Université de Lille

LEM - Lille Economie Management - UMR 9221

IÉSEG School of Management

ED SESAM-Sciences Economiques, Sociales, de l'Aménagement et du Management

Productivity gains decomposition and distribution of the price effects among stakeholders

Thèse en vue de l'obtention du titre de Docteur en Sciences Economiques

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2 juillet 2018

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Préparation de la thèse au LEM sur le site de l'IÉSEG School of Management, 3 Rue de la Digue, 59000 Lille

Acknowledgements

First, I would like to express my great acknowledgement to my supervisors Jean-Philippe Boussemart and Hervé Leleu for giving me the opportunity to undertake this PhD program. During all these years your expertise, patience, availability helped me a lot to advance through this challenging journey. Thanks for sharing with me your knowledge and experience, for giving me valuable advice and suggestions.

I would like to thank IESEG School of Management for providing me with excellent conditions to realize my PhD. I would like especially to thank its director, Jean-Philippe Ammeux, for hosting me at this institution through all my PhD and accepting me as a Teaching and Research Assistant.

I am also grateful to l'Université Catholique de Lille for giving me the financial support that allowed me to perform my research project. I am also acknowledged to LEM laboratory for providing me with financial resources to go to the conferences.

I thank as well my university of inscription, l'Université de Lille and l'Ecole Doctorale SESAM.

My special thanks go to Raluca Parvulescu, my colleague from the Economics department with who I had a chance to collaborate. Our discussions, your advice and support encouraged me a lot in difficult moments throughout this process.

I would like also to thank Albane Tarnaud with who I shared the office for more than two years. Thanks for helping me to integrate this school, for your support and availability. I am also grateful to Maia Gejadze for giving me some precious suggestions that made me advance significantly.

I cannot forget all my PhD colleagues (or already doctors) that made this journey more exciting: Cristina Ciobanu, Jenny Tran, Kristine Tamayo, Helen Cocco, Adrian Barragan Diaz, Minh Phan, Arno de Caigny, Khoi Pham, Annabelle Glaize, Marion Lauwers, Hélène Stefaniutyn, Salim Rostami, Zhiyang Shen, Libo Li, Steven Debaere, Stijn Geuens.

Finally, I would like to thank my parents and my sister for ongoing help and moral support during all these years.

General abstract

The financial performance of a firm depends both on its productive efficiency and the economic environment in which this firm performs its activity. Several seminal works studied the relationship between financial performance and productivity and the way the productivity gains are distributed among the beneficiaries. This thesis adds to the literature by developing new links between productive efficiency, financial performance and productivity gains distribution. Namely, this doctoral thesis contributes in the three following ways.

First, we propose an original way on how to decompose the profit gaps among firms at the cross-sectional level taking into account their productive inefficiency and then relating these productivity-based gaps to the price advantages/disadvantages of the firm's stakeholders. This methodological framework was applied to a sample of US banks over the period 2001-2012. The analysis was focused on the banks with positive profit gap over the whole period and before and after the crisis. The results showed that over the considered period performant banks benefitted from positive price environment but were inefficient in their productivity levels. The main providers of financial resources for performant banks were creditors and employees over the entire period and suppliers after the 2007-2008 financial crisis. Besides, a decrease in allocative inefficiency is observed after the crisis. Finally, a comparison analysis of commercial and savings banks revealed that the main source of price advantages was creditors for the former and suppliers for the latter over the whole period.

Second, we define an indicator of price environment for a firm comparing its distance to the volume- and value-based efficiency frontier. These price environment effects were computed for US industries from 1987 to 2014. The groups of industries with similar price effects evolutions were found and a panel model was performed to determine the influence of each stakeholder price effect on the global price effect. The results indicated that global mean price environment for all sectors was deteriorating over the entire period which can be related to the increasing degree of openness of US economy. This analysis showed that all specific input/output effects were statistically significant. A strong influence could be observed for capital, gross output and labor price environments. Intermediate inputs affected global price environment much less. Besides, structural breaks occurred in the beginning of 2000s and around 2005-2007 (the financial crisis).

Third, we suggest a decomposition of the overall technical inefficiency of firms at the aggregated level into two components: individual technical and individual structural inefficiencies. This decomposition was applied to the same data as for the second analysis. The convergence process was studied for both components and a panel procedure was used to link the two components changes to the changes of the stakeholder's price advantages/disadvantages. The results clearly confirm the convergence processes for both technological catching-up and input-output mixes. Using the link existing between TFP growth rate and price advantages/disadvantages, we then estimate the influence of technical and structural inefficiency changes on each stakeholder's compensation. A panel data regression revealed that customers and managers benefitted substantially from the two convergence processes while the opposite was found for suppliers. Employees and capital providers seemed not to be affected by technical inefficiency and input-output mixes convergence processes since their price changes seem essentially driven by the macro business cycle.

This essay tends to show that generation of productivity gains and their distribution are two sides of the same coin. The former is related to the economic analysis of TFP based on the estimation of a production technology while the latter deals with an accounting approach of business performance. By considering that firms should not only to be studied from the production side but also from their trading relationships, we contribute to build the bridge between economists and managers in the evaluation of business performances.

Keyword : Total Factor Productivity ; Productivity Accounting ; Price Advantages ; Data Envelopment Analysis ; Banks ; U.S. Industries ; Technical/Structural Inefficiencies ; Technological Catching-up.

Résumé général

Décomposition des gains de productivités et distribution des effets prix entre les parties prenantes

La performance financière d'une firme dépend à la fois de son efficacité productive et de l'environnement économique dans lequel cette firme exerce son activité. Quelques contributions importantes ont étudié la relation entre la performance financière et la productivité et la manière dont les gains de productivité sont répartis entre les bénéficiaires. Cette thèse enrichit cette littérature en développant de nouveaux liens entre l’efficacité productive, la performance financière et la distribution des gains de productivité. Trois contributions principales y sont développées.

Premièrement, nous proposons une manière originale de décomposer les écarts de profit entre les firmes au niveau spatial en tenant compte de leur inefficacité productive et en reliant ensuite ces écarts aux avantages/désavantages prix des parties prenantes de la firme. Ce cadre méthodologique a ensuite été appliqué à un échantillon de banques américaines sur la période 2001-2012. Les résultats ont montré que tout au long de la période considérée, les banques les plus performantes ont bénéficié d'un environnement de prix positif mais montraient des niveaux de productivité inférieurs. La source des avantages prix pour les banques proviennent essentiellement de leurs clients créditeurs et de leurs employés sur toute la période et de leurs fournisseurs après la crise financière de 2007-2008. En outre, une diminution de l'inefficacité allocative est observée après la crise. Enfin, une analyse comparative entre les banques commerciales et les caisses d'épargne a révélé que la principale source d'avantages prix était les prêteurs pour les premiers et les fournisseurs pour les seconds sur l'ensemble de la période.

Deuxièmement, nous définissons, d’un point de vue méthodologique, un indicateur de l'environnement prix pour une firme en comparant sa distance à la frontière d'efficacité estimée, d’une part, avec des volumes et, d’autre part, avec des valeurs. Cet effet d'environnement prix a été ensuite appliqué pour l’ensemble des industries américaines sur la période 1987-2014. Des groupes d'industries ayant des évolutions d’effets prix similaires ont été mis en évidence et un modèle de données de panel a été estimé pour déterminer l'influence de l’effet prix associé à chaque partie prenante sur l’effet prix global. Les résultats ont indiqué que l'environnement prix global moyen pour tous les secteurs se détériorait sur l’ensemble de la période, ce qui peut être lié au niveau d'ouverture croissant de l'économie américaine à la concurrence internationale. Cette analyse a aussi montré que tous les effets spécifiques à chaque input/output étaient statistiquement significatifs. Une forte contribution du capital, de la production et du travail sur l’effet prix globale a pu être observée. Les inputs intermédiaires ont beaucoup moins affecté l'environnement prix global. En outre, deux ruptures structurelles se sont produites au début des années 2000 et autour de 2005-2007 précédant juste la crise financière.

Troisièmement, nous développons une approche méthodologique pour décomposer l'inefficience technique globale des firmes à un niveau agrégé tel que le secteur ou l’économie en deux composantes : l’inefficacité technique individuelle et l’inefficacité structurelle. Cette décomposition a été appliquée aux mêmes données américaines que pour la deuxième analyse. Un processus de convergence a été étudié pour les deux composantes et une analyse de données de panel a été utilisée pour lier les changements des deux composantes aux changements des avantages/désavantages prix des parties prenantes. Les résultats confirment clairement les processus de convergence à la fois pour le rattrapage technologique et pour les processus de production (mix d’input/output). En utilisant le lien existant entre le taux de croissance de la productivité globale des facteurs (PGF) et les avantages/ désavantages prix, nous estimons ensuite l'influence des changements d'inefficiences technique et structurelle sur la rémunération de chaque partie prenante. Une régression en données de panel a révélé que les clients et les managers ont bénéficié substantiellement des deux processus de convergence alors que le contraire a été trouvé pour les fournisseurs. Les employés et les fournisseurs de capitaux n’ont pas été affectés par les processus de convergence de l'inefficacité technique et des processus de production car leurs avantages prix semblent essentiellement être reliés aux cycles macroéconomiques.

Au final, ce travail de thèse tend à montrer que la génération des gains de productivité et leur distribution entre les parties prenantes d’une firme sont les deux facettes d'une même pièce. La première est liée à l'analyse économique de la PGF basée sur l'estimation d'une technologie de production tandis que la seconde traite d'une approche comptable et managériale de la performance des firmes. En considérant que les firmes ne doivent pas seulement être étudiées du côté de la production mais aussi du point de vue des leurs échanges, nous contribuons à établir un pont entre les économistes et les gestionnaires dans l'évaluation des performances des firmes.

Mots clés : Productivité Globale des Facteurs ; Comptes de Surplus ; Avantages Prix ; Data Envelopment Analysis ; Banques ; Industries Américaines ; Inefficacité Technique/Structurelle ; Rattrapage Technologique.

Résumé substantiel

Le profit est habituellement considéré comme un indicateur privilégié de la performance économique d’une firme et la maximisation du profit est souvent utilisée comme l'hypothèse de base du comportement du producteur dans la théorie néoclassique. Mais, derrière cet indicateur de rentabilité globale, beaucoup d'autres sont aussi essentiels tels que les efficacités technique (utiliser les ressources de manière optimale), allocative (allouer efficacement les ressources en fonction de leur prix respectif), d’échelle (produire à la taille optimale), productive (maximiser la productivité), coût (minimiser les coûts) ou revenu (maximiser le revenu). En considérant tous ces concepts de performance productive, les décisions managériales sont prises et concernent l’ensemble des facteurs de production comme le travail, les équipements, les consommations intermédiaires, les actifs financiers... Même si la croissance de la productivité impacte fortement la rentabilité, celle-ci ne dépend pas uniquement de la performance productive. Elle est également influencée par l'environnement prix dans lequel une firme opère. Un environnement prix très avantageux peut conduire à des résultats financiers positifs, même sans efficacité productive. L'inverse est également possible : une firme efficace au niveau productif peut souffrir d'un environnement prix désavantageux. Au final, c’est à la fois la performance productive et l'environnement prix qui contribuent à la rentabilité.

D’un autre côté, la rentabilité d'une entreprise a une influence sur les échanges financiers avec ses parties prenantes : prêteurs, employés, propriétaires, clients et fournisseurs. Par conséquent, la performance d’une firme doit être analysée à travers deux dimensions principales, à savoir la production et les échanges. En effet, mesurer la performance productive n'est qu'un aspect de l'image globale. L'autre face concerne la répartition des gains de productivité entre les parties prenantes qui contribuent à l'activité de l'entreprise.

Dans cette thèse, nous avons développé de nouvelles façons de lier la productivité, la performance financière et l'environnement prix. Plus précisément, ce travail propose trois études sur la décomposition des gains de productivité et la répartition des effets prix entre les parties prenantes. Ces travaux sont tous basés sur une approche commune qui est l’estimation non paramétrique d’une technologie de production et développée dans le premier chapitre. Une première contribution de notre se situe dans le deuxième chapitre qui introduit une nouvelle décomposition de la différence de profit entre deux firmes et relie celle-ci à la définition des comptes de surplus. La seconde contribution est développée dans le troisième chapitre qui présente la définition d’un indicateur d'environnement prix global pour les firmes. Il est basé sur une comparaison de

l’efficience des firmes à partir de données en quantité et en prix et permet de définir un indicateur global mais aussi des indicateurs d’environnement prix spécifiques à chaque input/output et que nous pouvons directement reliés aux parties prenantes. Dans le quatrième et dernier chapitre, nous proposons une décomposition originale de l'inefficience technique agrégée au niveau sectoriel en deux composantes et relie leurs changements dans le temps aux changements des avantages prix des parties prenantes. Dans ce qui suit, nous présentons en quelques lignes des principales contributions et résultats des chapitres deux à quatre.

Le chapitre 2 décompose l’écart de profit mesuré entre deux firmes en trois effets « quantité » (inefficience technique, inefficacité allocative et effet taille) et un effet « prix » global ensuite réparti en avantages prix spécifique à chacune des parties prenantes. Au final, nous montrons comment le surplus de productivité qui est la somme effets quantités est distribué entre les parties prenantes à travers leurs avantages/désavantages prix respectifs, y compris le profit qui est la rémunération du propriétaire de la firme. En comparaison avec l'approche habituelle du CERC (Centre d’Etude des Revenus et des Coûts), l'originalité de notre travail réside dans le fait que les variations de profit sont étudiées dans une dimension spatiale plutôt que temporelle. L’objet de l’analyse est davantage orienté vers la comparaison des profits entre firmes plutôt que l’analyse de la variation du profit d’une firme dans le temps. Une autre spécificité de notre travail est que nous utilisons systématiquement les indicateurs de Bennet pour tous les effets quantité et prix. Cette méthodologie permet d'identifier à la fois les effets des différences d'efficacité technique, d'allocation des ressources et de taille ainsi que l'effet de différents environnements prix sur l'écart de profit entre deux entreprises. De plus, du point de vue des comptes de surplus, elle permet de déterminer les sources du surplus de productivité qui sont ensuite réparties entre les parties prenantes. Ce cadre a été appliqué à un échantillon de banques américaines sur la période 2001-2012. Nous avons réalisé notre analyse en termes de différences de taux de profit plutôt que de niveau de profit pour maitriser l’effet taille. Nous avons restreint notre analyse aux seules banques « performantes » qui montraient en moyenne un écart de taux de profit positif par rapport à l’ensemble des autres banques. Les résultats ont montré que tout au long de la période considérée, les banques les plus performantes ont bénéficié d'un environnement de prix positif mais montraient des niveaux de productivité inférieurs. La source des avantages prix pour les banques proviennent essentiellement de leurs clients créditeurs et de leurs employés sur toute la période et de leurs fournisseurs après la crise financière de 2007-2008. En outre, une diminution de l'inefficacité allocative est observée après la crise. Enfin, une analyse comparative entre les banques commerciales et les caisses d'épargne a révélé que la principale source d'avantages prix était les prêteurs pour les premiers et les fournisseurs pour les seconds sur l'ensemble de la période.

Dans le chapitre 3, nous définissons, d’un point de vue méthodologique, un indicateur de l'environnement prix pour une firme en comparant sa distance à la frontière d'efficacité estimée, d’une part, avec des volumes et, d’autre part, avec des valeurs. La comparaison des distances à ces deux frontières conduit à définir un indicateur d'environnement prix en considérant à la fois les quantités et les prix comme des variables de décision. Cette méthodologie peut être utilisée pour une mesure globale de l'environnement prix mais aussi appliquée pour définir un environnement prix spécifique à chaque input et output. Cet effet d'environnement prix a été ensuite appliqué pour l’ensemble des industries américaines sur la période 1987-2014. Des groupes d'industries ayant des évolutions d’effets prix similaires ont été mis en évidence et un modèle de données de panel a été estimé pour déterminer l'influence de l’effet prix associé à chaque partie prenante sur l’effet prix global. Les résultats ont indiqué que l'environnement prix global moyen pour tous les secteurs se détériorait sur l’ensemble de la période, ce qui peut être lié au niveau d'ouverture croissant de l'économie américaine à la concurrence internationale. Cette analyse a aussi montré que tous les effets spécifiques à chaque input/output étaient statistiquement significatifs. Une forte contribution du capital, de la production et du travail sur l’effet prix globale a pu être observée. Les inputs intermédiaires ont beaucoup moins affecté l'environnement prix global. En outre, deux ruptures structurelles se sont produites au début des années 2000 et autour de 2005-2007 précédant juste la crise financière.

Dans le dernier et quatrième chapitre, nous développons une approche méthodologique pour décomposer l'inefficience technique globale des firmes à un niveau agrégé tel que le secteur ou l’économie en deux composantes : l’inefficacité technique individuelle et l’inefficacité structurelle. Cette décomposition a été appliquée aux mêmes données américaines que pour la deuxième analyse. Un processus de convergence a été étudié pour les deux composantes et une analyse de données de panel a été utilisée pour lier les changements des deux composantes aux changements des avantages/désavantages prix des parties prenantes. Les résultats confirment clairement les processus de convergence à la fois pour le rattrapage technologique et pour les processus de production (mix d’input/output). En utilisant le lien existant entre le taux de croissance de la productivité globale des facteurs (PGF) et les avantages/ désavantages prix, nous estimons ensuite l'influence des changements d'inefficiences technique et structurelle sur la rémunération de chaque partie prenante. Une régression en données de panel a révélé que les clients et les managers ont bénéficié substantiellement des deux processus de convergence alors que le contraire a été trouvé pour les fournisseurs. Les employés et les fournisseurs de capitaux n’ont pas été affectés par les processus de convergence de l'inefficacité technique et des processus de production car leurs avantages prix semblent essentiellement être reliés aux cycles macroéconomiques.

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General introduction

Profit is usually considered as the economic performance indicator and the profit maximization is often used as the assumption of a producer behavior in neoclassical theory. But behind this global profitability indicator, many other factors are underlying such as technical, allocative, scale, scope, cost or revenue efficiencies. Underlying all these concepts of productive performance, management decisions are made concerning the best allocation of resources such as employees, equipment, intermediate inputs, and financial assets. While productivity growth strongly impacts profitability, the latter does not only depend on productive performance. It is also influenced by price environment in which a firm operates. A very advantageous price environment can lead to positive financial results even without productive efficiency. The inverse is also possible: a productively efficient firm can suffer from a disadvantageous price environment. Finally, both productive performance and price environment contribute to the profitability.

Moreover, profitability of a firm has a financial influence on its stakeholders: lenders, employees, owners, customers and suppliers. Consequently, firm performance should be analyzed through two main dimensions namely production and transaction. Indeed measuring productive performance is only one side of the global picture. The other part concerns the distribution of productivity improvements among the stakeholders who contribute to the firm activity.

From a more global perspective, Davis (1947, 1955) noticed that economic progress not only depends on productivity growth but also on the distribution of the productivity gains among all participants of society. Kendrick (1961) and, Kendrick and Sato (1963) initiated studies on analyzing generation and distribution of Total Factor Productivity (TFP) growth by combining price and quantity changes simultaneously. Their results show that productivity changes can be gauged either from quantity changes or from price variations. Nevertheless, most of studies succeeding these pioneering works rest at a macro or sectoral level (Hulten et al., 2001).

However, this concern is considered as a key issue in performance analysis at the micro level. Davis (1947, 1955) and Kendrick (1961) also studied the relationship between productivity changes and individual financial performance by linking quantity, price and profitability variations. In this way, they explored the distribution of the returns of productivity growth among the different participants contributing to the firm' activities. Davis (1947, 1955) attributed a monetary value to productivity change and shared it between six major stakeholders while

Kendrick (1961) explored the distribution of productivity gains among clients, suppliers, company owners and government at a durable manufacturing corporation. More precisely in the case of productivity growth, the consumers possibly will get benefit from price decreases or higher quality products, employees could earn greater compensation, intermediate input providers may receive higher prices, stockholders may obtain greater dividends, companies themselves could improve retained earnings and finally, the government may benefit from higher taxes. In the same vein, a large number of studies underlining the connection between generation and distribution of productivity gains was developed by a group of French economists. Initiated by Vincent (1968) and precisely settled through the document edited by the Centre d'Etudes des revenus et des Coûts (CERC, 1980), the methodology was applied to analyze the performances of several French public firms such as Electricité de France (EDF), Gaz de France (GDF), les Charbonnages de France and Société Nationale des Chemins de Fer Français (SNCF).

More recently, TFP analysis has known a significant revival with innovative researches (Fried et al., 2008). More particularly, greater attention has been focused on the distribution of the gains from TFP through price effect components at the firm level. Following this way, Grifell-Tatjé and Lovell (2015) studied the link between productivity and financial performance indicators, the sources of productivity growth and the way the benefits of productivity growth are distributed and quantify a complete analytical framework within each of these aspects. They refer to the models and extensions treating the relation between productivity and profitability and the way the fruits of the productivity growth are distributed among the beneficiaries developed by Davis (1955), Kendrick and Creamer (1961), Vincent (1968) and CERC (Centre d'Etudes des Revenus et des Coûts) institution (1980). They underline that a larger amount of information can be extracted by economists from financial accounts to analyze business performance.

This research takes place in the literature on productivity gains decomposition and distribution of the price effects among stakeholders establishing the formal link between productivity change and profitability.

The thesis is based on three main contributions developed throughout four chapters.

First, we start with the decomposition of profit differences into quantity and price effects. Traditionally a multiplicative decomposition is made in the time dimension where profit change among two periods for a specific firm is analyzed. In our work, we are more interested in the spatial dimension and a new and original additive decomposition of profit differences is proposed at the cross-sectional level. In this framework, the quantity effect of the profit decomposition reveals productivity gaps among firms which can be assimilated to the concept of productivity surplus (PS) as introduced by the CERC (1980). PS is further decomposed into three effects related to the sources of productive inefficiency namely technical, allocative and size effects. Then PS is equated to the sum of price advantages (PA) associated with each stakeholder and computed as the price differences among firms.

In that perspective, the first chapter is devoted to methodological aspects of production theory, modelling production technologies through distance functions, defining productivity and efficiency measures and estimating them by a nonparametric DEA framework. In addition, methodological aspects of surplus accounts are covered to analyze generation and distribution of productivity gains among stakeholders. Numerical examples and study cases are systematically developed to show the operational implementation of these concepts and tools for managers and practitioners. By integrating this two strands of literature, we finally propose a methodological contribution to decompose profit differential between two firms into quantity and price effects. Based on this contribution, chapter two offers a real-world application on U.S. small and medium banks over the period from 2001 to 2012. Price advantages of each stakeholder (creditors, employees, suppliers, government, borrowers, financial market participants and, commission and fee payers) are decomposed and analyzed. Finally, we compared surplus productivity accounts for banks with positive profit rate gap between the two periods before and after the 2007-2008 crisis.

Second, we develop a new methodology for estimating the impact of price environment on firm performance. Starting with a traditional definition of a production technology as developed by Shephard (1953, 1970), Koopmans (1951), Debreu (1951) and Farrell (1957), we extend the Data Envelopment Analysis (DEA) framework to introduce a value efficiency measure which is compared to the technical efficiency to define a price environment indicator. The latter distinguishes itself from the usual allocative efficiency since it does not require resource reallocation at the firm level. It is based on a comparison of prices among peers. In chapter three, we propose a price environment indicator for a Decision-Making Unit (DMU) taking into account the quantity-price correspondences. Two technologies are defined: one formed with observed quantities and the other constituted with observed values. Efficiency scores under these two technologies are estimated and then ratios of value efficiency scores to volume efficiency scores are computed. Obtained ratios are interpreted as indicators of positive or negative price environment for a DMU. We employ Shephard's output distance function to retrieve technical efficiency scores under both value and quantity technologies. Such indicators can be implemented for measuring the global price environment taking into account all output and input prices simultaneously or for estimating specific output or input price effects. This methodology is applied to all 63 U.S. industries over the period 1987-2014.

Third, we investigate the sources of price advantage changes for stakeholders related to productivity catching up. We separate efficiency gaps into two components: a technical efficiency effect taking into account size heterogeneity and a structural component which highlights the impacts of an input-output deepening or expanding effect on technological transfer over time. This original decomposition serves as the basis of the final chapter in which the technical catching-up and the convergence of input-output mixes among the US industries over 1987-2014 is analyzed. After demonstrating the equality between TFP growth rate and the sum of weighted price advantages, we propose a panel data analysis to estimate the influence of technical and structural inefficiency variations on the price advantages changes for each stakeholder (clients, suppliers, employees, capital providers and managers). An application analyzes input-output ratio convergence and technical efficiency catching-up among 63 North American industries over the period 1987-2014.

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Chapter 1

Efficiency, productivity accounting and profit decomposition

1 Introduction

In this chapter we present the methodological settings used in chapters 2, 3 and 4. In the first part of the second section, we focus on efficiency and productivity measures and on how these measures can be estimated. For this, we follow the literature developed by Shephard (1953, 1970), Koopmans (1951) and Debreu (1951). First, we introduce the definition of a production possibility set based on the underlying assumptions. Second, we define the distance function as the measurement tool over production sets providing an equivalent representation of the technology. Based on this approach, a nonparametric estimator (Data Envelopment Analysis, DEA) of the distance function is presented in the second part of the second section. DEA constructs an efficient frontier based on a sample of observed Decision Making Units (DMUs) and estimates inefficiency scores. In this chapter, only nonparametric deterministic framework is considered, other possibilities of estimating distance functions are stated but not developed.

In the third section, we develop the theory behind the surplus accounting method. First, it gives a general idea of the methodology and its objective and then presents the Productivity Surplus (PS) estimation itself. It provides the classical CERC's approach (CERC, 1980) with Laspeyres-and Paasche based surplus accounts for both PS and Price Advantages (PA) related to the different stakeholders participating in the production process. Then, this methodology is extended to the Bennet productivity and price advantage indicators. Furthermore, an explicit link between Total Factor Productivity (TFP) changes and PS is established. A numerical example illustrates a practical implementation of the CERC productivity methodology. Finally, a real data application analyzes and compares productivity gains evolution and their distribution over time between US automobile sector and the whole US economy.

In the fourth section, we propose a new methodology to decompose a profit gap between two firms in quantity and price effects through a Bennet approach. Then, three components of the quantity effect are identified namely technical, allocative and size indicators. As to the price effect, it is decomposed among stakeholders and the link to surplus accounting is established. Finally, we introduce a ratio instead of the usual additive decomposition for this methodology in order to solve the commensurability issue of profit comparisons between firms.

2 Efficiency and productivity

2.1 Production technologies

The seminal works of Koopmans (1951), Debreu (1951), Shephard (1953), and Farrell (1957) have developed the basis of the Neo-Walrasian production theory based on production sets. To define a basic production technology, we assume that decision making units (DMUs) have N number of inputs (x) that can be used to produce M number of outputs (y).

Understanding theoretical production principles behind productive reality are crucial to model production functions.

2.1.1 Definition of production sets based on quantities

General definitions

We can consider a technology as a process of transformation of a number of inputs into a number of outputs. We suppose that this process is an unknown model (black box) for economists and we only know that specific outputs can be produced using specific inputs. Examples of technology can be: agriculture (use of yields, fertilizers, irrigation, mechanization, pesticides and animal feed to produce meat, milk, eggs and cereals); health (use of hospitals, facilities, equipment, expertise, people and materials to produce goods and services in healthcare system); finance (use of customer deposits, employees, structures and equipment to produce loans, marketable securities and investments and services).

The classical production possibility set (or technology) can be defined as follows:

$$T = \left\{ (\mathbf{x}, \mathbf{y}) \in R_{+}^{N+M} : \mathbf{x} \text{ can produce } \mathbf{y} \right\} \tag{1}$$

It can be illustrated as in Figure 1 where one input x is used to produce one output y. T(x,y) represents the set of all feasible production plans. The boundary of this set gives the maximum output that can be produced for each level of input. It can be understood as the best practices and it is usually named as the efficient frontier. Production plans inside the set are feasible but not efficient.

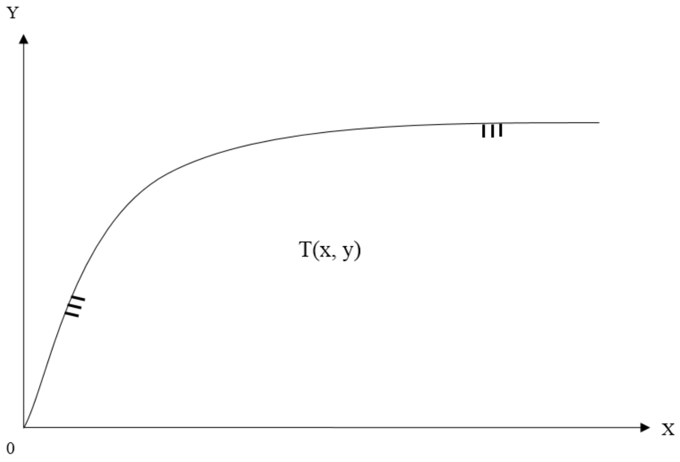

Figure 1 Production possibility set

In case of multiple outputs and/or multiple inputs, the production technology can also be represented by an output and/or an input correspondence. The output set is defined by all possible output combinations that can be produced by a given level of inputs. The output correspondence is defined as:

$$P(\mathbf{x}) = \left\{ \mathbf{y} \in R_{+}^{M} : (\mathbf{x}, \mathbf{y}) \in T \right\} \tag{2}$$

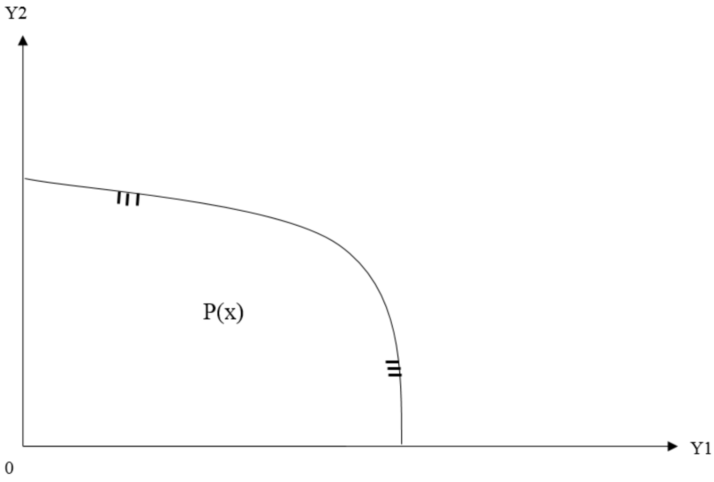

Figure 2 Output correspondence

Figure 2 illustrates the output set P(x) when two outputs (y1 and y2) can be produced from a vector of inputs (x). Feasible output combinations are inside P(x) and the boundary defines the efficient frontier. Similarly, the production technology can be characterized by an input set, namely all possible input combinations that can produce a given level of outputs. The input correspondence is defined as:

$$L(\mathbf{y}) = \left\{ \mathbf{x} \in R_{+}^{N} : (\mathbf{x}, \mathbf{y}) \in T \right\} \tag{3}$$

An illustration of input sets is given in Figure 3.

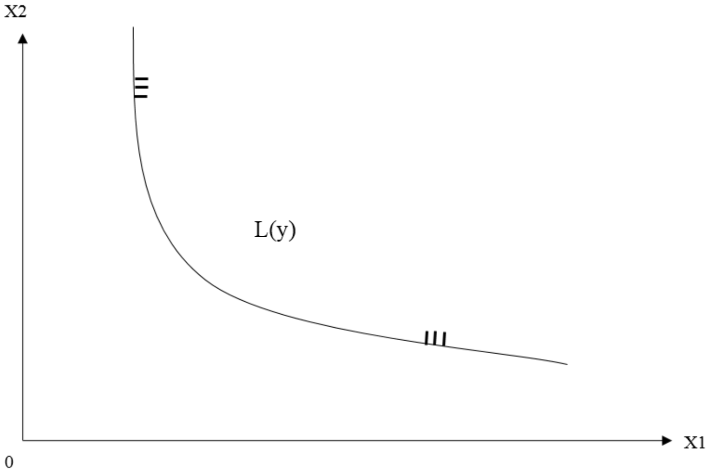

Figure 3 Input correspondence

A key theoretical result is that both output correspondence (P(x)) and input correspondence (L(y)) are equivalent representation to the production possibility set (T(x,y)). Therefore, economists can work on one or another representation which is the most appropriate to the context.

Axioms

The definition of a production technology given so far is general and only determines a global analysis frame for a certain number of inputs that can be transformed into outputs using the specific technology.

To give more structure to a considering set and, especially, to ensure the reliability of the modelling of the transformation of inputs into outputs, a certain number of axioms need to be defined. They are intended to give an economic sense to input-output sets. They are often called in economics “regularity conditions”.

A first axiom asserts that no productions can be made without using any resources. It is called the « No free lunch » axiom and can be expressed as follows:

$$\text{If } (\mathbf{x}, \mathbf{y}) \in T \text{ and } \mathbf{x} = 0, \text{ then } \mathbf{y} = 0 \tag{4}$$

Axiom of free disposability of inputs and outputs is the capacity to stock, eliminate and waste factors and productions. Formally, free disposability is defined as follows for inputs:

$$\text{If } (\mathbf{x},\mathbf{y}) \in T \text{ and } (\mathbf{x}',\mathbf{y}) \geq (\mathbf{x},\mathbf{y}) \text{ then } (\mathbf{x}',\mathbf{y}) \in T \tag{5}$$

The equation (5) specifies that if it is possible to produce a certain amount of outputs (y) using a given amount of inputs (x), then we hypothesize that a firm will be able to produce the same amount of output using more inputs (x'). This definition does not consider the case of congestion as when the excessive use of inputs can affect negatively the production of outputs.

Following the same logic, the free disposability of outputs can be presented as follows:

$$\text{If } (\mathbf{x},\mathbf{y}) \in T \text{ and } (\mathbf{x},\mathbf{y}') \leq (\mathbf{x},\mathbf{y}) \text{ then } (\mathbf{x},\mathbf{y}') \in T \tag{6}$$

The expression (6) underlines that if a firm produces a certain amount of outputs using a given amount of inputs, it can also produce less using the same amount of inputs. Otherwise speaking, the waste of outputs is allowed.

A free disposability of outputs can be interpreted in Figure 4.

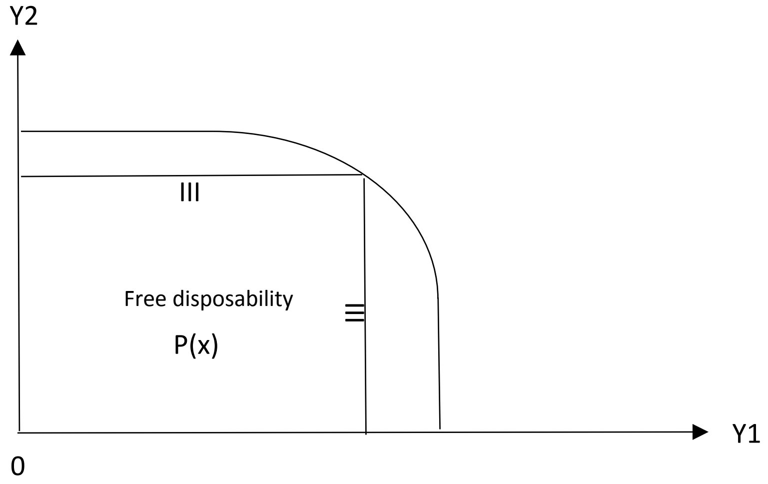

Figure 4 Free disposability of outputs

The convexity axiom allows us to distinguish two types of technologies: DEA technology which supposes convexity of production set and FDH (Free Disposal Hull) that does not suppose convexity. If only two dimensions are considered, one can define easily if the set is convex: it is impossible to link any two points of a convex set with a line not completely included in the set. To define the convexity in more than two dimensions, one needs to consider a more general definition. A set is convex if all the combinations of vectors belonging to the set also belong to the set. This axiom can be expressed as:

$$\text{If } (\mathbf{x}_k, \mathbf{y}_k) \in T, k = 1, \ldots, K, \text{then} \sum_k \lambda_k (\mathbf{x}_k, \mathbf{y}_k) \in T \text{ with } \lambda_k \geq 0 \ \forall k \text{ and} \sum_k \lambda_k = 1 \qquad (7)$$

A convex production frontier is displayed in Figure 5.

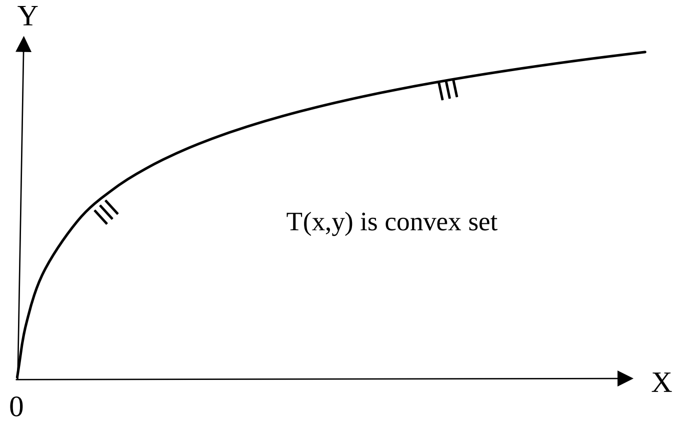

Figure 5 Convexity

An alternative to convex sets is proposed by Deprins et al. (1984) as Free Disposal Hull (FDH). The FDH set is non-convex and can be figured in Figure 6. The frontier of the FDH set has a staircase shape. FDH is only based on the free disposability assumption.

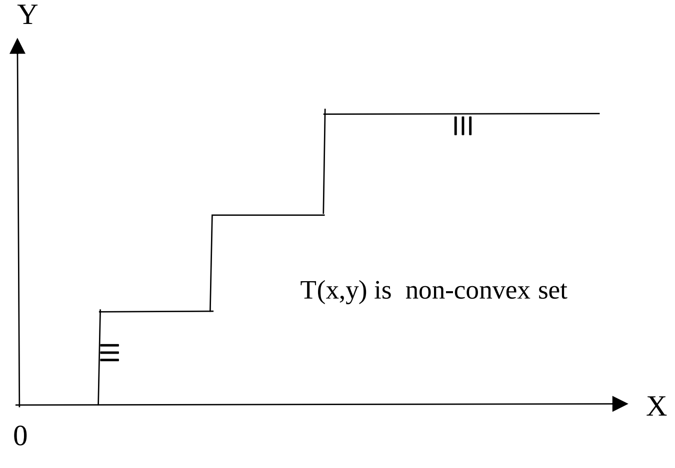

Figure 6 Free Disposal Hull

Moreover, the axiom of returns to scale implies the rate of change in outputs to inputs. Constant returns to scale (CRS) assume that all outputs are expended or reduced by a proportional increase or decrease in all inputs. Non-increasing returns to scale (NIRS) show outputs are scaled less than or equal to inputs. Non-decreasing returns to scale (NDRS) indicate outputs are scaled more than or equal to inputs. If none of these cases hold, the technology is characterized by variable returns to scale (VRS). The demonstrations of CRS, NIRS, NDRS, and VRS are presented in Figures 7, 8, 9 and 10 respectively.

The first theoretical developments in DEA, namely the model of Charnes, Cooper et Rhodes (CCR, Charnes et al. (1978)) are based on CRS hypothesis. It can be formulated as follows:

$$\text{If } (\mathbf{x},\mathbf{y}) \in T \text{ and } \lambda \geq 0, \text{ then } (\lambda\mathbf{x}, \lambda\mathbf{y}) \in T$$

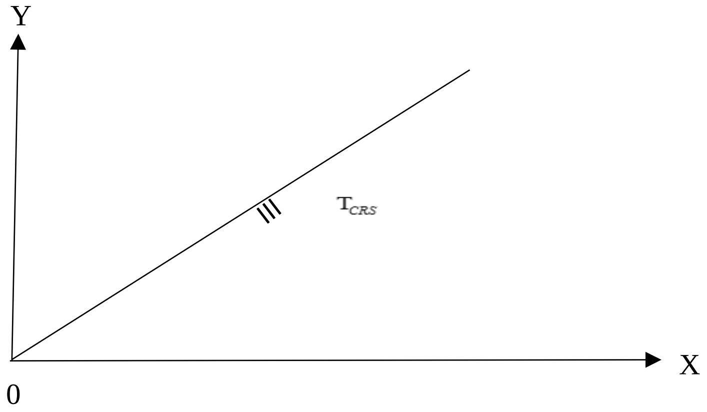

Figure 7 Constant returns to scale

Non-increasing returns to scale (NIRS) suppose:

$$\text{If } (\mathbf{x},\mathbf{y}) \in T \text{ and } 0 \leq \lambda \leq 1, \text{ then } (\lambda\mathbf{x}, \lambda\mathbf{y}) \in T$$

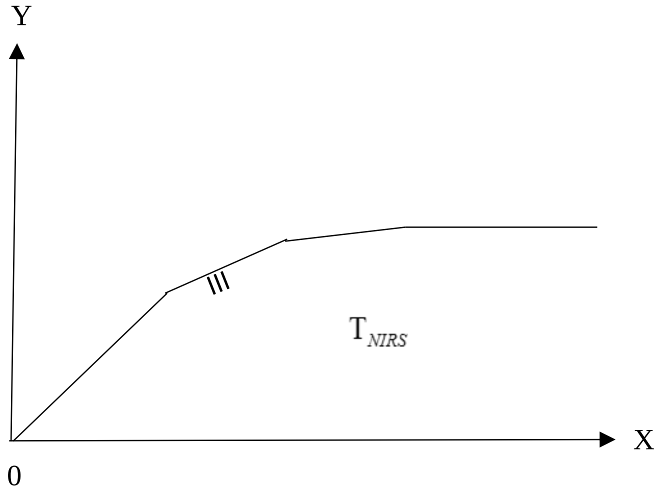

Figure 8 Non-increasing returns to scale

The mathematical expression for non-decreasing returns to scale (NDRS) is the following:

If $(\mathbf{x},\mathbf{y}) \in T$ and $\lambda \geq 1$, then $(\lambda\mathbf{x}, \lambda\mathbf{y}) \in T$

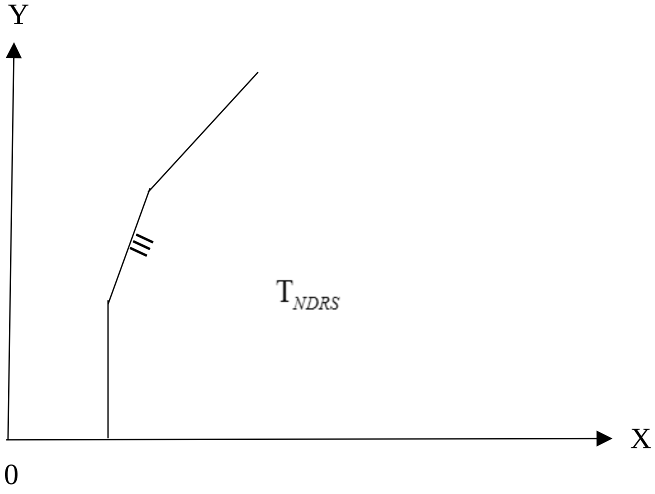

Figure 9 Non-decreasing returns to scale

Variable returns to scale (VRS) were introduced by Banker, Charnes et Cooper (1984) and hold when none of the previous models are valid.

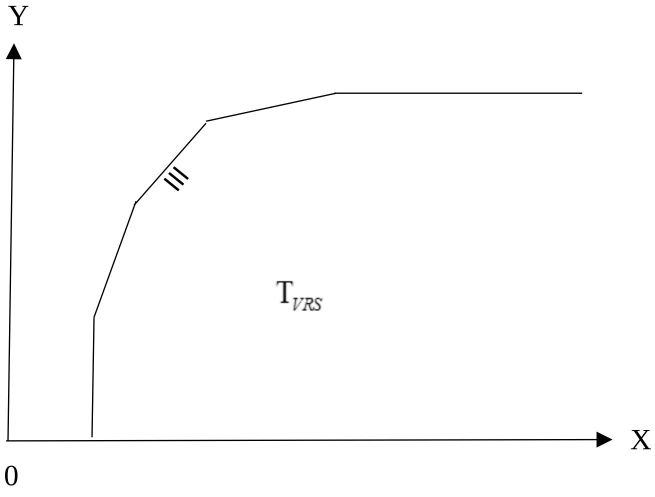

Figure 10 Variable returns to scale

2.1.2 Definition of the distance function: an equivalent representation of the production set

The axioms presented above allow us to characterize production set and ensure economic regularity conditions. However, it is merely limited to the knowledge that a production plan does or does not belong or not to the production set. Shephard (1953) was the first to introduce the notion of distance function. This tool will be the basis for calculating the production sets used here. Shephard proved that the distance function is an equivalent representation of the production possibility set. Therefore, economists can define production technologies through the distance function. The distance function is basically a tool to measure the distance from any production vector to the boundary of the production set. However, we first need to define a direction in which the distance is measured. The natural distance defined by Shephard (1970) is the output direction which is the basis of the output distance function which is formulated as:

$$D_{output}(\mathbf{x},\mathbf{y}) = \min\left\{ \theta \in R_+ : (\mathbf{y}/\theta) \in P(\mathbf{x})\right\} \tag{8}$$

where θ is the adjustment factor measuring the “distance” to the boundary of P(x). The interpretation of the distance function is straightforward. If we consider the boundary as the efficient frontier, the best practice, then the distance function can be interpreted as a measure of efficiency. The choice of the output direction also leads to a relevant interpretation. The distance function can be interpreted as the maximum increase of outputs allowed by the production technology given the level of input considered. This is the concept of technical efficiency, namely the maximum value that outputs can proportionally achieve at given inputs level. In Figure 11, points A and B are both on the boundary of the production set P(x). They are part of the efficient frontier and define the best practices. On the contrary, point C is located inside the production possibility set and represents an inefficient unit. For the same level of input x, C could achieve more outputs y1 and y2. The output distance function allows to compute this inefficiency. It is equal to OC/OB and is less than 1. Therefore, the distance function $\theta = OC/OB \leq 1$ can be interpreted as the technical efficiency. A point on the efficient frontier like A or B has a distance function equal to 1 and is considered as 100% efficient.

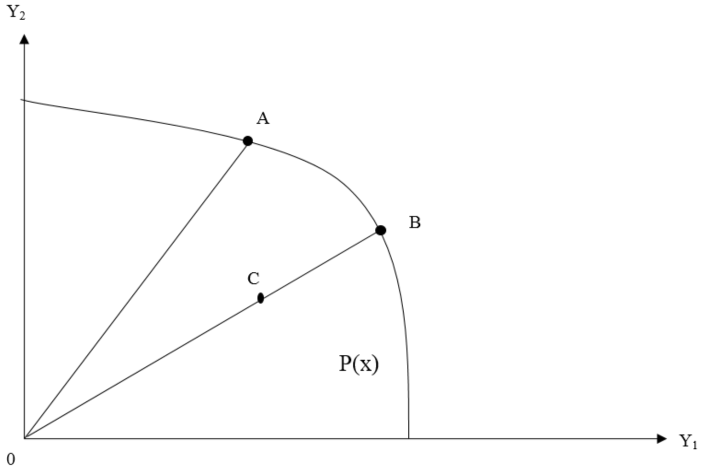

Figure 11 Shephard output distance function

Obviously, the output direction is only one direction among others to compute the distance from a production plan to the boundary of the production set. Any chosen direction will lead to a new distance function. The symmetrical choice to the output distance function is the input distance function. Here the economic interpretation is the possible reduction of inputs given a fixed level of outputs. The Shephard input distance function is defined as:

$$D_{input}(\mathbf{y},\mathbf{x}) = \max\left\{ \varphi \in R_+ : (\mathbf{x}/\varphi) \in L(\mathbf{y})\right\} \quad (9)$$

where φ implies the possible decrease in inputs at given outputs level. The Shephard input distance function seeks the radial maximum reduction in inputs. As shown in Figure 12, points A, B and C have the same level of outputs, and C is not on the frontier thus expending more inputs than A and B to achieve the same level of production. The distance function φ is equal to OC/OA and is greater than 1. In this case the technical efficiency is usually defined as the inverse of the distance function and is equal to OA/OC and is smaller than 1.

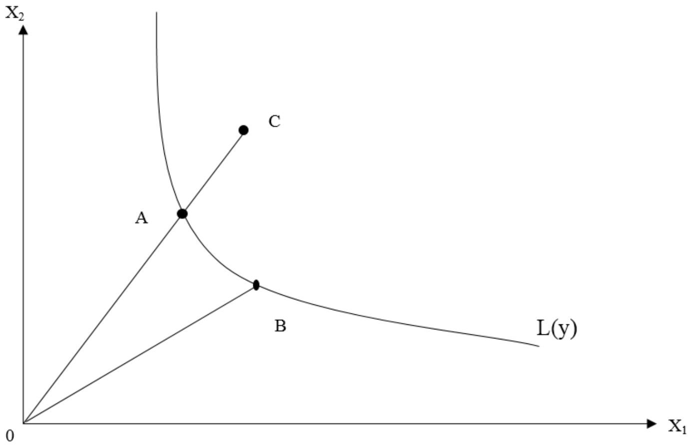

Figure 12 Shephard input distance function

Chambers et al. (1996) introduced the general case of any direction and called it directional distance function (DDF) which can increase outputs and reduce inputs simultaneously. DDF is defined as:

$$D_{DDF}(\mathbf{x},\mathbf{y};\mathbf{g}_{\mathrm{x}},\mathbf{g}_{\mathrm{y}}) = \max\left\{ \delta \in R_{+} : (\mathbf{x}-\delta\times\mathbf{g}_{\mathbf{x}},\mathbf{y}+\delta\times\mathbf{g}_{\mathbf{y}}) \in T\right\} \quad (10)$$

Where $(\mathbf{g}_{\mathrm{x}},\mathbf{g}_{\mathrm{y}}) \geq 0$ and $(\mathbf{g}_{\mathrm{x}},\mathbf{g}_{\mathrm{y}}) \neq 0$ are directional vectors of inputs and outputs, δ measures the maximum possibility of simultaneously increasing outputs and decreasing inputs. Compared to Shephard distance functions, directional distance functions are more flexible in choosing objective directions as illustrated in Figure 13.

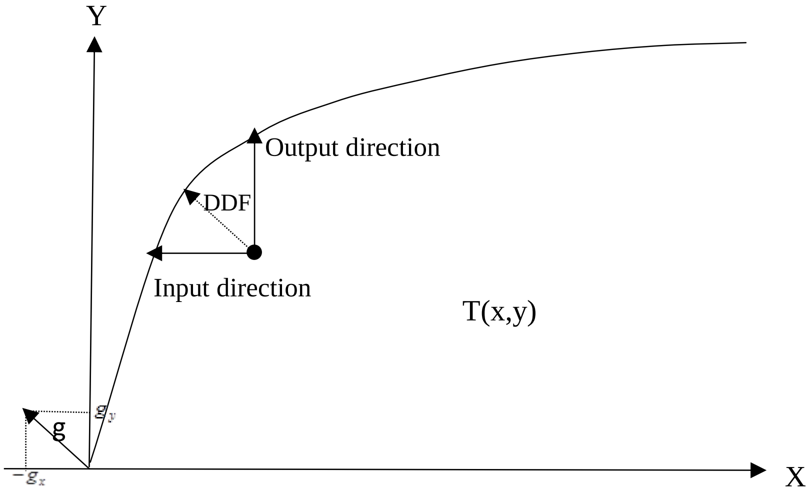

Figure 13 Directional distance function

Obviously, the output and input distance functions are particular cases of this more general definition.

2.1.3 Measures of productive, technical and scale efficiencies

Definition of efficiency concepts

As it was shown in the previous paragraph, the construction of efficient frontier is based on several economic and mathematical axioms and determines the upper envelop of production possibility set. To characterize the level of inefficiency of firms that are not on the frontier and the nature of this inefficiency, Farrell (1957) established the bases of the theoretical frame inspired also by Koopmans (1951) and Debreu (1951).

Technical efficiency is the capacity of a firm to eliminate waste. It can be achieved either by maximizing output production using the given amount of input or by minimizing the input usage given the amount of output. The input-oriented technical efficiency is the function inverse to the input distance function (9): $TE_{input} = 1 / D_{input}(\mathbf{y},\mathbf{x})$. The output-oriented technical efficiency is the function inverse to the output distance function: $TE_{output} = 1 / D_{output}(\mathbf{x},\mathbf{y})$.

Technical efficiency can be estimated under the different axioms related to the technologies, in particular the returns to scale axioms. The technical efficiency evaluated with CRS technology is referred to as productive efficiency. The technical efficiency estimated with VRS technology is considered as (pure) technical efficiency. The ratio between productive efficiency (CRS technical efficiency) and pure technical efficiency (VRS technical efficiency) is interpreted as scale efficiency: $\frac{TE^{CRS}_{input(output)}}{TE^{VRS}_{input(output)}}$. Scale efficiency shows the extent a firm is far from the "most productive scale size". In the single input/single output context, most productive scale size is characterized by the maximum output to input ratio that is the maximum average product.

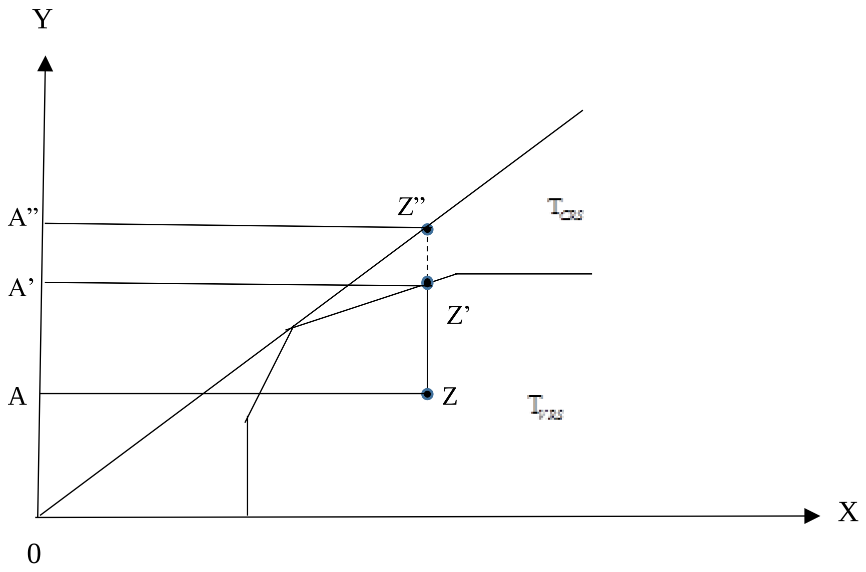

Figure 14 Productive, technical and scale efficiencies

In Figure 14, Z is an inefficient point. Its projection onto VRS technology in output direction gives output technically efficient point Z'. The projection of Z onto CRS technology in output direction results into technically efficient point Z". Thus, technical efficiency which is related to VRS frontier is measured as 0A/0A'. Productive efficiency which is related to CRS frontier is equal to

0A/0A”. Finally, scale efficiency which is the ratio between productive and technical efficiencies is equal to 0A’/0A”.

2.1.4 Measures of economic efficiency: cost, revenue, profit and allocative efficiency

So far, efficiency was only based on quantities and is related to the objective of avoiding waste in inputs and outputs. Economic efficiency introduces prices in the analysis where the economic objectives of producers are now profit maximization or cost minimization. Let us consider a vector of inputs prices $w \in R_+^n$. The cost function is defined as $C^*(y,w) = \min\{wx \mid x \in L(y)\}$. This corresponds to the minimum expenditure required to produce output vector y at input prices w .

Cost efficiency can be defined as the ratio of minimum cost to observed cost. It indicates to which extend a production unit minimizes the cost given an output vector y and input prices w : $CE = C^*(y,w) / C^{obs}(y,w) \leq 1$.

Let us now consider a vector of output prices $p \in R_+^m$. The revenue function is defined as $R^*(x,p) = \max\{py \mid y \in P(x)\}$. This corresponds to the maximum revenue that can be achieved using input vector x at output prices p.

In the same logic, revenue efficiency can be defined as the ratio of maximum revenue to observed revenue, that is the extent a production unit maximizes the revenue given an input vector x and output prices p : $RE = R^*(x,p) / R^{obs}(x,p) \geq 1$.

Given inputs prices $w \in R_+^n$ and output prices $p \in R_+^m$, the maximum attainable profit can be computed as: $\Pi^*(p,w) = max\{ p^T y - w^T x \mid (x,y) \in T)$. While revenue and cost functions are well defined under all returns to scale assumptions, the profit function merits some comments. First, it is defined by a difference and does not prevent negative values. Second, under constant returns to scale, it is well known that the maximum profit is either zero or infinite. In general, the profit function is well defined for non-increasing returns to scale. Whenever the observed profit and maximum profit are both non negative, a well-defined profit efficiency can be computed as a ratio of observed profit to maximum profit: $\Pi E = \Pi^{obs} / \Pi^* \leq 1$.

Cost, revenue and profit efficiency can be understood as the best allocation of input and output quantities given a set of prices on the production frontier. Therefore, it comprises two components: technical efficiency and allocative efficiency. Since we defined properly technical efficiency and economic efficiency (cost, revenue or profit), allocative efficiency is generally computed as a residue.

For cost minimization, allocative efficiency can be retrieved as a ratio of cost efficiency and technical input efficiency: $AE_I = CE / TE_I$. It measures the extent a technically efficient point fails to achieve the minimum cost because of inefficient allocation of resources.

For a revenue maximization framework, allocative efficiency can be evaluated as a ratio of revenue efficiency and technical output efficiency: $AE_O = RE / TE_O$. This ratio measures how far the technically efficient point is from the point with maximum revenue because of inefficient allocation of outputs.

Finally, in the profit context, allocative efficiency is computed as the ratio of technical profit to maximum profit: $AE_\Pi = \Pi' / \Pi^*$, where Π' is the profit at the technically efficient point.

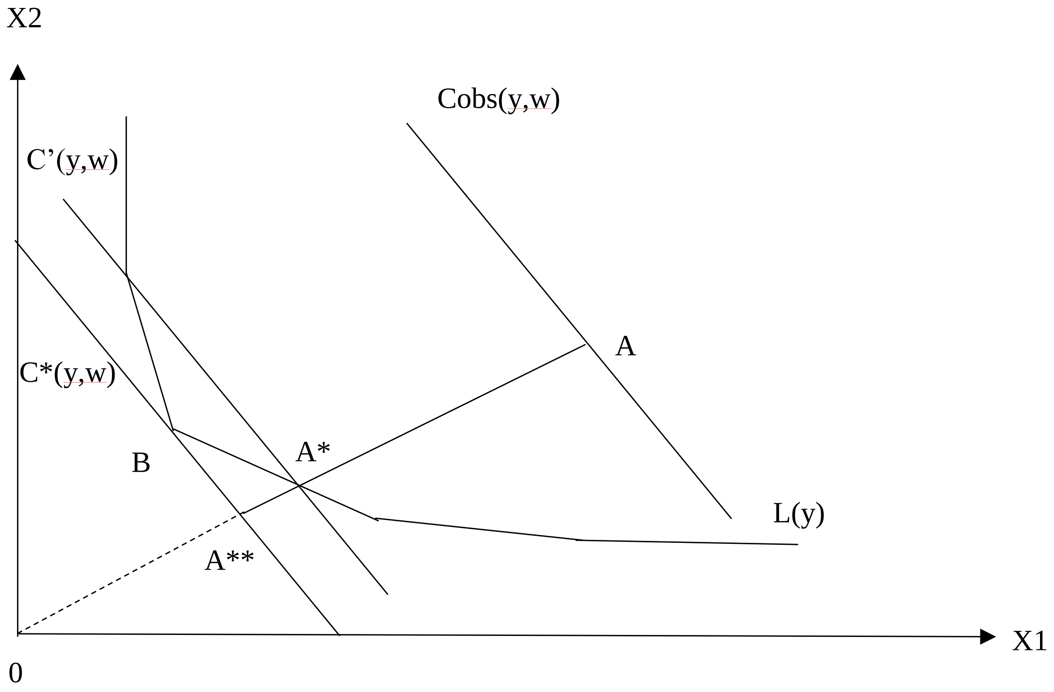

Figure 15 Cost efficiency and allocative efficiency in inputs

Figure 15 illustrates cost efficiency and its decomposition. Cost efficiency is equal to C*(y,w)/Cobs(y,w) = wx*/wx = 0A**/0A. Input technical efficiency is given by 0A*/0A. Thus, allocative input efficiency is equal to 0A**/0A*. This corresponds to the adjustment in input mix that a firm needs to make from a technically efficient point A* to the cost efficient point B.

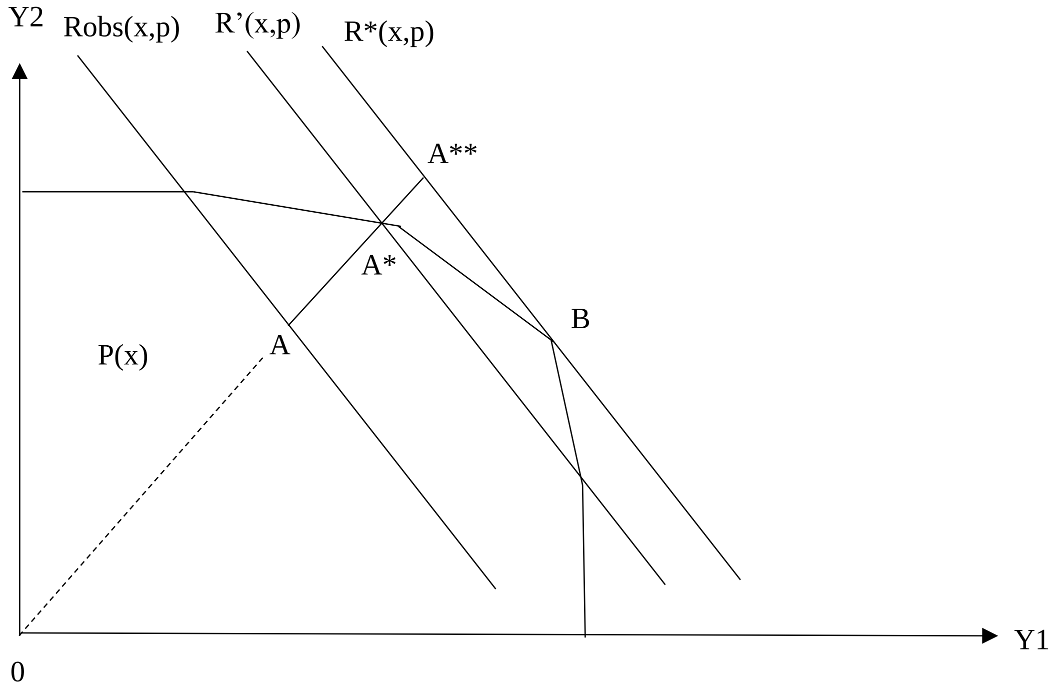

Figure 16 Revenue efficiency and allocative efficiency in outputs

In Figure 16, the revenue efficiency corresponds to the ratio Robs(x,p)/R*(x,p) = py/py* = 0A/0A**. Output technical efficiency is equal to 0A/0A*. Thus, allocative output efficiency is given by 0A*/0A**. It corresponds, in Figure 16, to the adjustment in output mix that a firm needs to make from a technically efficient point A* to the revenue efficient point B.

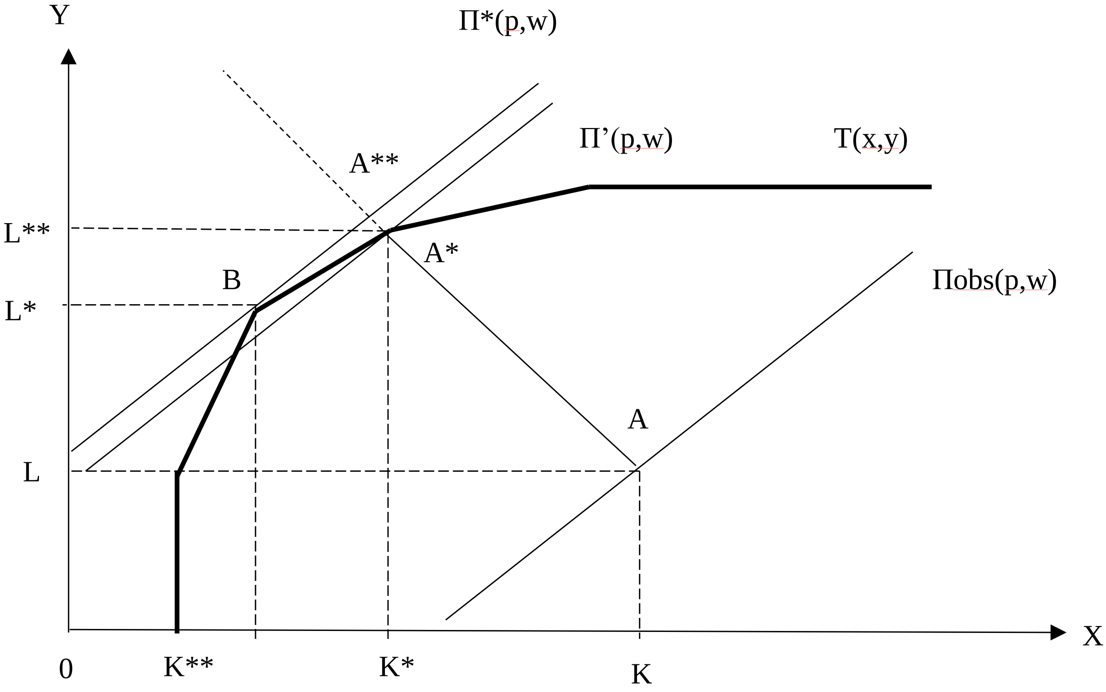

Figure 17 Profit efficiency and allocative profit efficiency

In Figure 17, the profit efficiency corresponds to the ratio Πobs(p,w)/Π*(p,w) = (py-wx)/(py*-wx*) = (p0L-w0K)/(p0L**-w0K**). Technical profit is equal to py'-wx' = p0L*-w0K*. Thus, allocative profit efficiency is given by (py'-wx')/(py*-wx*) = (p0L*-w0K*)/(p0L**-w0K**). It corresponds, in Figure 17, to the adjustment in output and input mix that a firm needs to make from a technically efficient point A* to the maximum profit point B.

2.2 Estimations of distance function

Both "parametric" and "nonparametric" estimations are popular in the produiction literature. Historically parametric approaches are related to the econometric approach where a functional form is given for the technology and estimation is based on estimators like OLS (Ordinary Least Squares) or ML (Maximum likelihood). Nonparametric estimations are usually referred to DEA (Data Envelopment Analysis) estimators which are computed with a linear programming framework. Another aspect is deterministic or stochastic nature of estimation. Generally, econometric approaches comprise an error term and are stochastic by nature. On the other hand, linear programming methods are generally deterministic.

The main difference between parametric and nonparametric approaches is whether a global functional forms of production technologies can be predefined or not. For the former, many forms can be found in the literature: Cobb-Douglas, CES (Constant Elasticity of Substitution), translog or quadratic functional forms are some examples. After the functional forms are determined, stochastic frontier analysis (SFA) is usually employed to estimate parameters of production functions or distance functions. Since in this thesis we only use nonparametric estimations, we do not discuss parametric models in depth.

2.2.1 Nonparametric estimation by Data Envelopment Analysis

Besides parametric models, nonparametric DEA approaches are also usually employed to estimate the production frontier. Compared to SFA, DEA does not require a global predefined functional form and a local piecewise linear production frontier is created on combinations of the best observed practices, due to an optimization of a linear program.

As it was outlined in the previous section, Shephard (1953, 1970) introduced input- and output-oriented distance functions. Farrell (1957) was the first to introduce a linear programming framework for the special case of one output. In 1978, Charnes, Cooper and Rhodes generalized to multiple outputs and presented the DEA model (CCR model) that allowed to empirically estimate the distance function under the constant returns to scale. Banker, Charnes and Cooper (1984) extended the CCR model to accommodate the technologies with variable returns to scale (BCC model). Let us consider a sample of K observed DMUs, $k=\{1,\dots,K\}$ which use a vector of N inputs $\mathbf{x}=\left(x^1,\dots,x^N\right)\in R_+^N$ to produce a vector of M outputs $\mathbf{y}=\left(y^1,\dots,y^M\right)\in R_+^M$. The envelopment form for BCC output oriented model is:

$$
\begin{aligned}
&1/D_{output}(\mathbf{x}_o,\mathbf{y}_o)=\underset{\lambda,\theta_o}{Max}\ \theta_o\\
&\sum_{k=1}^{K}\lambda_k y_k^m \geq \theta_o y_o^m,\ \forall m=1,\dots,M\\
&\sum_{k=1}^{K}\lambda_k x_k^n \leq x_o^n,\ \forall n=1,\dots,N \qquad\qquad \text{(LP1)}\\
&\sum_{k=1}^{K}\lambda_k =1\\
&\lambda_k \geq 0,\ \forall k=1,\dots,K
\end{aligned}
$$

Vector θ_o measures technical inefficiency. The left-hand side of the first two constraints and the third constraint specify the underlying technology. The right-hand side identifies the evaluated DMU. The constraint $\sum_{k=1}^{K}\lambda_k = 1$ refers specifically to the VRS technology.

The envelopment form for BCC input oriented model is:

$$
\begin{aligned}
& 1/D_{input}(\mathbf{x}_o,\mathbf{y}_o) = \underset{\boldsymbol{\lambda},\theta_o}{Min}\ \theta_o \\
& \sum_{k=1}^{K}\lambda_k y_k^m \geq y_o^m,\ \forall m = 1,\ldots,M \\
& \sum_{k=1}^{K}\lambda_k x_k^n \leq \theta_o x_o^n,\ \forall n = 1,\ldots,N \\
& \sum_{k=1}^{K}\lambda_k = 1 \\
& \lambda_k \geq 0,\ \forall k = 1,\ldots,K
\end{aligned}
\qquad \text{(LP2)}
$$

The models presented above are radial models and thus suppose proportional augmentation in all outputs or proportional reduction in all inputs. Efficient DMUs obtain a score of 1. However, efficiency is defined here as weak efficiency since efficient DMUs obtained using these models can be not Pareto-efficient. One can deal with this problem by resolving two-stage model in which in the first stage (LP1) or (LP2) model is solved and in the second stage the slack-maximizing model is solved.

Besides, several non-radial models were elaborated to eliminate slacks. Among them additive model (Charnes et al., 1985), Russell measure of efficiency (Fare and Lovell, 1978), range-adjusted measure of efficiency (Cooper et al., 1999), the slack-based measure of efficiency (Tone, 1993, 2001), the geometric distance function efficiency measure (Portela and Thanassoulis, 2002, 2005, 2007).

The other two models are hyperbolic (Fare et al., 1985) and directional distance (Chambers et al., 1996, 1998) efficiency models. Their specification is that they do not necessary project a DMU on the Pareto-efficient production frontier but allows simultaneous changes in both inputs and outputs (non-oriented models). We specify here only the directional distance model since it will be used later throughout this document:

$$
\begin{aligned}
& D_{DDF}(\mathbf{x}_o,\mathbf{y}_o;\mathbf{g}^N,\mathbf{g}^M) = \underset{\boldsymbol{\lambda},\theta_o}{Max}\ \theta_o \\
& \sum_{k=1}^{K}\lambda_k y_k^m \geq y_o^m + \theta_o g^m,\ \forall m = 1,\dots,M \\
& \sum_{k=1}^{K}\lambda_k x_k^n \leq x_o^n - \theta_o g^n,\ \forall n = 1,\dots,N \qquad \text{(LP3)} \\
& \sum_{k=1}^{K}\lambda_k = 1 \\
& \lambda_k \geq 0,\ \forall k = 1,\dots,K
\end{aligned}
$$

The directional distance function allows to specify different directions towards the production frontier. These can be observed inputs and outputs, average observed inputs and outputs, sum of observed inputs and outputs, etcetera. Moreover, the directional distance function can be reduced to the traditional Farrell input or output efficiency measures.

Aside from the DEA model for estimating technical efficiency, there are DEA models for minimizing cost, maximizing revenue, and maximizing profit. Considering further implications, we report here only the profit maximizing DEA model:

$$
\begin{aligned}
& \Pi(p_o,w_o) = \underset{\boldsymbol{\lambda},\tilde{\mathbf{y}},\tilde{\mathbf{x}}}{Max}\ \sum_{m=1}^{M} p_o\tilde{y}^m - \sum_{n=1}^{N} w_o\tilde{x}^n \\
& \sum_{k=1}^{K}\lambda_k y_k^m \geq \tilde{y}^m,\ \forall m = 1,\dots,M \\
& \sum_{k=1}^{K}\lambda_k x_k^n \leq \tilde{x}^n,\ \forall n = 1,\dots,N \qquad \text{(LP4)} \\
& \sum_{k=1}^{K}\lambda_k = 1 \\
& \lambda_k \geq 0,\ \forall k = 1,\dots,K
\end{aligned}
$$

DEA models for technical efficiency (LP1 to LP3) are linear programs and as such can be written in a dual form. Often, the latter gives an interesting economic interpretation in terms of shadow prices. In Table 1, we present the primal (LP3) and dual (LP3') models for the directional distance function under VRS. We also present the dual model in an equivalent form (LP3'') (see Leleu (2009) for its derivation) that allows a natural economic interpretation.

Table 1 Primal and dual directional DEA models under variable returns to scale

Primal DDF (LP3)	Dual DDF (LP3’)
$D_{DDF}(\mathbf{x}_o,\mathbf{y}_o;\mathbf{g}^N,\mathbf{g}^M)=\underset{\boldsymbol{\lambda},\theta}{Max}\ \theta$ $\sum_{k=1}^{K}\lambda_k y_k^m \geq y_o^m+\theta g^m,\ \forall m=1,\ldots,M$ $\sum_{k=1}^{K}\lambda_k x_k^n \leq x_o^n-\theta g^n,\ \forall n=1,\ldots,N$ $\sum_{k=1}^{K}\lambda_k=1$ $\lambda_k \geq 0,\ \forall k=1,\ldots,K$	$D_{DDF}(\mathbf{x}_o,\mathbf{y}_o;\mathbf{g}^N,\mathbf{g}^M)=\underset{\mathbf{u},\mathbf{v},z}{Min}\ \text{-}\sum_{m=1}^{M}u^m y_o^m+\sum_{n=1}^{N}v^n x_o^n+z$ $\text{-}\sum_{m=1}^{M}u^m y_k^m+\sum_{n=1}^{N}v^n x_k^n+z \geq\ 0,\ \forall k=1,\ldots,K$ $\sum_{m=1}^{M}u^m g^m+\sum_{n=1}^{N}v^n g^n=1$ $u^m \geq 0,\ \forall m=1,\ldots,M$ $v^n \geq 0,\ \forall n=1,\ldots,N$
	Dual DDF rewritten (LP3’’)
	$D_{DDF}(\mathbf{x}_o,\mathbf{y}_o;\mathbf{g}^N,\mathbf{g}^M)=\underset{\mathbf{u},\mathbf{v},\pi}{Min}\ \pi$ $\left(\sum_{m=1}^{M}u^m y_k^m-\sum_{n=1}^{N}v^n x_k^n\right)-\left(\sum_{m=1}^{M}u^m y_o^m-\sum_{n=1}^{N}v^n x_o^n\right)\leq \pi\ \ 0,\ \forall k=1,\ldots,K$ $\sum_{m=1}^{M}u^m g^m+\sum_{n=1}^{N}v^n g^n=1$ $u^m \geq 0,\ \forall m=1,\ldots,M$ $v^n \geq 0,\ \forall n=1,\ldots,N$

LP3’’ is very convenient for an economic interpretation. Variables $\mathbf{u}$ and $\mathbf{v}$ are respectively the shadow prices of outputs and inputs. Therefore the quantities $\sum_{m=1}^{M}u^m y_k^m$ and $\sum_{n=1}^{N}v^n x_k^n$ can be interpreted as the shadow revenue and the shadow cost of the DMU k. Then, the quantity $\left(\sum_{m=1}^{M}u^m y_k^m-\sum_{n=1}^{N}v^n x_k^n\right)$ is simply interpreted as the shadow profit for DMU k. Therefore, the LHS of the first set of constraints of LP’’ is simply the difference between the shadow profits of each DMU k and the evaluated DMU o. The RHS π is therefore an upper bound for the shadow profit inefficiency of DMU o. Finally, the objective of LP3’’ is to find the best set of shadow prices that minimizes the shadow profit inefficiency of the evaluated DMU. Lastly, it is obvious to see that a shadow profit is homogenous of degree 1 in prices (i.e. if prices are multiplied by two, then the shadow profit is multiplied by two) and we require normalization on shadow prices. This is the role of the last constraint in LP3’’. Interestingly, this corresponds to the choice of the direction in the primal problem. Therefore we clearly see that a choice of a direction (radial input, radial output, directional…) is equivalent to a choice of normalization of the shadow prices in the dual.

2.2.2 A numerical example

Let us consider a simple case of 6 DMUs which use one input and produce one output (Table 2).

Table 2 Input and output data for the 6 DMUs

DMU	X	Y
1	3	2
2	4	4
3	6	6,5
4	10	7
5	8	7
6	5	3

The following figure presents the production possibility set and the efficient frontier. Clearly, DMU 1 to 5 are efficient and composed the frontier. DMU 6 is inefficient and is located inside the production set.

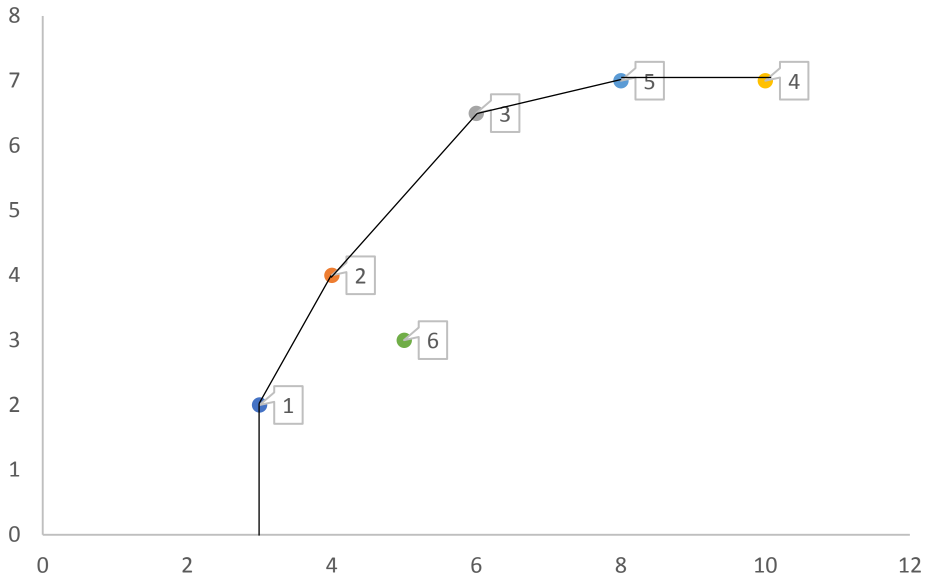

Figure 18 Representation of input and output data

We adapt LP3 and LP3'' to this particular case and write them as LPex and LPex''. When evaluating DMU 6, we use DMU6 input-output vector as the direction. LPex is as follows:

$$D_{DDF}(x_{DMU6}, y_{DMU6}; x_{DMU6}, y_{DMU6}) = \underset{\lambda,\theta}{Max}\ \theta$$

$$\begin{aligned}
&\sum_{k=1}^{6} \lambda_k y_k \geq y_{DMU6} + \theta y_{DMU6},\\
&\sum_{k=1}^{6} \lambda_k x_k \leq x_{DMU6} - \theta x_{DMU6},\\
&\sum_{k=1}^{6} \lambda_k = 1\\
&\lambda_k \geq 0, \forall k = 1,\ldots,6
\end{aligned} \qquad \text{LPex}$$

The optimal solution for LPex is:

$\theta^* = 0,23$

$$\begin{aligned}
\lambda^*_{DMU1} &= 0,15\\
\lambda^*_{DMU2} &= 0,85\\
\lambda^*_{DMU3} &= 0\\
\lambda^*_{DMU4} &= 0\\
\lambda^*_{DMU5} &= 0\\
\lambda^*_{DMU6} &= 0
\end{aligned}$$

We provide a SAS code for LPex in Appendix 1.

The efficiency score of DMU6 is given by $\theta^* = 0,23$. Thus, this DMU can decrease the input use by 23% and increase the output production in the same time by 23%.

With optimal values of λ we can compute the efficient level of input and output for DMU6:

$$\begin{aligned}
x^*_{DMU6} &= \lambda^*_{DMU1} * x_{DMU1} + \lambda^*_{DMU2} * x_{DMU2} = 0,15*3 + 0,85*4 = 3,85\\
y^*_{DMU6} &= \lambda^*_{DMU1} * y_{DMU1} + \lambda^*_{DMU2} * y_{DMU2} = 0,15*2 + 0,85*4 = 3,70.
\end{aligned}$$

Since we have only one input and one output, there are no slacks on the constraints. Therefore, we can also compute the efficient input output vector with the optimal value of θ as:

$$\begin{aligned}
x^*_{DMU6} &= x_{DMU6} - \theta^* x_{DMU6} = 5 - 0,23*5 = 3,85\\
y^*_{DMU6} &= y_{DMU6} + \theta^* y_{DMU6} = 3 + 0,23*3 = 3,70.
\end{aligned}$$

We now present LPex'' with the same direction to get the dual optimal values:

$$\begin{aligned}
&D_{DDF}(x_{DMU6}, y_{DMU6}; x_{DMU6}, y_{DMU6}) = \underset{u,v,\pi}{Min}\ \pi\\
&\left(u\ y_k - v\ x_k\right) - \left(u\ y_{DMU6} - v\ x_{DMU6}\right) \leq \pi\ 0, \forall k = 1,...,6\\
&u\ y_{DMU6} + v\ x_{DMU6} = 1 \qquad\qquad\qquad\qquad \text{LPex''}\\
&u\ \geq 0\\
&v\ \geq 0
\end{aligned}$$

At the optimal of the above program:

$\pi^* = 0,23$

$u^* = 0,077$
$v^* = 0,154$

As for LPex, we provide a SAS code for LPex'' in Appendix 1.

The efficiency score of DMU6 is given by $\pi^* = 0,23$. We verify that the dual objective function is equal to the primal: $\pi^* = \theta^*$.

With optimal values of the shadow prices, we can compute the marginal productivity as $\frac{v^*}{u^*} = \frac{0,154}{0,077} = 2$. This marginal productivity gives the slope of the efficient facet where DMU 6 is projected.

The summary of results is in Tables 3 and 4, and Figure 19.

Table 3 Results for LP3

Efficiency score	Activity variables		Efficient input and output	
θ^*	λ^*_{DMU1}	λ^*_{DMU2}	x^*_{DMU6}	y^*_{DMU6}
0,23	0,15	0,85	3,85	3,7

Table 4 Results for LP3''

Shadow profit	Shadow prices		Marginal productivity
π^*	u*	v*	v*/u*
0,23	0,077	0,154	2

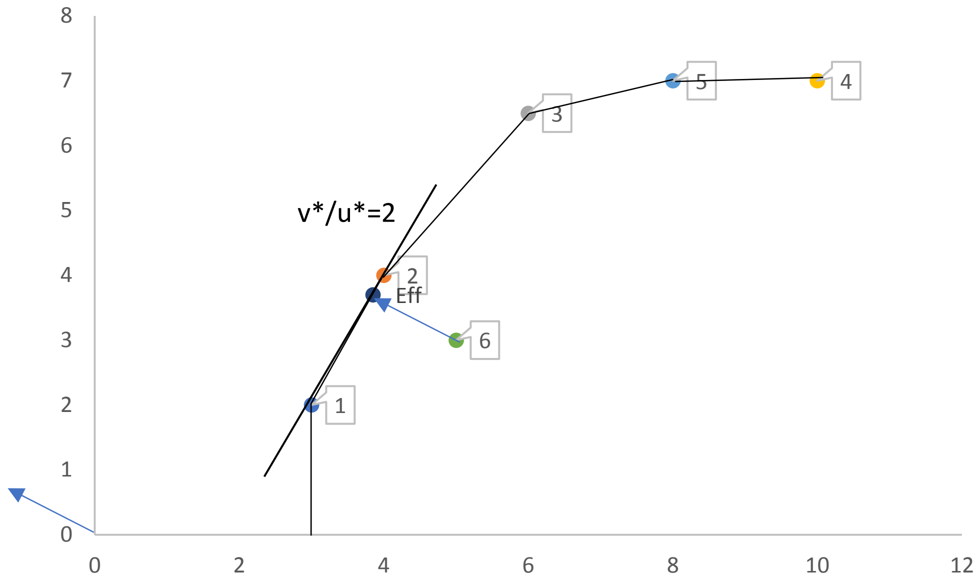

Figure 19 Illustration of primal and dual directional distance functions

3 Generation and distribution of productivity gains over time

3.1 Surplus account

3.1.1 Introduction to surplus account

Surplus accounting methods decompose interannual variations of different income statement items into two separate effects: the volume (or quantity) effect and the price effect. Using these decompositions, it measures the productive performances of a firm through total factor productivity (TFP) gains. Besides, the surplus accounting method assesses the distribution of new resources generated by these TFP gains among the different stakeholders involved in the production process and financial exchanges of the firm (customers, suppliers, capital providers, employees, government).

One of the important characteristics of the surplus accounting method is that the analysis of productive performances indicators and their distribution is based on all elements of the income statement and thus includes all the most important dimensions of a firm. In this sense, the method complements (but not replaces) the standard criteria of performance evaluation mostly based on partial indicators (operating income, some financial ratios or productivity indexes regarding one or another inputs).

As a result, the surplus accounting method provides a systematic analysis of the past and future decisions of a firm: productive performance, commercial policy, wage management, procurement policy, investment and financial strategy, etcetera. The method favors a total productivity approach and gives a synthetic evaluation of these policies taking into account economic and social constraints.

The method seems to be straight forward, but its operational implementation for decision making can represent some practical difficulties. One of them is linked to an information system of a firm and another one is caused to the conceptual issues of objective dissociation between quantity and price effects for a number of value items in the income statement. This incites a firm to use this tool to render its functioning more useful and efficient.

The implementation of the surplus accounting method was historically developed in France for four large national companies (SNCF, Gaz de France, Charbonnages de France and Electricité de France) by the *Centre d'Etudes des Revenus et des Coûts* at the beginning of 1970s[1]. Grifell-Tatjé

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[1] Cf. CERC documents n°55 /56 1980, La Documentation Française.

and Lovell in their *Productivity Accounting* (2015) provide the latest review of the surplus accounting models and their extensions.

We first present the objective of the surplus accounting method (section 1). In the second section, a firm is viewed as an organization of production and exchange of values among different stakeholders. The third section highlights some methodological aspects to make the method clear. The fourth section develops a fictitious example about a manufacturing firm to illustrate, in a concrete way, the type of results and conclusions the method is able to establish. Finally, the fifth section presents a real world case study developed for the whole US automobile industry for the period 1987-2014.

This case study has a double interest. On the one hand, using real long-term data (28 years), it shows the relevance of a chronological analysis of productivity gains and their distribution among the stakeholders of the related sector. Finally, comparing the results of automobile industry with those established for the entire US economy (all activity areas) allows both to contrast their productive performances and their distribution structure.

3.1.2 Objective

The surplus accounting method aims at evaluating the evolution of a firm’s performance between two accounting periods. This measure considers all the factorial resources and products of a firm and provides a global (and not partial) performance indicator. Moreover, it links explicitly the performance evolution to variations of remunerations and/or price advantages of the stakeholders (for example, suppliers, customers, employees, capital providers).

Through this approach, a firm is considered as an organization which produces and exchanges with different stakeholders. Therefore, the performance of a firm can be analyzed under two key dimensions: the production and the exchange activities. Indeed, using factor resources like raw materials, labor, equipment, financial capital, a firm produces goods and services. Simultaneously, this production activity generates exchange flows among different stakeholders involved in the firm activity (sales, purchasing, investments, employees’ remuneration, dividend payments, loan repayments, and taxes). The analysis of these production and exchange processes requires using a large amount of technical and accounting information.

The balance sheets, the income statements, the cost accounting, and more specific dashboards provide a good deal of operational information for decision makers concerning general functioning and profitability of a firm. Nevertheless, they insufficiently link the two main dimensions of a firm activity: **value creation** and **revenue distribution**. They rely on the evaluation criteria which mostly remain limited by using partial indicators such as profit, financial ratios or basic productivity indicators.

In a period of limited growth and price volatility which renders the management of a firm more difficult, productivity gains appear more and more necessary to satisfy a number of economic and social objectives (profitability, employment maintaining, and pollution reduction). In this sense by linking price changes to productivity gains, this method seems to be very useful to firms' decision-making strategic choices.

3.1.3 Splitting the created value into volume and price effect

More precisely, thanks to the decomposition of the value variations of different profit and loss account items into volume (quantity) and price effects, this method estimates total factor productivity gains and distributes them among different agents of a firm over time.

Let us consider two profit and loss accounts at periods *s* and *t* (*t*>*s*). The performance of a firm between the two periods can be evaluated by the change in operating income or profit. This profit change is explained jointly by quantities and prices variations of goods and services which are produced or used. If we consider that quantity variations reflect more particularly internal decisions of a firm while price variations result more from market business cycles or from commercial negotiations with its partners, it appears relevant to decompose profit changes between these two quantity and price effects. It is now possible to split what in profit changes are due to strategic choices and what is caused by more exogenous factors.

By grouping quantity and price changes into two synthetic distinct terms, the equation of profit change becomes:

$$\begin{aligned} \Delta\Pi &= \Pi_t - \Pi_s \\ \Delta\Pi &= \mathbf{p_t Y_t} - \mathbf{w_t X_t} - \mathbf{p_s Y_s} - \mathbf{w_s X_s} \\ \Delta\Pi &= \mathbf{p_s \Delta Y} - \mathbf{w_s \Delta X} + \mathbf{\Delta p Y_t} - \mathbf{\Delta w X_t} \end{aligned} \tag{11}$$

where :

$\mathbf{Y_t}$ = column vector of quantity for outputs, $o = 1, 2, ..., O$ at period t

$\mathbf{X_t}$ = column vector of quantity for inputs, $i = 1, 2, ..., I$ at period t

$\mathbf{\Delta Y}$ = column vector of quantity change for outputs between periods t and s,

$\mathbf{\Delta X}$ = column vector of quantity change for inputs between periods t and s,

$\mathbf{p_s}$ = line vector of outputs' unit prices at period s

$\mathbf{w_s}$ = line vector of inputs' unit prices at period s

The first term in the right hand side of (11) measures the gap between price weighted changes in output and input quantities. It is referenced to as a total factor productivity surplus (***PS)***. The second term evaluates quantity weighted changes in output and input prices. It is referenced to as a price advantage component (***PA)***.

It is important to highlight that all the quantity and/or price variables which include all the credit or debit elements of profit and loss account. Thus, this is an overall measure which encompasses all the dimensions of a firm. As a result, ***PS*** is an overall factor productivity measure which differs from a partial productivity indicator by the fact that it considers simultaneously all the resources flows involved in a production process.

If we consider again the equation (11) and move the price changes term into the left hand side, we obtain:

$$-\mathbf{\Delta p Y_t} + \mathbf{\Delta w X_t} + \Delta\Pi = \mathbf{p_s \Delta Y} - \mathbf{w_s \Delta X} \qquad (12)$$

We can associate the profit change to a remuneration change $\Delta w_M X_{M,t}$ of a specific input called "managerial input" X_M which evaluates the ability of a firm to generate a financial surplus after covering all the costs such as intermediate inputs, labor and fixed capital and other financial elements including dividends, interest costs, and taxes. This net operating income approximates the managers' remunerations for risk taking in business activities. In this case, the left hand side term of equation (12) can be written as the sum of remunerations changes or price advantages ***(PA)*** of different stakeholders of a firm (including the managerial factor):

$$\left[-\mathbf{\Delta p Y_t} + \mathbf{\Delta w X_t} + \Delta w_M X_{M,t}\right] = PA$$

The negative sign attributed to changes in selling prices in the previous expression shows explicitly the fact that an output price reduction is advantage for the customers. In the same manner, the positive sign before input price changes enables attribution to a price or remuneration advantage in case of price increase.

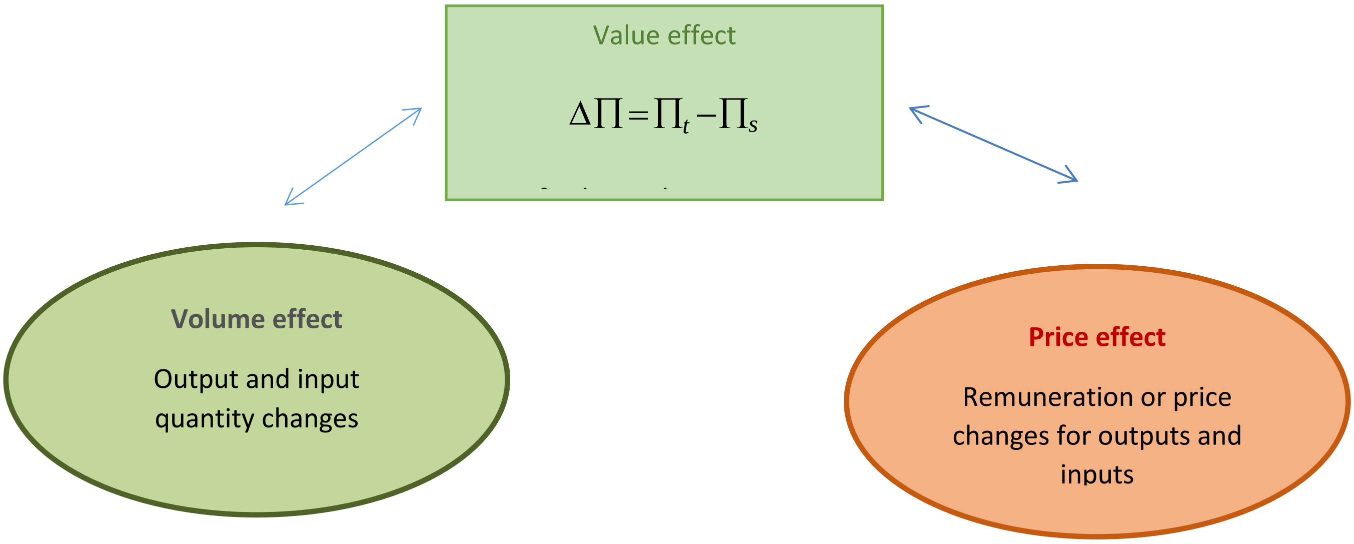

Figure 20 Decomposition of profit change into quantity effect and price effect

3.2 Productivity Surplus and TFP

Based on the aforementioned considerations, one can establish the fundamental equilibrium which is common to any analysis of overall productivity: the Total Productivity Surplus is necessarily equal to the sum of the Price Advantages distributed by a firm among its different partners:

$$\mathbf{-\Delta pY_t + \Delta wX_t} + \Delta\Pi = \mathbf{p_s\Delta Y - w_s\Delta X}$$

$$PA = TPS \tag{13}$$

Price Advantages = Total Productivity Surplus

The equilibrium (13) relies on the following simple idea: **“The total amount of remuneration or price advantages that a firm is able to distribute must be strictly equal to its generated productivity gains”.**

The decomposition of each term of ***PA*** reallocates to each stakeholder its own price advantage. It allows, thus, to identify the recipients or the losers in the sharing of ***PS***.

In equation (11), quantity changes are weighted by prices of initial period *s* and price changes are weighted by quantities of final period t. That is, PS is defined in a Laspeyres way and PA is defined in a Paasche way. We can similarly determine PS in a Paasche way and PA in a Laspeyres way as follows:

$$\Delta\Pi = \mathbf{p_t\Delta Y - w_t\Delta X} + \mathbf{\Delta pY_s - \Delta wX_s} \tag{14}$$

where :

$\mathbf{Y_s}$ = column vector of quantity for outputs, $o = 1, 2, ..., O$ at period s

$\mathbf{X_s}$ = column vector of quantity for inputs, $i = 1, 2, ..., I$ at period s

$\mathbf{\Delta Y}$ = column vector of quantity change for outputs between periods t and s,

$\mathbf{\Delta X}$ = column vector of quantity change for inputs between periods t and s,

$\mathbf{p_t}$ = line vector of outputs' unit prices at period t

$\mathbf{w_t}$ = line vector of inputs' unit prices at period t

If we consider the arithmetic average of the Laspeyres and Paasche PS expressions and Laspeyres and Paasche PA expressions, we obtain an equivalent to (11) and (14) profit change decomposition as a Bennet additive indicator:

$$\Delta\Pi = \left[\frac{1}{2}(\mathbf{p_s} + \mathbf{p_t})\mathbf{\Delta Y} - \frac{1}{2}(\mathbf{w_s} + \mathbf{w_t})\mathbf{\Delta X}\right] + \left[\frac{1}{2}(\mathbf{Y_s} + \mathbf{Y_t})\mathbf{\Delta p} - \frac{1}{2}(\mathbf{X_s} + \mathbf{X_t})\mathbf{\Delta w}\right] \quad (15)$$

The profit change decomposition defined in (15) does not need any arbitrary choice between the two periods.

We can now express the equality (13) in a Bennet way:

$$\left[\frac{1}{2}(\mathbf{Y_s} + \mathbf{Y_t})\mathbf{\Delta p} - \frac{1}{2}(\mathbf{X_s} + \mathbf{X_t})\mathbf{\Delta w}\right] + \Delta\Pi = \left[\frac{1}{2}(\mathbf{p_s} + \mathbf{p_t})\mathbf{\Delta Y} - \frac{1}{2}(\mathbf{w_s} + \mathbf{w_t})\mathbf{\Delta X}\right]$$

$$PA = TPS \quad (16)$$

Price Advantages = Total Productivity Surplus

In equation (16), PS is determined in level terms. However, we can relate it to the Solow technical change residual expressed in terms of growth rates.

Let us consider a multiple-output and multiple-input production function:

$$F(\mathbf{y}, \mathbf{x}, t) = 0 \quad (17)$$

where:

t is a time trend,

$\mathbf{x}$ is an input vector, $\mathbf{x} = (x_1, x_2, ..., x_i, ..., x_I)$,

$\mathbf{y}$ is an output vector, $\mathbf{y} = (y_1, y_2, ..., y_o, ..., y_O)$.

The Solow residual expresses the TFP change over time as weighted output variations not explained by weighted input changes:

$$\frac{\Delta TFP}{TFP} = \sum_{o=1}^{O} \alpha_o \frac{\Delta y^o}{y^o} - \sum_{i=1}^{I+1} \beta_i \frac{\Delta x^i}{x^i} \tag{18}$$

where $\boldsymbol{\alpha}$ represents the vector of *O* output elasticities in total revenue and $\boldsymbol{\beta}$ the vector of *I+1* input elasticities in total cost.

Assuming that output and input prices are equal to marginal costs and marginal productivity levels respectively, we can estimate TFP growth rate as a Törnqvist index

$$\frac{\Delta TFP}{TFP} = \sum_{o=1}^{O} \left(\frac{p^o y^o}{\sum_{o=1}^{O} p^o y^o} \right) \frac{\Delta y^o}{y^o} - \sum_{i=1}^{I+1} \left(\frac{w^i x^i}{\sum_{i=1}^{I+1} w^i x^i} \right) \frac{\Delta x^i}{x^i}$$

$$\text{with: } \alpha_0 = \frac{p^o y^o}{\sum_{o=1}^{O} p^o y^o} \text{ and } \beta_i = \frac{w^i x^i}{\sum_{i=1}^{I+1} w^i x^i}$$

Therefore, if we replace $\frac{p^o y^o}{\sum_{o=1}^{O} p^o y^o} \frac{\Delta y^o}{y^o}$ by $\frac{p^o \Delta y^o}{\sum_{o=1}^{O} p^o y^o}$ and $\frac{w^i x^i}{\sum_{i=1}^{I+1} w^i x^i} \frac{\Delta x^i}{x^i}$ by $\frac{w^i \Delta x^i}{\sum_{i=1}^{I+1} w^i x^i}$, we can estimate TFP growth rate as :

$$\frac{\Delta TFP}{TFP} = \frac{\sum_{o=1}^{O} p^o \Delta y^o - \sum_{i=1}^{I+1} w^i \Delta x^i}{\sum_{o=1}^{O} p^o y^o} \tag{19}$$

Consequently, the TFP growth rate is equal to the productivity surplus divided by the total output value. Moreover, we can establish the link between TFP growth rate and price advantage changes. Since $PS = PA$, TFP growth rate is equal to the aggregation of price advantages (disadvantages) divided by the total output value:

$$\frac{\Delta TFP}{TFP} = \frac{PS}{\sum_{o=1}^{O} p^o y^o} = \frac{PA}{\sum_{o=1}^{O} p^o y^o} = \frac{-\sum_{o=1}^{O} \Delta p^o y^o + \sum_{i=1}^{I+1} \Delta w^i x^i}{\sum_{o=1}^{O} p^o y^o}$$

If we consider that negative price advantages are net contributions from the concerned partners, we can cumulate them to ***PS*** (if it is positive). We then obtain the total amount of new resources that the beneficiaries of positive advantages will share. A firm can also register the productivity losses (PS<0). These losses should be balanced by additional deductions from some stakeholders which experience price disadvantages. Thus, the absolute PS value represents the amount that should be financed supplementary. It is now possible to construct the balanced surplus account as follows (Table 5):

Table 5 Surplus productivity account

Uses (Price advantages or productivity losses)	Resources (Price disadvantages and/or productivity gains)
$\lvert \mathrm{PS} \rvert$ if $\mathrm{PS} < 0$	PS if PS > 0
PA_o if $\mathrm{PA}_o > 0$	$\lvert \mathrm{PA}_o \rvert$ if $\mathrm{PA}_o < 0$
PA_i if $\mathrm{PA}_i > 0$	$\lvert \mathrm{PA}_i \rvert$ if $\mathrm{PA}_i < 0 \ldots$
Total of Uses	Total of Resources

PA_o = Price advantage for client of o product and PA_i = Price advantage of factor or supplier i

Figure 21 gives a schematic representation of surplus account.

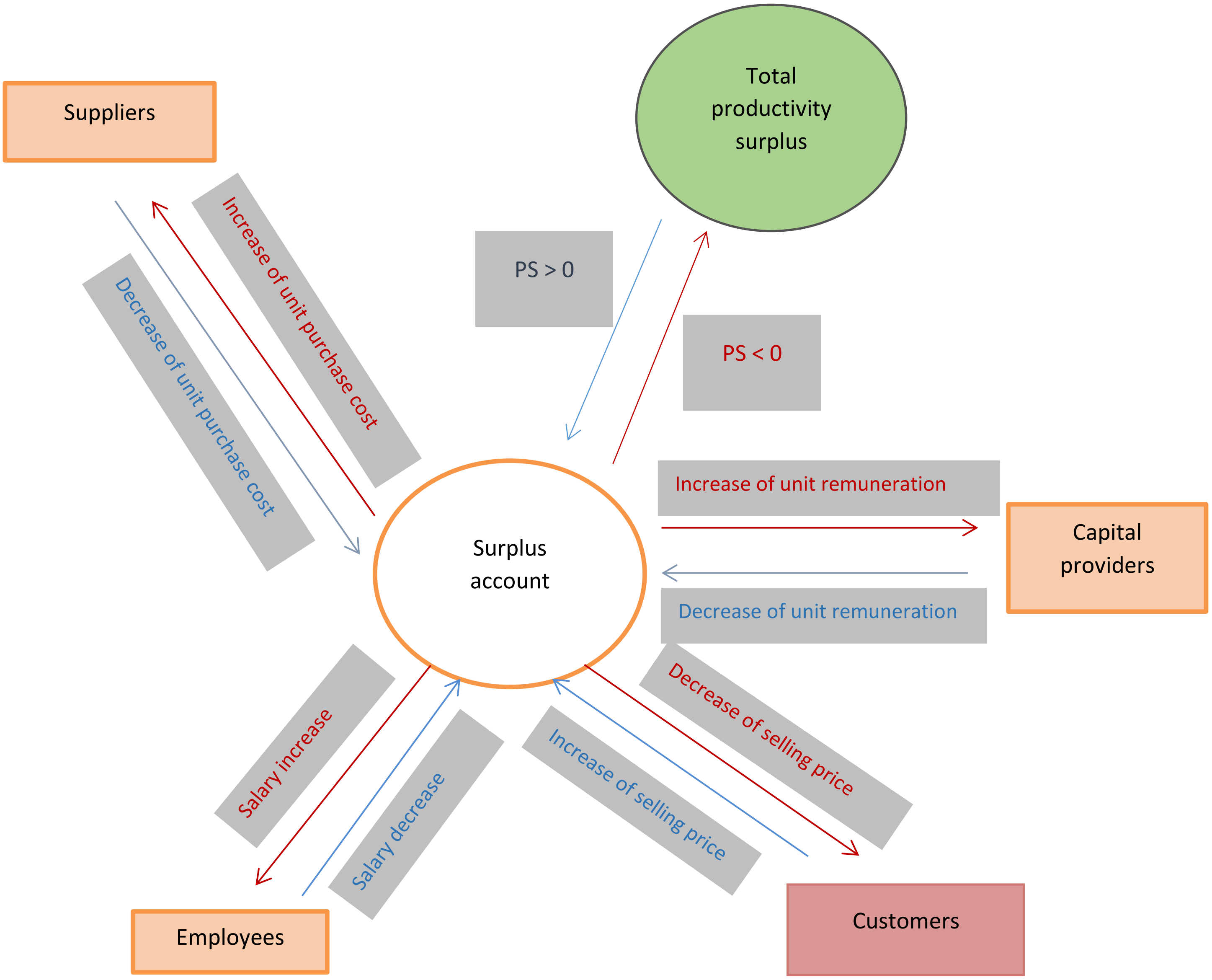

Figure 21 Schematic representation of surplus account

3.3 Empirical illustration

3.3.1 Data

This paragraph develops an illustrative example to clarify the productivity surplus approach. Suppose a firm which produces two manufactured goods. The simplified accounts for years 1 and 2 are listed in the Tables 6 and 7[2]. Value data are expressed in euros. We assume that due to internal

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[2] The numbers in these tables are completely fictitious and do not correspond to any real firm.

information system of the firm, it is possible to attribute the quantity and price levels to each income statement value. The quantities of products 1 and 2 as well as raw materials are measures in kilos; the energy quantity is measured in cubic meters; the amount of work is estimated in hours; and the use of productive capital is evaluated is a number of machines. The monetary erosion (general increase of prices) is established at 2,5% between the two periods.

Table 6 Income statement for year 1 at nominal prices

DEBIT	Quantity	Price	Value (€)	CREDIT	Quantity	Price	Value (€)
Purchase of raw materials	100 000	1 000	100 000 000	Sales of product 1	1 000 000	80	80 000 000
Energy cost	10 000 000	5	50 000 000	Sales of product 2	2 000 000	120	240 000 000
Staff cost	3 000 000	15	45 000 000				
Financial Cost	150 000 000	5%	7 500 000				
Amortization	20	3 000 000	60 000 000				
Corporate tax	57 500 000	33%	18 975 000				
Operating profit			*38 525 000*				
Dividends	500 000 000	5%	25 000 000				
Retained earnings	1	13 525 000	13 525 000				
Total			320 000 000	Total			320 000 000

Table 7 Income statement for year 2 at nominal prices

DEBIT	Quantity	Price	Value (€)	CREDIT	Quantity	Price	Value (€)
Purchase of raw materials	102 000	900	91 800 000	Sales of product 1	900 000	75	67 500 000
Energy cost	10 500 000	5,5	57 750 000	Sales of product 2	2 200 000	125	275 000 000
Staff cost	2 800 000	17	47 600 000				
Financial Cost	170 000 000	5,25%	8 925 000				
Amortization	19	3 421 052,63	65 000 000				
Corporate tax	71 425 000	35%	24 998 750				
Operating profit			*46 426 250*				
Dividends	550 000 000	5,50%	30 250 000				
Retained earnings	1	16 176 250	16 176 250				
Total			342 500 000	Total			342 500 000

3.3.2 The total productivity surplus computed with a Laspeyres approach

The first stage consists in estimating the Total Factor Productivity Surplus following a Laspeyres approach. Table 8 contains the quantity variations between the two periods and the prices of year 1. The difference between the credit total and the debit items equals to the level of PS^L. If we divide PS^L by the production level of year 1, we obtain the surplus rate which can be assimilated into the evolution of total factor productivity in percentage terms. In this example, a firm achieved a surplus of about 11 million €. This is due to the fact that its production increased by 16 million € while its costs, computed at constant prices, raised only at about 5 million € over the same period. This performance represents total factor productivity progress of 3,4%:

$$PS^L = \left[\mathbf{p_s \Delta Y - w_s \Delta X}\right] = 16\ 000\ 000 - 5\ 142\ 317 = 10\ 857\ 683$$

$$\frac{\Delta TFP}{TFP} = \frac{PS^L}{\mathbf{p_s Y_s}} = \frac{10\ 857\ 683}{320\ 000\ 000} = 3{,}4\%$$

Table 8 Computation of Total factor productivity surplus with Laspeyres approach

DEBIT	Δ Quantity	Price of year 1	Value (€)	CREDIT	Δ Quantity	Price of year 1	Value (€)
Purchase of raw materials	2 000	1 000	2 000 000	Sales of product 1	-100 000	80	-8 000 000
Energy cost	500 000	5	2 500 000	Sales of product 2	200 000	120	24 000 000
Staff cost	-200 000	15	-3 000 000				
Financial Cost	15 853 659	5%	792 683				
Amortization	-1	3 000 000	-3 000 000				
Corporate tax	12 182 927	33%	4 020 366				
Dividends	36 585 366	5%	1 829 268				
Retained earnings	0	13 525 000	0				
Productivity Surplus			**10 857 683**				
Total			16 000 000	Total			16 000 000

3.3.3 The surplus account

The second stage focuses on the distribution of new resources of the firm among different partners. Table 9 contains the price variations between the two periods and the quantities of year 2. The first new resource is generated by the positive productivity gains (PS>0); the other contributions come from the stakeholders which suffered from negative price advantages (customers of products 2 and suppliers of raw materials). Taking into account the inflation rate of 2,5%, these negative price advantages are computed through a Paasche approach as:

$$PA^{P}_{\text{customers 2}} = -\left(\frac{125}{1,025} - 120\right) * 2\,200\,000 = -4\,292\,683$$

$$PA^{P}_{\text{Sup. raw materials}} = \left(\frac{900}{1,025} - 1000\right) * 102\,000 = -12\,439\,024$$

The sum of these contributions $PS^{L} + \left|PA^{P}_{\text{customers 2}}\right| + \left|PA^{P}_{\text{Sup. raw materials}}\right| = 27\,589\,390$ is distributed among the partners which benefitted from positive price advantages (customers of products 1, energy suppliers, employees, lenders, government, shareholders and the firm for its amortization management and its profitability). Table 9 includes all items of this surplus account.

Table 9 Surplus account with Paasche approach

DEBIT	Quantity of year 2	Δ Price	Value (€)	CREDIT	Quantity of year 2	Δ Price	Value (€)
Customers of product 1	900 000	6,83	6 146 341	Productivity Surplus			10 857 683
Energy suppliers	10 500 000	0,37	3 841 463	Customers of product 2	2 200 000	1,95	4 292 683
Employees	2 800 000	1,59	4 439 024	Suppliers of raw materials	102 000	121,95	12 439 024
Lenders	165 853 659	0,25%	414 634				
Fixed capital	19	337 612,32	6 414 634				
Government	69 682 927	2,00%	1 393 659				
Shareholders	536 585 366	0,50%	2 682 927				
Profitability	1	2 256 707,32	2 256 707				
Total			27 589 390	Total			27 589 390

In this example, we estimated productivity surplus using the Laspeyres approach and surplus account using the Paasche approach. We can use a Bennet approach which allows us not to choose between the two periods. Table 10 provides the surplus account with a Bennet productivity surplus evaluated at 11 518 989 €. This performance represents total factor productivity progress of 3,6%:

$$PS^{B} = \left[\frac{\mathbf{p_t} + \mathbf{p_s}}{2} \mathbf{\Delta Y} - \frac{\mathbf{w_t} + \mathbf{w_s}}{2} \mathbf{\Delta X} \right] = 11\,518\,989$$

$$\frac{\Delta TFP}{TFP} = \frac{PS^{B}}{\mathbf{p_s Y_s}} = \frac{11\,518\,989}{320\,000\,000} = 3{,}6\%$$

Table 10 Surplus account with Bennet approach

DEBIT	Average of quantities for year 1 and 2	Price	Value (€)	CREDIT	Average of quantities for year 1 and 2	Price	Value (€)
Customers of product 1	950 000	6,83	6 487 805	Productivity Surplus			11 518 989
Energy suppliers	10 250 000	0,37	3 750 000	Customers of product 2	2 100 000	1,95	4 097 561
Employees	2 900 000	1,59	4 597 561	Suppliers of raw materials	101 000	121,95	12 317 073
Lenders	157 926 829,27	0,25%	394 817				
Fixed capital	19,50	337 612,32	6 583 440				
Government	63 591 463,41	2,00%	1 271 829				
Shareholders	518 292 682,93	0,50%	2 591 463				
Profitability	1	2 256 707,32	2 256 707				
Total			27 933 623	Total			27 933 623

If we divide the debit and credit value items to the total of the new resources (27 933 623 €), we obtain the distribution structure between different stakeholders (Figure 22). It gives the appreciation of relative levels of winners and losers when the firm is growing.

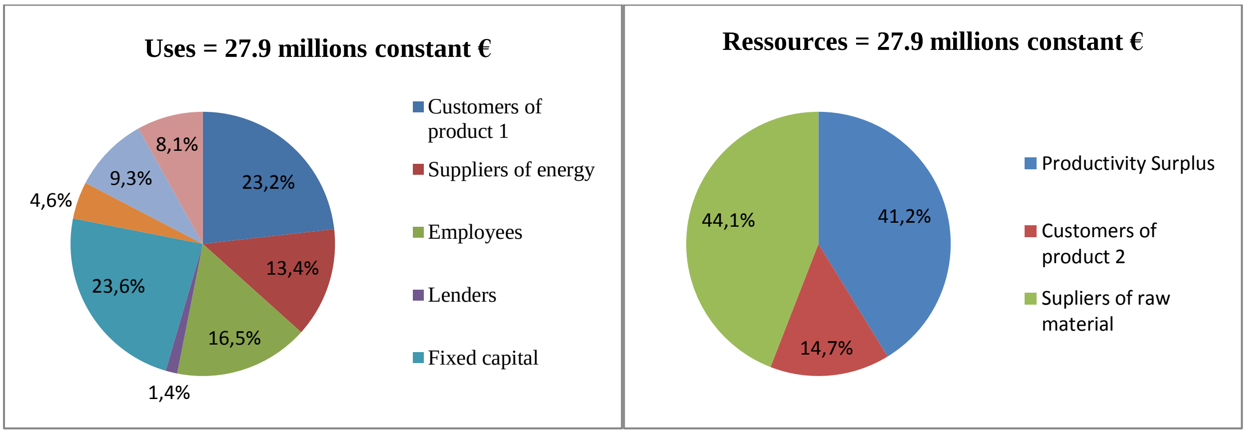

Figure 22 Distribution structure of surplus account with Bennet approach

3.4 A real world application: the U.S. automobile industry and the U.S. economy

This method can be used not only at the micro-economic level of a decision unit like a firm but also can be extended at a more aggregated scale as an industry or a whole economy. In this perspective, we use data from *Bureau of Economic Analysis (BEA)* to compare performances of the automobile industry and the US macro economy over the last 28 years. The implementation of this method aims at showing the impact of the recent financial crisis on productivity changes and how these changes were distributed among five main partners namely: customers, employees, suppliers and/or subcontractor, fixed capital providers (equipment, structure and intellectual property products) and profitability (shareholders, lenders, firms,…).

Profitability is assessed by the Net Operating Surplus obtained as the difference between Gross Operating Surplus (gross remuneration of capital and unpaid work) and depreciation cost of fixed capital (economic amortizations linked to equipment, software, buildings and intellectual property products). Computations explicitly took into account the monetary erosion deflating values and prices of all operating account items by the Gross Domestic Product price index. The following results are expressed in US dollar (base year 100 = 2009).

3.4.1 The productivity gains over 1987-2014

The productivity surplus of the US automobile industry has reached a cumulative of 111,1 billion dollars over the period 1987-2014 that represents approximately 4 billion per year. Given the average annual production level of 416,7 billion dollars, these productivity gains can be considered as sufficient. The average annual increase rate of the productivity gains is 1,05%. This relative performance of the automobile sector is higher compared to the whole US economy (0,78%) but less than the average trend for the manufacturing sector (1,18%). The highest trend among the manufacturing industries is set for “computer and electronic products” (7,49%) sector followed by “miscellaneous manufacturing” (1,27%).

Apart from the average trend of these last 28 years, we can notice very distinctive cyclical changes. Figure 23 clearly shows the 4 periods of productive crisis of the US automobile builders: 1988-1991 with the average annual decrease rate of -1,9%, 1994-1996 (-1,7%), 2000-2001 (-1,5%) and 2007-2009 (-5,8%). Between 2009 and 2011 the growth rate increases sharply at 10,3% and allows them to overcome the lower before crisis productivity level.

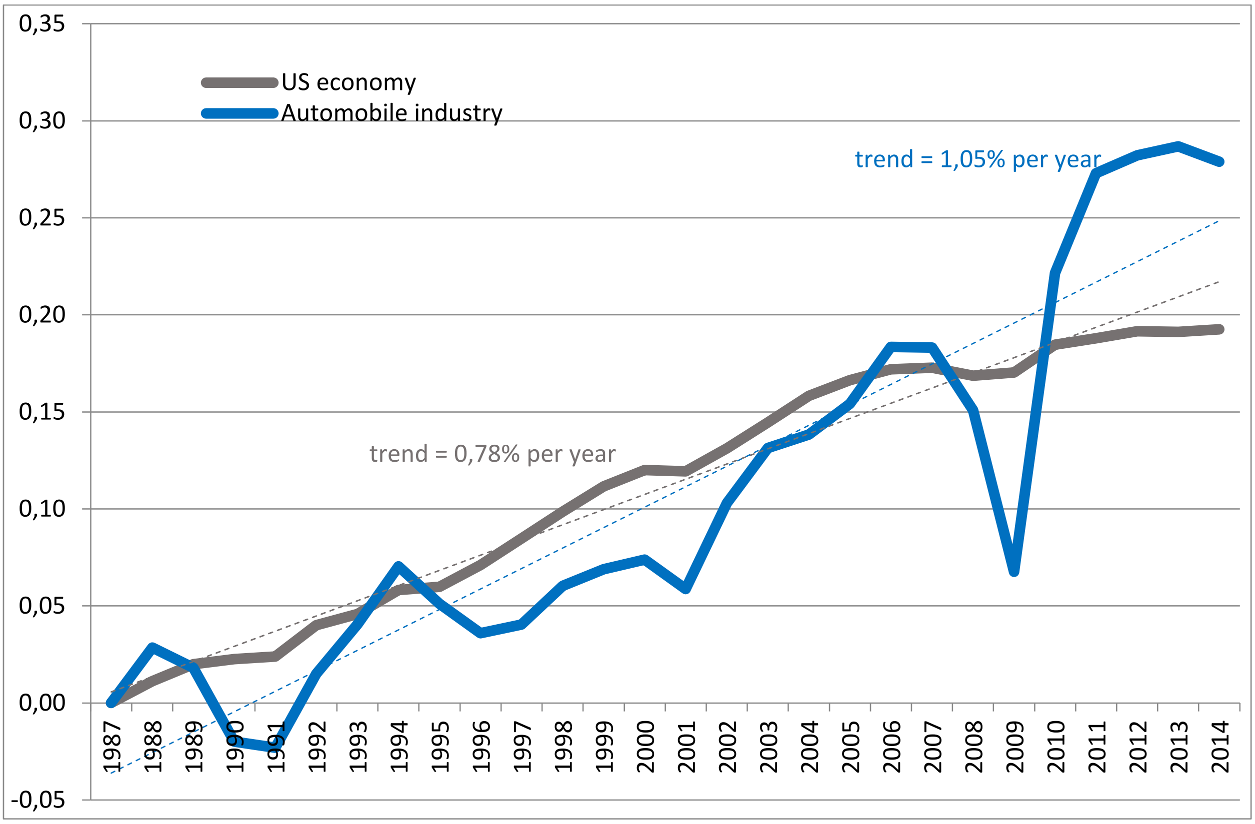

Figure 23 Total factor productivity evolution. Comparison between US automobile industry and the whole US economy (in logarithm, base year 100 = 2009)

3.4.2 Losers and winners in the repartition of the productivity gains

The total resources of the cumulated surplus account over the whole period have reached the amount of 195 billion dollars. The productivity gains of 111,1 billion dollars represent 57% of the total of the surplus account and thus make the main contribution to the resources part. Suppliers bring the second major resource contribution of 79,7 billion dollars (41%). The capital providers contribute only 4,3 billion dollars (2%) to the resources part.

Regarding the stakeholders which benefit from the price advantages and/or remunerations, customers are clearly the big winners of this distribution. They use 179 billion dollars (91,9%) of the total resources. Automobile firms gain cumulated profit of 10,5 billion dollars which constitutes 5,4% of the uses part of the surplus account. Employees use the smallest part of the resource table in amount of 5,3 billion dollars (2,7%).

Figures 24 and 25 compare the distribution structure of the automobile industry and of all sectors of the US economy. On this point, the automobile industry is in a very contrasted position compared to the US economy. Namely, the two biggest winners from the growth distribution for the US economy are employees and automobile firms. Moreover, clients suffer from the unfavorable relative price changes whereas suppliers, on the contrary, benefit from positive price advantages.

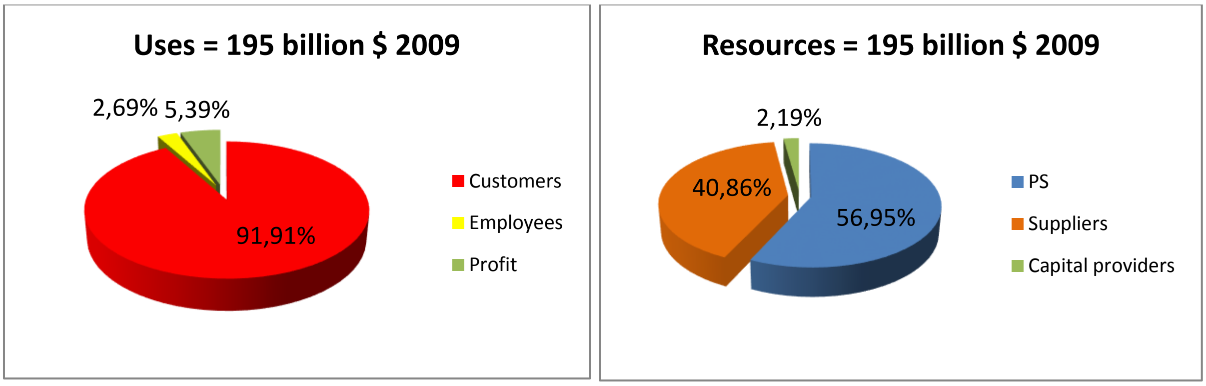

Figure 24 Distribution structure of the surplus account for the US automobile industry

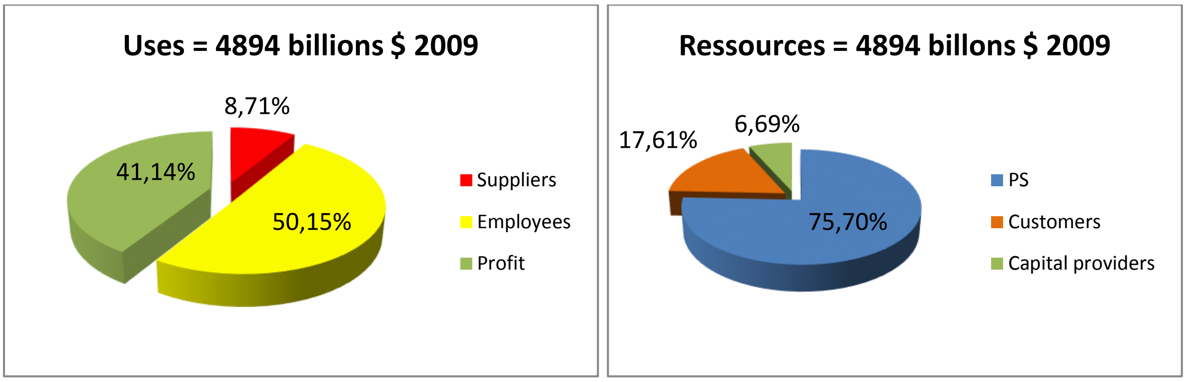

Figure 25 Distribution structure of the surplus account for the whole US industry

Figure 26 traces the chronology of the distribution indicators and productivity gains. The results show eloquently that automobile buyers benefitted from substantial price advantages to the detriment of builders and suppliers.

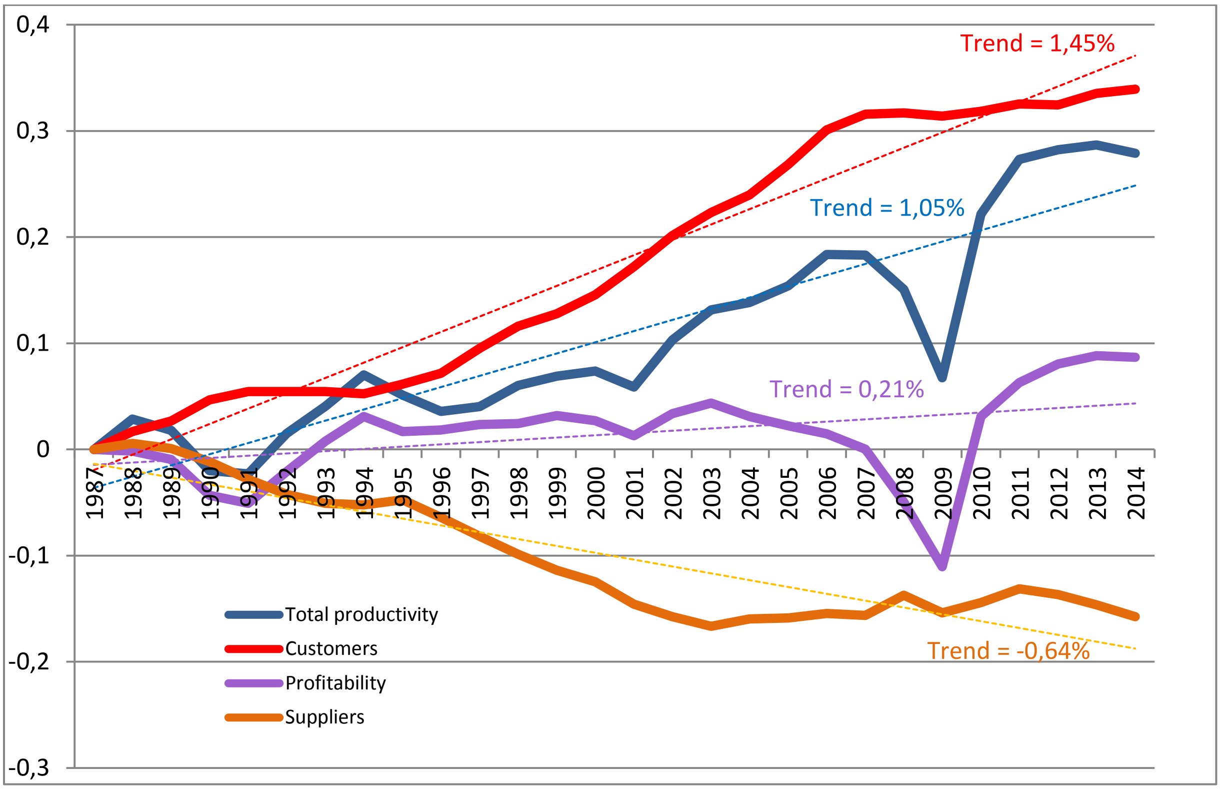

Figure 26 Comparison of the evolutions of the Total Factor Productivity and the Price Advantages for customers, suppliers, and firms' profitability of the US automobile industry (in logarithm, base year 100 = 1987).

3.4.3 Conclusion

It is evident that this real-world application developed at the sector level can be implemented to any firm considered as a whole or by separating its different institutions. The level of detail of income statement condition the number of partners explicitly. For example, it can be necessary to distinguish among the partner "customers", groups of buyers by different products. To estimate the real advantages that employees receive, it would be justified to separate increases in the labor cost into salaries and social contributions and reallocate the latter to the partner "Government". In the same way, it is sometimes useful to dissociate different capital providers: lenders (banks, bondholders), shareholders (dividends) and the firm itself (undistributed net income).

All these refinements need to be defined depending on the managers. Certainly, the operational implementation of this analysis tool is reliant on the information system of the firm.

4 Quantity and price effects in profit differential between two firms

4.1 Decomposition of the profit differential between two firms

Consider a firm A which produces an output vector Y_A from an input vector X_A. Suppose that R_A is an output price vector of a firm A and W_A is an input price vector of a firm A. The profit of a firm A can be defined as:

$$\Pi_{Q_A}^{P_A} = R_A Y_A - W_A X_A$$

where $P_A = (R_A, W_A)$ is a price vector of a firm A and $Q_A = (Y_A, -X_A)$ is a quantity vector of a firm A.

Consider now a firm B . Similarly as with a firm A, the profit of a firm B can be defined as:

$$\Pi_{Q_B}^{P_B} = R_B Y_B - W_B X_B$$

where $P_B = (R_B, W_B)$ is a price vector of a firm B and $Q_B = (Y_B, -X_B)$ is a quantity vector of a firm B.

The difference between the two profits is equal to:

$$\left(\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B}\right) = \left[\left(R_A Y_A - W_A X_A\right) - \left(R_B Y_B - W_B X_B\right)\right] \tag{20}$$

Bennet indicators and profit gap decomposition into quantity effect and price effect

We can decompose the profit gap between the two firms according to Bennet decomposition (Bennet, 1920):

$$\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} = \frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_A}\right) + \left(\Pi_{Q_A}^{P_B} - \Pi_{Q_B}^{P_B}\right)\right] + \frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A} - \Pi_{Q_A}^{P_B}\right) + \left(\Pi_{Q_B}^{P_A} - \Pi_{Q_B}^{P_B}\right)\right] \tag{21}$$

where

$\frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_A}\right) + \left(\Pi_{Q_A}^{P_B} - \Pi_{Q_B}^{P_B}\right)\right]$ (22) is a profit gap between firms A and B due to the differences in their quantities or **quantity effect**

$\frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A} - \Pi_{Q_A}^{P_B}\right) + \left(\Pi_{Q_B}^{P_A} - \Pi_{Q_B}^{P_B}\right)\right]$ (23) is a profit gap between firms A and B due to the differences in their price systems or **price effect**.

The Bennet decomposition allows us not to favor both the quantities of any of the firms and price systems of any of the firms.

In (22) we first measure the profit difference between the two firms with the price system of firm A. Then, we measure the profit difference between the two firms with the price system of firm B. Each component is a quantity effect since the price systems are held constant. We can average the two components in order not to have a preference of any of the price systems.

Similarly in (23), we first measure the profit difference between the two firms with the quantities of firm A. Then, we measure the profit difference between the two firms with the quantities of firm B. Each component is a price effect since the quantities remain constant. As well as for the quantity effect, we average the two components not to opt between the two quantity vectors.

4.2 Technical, Allocative and Size Efficiency effects

Boussemart et al. (2013) proposed a decomposition of the quantity effect (22) into four components: technical, allocative, technological dominance and size differential. We will follow their steps but we introduce a new decomposition entirely based on Bennet components.

Before going into our new decomposition we note a difference in the technological assumptions made in Boussemart et al. (2013) and in our framework. While they assume a different and specific technology for firms A and B, we consider the same technology for all firms. As such we suppose that all firms use the same technology and belong to the same production set. We therefore limit our study by considering only short term run. We suppose that firms cannot adapt a more efficient technology and stay within their current production frontier. The technological effect thus disappears in our framework and we therefore end up with a decomposition of the quantity into three components: technical, allocative and size.

The main difference between Boussemart et al. (2013) decomposition and ours is the way of computing each component. Boussemart et al. (2013) compute the technical and allocative efficiency effects only with the price system of the evaluated firm. We measure these effects as Bennet indicators. Thus, we use both price systems of firms A and B and then average the two results. Another difference is that Boussemart et al. (2013) while using the Bennet indicator to compute the size effect, consider the profit maximizing benchmarks of each firm situated on the technology of the comparable firm. Since we assume the same technology for all firms, we estimate the size effect with the profit maximizing benchmarks situated on the same technology for firms A and B.

In what follows, we first give the notations we use in order to describe the quantity effects. Then, we give a description of each quantity effect in accordance to Boussemart et al. (2013) and to the assumptions we made above.

4.2.1 Notations

1) Efficient frontier T^{VRS}. We use a directional distance function (Chambers et al., 1996, 1998) to construct an efficient frontier. We decide to select an output direction and to use a vector of mean outputs $\overline{\boldsymbol{Y}}$ as a directional vector: g=(0, $\overline{\boldsymbol{Y}}$). Thus the directional distance model can be written as follows:

$$
\begin{aligned}
\max \quad & \delta_0 \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \delta_o \overline{y}_r,\ r = 1,\dots,s, \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io},\ i = 1,\dots,m, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0,\ \forall j.
\end{aligned}
$$

2) First benchmark Q_1 is a technically efficient point obtained by projection of the observed DMU on the efficient frontier using the above model. We use the efficient referents found by resolving the above model to calculate technically efficient outputs:

$$y_{ro}^{*} = \sum_{j=1}^{n} \lambda_j^{*} y_{rj},\ r = 1,\dots,s.$$

We will denote by Y_1 technically efficient vector of outputs for each firm.

Thus, our first benchmark is a vector $Q_1 = (Y_1, -X)$.

3) Second benchmark Q_2 is a maximum profit efficient point. A firm, in order to maximize its profit, eliminates its allocative inefficiency by optimizing the distribution of the resources using the observed price system. To calculate this point, we use the following model:

$$\begin{aligned}
\max \quad & \Pi = \sum_{r=1}^{s} p_{ro} y_{ro} - \sum_{i=1}^{m} w_{io} x_{io} \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro},\ r = 1,\ldots,s, \\
& \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io},\ i = 1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0,\ \forall j.
\end{aligned}$$

We will denote by X_2 maximum profit efficient vector of inputs and by Y_2 maximum profit efficient vector of outputs. Thus, our second benchmark is a vector $Q_2 = (Y_2, -X_2)$.

4.2.2 Decomposition of the quantity effect

In a short run, a firm, in order to increase its profit, should first reach the efficient frontier and thus eliminate its technical inefficiency. If we consider, for example, firm A, the influence from eliminating its technical inefficiency on its profit can be expressed by $\Pi_{Q_A}^{P_A} - \Pi_{Q_{A_1}}^{P_A}$. For firm B, the influence from eliminating its technical inefficiency on its profit rate can be expressed in an analogous way: $\Pi_{Q_B}^{P_B} - \Pi_{Q_{B_1}}^{P_B}$. The difference between the profits of firms A and B caused by the difference between their technical inefficiencies can be written as $\left(\Pi_{Q_A}^{P_A} - \Pi_{Q_{A_1}}^{P_A}\right) - \left(\Pi_{Q_B}^{P_B} - \Pi_{Q_{B_1}}^{P_B}\right)$. This is the approach presented in Boussemart et al. (2013). However, the technical efficiency differential for firm A is only computed with the price system of A and the effect for firm B is also computed with its own price system. Each element is price specific. We propose here to eliminate the reference to a specific price system by using both price systems and averaging the result, keeping the philosophy of the Bennet approach. Thus, the technical efficiency differential effect can be expressed as: $\frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A} - \Pi_{Q_{A_1}}^{P_A}\right) + \left(\Pi_{Q_A}^{P_B} - \Pi_{Q_{A_1}}^{P_B}\right)\right] - \frac{1}{2}\left[\left(\Pi_{Q_B}^{P_A} - \Pi_{Q_{B_1}}^{P_A}\right) + \left(\Pi_{Q_B}^{P_B} - \Pi_{Q_{B_1}}^{P_B}\right)\right]$.

After eliminating technical inefficiency, a firm is interested in maximizing its profit using the information about the observed price system. That means that a firm can reallocate its inputs and its outputs and thus maximize its profit. The effect from reallocating the quantities given the existing price system on the profit of firm A can be expressed as $\Pi_{Q_{A_1}}^{P_A} - \Pi_{Q_{A_2}}^{P_A}$. The same effect for firm B is as follows: $\Pi^{P_B}_{Q_{B_1}} - \Pi^{P_B}_{Q_{B_2}}$. The difference between the profits of the two firms A and B caused by the difference in their allocative inefficiencies can be represented as $\left(\Pi^{P_A}_{Q_{A_1}} - \Pi^{P_A}_{Q_{A_2}}\right) - \left(\Pi^{P_B}_{Q_{B_1}} - \Pi^{P_B}_{Q_{B_2}}\right)$. This is the allocative component of Boussemart et al. (2013). As well as for the technical efficiency differential effect, we will apply Bennet indicator decomposition again. We will first calculate this term with the prices of firm A and then with the prices of firm B. We average the two components to obtain the allocative efficiency effect:

$$\frac{1}{2}\left[\left(\Pi^{P_A}_{Q_{A_1}} - \Pi^{P_A}_{Q_{A_2}}\right) + \left(\Pi^{P_B}_{Q_{A_1}} - \Pi^{P_B}_{Q_{A_2}}\right)\right] - \frac{1}{2}\left[\left(\Pi^{P_A}_{Q_{B_1}} - \Pi^{P_A}_{Q_{B_2}}\right) + \left(\Pi^{P_B}_{Q_{B_1}} - \Pi^{P_B}_{Q_{B_2}}\right)\right].$$

As we consider one technology and the short run period, the only quantity effect that can influence the difference between the profits of the two firms when they have both maximized their profits is caused by the different positions of the two firms on the production frontier. For

firms A and B this difference can be expressed as $\Pi^{P_A}_{Q_{A_2}} - \Pi^{P_B}_{Q_{B_2}}$. Once again, we apply the Bennet approach by using both price systems and then averaging: $\frac{1}{2}\left[\left(\Pi^{P_A}_{Q_{A_2}} - \Pi^{P_A}_{Q_{B_2}}\right) + \left(\Pi^{P_B}_{Q_{A_2}} - \Pi^{P_B}_{Q_{B_2}}\right)\right]$. We will refer this effect to as size differential effect.

If we sum up all the quantity effects determined above, we will obtain the whole quantity effect:

$$\begin{aligned}
&\frac{1}{2}\left[\left(\Pi^{P_A}_{Q_A} - \Pi^{P_A}_{Q_{A_1}}\right) + \left(\Pi^{P_B}_{Q_A} - \Pi^{P_B}_{Q_{A_1}}\right)\right] - \frac{1}{2}\left[\left(\Pi^{P_A}_{Q_B} - \Pi^{P_A}_{Q_{B_1}}\right) + \left(\Pi^{P_B}_{Q_B} - \Pi^{P_B}_{Q_{B_1}}\right)\right] + \\
&\frac{1}{2}\left[\left(\Pi^{P_A}_{Q_{A_1}} - \Pi^{P_A}_{Q_{A_2}}\right) + \left(\Pi^{P_B}_{Q_{A_1}} - \Pi^{P_B}_{Q_{A_2}}\right)\right] - \frac{1}{2}\left[\left(\Pi^{P_A}_{Q_{B_1}} - \Pi^{P_A}_{Q_{B_2}}\right) + \left(\Pi^{P_B}_{Q_{B_1}} - \Pi^{P_B}_{Q_{B_2}}\right)\right] + \\
&\frac{1}{2}\left[\left(\Pi^{P_A}_{Q_{A_2}} - \Pi^{P_A}_{Q_{B_2}}\right) + \left(\Pi^{P_B}_{Q_{A_2}} - \Pi^{P_B}_{Q_{B_2}}\right)\right] = \\
&\frac{1}{2}\left[\left(\Pi^{P_A}_{Q_A} - \Pi^{P_A}_{Q_B}\right) + \left(\Pi^{P_B}_{Q_A} - \Pi^{P_B}_{Q_B}\right)\right]
\end{aligned} \tag{24}$$

4.3 Distribution of efficiency gains among stakeholders

The CERC methodology (CERC's Documents, 1980), discussed in the third section of the first chapter, considers the repartition of productivity surplus among different stakeholders supposing two periods of one firm. In this section, we will apply the CERC methodology considering two firms and one time period.

We can decompose the price effect (23) into input and output stakeholders as follows:

$$\begin{aligned}&\frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A}-\Pi_{Q_A}^{P_B}\right)+\left(\Pi_{Q_B}^{P_A}-\Pi_{Q_B}^{P_B}\right)\right]=\\&\frac{1}{2}\left[\sum_o\left(\left(R_o^AY_o^A-R_o^BY_o^A\right)+\left(R_o^AY_o^B-R_o^BY_o^B\right)\right)\right]-\frac{1}{2}\left[\sum_i\left(\left(W_i^AX_i^A-W_i^BX_i^A\right)+\left(W_i^AX_i^B-W_i^BX_i^B\right)\right)\right]\end{aligned}\tag{25}$$

According to the CERC methodology, the price effect in (25) taken with negative sign plus profit gap between the two firms A and B corresponds to the sum of the price advantages (PA):

$$\begin{aligned}&\Sigma PA=-\frac{1}{2}\left[\sum_o\left(\left(R_o^AY_o^A-R_o^BY_o^A\right)+\left(R_o^AY_o^B-R_o^BY_o^B\right)\right)\right]+\\&\frac{1}{2}\left[\sum_i\left(\left(W_i^AX_i^A-W_i^BX_i^A\right)+\left(W_i^AX_i^B-W_i^BX_i^B\right)\right)\right]+\\&\left(\Pi_{Q_A}^{P_A}-\Pi_{Q_B}^{P_B}\right)\end{aligned}\tag{26}$$

The first component in (26) corresponds to the sum of the advantages/disadvantages of customers of firms A and B. The second component corresponds to the sum of the advantages/disadvantages of suppliers, employees and capital providers of firms A and B. The third component is the profit gap between firms A and B and can be interpreted as price advantage for the management board of firm A if this gap is positive and as price advantage for the management board of firm B otherwise.

The quantity effect in (22) corresponds to productivity surplus (PS):

$$PS=\frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A}-\Pi_{Q_B}^{P_A}\right)+\left(\Pi_{Q_A}^{P_B}-\Pi_{Q_B}^{P_B}\right)\right]\tag{27}$$

Thus, we can write the fundamental equality of CERC's methodology as follows:

$$\begin{aligned}&\frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A}-\Pi_{Q_B}^{P_A}\right)+\left(\Pi_{Q_A}^{P_B}-\Pi_{Q_B}^{P_B}\right)\right]=\\&-\frac{1}{2}\left[\sum_o\left(\left(R_o^AY_o^A-R_o^BY_o^A\right)+\left(R_o^AY_o^B-R_o^BY_o^B\right)\right)\right]+\\&\frac{1}{2}\left[\sum_i\left(\left(W_i^AX_i^A-W_i^BX_i^A\right)+\left(W_i^AX_i^B-W_i^BX_i^B\right)\right)\right]+\\&\left(\Pi_{Q_A}^{P_A}-\Pi_{Q_B}^{P_B}\right)\end{aligned}\tag{28}$$

The left-hand side of (28) estimates the influence of the differences in quantities between firms A and B weighted by the average prices on the difference in their profits. It corresponds to the difference in the corresponding productivities and thus can be interpreted as productivity surplus between firms A and B.

The first two components in the right-hand side of (28) estimate the influence of the differences in prices between firms A and B weighted by the averaged quantities on the difference in their profits. It can be seen as a sum of price advantages (or disadvantages) for every stakeholder of firms A and B. Depending on whether the difference in the outputs prices between firms A and B is positive or negative for each stakeholder, it will be a price disadvantage or a price advantage for the corresponding customer of firm A. Similarly, depending on whether the difference in the inputs prices between firms A and B is positive or negative for each stakeholder, it will be a price advantage or a price disadvantage for the corresponding supplier of firm A.

The third component on the right-hand side of (28) is the difference between the profits of firms A and B and can be considered as the difference in the corresponding prices for management of firms A and B.

If we decompose the surplus productivity according to (24), we obtain:

$$\begin{aligned}
&\frac{1}{2}\left[\left(\Pi_{Q_A}^{P_A}-\Pi_{Q_{A_1}}^{P_A}\right)+\left(\Pi_{Q_A}^{P_B}-\Pi_{Q_{A_1}}^{P_B}\right)\right]-\frac{1}{2}\left[\left(\Pi_{Q_B}^{P_A}-\Pi_{Q_{B_1}}^{P_A}\right)+\left(\Pi_{Q_B}^{P_B}-\Pi_{Q_{B_1}}^{P_B}\right)\right]+\\
&\frac{1}{2}\left[\left(\Pi_{Q_{A_1}}^{P_A}-\Pi_{Q_{A_2}}^{P_A}\right)+\left(\Pi_{Q_{A_1}}^{P_B}-\Pi_{Q_{A_2}}^{P_B}\right)\right]-\frac{1}{2}\left[\left(\Pi_{Q_{B_1}}^{P_A}-\Pi_{Q_{B_2}}^{P_A}\right)+\left(\Pi_{Q_{B_1}}^{P_B}-\Pi_{Q_{B_2}}^{P_B}\right)\right]+\\
&\frac{1}{2}\left[\left(\Pi_{Q_{A_2}}^{P_A}-\Pi_{Q_{B_2}}^{P_A}\right)+\left(\Pi_{Q_{A_2}}^{P_B}-\Pi_{Q_{B_2}}^{P_B}\right)\right]= \qquad (29)\\
&-\frac{1}{2}\left[\sum_o\left(\left(R_o^A Y_o^A-R_o^B Y_o^A\right)+\left(R_o^A Y_o^B-R_o^B Y_o^B\right)\right)\right]+\\
&\frac{1}{2}\left[\sum_i\left(\left(W_i^A X_i^A-W_i^B X_i^A\right)+\left(W_i^A X_i^B-W_i^B X_i^B\right)\right)\right]+\\
&\left(\Pi_{Q_A}^{P_A}-\Pi_{Q_B}^{P_B}\right)
\end{aligned}$$

The equality (29) allows us to identify what quantity effect contributes to the difference in the productivity between firms A and B and how this difference is distributed among different stakeholders of firms A and B.

4.4 Explaining the profit rate differential between banks

As banks can differ largely by their size, we prefer to analyze profit rate gaps rather than profit gaps in order to have comparable measures regardless of the size of the banks.

Thus, similar to the decomposition of the profit gap expressed in values in (21), we can decompose profit rate gap as follows:

$$\frac{\Pi_{Q_A}^{P_A}}{\mathbf{R_A Y_A}}-\frac{\Pi_{Q_B}^{P_B}}{\mathbf{R_B Y_B}}=\frac{1}{2}\left[\left(\frac{\Pi_{Q_A}^{P_A}}{\mathbf{R_A Y_A}}-\frac{\Pi_{Q_B}^{P_A}}{\mathbf{R_A Y_B}}\right)+\left(\frac{\Pi_{Q_A}^{P_B}}{\mathbf{R_B Y_A}}-\frac{\Pi_{Q_B}^{P_B}}{\mathbf{R_B Y_B}}\right)\right]+\frac{1}{2}\left[\left(\frac{\Pi_{Q_A}^{P_A}}{\mathbf{R_A Y_A}}-\frac{\Pi_{Q_A}^{P_B}}{\mathbf{R_B Y_A}}\right)+\left(\frac{\Pi_{Q_B}^{P_A}}{\mathbf{R_A Y_B}}-\frac{\Pi_{Q_B}^{P_B}}{\mathbf{R_B Y_B}}\right)\right] \quad (30)$$

where

$$\frac{1}{2}\left[\left(\frac{\Pi_{Q_A}^{P_A}}{\mathbf{R_A Y_A}}-\frac{\Pi_{Q_B}^{P_A}}{\mathbf{R_A Y_B}}\right)+\left(\frac{\Pi_{Q_A}^{P_B}}{\mathbf{R_B Y_A}}-\frac{\Pi_{Q_B}^{P_B}}{\mathbf{R_B Y_B}}\right)\right]$$ (31) is a profit rate differential between firms A and B due to the differences in their quantities or quantity effect,

$$\frac{1}{2}\left[\left(\frac{\Pi_{Q_A}^{P_A}}{\mathbf{R_A Y_A}}-\frac{\Pi_{Q_A}^{P_B}}{\mathbf{R_B Y_A}}\right)+\left(\frac{\Pi_{Q_B}^{P_A}}{\mathbf{R_A Y_B}}-\frac{\Pi_{Q_B}^{P_B}}{\mathbf{R_B Y_B}}\right)\right]$$ (32) is a profit rate differential between firms A and B due to the differences in their price systems or price effect.

If we decompose the quantity effect (31) into technical efficiency differential, allocative efficiency differential and size differential effects and if we decompose the price effect (32) among the stakeholders, we can formulate the CERC's fundamental equality with decomposed productivity surplus in rates as follows:

$$\begin{aligned}
&\frac{1}{2}\left[\left(\frac{\Pi_{Q_A}^{P_A}}{\mathbf{R_A Y_A}}-\frac{\Pi_{Q_{A_1}}^{P_A}}{\mathbf{R_A Y_{A_1}}}\right)+\left(\frac{\Pi_{Q_A}^{P_B}}{\mathbf{R_B Y_A}}-\frac{\Pi_{Q_{A_1}}^{P_B}}{\mathbf{R_B Y_{A_1}}}\right)\right]-\frac{1}{2}\left[\left(\frac{\Pi_{Q_B}^{P_A}}{\mathbf{R_A Y_B}}-\frac{\Pi_{Q_{B_1}}^{P_A}}{\mathbf{R_A Y_{B_1}}}\right)+\left(\frac{\Pi_{Q_B}^{P_B}}{\mathbf{R_B Y_B}}-\frac{\Pi_{Q_{B_1}}^{P_B}}{\mathbf{R_B Y_{B_1}}}\right)\right]+\\
&\frac{1}{2}\left[\left(\frac{\Pi_{Q_{A_1}}^{P_A}}{\mathbf{R_A Y_{A_1}}}-\frac{\Pi_{Q_{A_2}}^{P_A}}{\mathbf{R_A Y_{A_2}}}\right)+\left(\frac{\Pi_{Q_{A_1}}^{P_B}}{\mathbf{R_B Y_{A_1}}}-\frac{\Pi_{Q_{A_2}}^{P_B}}{\mathbf{R_B Y_{A_2}}}\right)\right]-\frac{1}{2}\left[\left(\frac{\Pi_{Q_{B_1}}^{P_A}}{\mathbf{R_A Y_{B_1}}}-\frac{\Pi_{Q_{B_2}}^{P_A}}{\mathbf{R_A Y_{B_2}}}\right)+\left(\frac{\Pi_{Q_{B_1}}^{P_B}}{\mathbf{R_B Y_{B_1}}}-\frac{\Pi_{Q_{B_2}}^{P_B}}{\mathbf{R_B Y_{B_2}}}\right)\right]+\\
&\frac{1}{2}\left[\left(\frac{\Pi_{Q_{A_2}}^{P_A}}{\mathbf{R_A Y_{A_2}}}-\frac{\Pi_{Q_{B_2}}^{P_A}}{\mathbf{R_A Y_{B_2}}}\right)+\left(\frac{\Pi_{Q_{A_2}}^{P_B}}{\mathbf{R_B Y_{A_2}}}-\frac{\Pi_{Q_{B_2}}^{P_B}}{\mathbf{R_B Y_{B_2}}}\right)\right]= \qquad (33)\\
&-\frac{1}{2}\left[\sum_o\left(\left(\frac{R_o^A Y_o^A}{\mathbf{R_A Y_A}}-\frac{R_o^B Y_o^A}{\mathbf{R_B Y_A}}\right)+\left(\frac{R_o^A Y_o^B}{\mathbf{R_A Y_B}}-\frac{R_o^B Y_o^B}{\mathbf{R_B Y_B}}\right)\right)\right]+\\
&\frac{1}{2}\left[\sum_i\left(\left(\frac{W_i^A X_i^A}{\mathbf{R_A Y_A}}-\frac{W_i^B X_i^A}{\mathbf{R_B Y_A}}\right)+\left(\frac{W_i^A X_i^B}{\mathbf{R_A Y_B}}-\frac{W_i^B X_i^B}{\mathbf{R_B Y_B}}\right)\right)\right]+\frac{\Pi_{Q_A}^{P_A}}{\mathbf{R_A Y_A}}-\frac{\Pi_{Q_B}^{P_B}}{\mathbf{R_B Y_B}}
\end{aligned}$$

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Chapter 2

A case study on U.S. banks industry

1 Introduction

The world financial crisis of 2007-2008 raised a question about the way banks create their value and how this value is distributed among stakeholders. What if they preferred the revenues from financial markets to their traditional less risky activities? What if they suffered from their too greedy shareholders? In fact, every bank is exposed differently to the crisis depending of its activities. A full-service bank, retail bank and investment bank at the same time, can, for example, compensate its losses from investment activities by other activities. Profitability analysis resulting from productive performance of banks allows us to understand the competitive advantages of every bank and their sensitivity to the price environments they face. This analysis also favors economic measures from actual activity of banks instead of evaluation by markets.

In this case study, we implement the concepts and methods developed in the fourth section of the first chapter on a sample of small to medium sized American banks from 2001 to 2012. To measure the profit gaps between banks and their distribution among stakeholders, we considered a bank as an organizational model that articulates factorial resources to generate products. The underlying technology includes 3 output variables (borrowers, financial market participants, commission and fee payers) and 4 input variables (creditors, employees, suppliers and government). We chose the approach based on profit rate differential. Indeed, while a bigger bank will obviously make more profit than a smaller one, the latter could be more efficient in terms of profitability. We thus computed for each bank its profit rate gaps towards all banks, 3 quantity effects (technical inefficiency, allocative inefficiency and size) and price effect decomposed into banks' stakeholders. We then aggregated the obtained results to be able to analyze banks at individual and global levels over the period but also before and after the financial crisis.

Our objective for this empirical application is twofold. At the methodological level, our study justifies the benefit to combine the surplus accounting approach and productive inefficiencies measures approach. This methodology can capture the way a bank, through its activities, generates and distributes its value. At the empirical level, we show what quantity and price effects globally ensure positive profitability for US banks over 2001-2012, how the influence of these effects changed after the financial crisis, what the difference is between commercial and savings banks in terms of their productive efficiency and price advantages and finally what different groups of banks according to their surplus account structure could be identified in the before and after crisis periods.

This case study is organized as follows. Section 2 provides a brief review of the banking sector in the US. Section 3.1 explains data collection and treatment and computation of variables for analysis. Section 3.2 gives our results and analysis. We perform our analysis at 3 levels. First, section 3.2.1 presents a comparison analysis of two banks with different surplus structures. In section 3.2.2 we continue with a global analysis of performant banks in terms of their mean profit rate gap over the entire period. In section 3.2.3 we propose an analysis of performant banks before and after the 2007-2008 crisis which we split into two parts: in section 3.2.3.1 we give details of a cluster analysis and then we use the obtained clusters in section 3.2.3.2 for a canonical discriminant analysis. Finally section 4 presents our main conclusions.

2 A brief review of the banking sector in the U.S.

There are 3 main types of banks in the US: commercial banks, savings institutions, and credit unions. Initially, commercial banks were aimed to provide services for businesses. Savings institutions were established to provide low-income individuals with access to banking services like saving accounts and residential loans. Credit unions were initiated by people sharing a common bond to provide emergency loans for those who could not get them from traditional banks. Currently, however, there is not much difference between the types of banks in US.

A dual banking system characterizes the US. A commercial bank chooses to be federal-chartered (national bank) or state-chartered (state bank). That means that the commercial banks receive a charter (a document that enables a bank to conduct banking activity) either from the federal government or from a state government.

If a commercial bank is federal-chartered, it is subject to three regulators: Comptroller of the Currency (an office in the US Treasury Department), Federal Deposit Insurance Corporation (FDIC) and Federal Reserve System (Fed). The Comptroller of the Currency regulates national commercial banks and grants charters on behalf of the US federal government. It also obliges national commercial banks to be members of the Federal Reserve and Federal Deposit Insurance Corporation. FDIC is an insurance agency and Federal Reserve System is the central bank of the United States and the lender of the last resort.

A state commercial bank is regulated by a state government agency and can be required in some states to join the Fed and/or FDIC.

Savings institutions and credit unions have their specific regulators. In the same way as commercial banks they can be federal-chartered or state-chartered financial institutions. Savings institutions are regulated by the Federal Home Loan Bank System (FHLBS) which is the US government agency like the Federal Reserve. FDIC insures deposits at savings institutions. Regarding credit unions, they are chartered from the National Credit Union Administration which grants charters on behalf of the federal government. The National Credit Union Administration also insures the deposits at credit unions.

The US banks differ from banks of other industrialized countries in that they are more numerous and smaller. One of the reasons of it was the McFadden Act[3]. This act forbade a commercial bank to open a branch in another state and thus kept small inefficient banks in business. Moreover, some states had imposed unit banking. Unit banking restricts a bank to a single geographical location, such as one city. Currently, no state forces unit banking.

As of June 30, 2016 the five largest banks in the United States by total assets are JPMorgan Chase, Bank of America, Wells Fargo, Citigroup, and Goldman Sachs (Wikipedia).

3 An application to US banks over 2001-2012

3.1 Data and the underlying technology

In this section, we will explain the way we obtained homogenous samples of US banks over the period 2001–2012 and how the variables for analysis were defined and computed.

In the banking activity, we can identify seven kinds of stakeholders. The four stakeholders that correspond to inputs are: creditors, employees, suppliers and government. Creditors include clients that bring deposits to a bank as well as other financial institutions that lend short-term loans to a bank. Suppliers are clients that are related to the operational activities of the bank such as occupancy expenses, depreciation and amortization, marketing and all other operating expenses. We consider the government as input because it creates the necessary economic and regulatory

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[3] *"The McFadden Act is a United States federal law, named after Louis Thomas McFadden, member of the United States House of Representatives and Chairman of the United States House Committee on Banking and Currency, enacted in 1927 from recommendations made by former Comptroller of the Currency Henry May Dawes."* (Wikipedia)

conditions to insure the banking activity (CERC's Documents, 1980). The three stakeholders that correspond to outputs are: borrowers, financial market participants and commission and fee payers. Borrowers include clients that borrow loans from a bank as well as other financial institutions to which the bank lend short-term interest-earning loans. Financial market participants include clients related to market-driven activities as well as clients related to investment activities (investment securities, mortgage-backed securities, trading securities and dividend income from equity securities). Commission and fee payers include clients who pay for service charges, brokerage, origination and servicing of mortgage loans, credit card receivables, automobile loans, and other consumer and commercial loans.

All data was collected from Bloomberg database. Bloomberg is a provider of historical and real-time financial market and economic data for all sectors worldwide. All volumes were taken from annual fiscal year balance sheets. The quantities for Employees were taken from Reference items of annual fiscal year balance sheets and correspond to the number of full time equivalents. All values were taken from annual fiscal year income statements. Prices are computed as ratios of values to volumes (quantities). All values and volumes are expressed in millions of dollars.

Table 11 Variables composition in values, volumes (quantities) and prices

Variables	**Volumes (Quantities)**	**Values**	**Prices**	**Stakeholders**
Inputs	Customer Deposits + Short-Term Borrowing	Interest Expense	Value/ Volume (Quantity)	Creditors
	Number of Employees	Personnel Expenses		Employees
	Total Assets - Customer Deposits	Non-interest expense - Personnel Expenses		Suppliers
	Pretax income	Income Tax Expenses		Government
Outputs	Total Loans + Interbank assets	Interest Income - Provision for loan losses		Borrowers
	Marketable Securities and Other Short-Term Investments	Trading Account Profits (Losses) + Investment Income(Losses)		Financial market participants
	Customer Deposits + Total Loans	Commissions and Fees Earned + Other Operating Income (Losses)		Commission and Fee payers

We collected data year by year and for all US banks available each year at Bloomberg. As existing banks could merge, be acquired or close and new banks could open, we obtained different samples of US banks each year.

Because we selected all available US banks each year, our samples could consist at the same time of giants like Bank of America or JP Morgan and small community retail banks. In order to create a homogenous sample for each year and keep only comparable small to medium sized US banks, we applied a FAST-MCD (Minimum Covariance Determinant) algorithm (Rousseeuw et Van Driessen, 1999) to each year samples. This algorithm finds a subset of points h out of n observations with the minimum covariance matrix determinant. Subset h was set to (n+p+1)/2 where p is the number of variables. This is a default value for h of this algorithm and corresponds to a high breakdown point. The location and scatter parameters estimated for this subset h are robust estimators. Robust distances which correspond to standardized values from multivariate normal distribution are then calculated for every observation in the subset *h* according to the following formula:

$$RD(x_i) = \left[\left(x_i - T_h \right)^T S_h^{-1} \left(x_i - T_h \right) \right]^{1/2},$$

where T_h and S_h are robust location and scatter parameters.

The squared robust distances RD_i^2 are then compared to chi-square distribution with p degrees of freedom. We implemented this algorithm to both volumes (quantities) and prices. Thus, the number of variables was set to 14. The outlier detection threshold was set to 97,5% quantile of chi-square distribution.

Table 12 Selection of banks in final samples, 2001-2012

	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Initial number of banks	186	187	205	337	455	460	468	458	451	431	415	370
Final number of banks	93	90	102	166	228	220	224	223	223	215	202	174

The descriptive statistics for the variables in obtained samples are in Tables 13 and 14.

Table 13 Mean values for all variables, 2001-2012

	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Creditors (Volume)	731,12	670,66	742,08	370,57	370,75	397,5	431,88	455,62	498,35	506,68	561,02	785,43
Employees (Volume)	305,9	265,21	276,7	141,83	134,69	137,05	144,63	144,35	148,46	148,04	161,39	209,64
Suppliers (Volume)	197,43	167,79	196,16	95,36	92,59	98,21	117,78	131,48	125,24	120,65	127,84	182,02
Government (Volume)	14,35	13,49	14,61	5,95	6,14	6,52	6,12	3,15	0,71	2,27	3,88	9,5
Borrowers (Volume)	612,85	549,59	616,71	319,59	323,39	351,27	391,06	414,78	439,26	437,18	466,62	642,24
Market participants (Volume)	181,41	177,97	187,32	89,52	80,58	81,19	87,71	95,62	107,83	116,82	141,52	210,62
Commision and Fees payers (Volume)	1280,15	1171,95	1299,05	661,51	667,1	720,48	792,31	833,44	904,22	909,72	993,19	1374,19
Creditors (Price)	0,0391	0,0249	0,0184	0,0182	0,0222	0,0303	0,0356	0,0295	0,0221	0,0166	0,0124	0,0089
Employees (Price)	0,0463	0,049	0,0532	0,0508	0,0535	0,0562	0,0591	0,0614	0,0627	0,0646	0,0663	0,0703
Suppliers (Price)	0,0705	0,0748	0,0682	0,0691	0,0697	0,0682	0,0641	0,0635	0,0796	0,0864	0,0915	0,0848
Government (Price)	0,3115	0,3096	0,3123	0,2968	0,303	0,3034	0,2902	0,2805	0,2226	0,2258	0,2029	0,2308
Borrowers (Price)	0,0774	0,0637	0,0581	0,0557	0,0608	0,0685	0,07	0,0571	0,0464	0,0459	0,046	0,046
Market participants (Price)	0,0678	0,06	0,0527	0,045	0,0477	0,0532	0,056	0,0549	0,0565	0,0491	0,0398	0,0404
Commision and Fees payers (Price)	0,0066	0,0067	0,0069	0,0054	0,0049	0,0048	0,0049	0,0049	0,0047	0,0047	0,0045	0,0049

Table 14 Coefficients of variation (standard deviation/mean, in %) for all variables, 2001-2012

	2001	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Creditors (Volume)	59,59	55,42	60,18	52,05	58,1	56,78	58,19	58,08	59,77	57,4	59,36	66,89
Employees (Volume)	59,22	54,35	60,55	56,69	62,68	60,69	62,08	63,74	64,91	63,45	62,84	69,89
Suppliers (Volume)	72,05	60,87	70,73	63,85	66,27	62,67	64,96	67,38	66,72	67,34	69,32	75,37
Government (Volume)	67,71	74,55	70,94	63,9	76,03	74,46	84,57	197,54	1082,08	332,77	175,79	106,03
Borrowers (Volume)	58,21	53,54	59,28	53,85	59,52	57,18	58,74	57,77	61,22	57,81	59,06	67,15
Market participants (Volume)	71,69	74,44	76,96	74,8	76,36	73,52	79,54	86,4	81,74	87,87	83,82	88,47
Commision and Fees payers (Volume)	57,96	54,41	59,32	52,12	58,37	57,19	58,21	57,58	60,34	57,2	59,43	67,41
Creditors (Price)	25,6	32,42	32,89	30,33	26,19	23,32	20,56	25,69	32,43	34,32	36,84	43,59
Employees (Price)	19,58	20,67	22,18	20,1	21,23	21,84	21	22,28	19,51	20,62	21,88	20,78
Suppliers (Price)	50,15	53,28	45,22	43,04	44,82	43,17	42,84	45,52	47,22	42,72	49,94	45,81
Government (Price)	17,55	16,04	17,2	19,66	23,69	22,5	29,64	64,32	108,93	91,45	99,14	57,48
Borrowers (Price)	8,67	11,35	10,89	10,54	9,76	11,24	10,45	14,26	23,25	23,62	17,78	14,7
Market participants (Price)	19,58	20,67	22,18	20,1	21,23	21,84	21	22,28	19,51	20,62	21,88	20,78
Commision and Fees payers (Price)	44,83	40,6	39,57	46,83	54,98	49,46	52,52	55,76	57,16	59,58	57,23	55,69

Mean coefficient of variation for all variables is equal to 57%. Coefficient of variation around 50% shows that the data are homogenous. We only observe high values for coefficient of variation for variable ‘Government’ in volume (‘Pretax income’) over the period 2008-2012 and ‘Government’ in price (‘Tax expenses’) over the period 2009-2011. These increases are obviously due to the financial crisis of 2007-2008.

The total number of distinct banks across all periods are equal to 430. Table 15 shows that most banks in our sample are commercial banks. These banks provide a wide range of commercial banking services to individuals and small to medium sized businesses including a variety of deposit accounts; commercial, consumer and real estate loans, as well as investment services. Around 20% (91 banks) of our distinct sample consists of savings banks. The difference of these banks from commercial banks is that their loan portfolio primarily consists of residential mortgage loans, including one- to four-family loans, multi-family loans, home equity loan, residential construction loans and commercial real estate loans. Their investment portfolio in many cases consists of mortgage-backed securities, the United States Government and federal agency securities.

Table 15 Business activity segmentation of US banks for the period 2001-2012

Product segment	**Number of banks**
Commercial Banking	339
Savings Institution	91
Total number of distinct banks	**430**

3.2 Results and analysis

In order to estimate and decompose profit rate gap between each pair of banks in each of the 12 samples, the following steps were fulfilled:

- For each bank in a sample a difference between its observed profit rate and all banks' observed profits rates were calculated;
- For each bank in a sample a technically efficient benchmark was found (a SAS code is given in Appendix 2);
- For each bank in a sample a profit maximizing benchmark was found (a SAS code is given in Appendix 2);
- For each bank in a sample a technical efficiency effect, an allocative efficiency effect and a size effect with respect to all banks in a sample were calculated (chapter 1 section 4.4);
- For each bank in a sample a price effect (decomposed into 7 stakeholders) with respect to all banks in a sample was calculated (chapter 1 section 4.4).

Thus, for each of 12 years we obtained 11 symmetrical matrices: 1 matrix of observed profit rates differences, 3 matrices each corresponding to 1 of 3 quantity effects (technical, allocative and size) and 7 matrices of price effect decomposed by 7 stakeholders (Creditors, Employees, Suppliers, Government, Borrowers, Financial market participants and Commissions and Fees payers).

If we consider any of these matrices, each cell corresponds to profit rate difference or to one of the 10 effects (3 quantity effects and 7 price effects) between the bank in a row and the bank in a column.

3.2.1 A detailed analysis for two banks: The Lorain National Bank and Northrim Bank

There are two possibilities to analyze the obtained matrices:

1. We choose any bank in a row and we compare it to another bank in a column;
2. We choose any bank in a row and we compare it to the whole sample, which means to all banks in columns.

We decide to perform the second type of analysis.

To obtain a mean value of profit rate gap and its components for each bank in each year sample, we aggregated each of the 11 matrices to a column by calculating the mean value of each row. We then merged the obtained columns for each year. The resulting **aggregated matrices** represent, for each year, a list of all banks in rows and mean values of profit rate gap/components of profit rate gap between the bank in a row and all banks in a sample in columns.

There are only 5 banks that are present all over the period 2001-2012. To illustrate the evolution of mean profit rate gap and its decomposition into quantity and price effects in a larger sample of banks, we considered the 35 banks that are present *at least* between 2004 and 2012 (some of them can be present before 2004). The results are presented in Appendix 3. Here we focus on a detailed analysis for the following 2 banks over the period 2001-2012:

- LNB Bancorp, Inc. which is the holding company for The Lorain National Bank;
- Northrim BanCorp Inc. which is the holding company for Northrim Bank.

In this section, we mostly focus on the evolution of the mean profit rate gap and its components over the considering period and we will give an idea on how interpret the values of obtained rates in the section 3.2.3 of this chapter.

We start with the analysis of the profit rate gap of The Lorain National Bank over the period 2001-2012. Figure 27 presents the global evolution of the profit rate differential over the period and its decomposition into the price and quantity effects. In 2001, the profit rate differential between this bank and all other banks in the sample was on average 2,4%. This means that The Lorain National Bank was more profitable that the other banks and this performance came from a good price environment (+12%). The productive performance was worse than the average bank (-9,6%). On the whole, the profit rate differential decreased between 2003 and 2009 and recovered after 2009. This global evolution can be explained by contrasted price and quantity effects. Clearly, price and quantity effects show more variability than the global evolution. While the quantity effect is generally increasing over the period (the trend is equal to 2,1% and is significant at 5%), the price effect is generally decreasing over the period (the trend is equal to -2,1% and is significant at 5%).

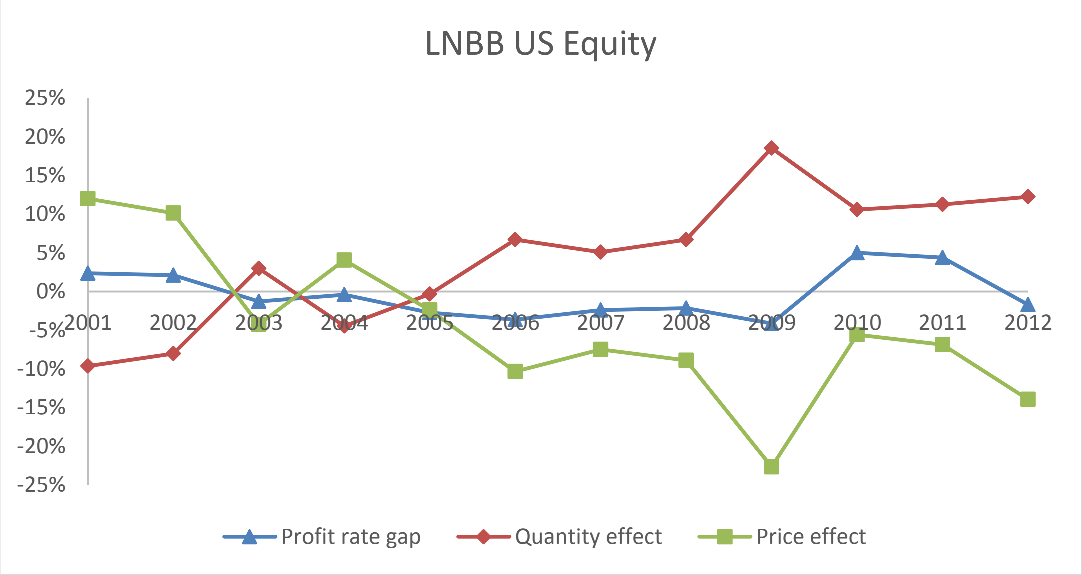

Figure 27 Mean profit rate gap and its decomposition into quantity and price effects over the period 2001-2012: *The Lorain National Bank.*

Beyond Figure 27, we can go further into details and decompose the quantity and price effects. This is presented in Table 16. The increase of quantity effect is mainly due to a global increase of the technical efficiency differential effect over the period (the trend is equal to 0,94% and is significant at 5%). The allocative efficiency differential effect due to a moderate increase after a sharp drop in 2002, has a trend not significantly different from 0. Size differential effect alternates peaks and drops over the whole period and has a trend not significantly different from 0. It means that The Lorain National Bank enhances its technical efficiency (the proximity to the production frontier) compared to the sample over the period.

The decrease of price effect is mainly caused by the decrease of the suppliers price effect (the trend is equal to -1,02% and is significant at 5%) and to a lesser extent by the decrease of Borrowers price effect and Creditors price effect (the corresponding trends are -0,4% and -0,7% and both are significant at 5%). It means that suppliers, borrowers and creditors of The Lorain National Bank could benefit from increasing price advantages over 2001-2012 compared to the sample. We notice as well that the commission and fee payers price effect is greater than 4% over the whole period. It means that the commission and fee payers of The Lorain National Bank paid the services at a greater rate compared to the sample over the whole considering period. The only price effect that increases over the whole period is financial market participants price effect (the trend is equal to 0,5% and p-value is equal to 0,0066). This means that the financial market participants of The Lorain National Bank buy their securities and other related products and services at an increasing rate compared to the sample. Employees price effect fluctuates around 0 over the entire period (the trend is not significant at 5%). Government price effect is very close to 0 over the period which means that there is almost no difference between income tax rates of The Lorain National Bank and other banks in the sample.

Table 16 Evolution of the mean profit rate gap and its decomposition into ten components over the period 2001-2012: *The Lorain National Bank.*

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Profit_dif	**2,4%**	**2,1%**	**-1,3%**	**-0,4%**	**-2,7%**	**-3,7%**	**-2,4%**	**-2,2%**	**-4,2%**	**5%**	**4,4%**	**-1,7%**
Tech_eff	-5,8%	-2%	0,2%	-2,2%	2,5%	2,3%	1%	6,1%	6,5%	5,6%	5,7%	4,1%
Alloc_eff	2,3%	-8,4%	-2,2%	-0,9%	1,7%	0,4%	3,1%	-0,8%	0%	2,6%	5,3%	0,5%
Size_eff	-6,1%	2,4%	5,1%	-1,4%	-4,6%	4%	1%	1,3%	12%	2,5%	0,2%	7,6%
Quantity effect	**-9,6%**	**-8%**	**3%**	**-4,5%**	**-0,3%**	**6,7%**	**5,1%**	**6,7%**	**18,5%**	**10,6%**	**11,2%**	**12,3%**
Borrowers	-2,7%	-3,2%	-3,1%	-3,8%	-3%	-3,3%	-4,9%	-4,5%	-9,4%	-4,7%	-6,5%	-6,7%
Financial_mkt	-2,3%	-2,4%	-2,2%	-6%	-4,2%	-2,3%	0,2%	-1%	3,1%	-0,3%	1,2%	2,2%
Commision_Fee	5%	5,6%	5,3%	9,8%	7,2%	5,6%	4,8%	5,5%	6,4%	5%	5,2%	4,4%
Creditors	11%	6,8%	4,4%	7%	5,4%	3,4%	1,3%	0,8%	-2,4%	5,5%	2,5%	0,8%
Employees	4,6%	7,4%	-4,1%	0,5%	-0,9%	-3%	-1,7%	1,2%	-0,9%	1,4%	-0,9%	-1,7%
Suppliers	-3,6%	-4,3%	-5%	-3,7%	-7,8%	-11,9%	-7,7%	-12%	-20,6%	-13%	-9,1%	-12,7%
Government	-0,05%	0,3%	0,4%	0,3%	1%	1,2%	0,6%	1,2%	1,2%	0,4%	0,7%	-0,3%
Price effect	**12%**	**10,1%**	**-4,3%**	**4,1%**	**-2,4%**	**-10,4%**	**-7,5%**	**-8,9%**	**-22,7%**	**-5,6%**	**-6,9%**	**-13,9%**

We now move to the analysis of the profit rate gap for Northrim Bank over the period 2001-2012 (Figure 28). The mean profit rate gap of Northrim Bank globally increased over the period 2001-2012 (the trend is equal to 1,5% with p-value 0,0002). This increase is due to the increase of the price effect (the trend is 3,1% and p-value is less than 0,0001). On the contrary, the quantity effect decreased (the trend is -1,6% with p-value equal to 0,0005). The more intensive growth of the price effect compared to the more moderate decline of the quantity effect causes the profit rate gap to grow. However, this evolution maintains only until 2009. After 2009 both the quantity and the price effects change their trends which decelerated the profit rate gap increase. In 2011, the price effect fell sharply from 19,4% to 7,5% which caused the decrease of profit rate gap from 15,7% to 9%. Thus, Northrim Bank compared to The Lorain National Bank has the opposite position towards the sample: it has declining productive efficiency towards the sample but benefits from the increasingly advantageous economic environment.

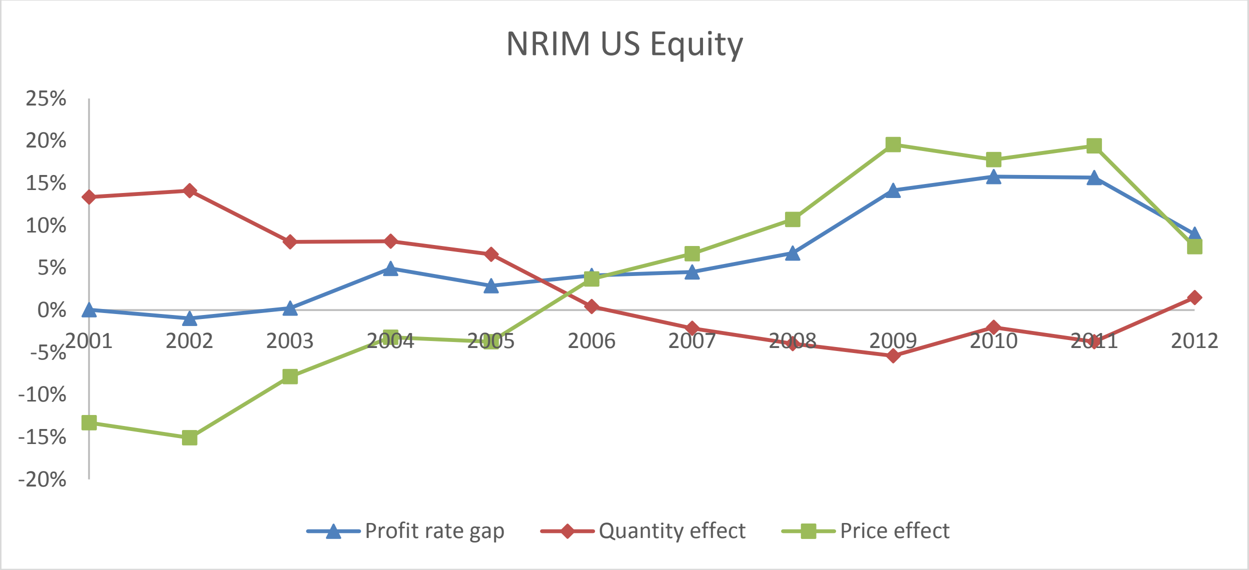

Figure 28 Mean profit rate gap and its decomposition into quantity and price effects over the period 2001-2012: *Northrim Bank.*

The decline of the quantity effect of Northrim Bank (Table 17) is caused by the decrease of its technical efficiency differential effect (the trend is equal to -0,57% and it is significant at 5%) and by the decrease of the size differential effect (the trend is equal to -1,2% and it is significant at 5%). The allocative effect and the size effect have similar mutually inverse patterns but the allocative effect is not significantly different from 0. This means that Northrim Bank has decreasing technical efficiency and size effects compared to the sample but globally has the same allocative efficiency as the sample under consideration.

The evolution of the price effect is explained by the increase of the creditors' price effect, the suppliers' price effect and the commission and fee payers' price effect (the corresponding trends are 1,04%, 1,9% and 1,4% and they are all significant at 5%). It means that creditors and suppliers were paid for their services by Northrim Bank at a decreasing rate as compared to the sample. While commission and fee payers paid for the services of Northrim Bank at a increasing rate as compared to the sample. The increase for the creditors' price effect, nevertheless, was maintained only until 2009. After 2009 the creditors' price effect decreases steadily. The borrowers' and financial market participants' price effects have slight decreasing trends (-0,68% and -0,74% and both are significant at 5%). It means that borrowers and financial market participants buy loans and investment securities of Northrim Bank at a reduced rate over 2001-2012 compared to the sample. Similar to The Lorain National Bank, the employees' and government's price effects of Northrim Bank are close to zero over the entire period (the trend for employees is not significant and the trend for government is significant but very small: 0,1% per year). This means that employees of Northrim Bank and the government were paid for their services at the similar rate as other banks in the sample.

Table 17 Evolution of the mean profit rate gap and its decomposition into ten components over the period 2001-2012: *Northrim Bank*.

	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Profit_dif	**0,1%**	**-1,0%**	**0,2%**	**4,9%**	**2,9%**	**4,1%**	**4,5%**	**6,7%**	**14,1%**	**15,8%**	**15,7%**	**9,0%**
Tech_eff	4,1%	3,8%	0,3%	4,4%	5,2%	1,4%	2,2%	-2,2%	-1,4%	-0,7%	-0,3%	-2,3%
Alloc_eff	-6,6%	6,4%	-9,9%	5,4%	6,3%	3,1%	-5,6%	-3,5%	-4,1%	-3,7%	-2,8%	8,2%
Size_eff	15,9%	3,9%	17,7%	-1,7%	-4,9%	-4,1%	1,3%	1,8%	0,1%	2,4%	-0,7%	-4,5%
Quantity effect	**13,3%**	**14,1%**	**8,1%**	**8,1%**	**6,6%**	**0,4%**	**-2,1%**	**-4,0%**	**-5,4%**	**-2,0%**	**-3,7%**	**1,5%**
Borrowers	8,7%	10,0%	10,9%	7,4%	4,1%	3,8%	2,3%	-0,1%	4,4%	4,1%	4,1%	3,0%
Financial_mkt	-5,2%	-2,5%	-2,0%	-0,2%	0,7%	-2,2%	-3,5%	-4,0%	-7,8%	-7,8%	-10,3%	-9,6%
Commision_Fee	-3,5%	-7,5%	-8,9%	-7,2%	-4,8%	-1,7%	1,2%	4,1%	3,4%	3,7%	6,3%	6,6%
Creditors	9,7%	9,4%	10,9%	12,2%	7,7%	9,3%	15,6%	23,2%	27,7%	21,2%	17,0%	12,2%
Employees	-3,2%	-4,5%	-0,9%	-2,8%	-2,7%	-1,5%	-1,0%	-1,7%	1,5%	-2,8%	-2,7%	-6,5%
Suppliers	-18,6%	-17,5%	-15,4%	-11,1%	-7,7%	-3,9%	-7,0%	-10,4%	-9,2%	-0,2%	5,6%	3,0%
Government	-1,2%	-2,5%	-2,4%	-1,5%	-1,0%	-0,3%	-0,9%	-0,4%	-0,4%	-0,4%	-0,6%	-1,2%
Price effect	**-13,3%**	**-15,1%**	**-7,9%**	**-3,2%**	**-3,7%**	**3,7%**	**6,7%**	**10,7%**	**19,6%**	**17,8%**	**19,4%**	**7,5%**

3.2.2 A global analysis for performant banks

After a focus on two banks, we now propose a more general approach for interpreting the results at a more aggregated level. The question we have is what factors describe **performant banks** that have positive mean profit rate gaps towards the sample in each year?

To obtain the corresponding data for performant banks, we selected from *the aggregated matrices* (chapter 2 section 3.2.1) only banks with positive mean profit rate gap towards the sample (with profit rate gap greater than 1% to not consider banks with profit rate gap too close to zero) and we computed mean values for the profit rate gap and ten components of these banks in each year sample. We first give an overall picture of the evolution of the profit rate differential for these performant banks. We also present the result for the two types of banks, namely commercial and savings banks.

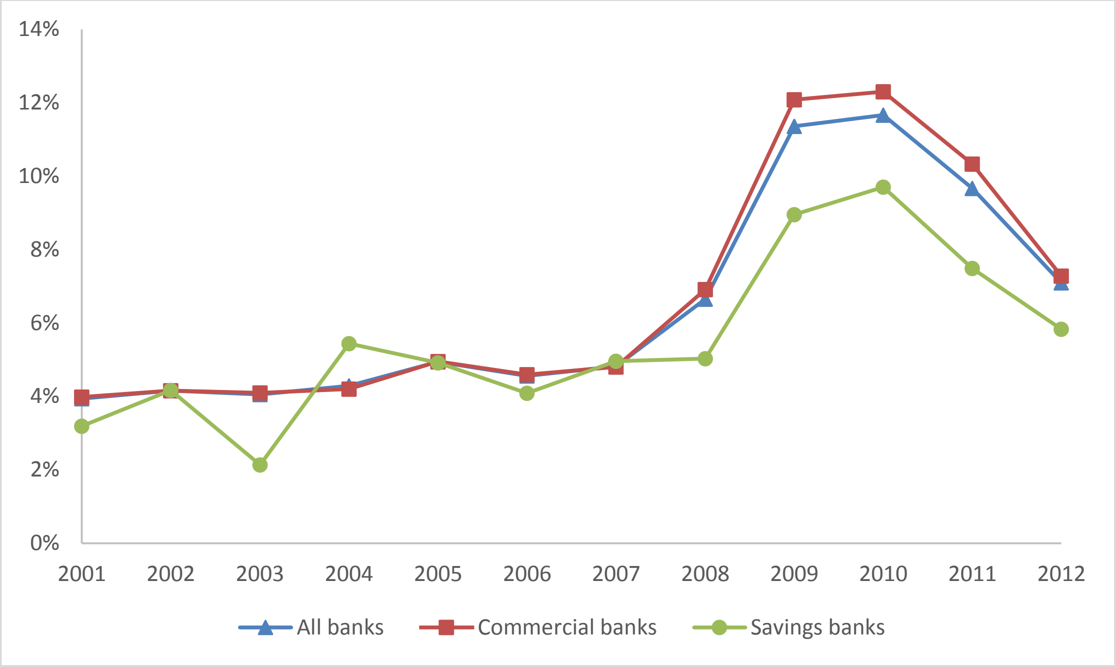

Figure 29 Evolution of the mean profit rate differential for performant banks (total, commercial and savings)

Overall, the mean profit rate differential of performant banks is stable at around 4-5% over the period 2001-2007. Figure 29 clearly shows that the gap between performant banks and all other banks increased during the financial crisis and has decreased since 2010. Commercial and savings banks show the same evolution, especially before the crisis, but commercial banks seem to be more performant after the crisis.

We continue with the decomposition of the mean profit rate gap into quantity and price effects. The evolution of the mean profit rate gap for performant banks can be clearly divided into two periods: before 2007 and after 2007 (Figure 30). That is, the mean profit rate gap before crisis was maintained at a level of 4-5% and after crisis it increased to 11,7% in 2010 and then declined to 7% in 2012. This finding means that the crisis of 2007-2008 provoked a greater differentiation among banks in terms of profit rate gaps. This is mainly due to the evolution of price effect which is positive over the whole period and has similar to the profit rate differential evolution pattern. Namely, the price effect is stable at a mean rate of 6% until 2006, then it rises to 15% in 2009 and falls to 6,4% in 2012. The quantity effect is stable until 2006 at a mean rate of -1,6%, then it decreases to -5,6% in 2008 and then increases up to 0,71% in 2012. To sum up, performant banks benefitted from more advantageous economic environment over the whole period. As to quantity effect, the crisis of 2007-2008 incited the improvement in the productive efficiency of performant banks.

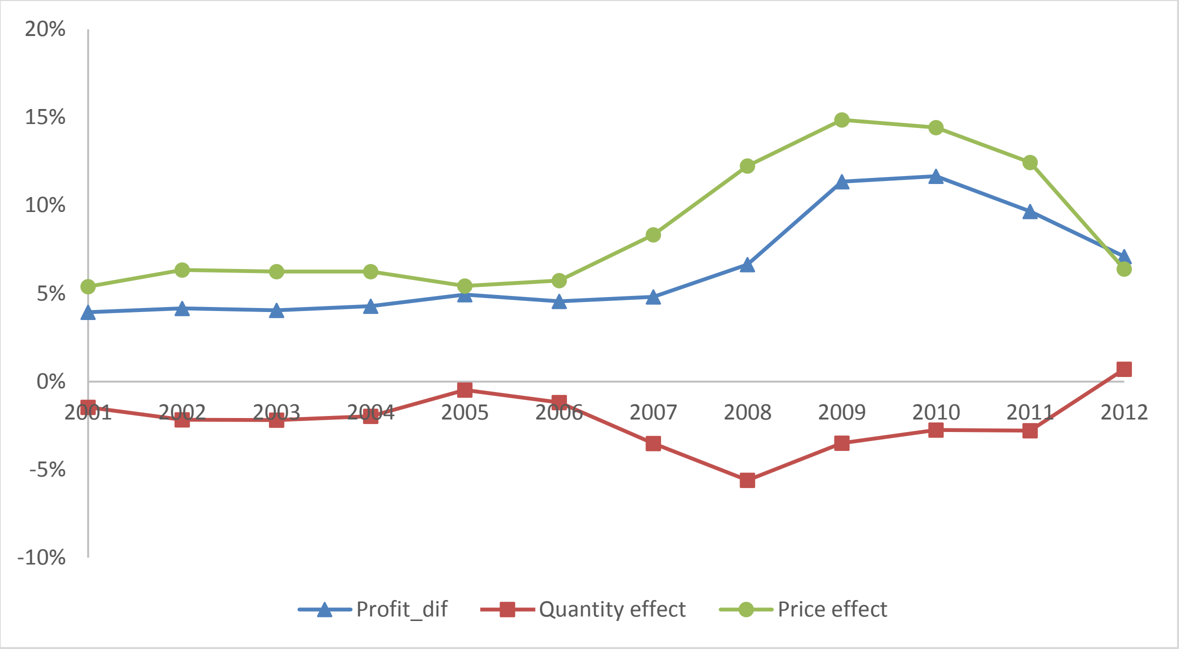

Figure 30 Mean profit rate gap and its decomposition into quantity and price effects over the period 2001-2012 for performant banks

The relatively stable evolution of the quantity effect until 2006 is caused by the contrasted evolutions of the allocative effect and the size effect (Table 18). After 2006, the allocative effect drops sharply to -4,6% in 2008 and then increases up to 2,8% in 2012 which is the main cause of the evolution of the quantity effect. Both trends for the allocative efficiency effect and the size effect are not significant at 5%. The technical efficiency differential effect is stable and close to 0 over the whole period (the trend is not significant at 5% and the average value is equal to 0,05%). This means that the crisis of 2007-2008 forced performant banks to improve the allocation of their inputs and outputs given the economic environment.

The positive price effect over the sampled period is mainly induced by the creditors' and employees' price effects over the entire period and to a large extent to the suppliers' price effect after 2007. The average value for the Creditors price effect is 4% and the trend is not significantly different from zero. The average value for the employees' price effect is equal to 2% and the trend is not significantly different from zero. The average value for Suppliers price effect before 2008 is equal to 0,4% and the trend is not significantly different from 0. Since 2008, the suppliers' price effect increases up to 7,4% in 2011 and then declines to 5,1% in 2012. The mean values for the borrowers', the financial market participants', the commission and fee payers' and the government's price effects are 0,25%, -0,61%, 0,36% and 0,07% respectively and their trends are very low (0,2% for borrowers, significant at 5%; -0,16% for financial market participants, significant at 5%; not significant at 5% trend for commission and fee payers; -0,04% for government, significant at 5%). To sum up, we can see that creditors, employees and since 2008 suppliers of performant banks were paid for their services and products less compared to banks with negative profit rate gap and this determined the more advantageous price environment for performant banks.

Table 18 Evolution of mean profit rate gap and its decomposition into ten components over the period 2001-2012 for performant banks

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Profit_dif	**3,9%**	**4,2%**	**4,1%**	**4,3%**	**4,9%**	**4,6%**	**4,8%**	**6,7%**	**11,4%**	**11,7%**	**9,7%**	**7,1%**
Tech_eff	0,5%	0,3%	0,1%	-0,1%	0,0%	0,0%	-0,2%	-0,7%	0,2%	0,0%	0,2%	0,3%
Alloc_eff	-0,7%	-0,9%	3,2%	-0,7%	1,2%	0,3%	-3,3%	-4,6%	-4,4%	-1,5%	-0,3%	2,8%
Size_eff	-1,3%	-1,5%	-5,5%	-1,2%	-1,7%	-1,5%	0,1%	-0,3%	0,7%	-1,3%	-2,6%	-2,4%
Quantity effect	**-1,5%**	**-2,2%**	**-2,2%**	**-2,0%**	**-0,5%**	**-1,2%**	**-3,5%**	**-5,6%**	**-3,5%**	**-2,8%**	**-2,8%**	**0,7%**
Borrowers	-0,7%	-0,3%	0,2%	-1,0%	0,2%	-0,4%	-0,6%	-0,6%	1,8%	2,0%	1,8%	0,5%
Financial_mkt	0,2%	-0,2%	-0,4%	0,5%	-0,2%	0,0%	-0,7%	-0,8%	-1,6%	-1,6%	-1,3%	-1,1%
Commision_Fee	0,5%	0,5%	0,1%	0,5%	0,0%	0,3%	1,3%	1,4%	-0,2%	-0,3%	-0,5%	0,6%
Creditors	5,9%	2,3%	3,0%	2,7%	4,2%	4,5%	6,0%	5,8%	5,2%	3,9%	3,2%	2,4%
Employees	0,1%	1,7%	2,4%	1,6%	1,1%	1,2%	2,7%	3,5%	4,0%	3,3%	1,8%	-0,7%
Suppliers	-0,7%	2,0%	0,5%	1,5%	0,1%	0,0%	-0,5%	2,9%	6,1%	7,1%	7,4%	5,1%
Government	0,1%	0,3%	0,3%	0,4%	0,0%	0,0%	0,2%	0,1%	-0,4%	0,2%	0,1%	-0,4%
Price effect	**5,4%**	**6,3%**	**6,2%**	**6,2%**	**5,4%**	**5,7%**	**8,3%**	**12,2%**	**14,9%**	**14,4%**	**12,5%**	**6,4%**

3.2.3 An analysis of performant banks before and after the financial crisis

We could see above that the evolution of the mean profit rate gap for performant banks changed after the crisis of 2007-2008. We thus decided to study separately the banks with positive profit rate gap before and after the 2007-2008 crisis.

To prepare the data for this analysis, we considered again the *aggregated matrices* (chapter 2 section 3.2.1). We first separated banks into two periods: before crisis (until 2008) and after crisis (since 2008). In the both periods, we selected all banks with positive mean profit rate gap. If banks had positive mean profit rate gap only once across the before or after crisis period, we kept their values for mean profit rate gap and the ten components. If some banks had positive mean profit rate gap for several years across the before or after crisis period, we computed mean values for their mean profit rate gap and the ten components in these years.

To compare performant banks before and after crisis, we will study their mean surplus accounts according to Table 5 of the section 3.2 of the first chapter (uses-resources table) and to equation

(33) of the first chapter. We first give the general concept on how to interpret the elements of the uses-resources table. We then study the differences between surplus accounts in the before and after crisis periods.

Table 19 represents the surplus account with decomposed quantity effect (technical efficiency, allocative efficiency and size effects) which corresponds to productivity surplus and decomposed price effect which corresponds to price advantages/disadvantages related to each stakeholder. As an example, we can interpret the components of productivity surplus in this table as follows: size effect of 1,04% in the uses part of the table means that on average for performant banks the influence of the difference between their size inefficiency and other banks' size inefficiency on the difference between their profit rates and other banks' profit rates is negative (performant banks are on average size inefficient compared to other banks) and is equal to -1,04%. In the same manner, as an example, we can interpret the components of price effect in this table as follows: the creditors' price effect of 3,72% in the Resources part of the table means that, on average, for performant banks the influence of the difference between their deposit rates and other banks' deposit rates on the difference between their profit rates and other banks' profit rates is positive (performant banks have on average lower deposit rates compared to other banks) and is equal to 3,72%. If we sum up all decomposed quantity and price effects (taking the effects from Resources part with a positive sign and the effects from uses part with a negative sign), we obtain the mean profit rate differential for performant banks of 4,2%.

The main source of price advantages for performant banks before the crisis are creditors and employees (Table 19) which corresponds to the previous analysis by years. It is worth noting that all stakeholders except borrowers are sources of price advantages for performant banks. These price advantages together with little technical efficiency ensure positive profit rate gap of 4,2% and compensate allocative and size inefficiency as well as borrowers price disadvantage.

Table 19 Surplus account with productivity surplus decomposition for performant banks before the crisis (194 banks)

Uses			**Resources**
Alloc_eff	0,72%	Tech_eff	0,25%
Size_eff	1,04%	Financial_mkt	0,01%
Borrowers	0,43%	Commision_Fee	0,42%
Profit_dif	**4,2%**	Creditors	3,72%
		Employees	1,35%
		Suppliers	0,54%
		Government	0,11%
Total	**6,4%**	**Total**	**6,4%**

In the after crisis period creditors and employees remain an important source of price advantages for performant banks (Table 20). But what makes a notable difference compared to the before crisis period is that suppliers became a large source of price advantages for performant banks. It means that the positive effect of the difference between prices at which performant banks pay to their suppliers and prices at which other banks pay to their suppliers on the profit rate gap between performant and other banks increased in the after crisis period up to 5,54%. This means that suppliers of performant banks in the after crisis period were payed at a much lower prices compared to the before crisis period. Another difference is that all the remaining stakeholders changed their positions in the uses-resources table. Thus, borrowers of performant banks in the after crisis period lost their price advantages and paid for their loans at a higher interest rate compared to borrowers of other banks. On the contrary, financial market participants and commission and fee payers of performant banks in the after crisis period found themselves in a more advantageous economic environment compared to other banks. In the after crisis period performant banks also paid higher taxes compared to other banks. The positions of quantity effect components in the uses-resources table after the crisis did not change. The only difference is that in the after crisis period the negative effect of the difference between allocative inefficiency of performant banks and allocative inefficiency of other banks on the profit rate gap between performant and other banks increased compared to size effect.

Table 20 Surplus account with productivity surplus decomposition for performant banks after the crisis (257 banks)

Uses			**Resources**
Alloc_eff	1,6%	Tech_eff	0,28%
Size_eff	0,52%	Borrowers	0,94%
Financial_mkt	0,64%	Creditors	3,2%
Commision_Fee	0,31%	Employees	1,9%
Government	0,04%	Suppliers	5,54%
Profit_dif	**8,75%**		
Total	**11,87%**	**Total**	**11,87%**

We will now provide the similar analysis for performant commercial and savings banks.

If we compare Tables 21 and 22, we notice that, on average, performant commercial and savings banks had completely different surplus accounts in the before crisis period. Namely, performant savings banks were better in all types of efficiency (technical, allocative and size). As to stakeholders’ price advantages, they all have “mirroring” positions in the two surplus accounts. Thus, performant commercial banks benefitted of price advantages from commission and fee payers, creditors, employees and the government in the before crisis period. Performant savings banks, on the contrary, enjoyed price advantages from borrowers, financial market participants and suppliers. We also observe that performant commercial banks have a high positive creditors’ price effect relative to all other effects and performant savings banks have a high positive suppliers’ price effect relative to all other effects.

Table 21 Surplus account with productivity surplus decomposition for performant commercial banks before the crisis (177 banks)

Uses			**Resources**
Alloc_eff	0,91%	Tech_eff	0,15%
Size_eff	1,34%	Commision_Fee	0,94%
Borrowers	0,67%	Creditors	4,43%
Financial_mkt	0,27%	Employees	1,80%
Suppliers	0,16%	Government	0,21%
Profit_dif	**4,18%**		
Total	**7,52%**	**Total**	**7,52%**

Table 22 Surplus account with productivity surplus decomposition for performant savings banks before the crisis (17 banks)

Uses			**Resources**
Commision_Fee	4,94%	Tech_eff	1,36%
Creditors	3,64%	Alloc_eff	1,23%
Employees	3,38%	Size_eff	2,09%
Government	0,96%	Borrowers	2,09%
Profit_dif	**4,45%**	Financial_mkt	2,86%
		Suppliers	7,74%
Total	**17,37%**	**Total**	**17,37%**

In the after crisis period, performant commercial and savings banks have, on average, similar productive efficiency (the components of quantity effect have equivalent positions in the uses-resources table). As to stakeholders, we remark that borrowers, employees and suppliers are, on average, sources of price advantages for both performant commercial and savings banks but suppliers' price effect on the profit rate gap of performant savings banks is much higher compared to performant commercial banks. Besides, we note that in the after crisis period performant commercial banks kept the highest positive creditors' price effect among all effects and performant savings banks keep the highest positive suppliers' price effect among all effects. Finally, performant commercial banks are, on average, a bit more profitable compared to performant savings banks after crisis. This is because performant savings banks counterbalance their considerable price advantage from suppliers by relatively high price disadvantages coming from creditors and commission and fee payers as well as by higher compared to performant commercial banks allocative inefficiency.

Table 23 Surplus account with productivity surplus decomposition for performant commercial banks after the crisis (201 banks)

Uses			**Resources**
Alloc_eff	1,15%	Tech_eff	0,03%
Size_eff	0,56%	Borrowers	0,35%
Financial_mkt	1,10%	Commision_Fee	0,75%
Profit_dif	**9,05%**	Creditors	5,29%
		Employees	2,27%
		Suppliers	3,06%
		Government	0,11%
Total	**11,86%**	**Total**	**11,86%**

Table 24 Surplus account with productivity surplus decomposition for performant savings banks after the crisis (56 banks)

Uses			**Resources**
Alloc_eff	3,23%	Tech_eff	1,18%
Size_eff	0,40%	Borrowers	3,06%
Commision_Fee	4,09%	Financial_mkt	1,03%
Creditors	4,29%	Employees	0,54%
Government	0,58%	Suppliers	14,45%
Profit_dif	**7,67%**		
Total	**20,26%**	**Total**	**20,26%**

3.2.3.1 A cluster analysis of performant banks before and after the crisis

To discover possible groups (clusters) of banks with different profit gap decomposition before and after crisis, we performed a cluster analysis using Ward's method (Ward, 1963).

For this analysis, we used the same data as in the previous chapter. That is, we selected from *aggregated matrices* banks with positive mean profit rate gap before and after crisis and ten variables: three quantity (productivity) effects and seven price effects corresponding to seven stakeholders.

We did not use principal component analysis prior to cluster analysis because all variables are poorly or not correlated. However, since all variables do not have equal or similar variances, we need to transform them. We performed standardization using MAXABS method (with zero location and maximum absolute value scale) for the before crisis period and MAD method (with median location and median absolute deviation from median scale) for the after crisis period prior to cluster analysis. We selected this standardization methods for the greatest evidence in the choice of the number of clusters using Ward's clustering method (Figure 31 and Figure 32). Besides, they ensured the most balanced separation of data among clusters (Table 25).

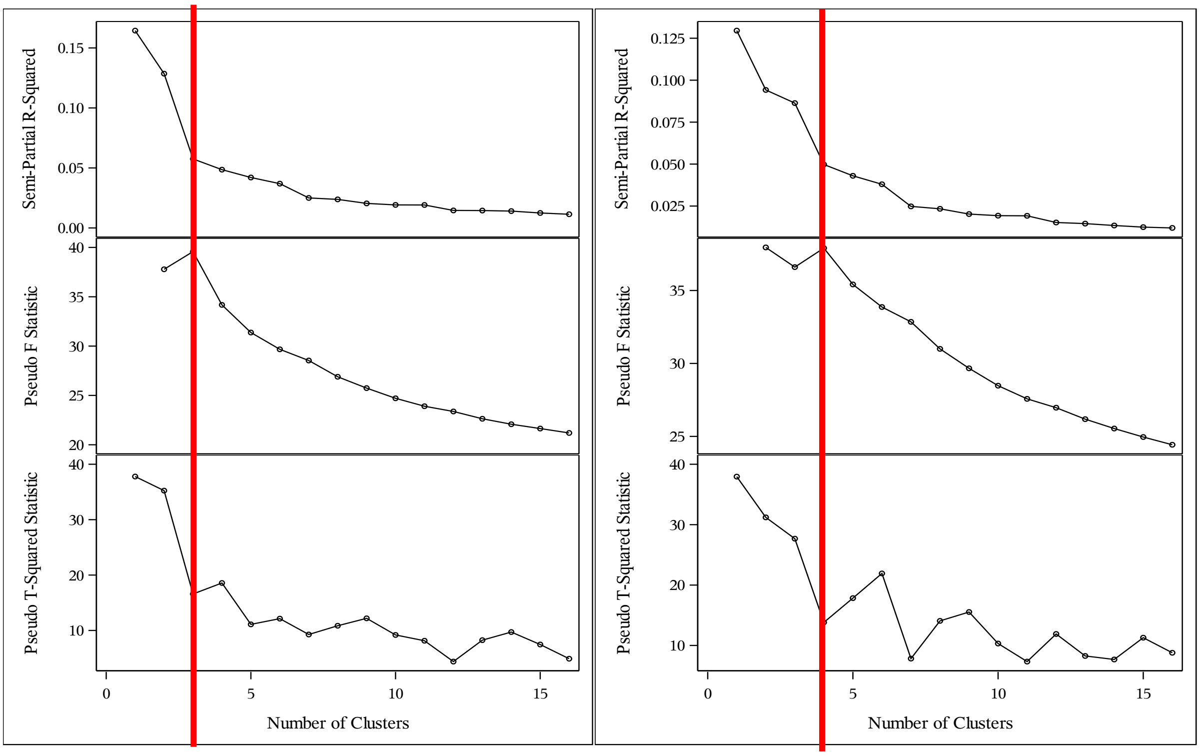

Figure 31 Criteria to select clusters in the before (on the left) and in the after (on the right) crisis period

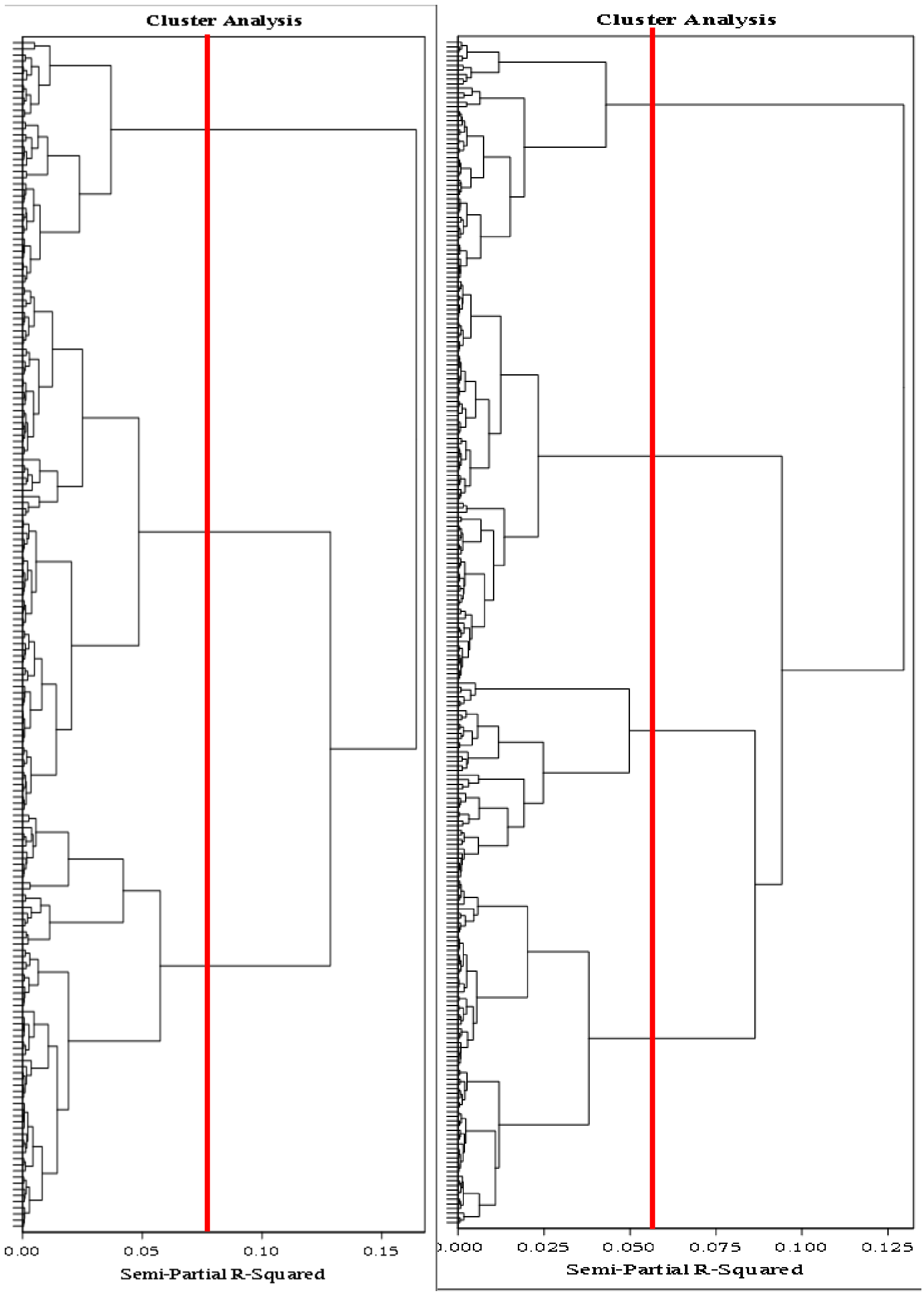

Figure 32 Dendrogram for the before crisis period (on the left) and for the after crisis period (on the right)

Table 25 Frequency of the retained clusters before and after the crisis

	Before crisis period			After crisis period			
	Cluster 1	Cluster 2	Cluster 3	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Frequency	86	40	68	87	52	75	43
Proportion	(44,3%)	(20,6%)	(35,1%)	(33,9%)	(20,2%)	(29,2%)	(16,7%)

3.2.3.2 Explaining clusters by a canonical discriminant analysis

To facilitate the interpretation of the obtained clusters and provide some visual interpretation, we performed a canonical discriminant analysis. We used the CANDISC procedure of SAS/STAT software (SAS/STAT, 14.3). Canonical discriminant analysis (CDA) is a dimension-reduction technique similar in some way to principal component analysis. But if principal component analysis is used prior to cluster analysis, canonical discriminant analysis is implemented after the cluster analysis. The benefit of it is that CDA uses the information about the clusters in order to find canonical variables which are linear combinations of the original variables that ensure the best separation between clusters. Given our data, 2 canonical variables were extracted in the before crisis period and 3 canonical variables were extracted in the after crisis period. We provide the details of the analysis in the Appendix 2.

Figure 33 shows the relative position of clusters in the space of the two first canonical variables. The first canonical variable separates between the second and the third clusters. Whereas the second canonical variable separates between the first and the second clusters. This relative position in the canonical variables space determines the differences between mean surplus accounts of the 3 clusters in the before crisis period (Tables 26, 27, 28). We highlighted the observed variables correlated to the first canonical variable in blue and the observed variables correlated to the second canonical variable in red. The first cluster is positively correlated to the second canonical variable and is not correlated to the first canonical variable. This means that the first cluster of performant banks before the crisis is characterized by the strong positive creditors' price effect and strong negative suppliers' price effect on the mean profit rate gap. The second cluster is positively correlated to the first canonical variable and is negatively correlated to the second canonical variable. Thus, on the one hand, results demonstrate a positive size and borrowers' price effects and negative employees' and the government price effects, and, on the other hand, as opposed to the first cluster, a positive suppliers' price effect and negative creditors' price effect on the mean profit rate gap. Concerning the third cluster, it is negatively correlated to the first canonical variable and is not correlated to the second canonical variable. This leads to the opposite to the second cluster situation: positive employees' and the government price effects and negative size effect and borrowers' price effect on the mean profit rate gap. We notice as well that even if the technical efficiency effect and the allocative efficiency effect were not retained as main explanatory variables in cluster separation, the second and the third clusters have opposite positions of all productivity effects in the uses-resources table. Indeed, the second cluster is characterized by complete productive efficiency while the third cluster has all its productivity effects negative.

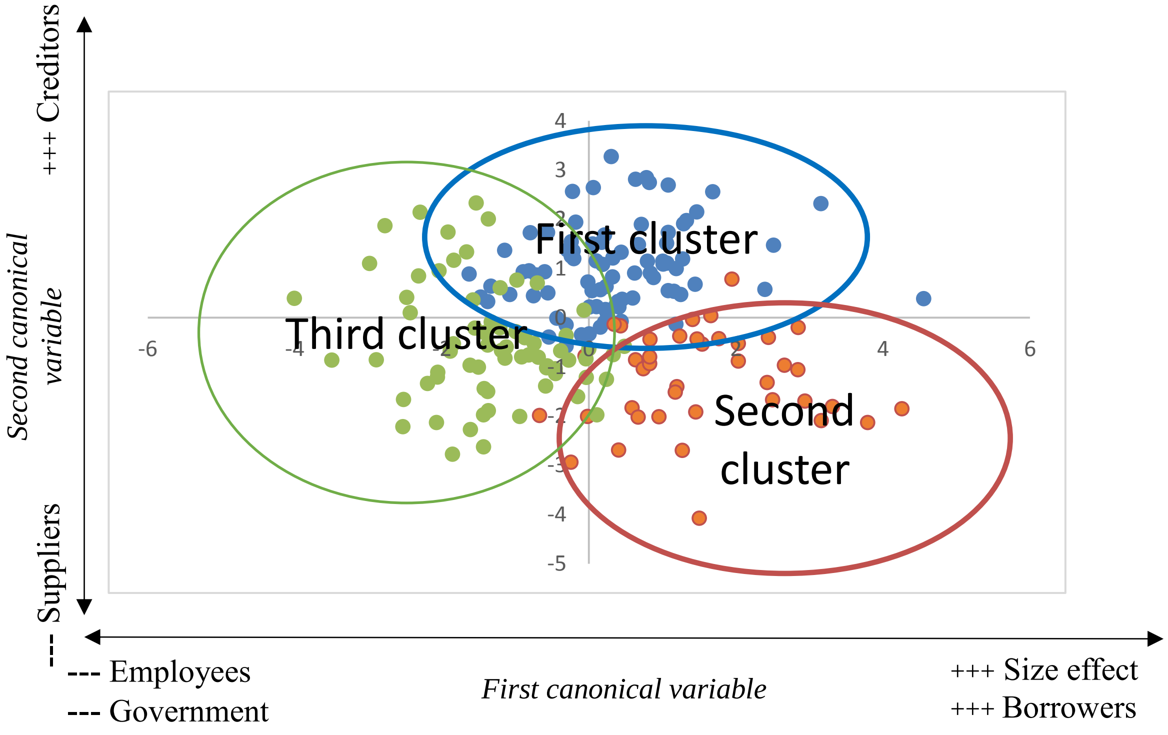

Figure 33 Representation of 3 clusters before the crisis at canonical variables space

Table 26 Surplus account for the first cluster before the crisis: mean values for profit rate gap and ten components

Uses			**Resources**
Alloc_eff	0,46%	Tech_eff	0,6%
Size_eff	0,0005%	Commision_Fee	1,08%
Borrowers	0,69%	Creditors	8,96%
Financial_mkt	0,39%	Employees	0,6%
Suppliers	4,86%		
Government	0,33%		
Profit_dif	**4,51%**		
Total	**11,24%**	**Total**	**11,24%**

Table 27 Surplus account for the second cluster before the crisis: mean values for profit rate gap and ten components

Uses			**Resources**
Commision_Fee	5,3%	Tech_eff	2,3%
Creditors	4,3%	Alloc_eff	2,4%
Employees	2,4%	Size_eff	1,7%
Government	0,88%	Borrowers	4,4%
Profit_dif	**4,1%**	Financial_mkt	0,91%
		Suppliers	5,2%
Total	**17%**	**Total**	**17%**

Table 28 Surplus account for the third cluster before the crisis: mean values for profit rate gap and ten components

Uses			**Resources**
Tech_eff	1,4%	Commision_Fee	3%
Alloc_eff	2,9%	Creditors	1,8%
Size_eff	4%	Employees	4,5%
Borrowers	2,9%	Suppliers	4,6%
Financial_mkt	0,02%	Government	1,3%
Profit_dif	**3,9%**		
Total	**15%**	**Total**	**15%**

We are now interested in the composition of clusters depending on the types of banking. Table 29 indicates that although commercial banks are in the majority in all clusters, savings banks are mostly presented in the second cluster. This is consistent with Table 22 which states the surplus account of performant savings banks in the before crisis period. Indeed, the two tables have an identical structure.

Table 29 Cluster composition by type of banking before the crisis

CLUSTER	Nb of banks	Type of banking
1	84	Commercial Banking
1	2	Savings Institution
2	28	Commercial Banking
2	12	Savings Institution
3	65	Commercial Banking
3	3	Savings Institution

Figures 34 and 35 give the visual representation of the four clusters after crisis in the space of the first two canonical variables and of the first and third canonical variables respectively. The first canonical variable separates between the second and the fourth clusters, the second canonical variable separates between the first and third clusters and the third canonical variable separates between the first and second cluster. This helps us to reveal the differences between the mean surplus accounts of the four clusters after crisis (Tables 30, 31, 32, 33). The observed variables that are linked to the first canonical variables are highlighted in green, the observed variables that are related to the second canonical variable are highlighted in red and the observed variables that linked to the third canonical variable are highlighted in blue. The first cluster is negatively correlated to the second and to the third canonical variables and not correlated to the first canonical variable. This means that the first cluster of performant banks in the after crisis period is distinguished from the third cluster by positive creditors' price effect and from the second cluster by negative allocative inefficiency effect and positive employees' price effect on the profit rate gap. The second cluster is positively correlated to the first and to the third canonical variables and not correlated to the second canonical variable. As a result, the second cluster is characterized by negative financial market participants' and employees' price effects and positive allocative inefficiency effect on the profit rate gap. The third cluster is only positively correlated to the second canonical variable and not correlated to the to the first and third canonical variables. Consequently, as opposed to the first cluster, the third cluster has negative creditors' price effect on the profit rate gap. Finally, the fourth cluster is only negatively correlated to the first canonical variable.

Therefore, the fourth cluster differs from the second cluster by the positive financial markets participants' price effect. We notice as well that the first and the second clusters differ by their productive efficiency: the first cluster has all its productivity effects in the uses part of the surplus account and the second cluster has all its productivity effects in the resources part of the surplus account. Another distinction is that the third cluster has a very high (16%) positive suppliers' price effect and the highest among the clusters profit rate gap (10%).

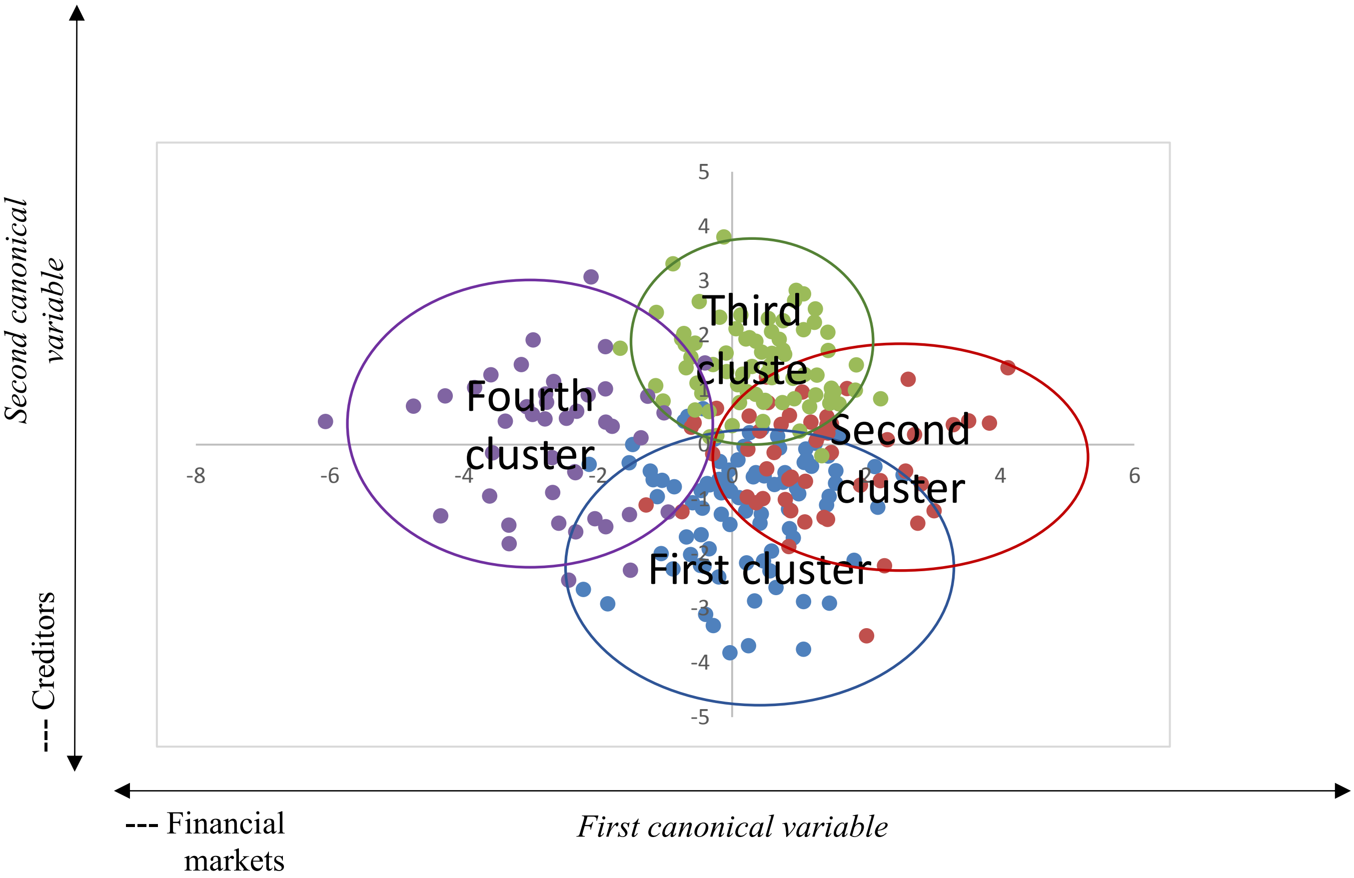

Figure 34 Representation of 4 clusters after the crisis at the first two canonical variables space

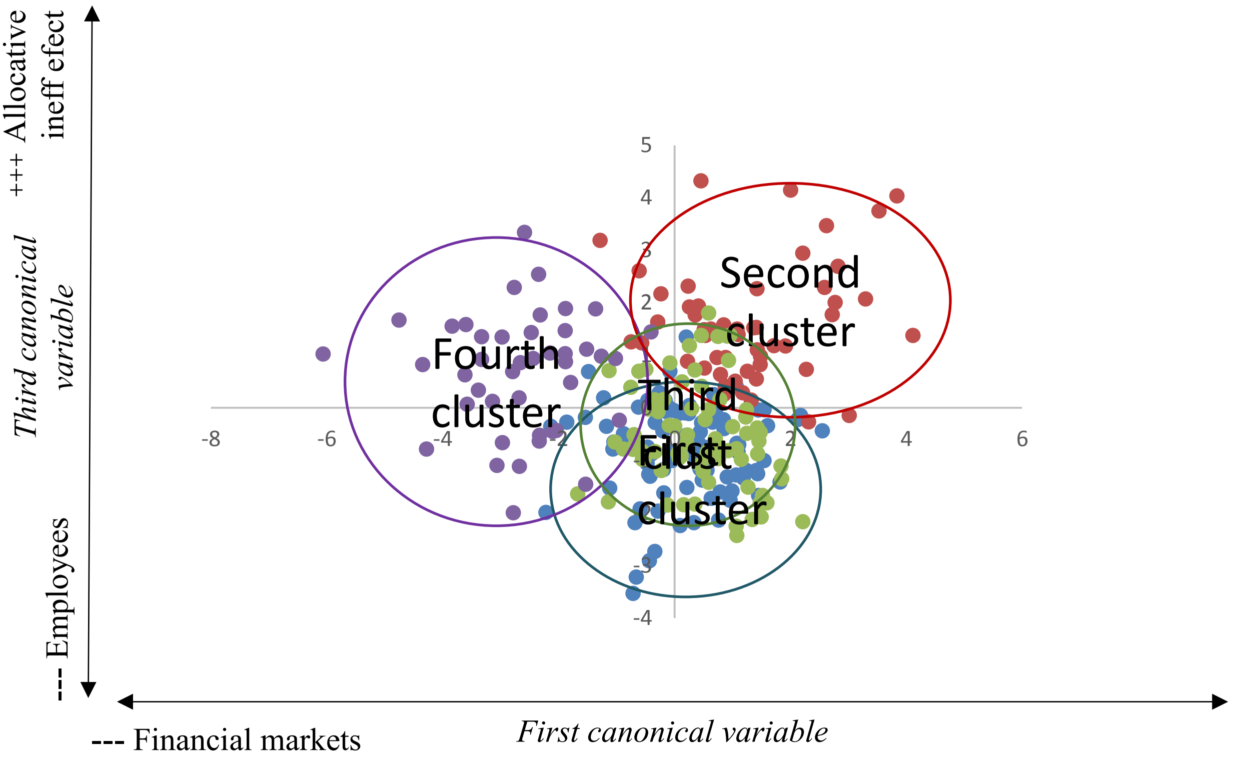

Figure 35 Representation of 4 clusters after the crisis at the first and third canonical variables space

Table 30 Surplus account for the first cluster after the crisis: mean values for profit rate gap and ten components

Uses			**Resources**
Tech_eff	0,82%	Commision_Fee	5,65%
Alloc_eff	6,42%	Creditors	7,8%
Size_eff	0,57%	Employees	6,56%
Borrowers	3%	Suppliers	1,81%
Financial_mkt	2,65%	Government	0,18%
Profit_dif	**8,55%**		
Total	**22%**	**Total**	**22%**

Table 31 Surplus account for the second cluster after the crisis: mean values for profit rate gap and ten components

Uses			**Resources**
Financial_mkt	3,64%	Tech_eff	3,29%
Commision_Fee	4,44%	Alloc_eff	3,89%
Employees	4,29%	Size_eff	5,62%
Suppliers	3,4%	Borrowers	8,08%
Government	0,72%	Creditors	3,55%
Profit_dif	**7,94%**		
Total	**24,43%**	**Total**	**24,43%**

Table 32 Surplus account for the third cluster after the crisis: mean values for profit rate gap and ten components

Uses			**Resources**
Alloc_eff	2,3%	Tech_eff	1,83%
Size_eff	3,86%	Borrowers	3,29%
Financial_mkt	0,69%	Employees	1,78%
Commision_Fee	2,6%	Suppliers	15,96%
Creditors	2,64%		
Government	0,68%		
Profit_dif	**10,08%**		
Total	**22,86%**	**Total**	**22,86%**

Table 33 Surplus account for the fourth cluster after the crisis: mean values for profit rate gap and ten components

Uses			**Resources**
Tech_eff	3,83%	Alloc_eff	2,71%
Size_eff	2%	Financial_mkt	7,15%
Borrowers	3,8%	Creditors	3,67%
Commision_Fee	3,35%	Employees	0,15%
Profit_dif	**7,83%**	Suppliers	5,73%
		Government	1,43%
Total	**20,85%**	**Total**	**20,85%**

As for the before crisis period, we want to know the composition of clusters depending on the types of banking activity. Table 34 shows that commercial banks are again presented in the majority in all clusters. As to savings banks, they are mostly presented in the third cluster. This is coherent with Table 24 which gives the surplus account of performant savings banks in the after crisis period. If we compare Table 24 and Table 32, we notice that two tables are almost the same: they differ only by the position of Financial market participants in the uses-resources table.

Table 34 Cluster composition by type of banking after the crisis

CLUSTER	Nb of banks	Type of banking
1	77	Commercial Banking
1	10	Savings Institution
2	43	Commercial Banking
2	9	Savings Institution
3	44	Commercial Banking
3	31	Savings Institution
4	37	Commercial Banking
4	6	Savings Institution

At last, if we compare all clusters before the crisis and after the crisis, we note that one group of banks kept its surplus account structure through the whole considering period. Indeed, the third cluster in the before crisis period has the same structure as the first cluster in the after crisis period although with different ‘weights’. Moreover, banks with these surplus accounts have all their productivity effects negative.

4 Conclusion

In this case study we applied the methodology developed in the fourth section of the first chapter to a sample of US banks over the period 2001-2012. We used the profit rate differential approach. We computed for each bank in a sample, its profit rate gaps relatively to all banks and ten components which may cause these rate gaps (technical inefficiency, allocative inefficiency and size effects). The analysis of banks with a positive mean profit rate gap (performant banks) revealed that globally performant banks are inefficient in terms of their productivity compared to other banks but take benefit from very advantageous price environment from 2001 to 2012. These price advantages come mainly from their creditors and employees over the entire period and from suppliers after the financial crisis. We note as well, that even though after the crisis quantity effect continues to impact negatively the mean profit rate gap of performant banks, a considerable improvement in their allocative efficiency is stated. An analysis of performant commercial and savings banks showed that performant commercial banks enjoy price advantages mostly from their creditors whereas performant savings banks are favored with price advantages from their suppliers over the whole period. Another result is that before the crisis performant savings banks have, on average, better productive efficiency compared to other banks as opposed to performant commercial banks but lose that distinction after the crisis. A cluster analysis allowed us to discern very contrasting structures of surplus accounts both before and after the crisis. In addition, we observe that the majority of performant savings banks was placed in one cluster before and after the crisis showing that they mostly have some common specific structure.

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Chapter 3

Measuring the effects of price environment: An application to U.S. industries

This part is based on a paper presented in an internal Economics seminar at University of Lille 1, march 2017 and at the 15th EWEPA (European Workshop on Efficiency and Productivity Analysis) Conference in London, June 2017. The paper is currently under revision at *Revue d'Economie Politique* (CNRS 2).

1 Introduction

In traditional data envelopment analysis (DEA) literature, efficiency is analyzed through benchmarking the best observed practices in terms of technical, scale, or productive efficiency, all of which are based on physical quantities. Farrell (1957) introduced the concepts of price and overall efficiencies to define producers' ability to optimally allocate their resources given their own respective prices. While the underlying behavioral assumption in this seminal work was cost minimization, Färe et al. (1994) extended this approach to revenue and/or profit maximizations. Allocative efficiency is usually computed indirectly as a residue of the overall and technical efficiencies[4].

In all of these approaches, the main objective remains identifying the best allocation of output and/or input in terms of physical quantities. Allocative efficiency is defined as a reallocation of physical resources. Therefore, this measure is relevant for medium to long-run analysis. Moreover, prices are indeed used, but genuine price effect measured through comparisons among producers' prices is not achieved. Profit maximization uses a firm's own price system to determine the optimal production plan, but the decision variables in the optimization program remain output/input quantities only.

Tone (2002) made a step further in this direction by defining a new form of allocative efficiency based on a "value" technology (see also the response from Färe and Grosskof, 2006 who propose to maintain the quantity technology frame but instead of Farrell measures, propose a directional

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[4] Departing from this line of research, Bogetoft et al. (2006) proposed a direct measure for allocative efficiency without supposing technical efficiency. This method can be justified when reallocating resources is more convenient than improving technical performances.

distance function approach). This work was pursued by Sahoo et al. 2014. For example, in Sahoo et al. (2014) the assumption that DMUs are price takers is dropped and a “value”-based directional distance function is proposed on the basis of a technology set that includes all feasible (input) costs and (output) revenues. Portela and Thanassoulis (2014) analyzed cost efficiency by considering both prices and input quantities to be decision variables. An input-price cost saving component was then integrated into the decomposition of this cost efficiency measure using the Bennet-type indicator. Although these measures clearly improve standard allocative efficiency, they still assume that resource reallocation at the firm level.

At a more practical level, “value” based technology is often used when quantity-based data are not available. It is well known that when the decision-making units (DMUs) face the same prices, these two sets of scores (quantity-based and value-based) are identical (Färe and Grosskopf, 1985). However, when the prices faced by the DMUs are different, these scores differ as well (Färe *et al*. 1990, Banker *et al*. 2007, Portela 2014). In the latter case, Cross and Färe (2008) showed that the input-oriented radial value-based efficiency score can be multiplicatively decomposed as a purely technical efficiency score, a technology effect and a firm effect.

In this article, we assume that firms seek the optimal price and quantity matching for a given production structure. Though more conservative than the allocative efficiency measure, our price effect concept has a practical application for decision makers in the short run. We consider that firms make their decisions by considering price and quantity matching in terms of values. Conducting a DEA analysis with quantity and value variables allows us to take this price-quantity matching into account. Then, by comparing the latter with traditional quantity-based measures, a price effect can be derived and interpreted as a favorable or unfavorable price environment for firms compared to their peers.

Our methodology is based on calculating DEA output efficiency scores using both quantity- and value-based data. This makes our analysis relevant for market contexts in which firms are not perfect price takers. In our analysis, we consider the ratio between i) technical efficiency scores calculated with quantity data and ii) value efficiency scores calculated with value data. We show that this indicator has a meaningful economic interpretation in terms of the price environment for evaluated DMUs. Thus, if this ratio is higher than the unit, the evaluated DMU benefits from a positive price environment as its distance to the benchmark is lower under the “value technology” than the distance estimated with the initial “quantity technology.” Conversely, when the ratio of the two scores is lower than the unit, we infer that the evaluated DMU has been subject to a disadvantageous price environment. By considering all output and input in both value and quantity, we obtain a total price effect. By focusing on some specific output or input, this methodology can also be extended to compute output- and input-specific price effects.

We apply this methodology to a dataset containing the 63 industries that comprised the U.S. economy from 1987 to 2014. The production technology is defined by one output (gross output) and three inputs: intermediate inputs, labor, and capital (equipment, structures, and intellectual property products). For each industry, all of these components are expressed in either current value terms or volume terms. On this basis, we can compute the price effect for each sector following the methodology presented above. The obtained price effects serve as a basis for a cluster analysis aimed at identifying specific reactions in the U.S. industries. Moreover, we used the output-specific and input-specific price effects in a panel model to explain the total price effect at the industry level.

The rest of this article is organized as follows. Section 2 presents the methodological issues related to the price effect. Section 3 presents the data used for our empirical work and the obtained results. Finally, Section 4 gives our main conclusions.

2 Methodology

Formally, let $\mathbf{x}^q \in R_{I,+}$ denote the vector of input quantities and $\mathbf{y}^q \in R_{O,+}$ denote the vector of output quantities for an observed DMU. All DMUs are assumed to face the same technology represented by the production set T^q[5]:

$$T^q = \left\{(\mathbf{x}^q, \mathbf{y}^q) : \mathbf{x}^q \text{ can produce } \mathbf{y}^q\right\} \tag{1}$$

Gaps between observed production plans and the technology's boundaries are measured using the following Shephard output distance function:

$$D_o^{T^q}(\mathbf{x}^q, \mathbf{y}^q) = \inf_{\theta^q}\left\{ \theta^q : (\mathbf{x}^q, \frac{\mathbf{y}^q}{\theta^q}) \in T^q \right\} \tag{2}$$

Note that this output distance function is reciprocal to Farrell's (1957) output-oriented measure of technical inefficiency and $D_o^{T^q}(\mathbf{x}^q, \mathbf{y}^q) \leq 1 \Leftrightarrow (\mathbf{x}^q, \mathbf{y}^q) \in T^q$.

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[5] In x^q, y^q, T^q, q stands for quantity.

Based on the nonparametric literature on activity analysis, an operational definition of T^q in (1) is specified given a set of observed DMUs and a list of axioms. Thus, the two main assumptions structuring T^q for estimation purposes are convexity and free disposability of inputs and outputs. Under constant returns to scale (CRS), $T^{q,CRS}$ is defined as:

$$\begin{aligned} T^{q,CRS} = \Big\{ & (\mathbf{x}^q, \mathbf{y}^q) : \mathbf{x}^q \in R^I_+, \mathbf{y}^q \in R^O_+, \sum_{n=1}^{N} \mu_n y^q_{o,n} \geq y^q_o, o = 1, \ldots, O, \\ & \sum_{n=1}^{N} \mu_n x^q_{i,n} \leq x^q_i, i = 1, \ldots, I, \mu_n \geq 0, n = 1, \ldots, N \Big\} \end{aligned} \tag{3}$$

Thus, for any evaluated DMU "*a*," its technical efficiency score $\beta^q_a = \frac{1}{\theta^q_a}$ is defined by the distance function $D^{T^{q,CRS}}(\mathbf{x}^q_a, \mathbf{y}^q_a)$ and is estimated by the following linear program (LP1):

$$\begin{aligned} & \frac{1}{D^{T^{q,CRS}}(\mathbf{x}^q_a, \mathbf{y}^q_a)} = \max_{\boldsymbol{\mu}, \beta^q_a} \ \beta^q_a \\ & s.t. \ \sum_{n=1}^{N} \mu_n y^q_{o,n} \geq \beta^q_a y^q_{o,a} \quad \forall o = 1, \cdots, O \\ & \quad \sum_{n=1}^{N} \mu_n x^q_{i,n} \leq x^q_{i,a} \quad \forall i = 1, \cdots, I \\ & \quad \mu_n \geq 0 \ \ \forall n = 1, \ldots, N \end{aligned} \tag{LP1}$$

Traditionally, emphasis of optimal allocation of resources given each producer's respective system of price was realized through the measure of allocative efficiency. This measure was obtained as a residue between revenue (or cost) maximization (minimization) and the technical efficiency obtained with physical quantity data. However, allocative efficiency measure is not a proper measure for gauging the price environment of a DMU. First, when computing allocative efficiency measure, decision variables are always physical quantities (outputs produced and/or inputs used). In this sense, work of Färe *et al.* (1990), Banker *et al.* (2007), Cross and Färe (2008) and Portela (2014) have pointed out that when data does not exist in physical quantities and value-data have to be used instead, cost minimization (or revenue maximization) measures differ from the standard measures that would have been obtained with physical quantity data. Second, within this frame of research, an evaluated DMU's price system is never compared to the prices observed for the rest of the DMUs. Third, since allocative efficiency measure is based on the system of relative prices of the evaluated DMU, an equiproportionate change in the evaluated DMU's price system would not modify the value of the allocative efficiency for that DMU (Tone, 2002, Sahoo et al., 2014). Finally, while allocative efficiency measure has been improved in Tone (2002) by considering technical and cost efficiency under a "value" technology, this new measure is relevant for medium to long run analysis when reallocation of inputs and/or outputs at the firm level is possible.

Our proposed measure for evaluating DMUs' price environments does not assume that DMUs reallocate the input/output mix. This assumption makes sense in the short-run when we can expect that DMUs cannot modify immediately their mix. Thus, for a given structure of outputs produced and inputs used, we are measuring whether the DMU's prices, compared to the prices of their peers, contribute to a favorable environment or not. In our methodology, output values (hence prices) are used as decision variables and an inter-DMU comparison of prices is realized. Finally, an equiproportionate change in the prices of the evaluated DMU will modify the value of the price effect calculated.

We introduce in what follows the "value" technology faced by DMUs. Let $\mathbf{x}^v \in R_+^I$ and $\mathbf{y}^v \in R_+^O$ denote the vector of input and output quantities multiplied by their respective prices for an observed DMU.

$$\mathbf{x}^v = \begin{pmatrix} x_1^v \\ x_2^v \\ \dots \\ x_I^v \end{pmatrix} = \begin{pmatrix} w_1 x_1^q \\ w_2 x_2^q \\ \dots \\ w_I x_I^q \end{pmatrix} \text{ and } \mathbf{y}^v = \begin{pmatrix} y_1^v \\ y_2^v \\ \dots \\ y_O^v \end{pmatrix} = \begin{pmatrix} p_1 y_1^q \\ p_2 y_2^q \\ \dots \\ p_O y_O^q \end{pmatrix}$$

All DMUs are assumed to face the same "value technology," as represented by the production set T^v:

$$T^v = \left\{ (\mathbf{x}^v, \mathbf{y}^v) : \mathbf{x}^v \text{ can produce } \mathbf{y}^v \right\} \qquad (1)$$

Similarly, distances between observed production plans and the technology's boundaries are measured through the following output distance function:

$$D_o^{T^v}(\mathbf{x}^v, \mathbf{y}^v) = \inf_{\theta^v} \left\{ \theta^v : (\mathbf{x}^v, \frac{\mathbf{y}^v}{\theta^v}) \in T^v \right\} \qquad (2)$$

Again, $D_o^{T^v}(\mathbf{x}^v, \mathbf{y}^v) \le 1 \Leftrightarrow (\mathbf{x}^v, \mathbf{y}^v) \in T^v$, and under CRS, the production possibility set is defined as:

$$\begin{aligned} T^{v,CRS} = \Big\{ & (\mathbf{x}^v, \mathbf{y}^v) : \mathbf{x}^v \in R_+^I, \mathbf{y}^v \in R_+^O, \sum_{n=1}^{N} \lambda_n y_{o,n}^v \ge y_o^v, o = 1, \dots, O, \\ & \sum_{n=1}^{N} \lambda_n x_{i,n}^v \le x_i^v, i = 1, \dots, I, \lambda_n \ge 0, n = 1, \dots, N \Big\} \end{aligned} \qquad (3)$$

Thus, for any evaluated DMU "*a*," its value efficiency score $\beta_a^v = \frac{1}{\theta_a^v}$ is defined by the distance function in the value space $D^{T^{v,CRS}}(\mathbf{x}_a^v, \mathbf{y}_a^v)$ and is estimated by the following linear program (LP2):

$$\begin{aligned}
&\frac{1}{D^{T^{v,CRS}}(\mathbf{x}_a^v,\mathbf{y}_a^v)} = \max_{\boldsymbol{\lambda},\beta_a^v} \beta_a^v \\
&s.t.\ \sum_{n=1}^{N} \lambda_n y_{o,n}^v \geq \beta_a^v y_{o,a}^v \quad \forall o = 1,\cdots,O \\
&\sum_{n=1}^{N} \lambda_n x_{i,n}^v \leq x_{i,a}^v \quad \forall i = 1,\cdots,I \\
&\lambda_n \geq 0 \quad \forall n = 1,\ldots,N
\end{aligned} \qquad \text{(LP2)}$$

In order to link quantity and value technologies, LP2 can be rewritten by decomposing values into their quantity and price components. Algebraically, LP2 is equivalent to LP3:

$$\begin{aligned}
&\frac{1}{D^{T^{v,CRS}}(\mathbf{x}_a^v,\mathbf{y}_a^v)} = \max_{\boldsymbol{\lambda},\beta_a^v} \ \beta_a^v \\
&s.t.\ \sum_{n=1}^{N} \lambda_n \left(\frac{p_{o,n}}{p_{o,a}}\right) y_{o,n}^q \geq \beta_a^v y_{o,a}^q \quad \forall o = 1,\cdots,O \\
&\sum_{n=1}^{N} \lambda_n \left(\frac{w_{i,n}}{w_{i,a}}\right) x_{i,n}^q \leq x_{i,a}^q \quad \forall i = 1,\cdots,I \\
&\lambda_n \geq 0 \quad \forall n = 1,\ldots,N
\end{aligned} \qquad \text{(LP3)}$$

First, we note that if all DMUs face the same price system, each price ratio in LP3 is equal to the unit and LP3 collapses in LP1, showing the known result that, in this particular case, the quantity-based and value-based efficiency scores are identical.

Compared to LP1, LP3 estimates a technical efficiency score in the quantity space but modifies the technology $T^{q,CRS}$ by replacing initial input and output levels by quantity terms weighted with their corresponding prices relative to those of the evaluated DMU "*a*."

The ratio between the two scores $\frac{\beta_a^q}{\beta_a^v}$ has a meaningful interpretation in terms of relative price environment for the evaluated DMU. If $\frac{\beta_a^q}{\beta_a^v} > 1$, DMU "*a*" benefits from a positive price environment as its distance to the benchmark is lower under the "value technology" than its correspondent distance estimated with the initial "quantity technology." On the contrary, for $\frac{\beta_a^q}{\beta_a^v} < 1$, DMU *a* suffers from a disadvantageous price environment as its distance to the benchmark under the "quantity" technology is lower than its distance to the "value" technology. When $\frac{\beta_a^q}{\beta_a^v} = 1$, no relative price environment can be detected.

By considering the different price components one by one, specific value technology can be defined relative to the chosen value output or input variable. For example, let us define the specific value technology for output *l*:

$$T_l^{v,CRS} = \left\{ (\mathbf{x}^q, \mathbf{y}^q, y_l^v) : \mathbf{x}^q \in R_+^I, \mathbf{y}^q \in R_+^{O\setminus\{l\}}, y_l^v \in R_+, \sum_{n=1}^{N} \lambda_n y_{o,n}^q \geq y_o^q, o \in \{1,...,O\} \setminus \{l\}, \sum_{n=1}^{N} \lambda_n y_{l,n}^v \geq y_l^v \right.$$
$$\left. \sum_{n=1}^{N} \lambda_n x_{i,n}^q \leq x_i^q, i = 1,...,I, \lambda_n \geq 0, n = 1,...,N \right\}$$

Therefore, for any evaluated DMU *a* its specific output (*l*) value efficiency score is defined by the distance function in the value space relative to output *l* and is estimated by the following linear program (LP4):

$$\begin{aligned}
&\frac{1}{D^{T_l^{v,CRS}}(\mathbf{x}_a^q, \mathbf{y}_{-l,a}^q, y_{l,a}^v)} = \max_{\boldsymbol{\lambda}, \beta_{l,a}^v} \beta_{l,a}^v \\
&s.t. \ \sum_{n=1}^{N} \lambda_n y_{l,n}^v \geq \beta_{l,a}^v y_{l,a}^v \quad \text{for output } l \\
&\sum_{n=1}^{N} \lambda_n y_{o,n}^q \geq \beta_{l,a}^v y_{o,a}^q \quad \forall o \in \{1,...,O\} \setminus \{l\} \qquad \text{(LP4)} \\
&\sum_{n=1}^{N} \lambda_n x_{i,n}^q \leq x_{i,a}^q \quad \forall i = 1,\cdots,I \\
&\lambda_n \geq 0 \quad \forall n = 1,...,N
\end{aligned}$$

The ratio between the two scores $\frac{\beta_a^q}{\beta_{l,a}^v}$ measures the output (*l*) specific price effect for the evaluated DMU. If $\frac{\beta_a^q}{\beta_{l,a}^v} > 1$, then DMU *a* benefits from a favorable price environment for its output *l*, as its distance to the benchmark is lower under the "specific value technology" $T_l^{v,CRS}$ than its corresponding distance estimated with the initial "quantity technology." In the same way, we can obtain a specific price effect for any input *j* by replacing the quantity variable x_j^q with its corresponding value component x_j^v.

An alternative approach could be to consider a non-radial efficiency measure when considering an input *j* or output *l* and to compute specific quantity and value scores for this dimension.

3 Empirical analysis: data and results

3.1 Data

This analysis focuses on the evolution of price-environment effects concerning 63 industries of the U.S. economy. Data was collected from the Bureau of Economic Analysis (BEA) website (http://www.bea.gov/). For each industry, we dispose of current values (expressed in current U.S. dollars) and quantity indexes (base year 100=2009) of their gross output net of taxes on production-less subsidies, intermediate inputs, labor (compensation of employees), and consumption of fixed capital (equipment, structures, and intellectual property products). Volume of taxes and subsidies on production are directly linked to their related quantity output indexes. Labor quantity is estimated in a full-time equivalent employee. Volume of capital consumption (sum of equipment, structure, and intellectual property products) is calculated by the cost depreciation at a constant price. Thus, for each sector, we can compute both the value and volume for each variable stated.

3.2 Results

The methodology presented in the previous section was applied in order to determine the (total) price environment for the U.S. economy. After analyzing this, we present a cluster analysis regarding the different evolutions of the U.S. sectors. Finally, we present a panel model for decomposing the total price effect into an output and three input-specific price effects.

3.2.1 Total price environment for the U.S. economy

In order to determine the price environment for the U.S. economy, we computed technical efficiency scores using volume-based data and value efficiency score using value-based data. Their respective evolutions for the U.S. economy[6] are represented in Figure 36. Given that the base year for calculating the volume indexes was 2009, we expect the two measures of technical efficiency to be equal to one another for this year. Thus, our interpretations make sense relatively to the year 2009.

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[6] Technical and value inefficiency scores for the U.S. economy have been computed for the aggregate production plan (as the sum of the 63 industries).

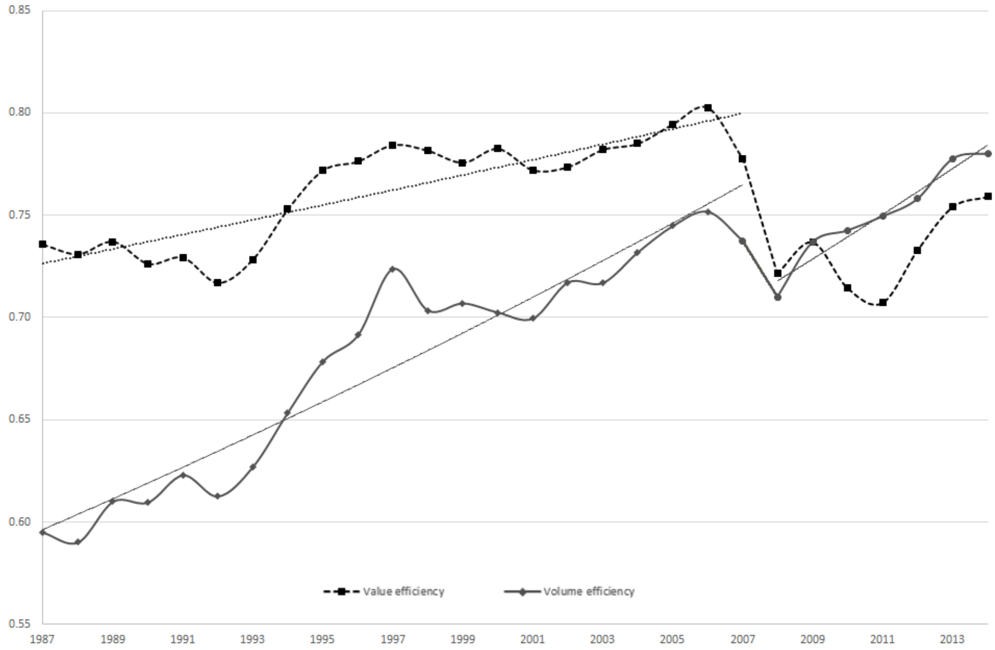

Figure 36 Evolution (and trend) of the U.S. economy's value and volume efficiency

The evolutions of the two efficiency scores differ significantly. While "value" efficiency followed a relatively flat path (time trend for the whole series is not significant), "volume" efficiency, in contrast, followed an increasing path (time trend is 0,98% and is sig. at <5%). Moreover, we notice a significant change in the trend in 2007. Thus, for value efficiency, the trend is 0,48% for 1987–2007 (sig. at <5%) and it becomes non-significant afterwards. Volume efficiency followed a positive and significant trend of 1,25% (sig. at <5%) before 2007, which increased to 1,48% afterwards. Thus, the financial crisis in 2007 seems to have only punctually affected the evolution of volume efficiency and its long-term growth. These evolutions point toward a probable deterioration of the price environment all along the period.

U.S. economy's price environment is presented in Figure 37. As suggested by the interpretation of volume and value efficiency in Figure 36, the trend of the price effect for 1987 –2014 is clearly decreasing. Over the period of study, the U.S. economy has been subject to increased pressure for competitiveness, especially through exports and imports (captured in our model by output and input prices, respectively). The deterioration observed in the price environment could be associated to the increasing degree of openness for the U.S. economy. This conjecture seems to be verified in Figure 37 where we clearly see a negative correlation between price advantages and the economy's openness, defined as the ratio between the sum of exports and imports to the GDP. This indicator has recorded a rise during the early 2000s exposing the U.S. economy to increased competitiveness. These evolutions seem to have had two effects: an increase in technical efficiency as measured by volume data (Figure 36) and a decrease in the price environment of the U.S. economy (Figure 37).

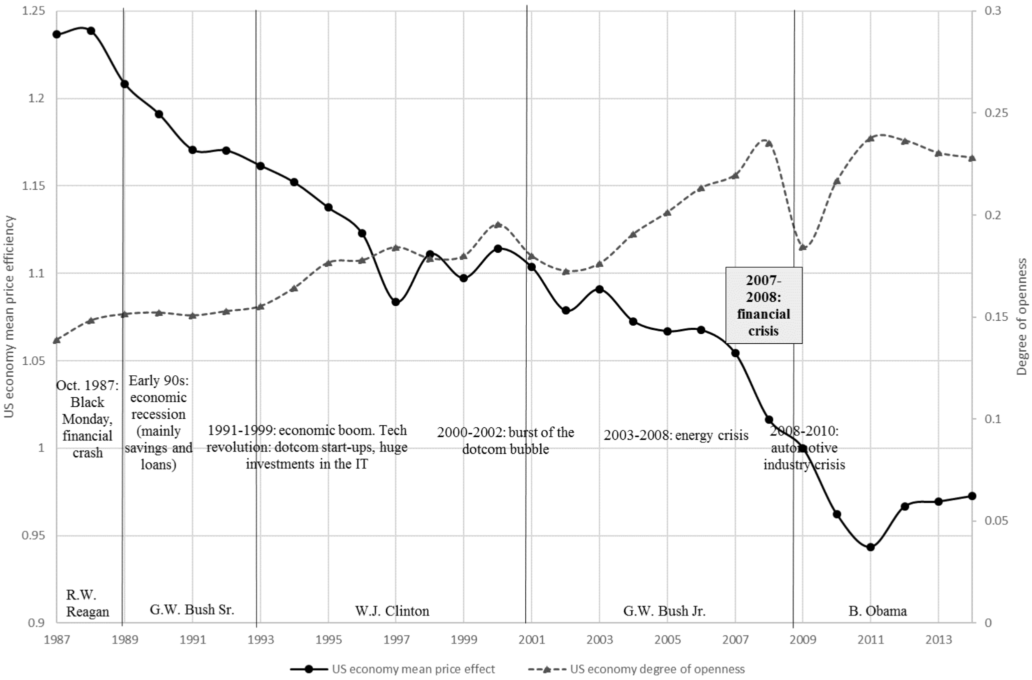

Figure 37 The U.S. economy’s total price effect and degree of openness

The U.S. economy’s price environment is computed as the ratio between the volume efficiency and the value efficiency for the aggregate (sum geometric mean of sum of the 63 industries composing the U.S. economy) production plan. A value of the price environment above 1 is interpreted in this context as a sign that the U.S. economy has enjoyed a positive price-environment relative to 2009· This indicator has followed a declining trend (-0,6% per year compared to 2009, sig. at < 5%). However, an accelerated price effect deterioration for the U.S. economy occurred following the recession in the early 2000s. Indeed, the time trend was -0,35% for 1987–2001 and deteriorates to -0,98% afterwards (sig. at <5% in both cases). Compared to the 2001 recession, the subprime crisis seems to have had a less serious influence on this indicator. While price environment has severely deteriorated up to 2011 (and relative to 2009), for the last three years in our analysis, the mean level seems to have recovered.

3.2.2 A cluster analysis of the industries' total price environments

Based on the time trend for the price effect, sectors can be divided into three groups: the ones for price environment was favorable (the trend is positive and significant), the ones for which the price environment was unfavorable (the trend is negative and significant), and finally, the ones for which price environment was neutral (non-significant time trend).[7] The first category has recorded a period mean trend of 0,76% and represents on average 13,11% of the gross output. The second category has recorded a period mean trend of -1,18% and represents altogether 45,22% of the gross output of the U.S. economy. The last category comprises 41,27% of the total output of the U.S. economy.

Table 41 Typology of the U.S. industries according to their time trend for their price environments

	Positive and significant trend for price environment	Non- significant trend for price environment	Negative and significant trend for price environment
Number of industries	6	23	34
Mean trend (%)	0,76	n.s.	-1,18
Sum of mean industry gross output in total gross output	13,11%	41,27%	45,22%

However, the common trend can conceal some contrasting evolutions across industries. Therefore, in what follows, we propose a time series similarity analysis (Barry and Linoff, 1997; Leonard et al., 2008) using absolute deviation as a distance measure. This analysis allowed us to identify four main clusters representing 47 sectors.[8] For each cluster, a series of indicators related to their structure were calculated: the three input cost shares relative to the gross output[9] and a capital deepening ratio (capital depreciation divided by employee compensation). These ratios characterize heterogeneous production processes and contrast industries from the point of view of their input intensity.

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[7] For three of the industries in the analysis, the efficiency scores are equal to 1 in every year. This is because these three sectors, *Funds, trusts, and other financial vehicles*, *Real estate* and *Legal services* are not comparable to the others.

[8] The remaining 16 sectors are too atypical and cannot be integrated into a homogeneous cluster.

[9] All variables are taken in current values.

To further investigate the differences between clusters, we performed four one-way ANOVAs for repeated measures. The purpose of these analyses is to verify whether or not there is a link between the variable cluster consisting of four categories and each of the four response variables mentioned above. If a classic ANOVA tests for the difference among group means at one moment in time, a repeated measures ANOVA considers the evolution of group means over time. Thus, this analysis tests for both overall differences in group mean repeated measures for a specific time period (between group analysis) and for differences in the evolution of group mean repeated measures over time (within group analysis).

The analysis between groups (Table 42) indicates whether there is a global significant difference between cluster mean repeated measures for each response variable. We notice at least one significant difference among the cluster mean repeated measures for the four variables selected (the F-tests performed between the clusters on each of the four indicators are all significant at <5%).

Table 42 Tests of hypotheses for between-subject effects

	Cluster 1	**Cluster 2**	**Cluster 3**	**Cluster 4**	**F-test (p-value)**
Mean of intermediate inputs to gross output (in %)	0,499	0,382	0,413	0,365	**3,28** *(0,029)*
Mean of employee compensation to gross output (in %)	0,324	0,478	0,409	0,403	**3,19** *(0,033)*
Mean of capital depreciation to gross output (in %)	0,066	0,045	0,09	0,211	**7,39** *(0,0004)*
Mean of capital deepening (in %)	0,239	0,094	0,301	0,666	**3,45** *(0,0243)*

The within-group analysis concerns how the repeated measures evolve over time. It provides two types of tests. First, it indicates whether the same pattern of increase or decrease can be observed for all clusters. According to Table 43, there is a significant pattern of decrease for all clusters regarding employee compensation to gross output (the F-test is sig. for this variable).

Second, this analysis indicates whether there is a significant interaction between the evolutions of repeated measures over time and cluster membership. More specifically, this analysis shows whether repeated measures for one cluster increase/decrease faster than for another cluster. For our dataset, this has been the case for at least one cluster concerning the evolution of capital depreciation to gross output (the F-test is sig. for this variable). Indeed, Figure 40 shows that for cluster 4, this variable has been declining at a more rapid pace than for the other clusters.

Table 43 Univariate hypothesis tests for within-subject effect

	Intermediate inputs to gross output	**Employee compensation to gross output**	**Capital depreciation to gross output**	**Capital deepening**
	F-test *(adj. Pr>F, H-F-L)*[*]	F-test *(adj. Pr>F, H-F-L)*	F-test *(adj. Pr>F, H-F-L)*	F-test *(adj. Pr>F, H-F-L)*
Year	1,72 *(0,1684)*	**7,69** ***(0,0005)***	1,22 *(0,2966)*	2,25 *(0,1349)*
Year* cluster	0,61 *(0,5157)*	1,2 *(0,3082)*	**3,07** ***(0,0125)***	1,16 *(0,338)*

[*]: H-F-L stands for the Huynh-Feldt-Lecoutre adjustment

Evolutions of the four indicators for each cluster are plotted in Figures 38–41 below.

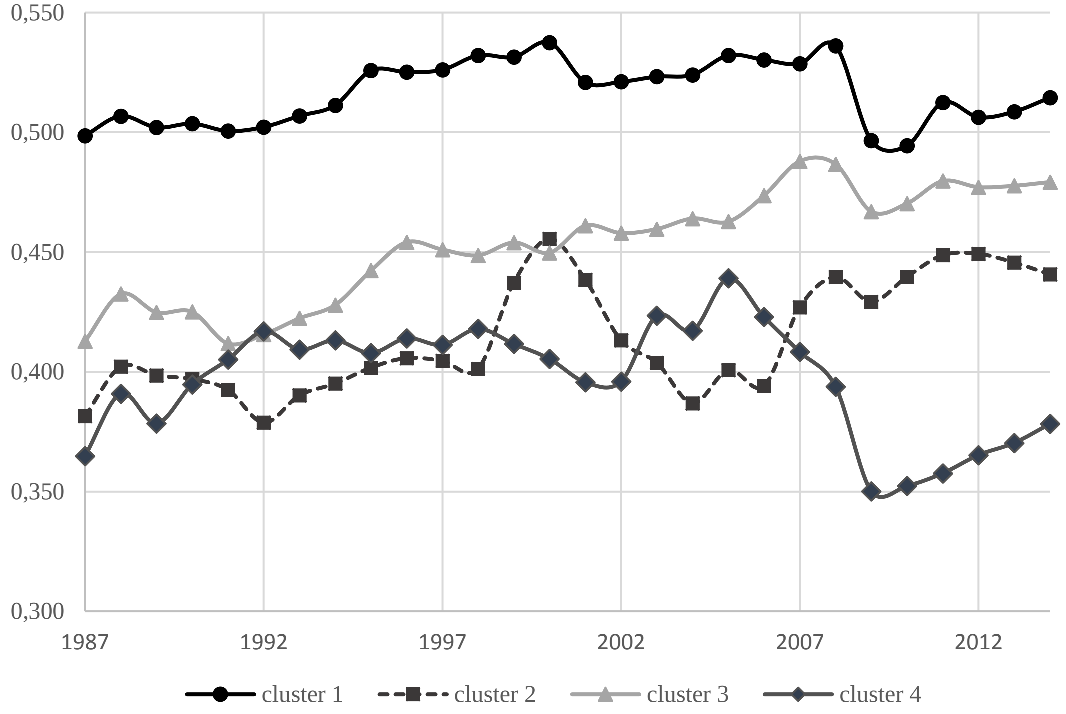

Figure 38 Evolutions of the cluster average for intermediate inputs share

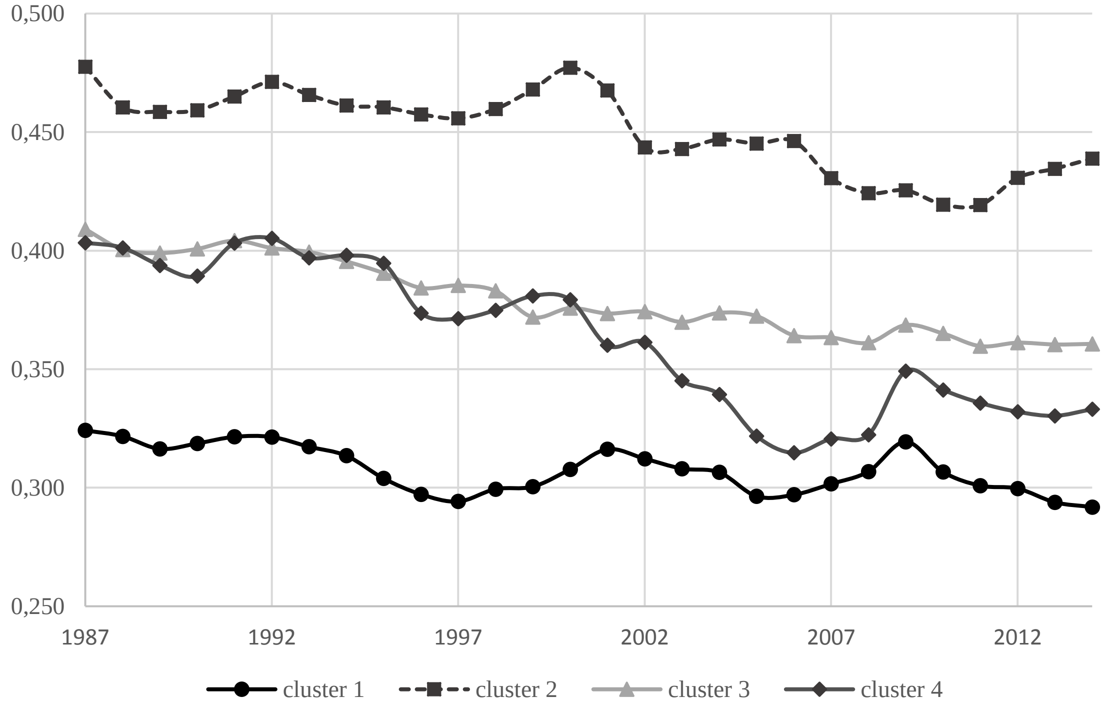

Figure 39 Evolutions of the cluster average for labor cost share

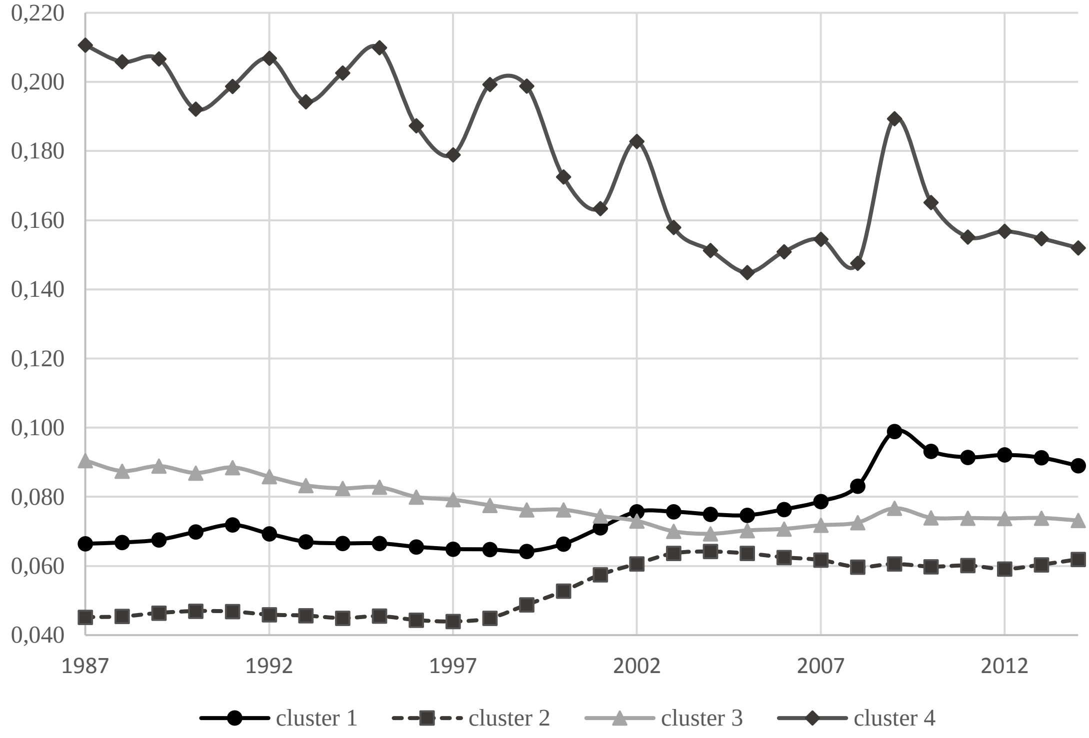

Figure 40 Evolutions of the cluster average for capital depreciation share

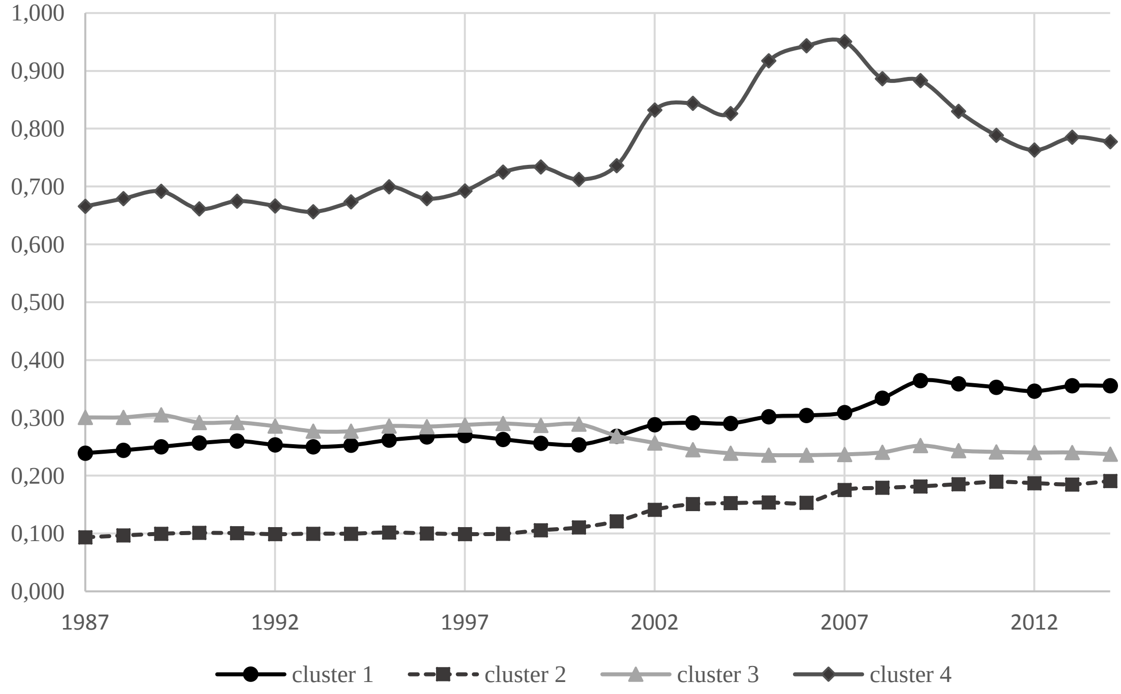

Figure 41 Evolutions of the cluster average for capital deepening

Cluster 1 is the most intensive in terms of intermediate inputs. At the same time, this cluster's mean for employee compensation to gross output is the lowest. Finally, we notice an increasing ratio of capital depreciation to gross output in the last period of the 2000s. Cluster 2 seems to have the highest average for the employee compensation ratio, although the recession of the early 2000s has negatively influenced this ratio. We notice that, at the same time, the capital depreciation to gross output ratio for this cluster is the lowest one. Not surprisingly, the cluster's mean for capital deepening is also the lowest. However, starting with 2002, the mean for capital deepening is twice the mean obtained at the beginning of the period. Although cluster 3 does not seem to dominate the others for any of the variables studied, the intermediate inputs in the gross output has increased over time (its mean is also the second highest). Moreover, the ratio of employee compensation to gross output significantly increases after the early 2000s' recession. The mean of capital deepening is highest for cluster 4. At the same time, this cluster also has the highest mean for its capital depreciation to gross output ratio, along with a relatively high mean for employee compensation to gross output.

Figures 42–45 graphically represent the evolution of the standardized natural logarithm of the price effect for each of the four clusters. This will allow us to identify the specific evolution for each cluster and to identify the main events shaping the price effect. Moreover, one or two representative sectors have been identified in each cluster.

Cluster 1 comprises the largest part of the total mean weighted gross output (37,17%), and it largely corresponds to the group of sectors that have obtained a negative and significant trend for their price effect throughout the entire period. Indeed, with one exception,[10] all industries belonging to this cluster observed a significant declining trend for their price effect between 1987 and 2007. The subprime crisis in 2008 had a mitigated effect on the sectors. For most of them, the trend is not significant after 2008. However, for six others, the trend is significantly negative.[11] Recall that this was the most intensive cluster in intermediate inputs. The increasing competition of developing countries may have contributed to the constantly deteriorating price environment for this cluster. Moreover, as its industries become more dependent on capital over the last period of the 2000s, the subprime effect may have contributed to further deteriorating price effects. The price effects for "*computer and electronic products*" has been monotonically decreasing

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[10] The exception concerns the sector *Accommodation* for which the time trend of the total price effect is not significant.

[11] The excepted sectors (*Computer and electronic products, Apparel and leather and allied products, Printing and related support activities, Publishing industries (includes software), Miscellaneous professional, scientific, and technical services and Administrative and support services*) seem to have been negatively affected by the crisis as their price effect has registered a negative trend even after 2008.

throughout this entire period. For “*motor vehicles, bodies and trailers, and parts*,” the trend is also negative, but less smooth.

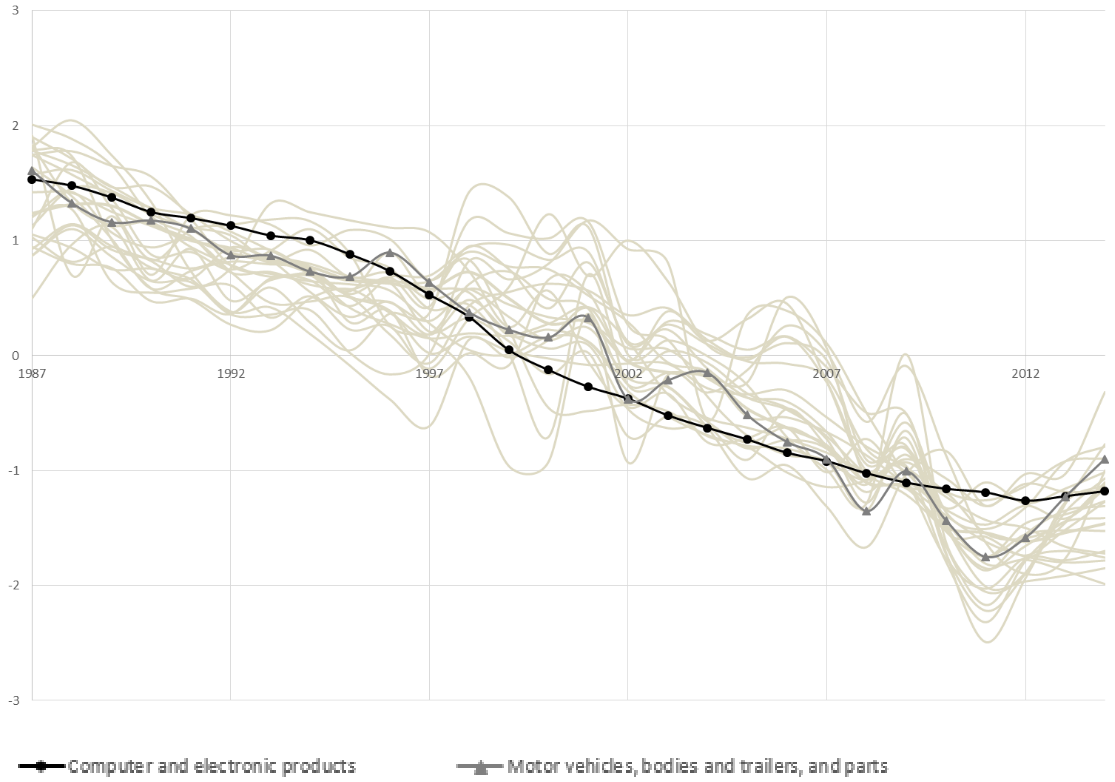

Figure 42 Standardized Ln of the global price-environment effect for industries in cluster 1 (focus on “computer and electronic products” and “motor vehicles, bodies and trailers, and parts”)

Cluster 2 is comprised of industries that altogether represent 11,64% of the sum of mean weighted gross output. All sectors in this cluster, which is the most labor-intensive one, seem to have been affected by the early 2000s’ recession. Indeed, the time trend of the total price environment is positive from 1987–2001, and it starts becoming negative in 2002. However, there is a change in this cluster’s evolution starting with the early 2000s, which was primarily triggered by a significant decline in the share of labor cost and an increase in the share of capital depreciation relative to the gross output. Consequentially, the capital deepening ratio after 2001, and until the end of the period of analysis is twice what it is as at the beginning of the period. One may wonder whether this evolution is due to faster info trends for companies in this cluster. Indeed, as long as labor dominated the economic environment of these sectors, the concerned industries clearly enjoyed a favorable total price effect. However, with the rise in capital deepening, the price environment deteriorated.

For two sectors of this cluster—“*information and data processing services*” and “*ambulatory health care services*”—the period of 1992–2003, -2006 for the latter, is characterized by an increasing trend for the price effect that reached its peak in 1998 and 2000, respectively. Afterwards, total price effect engaged in a declining trend. This decrease was further accentuated by the 2007 subprime recession. While the first sector seems to be slightly recovering from this recession, the latter seems to continually suffer from these negative effects all the way to 2014.

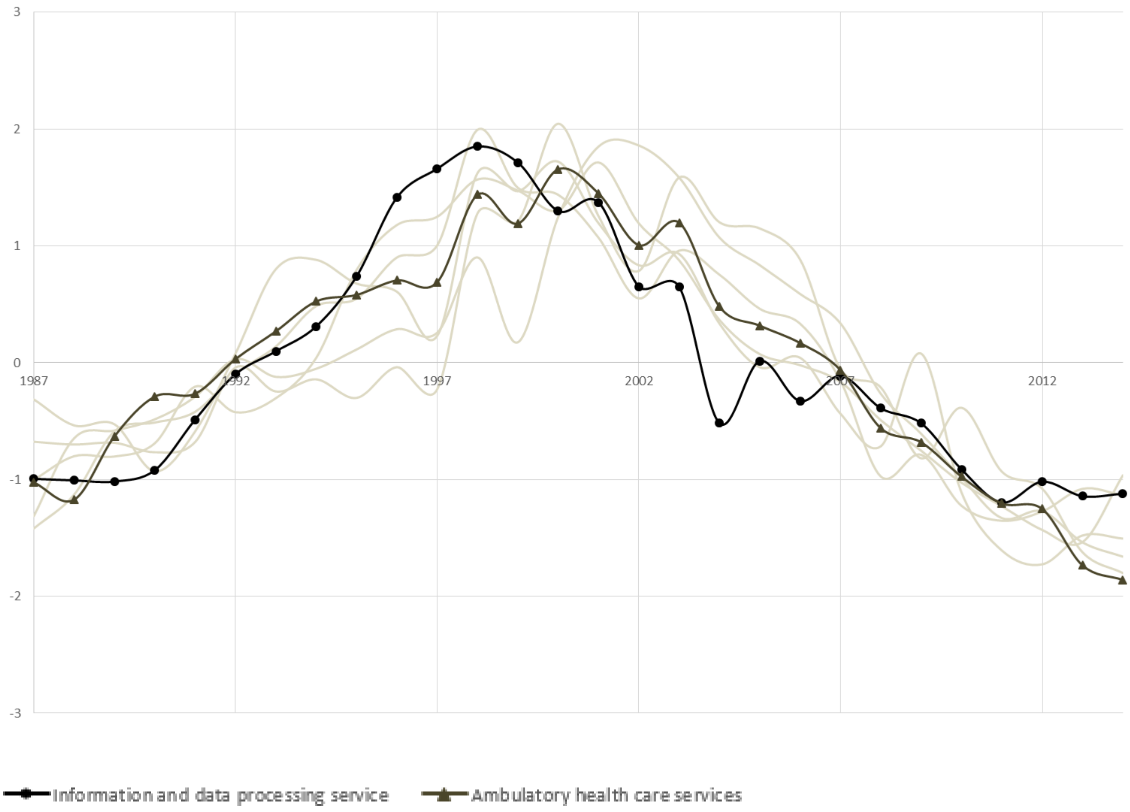

Figure 43 Standardized Ln of the global price-environment effect for industries in cluster 2 (focus on "information and data processing services" and "ambulatory health care services")

Cluster 3 regroups sectors that constitute approximately 15% of the total U.S. gross output. This cluster is characterized by a price environment evolution that is comparable with the one presented in the previous cluster. However, what makes this cluster unique is that, for the period of 1987–2001, the trend of the total price effect is not significantly different from 0.[12] As was the case for the previous cluster, the time trend after 2001 becomes significantly negative. Recall that this cluster was relatively intensive in intermediate inputs and, after the early 2000s, in labor. The increasing global competition and employee compensation may explain the decline in the price environment from the early 2000s onwards.

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[12] This is true for all sectors but two: *Federal Reserve banks, credit intermediation, and related activities* and *Waste management and remediation services* for which the trend was positive and respectively negative.

"*Federal Reserve banks, credit intermediation and related activities*" represent a slight exception in this cluster. Indeed, it is the only industry for which the price environment deteriorated from 1987 to 1990, which may correspond to post "Black Monday" effects.[13] However, the series recovers after 1990 and, with the exception of 1993, it followed an increasing path through to 1998. The 2007 subprime crisis sped up the indicator's decline. However, this indicator began recovering for the last two years in the analysis.

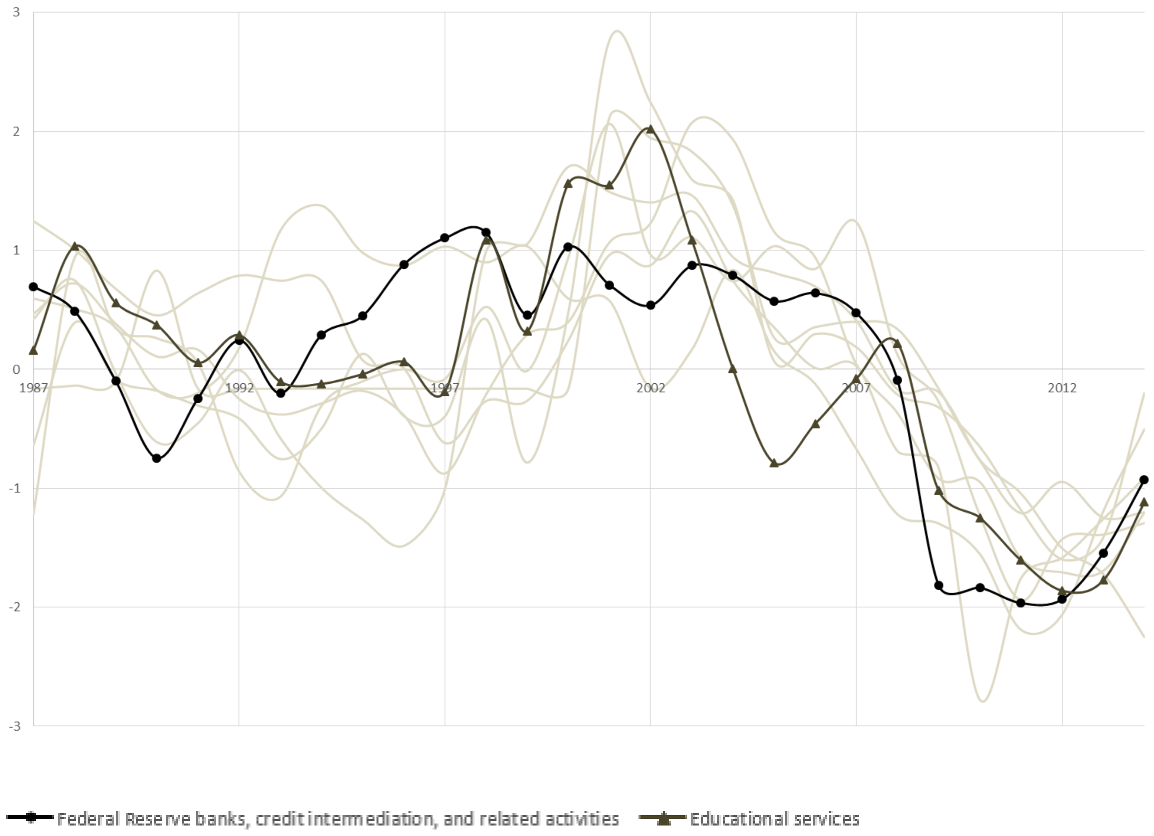

Figure 44 Standardized Ln of the global price-environment effect for industries in cluster 3 (focus on "federal Reserve banks, credit intermediation and related activities" and "educational services")

Cluster 4 constitutes 12,55% of the total U.S. gross output. The price effect trend for these industries is non-significant[14] before 1993 and positive over 1993–2007. These sectors have all been influenced by the subprime crisis, even though they seem to have experienced a recovery in

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[13] "*In finance, Black Monday refers to Monday, October 19, 1987, when stock markets around the world crashed, shedding a huge value in a very short time. The crash began in Hong Kong and spread west to Europe, hitting the United States after other markets had already declined by a significant margin. The Dow Jones Industrial Average (DJIA) fell exactly 508 points to 1,738.74 (22.61%).*" (Wikipedia)

[14] With one exception, *Construction*, for which the trend was negative over 1987-1992

the following years.[15] Recall that this is the most capital intensive cluster and the one with the highest capital deepening. It is therefore not surprising that these industries have reaped the benefits before the subprime crisis.

For "*oil and gas extraction*" industry, we notice a cyclical evolution during 1987–2014. Where "*construction*" is concerned, after a short period of decline, price environment improved from 1992 to 2007. The subprime crisis negatively affected the industry's price environment.

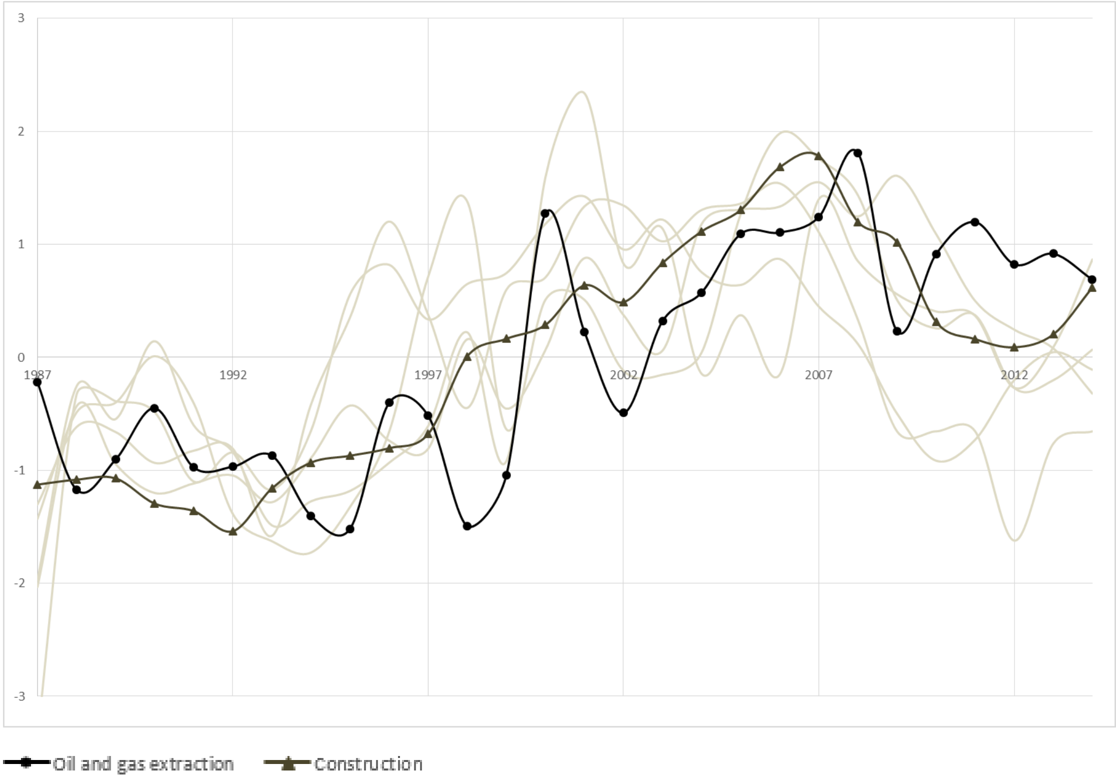

Figure 45 Standardized Ln of the global price-environment effect for industries in cluster 4 (focus on "oil and gas extraction" and "construction")

3.2.3 The decomposition of the total price environment into individual output and input price effects

Besides the total price effect, the methodology developed in the previous section has also allowed us to calculate for each industry an individual output price effect and three individual input price effects (for intermediate inputs, labor, and capital). In order to explain the relationship between

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[15] The trend is not significant except for industries *Support activities for mining* and *Management of companies and enterprises*.

total effect and its components, we constructed the following model in which total effect logarithm is a function of the individual effects. Our panel model includes time dummy variables in order to control for the U.S. economy business cycles, as well as sector dummy variables to control for the heterogeneity between industries. This model is given by:

$$\begin{aligned}
&LnET_{i,t} = \alpha_i + \beta_1 LnEO_{i,t} + \beta_2 LnEII_{i,t} + \beta_3 LnEL_{i,t} + \beta_4 LnEK_{i,t} + T_t + \mu_{i,t}, \\
&\text{with } t \in \{1987, 1988, ..., 2013\} \\
&i \in \{1, 2, ..., 63\} \\
&ET_{i,t} = \text{total price effect of industry } i \text{ in year } t \\
&EO_{i,t} = \text{output price effect of industry } i \text{ in year } t \\
&EII_{i,t} = \text{intermediate inputs price effect of industry } i \text{ in year } t \\
&EL_{i,t} = \text{labor price effect of industry } i \text{ in year } t \\
&EK_{i,t} = \text{capital price effect of industry } i \text{ in year } t \\
&T_t = \text{dummy variable for each year } t \\
&\alpha_i = \text{dummy variable for each industry } i
\end{aligned} \tag{4}$$

Table 44 The entire period (1987–2014) regression model results

	Coefficient	**STD. ERROR**	**T-RATIO**	**P-VALUE**
LnEO	0,683033	0,00895178	76,3014	<0,0001
LnEII	0,16206	0,0164142	9,8732	<0,0001
LnEL	0,631836	0,0374921	16,8525	<0,0001
LnEK	0,841231	0,044547	18,8841	<0,0001

First of all, we notice that all individual price effects are positive and significant (Table 44). Thus, as expected, individual price effects positively contribute to the total price effect. Notice that in our model, each coefficient can be interpreted as an elasticity. The strongest elasticity is related to the capital input (0,84), followed by output (0,68) and labor (0,63). Finally, the intermediate inputs' individual price effect is the weakest (0,16).

We analyze the structural change of the panel model regressors over time. In order to determine that the structural break occurs neither too close to the beginning nor too close to the end of the period, we focus on 1991–2010. For this, for each year T_0 in this period, we compute an unrestricted model in which the slopes of the explanatory variables have to be different between the two periods: before T_0 (BT_0) and after T_0 (AT_0). The unrestricted model is given by:

$$\begin{aligned}
&LnET_{i,t} = \alpha_i + \beta_j^{BT_0} X_{j,i,t\le T_0} + \beta_j^{AT_0} X_{j,i,t>T_0} + T_t + \mu_{i,t}, \\
&\text{with } X_j \in \{LnEO, LnEII, LnEL,\ LnEK\}, \\
&\text{where} \\
&T_0 \in \{1991, 1992, ..., 2010\} \\
&t \in \{1987, 1988, ..., 2014\} \\
&i \in \{1, 2, ..., 63\}
\end{aligned} \tag{5}$$

For each model in (8) and for all $\mathrm{T}_0 \in \{1991, ..., 2010\}$, the following F-test was applied:

$$\begin{cases} H_0 : \forall j \in \{EO, EII, EL, EK\}, \beta_j^{BT_0} = \beta_j^{AT_0} \\ H_1 : \exists\ j \in \{EO, EII, EL, EK\}, \beta_j^{BT_0} \neq \beta_j^{AT_0} \end{cases} \tag{6}$$

The corresponding F-statistic is given by:

$$F = \frac{(RSS_R - RSS_U)/q}{RSS_U / df_U}$$

$$where\, q = df_R - df_U = k\,(number\ of\ regressors),$$

$$df_U = NT - 2k - N - (T-1)$$

RSS_R Residual sum of squares for the restricted model (eq.7)

RSS_U Residual sum of squares for the unrestricted model (eq. 8)

Figure 46 reveals two periods of major structural change. The first occurred in the early 2000s, while the second one took place around the subprime crisis (2005–2007).

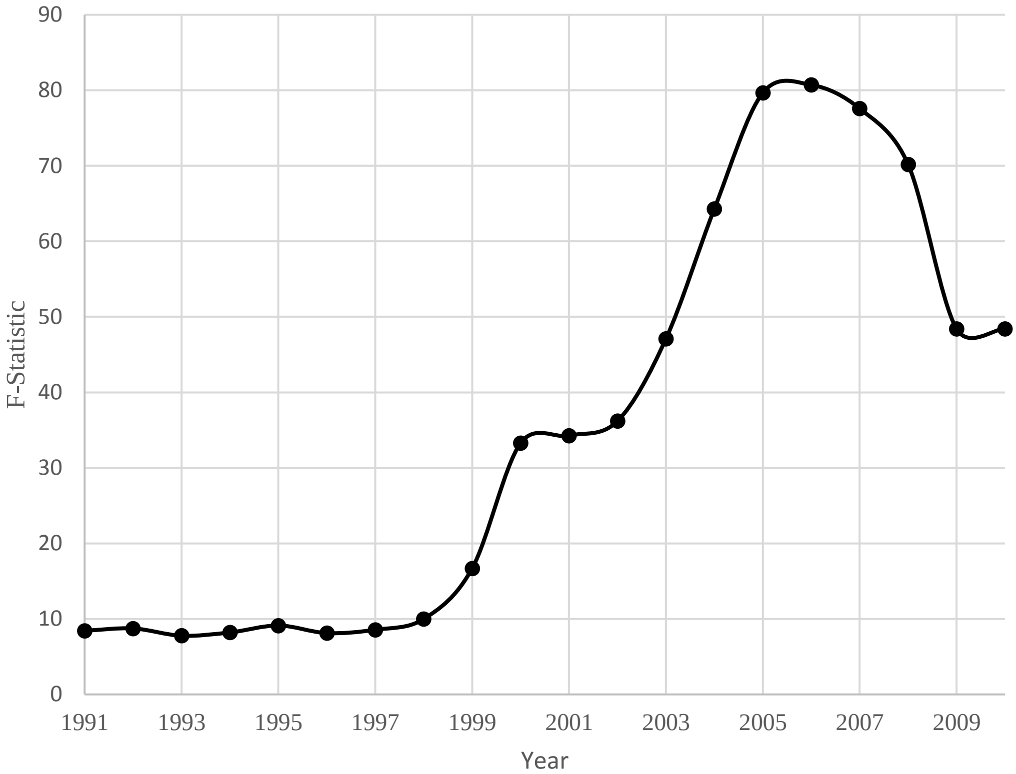

Figure 46 Plot of the F-statistic for the structural break year varying between 1991 and 2010

On this basis and given the known short-run cycles for the U.S. economy, we propose dividing the analysis period into three sub-periods: 1987–2001 (period 1), 2002–2007 (period 2) and 2008–2014 (period 3). The corresponding fixed effects panel model is then:[16]

$$
\begin{aligned}
& LnET_{i,t} = \alpha_i + \beta_1^C LnEO_{i,t} + \beta_2^C LnEII_{i,t} + \beta_3^C LnEL_{i,t} + \beta_4^C LnEK_{i,t} + T_t + \mu_{i,t}, \\
& \text{with } C \in \{P1; P2; P3\}, \\
& \text{where } P1 = \begin{cases} 1 \; if \; t < 2002 \\ 0, if \;\; not \end{cases} \\
& P2 = \begin{cases} 1 \; if \; 2002 \le t < 2008 \\ 0, if \;\; not \end{cases} \qquad (7) \\
& \text{and } P3 = \begin{cases} 1 \; if \; 2008 \le t \le 2014 \\ 0, if \;\; not \end{cases} \\
& t \in \{1987, 1988, ..., 2014\} \\
& i \in \{1, 2, ..., 63\}
\end{aligned}
$$

---

[16] T_{2014} was excluded from this regression in order to avoid perfect multicollinearity.

Table 45 The results for the sub-periods regression model

	Period 1 (1987–2001)				Period 2 (2002–2007)				Period 3 (2008–2014)			
	Coeff	std. err	t-ratio	p-value	Coeff	std. err	t-ratio	p-value	Coeff	std. err	t-ratio	p-value
LnEO	0,71	0,01	83,09	0	0,78	0,02	33,47	<0,0001	0,68	0,04	18,2	<0,0001
LnEII	0,08	0,02	5,08	<0,0001	0,27	0,04	7,47	<0,0001	0,97	0,06	15,57	<0,0001
LnEL	0,61	0,04	17,34	<0,0001	0,53	0,15	3,6	0,0003	0,44	0,19	2,37	0,018
LnEK	0,83	0,04	19,88	<0,0001	0,59	0,19	3,09	0,0021	0,53	0,38	1,4	0,1615

As expected, the F-statistic (8,1662) = 39,63 computed to compare the initial (restricted) model (eq. 7) and the model with two structural changes (eq. 10) is significant. Individual price effects in each period are positive and, with the exception of the capital price effect in the third period, they are all significant (Table 45). Nevertheless, their respective contributions to the price effect change over time. The most remarkable evolution is that of capital price effect, which decreases throughout the first two periods (0,83 in the first period and 0,59 in the second period) to become not significant in the third period. Conversely, the elasticity of the intermediate inputs price effect is significant and increasing from 0,08 in the first period to 0,27 in the second period and 0,97 in the third period. The output price effect and labor price effect seem to be quite stable during the entire period (0,71, 0,78, and finally, 0,68 for the output and 0,61, 0,53 and 0,44 for labor, respectively).

Thus, prior to the subprime crisis, the sectors' main source of price effects resided in the capital price. After the subprime crisis, industries' price effects were increasingly influenced by the specific intermediate inputs price. The subprime crisis does not seem to have structurally modified the output price contribution and, respectively, the labor price. These two effects have a relatively high and constant contribution to the sectors' total price effects.

4 Conclusions

This article proposed a new methodology for computing price effect, defined as the DMU's ability to profit from positive market opportunities. Contrary to the previous literature, we explicitly compare prices among DMUs. Our methodology is simple, straightforward, and easy to implement. It is based on computing efficiency scores using either quantity-based or value-based data. The ratio between these two scores gives the total price effect. Moreover, we show that this methodology can be extended in order to compute output- or input-specific price effects.

This approach was applied to a dataset involving U.S. industries from 1987–2014. We demonstrated that the U.S. economy has been characterized by two contrasting evolutions. While the mean technical efficiency has (almost) continuously improved, the value efficiency has followed a much flatter path. Consequently, the mean price effect has deteriorated over the entire period, a phenomenon which, in our opinion, cannot be isolated from the U.S. economy's growing degree of openness over the same time period. In addition, the clustering analysis performed on the individual sectors has shown that while a majority of industries have suffered from a declining price effect throughout this period, some others have experienced more contrasting evolutions. The input shares relative to the gross output value (intermediate input-, labor- or capital-intensity) seem to play an important role in these evolution patterns. Finally, the panel regression model revealed that two major events, namely, the early 2000 crisis and the 2007 subprime crisis, negatively influenced the capital specific price effect and played a positive role for intermediate inputs.

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Chapter 4

Technological catching-up and growth convergence among US industries

This part has been published as a working paper of LEM N° 2017-23 (http://lem.cnrs.fr/IMG/pdf/dp2017-23.pdf) and IESEG School of Management (IÉSEG Working Paper, 2017-EQM-07). The paper was submitted to *Economic Modelling* (CNRS 2) in November 2017.

1 Introduction

Most of previous studies about productive performance simply highlight technical efficiency at the individual level but pay less or no attention to a structural effect at a more aggregated level. Indeed, national productivity changes arises from two origins. First, a technological catching-up effect related to the fact that less productive industries put in extra effort to grow faster than the leading sectors. Second, a convergence process in output and input ratios through transfer of resources among industries do occur over time. The latter is connected to an output/input deepening or expanding effect and diminishing returns of the production technology.

This paper attempts to analyze the efficiency convergence process within a group of 63 industries which cover the whole U.S. economy over the period of 1987−2014. We intend to bring an original decomposition for productive performance growth at the macro level by splitting overall efficiency evolution into two components: technical and structural changes. The structural effect measures the differences in the input and output mixes among industries impacting productivity ratios at the macro level. Over time, a decrease of this effect means that input and output combinations are becoming more homogenous among the different industries which contributes to improve the productivity level for the global economy. The technical effect measures an efficiency gap between the evaluated sector and its benchmark located on the production frontier. Its reduction over time discloses a technological catching-up process: the inefficient industry has reached the benchmark progressively.

Relying on the convergence literature, two simultaneous processes support income convergence between countries: a capital deepening effect and a technological transfer/diffusion related to Total Factor Productivity (TFP) gaps. Through the initial Solow's Model, the neoclassical standard theory devoted most attention to the first process. Assuming an exogenous and non-costly technological progress, technology adoption issues were not explicitly taken into account. For Solow (1994) this restrictive hypothesis was necessary at that initial step of progress of the growth theory. Later, the identical production technologies assumption was rejected by authors such as Jorgenson (1995) or Durlauf and Johnson (1995). In the same vein, a less drastic approach adopted by Abramovitz (1986) considers a common available technology among economies which may diverge in their capability to join and use it. As a result, the concept of "social capabilities" was introduced to explain different productivity levels between countries and concern in cross-country TFP gaps has become a major issue to investigate economic growth (Islam, 2003).

As empirical measure of technology can be linked to TFP estimations, the concept of TFP-convergence investigates whether production plans such as industries are capable to catch up in terms of highest observed TFP levels. Most of empirical studies concerning TFP convergence have focused on international comparison of TFP and have shown that differences in technology are related to gaps in TFP levels. For example, through a regression of the productivity growth rates with the initial TFP levels of fifteen OECD nations, Dowrick and Nguyen (1989) analyzed TFP-convergence. Substantial signs of TFP catch-up among developed nations are established. However, their restraining hypothesis of a single capital-output ratio for all countries is a main issue. Wolff (1991) developed a TFP catching-up equation including a capital/labor ratio growth rate on the G-7 countries in order to study the relation between technical change and capital deepening effects. He found a positive influence of capital accumulation on TFP catch-up. Later, Dougherty and Jorgenson (1997) revealed a process of sigma-convergence of TFP levels among the G-7 through a significant reduction of their coefficients of variation over time.

TFP growth due to the interaction between technological adoption and capital accumulation was mainly studied for East Asian economies during the nineties. Several of authors (Young, 1992, 1994, 1995; Kim and Lau 1994) found that TFP growth did not play a major effect on their economic expansion. As a result, Krugman (1994) deduced that East Asian development should mainly result from factor accumulation. Nevertheless, Collins et al. (1996) and Klenow and Rodriguez (1997) showed more substantial role of TFP growth for some East Asian economies such as Singapore

While a huge literature was devoted to productivity convergence at country level, the sources of these aggregate productivity changes at the industry level remain largely unstudied. Through data on sectors, Bernard and Jones (1996) analyzed the sources of aggregate labor productivity convergence among the U.S. states over the period 1963-1989. They estimated the individual sectors' contribution to aggregate convergence. Their main result is that productivity growth in the manufacturing industries explained the main part of private non-farm productivity growth. Focusing on productivity changes by sector from 1963-1989, Barro and Sala-i-Martin (1991) pointed out that convergence was happening in all industries, although this process has been more significant in manufacturing than in other types of industries. They also established a break of macro-convergence after the early 70s mainly due to price changes in oil industry. More recently, Cardarelli and Lusinyan (2015) studied the aggregate US TFP slowdown using TFP estimators across U.S. states over the last two decades. They revealed that the deceleration of TFP growth was quite common among the states and not correlated to the presence of IT producing or using industries. Gaps in production efficiency across U.S. states are mainly explained by differences in investment rates of education and R&D.

Estimating productivity gains and their distribution among inputs and outputs for 63 American industries over the period 1987-2012, Boussemart and al. (2017) showed that TFP of US industries increased at an average trend of 0,8% and highlighted that employees and firms' profitability were the winners while clients and fixed capital providers were the losers in the distribution of productivity gains. Beyond these global results, TFP growth rates have been significantly different between the 63 industries over the last 26 years. Clearly, the computer and electronic products industry had the highest level of TFP growth (7,48%) followed by other sectors such as support activities for mining and wholesale trade (2,32% and 2,09%) while the oil and gas extraction industry registered the lowest performance (-1,22%).

Yet, most of studies about TFP growth present several caveats. First, they need to define a technological leader a priori (generally the US) instead of letting data choose the benchmark to reach. Second, the technology estimation requires a particular functional form (Cobb–Douglas, CES, Translog…). Third, the constant returns to scale assumption does not take into account size heterogeneity across production plans and may bias TFP indexes and the underlying catching-up process.

To avoid the first two drawbacks, the catching-up mechanism was re-examined with a new methodology by Kumar and Russell (2002) which did not impose any functional form on the production frontier, nor any hypothesis for the market structure. In addition, they did not choose a specific country as the world leader and allow for eventual technical and/or allocative inefficiencies for economies. Through productivity indexes estimated with a non-parametric method, the catching-up hypothesis across 57 poor and rich nations was investigated. More precisely, they decomposed variations of the cross-country distribution of labor productivity in dissimilarities in levels of technology and technical changes over time. They showed how much of income convergence was due to technological transfer or to alignment in capital/labor intensities. They settled an evident technological catch-up, as most of countries have moved closer to the production frontier, non-neutral technological progress and a dominant role of capital deepening effect compared to technological catch-up inducing both growth and income divergence between countries.

A Data Envelopment Analysis (DEA) approach was also used by Christopoulos (2007) to study the effect of human capital and international opening (i.e.: economic globalization) on efficiency within a group of 83 developed and less developed nations. He confirmed that more openness improves significantly countries' productive performances while human capital does not impact the efficiency to a great degree. Nonetheless, a constant returns to scale hypothesis still characterized the underlying technology.
Relaxing this restrictive constant returns to scale assumption for the technology, Färe et al. (1994) decomposed productivity growth in a technical progress effect and an efficiency change component that was referred to a catching-up process for 17 OECD countries over the period 1979-1988. Additionally, the catching-up component was split into two terms: a pure technical efficiency changes and a scale efficiency change. Their results showed that Japan obtained the highest TFP growth rate.

Using such a non-parametric programming framework, our study analyzes both input-output ratio convergence and TFP catching-up among 63 North American industries over the period 1987-2014. Compared to most of studies on convergence cited above, one empirical contribution of our research is to analyze the catching-up process at the sectoral level within the US economy. We first separate efficiency gaps into two components: a technology effect taking into account industry size heterogeneity by relaxing the constant return to scale assumption and a structural element which highlights the impacts of an input-output deepening or expanding effect on technological transfer over time. The convergence processes on each of them are analyzed. Secondly, following Boussemart et al. (2017) who interrelated the distribution of TFP changes between inputs and outputs, we perform a panel data analysis to explain the input and output price evolutions by the changes of technical efficiencies and input-output mixes.

This paper is organized as follows. In the next section we use a directional distance function to define the production frontier and evoke the measures of technical and structural effects which may impact the convergence process within a set of units. Section 3 presents data and the underlying technology and discusses the results. Finally, we give the main conclusions in the last section.

2 Analyzing convergence process with directional distance functions

We aim at estimating both a technical catching-up effect between observed production plans of industries and their maximum achievable levels of TFP and a convergence process of input-output ratios among industries. A technical catching-up process reveals the ability to fit the current technology and a structural convergence process considers the diversity across industries regarding their respective input or output intensity evolutions. This latter can be related to an input/output deepening or expanding effect.

In the followings paragraphs, the concepts of technical catching-up and structural convergence are defined. Moreover, methodological tools to measure these effects are developed.

2.1 Definition and measure of a technical catching-up process

A technical catching-up process happens when less efficient industries tend to catch up more efficient ones over time. In this case, the inefficient industries are overtaking the efficient sectors which have retained leadership positions. Thus, one can observe a convergence process to the efficient frontier if the technical inefficiency level is decreasing over time. Less efficient industries can progressively adopt technological innovations, managerial procedures, or organizational capabilities from the most productive ones.

Traditionally, in the literature, technological adoption is viewed as comparison of TFP levels across sectors or countries and testing an inverse link between TFP growth rates and their original levels. Convergence process occurs if industries with the smallest TFP levels display the highest growth rates. The assumption of constant returns to scale (CRS) is necessary since the best production plan, set as a benchmark for all sectors, has the maximal observed productivity. Nevertheless, if the CRS assumption is not fulfilled and increasing and/or decreasing returns to scale (variable returns to scale, VRS) appear to be more appropriate, the maximal feasible productivity level may not correspond to the maximal observed productivity level and should be estimated for each sector relatively to its own size. Indeed, this size is constrained by the industry's scale of operations which can be considered as quasi-fixed in the short-run. In fact, if a CRS technology is retained while a VRS is more faithful to the data, the analysis of technological transfer can lead to substantial bias. Indeed, one can observe a divergence in productivity levels among industries when they achieve the production frontier and contribute to a technological catching-up process as it is shown in Figure 46.

Let us consider 3 sectors A, B and C which use one input (X) to produce one output (Y) under variable return to scale (Figure 46). One can observe that sectors B and C characterized by a similar productivity level are inefficient while industry A is efficient and has the most productive scale size (mpss). If we suppose that sector A is a benchmark for all industries, we implicitly assume a CRS technology. That is, if sectors B and C could achieve B** and C**, TFP convergence will occur as all industries will reach the same maximal productivity level. However, if the true VRS technology holds, sectors B and C will be only capable to achieve B* and C* for which productivity divergence is observed. Since B and C will never be able to achieve B** and C**, one can draw a conclusion about divergence of productivity levels between the industries even if they have reached their respective maximum feasible productivity levels located on the VRS production frontier. In that case, their technical inefficiencies have decreasing over time denoting a clear technical catching up process.

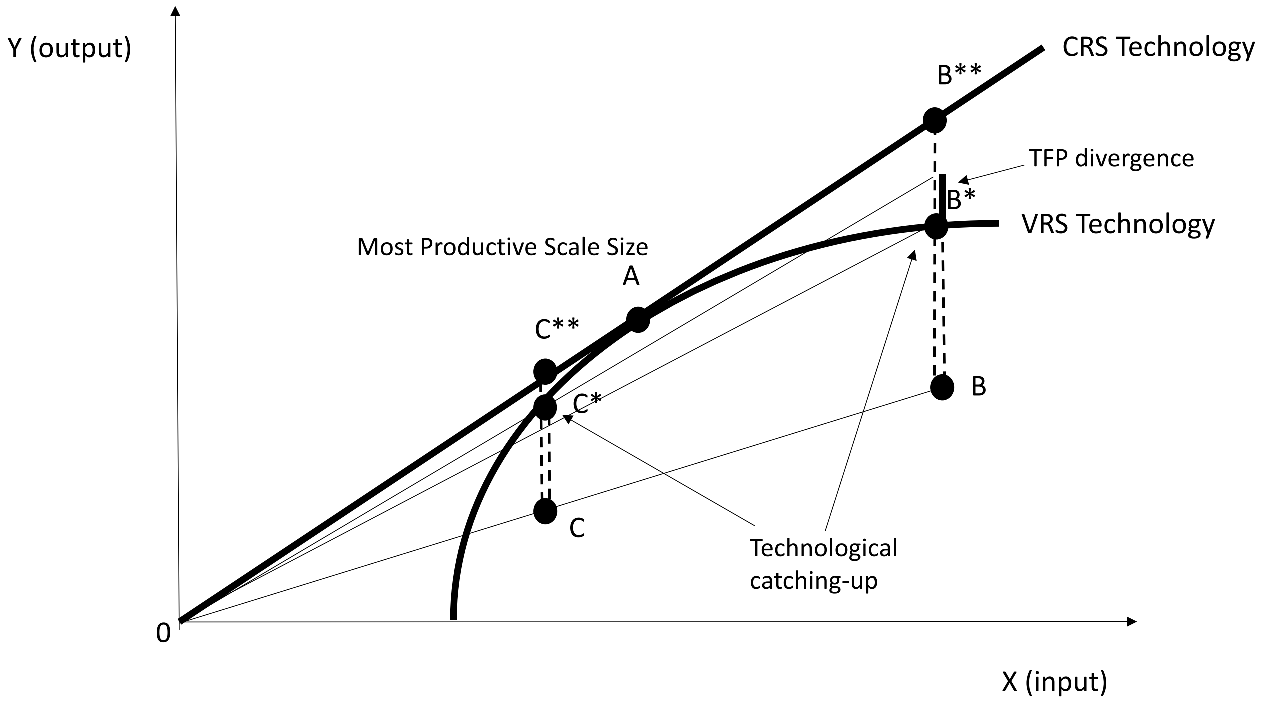

Figure 46 Maximal observed productivity level under CRS assumption versus maximal feasible productivity level under VRS assumption

In order to measure technical inefficiency, we develop an activity analysis model assuming that all industries face the same VRS technology in the sense that they are able to produce a common output such as gross output from similar resources such as fixed capital, labor and intermediate inputs:

In a more general way, let us consider a vector of inputs $\mathbf{x} \in R_+^N$ and a vector of outputs $\mathbf{y} \in R_+^M$ for an observed industry or DMU (decision making unit). At time *t*, the technology can be simply defined by the production set which includes all the feasible production plans:

$$T_{VRS}^t = \left\{ (\mathbf{x}^t, \mathbf{y}^t) : \mathbf{x}^t \text{ can produce } \mathbf{y}^t \right\} \tag{1}$$

To better structure and clarify the definition of T_{VRS}^t, we consider two assumptions on the production possibility set: free disposability of inputs and outputs and convexity. Now from a sample of K observed DMUs, we achieve an operational definition of T_{VRS}^t as:

$$\begin{aligned} T_{VRS}^t = \Bigg\{ & (\mathbf{x}^t, \mathbf{y}^t) : \mathbf{x}^t \in R_+^N, \mathbf{y}^t \in R_+^M, \sum_{k=1}^{K} \mu_k y_k^{m,t} \geq y^{m,t}, m = 1,\dots, M, \\ & \sum_{k=1}^{K} \mu_k x_k^{n,t} \leq x_k^{n,t}, n = 1,\dots, N, \sum_{k=1}^{K} \mu_k = 1, \mu_k \geq 0, k = 1,\dots, K \Bigg\}. \end{aligned} \tag{2}$$

We measure the gaps between any DMU and the technology frontier at time *t* using a directional distance function:

$$\vec{D}_{T^t}(\mathbf{x}^t, \mathbf{y}^t; \mathbf{g}_{\mathbf{x}}^t, \mathbf{g}_y^t) = \sup_{\lambda^t} \left\{ \lambda^t \in \Re_+ : (\mathbf{x}^t + \lambda^t \cdot \mathbf{g}_{\mathbf{x}}^t, \mathbf{y}^t + \lambda^t \cdot \mathbf{g}_y^t) \in T_{VRS}^t \right\}, \tag{3}$$

where $\mathbf{g}^t = (\mathbf{g}_{\mathbf{x}}^t; \mathbf{g}_{\mathbf{y}}^t) \in (-R_+^N; R_+^M)$ characterizes the direction of the projection onto the annual production frontier. In our analysis we define $\mathbf{g}_{\mathbf{x}}^t = 0$ and $\mathbf{g}_{\mathbf{y}}^t = \sum_{k=1}^{K} \mathbf{y}_k^t$. Therefore, the technical inefficiency for any evaluated DMU "*a*" can be estimated with the following linear program:

$$\begin{aligned} & I_a^{TECH,t} = \vec{D}_{T_{VRS}^t}(\mathbf{x}_a^t, \mathbf{y}_a^t; \mathbf{g}_{\mathbf{x}}^t, \mathbf{g}_{\mathbf{y}}^t), \forall a \in \{1, 2, \dots, \mathrm{K}\} \\ & \vec{D}_{T_{VRS}^t}(\mathbf{x}_a^t, \mathbf{y}_a^t; \mathbf{g}_{\mathbf{x}}^t, \mathbf{g}_{\mathbf{y}}^t) = \max_{\mu, \lambda_a^t} \ \lambda_a^t \\ s.t. \ & \sum_{k=1}^{K} \mu_k y_k^{m,t} \geq y_a^{m,t} + \lambda_a^t \mathbf{g}_{\mathbf{y}}^t \quad \forall m = 1, \cdots, M \\ & \sum_{k=1}^{K} \mu_k x_k^{n,t} \leq x_a^{n,t} + \lambda_a^t \mathbf{g}_{\mathbf{x}}^t \quad \forall n = 1, \cdots, N \\ & \sum_{k=1}^{K} \mu_k = 1 \\ & \lambda_a^t \geq 0 \\ & \mu_k \geq 0 \ \forall k = 1, \dots, K \end{aligned} \tag{LP1}$$

Considering the group of K industries or DMUs called “AGREG”, all individual technical inefficiencies can be summed up to obtain the technical inefficiency score at the aggregate level:

$$I_{AGREG}^{TECH,t} = \sum_{a=1}^{K} \vec{D}_{T_{VRS}^{t}}(\mathrm{x}_a^t, \mathrm{y}_a^t; \mathrm{g}_\mathrm{x}^t, \mathrm{g}_\mathrm{y}^t)\,. \tag{4}$$

Thus, a decrease with time of $I_{AGREG}^{TECH,t}$ will denote a general catching-up process to the maximal feasible productivity levels for the majority of industries (Figure 47).

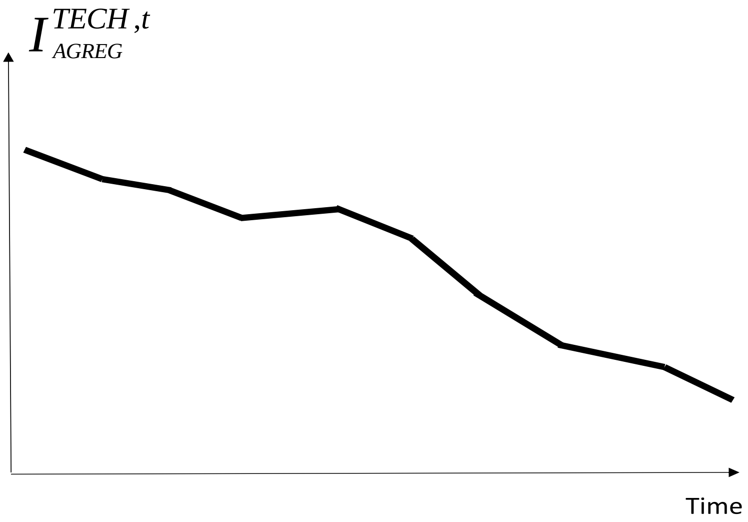

Figure 47 Illustration of a technological catching-up process

2.2 Definition and measure of a structural convergence process

If we consider a multi outputs-inputs technology, diversity in input and output allocations among industries can cause structural inefficiency (Ferrier et al. 2010). As we can observe at Figure 48, efficient sectors A and B which produce the same output level but with different input mixes, create technical inefficiency at the aggregate level. Related structural effects in the output and input-output spaces are displayed in Figures 49 and 50. Thus, differences in relative input and output endowments between the two technically efficient industries induce such a structural inefficiency.

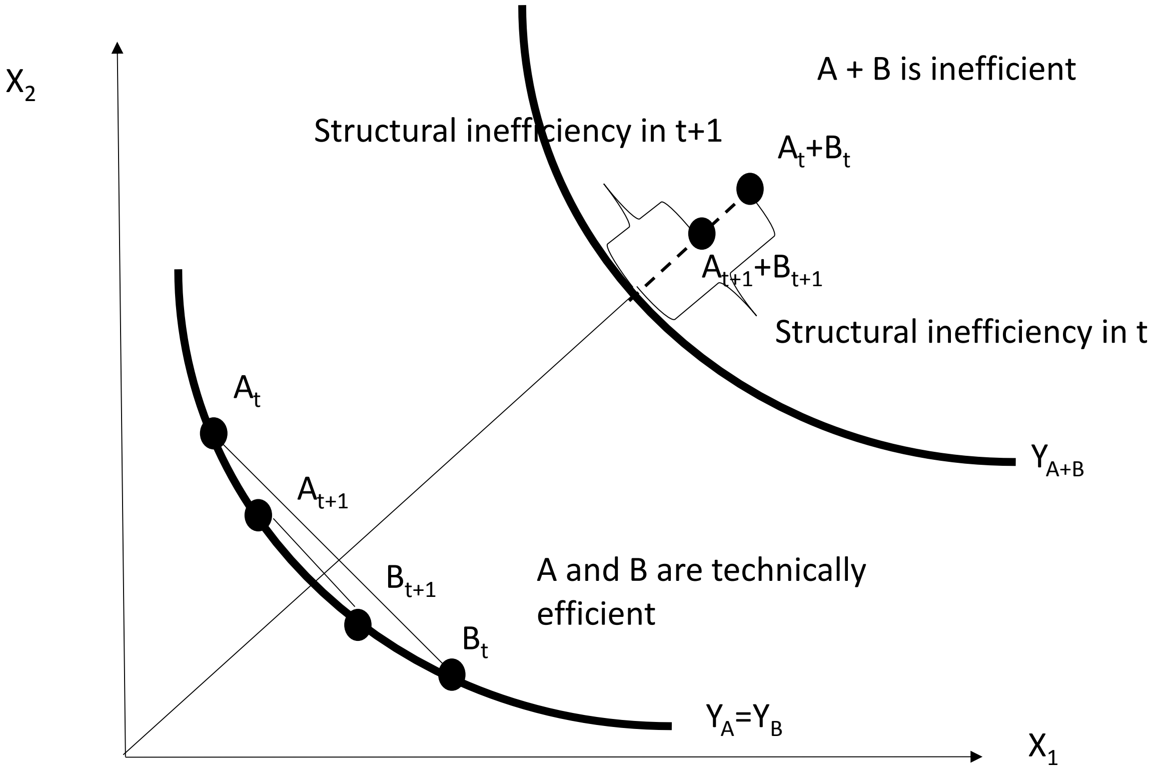

Figure 48 Structural inefficiency and convergence in the input space

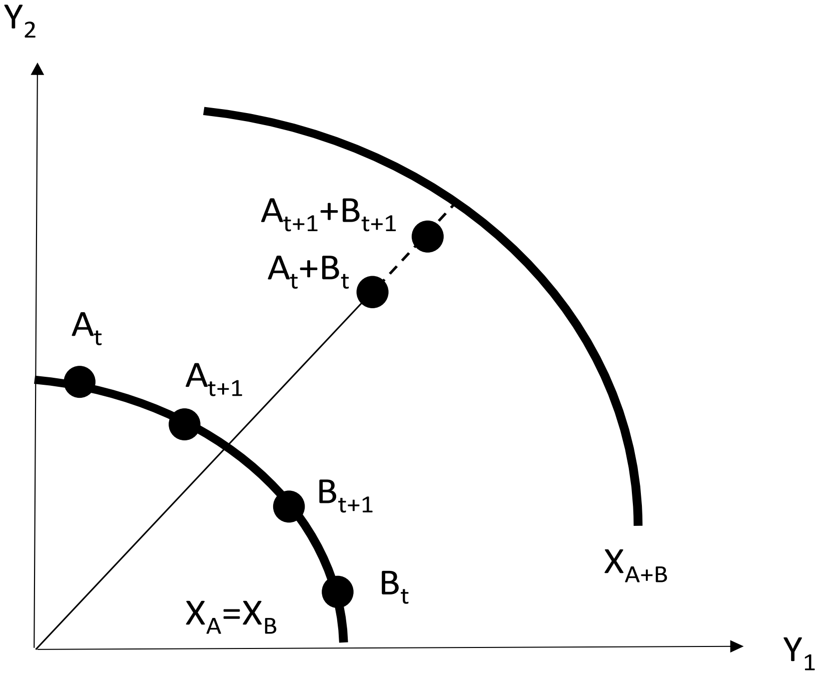

Figure 49 Structural inefficiency and convergence in the output space

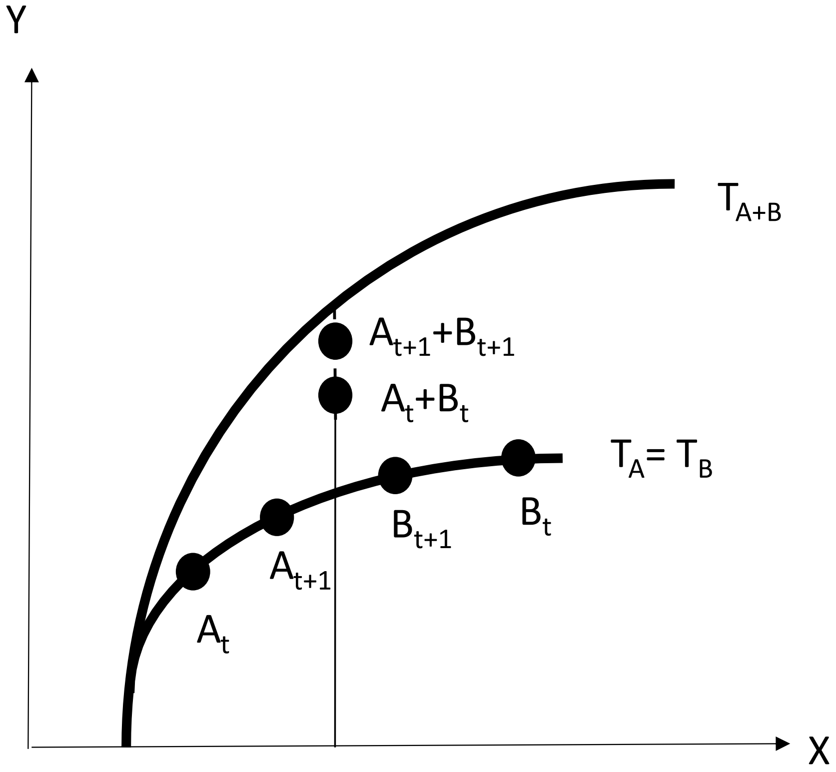

Figure 50 Structural inefficiency and convergence in the input-output space

In a perfect competitive market, a common given input-output price vector leads to industries to adopt identical input-output allocations. As a result, less resistances in the reallocation process reduce misallocation of resources and improve aggregate productivity. Consequently, in the spirit of Debreu's (1951) concerning coefficients of resource utilization, market allocation inefficiency can be revealed through the structural inefficiency. Thus, the decrease of this component over time is correlated to aggregate productivity growth at the group level since sectors homogenize their input-output allocations gradually disclosing a structural convergence process.

We intend to estimate structural inefficiency scores for all industries at the group and individual levels. To obtain the inefficiency scores at individual level, we first estimate structural inefficiency at the group level. As previously, we consider K industries or DMUs which constitute the total group AGREG and we suppose, in a formal way, that the group technology is the sum of the K DMUs technologies:

$$T^{AGREG,t}=\sum_{k=1}^{K}T^{t}. \qquad (5)$$

Li and Ng (1995) proved that under convexity assumption the VRS aggregate technology $T_{VRS}^{AGREG,t}$ is equal to K times the individual technology:

$$T_{VRS}^{AGREG,t} = \sum_{k=1}^{K} T_{VRS}^{t} = K \times T_{VRS}^{t}. \qquad (6)$$

We estimate first the overall inefficiency as the technical inefficiency for AGREG with the following linear program:

$$\begin{aligned}
&\vec{D}_{T_{VRS}^{AGREG,t}}\Big(\sum_{\mathbf{k=1}}^{\mathbf{K}} \mathbf{x}_k^t, \sum_{\mathbf{k=1}}^{\mathbf{K}} \mathbf{y}_k^t; \mathbf{g}_\mathbf{x}^t, \mathbf{g}_\mathbf{y}^t\Big) = \max_{\mu, \lambda_{AGREG}^t} \ \lambda_{AGREG}^t \\
&s.t.\ K\sum_{k=1}^{K} \mu_k y_k^{m,t} \geq \sum_{k=1}^{K} y_k^{m,t} + \lambda_{AGREG}^t \mathbf{g}_\mathbf{y} \quad \forall m = 1, \cdots, M \\
&K\sum_{k=1}^{K} \mu_k x_k^{n,t} \leq \sum_{k=1}^{K} x_k^{n,t} + \lambda_{AGREG}^t \mathbf{g}_\mathbf{x} \quad \forall n = 1, \cdots, N \\
&K\sum_{k=1}^{K} \mu_k = K \Leftrightarrow \sum_{k=1}^{K} \mu_k = 1 \\
&\lambda_{AGREG}^t \geq 0 \\
&\mu_k \geq 0 \ \ \forall k = 1, \ldots, K
\end{aligned} \qquad \text{(LP2)}$$

The linear program given above allows us to identify overall inefficiency which measures the technical efficiency of the aggregated production plan merging the K DMUs.

$$I_{AGREG}^{OVERALL,t} = \vec{D}_{T_{VRS}^{AGREG,t}}\Big(\sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{x}_k^t, \sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{y}_k^t; \mathrm{g}_\mathrm{x}^t, \mathrm{g}_\mathrm{y}^t\Big) \qquad (7)$$

The difference between the overall component and the sum of individual technical scores defines the structural inefficiency coming from the heterogeneity in relative input/output allocations among the K DMUs. Thus, structural inefficiency is defined at the group level:

$$I_{AGREG}^{STRUC,t} = I_{AGREG}^{OVERALL,t} - I_{AGREG}^{TECH,t} = \vec{D}_{T_{VRS}^{AGREG,t}}\Big(\sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{x}_k^t, \sum_{\mathrm{k=1}}^{\mathrm{K}} \mathrm{y}_k^t; \mathrm{g}_\mathrm{x}^t, \mathrm{g}_\mathrm{y}^t\Big) - \sum_{a=1}^{K} \vec{D}_{T_{VRS}^{t}}(\mathrm{x}_a^t, \mathrm{y}_a^t; \mathrm{g}_\mathrm{x}^t, \mathrm{g}_\mathrm{y}^t)$$

(8)

As a result, if structural inefficiency decreases over time, we observe an input/output mixes convergence process among the K DMUs (Figure 51).

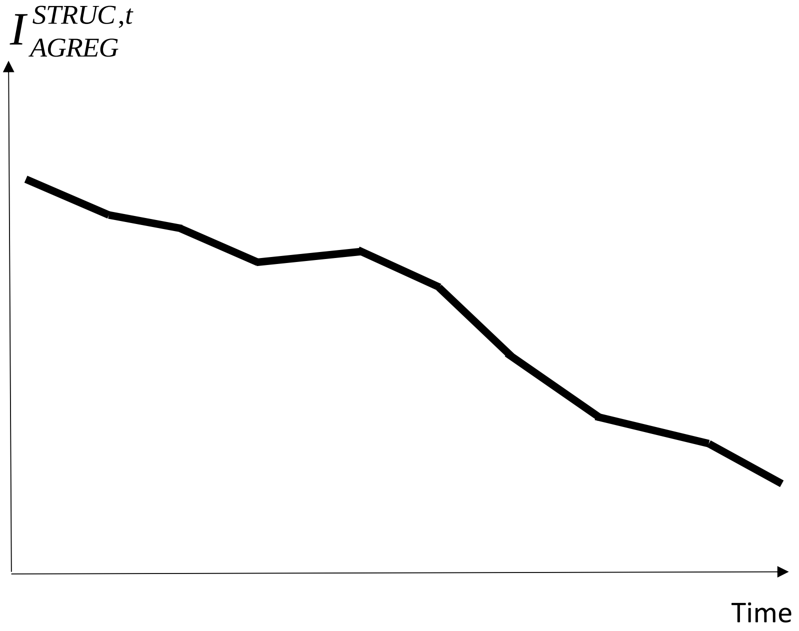

Figure 51 Illustration of a structural convergence process

While technical inefficiency can be retrieved directly at individual level through LP1, structural inefficiency is computed as a part of the overall inefficiency for the whole group. Nevertheless, we can allocate the overall inefficiency across DMUs by using the shadow prices derived in LP2 (Briec and al., 2003) in order to deduce the individual structural inefficiency:

$$I_{AGREG}^{OVERALL,t} = \sum_{a=1}^{K} I_a^{OVERALL,t} \Rightarrow I_a^{STRUC,t} = I_a^{OVERALL,t} - I_a^{TECH,t}$$

$$\text{and } I_{AGREG}^{STRUC,t} = \sum_{a=1}^{K} I_a^{STRUC,t} \tag{9}$$

3 Data and results

In order to analyze the technological catching-up and structural convergence processes among the industries, the previous models are now applied to a dataset on 63 different sectors covering the whole US economy over the period 1987-2014. In a second step, a panel data analysis is performed to explore the link between the changes in both technical catching-up and structural inefficiencies and the distribution of productivity gains among the different retained inputs and output.

3.1 Data

Data was collected from the Bureau of Economic Analysis (BEA) website (http://www.bea.gov/). For each industry, we have the current values (expressed in current U.S. dollars) and the quantity indexes (base year 100=2009) of their gross output net of taxes on production-less subsidies, intermediate inputs, labor (compensation of employees), and consumption of fixed capital (equipment, structures, and intellectual property products). The volume of taxes and subsidies on production directly link to their related quantity output indexes. The labor quantity is estimated in a full-time equivalent employee. The volume of capital consumption (the sum of equipment, structure, and intellectual property products) is calculated by the cost depreciation at a constant price. Thus, for each sector, we can compute both the value and the volume for each variable stated. Finally, the underlying technology is defined as a production function of one output (gross output) which depends on 3 inputs (intermediate inputs, labor and fixed capital).

3.2 Technical catching-up and structural convergence processes among US industries

Production frontiers are estimated year by year over the whole period. In a first step, we compute technical inefficiency scores at industry level using a directional distance function. The direction is defined as the sum of gross outputs of all industries. For each evaluated industry, efficiency scores reveal potential growth computed in terms of percentages of the total US gross output. Based on this common direction, the individual efficiency scores can be directly aggregated to each other. In a second step, we estimate overall inefficiency for all industries at the aggregate level. Finally, for each sector, the structural component is deduced through the difference between individual overall and technical inefficiencies. As the production frontier is year-specific, the number of efficient industries can change over time. Although some of them are always located on the production frontier. Table 46 list these stable efficient sectors over the whole period.

Table 46 Technically efficient industries over the period 1987-2014

Industry	**Sum of inefficiency scores over 1987-2014**
Legal services	0,00%
Funds, trusts, and other financial vehicles	0,00%
Real estate	0,00%
Construction	0,00%
State and local	0,00%
Petroleum and coal products	0,00%
Food and beverage and tobacco products	0,00%

The respective evolutions of technical, structural and overall inefficiencies for the sum of 63 industries are presented in Figure 52. All three types of inefficiencies demonstrate convergence processes over the period 1987-2014 at the macro level. The technical and structural inefficiency dynamics follow a similar pattern with average annual decrease rate of respectively 2,6% and 2,5%. The technical inefficiency evolution seems to be more fluctuated over the period 2006-2011. As a result, the overall inefficiency is decreasing over time with a trend of -2,5%.

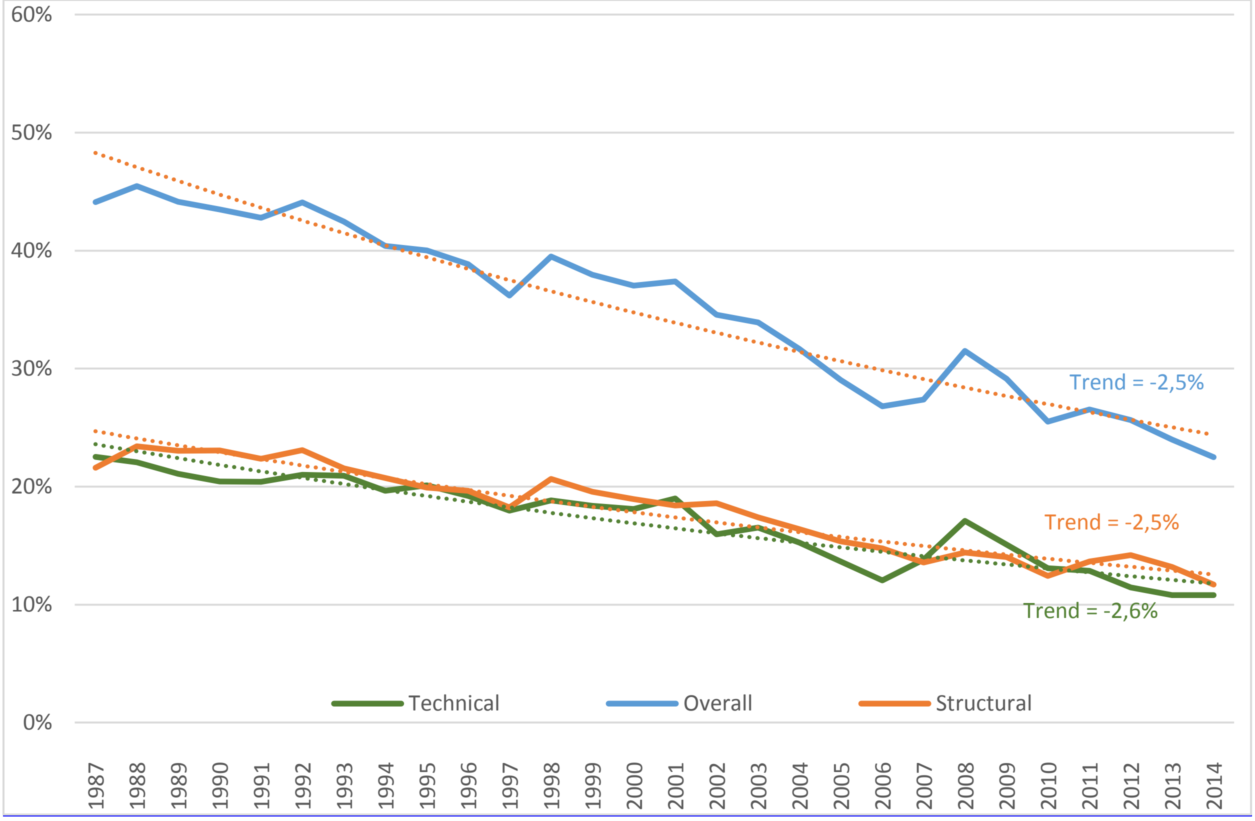

Figure 52 Evolution of the overall, technical and structural inefficiencies for the sum of 63 US industries over the period 1987-2014

Individual industries contribute to the technical catching-up and structural convergence processes differently. For instance, compared to computer systems design, chemical products and hospitals and nursing facilities, textile, electrical equipment industries and food services face significant and regular technical inefficiency decreases. Concerning the homogenization of input-output mixes, the individual effects seem more irregular. Examples of industries with ones of the most important convergence rates of technological catching-up effect and homogenization of input-output mixes over the considering period are given in Figures 53 and 55 respectively. Examples of industries without technological catching-up effect and input-output mixes homogenization are presented respectfully in Figures 54 and 56.

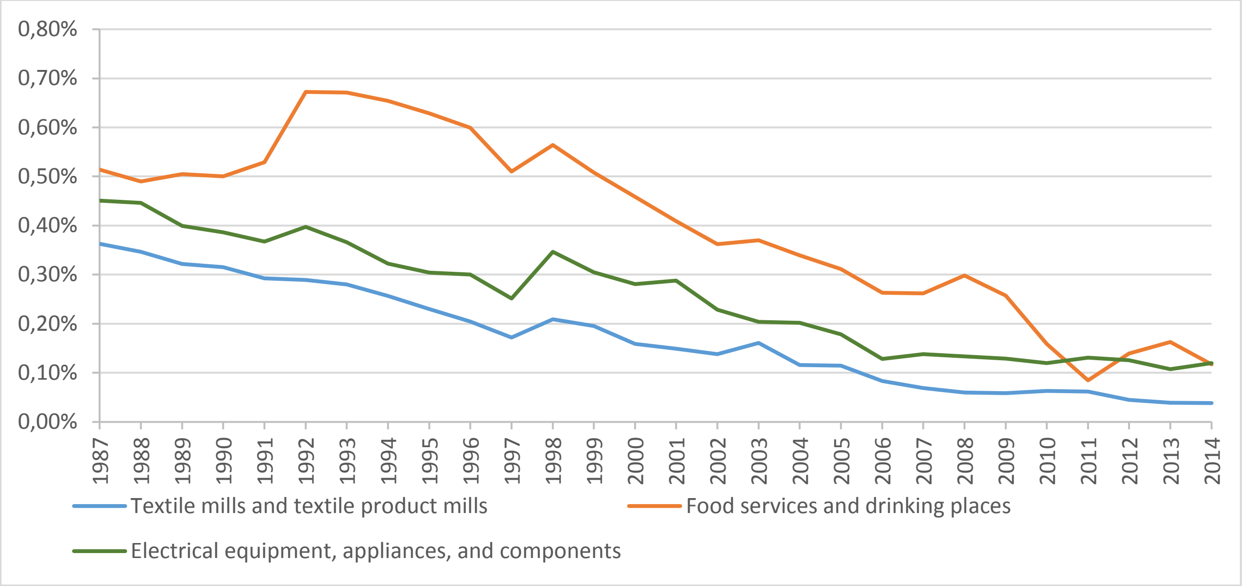

Figure 53 Examples of industries with significant technological catching-up effects

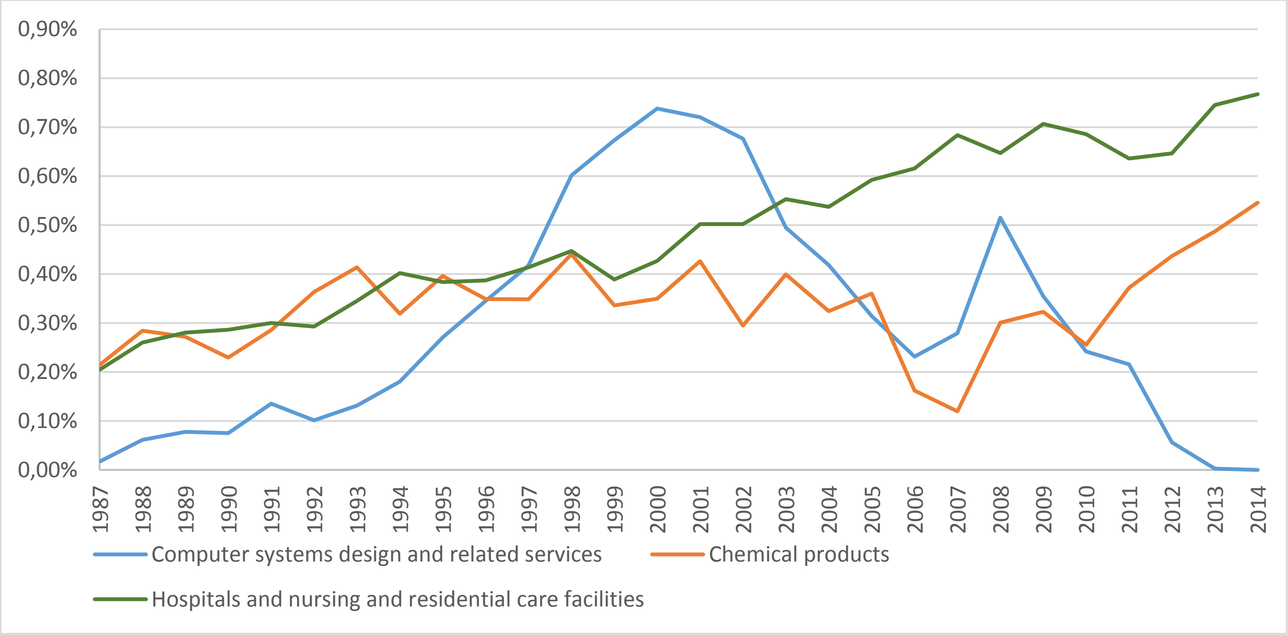

Figure 54 Examples of industries without technological catching-up effects

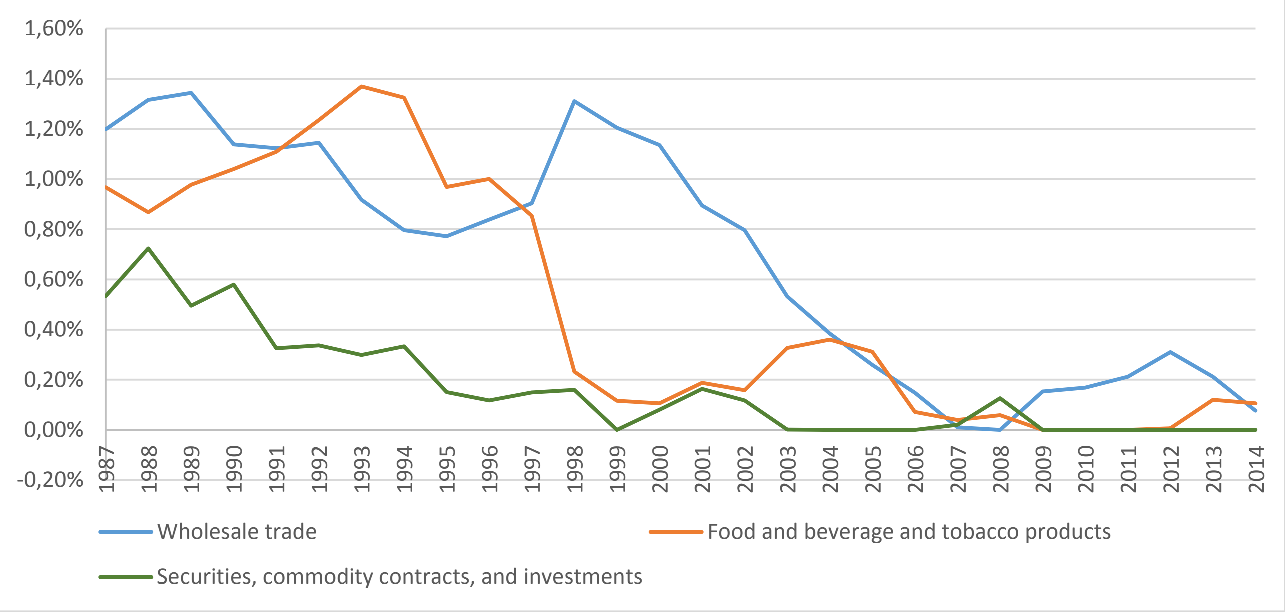

Figure 55 Examples of industries with homogenization of input-output mixes

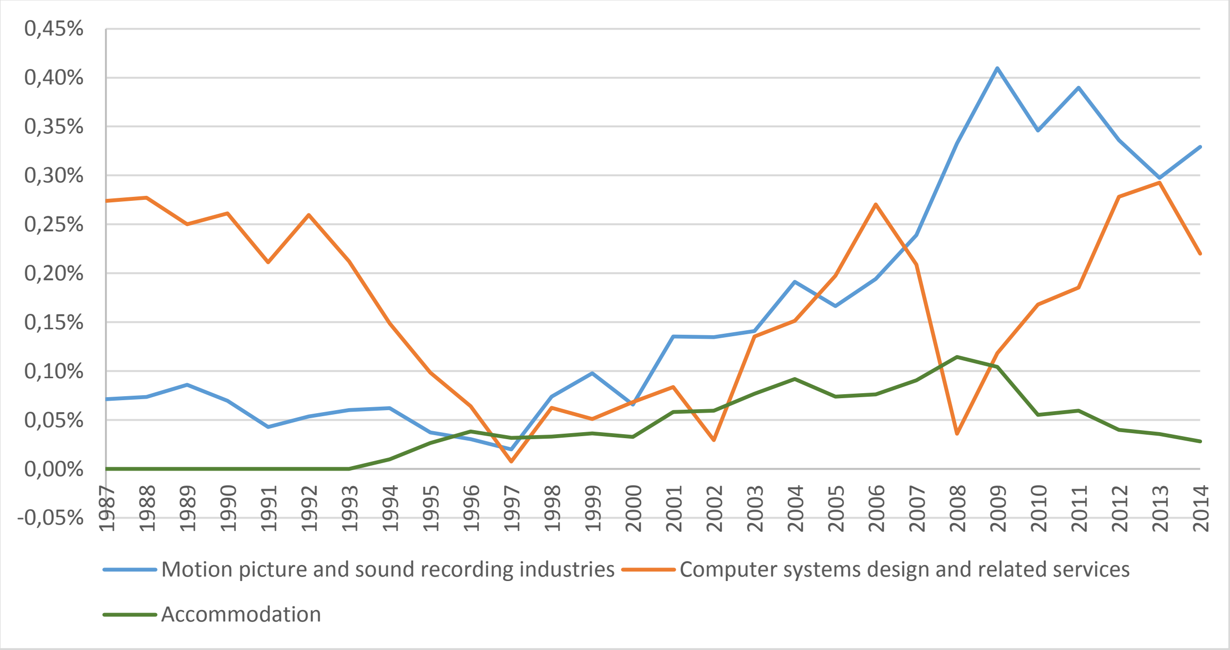

Figure 56 Examples of industries without homogenization of input-output mixes

Furthermore, the impacts of overall convergence process (technical + structural effects) on the US output growth can be estimated. By correcting all observed output levels with their respective annual inefficiency scores, one can compute the virtual output growth along the production frontier. Considering that technical and structural inefficiency reductions over time denote additional productivity gains for the US economy, the difference of the observed and the previous virtual growth rates gives an estimation of the impact of overall convergence process on the US growth which is around 0,64% (2,25% - 1,61%). These virtual and observed output changes are displayed in Figure 56.

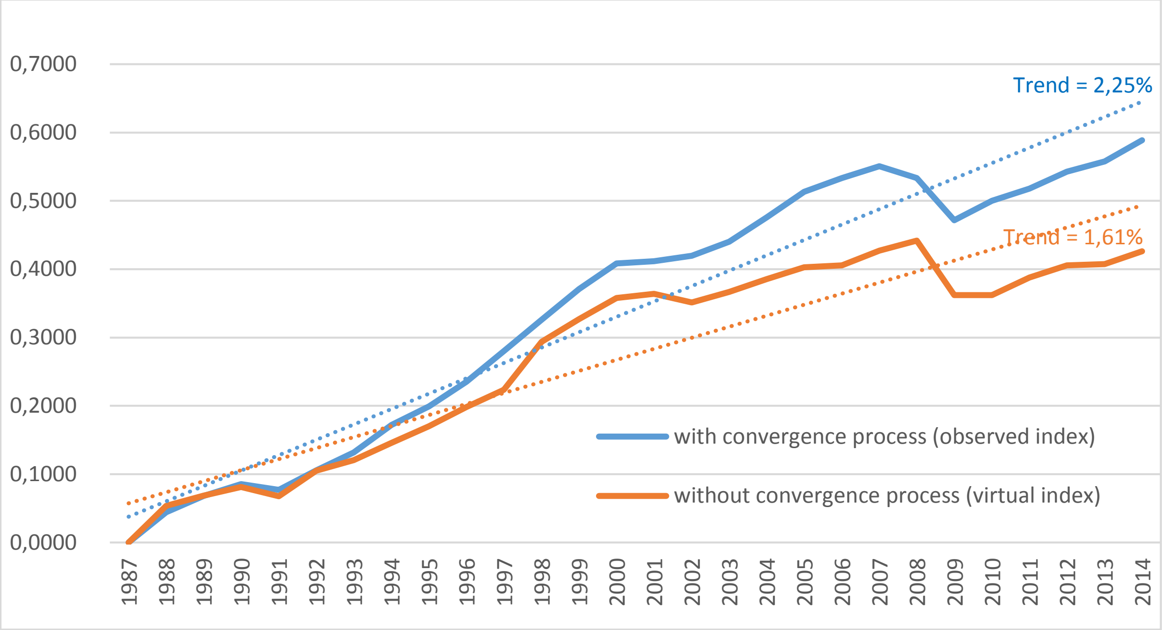

Figure 56 Gross-output in logarithm terms for all industries (100 =1987)

3.3 The linkage between stakeholder's price advantages and convergence processes

Technological catching-up and structural convergences processes by implying productivity gains at the industry and macro levels should be transferred to the different stakeholders (namely clients, intermediate input suppliers, employees, fixed capital suppliers and profitability). These productivity transfers may be related or not to the input/output price evolutions reflecting financial advantages or disadvantages that the stakeholders have benefitted or suffered over time. In this perspective, we investigate the long-run relationship between real output/input price evolutions and technical/structural efficiency scores at the industry level for the panel of 63 US industries over the period 1987-2014.

3.3.1 Definition of price advantages

Considering that a value change of any input *n* between two periods *t* and *t+1* can be split into Bennet price and quantity effects as:

$$\Delta(w^n x^n) = \left[\frac{1}{2}\left(x_t^n + x_{t+1}^n\right)\Delta w^n\right] + \left[\frac{1}{2}\left(w_{t+1}^n + w_t^n\right)\Delta x^n\right] \tag{10}$$

with $x_t^n =$ quantity of input *n* at period *t* and $w_t^n =$ price of input *n* at period *t*.

On the right hand side, the first bracket measures a price effect called the price advantage of the stakeholder related with input *n*. The price advantage or remuneration change over the two periods for any stakeholder is equal to the difference between the quantity weighted changes in its related input price. If $\Delta w^n > 0$ (price increasing between the two periods), the price advantage gives a positive remuneration change to the stakeholder. The second bracket measures a quantity effect related to the considered input. As a result, the combination of the price and quantity effects allows to retrieve the value change over the two periods.

In a similar way, a value change of any output *m* between periods t and t+1 is decomposed into Bennet price and a quantity effects as:

$$\Delta(p^m y^m) = \left[\frac{1}{2}\left(y_t^m + y_{t+1}^m\right)\Delta p^m\right] + \left[\frac{1}{2}\left(p_{t+1}^m + p_t^m\right)\Delta y^m\right] \tag{11}$$

with $\mathrm{y}_t^m =$ quantity of output *m* at period *t* and $p_t^m =$ price of output *m* at period *t*.

From equation (11), the price advantage related to the stakeholder or purchaser *m* is defined by $-\left[\frac{1}{2}\left(y_t^m + y_{t+1}^m\right)\Delta p^m\right]$ where the negative signs indicates a positive price advantage in case of a selling price decrease over the two periods.

3.3.2 Price advantages and TFP growth

Adopting a productivity accounting approach outlined by Grifell-Tatjé and Lovell (2015), we estimate productivity gains and their distribution among inputs and output for the US industries. In this perspective the traditional surplus accounting approach, initially developed by CERC[17] (1980), is performed in order to compute the respective stakeholders' price advantages related to the corresponding input and output price changes. These price advantages allow us to determine which stakeholders benefit or do not benefit from productivity gains over time.

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[17] Centre d'Etudes des Revenus et des Coûts

Furthermore, considering that the total output value is distributed into returns to the *n* different inputs, the following accounting identity holds for any particular industry:

$$\sum_{m=1}^{M} p_t^m y_t^m = \sum_{n=1}^{N} w_t^n x_t^n \tag{12}$$

Given the previous equation, changes in output and input values between periods t and t+1 can be measured in terms of changes in quantity and price components. Denoting that $p_{t+1}^m = (p_t^m + \Delta p^m)$, $y_t^m = (y_t^m + \Delta y^m)$, $w_{t+1}^n = (w_t^n + \Delta w^n)$ and $x_{t+1}^n = (x_t^n + \Delta x^n)$, after simplification and re-arrangement, equation (12) leads to equation (13):

$$\begin{gathered} \sum_{m=1}^{M} \frac{p_t^m + p_{t+1}^m}{2} \Delta y^m - \sum_{n=1}^{N} \frac{w_t^n + w_{t+1}^n}{2} \Delta x^n = -\sum_{m=1}^{M} \Delta p^m \, \frac{y_t^m + y_{t+1}^m}{2} + \sum_{n=1}^{N} \Delta w^n \, \frac{x_t^n + x_{t+1}^n}{2} \\ PS = PA \end{gathered} \tag{13}$$

In equation (13), the left hand side characterizes a productivity surplus (PS) defined as the difference between price weighted changes in output and input quantities while the right hand side aggregates the different stakeholders' price advantages (PA). Such price variations result in reallocations among stakeholders that are constrained by the productivity surplus level. More precisely, equation (13) ensures that the total amount of remuneration changes shared among the different agents (PA) cannot surpass the total productivity growth (PS).

Through equation (13), *PS* estimates productivity gains expressed in level terms (i.e. in dollars) which can also be directly linked to the usual Solow technical change residual as a measure of TFP changes defined in terms of relative growth rates (%).

Departing from the multi-output and multi-input production function:

$$\begin{gathered} F(\mathrm{y},\mathrm{x},t) = 0 \\ \text{with t a time trend} \\ \text{and x, y input and output vectors respectively} \\ \mathrm{x} = (x_1, x_2, ..., x_n, ..., x_N) \\ \mathrm{y} = (y_1, y_2, ..., y_m, ..., y_M). \end{gathered} \tag{14}$$

Supposing output prices equal to marginal costs and associating input prices to the marginal productivity levels, the Solow residual computed as a Törnqvist index of the TFP change over time is equal to the weighted output variations not explained by weighted input changes:

$$\frac{\Delta TFP}{TFP} = \sum_{m=1}^{M} \alpha_m \frac{\Delta y^m}{y^m} - \sum_{n=1}^{N} \beta_n \frac{\Delta x^n}{x^n} \tag{15}$$

where $\boldsymbol{\alpha}$ represents the vector of M output shares in total revenue and $\boldsymbol{\beta}$ the vector of N input shares in total cost.

Substituting $\alpha^m \frac{\Delta y^m}{y^m}$ by $\frac{p^m \Delta y^m}{\sum_{m=1}^{M} p^m y^m}$ and $\beta^n \frac{\Delta x^n}{x^n}$ by $\frac{w^n \Delta x^n}{\sum_{n=1}^{N+1} w^n x^n}$ and seeing that the total revenue equals the total cost ($\sum_{m=1}^{M} p^m y^m = \sum_{n=1}^{N+1} w^n x^n$), TFP growth rate can be measured as:

$$\frac{\Delta TFP}{TFP} = \frac{\sum_{m=1}^{M} p^m \Delta y^m - \sum_{n=1}^{N+1} w^n \Delta x^n}{\sum_{m=1}^{M} p^m y^m} \tag{16}$$

As a result, the TFP growth rate is just equal to the productivity surplus rate defined by the ratio between *PS* and the total output value. Additionally, an interesting link between TFP growth rate and price advantage changes can be proven. From the equality *PS* = *PA*, TFP growth rate is equivalent to the aggregation of price advantage ratios defined as the percentages between price advantages and the total output value):

$$\frac{\Delta TFP}{TFP} = \frac{PS}{\sum_{m=1}^{M} p^m y^m} = \frac{PA}{\sum_{m=1}^{M} p^m y^m} = \frac{-\sum_{m=1}^{M} \Delta p^m y_t^m + \sum_{n=1}^{N+1} \Delta w^n x_t^n}{\sum_{m=1}^{M} p^m y^m} \tag{17}$$

3.3.3 The model linking price advantages and technical and structural inefficiency scores

For each industry and each year, available data enable the establishment of the following balanced production account

Gross output value

=

Intermediates inputs + Compensation of employees + Depreciation of capital + Net operating Surplus

In this accounting identity, the net operating surplus can be equated to a cost which remunerates a virtual x^N including dividends, interest costs or managers' remunerations before tax. This specific cost gauges the capacity of an industry to achieve a financial surplus after covering the costs of intermediate consumptions and primary inputs (labor and fixed capital). Therefore, one can associate 5 different stakeholders: clients, suppliers of intermediate inputs, employees, suppliers of fixed capital and managers who are remunerated through the net operating surplus.

In this context, our model combines six equations. The first five equations are related to output/input price advantages. The sixth one refers to the previous equation (17) by linking the distribution of TFP gains between the five stakeholders. Consequently, the model can be described through the following simultaneous equations:

- one equation related to the gross output :

$$\left(-\frac{\Delta py}{py}\right)_a^{t+1/t} = \alpha\ \Delta I_a^{TECH,t+1/t} + \beta\ \Delta I_a^{STRUC,t+1/t} + d_a + f^t + \varepsilon_a^t \tag{18}$$

- four equations related to the inputs including the profitability:

$$\left(\frac{\Delta w^n x^n}{py}\right)_a^{t+1/t} = \alpha^n\ \Delta I_a^{TECH,t+1/t} + \beta^n\ \Delta I_a^{STRUC,t+1/t} + d_a^n + f^{n,t} + \varepsilon_a^{n,t} \tag{19}$$

- one equation linking the TFP growth rate to the different price advantages

$$\left(\frac{\Delta TFP}{TFP}\right)_a^{t+1/t} = \left(\frac{-\Delta py}{py}\right)_a^{t+1/t} + \sum_{n=1}^{4}\left(\frac{\Delta w^n x^n}{py}\right)_a^{t+1/t} \tag{20}$$

with:
$a = 1, 2, ..., 63$ industries
t = 1987, 1988, ..., 2014
$n =$ 1,2,3,4 inputs.

In case of technical catching-up and structural convergence processes, productivity gains occur. As a result, if the coefficients $\alpha, \alpha^n, \beta, \beta^n$ are negative, the productivity gains (derived from inefficiency decreases) exert positive effects on the stakeholders' price advantages.

From the last TFP identity, the error terms are assumed to be correlated across the other equations. This justifies to estimate them simultaneously through the seemingly unrelated regression procedure, proposed by Zellner (1962). The econometric results are presented in Table 47.

Results show that both technical and structural efficiency scores have positive effects on clients' price advantages and profitability besides negative effects on supplier's price advantages. Labor and fixed capital stakeholders seem to be not dependent on structural and technical efficiency scores. We also notice that compared to the structural component, technical inefficiency score has higher impact on buyers' price advantages and profitability. According to these results, it is obvious that over the last 28 years, industries with significant inefficiency decreases have profited clients and firms through lower output prices or higher profitability rates while relative compensations of the other inputs do not seem to be impacted by inefficiency changes over time. Intuitively, the price and cost convergence process related to the global mobility of production resources among industries could explain this point. Indeed, price changes for these inputs are mainly determined by their specific national market structure and their own availability in accordance with the macroeconomic business cycle. As a result, inefficiency reductions produced in a certain industry do not significantly impact the labor and fixed capital market prices. On the contrary, they should influence considerably final demand prices and profitability rates of the considered sector.

Table 47 SUR procedure results

Equation	Stakeholders	Coef.	T-stat	R2	DW
Output	**Clients**				
Tech. catching-up	a	**-18,72**	**-11,35**	**0,24**	1,83
Convergence process	b	**-4,03**	**-3,11**		
Intermediate inputs	**Suppliers**				
Tech. catching-up	a	**3,09**	**3,48**	**0,32**	2,06
Convergence process	b	**2,73**	**3,92**		
Labor	**Employees**				
Tech. catching-up	a	-0,4	-0,89	0,28	2,18
Convergence process	b	-0,48	-1,37		
Fixed Capital	**Capital providers**				
Tech. catching-up	a	-0,26	-1,56	0,15	0,77
Convergence process	b	0	0,01		
Profitability	**Managers**				
Tech. catching-up	a	**-13,21**	**-9,97**	**0,11**	2,2
Convergence process	b	**-5,65**	**-5,44**		

4 Conclusion

In this paper we propose to evaluate two types of inefficiency: technical inefficiency between an industry and its benchmark on the production frontier and structural inefficiency seen as heterogeneity between input-output mixes among sectors. We define these two inefficiencies both at individual and at group levels. Finally, we link these two inefficiency measures to the stakeholders' price advantages.

This analysis was applied to US industries from 1987-2014. The results clearly show that convergence is observed for both technical and structural efficiencies. This reveals that technological transfer and reallocation process among sectors generate significant productivity gains at the country level. We estimate the impact of these convergence processes on the US economy at around 0,64% of additional growth.

Then, a panel data analysis performed for 63 US industries over the considering period relates positive influence of the two convergence processes onto final demand prices and profitability and negative influence onto suppliers' prices. The clients and managers get significant benefit from efficiency gains which occur in their specific industries which is not the case for the suppliers. Finally, no link can be established between technological catching-up process and input-output mixes homogenization and employees or capital providers' remunerations. For these two stakeholders, it seems that their price changes essentially result from the macro business cycle and do not take benefit or disadvantage from sectoral efficiency gains.

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General conclusion

In this thesis we suggested new ways to link productivity, financial performance and price environment. More precisely, this document proposes three studies on productivity gains decomposition and distribution of the price effects among stakeholders. These works are all based on a common non parametric framework developed in the first chapter. As a main contribution of this research, the second chapter introduces a new decomposition of profit difference between two firms and links it to the surplus account methodology. As a second contribution, the third chapter presents a global price environment indicator based on price-quantities correspondences but also allows to define stakeholders' specific price indicators. Finally, the fourth chapter proposes an original decomposition of the aggregated technical inefficiency into two components and relates their changes over time to stakeholders' price advantages changes. In what follows, we provide some highlights of the main contributions and results.

Chapter 2 decomposes a profit gap between two firms into three quantity effects (technical inefficiency, allocative inefficiency and size component) and one price effect further split into stakeholders' price advantages. As a result, the sum of the quantity effects equaling the productivity surplus is distributed to the stakeholders through their respective price advantages/disadvantages including the profit gap related to the managers' compensations. Compared to the usual CERC's approach, the originality of our work is that profit variations are studied at a cross-sectional dimension rather than a temporal one. Another specificity is that we systematically use Bennet indicators for all quantity and price effects. This methodology allows to identify both the effects of differences in technical efficiency, resource allocation and size and the effect of different price environment on the profit gap between two firms. Moreover, when seen from the surplus accounting perspective, it allows to determine the sources of productivity surplus that is then distributed among stakeholders. This framework was applied to a sample of US banks over the period 2001-2012. We implemented our analysis in term of profit rate differences and focused on banks with positive mean profit rate gap (performant banks). The results showed that over the considering period performant banks benefitted from positive price environment but were inefficient in their productivity levels. The main providers of financial resources for performant banks were creditors and employees over the entire period and suppliers after the 2007-2008 financial crisis. Besides, a decrease in allocative inefficiency is observed after the crisis. Finally, a comparison analysis of commercial and savings banks revealed that the main source of price advantages was creditors for the former and suppliers for the latter over the whole period.

Chapter 3 proposes a relative measure of price environment among firms. It is based on two efficiency frontiers obtained with observed quantities and values respectively. Comparing distances towards these two frontiers leads to a definition of a price environment indicator considering both quantities and prices as decision variables. This methodology can be used both for a global measure of price environment and for a specific input or output price environment. The implementation used US industries over the period 1987-2014. The results indicated that global mean price environment for all sectors was deteriorating over the entire period which can be related to the increasing degree of openness of US economy. This analysis showed that all specific input/output effects were statistically significant. A strong influence could be observed for capital, gross output and labor price environments. Intermediate inputs affected global price environment much less. Besides, structural breaks occurred in the beginning of 2000s and around 2005-2007 (the financial crisis).

In chapter 4, we analyze the convergence process for technical inefficiency at the whole US economy level. We split the aggregated inefficiency evolution over time into two components: technological catching-up process and input-output homogenization process. The results clearly confirm the convergence processes for both technological catching-up and input-output mixes. Using the link existing between TFP growth rate and price advantages/disadvantages, we then estimate the influence of technical and structural inefficiency changes on each stakeholder's compensation. A panel data regression revealed that customers and managers benefitted substantially from the two convergence processes while the opposite was found for suppliers. Employees and capital providers seemed to not be affected by technical inefficiency and input-output mixes convergence processes since their price changes seem essentially driven by the macro business cycle.

In conclusion, this essay tends to show that generation of productivity gains and their distribution are two sides of the same coin. The former is related to the economic analysis of TFP based on the estimation of a production technology while the latter deals with an accounting approach of business performance. The main contributions of this work was first to relate these two aspects from a methodological point of view and then to show the usefulness of this joint analysis in terms of quantity and price effects for real world applications. Thanks to this analytical framework, we are now able to go beyond the simple measure of productive efficiency by relating its technical, allocative and size components to the stakeholders' advantages. By considering that firms should not only to be studied from the production side but also from their trading relationships, we contribute to build the bridge between economists and managers in the evaluation of business performances.

Appendix 1 Chapter 1

Implementation in SAS of the numerical example from section 2.2.2

a) Primal directional distance function

```
/*+-----------------------------------------------------------+
|        DEA - PRIMAL DIRECTIONAL DISTANCE FUNCTION (LP3)   |
+-----------------------------------------------------------+*/
options nosource nonotes;

%MACRO primal_DDF();
            /*we create tables of inputs and outputs*/
            data Inputs;
                        set banks.ex_herve (keep=DMU XVOL1);
            run;

            data Outputs;
                        set banks.ex_herve (keep=DMU YVOL1);
            run;

            /*we create a table lambda with only a DMU column to collect all lambda vectors
            calculated later*/
            proc sql ;
                        create table lambda as select Inputs.DMU from Inputs;
                        run;

                        /*we calculate number of banks*/
            proc sql ;
                        create table num_banks as select count(Inputs.DMU) as num_banks from Inputs;
                        run;

            DATA _NULL_;
                        SET num_banks;
                        CALL SYMPUT('num_banks', num_banks);
                        STOP;
            RUN;

            data Score_output;
                        set _NULL_;
            run;

            /*Loop Through DMUs*/


            %do LOOP_COUNT=1 %to &num_banks.;

                        proc optmodel printlevel=0;
                                    /*we declare variables and number of DMUs*/
                                    set Inputs=1.. 1;
                                    set Outputs=1.. 1;
                                    set <num> DMUs;

                                    /*we declare matrices of inputs, outputs*/
                                    number X{DMUs, Inputs};
                                    number Y{DMUs, Outputs};

                                    /*we read data into declared matrices*/
                                    read data Inputs into DMUs=[DMU]
{i in Inputs} < X[DMU, i]=col("XVOL"||i) >;
                                    read data Outputs into DMUs=[DMU]
{r in Outputs} < Y[DMU, r]=col("YVOL"||r) >;

                                    /*we declare variables that we consider for optimization*/
                                    var theta, lambda{DMUs}>=0;
```

```
                                        /*we declare variable that we want to optimize*/
                                        Max Efficiency_output=theta;

                                        /*we declare the constraints*/
                                        con Out {r in Outputs}: sum{j in DMUs}(Y[j, r]*lambda[j]) >=Y[&LOOP_COUNT,
                                                r]+theta*Y[&LOOP_COUNT, r];
                                        con In {i in Inputs}: sum{j in DMUs}(X[j, i]*lambda[j]) <=X[&LOOP_COUNT,
                                                i]-theta*X[&LOOP_COUNT, i];

                                        /*we declare a VRS constraint*/
                                        con VRS: sum{j in DMUs}(lambda[j])=1;
                                        solve;

                                        /*we retrieve lambda vectors for each DMU*/
                                        create data lambda_temp from [DMU]={j in DMUs: lambda[j].sol >=-1}
lambdadata_&LOOP_COUNT.=lambda;

                                        /*we retrieve efficiecny scores for each DMU*/
                                        create data Score_output_temp from DMU=&LOOP_COUNT
Efficiency_output=Efficiency_output;
                                quit;

                                /*we collect all lambda vectors*/
                                data lambda;
                                        merge lambda lambda_temp;
                                        by DMU;
                                run;

                                data Score_output;
                                        set Score_output Score_output_temp;
                                run;

                %end;
%MEND;

%primal_DDF();
```

b) Dual directional distance function

```
/*+-----------------------------------------------------------+
|     DEA - DUAL DIRECTIONAL DISTANCE FUNCTION (LP3")        |
+-----------------------------------------------------------+*/
options nosource nonotes;

%MACRO dual_DDF();
        /*we create tables of inputs and outputs*/
        data Inputs;
                set banks.ex_herve (keep=DMU XVOL1);
        run;

        data Outputs;
                set banks.ex_herve (keep=DMU YVOL1);
        run;

        /*we calculate number of banks*/
        proc sql ;
                create table num_banks as select count(Inputs.DMU) as num_banks from Inputs;
                run;

        DATA _NULL_;
                SET num_banks;
                CALL SYMPUT('num_banks', num_banks);
                STOP;
        RUN;
```

```
            /*Loop Through DMUs*/


            %do LOOP_COUNT=1 %to &num_banks.;

                        proc optmodel printlevel=0;
                                    /*we declare variables and number of DMUs*/
                                    set Inputs=1.. 1;
                                    set Outputs=1.. 1;
                                    set <num> DMUs;

                                    /*we declare matrices of inputs, outputs*/
                                    number X{DMUs, Inputs};
                                    number Y{DMUs, Outputs};

                                    /*we read data into declared matrices*/
                                    read data Inputs into DMUs=[DMU]
{i in Inputs} < X[DMU, i]=col("XVOL"||i) >;
                                    read data Outputs into DMUs=[DMU]
{r in Outputs} < Y[DMU, r]=col("YVOL"||r) >;

                                    /*we declare variables that we consider for optimization*/
                                    var pi, u{Outputs}>=0, v{Inputs}>=0;

                                    /*we declare variable that we want to optimize*/
                                    Min profit=pi;

                                    /*we declare the constraints*/
                                    con technology {k in DMUs}: (sum{m in Outputs}(Y[k, m]*u[m])-sum{n in
                                                Inputs}(X[k, n]*v[n]))-
(sum{m in Outputs}(Y[&LOOP_COUNT, m]*u[m])-sum{n in Inputs}(X[&LOOP_COUNT,
                                                n]*v[n])) <=pi;
                                    con linear: sum{m in Outputs}(Y[&LOOP_COUNT, m]*u[m]) +
sum{n in Inputs}(X[&LOOP_COUNT, n]*v[n])=1;
                                    solve with lp / solver=dual_spx;

                                    /*we retrieve optimized variable as well as shadow prices*/
                                    create data pi_temp from DMU=&LOOP_COUNT
pi=pi;
                                    create data u from [DMU]
u=u;
                                    create data v from [DMU]
v=v;
                        quit;

                        /*we collect all shadow profits*/
                        proc append base=profit data=pi_temp;
                        run;

            %end;
%MEND;

%dual_DDF();
```

Appendix 2 Chapter 2

SAS codes to compute technically efficient and profit maximizing benchmarks

a) Computation of technically efficient benchmark

```
/*+----------------------------------------------------------+
|        DEA - PRIMAL DIRECTIONAL DISTANCE FUNCTION        |
+-----------------------------------------------------------+*/
options nosource nonotes;

%MACRO primal_DDF();
        %do j=2001 %to 2012;

                /*we create tables of inputs and outputs*/
                data Inputs_&j.;
                        set banks.us_&j (keep=DMU XVOL1-XVOL4);
                run;

                data Outputs_&j.;
                        set banks.us_&j (keep=DMU YVOL1-YVOL3);
                run;

                /*we sort them to transpose and count their number*/
                proc sort data=Inputs_&j.;
                        by DMU;
                run;

                proc sort data=Outputs_&j.;
                        by DMU;
                run;

                proc transpose data=Inputs_&j. out=countins;
                run;

                data _null_;
                        set countins;
                        call symput('INP_COUNTER', _N_-1);
                run;

                proc transpose data=Outputs_&j. out=countouts;
                run;

                data _null_;
                        set countouts;
                        call symput('OUT_COUNTER', _N_-1);
                run;

                /*we create a table with 4 columns : DMU and mean values for each output*/
                proc sql ;
                        create table mean_outputs as select Outputs_&j..DMU,
                                sum(Outputs_&j..YVOL1)/count(Outputs_&j..DMU) as mean_outputs_1,
                                sum(Outputs_&j..YVOL2)/count(Outputs_&j..DMU) as mean_outputs_2,
                                sum(Outputs_&j..YVOL3)/count(Outputs_&j..DMU) as mean_outputs_3 from
                                Outputs_&j. as Outputs_&j.;
                        run;

                        /*we create a table lambda with only a DMU column to collect all lambda vectors
                        calculated later*/
                proc sql ;
                        create table lambda as select mean_outputs.DMU from mean_outputs as
                                mean_outputs;
                        run;

                        /*we calculate number of banks*/
                proc sql ;
```

```
            create table num_banks as select count(Inputs_&j..DMU) as num_banks from
                Inputs_&j.;
            run;

        DATA _NULL_;
            SET num_banks;
            CALL SYMPUT('num_banks', num_banks);
            STOP;
        RUN;

        /*Loop Through DMUs*/


        %do LOOP_COUNT=1 %to &num_banks.;

            proc optmodel printlevel=0;
                /*we declare variables and number of DMUs*/
                set Inputs=1.. &INP_COUNTER;
                set Outputs=1.. &OUT_COUNTER;
                set mean_outputs=1.. &OUT_COUNTER;
                set <num> DMUs;

                /*we declare matrices of inputs, outputs and mean outputs*/
                number X{DMUs, Inputs};
                number Y{DMUs, Outputs};
                number mean_Y{DMUs, mean_outputs};

                /*we read data into declared matrices*/
                read data Inputs_&j.
into DMUs=[DMU]
{i in Inputs} < X[DMU, i]=col("XVOL"||i) >;
                read data Outputs_&j.
into DMUs=[DMU]
{r in Outputs} < Y[DMU, r]=col("YVOL"||r) >;
                read data mean_outputs into DMUs=[DMU]
{r in mean_outputs} < mean_Y[DMU, r]=col("mean_outputs_"||r) >;

                /*we declare variables that we consider for optimization*/
                var theta, lambda{DMUs}>=0;

                /*we declare variable that we want to optimize*/
                Max Efficiency_output=theta;

                /*we declare the technology constraints*/
                con Out {r in Outputs}: sum{j in DMUs}(Y[j, r]*lambda[j]) >=Y[&LOOP_COUNT,
                    r]+theta*mean_Y[&LOOP_COUNT, r];
                con In {i in Inputs}: sum{j in DMUs}(X[j, i]*lambda[j]) <=X[&LOOP_COUNT, i];

                /*we declare a VRS constraint*/
                con VRS: sum{j in DMUs}(lambda[j])=1;
                solve;

                /*we retrieve lambda vectors for each DMU*/
                create data lambda_temp from [DMU]={j in DMUs: lambda[j].sol >=-1}
lambdadata_&LOOP_COUNT.=lambda;

                /*we retrieve efficiency score (theta)*/
                /*we retrieve efficiency scores for each DMU*/
                create data Score_output_temp from DMU=&LOOP_COUNT
Efficiency_output=Efficiency_output;
            quit;

            /*we collect all lambda vectors*/
            data lambda;
                merge lambda lambda_temp;
                by DMU;
            run;
```

```
          /*we collect all efficiency scores*/
          proc append base=Score_output data=Score_output_temp;
          run;

%end;

/*we merge lambda vectors and vectors of observed outputs*/
data temp_eff;
          merge lambda Outputs_&j.;
          by DMU;
run;

/*we multiply all lambda vectors to each of observed outputs*/
data temp_eff;
          set temp_eff;

          %do i=1 %to &num_banks.;
                    Yeff1_&i.=lambdadata_&i.*YVOL1;
                    Yeff2_&i.=lambdadata_&i.*YVOL2;
                    Yeff3_&i.=lambdadata_&i.*YVOL3;
          %end;
run;

/*we compute efficient vector for each output*/
/*first output*/
data Yeff1;
          set _NULL_;
run;

%do i=1 %to &num_banks.;

          proc sql ;
                    create table Yeff1_&i as select sum(temp_eff.Yeff1_&i) as yeff1 from
                              temp_eff;
          quit;

          data Yeff1;
                    set Yeff1 Yeff1_&i;
          run;

          data Yeff1;
                    set Yeff1;
                    unit=1;
          run;

%end;

/*second output*/
data Yeff2;
          set _NULL_;
run;

%do i=1 %to &num_banks.;

          proc sql ;
                    create table Yeff2_&i as select sum(temp_eff.Yeff2_&i) as yeff2 from
                              temp_eff;
          quit;

          data Yeff2;
                    set Yeff2 Yeff2_&i;
          run;

          data Yeff2;
                    set Yeff2;
                    unit=1;
          run;
```

```
%end;

/*third output*/
data Yeff3;
            set _NULL_;
run;

%do i=1 %to &num_banks.;

            proc sql ;
                        create table Yeff3_&i as select sum(temp_eff.Yeff3_&i) as yeff3 from
                                    temp_eff;
            quit;

            data Yeff3;
                        set Yeff3 Yeff3_&i;
            run;

            data Yeff3;
                        set Yeff3;
                        unit=1;
            run;

            proc sql ;
                        drop table Yeff1_&i;
                        drop table Yeff2_&i;
                        drop table Yeff3_&i;
            quit;

%end;

/*we merge the three efficient outputs into one table*/
data eff_outputs;
            merge Yeff1 Yeff2 Yeff3;
            by unit;
run;

/*we add DMU column*/
data eff_outputs (drop=unit);
            retain DMU;
            set eff_outputs;
            retain DMU 0;
            DMU=DMU+1;
run;

/*we sort efficient output vectors and efficiency scores to add them to report table*/
proc sort data=eff_outputs;
            by DMU;
run;

proc sort data=Score_output;
            by DMU;
run;

/*we create a report table with observed inputs, observed outputs, efficiency vector and
3 technically efficient output vectors*/
data banks.Report_&j.;
            merge Inputs_&j. Outputs_&j. Score_output eff_outputs;
            by DMU;
run;

/*we delete temporary tables*/
proc sql ;
            drop table Inputs_&j.;
            drop table Outputs_&j.;
            drop table Score_output;
            drop table eff_outputs;
            drop table countins;
```

```
                        drop table countouts;
                        drop table mean_outputs;
                        drop table num_banks;
                        drop table Score_output_temp;
                        drop table lambda;
                        drop table lambda_temp;
                        drop table temp_eff;
                        drop table Yeff1;
                        drop table Yeff2;
                        drop table Yeff3;
                        run;
                %end;
        %MEND;

        %primal_DDF();
```

b) Computation of profit maximizing benchmark

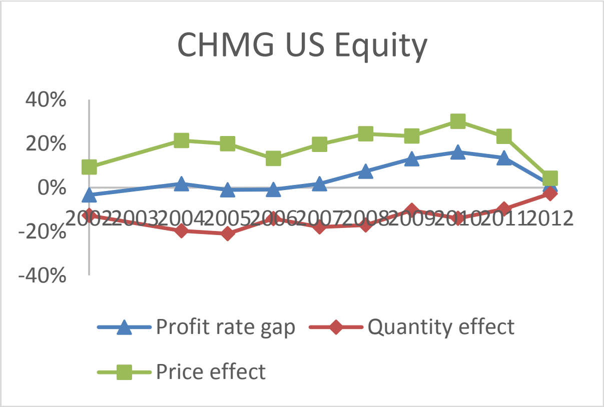
```
/*+-------------------------------------------------------------+
|     DEA – PROFIT MAXIMIZING MODEL       |
+-------------------------------------------------------------+*/
options nosource nonotes;

%MACRO dual_DDF();
        /*we create preliminary tables for shadow prices and shadow profit*/
        data u;
                set banks.us_2001;
                keep DMU;

                if DMU in (1, 2, 3) then
                        output;
        run;

        data v;
                set banks.us_2001;
                keep DMU;

                if DMU in (1, 2, 3, 4) then
                        output;
        run;

        data shadow_profits (keep=DMU);
                set banks.us_2001;
        run;

        %do j=2001 %to 2012;

                /*we create tables of inputs and outputs*/
                data Inputs_&j.;
                        set banks.us_&j (keep=DMU XVOL1-XVOL4);
                run;

                data Outputs_&j.;
                        set banks.us_&j (keep=DMU YVOL1-YVOL3);
                run;

                /*we sort them to transpose and count their number*/
                proc sort data=Inputs_&j.;
                        by DMU;
                run;

                proc sort data=Outputs_&j.;
                        by DMU;
                run;

                proc transpose data=Inputs_&j. out=countins;
```

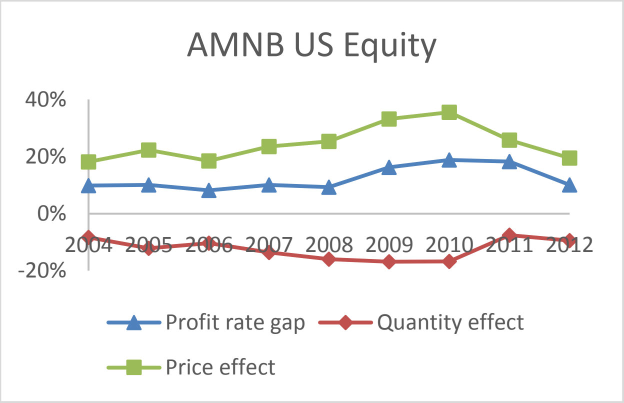
```
                run;

                data _null_;
                        set countins;
                        call symput('INP_COUNTER', _N_-1);
                run;

                proc transpose data=Outputs_&j. out=countouts;
                run;

                data _null_;
                        set countouts;
                        call symput('OUT_COUNTER', _N_-1);
                run;

                /*we create a table with 4 columns : DMU and mean values for each output*/
                proc sql ;
                        create table mean_outputs as select Outputs_&j..DMU,
                                sum(Outputs_&j..YVOL1)/count(Outputs_&j..DMU) as mean_outputs_1,
                                sum(Outputs_&j..YVOL2)/count(Outputs_&j..DMU) as mean_outputs_2,
                                sum(Outputs_&j..YVOL3)/count(Outputs_&j..DMU) as mean_outputs_3 from
                                Outputs_&j. as Outputs_&j.;
                        run;

                        /*we calculate number of banks*/
                proc sql ;
                        create table num_banks as select count(Inputs_&j..DMU) as num_banks from
                                Inputs_&j.;
                        run;

                DATA _NULL_;
                        SET num_banks;
                        CALL SYMPUT('num_banks', num_banks);
                        STOP;
                RUN;

                /*Loop Through DMUs*/


                %do LOOP_COUNT=1 %to &num_banks.;

                        proc optmodel printlevel=0;
                                /*we declare variables and number of DMUs*/
                                set Inputs=1.. &INP_COUNTER;
                                set Outputs=1.. &OUT_COUNTER;
                                set mean_outputs=1.. &OUT_COUNTER;
                                set <num> DMUs;

                                /*we declare matrices of inputs, outputs and mean outputs*/
                                number X{DMUs, Inputs};
                                number Y{DMUs, Outputs};
                                number mean_Y{DMUs, mean_outputs};

                                /*we read data into declared matrices*/
                                read data Inputs_&j.
into DMUs=[DMU]
{i in Inputs} < X[DMU, i]=col("XVOL"||i) >;
                                read data Outputs_&j.
into DMUs=[DMU]
{r in Outputs} < Y[DMU, r]=col("YVOL"||r) >;
                                read data mean_outputs into DMUs=[DMU]
{r in mean_outputs} < mean_Y[DMU, r]=col("mean_outputs_"||r) >;

                                /*we declare variables that we consider for optimization*/
                                var pi, u{Outputs}>=0, v{Inputs}>=0;

                                /*we declare variable that we want to optimize*/
                                Min profit=pi;
```

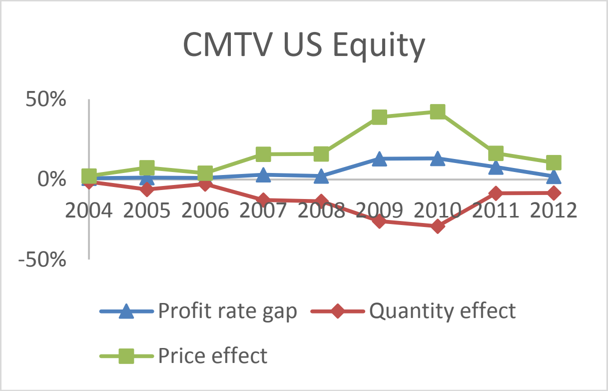
```
                                        /*we declare the constraints*/
                                        con technology {k in DMUs}: (sum{m in Outputs}(Y[k, m]*u[m])-sum{n in
                                                Inputs}(X[k, n]*v[n]))-
(sum{m in Outputs}(Y[&LOOP_COUNT, m]*u[m])-sum{n in Inputs}(X[&LOOP_COUNT,
                                                n]*v[n])) <=pi;
                                        con linear: sum{m in Outputs}(mean_Y[&LOOP_COUNT, m]*u[m])=1;
                                        solve with lp / solver=dual_spx;

                                        /*we retrieve optimized variable as well as shadow prices*/
                                        create data pi_temp from DMU=&LOOP_COUNT
pi=pi;
                                        create data u_&j from [DMU]
u_&j=u;
                                        create data v_&j from [DMU]
v_&j=v;
                                quit;

                                /*we collect all shadow profits*/
                                proc append base=profit_&j  data=pi_temp;
                                run;

                        %end;

                        data profit_&j (rename=(pi=pi_&j));
                                set profit_&j;
                        run;

                        /*we collect all shadow prices*/
                        data u;
                                merge u u_&j;
                                by DMU;
                        run;

                        data v;
                                merge v v_&j;
                                by DMU;
                        run;

                        proc sql ;
                                drop table inputs_&j;
                                drop table outputs_&j;
                                drop table u_&j;
                                drop table v_&j;
                        %end;
                %MEND;

                %dual_DDF();
```

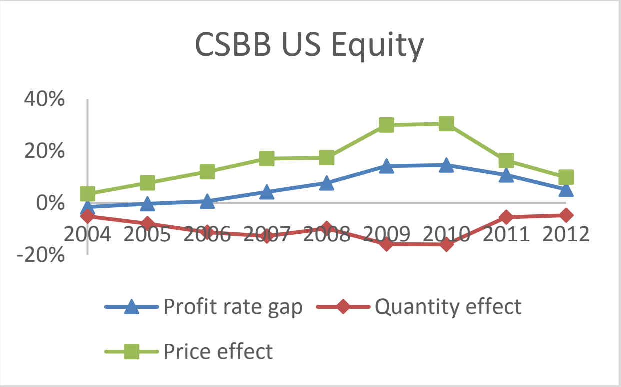
Appendix 3 Chapter 2

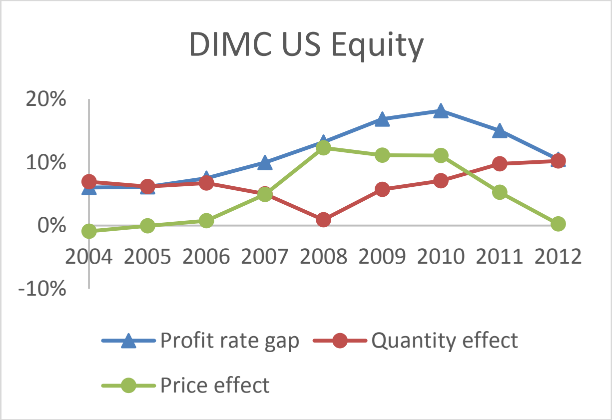
Profit rate gap evolution and its decomposition into quantity and price effects for a sample of 35 US banks presented over at least 2004-2012

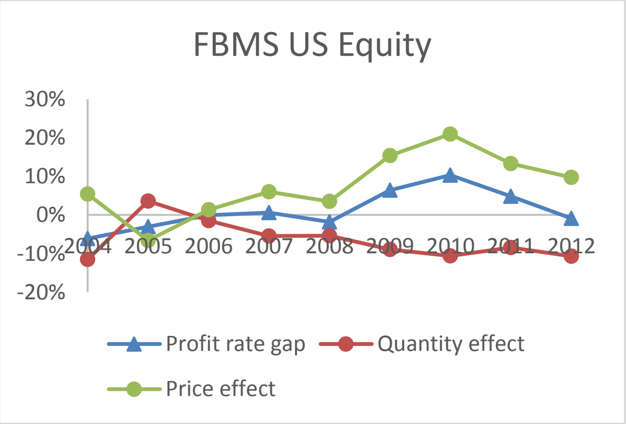


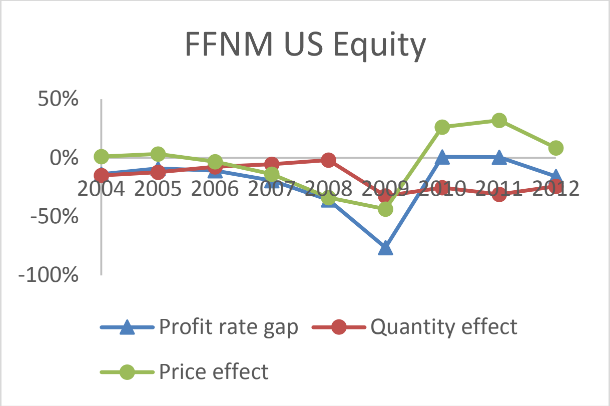


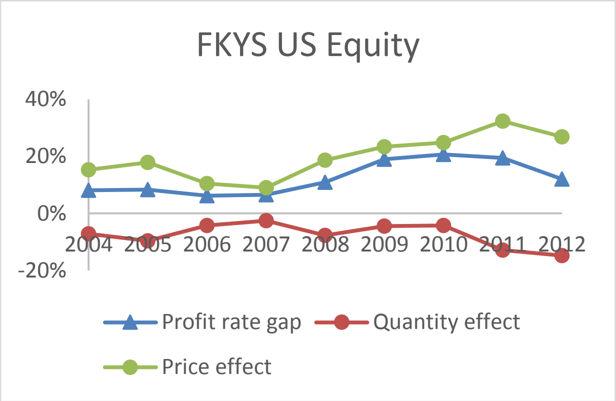









FFNM US Equity
50%
0%
-50%
-100%
2004 2005 2006 2007 2008 2009 2010 2011 2012
Profit rate gap
Quantity effect
Price effect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20%
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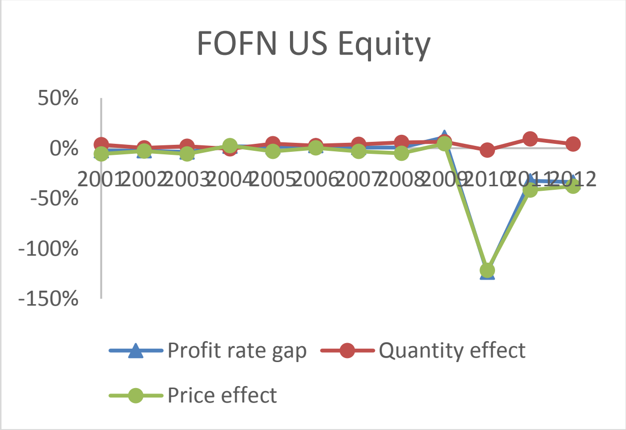
2004 2005 2006 2007 2008 2009 2010 2011 2012
Profit rate gap
Quantity effect
Price effect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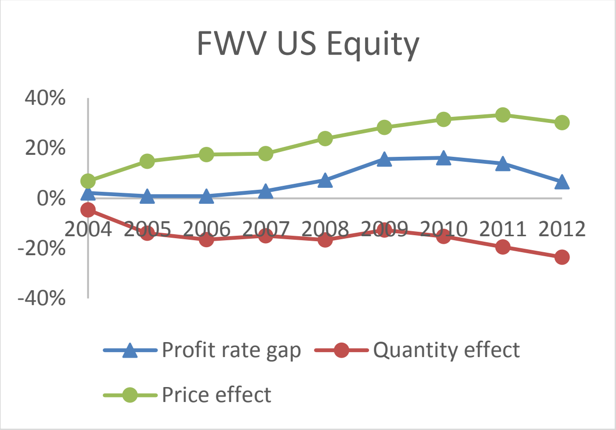
Profit rate gap
Quantity effect
Price effect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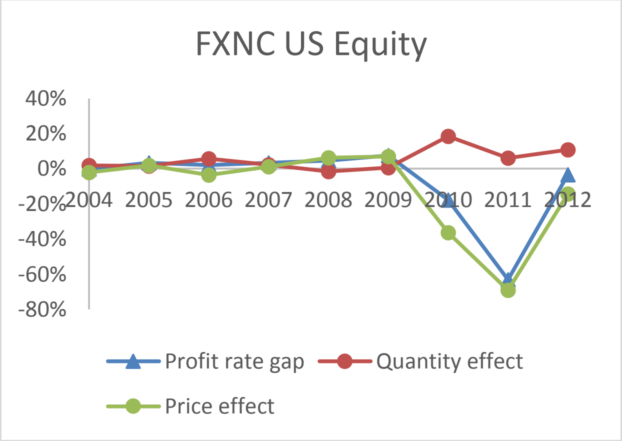
Quantity effect
Price effect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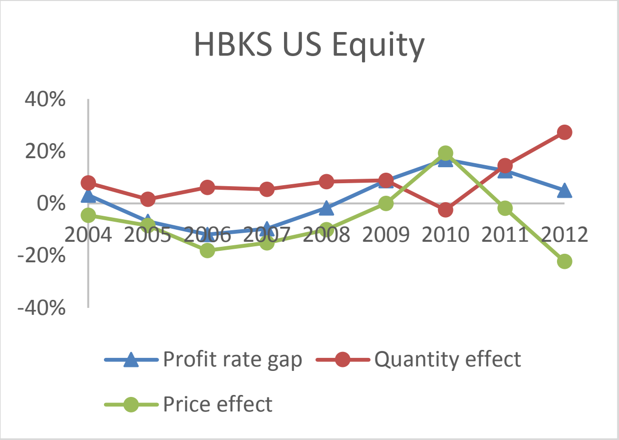
Price effect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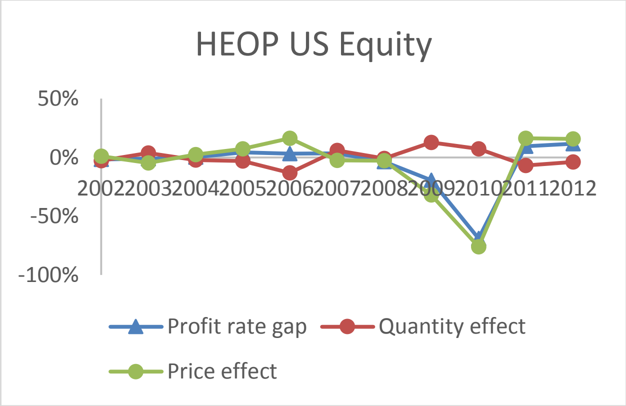
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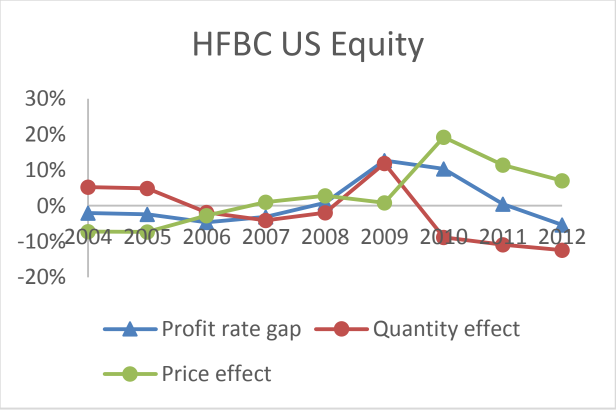
Profit rate gap
Quantity effect
Price effect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Profit rate gap
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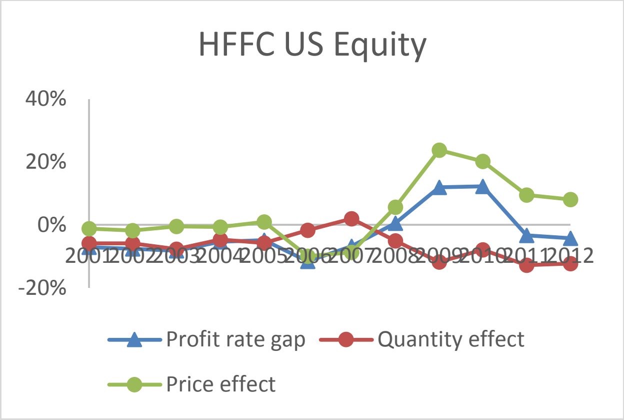
Price effect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Quantity effect
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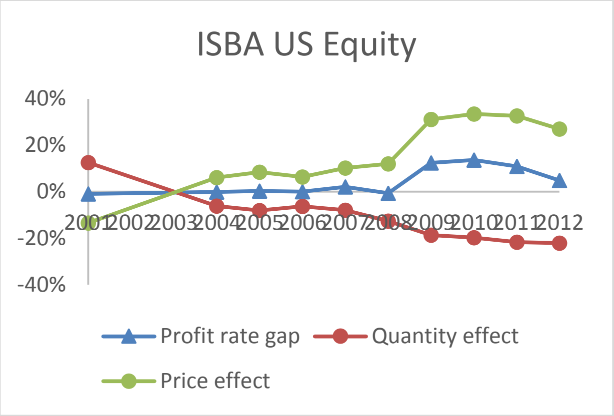
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Profit rate gap
Quantity effect
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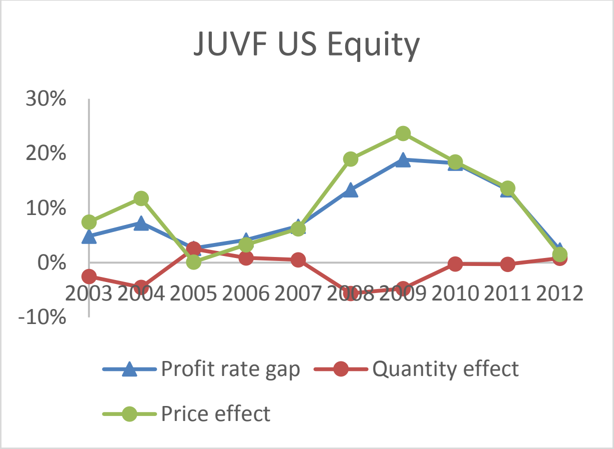
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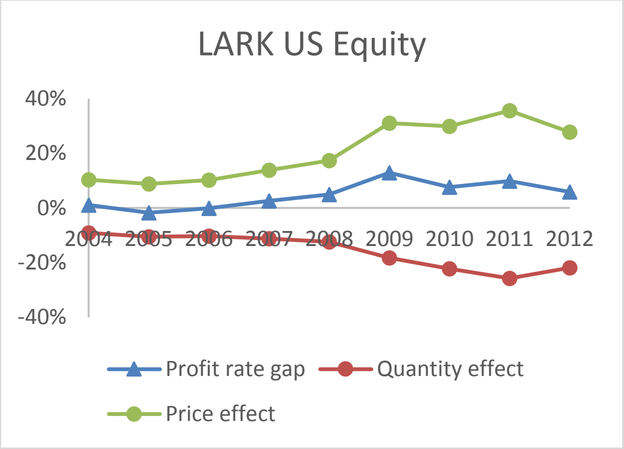
Price effect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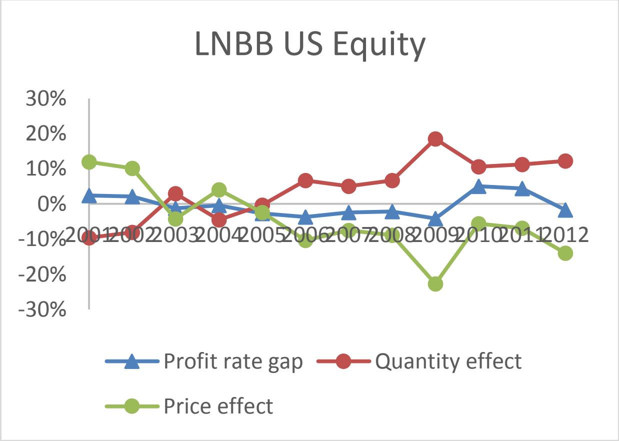
Quantity effect
Price effect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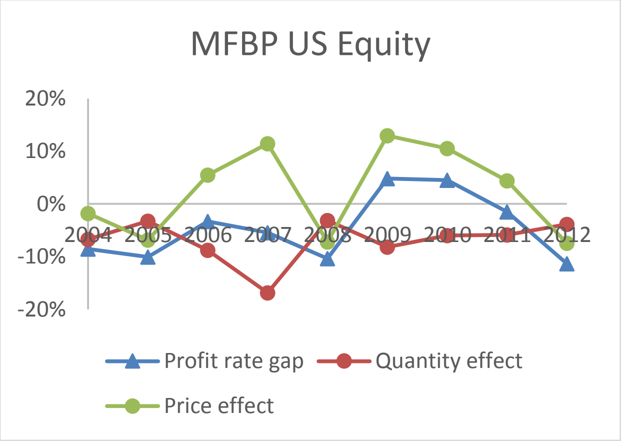
Price effect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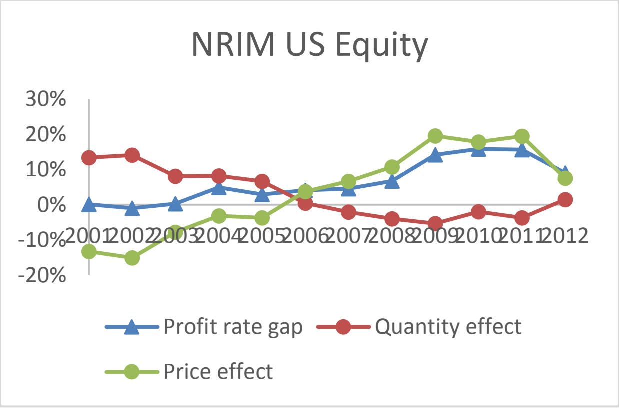
Quantity effect
Price effect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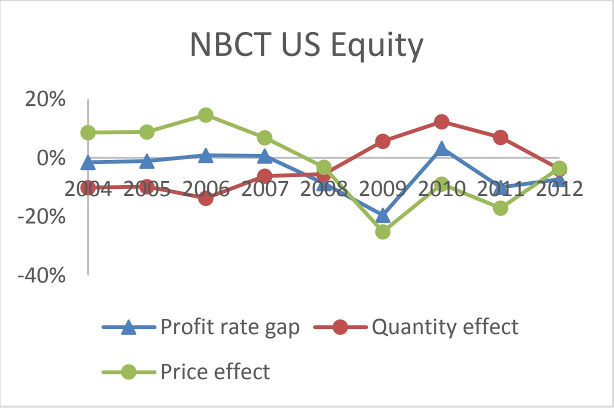
Profit rate gap
Quantity effect
Price effect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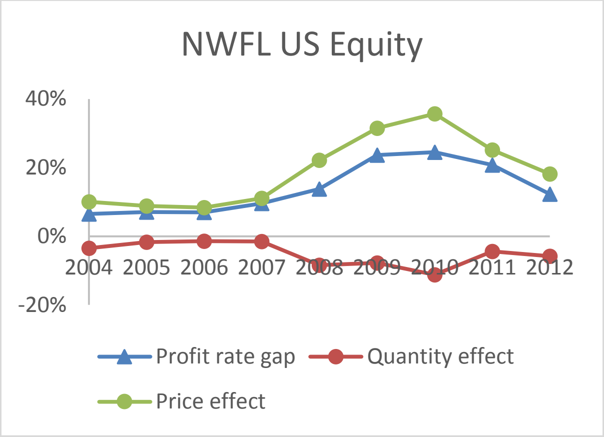
Profit rate gap
Quantity effect
Price effect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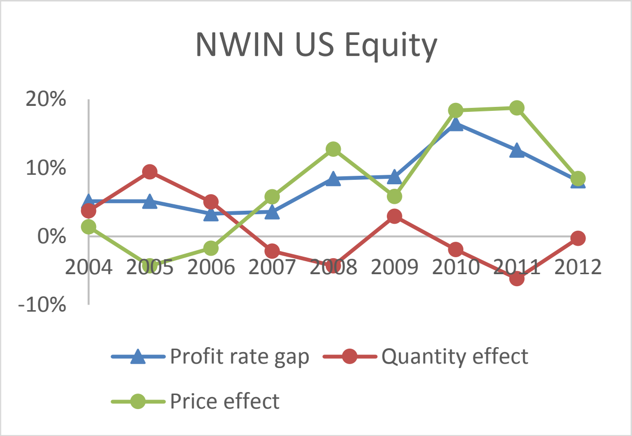
Profit rate gap
Quantity effect
Price effect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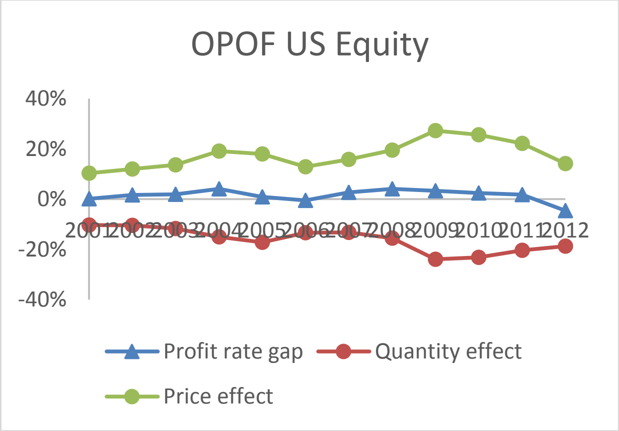
Profit rate gap
Quantity effect
Price effect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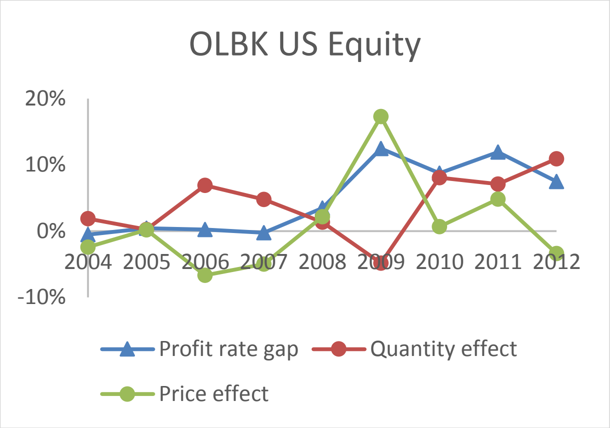
Profit rate gap
Quantity effect
Price effect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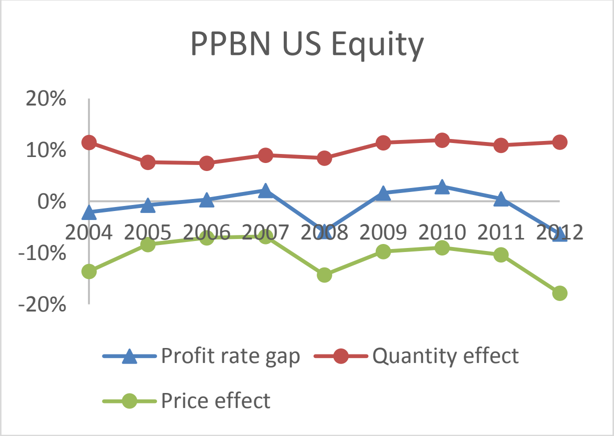
Profit rate gap
Quantity effect
Price effect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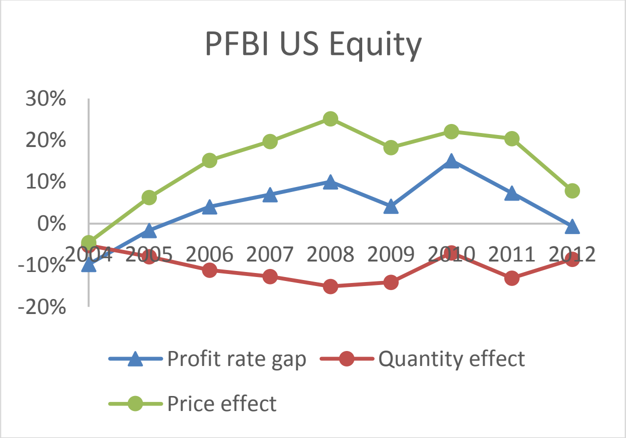
Profit rate gap
Quantity effect
Price effect


PFBI US Equity
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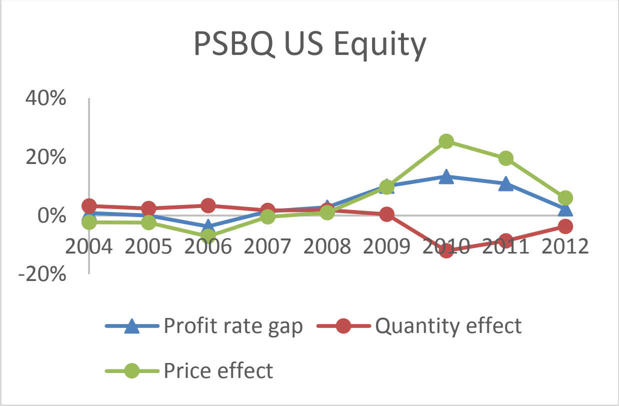
Profit rate gap
Quantity effect
Price effect


PSBQ US Equity
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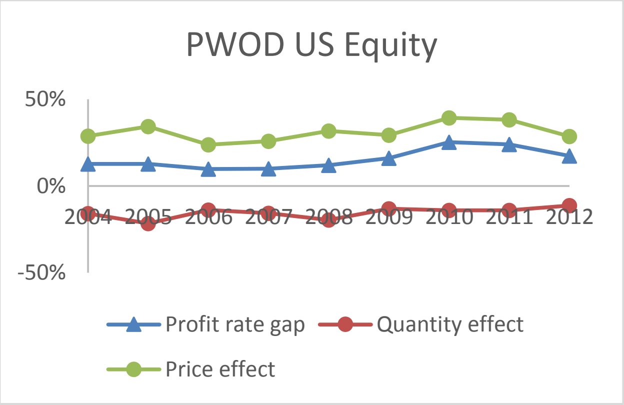
Profit rate gap
Quantity effect
Price effect


PWOD US Equity
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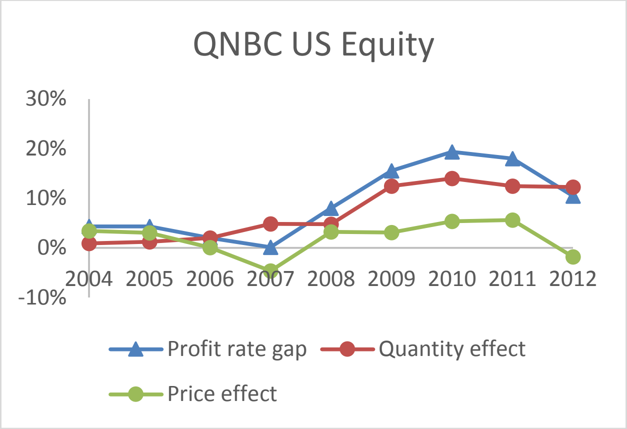
2004 2005 2006 2007 2008 2009 2010 2011 2012
Profit rate gap
Quantity effect
Price effect


QNBC US Equity
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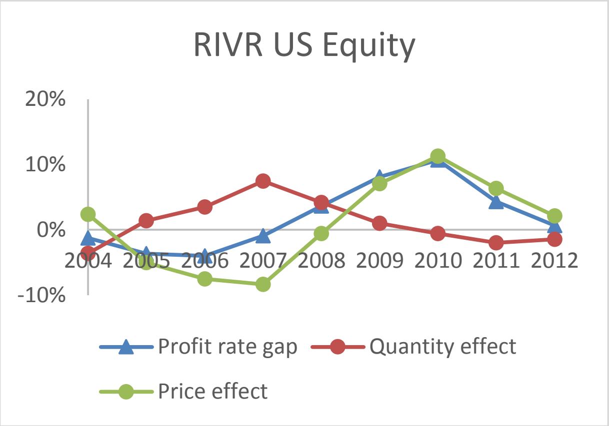
2004 2005 2006 2007 2008 2009 2010 2011 2012
Profit rate gap
Quantity effect
Price effect


RIVR US Equity
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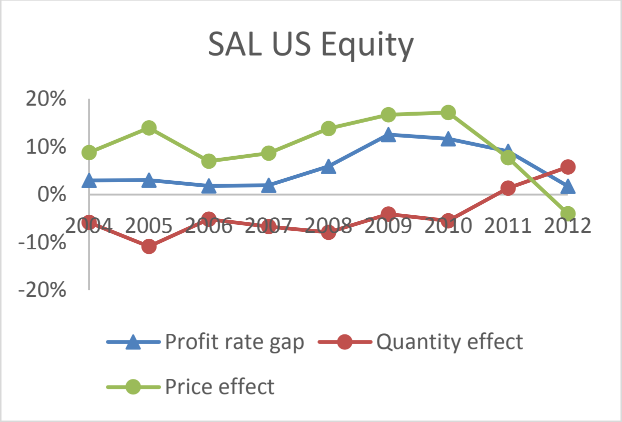
Profit rate gap
Quantity effect
Price effect


SAL US Equity
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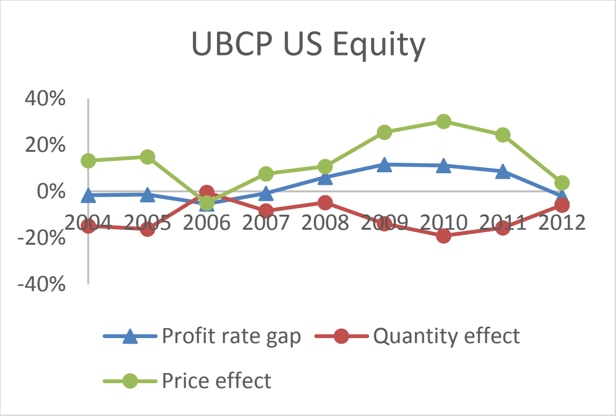
2004 2005 2006 2007 2008 2009 2010 2011 2012
Profit rate gap
Quantity effect
Price effect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20%
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-20%
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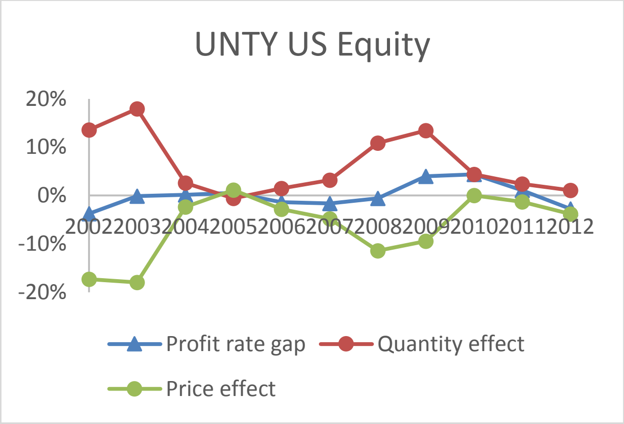
2004 2005 2006 2007 2008 2009 2010 2011 2012
Profit rate gap
Quantity effect
Price effect


UNTY US Equity
20%
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-10%
-20%
2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012
Profit rate gap
Quantity effect
Price effect

Appendix 4 Chapter 2

Canonical Discriminant Analysis

Canonical discriminant analysis is a dimension-reduction technique which is related to principal component analysis and canonical correlation (SAS Institute Inc. 2015. SAS/STAT® 14.1 User's Guide. Cary, NC: SAS Institute Inc.). It finds the canonical variables which are linear combinations of the observed variables and which separate the clusters the most. This analysis summarizes between-cluster variation similar to the way principal component analysis summarizes total variation. The first canonical variable is a linear combination of the original variables that has the maximum possible multiple correlation with the clusters. The second canonical variable is retrieved in the same way given that it is not correlated to the first canonical variable. The process of extracting canonical variables continues until their number equals the number of the original variables or the number of clusters minus one depending on what is less. Since we obtained 3 clusters in the before crisis period and 4 clusters in the after crisis period, 2 canonical variables were extracted before crisis and 3 canonical variables were extracted after crisis.

According to Table 35 the first canonical variable explains 57,4% of between-cluster variation and the second canonical variable explains 42,6% of between-cluster variation.

Table 35 Correlations between canonical variables and clusters and corresponding eigenvalues before the crisis

	Canonical Correlation	Adjusted Canonical Correlation	Eigenvalues			
			Eigenvalue	Difference	**Proportion**	Cumulative
1	0,734126	0,710965	1,1689	0,3007	**0,5738**	0,5738
2	0,681706	0,680158	0,8682		**0,4262**	1

Table 36 indicates that for both canonical variables, we reject the null hypothesis that there is no canonical correlation between the canonical variable and clusters.

Table 36 Tests on the absence of canonical correlations between canonical variables and clusters before the crisis

	Test of H0: The canonical correlations in the current row and all that follow are zero				
	Likelihood Ratio	Appr F value	Num DF	Den DF	**Pr > F**
1	0,24679377	20,60	18	366	**<,0001**
2	0,53527631	19,97	8	184	**<,0001**

Table 37 gives correlations between canonical variables and observed variables. We suppose that there is a strong correlation between observed variable and one of the canonical variables if the correlation coefficient is greater than 0,6 for one of the canonical variables and is less than 0,3 for all other canonical variables. Thus, we notice that the first canonical variable is positively correlated to size effect and Borrowers price effect and negatively correlated to Employees price effect and Government price effect. The second canonical variable is positively correlated to Creditors price effect and negatively correlated to Suppliers price effect. Other observed variables seem to be correlated to both canonical variables or not correlated to either of them and thus do not play an important role in separating between clusters.

Table 37 Correlations between canonical variables and observed variables before the crisis

Variable	First canonical correlation	Second canonical correlation
Tech_eff	0,421	-0,028
Alloc_eff	0,423	-0,077
Size_eff	**0,732**	0,112
Borrowers	**0,72**	-0,304
Financial_mkt	0,084	-0,178
Commision_Fee	-0,756	0,424
Creditors	-0,129	**0,917**
Employees	**-0,602**	0,016
Suppliers	-0,184	**-0,802**
Government	**-0,822**	-0,169

Table 38 indicates that the first canonical variable explains 43,9% of between-cluster variation, the second canonical variable explains 30,5% of between cluster variation and the third canonical variable explains 25,7% of between-cluster variation.

Table 38 Correlations between canonical variables and clusters and corresponding eigenvalues after the crisis

	Canonical Correlation	Adjusted Canonical Correlation	Eigenvalues			
			Eigenvalue	Difference	**Proportion**	Cumulative
1	0,783	0,766	1,58	0,483	**0,439**	0,439
2	0,723	0,705	1,097	0,171	**0,305**	0,743
3	0,693	,	0,926		**0,257**	1

Table 39 indicates that for all 3 canonical variables, we reject the null hypothesis that there is no canonical correlation between the canonical variable and clusters.

Table 39 Tests on the absence of canonical correlations between canonical variables and clusters after the crisis

	Test of H0: The canonical correlations in the current row and all that follow are zero				
	Likelihood Ratio	Appr F value	Num DF	Den DF	**Pr > F**
1	0,096	32,65	27	716,17	<**,0001**
2	0,248	31,04	16	492	<**,0001**
3	0,519	32,67	7	247	<**,0001**

Using the same threshold as for the before crisis period, we observe that the first canonical variable is negatively correlated only to Financial market participants price effect (Table 40). The second canonical variable is negatively correlated only to Creditors price effect. The third canonical variable is positively correlated to allocative inefficiency effect and is negatively correlated to Employees price effect. Other observed variables are either correlated to more than one canonical variable or are not strongly correlated to all of them. They thus do not help in separating between clusters.

Table 40 Correlations between canonical variables and observed variables after the crisis

Variable	Can1	Can2	Can3
Tech_eff	0,515	0,214	0,157
Alloc_eff	-0,108	0,215	**0,695**
Size_eff	0,362	-0,447	0,677
Borrowers	0,575	0,386	0,528
Financial_mkt	**-0,758**	0,232	0,125
Commision_Fee	0,054	-0,606	-0,662
Creditors	-0,058	**-0,606**	-0,027
Employees	-0,074	-0,257	**-0,666**
Suppliers	-0,111	0,664	-0,414
Government	-0,547	-0,241	-0,01