR
PLANT ADVANCED	1.0000	IVALIS	1.0000	LE NOBLE AGE	1.0000	CFAO	1.0000
EUTELSAT COMMUNIC.	1.0000	VALTECH	1.0000	PAREF	1.0000	CHARGEURS	1.0000
DASSAULT AVIATION	1.0000	CHINA SUPER POWER	1.0000	AFONE	1.0000	FIDUCIAL OFF.SOL.	1.0000
ST DUPONT	1.0000	LOREAL	1.0000	IRDNORDPASDE CALAIS	1.0000	CHAUF.URB.	1.0000
IRDNORDPASDECA LAIS	1.0000	FIDUCIAL OFF.SOL.	1.0000	FIDUCIAL OFF.SOL.	1.0000	ALTAREIT	1.0000
BAINS MER MONACO	0.9906	CNIM CONSTR.FRF 10	0.8216	MERCK AND CO INC	1.0000	BRICORAMA	1.0000
RAMSAY GEN SANTE	0.9483	SABETON	0.7942	SOFRAGI	0.9689	PRECIA	0.9032
DANONE	0.9466	SOFRAGI	0.7933	DANONE	0.9593	MALTERIES FCO-BEL.	0.8584
FREY	0.9413	IRDNORDPASDE CALAIS	0.7768	FONCIERE LYONNAISE	0.9335	EUROMEDIS GROUPE	0.7991
ESSO	0.9257	VILMORIN	0.7447	SIGNAUX GIROD	0.9155	INSTALLUX	0.7988
FONCIERE INEA	0.9031	EUROGERM	0.6763	SABETON	0.8978	CRCAM LANGUED CCI	0.7952
BIC	0.8963	GAMELOFT SE	0.5180	EXEL INDUSTRIES	0.8957	COURTOIS	0.7624
L'OREAL	0.8952	ZODIAC AEROSPACE	0.4936	BNP PARIBAS ACT.A	0.8819	CRCAM NORM.SEINE	0.7536
ESSILOR INTL.	0.8527	FONCIERE DE PARIS	0.4301	MONTEA C.V.A.	0.8534	SELECTIRENTE	0.7389
STEF	0.8160	STEF	0.4214	CFAO	0.8438	IRDNORDPASD ECALAIS	0.7166
SELECTIRENTE	0.8089	VIDELIO	0.4150	COURTOIS	0.8372	SCBSM	0.7105
ALTAREIT	0.7962	BIGBEN INTERACTIVE	0.4124	NETBOOSTER	0.8108	TURENNE INV	0.6923
VIEL ET COMPAGNIE	0.7816	FONCIERE INEA	0.3990	ADL PARTNER	0.7940	VEOLIA ENVIRON.	0.6684
FDL	0.7582	TFF GROUP	0.3963	SELECTIRENTE	0.7906	SI PARTICIPATIO NS	0.6649
IGE XAO	0.7479	BERNARD LOISEAU	0.3840	PRECIA	0.7886	CAPELLI	0.6516
MERCK AND CO INC	0.7466	EUROSIC	0.3828	GROUPE EUROTUNNEL	0.7873	SIDETRADE	0.6077
TURENNE INV	0.7439	DASSAULT SYSTEMES	0.3819	CRCAM TOURAINE CCI	0.7841	SMTPC	0.6055
ROTHSCHILD	0.7305	LEBON	0.3745	TFF GROUP	0.7721	SALVEPAR	0.6052
DEMOS	0.7213	MERCIALYS	0.3740	ROTHSCHILD	0.7356	MR BRICOLAGE	0.6036
KORIAN	0.6877	FONCIERE EURIS	0.3658	INFOTEL	0.7234	FLEURY MICHON	0.6033
FONCIERE EURIS	0.6849	SES	0.3640	BIOMERIEUX	0.7192	EFESO CONSULTING	0.5912
TOUAX	0.6763	BRICORAMA	0.3562	PROCTER GAMBLE	0.7111	ROUGIER S.A.	0.5740
FROMAGERIES BEL	0.6605	INSTALLUX	0.3463	SAFT	0.6881	NEURONES	0.5691
PROCTER GAMBLE	0.6578	PERRIER (GERARD)	0.3426	SIMO INTERNATIONAL	0.6777	CRCAM SUD R.A.CCI	0.5687

[23] Results of years 2011 and 2013 are not reported in tables for reason of presentation and are available upon request.

GRAND MARNIER	0.6468	SARTORIUS STED BIO	0.3425	BERNARD LOISEAU	0.6768	LAFUMA	0.5552
Mean	0.8322	Mean	0.5702	Mean	0.8482	Mean	0.7466

*CCR: DEA efficiency score

Table 4.4 presents the best 30 performers stocks in the sample using DEA cross-efficiency analysis. The peer-evaluation gives a unique order proving the discrimination power of cross-efficiency evaluation. In fact, only two firms (EUTELSAT COMMUNIC and DANONE) in 2010, three firms (SES, MAROC TELECOM and TIPIAK) in 2011, two firms (L'OREAL and IVALIS) in 2012, one firm (EULER HERMES GROUP) in 2013, one firm (MERCK AND CO INC) in 2014 and one firm (ALTAREIT) in 2015 got a cross-efficiency scores higher than 0.9. Moreover, 50% of assets have cross-efficiency scores less than 0.1705, 0.1646, 0.0840, 0.1788, 0.1853 and 0.1633 for the respective study years. Among them, the worst performer stock is HOLSFIND with a cross-efficiency score equal to 0.0006 in the year 2010. These results are quite similar to the CCR results. Therefore, it is of urgent need for these firms to take action to improve their efficiency.

Table 4.4: Portfolio selection based on DEA cross-efficiency evaluation

DMU 2010	Cross*	DMU 2012	Cross	DMU 2014	Cross	DMU 2015	Cross
EUTELSAT COMMUNIC.	0.9736	LOREAL	0.9575	MERCK AND CO INC	0.975	ALTAREIT	0.9788
DANONE	0.9024	IVALIS	0.9557	DANONE	0.8362	CFAO	0.7314
DASSAULT AVIATION	0.8661	CNIM CONSTR.FRF 10	0.7365	BNP PARIBAS ACT.A	0.7291	CRCAM NORM.SEINE	0.5922
ESSO	0.8525	SABETON	0.7353	LE NOBLE AGE	0.7256	CHARGEURS	0.5718
LOREAL	0.7493	VILMORIN	0.6828	SIGNAUX GIROD	0.6738	SELECTIRENTE	0.5378
ST DUPONT	0.7053	SOFRAGI	0.6793	SABETON	0.6561	CHAUF.URB.	0.525
FREY	0.7029	ZODIAC AEROSPACE	0.4392	PAREF	0.6457	INSTALLUX	0.5138
RAMSAY GEN SANTE	0.6968	TFF GROUP	0.3554	EXEL INDUSTRIES	0.6305	PRECIA	0.4755
ESSILOR INTL.	0.6823	BERNARD LOISEAU	0.3551	CRCAM TOURAINE CCI	0.6292	CRCAM LANGUED CCI	0.4686
SELECTIRENTE	0.6576	LEBON	0.353	NETBOOSTER	0.6081	LINEDATA SERVICES	0.4587
STEF	0.6364	MERCIALYS	0.3469	GROUPE EUROTUNNEL	0.6012	FLEURY MICHON	0.4584
MERCK AND CO INC	0.6342	DASSAULT SYSTEMES	0.3422	SOFRAGI	0.5986	SAINT GOBAIN	0.4451
BIC	0.6321	SES	0.3406	ALES GROUPE	0.5859	COURTOIS	0.4326
FONCIERE EURIS	0.5942	GAMELOFT SE	0.3308	CFAO	0.585	EUROMEDIS GROUPE	0.4302
ROTHSCHILD	0.5675	BRICORAMA	0.3245	MONTEA C.V.A.	0.5771	SMTPC	0.4279
PLANT ADVANCED	0.5448	INSTALLUX	0.3192	BIOMERIEUX	0.5771	EFESO CONSULTING	0.4238
CATERING INTL SCES	0.5419	SARTORIUS STED BIO	0.3161	ADL PARTNER	0.5731	VEOLIA ENVIRON.	0.4222
VIEL ET COMPAGNIE	0.5353	PERRIER (GERARD)	0.3138	TFF GROUP	0.5596	TURENNE INV	0.4208
CA TOULOUSE 31 CCI	0.5342	STEF	0.3103	BIC	0.5405	CRCAM SUD R.A.CCI	0.4145
IRDNORDPASDECA	0.5238	ADL PARTNER	0.3032	DASSAULT	0.5313	NEURONES	0.4115

LAIS				SYSTEMES			
FROMAGERIES BEL	0.5119	TRILOGIQ	0.2814	BERNARD LOISEAU	0.5294	LAFUMA	0.4109
GRAND MARNIER	0.5066	HOTELS DE PARIS	0.2809	SELECTIRENTE	0.5241	EURAZEO	0.406
SANOFI	0.5045	SIDETRADE	0.2798	COURTOIS	0.5219	SCBSM	0.4042
KORIAN	0.4971	RAMSAY GEN SANTE	0.2701	HSBC HOLDINGS	0.5159	SALVEPAR	0.3902
FONCIERE INEA	0.4909	ENVIRONNEMEN T SA	0.2696	PROCTER GAMBLE	0.5138	ATOS	0.3839
SOMFY SA	0.4837	ICADE	0.2623	ESPERITE	0.5035	PATRIMOINE ET COMM	0.3824
TURENNE INV	0.478	BIC	0.2509	SIMO INTERNATIONAL	0.4911	GROUPE EUROTUNNEL	0.3814
TOUPARGEL GROUPE	0.4755	BONDUELLE	0.2492	SAFT	0.4889	TERREIS	0.3793
GROUPE FLO	0.4744	PRODWARE	0.2453	CASINO GUICHARD	0.4788	THERMADOR GROUPE	0.376
BAINS MER MONACO	0.4701	DANONE	0.2447	EULER HERMES GROUP	0.4779	FONCIERE DE PARIS	0.3755
Mean	0.6142	Mean	0.4044	Mean	0.5961	Mean	0.4676

*Cross: the DEA cross-efficiency score

To measure the percentage increment of a stock j when shifting from peer-evaluation to self-evaluation, we compute the maverick index M_j proposed by Green et al. (1996) as follows:

$$M_j = \left(E_{jj} - \bar{E}_j\right) / \bar{E}_j \tag{4.7}$$

Where E_{jj} is the CCR score of stocks j determined through resolving model (1) and $\bar{E}_j$ is the average of all cross-efficiency scores E_{dj} determined by equation (4.3). A financial asset with a low maverick index benefits the least when moving from peer-evaluation to self-evaluation. Hence, a higher maverick index yields a poor performance. The maverick index of the thirty best performer assets using cross-efficiency evaluation are presented in Table 4.5. In 2010, ALTARAEIT which has 0.7962 self-evaluation has a low average score of only 0.2222 and at the same time the highest maverick index of 258.333%. It presents the percentage improvement when moving from peer-appraisal to self-appraisal. Such firm is called maverick or “false positive”. Indeed, besides providing a unique ranking among assets cross-efficiency analysis can be used to identify the maverick assets. We find that PLANT ADVANCED, GAMELOFT SE, NETBOOSTER, SOFRAGI and CHAUF.URB are cases of strong Mavericks with high values of 61.55%, 56.58%, 121.58%, 61.85% and 90.48% in 2010 to 2015 respectively. These firms don’t perform well.

Table 4.5: Maverick index based on cross-efficiency evaluation

DMU 2010	$M_j{}^*$	DMU 2012	M_j	DMU 2014	M_j	DMU 2015	M_j
PLANT ADVANCED	0.8355	LOREAL	0.0444	MERCK AND CO INC	0.0256	ALTAREIT	0.0217
EUTELSAT COMMUNIC.	0.0271	IVALIS	0.0464	DANONE	0.1473	CFAO	0.3672
DASSAULT AVIATION	0.1546	CNIM CONSTR.FRF 10	0.1156	BNP PARIBAS ACT.A	0.2096	CRCAM NORM.SEINE	0.2725
ST DUPONT	0.4178	SABETON	0.0801	LE NOBLE AGE	0.3782	CHARGEURS	0.7489
IRDNORDPASDECALAIS	0.9091	VILMORIN	0.0907	SIGNAUX GIROD	0.3587	SELECTIRENTE	0.3739
BAINS MER MONACO	1.1073	SOFRAGI	0.1679	SABETON	0.3684	**CHAUF.URB.**	**0.9048**
RAMSAY GEN SANTE	0.3610	ZODIAC AEROSPACE	0.1238	PAREF	0.5487	INSTALLUX	0.5548
DANONE	0.0490	TFF GROUP	0.1150	EXEL INDUSTRIES	0.4207	PRECIA	0.8995
FREY	0.3391	BERNARD LOISEAU	0.0813	CRCAM TOURAINE CCI	0.2462	CRCAM LANGUED CCI	0.6969
ESSO	0.0859	LEBON	0.0609	NETBOOSTER	0.3333	LINEDATA SERVICES	0.0552
FONCIERE INEA	0.8396	MERCIALYS	0.0781	GROUPE EUROTUNNEL	0.3095	FLEURY MICHON	0.3161
BIC	0.4180	DASSAULT SYSTEMES	0.1160	**SOFRAGI**	**0.6185**	SAINT GOBAIN	0.1403
L'OREAL	0.1947	SES	0.0686	ALES GROUPE	0.1414	COURTOIS	0.7624
ESSILOR INTL.	0.2497	**GAMELOFT SE**	**0.5658**	CFAO	0.4424	EUROMEDIS GROUPE	0.8576
STEF	0.2822	BRICORAMA	0.0978	MONTEA C.V.A.	0.4787	SMTPC	0.4151
SELECTIRENTE	0.2301	INSTALLUX	0.0848	BIOMERIEUX	0.2462	EFESO CONSULTING	0.3950
ALTAREIT	**2.5833**	SARTORIUS STED BIO	0.0834	ADL PARTNER	0.3854	VEOLIA ENVIRON.	0.5830
VIEL ET COMPAGNIE	0.4601	PERRIER (GERARD)	0.0916	TFF GROUP	0.3798	TURENNE INV	0.6453
FDL	0.8744	STEF	0.3581	BIC	0.0866	CRCAM SUD R.A.CCI	0.3719
IGE XAO	0.6083	ADL PARTNER	0.1121	DASSAULT SYSTEMES	0.2042	NEURONES	0.3830
MERCK AND CO INC	0.1772	TRILOGIQ	0.1010	BERNARD LOISEAU	0.2785	LAFUMA	0.3511
TURENNE INV	0.5562	HOTELS DE PARIS	0.0923	SELECTIRENTE	0.5086	EURAZEO	0.2934
ROTHSCHILD	0.2872	SIDETRADE	0.1857	COURTOIS	0.6041	SCBSM	0.7578
DEMOS	0.7814	RAMSAY GEN SANTE	0.1132	HSBC HOLDINGS	0.1313	SALVEPAR	0.5511
KORIAN	0.3834	ENVIRONNEMENT SA	0.0667	PROCTER GAMBLE	0.3840	ATOS	0.3631
FONCIERE EURIS	0.1527	ICADE	0.1545	ESPERITE	0.2930	PATRIMOINE ET COMM	0.4238
TOUAX	0.5107	BIC	0.1114	SIMO INTERNATIONAL	0.3799	GROUPE EUROTUNNEL	0.3888
FROMAGERIES BEL	0.2902	BONDUELLE	0.1044	SAFT	0.4074	TERREIS	0.1264
PROCTER GAMBLE	0.6387	PRODWARE	0.0751	CASINO GUICHARD	0.3487	THERMADOR GROUPE	0.2317
GRAND MARNIER	0.2767	DANONE	0.1467	EULER HERMES GROUP	0.1441	FONCIERE DE PARIS	0.3277

${}^*M_j$: The maverick index

Doyle and Green (1994a) propose the cross-efficiency evaluation to maximize discrimination among DMUs. However, it has been argued that cross-efficiency is not unique and secondary goals may be imposed. Liang et al. (2008b) show that whatever the secondary goal in use (arbitrary, aggressive or benevolent), the cross-efficiency scores leads to the same game cross-efficiency scores that is the Nash equilibrium solution. Using model (4.4), we compute the DEA game cross-efficiency scores of firms and we present the 30 best performer firms in Table 4.6.

Table 4.6 : Portfolio selection based on game cross-efficiency evaluation

DMU 2010	Game*	DMU 2012	Game	DMU 2014	Game	DMU 2015	Game
EUTELSAT COMMUNIC.	0.999	LOREAL	0.993	MERCK AND CO INC	1	ALTAREIT	1
DASSAULT AVIATION	0.9689	IVALIS	0.9918	DANONE	0.9069	CFAO	0.8052
DANONE	0.9372	CNIM CONSTR.FRF 10	0.7985	BNP PARIBAS ACT.A	0.824	CRCAM NORM.SEINE	0.6544
ESSO	0.9122	SABETON	0.7843	LE NOBLE AGE	0.8238	CHAUF.URB.	0.6528
LOREAL	0.8475	SOFRAGI	0.7544	SIGNAUX GIROD	0.783	CHARGEURS	0.6453
FREY	0.8365	VILMORIN	0.7304	EXEL INDUSTRIES	0.7515	PRECIA	0.6102
RAMSAY GEN SANTE	0.8283	ZODIAC AEROSPACE	0.4704	SABETON	0.7219	SELECTIRENTE	0.6009
ESSILOR INTL.	0.7836	TFF GROUP	0.3853	PAREF	0.7202	INSTALLUX	0.6008
ST DUPONT	0.7745	BERNARD LOISEAU	0.3796	CRCAM TOURAINE CCI	0.7132	CRCAM LANGUED CCI	0.5616
BIC	0.7707	DASSAULT SYSTEMES	0.3699	NETBOOSTER	0.696	EUROMEDIS GROUPE	0.5429
PLANT ADVANCED	0.7608	MERCIALYS	0.3694	SOFRAGI	0.6766	COURTOIS	0.5219
SELECTIRENTE	0.7453	LEBON	0.3693	ADL PARTNER	0.6615	FLEURY MICHON	0.5089
STEF	0.7405	SES	0.3603	GROUPE EUROTUNNEL	0.6562	TURENNE INV	0.5008
MERCK AND CO INC	0.7147	STEF	0.3603	CFAO	0.6492	BRICORAMA	0.4976
IRDNORDPASDE CALAIS	0.7126	GAMELOFT SE	0.3505	MONTEA C.V.A.	0.6434	VEOLIA ENVIRON.	0.4959
ROTHSCHILD	0.6679	BRICORAMA	0.3496	ALES GROUPE	0.6387	MALTERIES FCO-BEL.	0.4897
BAINS MER MONACO	0.6638	INSTALLUX	0.3392	BIOMERIEUX	0.6217	SCBSM	0.4872
FONCIERE EURIS	0.6633	SARTORIUS STED BIO	0.338	TFF GROUP	0.6166	SMTPC	0.4828
FONCIERE INEA	0.6632	PERRIER (GERARD)	0.3335	BERNARD LOISEAU	0.6077	LINEDATA SERVICES	0.4787
TURENNE INV	0.6118	ADL PARTNER	0.3286	SELECTIRENTE	0.5895	EFESO CONSULTING	0.478
KORIAN	0.6012	SIDETRADE	0.3113	COURTOIS	0.5886	SAINT GOBAIN	0.4734
CATERING INTL SCES	0.5989	TRILOGIQ	0.3031	DASSAULT SYSTEMES	0.5826	CRCAM SUD R.A.CCI	0.4719
VIEL ET COMPAGNIE	0.5946	HOTELS DE PARIS	0.3008	BIC	0.5798	LAFUMA	0.4688
FROMAGERIES BEL	0.5928	RAMSAY GEN SANTE	0.2928	ESPERITE	0.5788	NEURONES	0.4618
CA TOULOUSE 31 CCI	0.5914	ICADE	0.2891	PROCTER GAMBLE	0.5774	EURAZEO	0.4617
IGE XAO	0.5899	ENVIRONNEMENT SA	0.2846	SAFT	0.5665	SALVEPAR	0.4615
GRAND MARNIER	0.5883	BIC	0.2709	SIMO INTERNATIONAL	0.5661	PATRIMOINE ET COMM	0.4434
TOUPARGEL GROUPE	0.5557	BONDUELLE	0.268	CASINO GUICHARD	0.5498	CAPELLI	0.4401
TOUAX	0.5543	DANONE	0.2672	HSBC HOLDINGS	0.5493	ATOS	0.4356

FDL	0.5521	UNIBAIL-RODAMCO	0.261	ARCELORMITTAL	0.5487	FIDUCIAL OFF.SOL.	0.4349
Mean	0.7140	Mean	0.4335	Mean	0.6663	Mean	0.5389

*Game: DEA game cross-efficiency score

The average efficiency level of the sample firms is about 0.2342, 0.2260, 0.213, 0.2323, 0.2510 and 0.2070 for the respective study years. The year 2010 is marked by the lowest level of game cross-efficiency score (0.0008) registered by HOLSFIND, however the highest efficiency level is equal to one and was recorded by EULER HERMES GROUP, MERCK AND CO INC and ALTAREIT for 2013, 2014 and 2015 respectively.

In Table 4.7, we present Mavericks index based on game cross-efficiency evaluation MG_j. It measures the percentage increment of a stock j when shifting from game cross-efficiency peer-evaluation to self-evaluation. We compute the MG_j as follows:

$$MG_j = \left(E_{jj} - \alpha_j\right) / \alpha_j \tag{4.8}$$

Where E_{jj} is the CCR score of stocks j determined through resolving model (4.1) and α_j is the average game cross-efficiency score of DMU_j determined by resolving the model (4.4).

As defined by Essid et al. (2018), the maverick index based on game cross-efficiency evaluation is the percentage increment when shifting from game cross-efficiency evaluation to self-evaluation. BAINS MER MONACO, MGI DIGITAL GRAPHI, GAMELOFT SE, IRDNORDPASDECALAIS, SOFRAGI and FIDUCIAL OFF.SOL are cases of strong “false positive” with high values of 49.24%, 34.61%, 85.35%, 43.20% and 129.94% in 2010 to 2015 respectively. In mostly, mavericks using cross-efficiency are higher than mavericks using game cross-efficiency. Then, game cross-efficiencies can be used to overcome the problem of mavericks in a best way.

Table 4.7: Maverick index based on Game cross-efficiency evaluation

DMU 2010	MG_j *	DMU 2012	MG_j	DMU 2014	MG_j	DMU 2015	MG_j
EUTELSAT COMMUNIC.	0.0010	LOREAL	0.0070	MERCK AND CO INC	0.0000	ALTAREIT	0.0000
DASSAULT AVIATION	0.0321	IVALIS	0.0083	DANONE	0.0578	CFAO	0.2419
DANONE	0.0101	CNIM CONSTR.FRF 10	0.0290	BNP PARIBAS ACT.A	0.0703	CRCAM NORM.SEINE	0.1516
ESSO	0.0148	SABETON	0.0126	LE NOBLE AGE	0.2139	CHAUF.URB.	0.5319
LOREAL	0.0563	SOFRAGI	0.0516	SIGNAUX GIROD	0.1692	CHARGEURS	0.5497
FREY	0.1253	VILMORIN	0.0196	EXEL INDUSTRIES	0.1919	PRECIA	0.4802

RAMSAY GEN SANTE	0.1449	ZODIAC AEROSPACE	0.0492	SABETON	0.2437	SELECTIRENTE	0.2297
ESSILOR INTL.	0.0881	TFF GROUP	0.0285	PAREF	0.3885	INSTALLUX	0.3296
ST DUPONT	0.2912	BERNARD LOISEAU	0.0115	CRCAM TOURAINE CCI	0.0995	CRCAM LANGUED CCI	0.4159
BIC	0.1630	DASSAULT SYSTEMES	0.0324	NETBOOSTER	0.1649	EUROMEDIS GROUPE	0.4720
PLANT ADVANCED	0.3144	MERCIALYS	0.0124	**SOFRAGI**	**0.4320**	COURTOIS	0.4609
SELECTIRENTE	0.0854	LEBON	0.0141	ADL PARTNER	0.2003	FLEURY MICHON	0.1855
STEF	0.1020	SES	0.0102	GROUPE EUROTUNNEL	0.1998	TURENNE INV	0.3824
MERCK AND CO INC	0.0447	STEF	0.1696	CFAO	0.2998	BRICORAMA	1.0096
IRDNORDPASDE CALAIS	0.4033	**GAMELOFT SE**	**0.4778**	MONTEA C.V.A.	0.3264	VEOLIA ENVIRON.	0.3478
ROTHSCHILD	0.0937	BRICORAMA	0.0190	ALES GROUPE	0.0471	MALTERIES FCO-BEL.	0.7528
BAINS MER MONACO	**0.4924**	INSTALLUX	0.0209	BIOMERIEUX	0.1568	SCBSM	0.4584
FONCIERE EURIS	0.0326	SARTORIUS STED BIO	0.0132	TFF GROUP	0.2522	SMTPC	0.2542
FONCIERE INEA	0.3617	PERRIER (GERARD)	0.0272	BERNARD LOISEAU	0.1138	LINEDATA SERVICES	0.0112
TURENNE INV	0.2159	ADL PARTNER	0.0261	SELECTIRENTE	0.3412	EFESO CONSULTING	0.2368
KORIAN	0.1439	SIDETRADE	0.0658	COURTOIS	0.4223	SAINT GOBAIN	0.0721
CATERING INTL SCES	0.0233	TRILOGIQ	0.0222	DASSAULT SYSTEMES	0.0982	CRCAM SUD R.A.CCI	0.2050
VIEL ET COMPAGNIE	0.3145	HOTELS DE PARIS	0.0200	BIC	0.0129	LAFUMA	0.1842
FROMAGERIES BEL	0.1141	RAMSAY GEN SANTE	0.0269	ESPERITE	0.1248	NEURONES	0.2323
CA TOULOUSE 31 CCI	0.0464	ICADE	0.0475	PROCTER GAMBLE	0.2316	EURAZEO	0.1374
IGE XAO	0.2678	ENVIRONNEMENT SA	0.0105	SAFT	0.2146	SALVEPAR	0.3115
GRAND MARNIER	0.0994	BIC	0.0293	SIMO INTERNATIONAL	0.1971	PATRIMOINE ET COMM	0.2279
TOUPARGEL GROUPE	0.0763	BONDUELLE	0.0269	CASINO GUICHARD	0.1745	CAPELLI	0.4805
TOUAX	0.2201	DANONE	0.0501	HSBC HOLDINGS	0.0625	ATOS	0.2013
FDL	0.3733	UNIBAIL-RODAMCO	0.0451	ARCELORMITTAL	0.1085	**FIDUCIAL OFF.SOL.**	**1.2994**

* MG_j : The maverick index based on game cross-efficiency evaluation

As we have already mentioned, the arbitrary cross-efficiency approach is unstable and unpredictable depending on the software in use and the problem context compared to the Nash equilibrium score provided by the game cross-efficiency. Results of Wilcoxon Signed-Rank test presented in Table 4.8 proves that there are a significant difference between ranking derived by the DEA game cross-efficiency and the ranking obtained by the arbitrary cross-efficiency approaches.

Table 4.8 : Wilcoxon Signed-Rank test "Game cross-efficiency ranking" vs. "Arbitrary cross-efficiency ranking"

Wilcoxon	2010	2011	2012	2013	2014	2015
Statistic	127760***	129290***	125250***	125250***	126250***	125750***
P-value	2.20E-16	2.20E-16	2.20E-16	2.20E-16	2.20E-16	2.20E-16

Significance level 1%***. 5%**. 10%*

However, in portfolio selection field, a major drawback of game cross-efficiency approach is the lack of investor control over the expected portfolio return. Indeed, a rational investor would have a reserve utility, in other words he would want to set a minimum expected profit level. Wilcoxon Signed Rank test conducted between game cross-efficiency ranking and annual return based ranking allows drawing the conclusion that ranking based on return is significantly different than that based on game cross-efficiency (Table 4.9). That means the efficient stock may not be gainful and vice versa. This motivates our development of a PE DEA game cross framework to portfolio selection, in which we seek to maximize the overall portfolio efficiency score subject to a specific level of expected return.

Table 4.9: Wilcoxon Signed-Rank test "Game cross-Efficiency ranking" vs. "Annual return based ranking"

Wilcoxon	2010	2011	2012	2013	2014	2015
Statistic	92509***	118340***	82225***	74464***	97264***	77298***
P-value	2.20E-16	2.20E-16	1.331E-09	0.0002497	2.20E-16	8.649E-06

Significance level 1%***. 5%**. 10%*

To present different investor's preferences, we use three PE parameter levels (10%, 20% and 30%). We select the thirty best practices that compose an optimal portfolio from the point of view gains and efficiency at the same time. This portfolio achieves the higher average efficiency at the year 2014 whatever the PE tradeoff parameter value. Then, results are not sensitive enough to tradeoff parameter change. In Table 4.10, we present the portfolio selection results of PE DEA game cross-efficiency framework when the tradeoff[24] parameter is equal to 30%.

Table 4.10: Portfolio selection based on PE_30% evaluation

DMU2010	**PE _30%***	**DMU2012**	**PE _30%**	**DMU2014**	**PE _30%**	**DMU2015**	**PE _30%**
EUTELSAT COMMUNIC.	0.999	LOREAL	0.993	MERCK AND CO INC	1	ALTAREIT	1
DANONE	0.9372	IVALIS	0.9918	DANONE	0.9069	CFAO	0.8052
DASSAULT AVIATION	0.9689	CNIM CONSTR.FR F 10	0.7985	BNP PARIBAS ACT.A	0.824	CHARGEURS	0.6453
ESSO	0.9122	SABETON	0.7843	LE NOBLE AGE	0.8238	CHAUF.URB.	0.6528
L'OREAL	0.8475	VILMORIN	0.7304	SIGNAUX GIROD	0.783	PRECIA	0.6102
ST DUPONT	0.7745	SOFRAGI	0.7544	SABETON	0.7219	LINEDATA SERVICES	0.4787
ESSILOR INTL.	0.7836	ZODIAC AEROSPAC E	0.4704	CRCAM TOURAINE CCI	0.7132	FLEURY MICHON	0.5089
BIC	0.7707	TFF GROUP	0.3853	NETBOOSTER	0.696	VEOLIA ENVIRON.	0.4959

[24] Results of PE DEA Game cross-efficiency framework when $\gamma = 10\%$ and $\gamma = 20\%$ are given upon request.

FROMAGERIES BEL	0.5928	DASSAULT SYSTEMES	0.3699	GROUPE EUROTUNNEL	0.6562	ECA	0.4077
SOMFY SA	0.5508	SARTORIUS STED BIO	0.338	ALES GROUPE	0.6387	SARTORIUS STED BIO	0.3388
ACTEOS	0.2375	STEF	0.3603	MONTEA C.V.A.	0.6434	BENETEAU	0.1559
LECTRA	0.0229	MGI DIGITAL GRAPHI	0.2359	ADL PARTNER	0.6615	GROUPE GORGE	0.1491
HOLOSFIND	0.0008	ESKER	0.2254	SAFT	0.5665	SAFT	0.064
PHILIP MORRIS INTL	0.5293	VDI GROUP	0.0765	VDI GROUP	0.1389	STEF	0.0839
BURELLE	0.2385	EASYVISTA	0.0703	EASYVISTA	0.0637	LECTRA	0.0608
VDI GROUP	0.0368	PERNOD RICARD	0.1323	CELLECTIS	0.0378	CAPELLI	0.4401
EVERSET	0.0264	TOUAXBSA R0316	0.0147	PAREF	0.7202	TRIGANO	0.1798
FREY	0.8365	BERNARD LOISEAU	0.3796	EXEL INDUSTRIES	0.7515	FAURECIA	0.1051
RAMSAY GEN SANTE	0.8283	LEBON	0.3693	SOFRAGI	0.6766	MASTRAD	0.0287
SELECTIRENTE	0.7453	MERCIALYS	0.3694	BIOMERIEUX	0.6217	CRCAM NORM.SEINE	0.6544
MERCK AND CO INC	0.7147	SES	0.3603	BIC	0.5798	SELECTIRENT E	0.6009
ROTHSCHILD	0.6679	GAMELOFT SE	0.3505	SELECTIRENTE	0.5895	INSTALLUX	0.6008
PLANT ADVANCED	0.7608	INSTALLUX	0.3392	PROCTER GAMBLE	0.5774	COURTOIS	0.5219
GRAND MARNIER	0.5883	PERRIER (GERARD)	0.3335	CFAO	0.6492	EFESO CONSULTING	0.478
TOUAX	0.5543	TRILOGIQ	0.3031	TFF GROUP	0.6166	CRCAM LANGUED CCI	0.5616
FONCIERE EURIS	0.6633	BRICORAM A	0.3496	DASSAULT SYSTEMES	0.5826	SAINT GOBAIN	0.4734
CATERING INTL SCES	0.5989	ADL PARTNER	0.3286	BERNARD LOISEAU	0.6077	EUROMEDIS GROUPE	0.5429
IRDNORDPASDE CALAIS	0.7126	HOTELS DE PARIS	0.3008	COURTOIS	0.5886	NEURONES	0.4618
FONCIERE INEA	0.6632	RAMSAY GEN SANTE	0.2928	ESPERITE	0.5788	EURAZEO	0.4617
BAINS MER MONACO	0.6638	BIC	0.2709	SIMO INTERNATION AL	0.5661	MALTERIES FCO-BEL.	0.4897
Mean	0.6076	Mean	0.4026	Mean	0.6194	Mean	0.4352

* PE _30% : the game cross-efficiency score of stocks comprising the PE portfolio when PE tradeoff parameter is equal to 30%

This type of analysis does not allow taking into account the couple return and volatility that is essential to the decision making of the investor. Thus, in the next section we use the Sharpe ratio, developed by Sharpe (1963), to evaluate the performance of the constructed portfolios based on each model. The Sharpe ratio SR_Ω of a portfolio Ω describes how much excess return received for the extra volatility that investor endure for holding a riskier asset. It is determined as follow:

$$SR_\Omega = \left(R_\Omega - R_f\right) \big/ \sigma_\Omega \tag{4.9}$$

Where R_Ω is the portfolio geometric average annual return, R_f is he risk-free return and σ_Ω is the annualized excess return volatility. Nevertheless, the standard deviation as used in the Sharpe ratio does not account for extreme returns. Thus, we use the Studentized Circular Block Bootstrap (SCBB) developed by Ledoit and Wolf (2008), which takes into consideration the skewness, kurtosis and autocorrelation effects when comparing two Sharpe ratios to test whether the differences between performances of derived portfolios are statistically significant.

4.3 Why Profitability-Efficiency DEA game cross-efficiency approach instead of other frameworks ?

In a multi-criteria analysis, an investor who has a specific attitude towards risk, a degree of prudence and temperance looks undoubtedly to make profits. While efficient portfolio means the interference of minimizing the variance (risk) and kurtosis (temperance) on the one hand and maximizing the return (profit) and the skewness (prudence) on the other hand, a given level of portfolio profitability still not guaranteed via the DEA game cross-efficiency approach. Hence, we incorporate the portfolio efficiency in a PE framework (model 5). In this paragraph, we show that the PE methodology can be a promising tool for portfolio selection. In fact, we examine the 6-year performance of the portfolio generated through the PE DEA game cross-efficiency, the DEA game cross-efficiency and the arbitrary cross-efficiency. We compare the performance of different derived portfolios to the performance of benchmark portfolios that are four market indexes: (CAC40[25], AEX[26], BEL20[27] and PSI20[28]) over the test period starting from 2010 to 2015. These benchmark portfolios are the main national indices of the stock exchange group Euronext. The intent of this comparison is to show that the portfolio selected via the developed model outperforms the best Euronext indices and beat that of arbitrary model. It has also the highest level of gains and the minimum losses.

We present in Table 4.11 the geometric annualized excess return $\left(R_{\Omega}-R_{f}\right)$ of each portfolio over the 6-year study period. The "PE portfolio" with tradeoff parameter equal to 10%, 20% and 30% attains the highest geometric mean excess return respectively, 122.94%, 91.94% and 68.24%, which is quite higher than those of the CAC40 (0.66%), AEX (2.56%), BEL20 (4.48%), PSI20 (-9.39%), "arbitrary portfolio" (4.89%) and "game portfolio" (4.7%).

[25] The CAC40 is the index of top 40 performing stocks traded in Paris Exchange.
[26] The AEX (Amsterdam Exchange index) is a stock market composed of 25 securities, the most traded on the exchange.
[27] BEL20 is performing index of the best 20 companies traded at the Brussels Stock Exchange.
[28] PSI20 is performing index of the best 20 companies listed at stock exchange of Portugal.

Furthermore, the respective Sharpe ratios of the PE portfolio" (5.8942, 5.1616, and 4.0720) are quite higher than that of the market indices, CAC40 (0.0395), AEX (0.1674), BEL20 (0.3379) and PSI20 (-0.4884) on the one hand, and higher than Sharpe ratios of "arbitrary portfolio" (0.8961) and "game portfolio" (0.9059) on the other hand.

Table 4.11: Sharpe ratios of portfolios

	PE_10%	PE_20%	PE_30%	CAC.40	AEX	BEL20	PSI20	Game cross	Arbitrary cross
Annualized Return	1.2294	0.9194	0.6824	0.0066	0.0256	0.0448	-0.0939	0.0470	0.0489
Annualized Std Dev	0.2086	0.1781	0.1676	0.1661	0.1528	0.1327	0.1923	0.0519	0.0545
Annualized Sharpe	5.8942	5.1616	4.0720	0.0395	0.1674	0.3379	-0.4884	0.9059	0.8961

Moreover, we use the R implementation of Michael Wolf to test hypothesis presented in appendix B. We apply the test on pairs of monthly excess returns of portfolios. Table 4.12 summarizes the statistical tests resulting from the R code using the default parameter setting. Based on SCBB P-values results, we reject the null hypotheses which indicate that the Sharpe ratio of the "game portfolio" is significantly greater than that of CAC40 and PSI20 at 5% significance level and greater than that of BEL20 and AEX at 10% significance level. However, we fail to reject that versus the "arbitrary portfolio" with a P-value=0.3446 even though the rank test proves that the "game cross-efficiency ranking" and "arbitrary cross-efficiency ranking" are not identical for the test period as shown in Table 4.11. These further supports claim that the "game portfolio" outperforms some of Euronext benchmark market indices and provide a more stable and meaningful ranking that incorporate firms' behavior.

Table 4.12: Two sided Sharpe difference test: the Studentized Circular Block Bootstrap (B=10. M=4999)

		CAC 40	AEX	BEL20	PSI20	Game cross	Arbitrary cross
PE_γ=10%	Difference	1.1383	1.1039	1.059	1.2933	0.9101	0.9127
	P-value	**(0.0016) *****	**(0.004) *****	**(0.0078) *****	**(0.0018) *****	**(0.008) *****	**(0.0098) *****
PE_γ=20%	Difference	1,0804	1,046	1,0011	1,2354	0,8522	0,8548
	P-value	**(0.0104) ****	**(0.0204) ****	**(0.0258) ****	**(0.0084) *****	**(0.0346) ***	**(0.0598) ***
PE_γ=30%	Difference	0.9011	0.8667	0.8218	1.0561	0.6729	0.6755
	P-value	(0.1258)	(0.1826)	(0.332)	**(0.0692) ****	(0.378)	(0.3938)
Game cross	Difference	0.2282	0.1938	0.1489	0.3832	---------	0.0026
	P-value	**(0.087) ***	(0.2244)	(0.1722)	**(0.0788) ****	---------	(0.9398)
Arbitrary cross	Difference	0.2256	0.1912	0.1463	0.3806	-0.0026	---------
	P-value	**(0.0768) ***	(0.175)	(0.167)	**(0.0482) ****	(0.9392)	---------

Significance level 1%***. 5%**. 10%*

Table 4.12 proves the statistical significant difference between the performance of PE portfolio and other portfolios. In fact, based on the P-values we find that the Sharpe ratios of the "PE portfolio" when the PE tradeoff parameter is equal to 10% or 20% are significantly greater than the Sharpe ratios of all the market indices, "arbitrary portfolio" and "game portfolio". However, we fail to prove the significance of difference between the Sharpe ratio of "PE portfolio" when γ=30% and benchmark portfolios. We can infer from the failure to reject the null hypothesis that the Sharpe ratio difference for γ=30%, that as we increase the PE tradeoff parameter, we approach similar portfolio's characteristics to that of game and arbitrary portfolios. In order to deepen the analysis of portfolios' performance, we illustrate cumulative returns and losses in figures. We find that whatever the value of PE tradeoff parameter, the portfolio derived from the developed approach has the highest growth of hypothetical initial investment over the 6 years of study. Figures 4.1-4.3 present cumulative returns curves of PE portfolio comparing to other portfolios. It is obvious that PE portfolios' cumulative return is a lot larger than other portfolios including game and arbitrary cross portfolios. In fact, the black curve of PE portfolio grows exponentially and cumulative return reaches 120%, 50% and 20% in December 2015 when the PE tradeoff parameter is equal to 0.1, 0.2 and 0.3 respectively. However, other curves are close to zero during the whole period.

Figure 4.2: Cumulative return curves of portfolios ($\gamma = 20\%$)

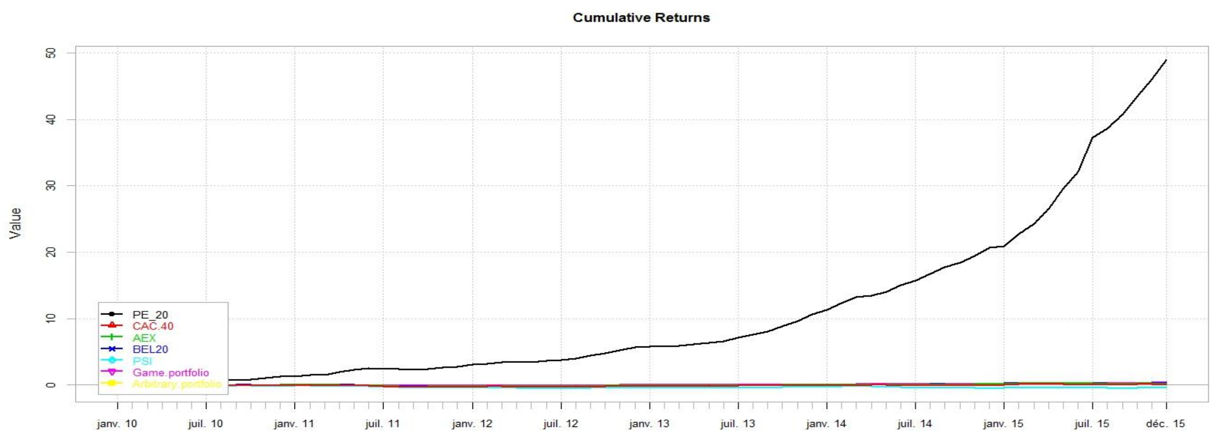
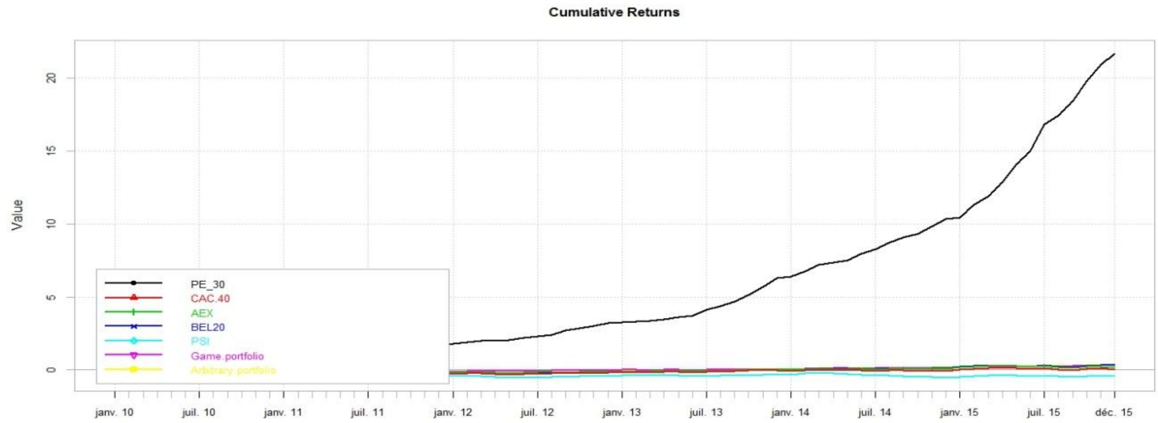


Figure 4.3: Cumulative return curves of portfolios ($\gamma = 30\%$)



To analyze losses, we present the drawdown curves and we find that whatever the PE parameter level, the PE portfolio has the less drawdown, thereafter the minimum amount of losses during the whole period of study. Figures 4.4-4.6 presents the drawdown curves of portfolios and the black one is the curve of PE portfolio when the tradeoff parameter. It is close to zero during the whole period except the period between May and September 2011 when it recorded a loss of about less than 10%. Moreover, game cross-efficiency portfolio has lower and more stable drawdown compared to arbitrary portfolio on the one hand, and it has the minimum amounts of loss and volatility compared to market portfolios on the other hand.

Figure 4.4: Drawdown curves of portfolios ($\gamma = 10\%$)

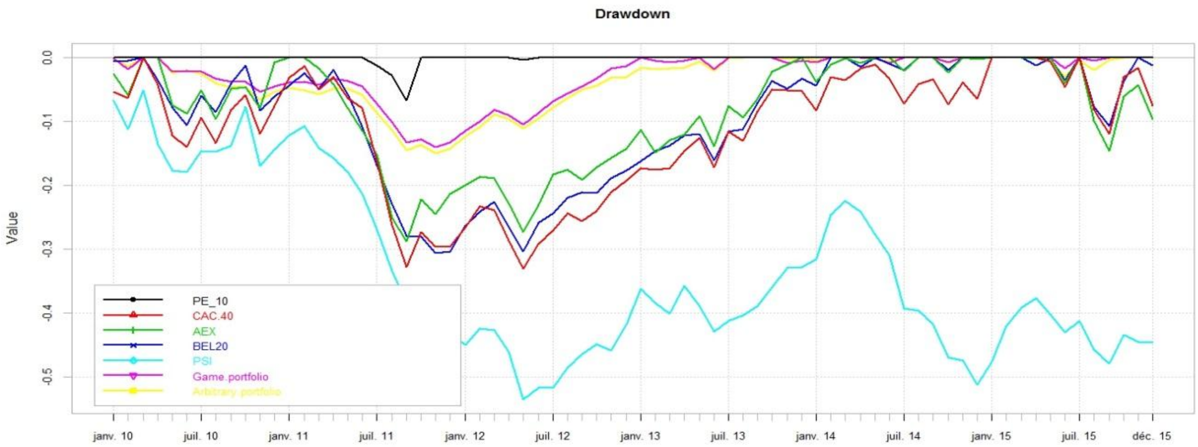


Figure 4.5: Drawdown curves of portfolios ($\gamma = 20\%$)

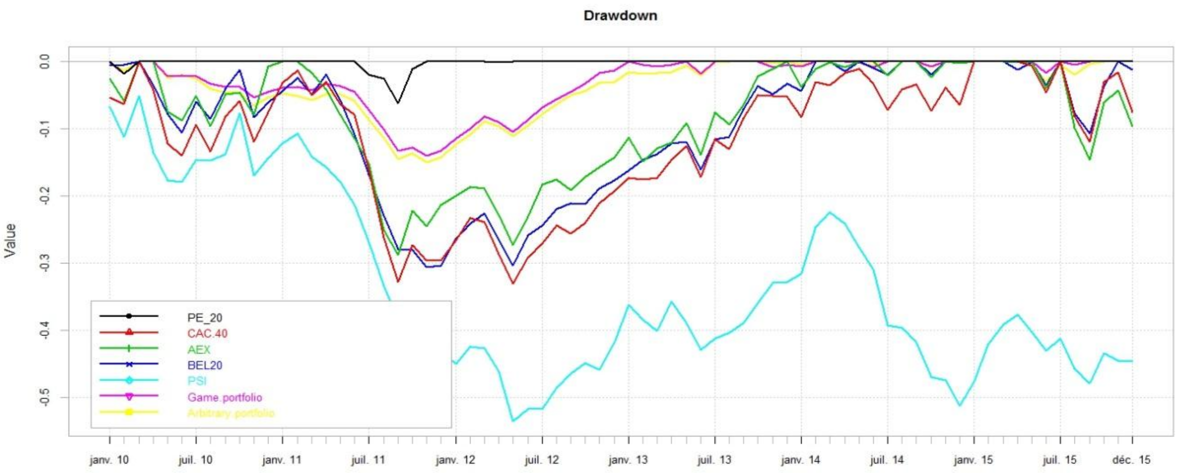
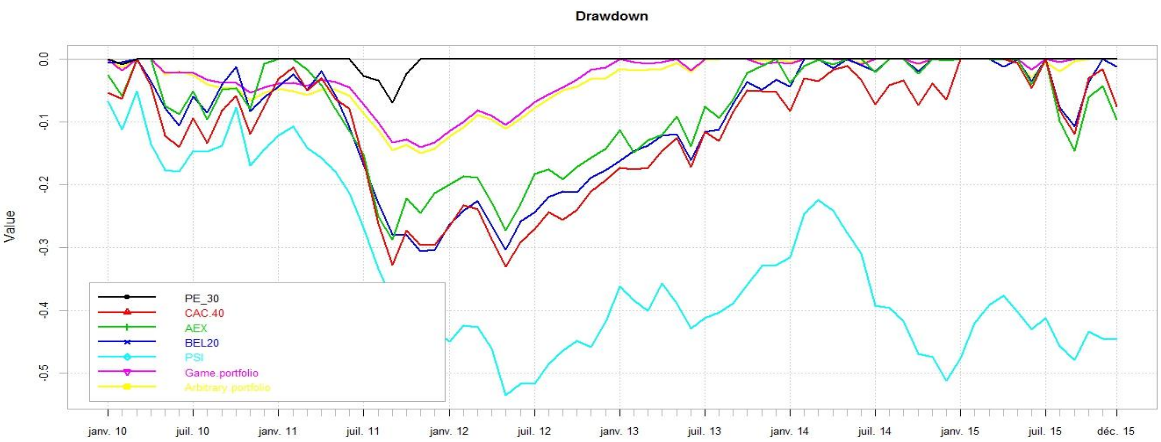


Figure 4.6: Drawdown curves of portfolios ($\gamma = 30\%$)



Our results show that the game cross-efficiency approach is more effective and stable than the one based on the simple use of cross-efficiency in portfolio evaluation area. Furthermore, whatever the investor' attitude, findings demonstrate the effectiveness of the developed PE DEA game cross-efficiency approach as tool to portfolio selection.

5. Conclusion

The most well-known two-moment decision model is that of modern portfolio theory which employs mean-variance analysis. Since that, there has been an ongoing debate as to the necessity of including higher moments of return distributions into the decision making process of investor. Therefore, several studies have investigated the efficiency of portfolio based on a multi-criteria analysis, whether variance, skewness and Kurtosis. However, while the efficiency of a portfolio guarantees a reasonable mediation between different criteria, it cannot in mostly guarantee a satisfactory level of profitability for the investor. Indeed, investors are interested in getting high returns but at the same time reducing their risks, but the financial assets that have the potential of bringing high returns typically also carry high risks of losing money. In this study, we have introduced profitability criterion in decision making process based on game cross-efficiency approach to portfolio selection. The developed model permits to investor to select financial assets ensuring an efficient portfolio with a certain level of gain. The empirical application has demonstrated that cross-efficiency approach is unstable and unpredictable depending on the software and problem context compared to the Nash equilibrium efficiency score provided by the game cross-efficiency. This problem is mainly due to DEA flexibility and the nature of the model which is fractional. We also provide novel way of selecting a PE portfolio and illustrate the proposed approach by applying it to financial asset portfolio selection in the Paris stock exchange and show that the selected portfolio yielded higher efficiency adjusted returns over other benchmark portfolios for 6-year sample period. The research can be extended by considering another criterion than that the profitability. Then, it would be a great interest to employ the proposed approach using another large data and taking into consideration taxes and transaction costs.

Conclusion and prospects for future research

Since the prediction of stock price returns is central to portfolio selection decision-making, many quantitative models have been proposed in the literature to aid the investor make the decision about investment, based on historical data.

Data Envelopment Analysis (DEA) models have been extensively used in performance appraisal in financial area. In the financial applications of DEA methodology, one particularly appealing idea is to gauge stock efficiency by using its higher order moments. In this dissertation, the developed models are based on stocks returns statistic distribution. In fact, DEA as a benchmarking tool can be used to work on historical data, which is used in portfolio analysis as an instrument for performance measurement.

While DEA provides each decision making unit (DMU) a good opportunity to self-evaluate its efficiency relative to other homogenous DMUs, DEA cross-efficiency evaluation suggests that each DMU is not only to be self-evaluated but also to be peer-evaluated. It can guarantee a unique ordering of the DMUs. Moreover, cross-efficiency evaluation eliminates unrealistic weight schemes without inputs and outputs weight restrictions. Despite the advantages of DEA cross-efficiency, it still rarely used in portfolio selection field. In this dissertation, we have built optimization models based on cross-efficiency approach and we have shown the effectiveness of these approaches as a tool for portfolio selection.

Basically, we contribute to literature by proposing a DEA cross-efficiency as a promising tool to portfolio selection.

Firstly, we incorporate the cross-efficiency approach into a Mean-Variance-Skewness-Kurtosis (MVSK) space to select portfolio, we determined two Mean-variance (MV) and Skewness-Kurtosis (SK) tradeoff parameters endogenously. We then took in consideration aversion degree, temperance and prudence of the investor. The developed methodology permits to obtain a well-diversified portfolio, which beats all benchmark portfolios. In fact, our findings confirm the intuition that higher order moments can significantly change the optimal portfolio selection. We also have shown that our framework is robust when tested using the Ledoit and Wolf (2008) Sharpe test against a benchmark portfolio. However, because the developed MVSK model is quartic, its application on large sample could not be easily performed. It may be possible to resolve the model by employing a heuristic approach in future research. Moreover, as we showed that investor has a general preference for odd moments and an aversion to even moments, a possible extension of the model to the fifth or higher order moments can be of great interest for future research.

Secondly, we proved that DEA game cross-efficiency could be a promising tool for evaluating financial assets by providing Nash equilibrium efficiency scores. In addition, arose from the DEA game cross-efficiency method, we developed a novel risk indicator in the portfolio area: the Maverick index. It presents the sensitivity level to environmental changes. We examine the 6-year performance of the portfolio generated by the developed model and we compared it to those of the four market indexes, CAC40, AEX, BEL20 and PSI20 over the study period starting from 2010 to 2015. Using Ledoit and Wolf (2008) Sharpe test we proved the robustness of our methodology. We have shown that incorporating DEA game cross-efficiency into Mean-Maverick space permits to obtain a well-diversified portfolio that is superior to all market portfolios. Further research can be performed using the developed framework by relaxing assumptions about taxes and transaction costs.

Finally, we attempted to solve the question of portfolio selection by supporting profitability criterion on the one hand and efficiency of financial assets on the other hand. We incorporated DEA game cross-efficiency model into Efficiency-Profitability space and we have shown that it presents a reliable framework to portfolio selection. We find that the derived portfolio has the highest growth of hypothetical initial investment and the minimum drawdown in comparison with market indices over the period. We have also shown that our portfolio beats all Benchmark portfolios in the Paris Stock exchange. The research can be extended by considering another criterion than that the profitability such as liquidity or marketability.

In overall, these methodologies provide more discrimination for financial assets by providing unique ranks in a first step and permit to select portfolio by underlying preferences of the decision-maker in a second step.

Despite the consistency of the developed models, further investigation on cross-efficiency evaluation based on directional distance functions (DDF) instead of DEA could be explored to portfolio performance evaluation. Thus, the cross-efficiency DDF aims at providing a peer-evaluation of financial assets based on measures that account for the inefficiency both in inputs and in outputs simultaneously. Furthermore, a peer-appraisal Free-

Disposal-Hull (FDH) cross-efficiency approach could be used to evaluate securities in future research.

Note that although the non-parametric deterministic frontier models rely on very few assumptions, they are very sensitive to extreme values and to outliers. Further investigation on the assessment of the statistical precision of the DEA cross-efficiency estimators could also be led, by implement the concept of the expected minimum input level (or output level) of order m of Cazals et al. (2002).

We also suggest for future investigation an hybrid analysis using DEA cross-efficiency approach and machine learning techniques to evaluate financial assets and to select portfolio.

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