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Galileo's Falling Bodies into the History of Physics and the Nature of Science as a Case Study

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This thesis is the fruit of six years of study and research in Physics Education and History of Physics as a PhD student-worker and Physics teacher at the *Liceo Scientifico "F. Sbordone"* in Italy.

The aim of the work was to investigate precisely what contribution the History of Physics could make to the Teaching of Physics. As an example, the falling motion of bodies and the motion of projectiles as studied by Galileo Galilei and as it can be presented in the first two years of the Liceo scientifico or at the beginning of the third year, in the curricular timetable, was chosen as a case study.

The first phase of the work consisted in analysing primary historical sources, volumes and manuscripts (such as Galileo's laboratory notes on Mechanics preserved in the *Biblioteca Nazionale Centrale di Firenze*) and secondary sources (the most recent interpretative analyses and educational applications) in order to identify an original interdisciplinary theoretical and experimental educational path to propose to the students in order to make them understand the fundamental characteristics of the experimental method, how a scientific theory is constructed and what the relationship between science and society is, in short what the Nature of Science is.

The most effective and modern teaching methods, such as learning by doing, inquiry-based learning and the replication of experiments of particular historical value, were used in the drafting of the teaching course. The selection of learning units was based on the analysis of students' main misconceptions of kinematics and dynamics as determined by the research literature and a first year of experimentation. Analysis of the writings of Galileo's forerunners - from Aristotle's critics such as Hipparchus and Philoponus to Tartaglia - also clarified the students' misconceptions and made it possible to identify Galileo's real contribution to the progress of physics. A new teaching method was developed, integrating the reading of the original texts into the process of predicting the development of a phenomenon and comparing it by means of special educational sheets.

The results of laboratory experiments on the motion of the pendulum, falling motion in fluids and especially falling motion along the inclined plane and parabolic motion were further confirmation of the effectiveness of Galileo's experiments, following Alexandre Koyré's (1953) critique of Platonism, Stillman Drake's interpretation of Galileo's laboratory notes (Galilei [1602-1637], 1979) and Thomas Settle's (1961) experiments.

An extensive three-year experimentation in teaching methodology was conducted. It involved first, second and third grade students in the scientific high school. In order to assess its effectiveness, entry and exit questionnaires were conducted for both knowledge and skills and motivational aspects. The last phase of the work consists of both quantitative and qualitative analysis of the work carried out, achieving interesting results both on the effectiveness of the educational path and on the organisation of the physics curriculum in high school.

The structure of the thesis follows the research work that has been carried out. After a general introduction on the development of physics education and Galileo's mechanics and mathematics, the thesis includes a first part on the analysis of historical sources from Aristotle to Galileo, a second part on the application of the history of physics to the teaching of physics with the design and implementation of an integrated course of theory and experiments, and a third part on the statistical analysis of students' answers to questionnaires and their interpretation.

Readers will find an appendix at the end of the thesis containing copyright permissions, a presentation of the lecture I gave on topics related to those dealt with in the thesis, and spreadsheets for data analysis.

References follows the *Springer Basic Style* both as cited in the running text and in the dedicated section. This section is composed of: *Primary Historical Sources*, *Sources on Research into Physics and Mathematics History* and *Sources on Research into Physics and Mathematics Education and Statistics*.

Finally, readers were provided with the analytical index of subjects and names.

INTRODUCTION

General Objectives

The overall objective of this work is to address the Nature of Science (NoS)–teaching of science (physics, physics–mathematics) and educational aspects of the teaching–learning of physics in high school and University, studying both didactic and pedagogical questions within history of science. The revolutionary change in perspective will situate the learning difficulties experienced by pupils at the centre of educational activity. Although there is a large corpus of literature on this topic, there is no general synthesis and the topic deserves to be explored again in the light of the results of recent Galilean studies and of history of science and connected educational considerations on the subject.

Specific Objectives

The particular objective is to examine essays within the birth of modern science: kinematics and early dynamics studies on writings of Galileo (several, s. references sections) and his predecessors (for example, s. Pisano and Capecchi 2016). The idea is to argument within NoS in order to understand how historical foundations of science can be used for teaching / pedagogical purposes; by proposing to the students an appropriate educational path in a physics measurement workshop while respecting the didactics of science (physics-mathematical modelling) by means of a historical interdisciplinary approach to the disciplines of science education. Particularly I have worked on the sources of Galileo's scientific works and the context of formation of his scientific personality through his critical emulation with his immediate predecessors in 16th-century (see works by Pisano; Abattouy; Renn; Heilbron; Vergara Caffarelli; et al. quoted in References sections). International collaboration was of great help; contacts were established, for example, with Prof. Mohammed Abattouy, Rabat Mohamed V University, Morocco, Prof. Laurence Viennot, University of Paris, France, Prof. Michael Segre, Chieti University, Italy, Prof. Marisa Michelini, University of Udine. Other contacts from educational and historical standpoint are in progress. In addition, of course, the whole of the international Thesis Monitoring Committee, with the different skills of its members in both the history of physics and physics education, made an important contribution to the thesis work.

1. General Introduction on Physics Education

Over many years the physics and its teaching–didactics have been experienced by students as obscure and complicated subjects. Its teaching was excessively abstract, occupied with idealized questions, almost entirely disconnected from daily life. The students rather than learning the method of scientific research, had the tendency to reproduce only the content of the scholastic manuals.

A profound renewal, from 1960, began first in the US and then around the world, with the PSSC (Physical Science Study Committee) course, a perfectly integrated system of “resources for the teaching of secondary school physics, including a text, laboratory guides, laboratory apparatus, closely related films, tests, and a growing collection of supplementary materials – the Science Study Series” (PSSC 1961, p. iii) – dealing with the role of science in society and the “fascinating stories of great discoverers and their discoveries” (Ivi, p. x).

This great enterprise, directed by Jerrold Zacharias (1905-1986), Professor at the Massachusetts Institute of Technology, was realized with the support of the American Institute of Physics, the American Association of Physics Teachers, and the National Science Teachers Association by extensive funding from various government foundations, including the National Science Foundation. This organization benefitted from the National Defense Education Act of 1958 issued following the launch of the Soviet Sputnik satellite and the discomfort it caused in the US scientific and military circles. The PSSC brought about real attention to the improvement of teaching, emphasizing a quality didactic approach, not a dogmatic one, that started from the facts experimentally observed but also took into account the cultural aspects of the teaching of physics. Unfortunately, the project did not have the desired results (see for example Jackson 1983 which quotes the thought of Zacharias). The causes were multiple. According to Arons (1983) the main reason lies in the training of elementary schoolteachers not being suitable for the development of the reform plan and more generally for the insufficient training of teachers with regard to the teaching resources that had been provided them. An equally important reason for the partial failure of the PSSC was the extreme autonomy in the choice of curricula by States, districts and individual American institutes. From 1975 funding returned to the level of the fifties, and the worrying hypothesis once more arose that the state of science education rendered the United States a Nation at Risk (NCEE, 1983) for the disadvantage that had accumulated in the scientific field.An ingenious solution was to continue to leave autonomy to educational institutions in the detailed choice of curricula but to set up National Science Education Standards for “science teaching, science content, professional development for teachers of science, assessment, science education programs and systems” (NRC 1996, p. 4).

More recently, a comprehensive update called Next Generation Science Standards (NRC 2013) was realized, based on the remarkable progress made by the scientific research community over the last fifteen years, and of course on the expansion of scientific knowledge. Other countries have also recently experienced a profound renewal of teaching due to research in physics education and the modernisation of curricula.

One example is the Italian state, where I use to teach (and research in) physics and mathematics for 30 year, in a Lyceum[1]. The reform of the secondary school curriculum (MIUR 2010) since 2010/11 has led to greater attention to the study of physics in scientific high schools: increasing the number of hours per week, providing the possibility of a written test of physics in the final state exams of scientific high school, encouraging an increase in the use of the laboratory, and introducing the study of twentieth-century physics to achieve an integral education of the citizen. Even in Italy, however, the detailed curricula disappeared to make room for the National Indications, a sort of recommendation of the educational approach to be followed and of the logical thread of the topics of study. For physics at scientific high school, being a subject characterizing the field of studies, a reference framework was created with prerequisites, essential contents, skills related to the contents and sectoral skills (MIUR Technical Panel 2015).

In England and Wales, since 1988, as a result of the Education Reform Act, the National Curriculum is in force, updated approximately every four years. This has standardized the curricula, leaving the individual schools the possibility to choose methodologies and textbooks. The National Curriculum has responded to the need for scientific training not only for those who might study science at University but for all young people up to the age of 16, providing specialist knowledge but also a generalist one for citizenship skills.

More technical, on the other hand, is the preparation of students up to the age of 18 who for the admission to university must obtain the General Certificate of Education: GCE Advanced Level (GCE A level) and GCE Advanced Supplementary (GCE AS level) or the International Baccalaureate (See for example AQA 2017).

In the first document of Science in the National Curriculum (NCC 1988) great importance was given to the teaching of the History and Philosophy of Science, indicating in detail the motivations:

> pupils should develop their knowledge and understanding of the ways in which scientific ideas change through time and how the nature of these ideas and the uses to which they are put are affected by the social, moral, spiritual and cultural contexts in which they are developed; in doing so, they should begin to recognise that while science is an important way of thinking about experience, it is not the only way (NCC 1989, p. 36).

In France the teaching of physics and chemistry at the Lyceum is centered on the acquisition of skills and on arousing the interest of the pupils. The teacher's freedom of teaching is guaranteed within some topics relevant to the orientation of the students and in the choice of levels of depth of various themes: colors and images, waves and matter, laws and models, health, the practice of sport, the universe, the challenges of the 21st century (Ministère de l'Éducation Nationale 2010a, 2010b, 2011).

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[1] *Liceo scientifico "F. Sbordone" di Napoli..*

The renewal of teaching may be based on Shulman's theoretical assumptions about *Pedagogical Content Knowledge* (Shulman 1987) and on the integration of the teacher's profession with educational concepts that take into account the *Model of Educational Reconstruction* (Duit 2006) and its latest developments.

Shulman argued that educational research on teacher training had until then unjustly ignored the means of facilitating students' understanding of the discipline. He argued that teachers should know how to structure and represent academic content for direct teaching to students, the misconceptions and difficulties encountered by students in learning a particular disciplinary content and the specific teaching strategies that can be used to meet the learning needs of students.

According to the *Educational Reconstruction Model* (ERM) attempts to improve didactics must attend either to problems related to the content of the discipline or to the learning needs of the students. The problems to be considered are closely related: the clarification and analysis of the subject matter (including key ideas of science and principles) and analysis of the dispositions of students and teachers in relation to the choice of contents, including interests, attitudes, skills and pre-instructional conceptions.

The students' learning difficulties are generally due to the lack of coordination between scientific learning and everyday experience (which is rather a matter of common sense and common sense knowledge). It will therefore be indispensable to refute knowledge based on common sense and to enable the acquisition of genuine scientific learning, referring to a historical approach of the teaching of scientific disciplines.

It is necessary to overcome the conceptual nodes. The conceptual nodes are the issues considered critical with respect to the ideas to be learned. They arise from the reasoning of common sense, unspoken natural reasoning with related elements of coherence. Examples of conceptual nodes in mechanics are the independence of the free fall motion from the mass, the proportionality between force and acceleration and not speed, etc.

Scientific teaching is interdisciplinary: science is one of the main disciplines of reference, but the skills of other different disciplines are necessary. The philosophy of science and the history of science provide models of thought to analyse (critically) the nature of science and the special contribution that science can give "to understand the world" (Duit 2006, p. 1).

Reference to the history of physics makes it possible to critically argue alternative hypotheses critically by stimulating the students to follow the reasoning of the scholars and by motivating them with comparison of theories that have been developed over time. Some spontaneous ideas are similar to outdated interpretive models, and these can become instruments for overcoming ideas of common sense (Viennot 1979a; Wandersee 1986): studying the theories of historical interest, the students would recognise their own conceptions and they would discuss and revise them, through processes of conceptual change (Posner, Strike, Hewson and Gertzog 1982) similar to those actually occurred in the history of physics. It is supposed that the individual cognitive development could somehow recapitulate the historical development of the science (See also Duhem 1906). Of course, it is not said that this process takes place for every single student. Furthermore, the history of the science can contribute to improving the understanding of the Nature of Science in its conceptual, procedural and contextual aspects, and of the relationships among science, technique and society (for example see: Teixeira, Greca and Freire, 2012; Maurines and Beaufils 2012).

In the section of the National Indications (MIUR 2010) for *Licei scientifici* entitled “General outlines and competences” we read that the student must acquire "awareness of the cognitive value of the discipline and of the link between the development of physical knowledge and the historical and philosophical context in which it has developed”. Moreover, in the *Learning outcomes* of the cultural, educational and professional profile of *Licei scientifici* we read that the students are required to

> […] be aware of the reasons which have produced the scientific and technological development over time, in relation to the needs and demands of knowledge of different contexts, with critical attention to application-technical and ethical dimensions of scientific achievement, highlighting the most recent […].[2]

Of course, the study of the history of science can foster a *Cultural Content Knowledge* (Galili 2008, 2012) directly related to the narrative component of the science. This includes the principal alternative historical theories that have preceded the actual structuring and – in the didactic context – also the common conceptions of the students. The inclusion into science teaching of debates and controversies situates the discipline in a vast context and qualifies it as cultural fact. Knowledge of the difficulties that historically were faced during the development of science and of the conceptual changes that occurred help to clarify what the conceptual nodes of the discipline are. Different methods can be used to include the history of science in the teaching of scientific subjects: an additional approach, which adds some didactic units related to science history episodes to a non-historical science program or an integrated approach according to which an entire science course is organized on historical bases; the storyline method, where history is used to create a narrative development within which to arrange the various topics or to structure the whole course. It can be very useful to develop some historical case studies of remarkable didactic, cultural, scientific and methodological meaning, or to use some models, examples, analogies, historical experiments to help the students to overcome specific wrong conceptions and conceptual difficulty. I agree on the obligation “to apply historical units - not only isolated lessons - in science teaching” (Heering 2000, p. 370; see also Cioci 2014b).

[2] Retrieved via: http://www.indire.it/lucabas/lkmw_file/licei2010///Profili1.pdf, p. 11. Website access date 30 April 2018. The English translation is mine

2. A specific introduction in my PhD work

The general aim of my PhD research was to re–examine the present state of historical Galilean science: selections of textual examples from mechanics (from historical and teaching standpoints) as found in *Dialogo Sopra i Due Massimi Sistemi del Mondo Tolemaico e Copernicano* (Galilei 1632 hereafter *Dialogo*) and in *Discorsi e Dimostrazioni Matematiche Intorno a Due Nuove Scienze Attenenti alla Meccanica e i Movimenti Locali* (Galilei 1638 hereafter *Discorsi*). I also intended to produce a historical–fundamental inquiry about how and to which Galileo's works fed on the ideas of the others contemporaries or that preceded Galileo. For example, I considered some aspects of Tartaglia's works as investigated by Pisano (see references sections).

I intend to carry out a class experimentation of the designed path with students of the Scientific Lyceum both in the theoretical and applied and experimental aspects.

In particular,

- I analyse primary sources on the subject. My work also involves a qualitative description of the mechanical concepts.
- I realize the aim of creating an educational path about Galileo's physics which can constitute a full course for high school or university students or even for physics teachers. These lessons, presented in an organic way, contain elements of physics and mathematics introduced by Galileo, enriched with a laboratory path with the experiments actually performed by Galileo, in an attempt to build a new model of the physical–and–mathematical Galilean framework. Some particularly interesting experiments in the history of physics were presented in a modern form, using technologically advanced tools, and then the didactic results obtained have been compared.
- I rethink the mathematics used by Galileo (geometry and proportions) and compare it with mathematics used today (algebra and analytic geometry) with respect to official physics and mathematics ministerial programs/curricula.

3. Galileo & Mechanics

Galileo is a central figure in the history of science. His greatness is based on his physical discoveries about motion, but also on the methodological innovations that led him to the fundamental results he achieved.

Scientific research, according to Galileo, must be based on two fundamental pillars: *sensible experiments* and *necessary demonstrations* (proofs). The use of experience had already been required by the Aristotelian school. In addition, Galileo foresaw the employment of tools that could perfect the senses of the experimenter, such as the telescope. The use of controlled tests, under special conditions, can actively interrogate nature. It must respond quantitatively through numerical measurements/modelling, confirming or disproving the physical laws written in mathematical form.

The study of the motion of free fall bodies is a particularly interesting example, full of didactic suggestions, that can be considered a prototype in which Galileo used the scientific method.

An important point of the scientific method is to discern the variables that determine the phenomenon in question. It is interesting to note that Galileo made several attempts to identify the laws that bind these variables: we will see how the scientist first misunderstood a linear relationship between speed and distance before formulating the space-temporal law that we all know today and the law of proportionality between speed and time (and hence between speed and the square root of the distance travelled).

In 1604, Galileo thought he could use the percussion method to get speed information. This involves dropping a small sphere from a certain height and assessing the depth of deformation that the body produces, striking a plane on which a layer of deformable substance such as wax has been placed. Galileo noticed that this deformation was bigger as was greater the height from which the sphere falls. Also assuming a connection between the deformation and the final velocity of the sphere, Galileo assumed that the latter was proportional to the height from which the body falls (however, the height is proportional to kinetic energy and hence to the square of speed).

Galileo demonstrated that the fall of the bodies in the vacuum does not depend on their weight as the Aristotelian school mistakenly believed as common sense suggests. At the time of Galileo technical progress did not allow the creation of the pneumatic vacuum needed for experimental testing. In 1638 in his *Discorsi*, Galileo proved it by a thought experiment, demonstrating how Aristotelian school conceptions on the subject lead to absurd conclusions.

Galileo proposed an experience based on an extrapolation process by dropping two bodies of different weight and of the same form in different fluids (e.g., oil, water and air), noting that the difference between the fall times of the two bodies decreases as decreases the resistance that the fluid opposes to the passage of the bodies. According to Galileo *we are justified in believing it highly probable that in a vacuum all bodies would fall with the same speed* (Galilei [1638], 1914).

In his works about mechanics such as the *Discorsi* and the *Dialogo*, Galileo made much use of exemplifications (i.e., thought experiments), analogies between similar physical systems and reasoning at the limit, all of which are useful tools in the heuristic method of Galileo for the acquisition of new knowledge. This thought experiment is an extrapolation from experiments made that expresses Galileo's attempt to remove more and more impediments to the movement. It anticipates an experiment which will then be actually realized (Camilleri 2015, p. 109).

One of the best–known examples of analogy (Duit 1991) employed by Galileo is certainly that of the study of the motion of free fall of bodies that, being too quick to be studied, was suitably slowed using an inclined plane. Thus, Galileo was able to undertake the study of the dependence of the space travelled by the time taken during accelerated motion, achieving one of the most important results in the history of science.

As indicated above, in *Discorsi* Galileo presents a series of theorems on naturally accelerated motion, typical of the free fall, in which a body *starting from rest, acquires, during equal-time intervals, equal increments of speed*. By means of purely geometric considerations, Galileo showed that the distance travelled in a naturally accelerated motion is equal to that travelled in a uniform motion having as its speed half of the final one achieved in the accelerated motion – that is equivalent to writing that the distances are proportional to the squares of the times (fig. 1).

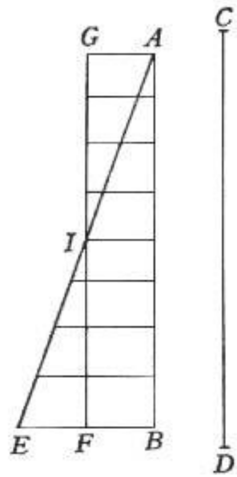

Fig. 1. Anticipating the Cartesian plane and the integral calculus, Galileo shows that the space travelled in a uniformly accelerated motion from rest represented by the area of the triangle AEB (with speeds continuously increased from A to EB) is equal to the space travelled in a uniform motion represented by the area of the rectangle GAFB (with speeds all equal to EB), distances both travelled at the same time CD. Galilei [1638], 1890-1909, vol. VIII, p. 208. Source: Bibliothèque nationale de France / gallica.bnf.fr.

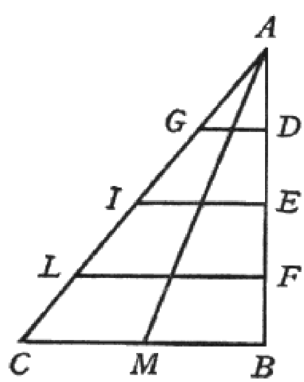

Fig. 2. Galileo showed that *if a body falls freely along smooth planes inclined at any angle whatsoever, but of the same height, the speeds with which it reaches the bottom are the same* and that the travel times are proportional to the lengths of the planes. Galilei [1638], 1890-1909, vol. VIII, p. 216. Source: Bibliothèque nationale de France / gallica.bnf.fr.

Galileo then related the fall of a body from the same height along an inclined plane with that occurring along the vertical direction, demonstrating geometrically that the two motions exhibit the same final speeds and that the times employed are directly proportional to the distances travelled (fig. 2).

Galileo therefore wanted to prove that naturally descending bodies follow a uniformly accelerated motion. The famous experiment of the inclined plane reported in the *Discorsi* performs this function. Galileo claims to have made a bronze ball descend along a clean and smooth canal, measuring time using a water clock.

The *constant-flow chronograph*, coupled with a fairly sensitive weight scale, was a particularly useful tool for Galileo because it can measure short intervals of time in a continuous manner in which the uncertainty of the measure is essentially due to the investigator's reflexes (Vergara Caffarelli 2009, p. 211-212).

In the letter to Giovan Battista Baliani dated 1 August 1639, Galileo emphasises the importance of this measuring instrument:

> It is true that we could move on to more accurate measurements having seen et observed what the flow of water through a thin pipe is, because by collecting it, and having weighed how much water passed, e.g., in a minute, we could then, by weighing the amount of water passed during the time of the descent along the groove, find the most exact measurement and the amount of that time[3]

In this same text Galileo also stresses the importance of using the pendulum for time measurements - especially in reference to the sidereal day:

> It remains now that we find the amount of time of the descents along the groove. We will get it by the admirable property of the pendulum, which is to make all its vibrations, large or small, under equal times. We seek, for once only, that two, three or four curious and patient friends, having set a fixed star against some stable sign, taken a pendulum of any length, go numbering its vibrations for all the time until the return of the same fixed star to the first place; and this will be the number of vibrations in 24 hours.[4]

Galileo announces his discovery of the pendulum isochronism in the letter to Guidobaldo del Monte dated 29 November 1602 (Galilei [1602], 1890-1909, vol. X, pp. 97-100), describing first in the *Dialogo*[5] but above all in *Discorsi*[6] some simple experiments with the pendulum that have great educational value.[7]

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[3] The English translation is mine. "Vero è che noi potremo passare a più esatte misure con havere veduto et osservato qual sia il flusso dell'acqua per un sottile cannello, perchè raccogliendola, et havendo pesata quanta ne passa, vgr., in un minuto, potremo poi, col pesare la passata nel tempo della scesa per il canale, trovare l'esattissima misura e quantità di esso tempo" (Galilei [1639], 1890-1909, vol. XVIII p. 76-77).

[4] The English translation is mine. "Resta hora che troviamo la quantità del tempo delle scese per il canale. Ciò otterremo dalla ammirabile proprietà del pendolo, che è di fare tutte le sue vibrationi, grandi o piccole, sotto tempi eguali. Si ricerca, pro una vice tantum, che dua, tre o quattro amici curiosi e pazienti, havendo appostata una stella fissa che risponda contro a qualche segno stabile, preso un pendolo di qualsivoglia lunghezza, si vadano numerando le sue vibrationi per tutto il tempo del ritorno della medesima fissa al primo luogo; e questo sarà il numero delle vibrationi di 24 hore" (Ivi, p. 76).

[5] Galileo [1632], 1890-1909, vol. VII pp. 475-477.

[6] Galileo [1638], 1890-1909, vol. VIII, pp. 128-130, 139-140.

[7] Pendulum properties can be studied as was done by Galileo through 3 simple experiments: two pendulums of different masses are made to oscillate together, with different amplitudes or different lengths, discovering that the oscillation time changes only in the last case (Galileo suggests a pendulum with a length of 4 times that of the other one,

In this letter, Galileo correlates the isochronism of the pendulum with that which takes place along inclined planes which constitute chords of a circumference having as a common end the highest or the lowest point of a circumference arranged vertically.

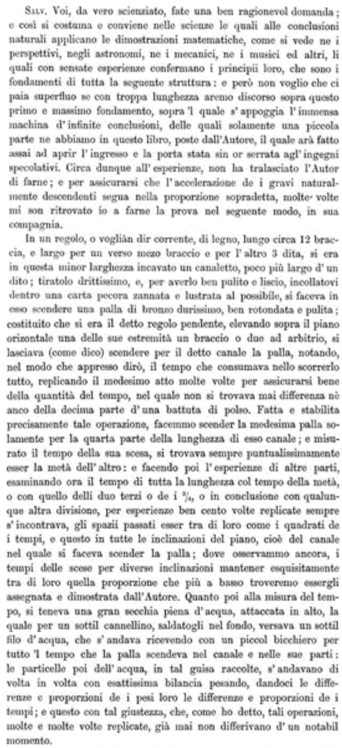
Galileo will geometrically demonstrate this last proposition and, going from the chords to the circumference arcs, will try to justify the pendulum isochronism. He will not be able to give a direct demonstration but only an experimental one, without realizing that this property is valid only for small oscillations.

As highlighted by Vergara Caffarelli (2005, p. 78) and then by Pisano and Cioci (2020a, p. 266), Galileo, in particular, would use the pendulum to get time intervals all equal to each other in which to experimentally verify the so-called law of odd numbers, theoretically obtained by him.[8]

Galileo - by using the mean value theorem - showed that a naturally accelerated motion occurs according to the so-called law of the odd number, which states that *a moving body starting from rest and acquiring velocity at a rate proportional to the time, will, during equal intervals of time, traverse distances which are related to each other as the odd numbers beginning with unity, 1, 3, 5, ...*, that is, measuring the spaces all from the resting position and the corresponding time intervals, *the spaces traversed are in the duplicate ratio of the times, i.e., in the ratio of the squares of the times* (Galilei [1638], 1914).

obtaining an oscillation time that is twice of the other).

[8] This hypothesis is based on practical considerations and on the statements of Galileo: "Therefore, the distances from the beginning of motion are as the squares of the times and, dividing the travelled spaces in equal times, are as odd numbers starting from the unit: that responds to what I have always said and observed with experiments" (The English translation is mine. "Le distanze dunque dal principio del moto sono come i quadrati de i tempi et, dividendo gli spazii passati in tempi eguali sono come i numeri impari ab unitate: che risponde a quello che ho sempre detto et con esperienze osservato") (Galilei 1602-1637, f. 128v).



Salv. Voi, da vero scienziato, fate una ben ragionevol domanda; e così si costuma e conviene nelle scienze le quali alle conclusioni naturali applicano le dimostrazioni matematiche, come si vede ne i perspettivi, negli astronomi, ne i mecanici, ne i musici ed altri, li quali con sensate esperienze confermano i principii loro, che sono i fondamenti di tutta la seguente struttura: e però non voglio che ci paia superfluo se con troppa lunghezza aremo discorso sopra questo primo e massimo fondamento, sopra 'l quale s' appoggia l' immensa machina d' infinite conclusioni, delle quali solamente una piccola parte ne abbiamo in questo libro, poste dall'Autore, il quale arà fatto assai ad aprir l' ingresso e la porta stata sin or serrata agl' ingegni specolativi. Circa dunque all' esperienze, non ha tralasciato l'Autor di farne; e per assicurarsi che l' accelerazione de i gravi naturalmente descendenti segua nella proporzione sopradetta, molte volte mi son ritrovato io a farne la prova nel seguente modo, in sua compagnia.

In un regolo, o vogliàn dir corrente, di legno, lungo circa 12 braccia, e largo per un verso mezo braccio e per l' altro 3 dita, si era in questa minor larghezza incavato un canaletto, poco più largo d' un dito; tiratolo drittissimo, e, per averlo ben pulito e liscio, incollatovi dentro una carta pecora zannata e lustrata al possibile, si faceva in esso scendere una palla di bronzo durissimo, ben rotondata e pulita; costituito che si era il detto regolo pendente, elevando sopra il piano orizontale una delle sue estremità un braccio o due ad arbitrio, si lasciava (come dico) scendere per il detto canale la palla, notando, nel modo che appresso dirò, il tempo che consumava nello scorrerlo tutto, replicando il medesimo atto molte volte per assicurarsi bene della quantità del tempo, nel quale non si trovava mai differenza nè anco della decima parte d' una battuta di polso. Fatta e stabilita precisamente tale operazione, facemmo scender la medesima palla solamente per la quarta parte della lunghezza di esso canale; e misurato il tempo della sua scesa, si trovava sempre puntualissimamente esser la metà dell' altro: e facendo poi l' esperienze di altre parti, esaminando ora il tempo di tutta la lunghezza col tempo della metà, o con quello delli duo terzi o de i $^{3}/_{4}$, o in conclusione con qualunque altra divisione, per esperienze ben cento volte replicate sempre s' incontrava, gli spazii passati esser tra di loro come i quadrati de i tempi, e questo in tutte le inclinazioni del piano, cioè del canale nel quale si faceva scender la palla; dove osservammo ancora, i tempi delle scese per diverse inclinazioni mantener esquisitamente tra di loro quella proporzione che più a basso troveremo essergli assegnata e dimostrata dall'Autore. Quanto poi alla misura del tempo, si teneva una gran secchia piena d' acqua, attaccata in alto, la quale per un sottil cannellino, saldatogli nel fondo, versava un sottil filo d' acqua, che s' andava ricevendo con un piccol bicchiero per tutto 'l tempo che la palla scendeva nel canale e nelle sue parti: le particelle poi dell' acqua, in tal guisa raccolte, s' andavano di volta in volta con esattissima bilancia pesando, dandoci le differenze e proporzioni de i pesi loro le differenze e proporzioni de i tempi; e questo con tal giustezza, che, come ho detto, tali operazioni, molte e molte volte replicate, già mai non differivano d' un notabil momento.

In a wooden beam, about 12 *braccia* long, half a *braccio* wide, and three finger-breadths thick, a groove was carved, a little more than one finger in breadth; having made this channel very straight, clean and smooth, and having covered it with parchment, which was also as smooth and polished as possible, a very hard bronze ball, well rounded and polished was made to descend in it. […] we made the same ball descend only one quarter of the length of this channel; and having measured the time of its descent, we found it precisely one-half of the former. We then tried other distances […] we always found that the spaces traversed were each other as the squares of the times […] For the measurement of time, we employed a large vessel of water placed in an elevated position. To the bottom of this vessel was soldered a thin pipe giving a narrow threat of water, which we collected in a small glass during the time of each descent, whether for the whole length of the canal or for a part of it. The water we collected in this way was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times.[9]

Fig. 3. Galileo's laboratory experiment of descent along an inclined plane (Galilei [1638], 1890-1909, vol. VIII, pp. 212 – 213). Source: Bibliothèque nationale de France / gallica.bnf.fr.

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9 The English translation is mine. "In un regolo, o vogliàn dir corrente, di legno, lungo circa 12 braccia, e largo per un verso mezo braccio e per l'altro 3 dita, si era in questa minor larghezza incavato un canaletto, poco più largo d'un dito; tiratolo drittissimo, e, per averlo ben pulito e liscio, incollatovi dentro una carta pecora zannata e lustrata al possibile, si faceva in esso scendere una palla di bronzo durissimo, ben rotondata e pulita; [...] facemmo scender la medesima palla solamente per la quarta parte della lunghezza di esso canale; e misurato il tempo della sua scesa, si trovava sempre puntualissimamente esser la metà dell'altro: e facendo poi l'esperienze di altre parti, [...] sempre s'incontrava, gli spazii passati esser tra di loro come i quadrati de i tempi, [...]. Quanto poi alla misura del tempo, si teneva una gran secchia piena d'acqua, attaccata in alto, la quale per un sottil cannellino, saldatogli nel fondo, versava un sottil filo d'acqua, che s'andava ricevendo con un piccol bicchiero per tutto'l tempo che la palla scendeva nel canale e nelle sue parti: le particelle poi dell'acqua, in tal guisa raccolte, s'andavano di volta in volta con esattissima bilancia pesando, dandoci le differenze e proporzioni de i pesi loro le differenze e proporzioni de i tempi" (Galileo [1638], 1890-1909, vol. VIII, pp. 212 – 213). Compare with the English translation by Henry Crew and Alfonso De Salvio (Galilei [1638], 1914, pp. 178-179) and with that of Stillman Drake (Galilei [1638], 1989, p. 170). In my opinion, Drake's translation of *Discorsi* is closer to the original text and more accurate from an epistemological point of view. Crew and De Salvio's translation is closer to the modern spoken language and for this reason more easily understood. I prefer the second one for educational purposes. I will use it to indicate the passages I refer to, together with the Italian edition of Galileo's works edited by Antonio Favaro (Galileo [1638], 1890-1909), and for short quotations. Even for quotations from Galileo's writings in Italian for my students, I used more modern language without altering the meaning of the texts. For the translation of the Latin passages in *Discorsi*, the Italian translation by Carugo and Geymonat (Galilei [1638], 1958) was very useful.

The emphasis given by Galileo to the notion of time is therefore achieved through two complementary approaches - mathematical and measuring - that have allowed us to consider time as the parameter by which it is possible to describe movement. This great innovation is at the base of the birth of the modern science of mechanics - which will be later developed by Newton - and of the pendulum clock construction, which was initially conceived by Galileo, thanks to the decisive contribution of Huyghens (Abattouy 1992, p. 119).[10]

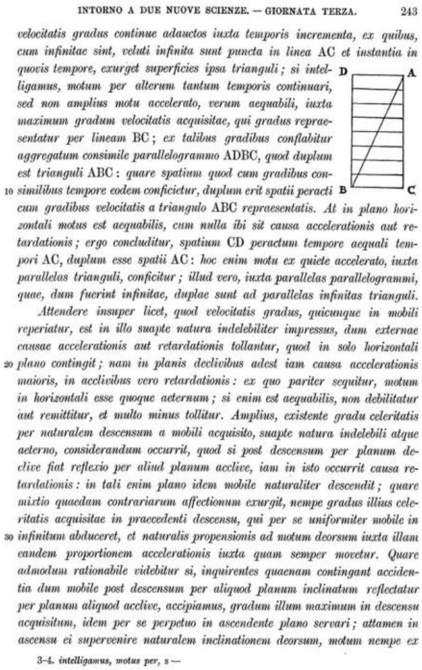
Among the historians and philosophers of science, a discussion has arisen about whether Galileo did or did not perform the experiments he describes in his works (Koyré 1939; Settle 1961). There are in fact some laws that Galileo could not directly verify. For example, one is the idealized formulation of the principle of inertia - at least for its reference to perpetual motion and infinity (see fig. 4).

This is the last and the most effective formulation of the principle of inertia by Galileo that the scientist used as a starting point to discuss parabolic motion, but it is not the best known. In a first version that we find in the Second Letter from Galileo Galilei to Marco Velseri (1612) (Galilei [1612], 1613)[11] and later

[10] Before Galileo there is no systematic mathematical study of motions as a function of time. We find time in the description of motion only in the formulation of Kepler's third law and in the mathematical description of the spiral as a function of time by Archimedes (Abattouy 1992, p 130). In fact, Archimedes defines the spiral as the composition of two motions, a uniform rectilinear motion and a uniform circular one: "If a straight line of which one extremity remains fixed be made to revolve at a uniform rate in a plane until it returns to the position from which it started, and if, at the same time as the straight line revolves, a point move at a uniform rate along the straight line, starting from the fixed extremity, the point will describe a spiral in the plane" (Archimedes [1897], 2009, p. 154). In *Discorsi*, in particular, Galileo highlights the differences between his approach and that of Archimedes which consist in the different applications of their theories to actual or natural phenomena and in their use of experimentation: "Although it is not unreasonable to arbitrarily invent some kind of motion and consider the properties that result from it (in fact, those who imagined spiral or conchoidal lines originating from certain motions have commendably demonstrated their properties from assumptions), […] since nature makes use of a certain kind of acceleration in descending bodies, we decided to study its properties, [… which …] seem to agree with and exactly correspond to those that natural experiments show to the senses" (Galileo [1638], 1890-1909, p. 197; Galilei [1638], 1914, p. 160). English translation is mine.

[11] "All external impediments removed, an heavy body on a spherical surface concentric to the Earth […]will retain that state in which it has been; that is, if it is placed in a state of stillness, in that it will be preserved, and if it is placed in movement, for example toward the west, it will likewise retain that movement: and so a ship, for example, having once received some impetus for the quiet sea, would move continually around our globe without ever stopping, and placed in it at rest, perpetually it would remain at rest, if in the first case all external impediments could be removed, and in the second case if some external cause of motion did not arise". The English translation is mine ("Rimossi tutti gl'impedimenti esterni, un grave nella superficie sferica e concentrica alla Terra […] in quello stato si conserverà nel qual una volta sarà stato posto; cioè se sarà messo in stato di quiete, quello conserverà, e se sarà posto in movimento, v. g. verso occidente, nell'istesso si manterrà: e così una nave, per essempio, avendo una sol volta ricevuto qualche impeto per il mar tranquillo, si moverebbe continuamente intorno al nostro globo senza cessar mai, e postavi con quiete, perpetuamente quieterebbe, se nel primo caso si potessero rimuovere tutti gl'impedimenti estrinseci, e nel secondo qualche causa motrice esterna non gli sopraggiungesse") (Galileo [1612], 1890-1909, vol. V, pp. 134-135). Compare with Galileo 1957, pp. 113–114. Galileo treats this topic in a similar but wordier way in *Dialogo* (Galileo [1632], 1890-1909, vol. VII, pp. 179-182).

in the *Dialogo*, he refers to a motion along a circumference concentric with the Earth. It was Newton in 1687 to give it a more rigorous formulation that also included the concept of force.[12]



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velocitatis gradus continue adauctos iuxta temporis incrementa, ex quibus, cum infinitae sint, veluti infinita sunt puncta in linea AC *et instantia in quovis tempore, exurget superficies ipsa trianguli; si intelligamus, motum per alterum tantum temporis continuari, sed non amplius motu accelerato, verum aequabili, iuxta maximum gradum velocitatis acquisitae, qui gradus repraesentatur per lineam* BC; *ex talibus gradibus conflabitur aggregatum consimile parallelogrammo* ADBC, *quod duplum est trianguli* ABC: *quare spatium quod cum gradibus consimilibus tempore eodem conficietur, duplum erit spatii peracti cum gradibus velocitatis a triangulo* ABC *repraesentatis. At in plano horizontali motus est aequabilis, cum nulla ibi sit causa accelerationis aut retardationis; ergo concluditur, spatium* CD *peractum tempore aequali tempori* AC, *duplum esse spatii* AC: *hoc enim motu ex quiete accelerato, iuxta parallelas trianguli, conficitur; illud vero, iuxta parallelas parallelogrammi, quae, dum fuerint infinitae, duplae sunt ad parallelas infinitas trianguli.*

Attendere insuper licet, quod velocitatis gradus, quicunque in mobili reperiatur, est in illo suapte natura indelebiliter impressus, dum externae causae accelerationis aut retardationis tollantur, quod in solo horizontali plano contingit; nam in planis declivibus adest iam causa accelerationis maioris, in acclivibus vero retardationis: ex quo pariter sequitur, motum in horizontali esse quoque aeternum; si enim est aequabilis, non debilitatur aut remittitur, et multo minus tollitur. Amplius, existente gradu celeritatis per naturalem descensum a mobili acquisito, suapte natura indelebili atque aeterno, considerandum occurrit, quod si post descensum per planum declive fiat reflexio per aliud planum acclive, iam in isto occurrit causa retardationis: in tali enim plano idem mobile naturaliter descendit; quare mixtio quaedam contrariarum affectionum exurgit, nempe gradus illius celeritatis acquisitae in praecedenti descensu, qui per se uniformiter mobile in infinitum abduceret, et naturalis propensionis ad motum deorsum iuxta illam eandem proportionem accelerationis iuxta quam semper movetur. Quare admodum rationabile videbitur si, inquirentes quaenam contingant accidentia dum mobile post descensum per aliquod planum inclinatum reflectatur per planum aliquod acclive, accipiamus, gradum illum maximum in descensu acquisitum, idem per se perpetuo in ascendente plano servari; attamen in ascensu ei supervenire naturalem inclinationem deorsum, motum nempe ex

3-4. *intelligamus, motus per,* s —

Fig. 4. Galileo’s formulation of the inertia principle Galilei [1638], 1890-1909, vol. VIII, p. 243. Source: Bibliothèque nationale de France / gallica.bnf.fr.

Furthermore, we can observe that whatever degree of velocity is applied to a body in motion, it is by its nature indelibly imprinted on the body, provided that the external causes of acceleration or retardation are removed; which happens only in the horizontal plane; for in descending planes a cause of acceleration is already present, while in ascending ones a cause of retardation is already present: from this it likewise follows that motion in the horizontal plane is also eternal; for if it is equable, it does not decrease or diminish, nor even less does it stop [...] by itself would lead the body to move endlessly with uniform motion[13]

The discovery and the interpretation in a modern era of a precious manuscript containing Galileo’s own lab notes enables us to conclude that he had actually performed remarkable experiments that would guarantee the whole system of his theory consistently based on the fundamental principles. A precious historical source for the reconstruction of Galileo's physics is the *Manuscript 72* (Galilei 1602-1637) that collects his lab notes on Mechanics written before 1609, when the scientist devoted all his attention to telescope and astronomical observations. The manuscript is held at *Biblioteca Nazionale Centrale di Firenze* and can be consulted in full on its website, at www.internetculturale.it,

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[12] See Newton [1687, 1729], 1934, p. 13. Drake, referring to Galileo's applications of the principle of inertia, argued that “He did not, as is sometimes stated, attribute the orbital motions of the planets to an inertial principle acting circularly. In fact, he did not attempt any explanation of the cause of planetary motions” (Drake in Galilei [1623], 1957, p. 113, ft. 8). This is a result that Drake attributes to Newton.

[13] English translation is mine. “Attendere insuper licet, quod velocitatis gradus, quicunque in mobili reperiatur, est in illo suapte natura indelebiliter impressus, dum externae causae accelerationis aut retardationis tollantur, quod in solo horizontali plano contingit ; nam in planis declivibus adest iam causa accelerationis maioris, in acclivibus vero retardationis : ex quo pariter sequitur, motum in horizontali esse quoque aeternum ; si enim est aequabilis, non debilitatur aut remittitur, et multo minus tollitur […] qui per se uniformiter mobile in infinitum abduceret” (Galilei [1638], 1890-1909, vol. VIII, p. 243). Compare with Galilei [1638], 1914, p. 215.

the web portal of catalogues and digital collections of Italian libraries, in the digital library of the *Museo Galileo* in Florence and of Max Planck Institute for the History of Science in Berlin (See Galilei [1602-1637], 1998, 1999). The historian of science Stillman Drake (1910–1993) published the most relevant manuscript papers in the 1970s, together with their interpretation (Galilei [1602-1637], 1979). These notes shed new light on the problems Galileo faced and on the experiments he had actually made.

The most significant experiment can be considered an indirect test of the principle of inertia. The top of sheet *116v* of the manuscript represents the trajectories of a ball that, after falling from an inclined plane, is deflected to move initially in the horizontal direction. The horizontal motion, ignoring air resistance, obeys the Galilean inertia principle that, in the absence of forces, the ball must continue to move with a straight line motion at constant speed. The ball in the vertical direction is subject to the weight force. The overall motion is the result of a composition of movements: one horizontal with constant speed and the other, vertical, naturally accelerated. Along a vertical line Galileo recorded the numbers 300, 600, 800, and 1000. They are the heights from which a ball is descending along an inclined plane (that has not been drawn). The unit of measure taken by Galileo is *the point*, which is equivalent to 0.9 mm. The table is placed at a height of 828 points. Along the horizontal axis are reported the distances in points from the vertical to which the ball touches the ground for the different heights of the inclined plane. The expected values of these *distances* are also indicated (with the deviations from the experimental values) calculated by Galileo, assuming the conservation of velocity in the horizontal direction.

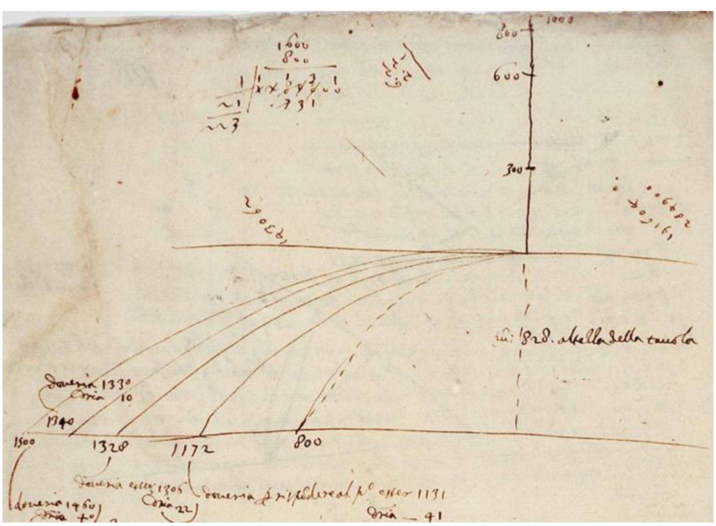
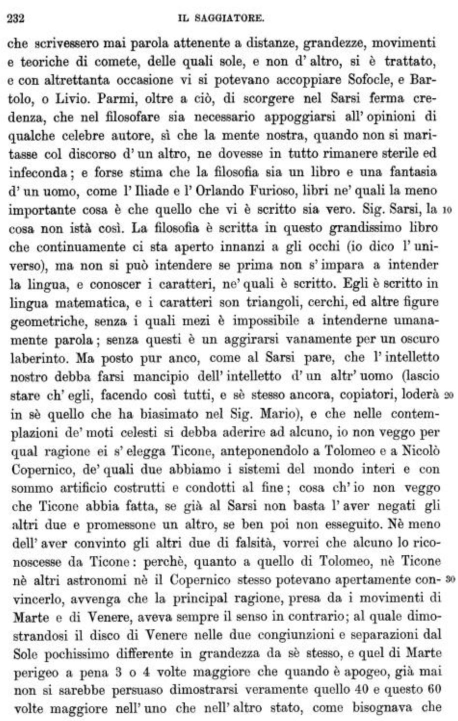


Fig. 5. The ranges of the projectile launched in the horizontal direction. Galilei 1602-1637, Ms. Gal. 72, detail of f. 116v. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.

In order to visualize the whole trajectories (not just the impact point with the ground), Galileo recorded the intersections of these with multiple parallel planes placed at different heights. As shown in the sheets *114v* and *81r*, he examined the behaviour of a ball that, after falling down along the inclined plane, instead of encountering a deflector at the end of its run, could proceed downward in an oblique direction. The ball, leaving the inclined plane, describes a curved trajectory until it strikes with a plane surface. Galileo recorded the impact points with the utmost precision possible for different planes in order to obtain a set of points in space whose interpolation assumes the shape of a curve, which nowadays is geometrically described by a parabola and analytically by a second order equation.

4. Galileo & Mathematics

The importance of mathematics in the physics of Galileo is emphasized in this passage of *Il Saggiatore* (Galilei 1623):



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che scrivessero mai parola attenente a distanze, grandezze, movimenti e teoriche di comete, delle quali sole, e non d' altro, si è trattato, e con altrettanta occasione vi si potevano accoppiare Sofocle, e Bartolo, o Livio. Parmi, oltre a ciò, di scorgere nel Sarsi ferma credenza, che nel filosofare sia necessario appoggiarsi all' opinioni di qualche celebre autore, sì che la mente nostra, quando non si maritasse col discorso d' un altro, ne dovesse in tutto rimanere sterile ed infeconda; e forse stima che la filosofia sia un libro e una fantasia d' un uomo, come l' Iliade e l' Orlando Furioso, libri ne' quali la meno importante cosa è che quello che vi è scritto sia vero. Sig. Sarsi, la cosa non istà così. La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi a gli occhi (io dico l' universo), ma non si può intendere se prima non s' impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto. Ma posto pur anco, come al Sarsi pare, che l' intelletto nostro debba farsi mancipio dell' intelletto d' un altr' uomo (lascio stare ch' egli, facendo così tutti, e sè stesso ancora, copiatori, loderà in sè quello che ha biasimato nel Sig. Mario), e che nelle contemplazioni de' moti celesti si debba aderire ad alcuno, io non veggo per qual ragione ei s' elegga Ticone, anteponendolo a Tolomeo e a Nicolò Copernico, de' quali due abbiamo i sistemi del mondo interi e con sommo artificio costrutti e condotti al fine; cosa ch' io non veggo che Ticone abbia fatta, se già al Sarsi non basta l' aver negati gli altri due e promessone un altro, se ben poi non esseguito. Nè meno dell' aver convinto gli altri due di falsità, vorrei che alcuno lo riconoscesse da Ticone: perchè, quanto a quello di Tolomeo, nè Ticone nè altri astronomi nè il Copernico stesso potevano apertamente convincerlo, avvenga che la principal ragione, presa da i movimenti di Marte e di Venere, aveva sempre il senso in contrario; al quale dimostrandosi il disco di Venere nelle due congiunzioni e separazioni dal Sole pochissimo differente in grandezza da sè stesso, e quel di Marte perigeo a pena 3 o 4 volte maggiore che quando è apogeo, già mai non si sarebbe persuaso dimostrarsi veramente quello 40 e questo 60 volte maggiore nell' uno che nell' altro stato, come bisognava che

Fig. 6. Galileo on the relationship between physic and mathematics. Galileo 1890-1909, vol. VI, p. 232. Source: Bibliothèque nationale de France / gallica.bnf.fr.

Philosophy is written in this very great book that continually stands open before our eyes (I say the universe), but it cannot be understood unless we first learn to understand the language, and know the characters, in which it is written. It is written in a mathematical language, and the characters are triangles, circles, and other geometrical figures, without which means it is impossible to humanly understand a word of it; without these it is a vain wandering through a dark labyrinth.[14]

[14] The English translation is mine. "La filosofia è scritta in questo grandissimo libro che continuamente ci sta aperto innanzi a gli occhi (io dico l'universo), ma non si può intendere se prima non s'impara a intender la lingua, e conoscer i caratteri, ne' quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro laberinto." (Galilei [1623], 1890-1909, vol. VI, p. 232). Compare with (Galilei 1957, pp. 237–238). See also the English translation by Edwin Arthur Burtt (1925), p. 64.

The mathematics referred to by Galileo is Euclidean geometry and its theory of proportions. The latter, in particular, will allow Galileo to put on quantitative bases some simple relations between the quantities involved in the description of the phenomena, and will also allow him to transform these relations in a geometric sense (Giusti 1993, pp 96-97).

While Galileo proposes to analyze the relations of proportionality between any type of homogeneous physical quantities, two by two, provided they were measurable, Euclid had only given his definition of proportionality for geometrical quantities or numbers. For these quantities he had constructed a theory that Galileo fully used and tried to reform to make it simpler and suitable for the description of physical phenomena (Ivi, chapters 3 and 4).[15] To this end, moreover, Galileo postulates the existence of the fourth proportional and makes extensive use of compound proportions. An example is that of Theorem IV, Proposition IV, about uniform motion:

> If two particles are carried with uniform motion, but each with a different speed, the distances covered by them during unequal intervals of time bear to each other the compound ratio of the speeds and time intervals (Galilei [1638], 1914, p. 157; Galilei [1638], 1890-1909, vol. VIII, p. 194).

that Galileo used to prove the proportionality of spaces with the squares of the times in naturally accelerated falling motion.[16]

Galileo demonstrates a great knowledge of proportions and their use, both theoretical and experimental. He defines uniform motion with its properties, demonstrates a series of theorems in order to found a new science of movement, verifies the proportionality between the spaces travelled and the squares of the times employed in naturally accelerated motion.

The use that Galileo makes of proportions is far from trivial. To indicate the proportionality between the times of fall and the square roots of the spaces travelled in a naturally accelerated motion, he makes use of the proportional mean:

> starting from any initial point, if we take any two distances, traversed in any time-intervals whatsoever, these time-intervals bear to one another the same ratio as one of the distances to the mean proportional of the two distances (Galilei [1638], 1914, p. 180; Galilei [1638], 1890-1909, vol. VIII, p. 214).

[15] In the last years of his life, Galileo dictated to his disciple Evangelista Torricelli (1608-1647), in the form of dialogue, his critiques of Euclid's theory of proportions to be added to the *Discorsi* as the fifth day. The Torricelli's manuscript Gal. 75 (Galileo, Torricelli and Viviani 1641-1679), to which further modifications were made, was published by Vincenzo Viviani (1622-1703) (Galileo et al., 1679). The text is reported in Volume VIII of the Works of Galileo (Galileo [1638], 1890-1909, vol. VIII, pp. 347-362). Enrico Giusti in his new edition of this manuscript tried to isolate Galileo's contribution from that of his disciples (Galileo, Torricelli and Viviani [1641-1679, 1690], 1993).

[16] Galilei considers naturally accelerated motion equivalent to a uniform motion with a speed equal to the mean speed, that in this case is half of the final speed, of the accelerated motion (Mean speed theorem also known as Merton's theorem from the Merton Collage, Oxford), and since the ratio between these velocities is equal to the ratios of the times, the ratio between the distances travelled is equal to the ratio of the squares of the times. Theorem II, Proposition II of naturally accelerated motion: "The spaces described by a body falling from rest with a uniformly accelerated motion are to each other as the squares of the time-intervals employed in traversing these distances" (Galilei [1638], 1914, p. 174; Galilei [1638], 1890-1909, vol. VIII, p. 209).

He uses proportions also for the study of the parabolic motion of the horizontally launched projectile that was examined in the preceding paragraph, measuring first the range in correspondence of a certain initial height and then calculating a new range in the hypothesis that the ranges (as the speeds in the horizontal direction) are proportional to the square roots of the heights (see Abattouy 1996b, p. 45).

However, it is mathematics more than logic that substantiates Galileo's demonstrations. Mathematics allows Galileo to surpass the common language in the description of physical laws and to pass, without fear of being deceived, from the first principles of a theory to its final consequences.

According to Geymonat ([1957] 1969), Galileo refers to the idea of limit even though it has not yet been codified, more than to the mathematical concepts of classical geometry. In fact, he makes an implicit use of it in dealing with quantities such as speed and acceleration, so that some of his calculations “[...] are equivalent to true derivations and integrations” (Ivi, p. 296).

In the first day of the *Discorsi*, Galileo deals with the problem of infinity, the degrees of infinity and indivisibles, debating the significant differences identified between actual infinity and potential infinity. He raises the difficult question of how to select one typed of infinity, the actual or potential one, a question which appears at the same time both philosophical and scientific (Cioci and Drago, 2017).

The paradoxes about infinity devised by Galileo are well known, as are the comparison between integers and their squares or the comparison between sets of points of two segments of different length.

In Galileo's *Discorsi,* for the purpose of understanding the problems of the continuum and of the constitution of matter, particularly interesting is the paradox of the Aristotle's wheel, so-called since it is presented in pseudo-Aristotelian *Mechanical Problems* ([appr. 330 bC], 1991). Galileo devises a model of continuum – and then a physical and mathematical model of the structure of matter through which he will give an interpretation of the phenomena of condensation and rarefaction – consisting of an infinite number of filled spaces alternated with the same number of empty spaces. In general, according to Galileo, a line (a surface or a solid) is built up of a subdivision of any finite number of quantified parts or of an infinite number of quantities not finite in size and, therefore, indivisible. The issue of continuum is of fundamental importance in Galileo's physics because he had “to present in some way as a plausible and justified the demonstrative processes and the representation with geometric quantities of velocity, with its endless moments of time and space, infinitely divisible” (Galluzzi 1979, p. 362).

While on the one hand Galileo introduces the discussion about actual infinity, on the other he does not bring his choice to a close because he decided not to use completely Cavalieri’s geometry of indivisibles in its entirety (the forerunner of modern integral calculus) in an operating mode, although he was well aware of the research of his pupil.

Galilei was confused about the paradoxes that arose from admitting actual infinity, and due to this, while strongly affirming his ideas on the philosophical level, he preferred to be more cautious from a mathematical point of view.

A particularly significant example – also for educational purposes in a Scientific Lyceum – is that of the bowl, a solid figure determined by the removal of a hemisphere from a cylinder also containing a cone. The cone and the bowl have the same volume. Taking the floor and sliding it upwards, but always keeping it parallel to itself, the parts where that plan is intersecting the cone and the bowl are always equal to each other.

The paradox arises because as the cutting plane nears the top, the intersection of the planes with the two solids degenerates into a circumference and in a single point, "It appears therefore that we may equate the circumference of a large circle to a single point". Bonaventura Cavalieri gave a justification to the paradox in a letter addressed to Galileo. He argued that, in the example of the *bowl*, both the circumference, and the point could, in a sense, be considered equal, by both having an area of zero.

The study at the Lyceum of these considerations of Galileo about infinity could be a good introduction to the teaching /learning of mathematical analysis that could be studied in a problematic way.

5. Galileo & Nature of Science

I focused on the historical foundation of science (physics and mathematics) related to Galileo's works and his heritage. For this, I used *Nature of Science* (Nos) taking into account the history, philosophy and epistemology of science. For example:

> The implications of nature of science, its history, philosophy, sociology and epistemology, for science education. The significance of models and modelling for science education as reflected in the particular importance attached to the use of metaphors, analogy, visualization, simulations and animations in science.[17]

Particularly, in my case, I avoided sociological studies.

The analysis of mechanics in Galileo's writings was used for teaching and pedagogical purposes, by proposing to the students an appropriate pedagogical path in a laboratory of physics measurement with an historical interdisciplinary approach to the disciplines of science teaching.

The main subjects of kinematics which are taught at the Lyceum are the fruits of Galileo's research: uniform motion and uniformly accelerated motion, the free fall motion in the void, parabolic movement, pendulum motion. A historical approach to the teaching of the physics of Galileo is therefore particularly effective at the Lyceum. The following are a number of examples which I have already done at *Liceo Scientifico "F. Sbordone" di Napoli*.

An educational path consists mainly of four experiments actually performed by Galileo. Each of them proposes to answer a problem and to solve a conceptual node.

[17] Retrieved via: http://www.esera2017.org/programme/strands/

1. The first question is: *does the free fall of bodies in a vacuum depend on the mass*? We drop small balls of different masses first in oil, then in water, then in air by extrapolating their behaviour in vacuum. I performed this experiment with my students, and during a training course for physics teachers. We can see that the two balls have different speeds in water. In air the difference between the two speeds is smaller. Galileo extrapolates that in the void (in which, in his time, he could not perform the experiment) the two balls have the same speed.
2. The second question is: how could we measure time if we were in Galileo's place? Let us swing two pendulum of different masses simultaneously, with different amplitude or with wires of different lengths. It is observed that the period of oscillation depends only on the length of the wire. Because it does not depend on the amplitude of the pendulum, oscillation during the motion allows us to use the oscillation period as a unit of time measurement. To then obtain a continuous measurement of time it is necessary to construct a water clock that is calibrated using the pendulum.
3. The third question is: how does the distance travelled depend on the *time employed?* We slow the free fall movement by using an inclined plane so that we can study motion. A moving body starting from rest and falling on an inclined plane travels during equal intervals of time distances which are related to each other as the odd numbers beginning with unity 1, 3, 5, 7. In fact, setting some bells in these positions, we hear sounds at equal intervals of time. Considering the total spaces travelled, they are in the ratio of the squares of the times.
4. The fourth question is: *is it possible to obtain a motion at constant speed when a body is not subject to forces?* If a ball is launched in air in horizontal direction, it describes a parabolic trajectory that is the composition of a uniform motion in the horizontal direction and a uniformly accelerated motion in the vertical direction where the force of gravity acts.[18] A small ball falls on an inclined plane and then it continues with a parabolic trajectory obtaining in the horizontal direction a uniform movement. We solved the equations of motion and compared the theoretical expected results with those experimental. The trajectory of the ball is reconstructed by measuring the position of the impact point with a horizontal plane placed at different heights.

The students showed great interest and active participation in the lessons, especially in finding and using the original Galilean texts in Italian (or English if the activity is multidisciplinary) and manuscripts containing Galileo’s essays on Mechanics.

[18] I made this experiment with my students of Lyceum, terminal cycle.

Thus, in this work I wanted the students to debate on an educational path based on the historical foundations of Galilean science, and to understand how to apply the experimental method for modelling and then determining a physical law.

I utilized inquiry-based education (for example *What would you do if you were in Galileo's place?*). The pupils actively contributed to the definition of the path. Inductive scientific methods of reasoning, ad absurdum proofs et al. were preferred.

I evaluated the impact of this approach by means of entry and exit questionnaires about the satisfaction and the preferences of pupils and teachers and about the students' knowledge. Special verifications appropriate to evaluate this educational path included exercises in historical text comprehension and a written report on laboratory activities.

An interesting innovation that could be followed in teaching this subject is to use the sensors integrated into smartphones, as I did recently (Cioci and Sapia 2015; Cioci 2017; see also Vogt and Kuhn 2012a, 2012b). One of these sensors, the accelerometer, can provide important information about the movement of the smartphone. By using the smartphone's acceleration sensor, it is relatively easy to capture the interest of students. Today, Galileo's experiments can be performed in a very simple and interesting way, but the experiments, reasoning and analogies identified by Galileo have a superior educational value, because they will help the students to understand the methods that science uses to go beyond the limits of knowledge already acquired.

6. Expected Impact

The expected impact of my thesis relates to both the history of physics and to physics education in terms of curriculum issues. I expect to shed new light on the experiments actually performed by Galileo that can be proposed in an educational path on kinematics and the initial elements of dynamics, valid for both university and upper secondary school. I hope, through a three-year didactic experimentation at *Liceo scientifico "F. Sbordone*" in Naples, to be able to demonstrate the effectiveness of a teaching method that includes the History of physics - and in particular the re-proposal of experiments of historical interest carried out, where possible, under the same conditions as they were performed at the time - in the learning process of inquiring. I also expect with this case study to be able to contribute to an understanding of students' learning difficulties in mechanics, so as to appropriately modulate the teaching-learning topics and their degree of depth between the first three years of upper secondary school teaching, also taking into account the mathematical tools possessed by the students. Another expected impact of this experimental educational path is to make students (and trainee teachers who wish to follow it) understand what the Nature of Science is and in particular how scientists work.

7. Publicising my research work

In the last few years, I have presented the preliminary results of my research at an international level.

On 31 March 2017, I took part in the study day “Une vision d'ensemble sur les didactiques, pédagogies et professionnalisations” with a paper entitled L'enseignement de la physique au lycée et le rôle de l'histoire des sciences : le cas de la physique galiléenne, which was held at the *Science de l'Homme et de la Societé (SHS)* Doctoral School at the *Université de Lille*.

In Lille, on 11 and 12 October 2017, I was also part of the organising committee of the *2nd International Summer School for Sciences, History and Philosophy of Sciences & Science Education (ISSHPSE)* hosted by the Inter-Divisional Teaching Commission (IDTC), the *Maison Européenne des Sciences de l'Homme et de la Société (MESHS)* and *the Ècole Doctorale SHS de l'Université de Lille.*

I personally presented my work in Pisa in 9 - 12 September 2019 at the 39th Congress of *SISFA (Società Italiana degli Storici della Fisica e dell'Astronomia)*, with a paper entitled *A Nature of Science Experiments Exploring on the Galilean Physics of the Motion* (fig.8).

Fig. 7. Photo of Galileo's house in Pisa. Vincenzo Cioci self-portrait. In the background you can see the Tower of Pisa where, according to tradition, Galileo carried out the famous experiments on falling bodies. Source: Photos taken by Vincenzo Cioci on the occasion of the 39th SISFA Congress.

I was also part of the organising committee of the training course for teachers entitled *Il concetto di Massa tra storia e didattica della fisica e dell'astronomia*, 11-13 September, Pisa, Dipartimento di Fisica 'Enrico Fermi'. Workshop promoted by SISFA, URDF of the University of Udine, MIUR, PNLS, in collaboration with AIF, INFN, Department of Physics of the University of Pavia., Department of Physics of the University of Pisa, DSFTA of the University of Siena (fig.9).

I presented the results of my research work with increasing depth in several meetings and congresses:

- the *9th International Conference of the European Society for the History of Science (ESHS)*, September 2020 organised by the Centre for the History of Universities and Sciences of the University of Bologna (CIS) and the Italian Society for the History of Science (SISS) from August 31 to September 3, 2020, with a report entitled *Exploring Sensory Galileo's Tools for Modelling Motion into History of Physics & Nature of Science* Panel Session Visualizing and Modelling Sensory Actions (VMSA) for Inquiring Science & Technology into History - 2

- the *40th National Congress of SISFA (Società Italiana degli Storici della Fisica e dell'Astronomia)*, Online September 7-11, 2020, in videoconference, with a report (together with Prof. Pisano) entitled *Reporting Experimental Results of a Galilean Teaching Case Study*.

- the Seminar of the *Groupe International de Recherche sur l'Enseignement de la Physique*/International Research Group on the Teaching of Physics (Girep) *Physics Teacher Education – What matters?* organized by the University of Malta, November 16-18, 2020, in videoconference, Work-group *In-service teacher professional learning strategies*, with a short report (together with Prof. Pisano) entitled *History of Physics as Teaching Framework for Phenomena, Experimenta & Modelling: Galilean Falling Bodies.*

- the *Seventh Congress of the French Society for the History of Science and Technology* during the eighth centenary of the School of Medicine in Montreal; in videoconference, from 21 to 23 April 2021, as responsible for symposium n. 25 entitled *NoS–Nature of Science Teaching: Measurement Tools of Physics from Galileo to the Present Day between Science and Technology*, with a contribution entitled *Galileo's Tools for the Study of Motion into History Physics and Nature of Science Teaching* (together with Professor Pisano).

- the *26th International Congress of History of Science and Technology (ICHST)*, July 25-31, 2021, symposium ID 60, "Designing curricula as an interdisciplinary programmed framework in the history of science & scientific–technical teaching", with a report entitled *A NoS Experimental Curriculum on motion: Galileo and His Contemporaries.*

Lastly, I extended this research beyond the context of Galileo's scientific and personal experience: I participated in the *2nd ESHS Early Career Scholars Conference Science and its Enemies: Exploring Conflicts and Alliances in the History of Science* from the 20th until the 22nd of September 2021. I contributed with an online presentation entitled *Galileo and Oppenheimer: history of two scientists fought but not won*. In the appendix of this thesis you can find the presentation given in this conference (Cioci 2021).

Fig. 8. The poster of the Workshop for the training of teachers at the Italian national level in which Cioci Vincenzo was part of SISFA organizing committee. With permission. Retrieved via Sisfa.org.

8. A Short Overview of the Thesis Project

Ph.D Candidate	Mr. Vincenzo CIOCI CIREL, *Université de Lille*, France
Ph.D Thesis Title **a.y. 2016-2022**	***Galileo's Falling Bodies into the History of Physics and the Nature of Science as a Case Study***
Ph.D Advisor	Prof. Dr. Raffaele PISANO, HDR Physicist, Professor of History of Physics, History of science & Education IEMN, CNRS–*Université de Lille*, France
Thesis Monitoring Committee[19] **[*Comité de suivi*]**	1. Prof. Olival Freire Jr., Institute of Physics, Post–graduate programme of Epistemology, History and Teaching of sciences, Bahia Federal University (**Brazil**) 2. Prof. Peter Heering, Institute of mathematic, scientific and technical literacy. Section of physics, its didactics and its history. European University of Flensburg (**Germany**) 3. Prof. Laurence Maurines, DidaScO, *SHS de l'université Paris-Saclay, Faculté des sciences d'Orsay Université Paris-Sud /Université Paris-Saclay, Orsay* (**France**) 4. Prof. Eilish Mcloughlin, Centre for the Advancement of STEM Teaching and Learning (CASTeL), Dublin City University, Dublin (**Ireland**) 5. Prof. Snezana Lawrence, Faculty of Health, Social Care & Education, Department of School of Education & Social Care, Cambridge, Anglia Ruskin University,

[19] The Thesis Monitoring Committee has a scientific role of support and advice. It is a benevolent mechanism that aims to support the doctoral student's research work. In accordance with current French rules and regulations, this committee ensures that the course is properly carried out, based on the doctoral charter and the training agreement. It evaluates, in an interview with the doctoral student, the conditions of his training and the progress of his research. It makes recommendations and sends a report on the interview to the director of the doctoral school, the doctoral student and the thesis director. It shall in particular take care to prevent any form of conflict, discrimination or harassment.

Cambridge (**The United Kingdom**)

6. Prof. Jim Ryder, School of Education, Faculty of Education, Social Sciences and Law, University of Leeds, Leeds (**The United Kingdom**)

Ph.D School — *École Doctorale des Sciences de l'Homme et de la Société, Université de Lille*, France

Area of Research — History of Physics, Nature of Science Teaching, Epistemology of Science (Physics and Mathematics), Physics education

Subject — History of Mechanics (Kinematics and Dynamics)

The Structure

- The first part deals with free-fall motion and parabolic motion as addressed by Galileo, setting his theories in the historical context of the interpretations of his predecessors.
- The second part concerns research in Physics Education. As an example of the application of the latest models and as a case study, an educational historical path on Galieo's physics (essentially kinematics and the early dynamics) is designed.
- The last part is about the analysis of Teaching-Learning both as NoS inquiring and Didactics and Epistemological inquiring for this case study.

Outline of the Problem — History of the discovery of free fall law and of parabolic motion. A case study concerning the inclusion of the teaching of the history of physics at high school (to students and teachers) in the curriculum and its influence on the understanding of the Nature of Science.

General Objectives — The overall objective of this work is to study the history of the discovery of free fall law and of parabolic motion, the contribution of Galileo and his precursors, as a relevant topic in the History of Science and Nature of Science Teaching (NoS) from an interdisciplinary point of view and to make a teaching proposal - which is widely tested - within the NoS Teaching framework.

Specific Objectives — To evaluate how kinematics and initial elements of dynamics can be taught

To develop an historical account of the discovery of the free fall law

	To construct a synthesis of the history of the subject to elaborate a teaching proposal within the NoS Teaching framework.
Research Questions	How can the curriculum/educational path be enriched so that students better understand the Nature of Science? What are the students' misconceptions and learning difficulties about free fall motion and projectile motion? What elements of Galileo's physics (both experimental and theoretical) can be deducted from his writings to develop an educational path aimed at overcoming the conceptual difficulties of the students? Is this educational path effective with regard to the concepts and skills acquired by the students? Did the students change their interest and way of looking at physics and more generally science?
Methodology of Research	Research and study based on primary sources, volumes, letters and laboratory notes Online Research, peer-reviewed papers, books, conferences, etc. Using established physics education research methodology guidelines Comparing the results of the entry and exit questionnaires about the motivations and interests of the students who participated in the learning activity, with the correct statistical method, the educational action carried out with the project will be causally analysed. Guided inquiry process is used for teaching / learning, but pupils can design variations of the experiments to be carried out as long as they are compatible with the aims of the project.
Expected Results	Showing that the history of free fall and of projectile motion can be taught both at the University (to teachers in training as well as future researchers) and to high school students with very satisfactory results Recognition of the role played by different scientists - beyond Galileo - in the discovery of the free fall law with the awareness that scientific results are the result of an incremental and gradual process Designing a comprehensive course on Galileo's physics (kinematics and first elements of dynamics) in both the theoretical and experimental parts

Influence on the teaching of mechanics at high school and university and on the interest and motivation of students towards the study of this theme

Perspectives

To develop interdisciplinarity which plays an extremely significant role in research and science education

In terms of teaching renewal, this can be addressed through theoretical assumptions, such as those related to the importance of the Nature of Science in the educational sciences. In fact, references to the history of physics can be used as a guide and provide a rich context for a critical analysis of different theories, allowing students to follow the reasoning of scientists and giving them motivation

To propose in the classroom, the reproduction of experiments of historical interest performed under the same conditions as at the time - if possible - or with modern instrumentation

The identification of an inquiry method for science teaching which is enriched by the consideration of the history of science within the learning process

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- *Biblioteca Nazionale Centrale di Roma*
- *Bibliothèque Nationale de France*
- Library of Congress

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- American Journal of Physics
- Annals of Science
- Bolletino di Storia delle Scienze Matematiche
- British Journal for the History of Mathematics
- Bulletin of Atomic Scientists,
- Foundation for Science Technology and Civilisation
- Humanistica Lovaniensia
- Isis
- Journal of College Science Teaching
- Journal of the American Statistical Association
- La Fisica nella Scuola
- Manuscripta. A journal for manuscript research.
- Physis
- Science education
- Science & Education

- Science in Context
- Scientific American
- Studies in Science Education
- The British Journal for the History of Science
- The Physics Teacher

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Online Resources:

- ✓ Academia
- ✓ Archive.org
- ✓ Città della Scienza
- ✓ Eric.ed.gov
- ✓ Europeana.eu
- ✓ Gallica (BNF)
- ✓ Girep.org
- ✓ HAL
- ✓ Internet Culturale. Cataloghi e collezioni digitali delle biblioteche italiane
- ✓ JSTOR
- ✓ Max Planck Institute for the History of Science
- ✓ *Museo Galileo – Istituto e Museo di Storia della Scienza* in Florence
- ✓ Springer
- ✓ ResearchGate
- ✓ Various University Presses (Cambridge, Pavia, Pisa, …)

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Figg. 1, 2, 3, 4, 6, 2.4, 2.8, 2.9, 2.10, 2.11, 2.12, 2.13, 2.14, 2.15, 2.18, 2.21. Galilei 1890-1909, Edizione Nazionale delle Opere di Galileo Galilei. A cura di Favaro A. Barbera, Firenze. Source: Bibliothèque nationale de France / gallica.bnf.fr. **Fig. 1.4** Oresme N (1400-1420) *Nicole Oresme, Traité de la sphère; Aristote, De caelo et de mundo*, traduction française par Nicole Oresme. Detail of f. 1r. Source: Bibliothèque nationale de France. Département des Manuscrits. Français 565 / gallica.bnf.fr. **Fig. 5.1.** My own representation obtained by modifying an image taken from Galileo's *Trattato di fortificazione* (Galilei 1890-1909, vol. II, p. 93). Source: Bibliothèque nationale de France / gallica.bnf.fr. The non-commercial re-use of this content accessible on the Gallica website is free and unrestricted but the mention of the source is mandatory. Commercial re-use of this content is subject to a fee and a licence. Researchers are exempted from any commercial use fee for their publications for academic purposes.

Figg. 5, 2.16, 2.17, 2.19, 2.22, 2.23, 2.24. Galilei

1602-1637, Ms. Gal. 72, folios 1r and 128v; parts of folios 81r, 107v, 114v, 116v, 117r, 152r and 175v. **Fig. 2.1, 2.2.** Galilei 1589-1592. Ms. Gal. 71. Details of folios 95r, 97r, 97v. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.[20] None of these images may be used for further reproduction and duplication by any means without the written permission of the *Biblioteca Nazionale Centrale di Firenze*. Reproductions for non-commercial publishing purposes are free, but with prior notification to the *Biblioteca Nazionale Centrale di Firenze* by electronic means.

Figg. 7, 5.5. Photos taken by Vincenzo Cioci.

Fig. 8. By concession of *SISFA (Società Italiana degli Storici della Fisica e dell'Astronomia)*. Retrieved via Sisfa.org.

Fig. 1.1. Santbech D (1561) *Problematum Astronomicorum et Geometricorum Sectiones Septem.* Detail of p. 212. Source: Public domain. Retrieved via Wikimedia Commons.

Fig. 1.2. H. Diels (1895) *Commentaria in Aristotelem graeca. Edita Consilio et Auctoritate. Academiae Litterarum Regiae Borussicae*. Berolini, Berlin. Vol. X. Cover and title page. Source: Public domain, University of Toronto - Robarts Library. Retrieved via Archive.org.

Fig. 1.3. Manuscript of Buridan's *Questiones super octo phisicorum libros Aristotelis*, copied in Padua. Source: Vat. Lat. 2163, f. 1r. © 1377 *Biblioteca Apostolica Vaticana*, Vatican City. Reproduced by concession of the *Biblioteca Apostolica Vaticana*, all rights reserved. Retrieved via Digi.vatlib.it.

Fig. 1.5. Politus B (ed.) (1505) *Questio de modalibus Bassani Politi* […], p. 56. Source: *Biblioteca Nazionale Centrale "Vittorio Emanuele II" di Roma*. Collocation 12. 27.H.7. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale "Vittorio Emanuele II" di Roma.* Image retrieved via Archive.org. Digitized by Google. This image cannot be used for further reproduction and duplication by any means without the written permission of the *Biblioteca Nazionale Centrale di Roma.*

Fig. 1.6. Duhem P (1913) *Etudes sur Léonard de*

[20] Ms. Gal. 72 can also be consulted on the web portal of catalogues and digital collections of Italian libraries – Internetculturale.it, on the website of the *Museo Galileo – Istituto e Museo di Storia della Scienza* in Florence and the Max Planck Institute for the History of Science in Berlin, where it is shown with several useful tools for its study.

Vinci. Les précurseurs parisiens de Galilée. Troisième série. Librairie scientifique A. Hermann et fils, p. 395. Source: Public domain, PIMS - University of Toronto. Retrieved via Archive.org.

Figg. 1.7, 1.8. Tartaglia (1537) *Nova Scientia inventa da Nicolo Tartalea.* Stephano da Sabio, Venezia. Book I, ff. 1r, 4r, 8v. Source: Public domain. The Dibner Library of the History of Science and Technology. Smithsonian Institution Libraries. Retrieved via Library.si.edu/digital-library.

Fig. 1.9. Tartaglia (1562) *Regola Generale di soleuare ogni fondata Naue & Nauilli con Ragione.* Title page. Source: Public domain, *Biblioteca Central Militar* / Virtual Library of the Spanish Ministry of Defense. Retrieved via Bibliotecavirtual.defensa.gob.es.

Figg. 2.3, 2.6, 2.7, 2.20, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 4.13, 5.2, 5.3, 5.4. Source: my own graphic production.

Fig. 2.5. Galilei (1638) *Discorsi e Dimostrazioni Matematiche intorno a due Nuove Scienze Attenenti alla Mecanica & I Movimenti Locali*. *In Leida. Appresso gli Elsevirii.* Frontispiece. Source: Public domain, via Wikimedia Commons.

Figg. 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, 4.14, 4.15. Source: my photos of the experiment and my own experiment images.

Fig. 5.5. My photo of the training course for teachers (with permission).

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CONTENTS

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A NOS INNOVATIVE EDUCATIONAL PATH

PART III – NOS STUDY CASE ABOUT THE KINEMATICS OF GALILEO

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Appendix

PART I

Galileo's Falling Bodies into the History of Physics

An outline

This part of the thesis is dedicated to the analysis of historical sources dealing with the fall of the bodies and the motion of the projectiles that will serve as a basis for designing an educational path for upper secondary school students (or for university) centred on some experiments of particular historical value.

CHAPTER I

The First Studies on the Fall of Bodies

Prologue I

The aim of this chapter is to deal with some issues that remained open in Aristotle's physics regarding the fall of bodies and the motion of projectiles. An important point is the problem of the existence of the void and its implications on the motion of bodies. Two other questions of primary importance for the understanding of the physical world are connected to this subject: a purely “dynamic” one, such as the relationship between motion and force, that is, how the motion of a projectile could continue after the application of the force that had caused it, and the other - a classic subject taught in high school - the existence of a relationship between the motion of free fall and the mass of the body.

In this chapter, we will start from the analysis of Aristotle's texts in which these themes are treated, we will see how they were discussed by Galileo's precursors - including Jean Buridan (c. 1301 - c. 1359/62), Nicole Oresme (c. 1322 – 1382), Nicolò Tartaglia (c. 1449 – 1557) - and what improvements in understanding the phenomena have been made. These themes are particularly important because - as we saw in the part of the introduction on the teaching of physics - they correspond precisely to the major difficulties encountered by students in dynamics.

1.1 Elements of Aristotelian physics

The thinking of Aristotle about the motion of bodies is found in *De Caelo* and in *Physica (Book IV and VIII)*. Aristotle had supposed that the local motion of bodies could be natural or violent (unnatural). Natural motion is that of bodies that move spontaneously towards their natural place. Violent motion is that determined by a force (Aristotle 1991, *De Caelo*, I, 8, 276a 22-27).

An example of natural motion is the free fall motion of heavy bodies towards the centre of the Earth. The relationships between the falling speed of bodies and their weight or the relationship of this speed with the characteristics of the medium in which it falls are described by Aristotle in the fourth book of *Physica* where he discusses the impossibility of the existence of a vacuum.

Aristotle points out the two reasons one body falls faster than another:

> either because there is a difference in what it moves through, […], or because, other things being equal, the moving body differs from the other owing to excess of weight or of lightness. Now the medium causes a difference because it impedes the moving thing, […] and especially a medium that is not easily divided, i.e. a medium that is somewhat *dense*[21] (Aristotle 1930, 1991, *Physica*, IV, 8, 215a 25-31)[22].

Therefore, according to Aristotle, the velocities of free-falling bodies - that can be compared because they "occupy time, and there is a ratio between any two times" (Ivi, 216a 10-11) - are in the same ratio of their weights, but they are in the inverse relationship to the density and resistance that the media oppose to their passage. Since it is not possible to calculate this ratio for any medium and the vacuum (being the second null) it would result that the body would move "through the void with a speed beyond any ratio" (Ivi, 216a 22-23). This would be impossible and would imply the non-existence of the void.[23]

An example of violent motion is the motion of projectiles. Aristotle believed that motion could occur as a result of a force continuously applied to the body. How then is it possible that this motion endures even after it has been thrown? His theory to justify violent motion includes the active role of the medium:

> things that are thrown move though that which gave them their impulse is not touching them, either by reason of mutual replacement, as some maintain, or because the air that has been pushed pushes them with a movement quicker than the natural locomotion of the projectile wherewith it moves to its proper place (Ivi, 215a 14-17)[24].

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[21] The italics are mine to indicate the identification between density and viscosity which is often a misconception even by students.

[22] The English translation of *Physica* in both the Ross (Aristotle 1930) and Barnes (Aristotle 1991) editions are by R. P. Hardie and R. K. Gaye.

[23] It makes us reflect that Aristotle arguing on the impossibility of the void, analysing the mechanism by which the body overcomes the resistance of the medium, also comes to affirm - thinking the absurd thing - that in the void bodies "which have a greater impulse either of weight or of lightness, if they are alike in other respects," would move with the same speed (Ivi, 216a 12-20).

[24] In the next paragraph, in order to make manifest the impossibility of the void, Aristotle

Therefore, there are two mechanisms identified by Aristotle to explain the motion of projectiles. The first is the theory of mutual replacement (or also called - from the Greek - antiperistasis) which Aristotle states but does not further defend and for which Marshall Clagett in his encyclopedic work *The Science of Mechanics in the Middle Age* (Clagett [1959], 1979, pp. 507-508) bore evidence to be from Plato (see *Timaeus*, 79-80, pp. 571-572). The medieval commentator of Aristotle, Simplicius (c. 490 – c. 560) and 14th-century philosopher John Buridan (see Clagett [1959], 1979, p. 508) explained that the motion of the projectile was due to the mutual replacement of the air that, pushed by the projectile, in turn pushed more air to take the place left free by the projectile, not allowing the formation of the vacuum, and that - attracted quickly - gives the necessary thrust for motion. The second always includes the thrust of the projectile by the air which is moved by the same mover that sets the body in motion. The mechanism is further explained by Aristotle in the eighth book of *Physica* in which he illustrates how the air in contact with the original motor moves the contiguous air up to the projectile and causing the continuous motion of the body with its thrusts (Aristotle 1991, *Physica*, VIII, 10, 266b.26-267b.19).

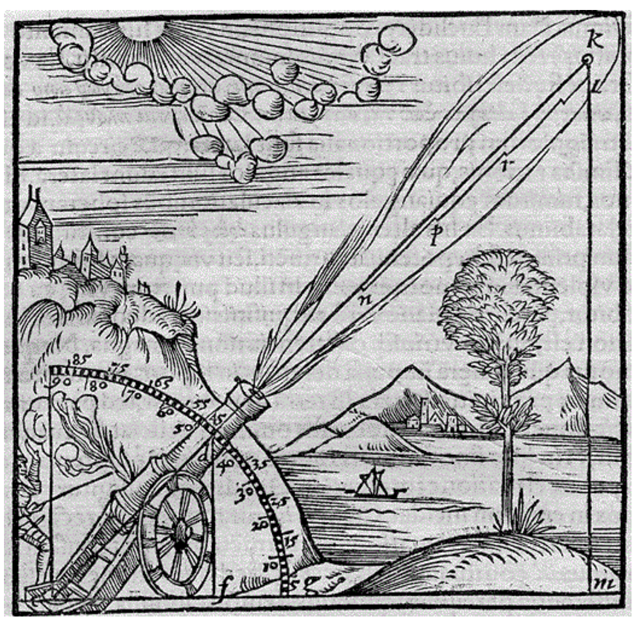

Fig. 1.1. Violent motion followed by natural motion with an attempt (lines n, p, r) to visualize the successive actions of the air that pushes the projectile upwards. Santbech D (1561) *Problematum Astronomicorum et Geometricorum Sectiones Septem.* Detail of p. 212. Source: Public domain. Retrieved via Wikimedia Commons.

states - in a hyperbolic way - what will later constitute the principle of inertia: "But in a void […] no one could say why a thing once set in motion should stop anywhere; for why should it stop here rather than here? So that a thing will either be at rest or must be moved ad infinitum, unless something more powerful gets in its way" (Ivi, 215a 18-21).

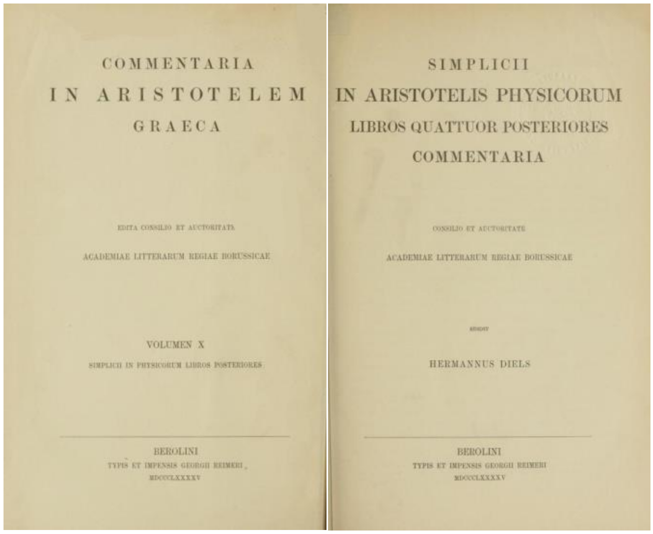
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Fig. 1.2. Cover and title page of Simplicius' writing *Aristotelis Physicorum Libros Quattuor Posteriores Commentaria*. In: H. Diels (1895) *Commentaria in Aristotelem graeca.* Edita Consilio et Auctoritate. Academiae Litterarum Regiae Borussicae. Vol. X. Berolini, Berlin. Source: Public domain, University of Toronto - Robarts Library. Retrieved via Archive.org.

1.2 Critics of Aristotle: Hipparchus and Philoponus

The first thinker to contradict Aristotle, of whom we know, was Hipparchus, a Greek astronomer and mathematician of the second century BC. None of his writing remain. Simplicius wrote that Hipparchus, in his work entitled *On Bodies Carried Down by Their Weight*, had stated that the motion of a body thrown upwards is characterized by the action of the upward projecting force that makes its effect felt for much of the motion and which does not require the propulsive action of the medium. He divided the motion into three phases. Until “the projecting force overpowers downward tendency of the projectile, and that to the extent that this projecting force predominates,” it moves more rapidly upwards (first phase); then, as this force decreases, this movement continues at a slower rate as long as the projectile falls down “under the influence of its own internal impulse,” (second phase) and as this force continues to decrease, the projectile moves more rapidly downwards, and with maximum velocity “when this force is completely lost” (third phase) (Clagett [1959], 1979, p. 543).

Stillman Drake, analysing this passage, comes to hypothesize that Hipparchus had identified a law of proportionality between the extent that the projecting force predominates on the force of gravity, and velocity (or its velocity variation) upwards. He also identifies a phase of accelerated motion until “the downward motion must become uniform at a speed determined only by Aristotle's cause of fall, the heaviness of the body” (Drake 1989, p.

9).

It was John Philoponus, a sixth-century contemporary of Simplicius, who in his *Aristotelis Physicorum Libros Commentaria* (Clagett [1959], 1979, pp. 508-509) explicitly criticized the Aristotelian theory concerning projectile motion. Regarding the mutual replacement theory, Philoponus considered it unreasonable to think that air could move in a circular way and support the motion of the projectile. Regarding the second mechanism, Philoponus judged it wrong to consider the action due to the thrust of the air because it meant not taking into due account the action of the force, without which, in fact, motion does not occur. Philoponus is also remembered because he was the first to have reported that he actually observed the fall of two bodies of different weights from the same height, refuting the position of Aristotle, many years before Galileo.

> […] our view may be corroborated by actual observation more effectively than by any sort of verbal argument. For if you let fall from the same height two weights [„,] you will see that the ratio of the times required for the motion does not depend on the ratio of the weights […] (Clagett [1959], 1979, p. 546).

In order to justify violent motion, without assuming the action of the medium, Philoponus postulates that an "incorporeal motive force is imparted by the projector to the projectile". His thought will have a great influence on Arab authors, especially on Avicenna in the eleventh century (Ivi, pp. 509-510).

1.3 Jean Buridan and the theory of *impetus*

In the fourteenth century, the philosopher Jean Buridan – considered the founder of the Parisian school which would also include Nicole Oresme and Albert of Saxony – brought forward the concept of force impressed on a body in violent motion, identifying it with the *impetus*. It is a "permanent quality" of the body, responsible for its motion, acquired by it as a result of the application of force - such as the magnetic properties that are acquired by iron after being close to a magnet - and which never runs out (it would last indefinitely) if not for of the resistance suffered or due to other tendencies to movement - for example the heaviness of the body - which give it a motion in another direction. An important aspect about this physical quantity is that Buridan introduces a semi-quantitative relationship: the *impetus* in fact increases, the greater are the mass of the body and its speed (Clagett [1959], 1979, pp. 522-523). Duhem wrote about this in his work *Le Système du Monde*, acknowledging Buridan's role as a forerunner of Galilean and modern physics.

> If Buridan had been forced to give a defined mathematical form to the relationship between impetus and speed, he would have without doubt chosen the simplest and most natural form, that of proportionality. So did Galileo and Descartes later. What the former will call *impeto*, what the latter will call momentum (*quantitas motus*), is

> Buridan’s *impetus*, which the scholastic treatises studied in their youth have made known to these two masters; but it is the Impetus mathematically determined by the hypothesis that is proportional to the speed […] Galileo's Mechanics is, can we say, the adult form of a living science of which Buridan's Mechanics was the larva.[25]

On the other hand, Clavelin (1974, p. 94) and Clagett ([1959], 1979, p. 523) agree on the different ontological meaning of Buridan's *impetus* and modern momentum. The first in fact should be understood as a quantity which, as an impressed force, is the cause of the continuous movement - the concept of inertia having not yet been fully realised - the second, on the other hand, is a dynamic attribute of motion.

[25] The translation of the passages into English is mine. “Si l'on eût contraint Buridan de donner une forme mathématique déterminée à la relation entre l'impetus et la vitesse, il eût choisi, sans doute, la forme qui se présentait plus simple et la plus naturelle, celle de la proportionnalité. Ainsi feront plus tard Galilée et Descartes. Ce appellera impeto, ce que celui-ci nommera comme que celui-là quantité de mouvement, c'est l'impetus de Buridan, les traités scolastiques étudiés en que deux maîtres mais c'est l'impetus mathématiquement déterminé leur jeunesse ont fait connaître à ces l'hypothèse qu'il est proportionnel à la vitesse” (Duhem 1958, pp. 215). “La Mécanique de Galilée, c'est, peut-on dire, la forme adulte d'une science ses vivante dont la Mécanique de Buridan était la larve” (Ivi, p. 200).

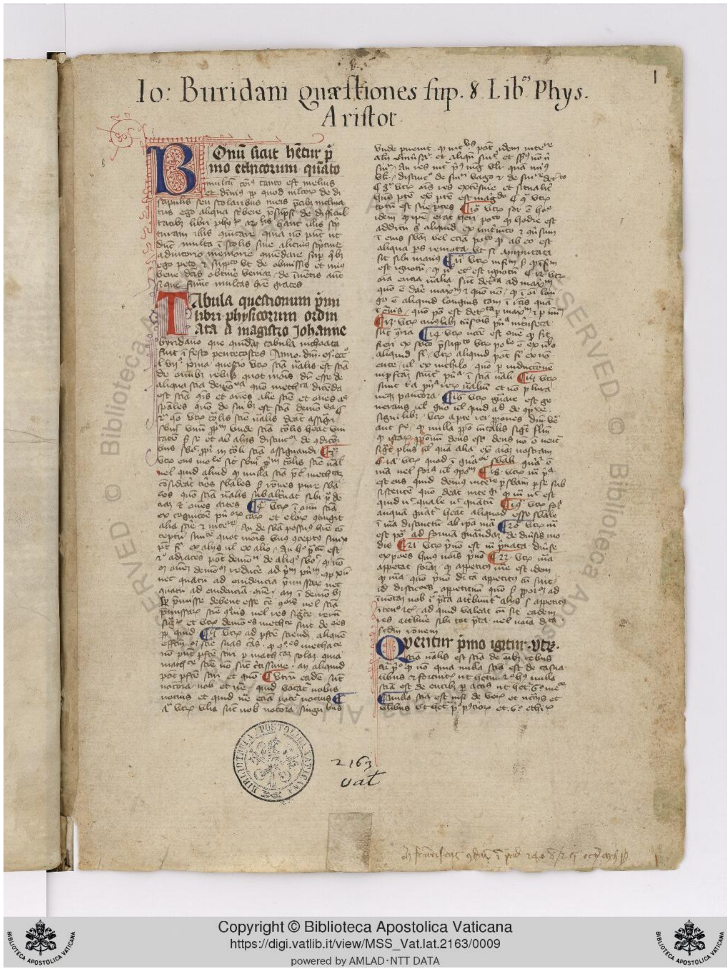


Fig. 1.3. Manuscript of Buridan's final version of the *Questiones super octo phisicorum libros Aristotelis*, Paris, 1509. Original work in c. 1357. Copied in Padua. Source: Reproduced with permission. Vat. Lat. 2163, f. 1r. © 1377 *Biblioteca Apostolica Vaticana*, Vatican City. Image retrieved via Digi.vatlib.it.[26]

1.4 Nicole Oresme and the Mean Speed Theorem

Great results were achieved on accelerated motion in the fourteenth century by the Paris and Oxford schools. Nicole Oresme, grand master of the College of Navarre at the University of Paris and later bishop of Lisieux and advisor to King Charles V, had formalized, ordered and developed what had been done before him (Clagett [1959], 1979, p. 347).

[26] For more details on this manuscript see: Clagett [1959], 1979, pp. 522, 645; Clagett and Murdoch 1959, p. 22; Marucchi 1985, p. 115; Maier 1951, p. 207, fn. 16.

Fig. 1.4. Oresme at work. The miniature is a detail of f. 1r of Oresme N (1400-1420) *Nicole Oresme, Traité de la sphère ; Aristote, De caelo et de mundo*, traduction française par Nicole Oresme. Source: Bibliothèque nationale de France. Département des Manuscrits. Français 565 / gallica.bnf.fr.

In his work *De Latitudinibus formarum*[27] (Oresme [1482,1486], 1505), he had illustrated the law of the spaces travelled by a body that moves with naturally accelerated motion.

> If a certain time has been divided into proportional parts (ie divided successively according to the powers of two), so that, in the first part of that time, a body moves with a certain speed; that, in the second, it moves twice as fast; in the third, three times more; and so on; the speed, always increasing in the same way, will be exactly four times the speed of the first part. In this way, in a whole hour, that body would cover a distance exactly four times that it travelled in the first proportional part, that is, in the first half hour; if, for example, in that first proportional part, it has travelled a length of one foot, in the remaining time it will travel three feet, and in the entire duration it will travel four feet.[28]

[27] Oresme did not publish any of his kinematic writings. The *De Latitudinibus formarum* was published in 1482, 1486, 1505 and 1515 but it could be an abridgment of the work of Oresme by another author. Extremely long quotations can be found in Duhem ([1913], 1984) and Clagett ([1959], 1979). Another collection of his writings is Oresme (1968).

[28] The English translation is mine. "Si un certain temps avait été ainsi divisé en parties proportionnelles ; qu'en la première partie de ce temps, un certain mobile se mût avec une certaine vitesse; qu'en la seconde, il se mût deux fois plus vite, en la troisième trois fois plus vite, et ainsi de suite, la vitesse croissant toujours de même, cette vitesse serait exactement quadruple de la hauteur de la première partie; en sorte qu'en l'heure entière, ce mobile parcourrait un chemin quadruple exactement de celui qu'il a parcouru en la première partie proportionnelle, c'est-à-dire en la première demi-heure; si, par exemple, en cette première partie proportionnelle, il a parcouru une longueur d'un pied, pendant le

Fig. 1.5. The representations of qualities. Plate from Oresme's *De latitudinibus formarum*. In: Politus B (ed.) (1505) *Questio de modalibus Bassani Politi* […], p. 56. Source: *Biblioteca Nazionale Centrale "Vittorio Emanuele II" di Roma*. Collocation 12. 27.H.7. Image retrieved via Archive.org. Digitized by Google. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale "Vittorio Emanuele II" di Roma*.

In *Tractatus de configurationibus qualitatum et motuum*, Oresme describes the patterns of physical qualities or properties (such as heat or speed). He distinguishes the *intensio* which is the degree of perfection of a quality (such as the degree of heat in each point) and the *extensio* which is the quantity, such as space or time, in which that quality is realized (such as for example the length of the heated iron bar). With clarifying intent, Oresme wanted to visualize these concepts with plane figures, what we now call rectangular coordinates. The *latitudo* represented *intensio* by a length proportional to it,

reste du temps, il parcourra trois pieds, et pendant la durée tout entière, il parcourra quatre pieds" (Duhem [1913], 1984, p. 393).

drawn perpendicular to the base, the *longitudo*, which represents the *extensio*. Oresme applied this representation to the analysis of local motion where the *latitudo* represents the speed, the *longitudo* represents the time and the area of the figure represents the distance travelled. The geometric shape of the figure obtained, according to Oresme, could correspond to one of the characteristics of the quality itself. Oresme defined a uniform quality (corresponding to a motion with a constant speed) as that which is represented by a line parallel to the longitude, and which differs from that characterized by a variable quality. Uniformly dissimilar qualities (corresponding to uniformly accelerated motions) are represented by a straight line inclined with respect to the longitude axis, moreover he described numerous cases of differently dissimilar qualities (corresponding to the various motion), extending this representation also to three dimensional figures (Clagett [1959], 1979, pp. 347-357).

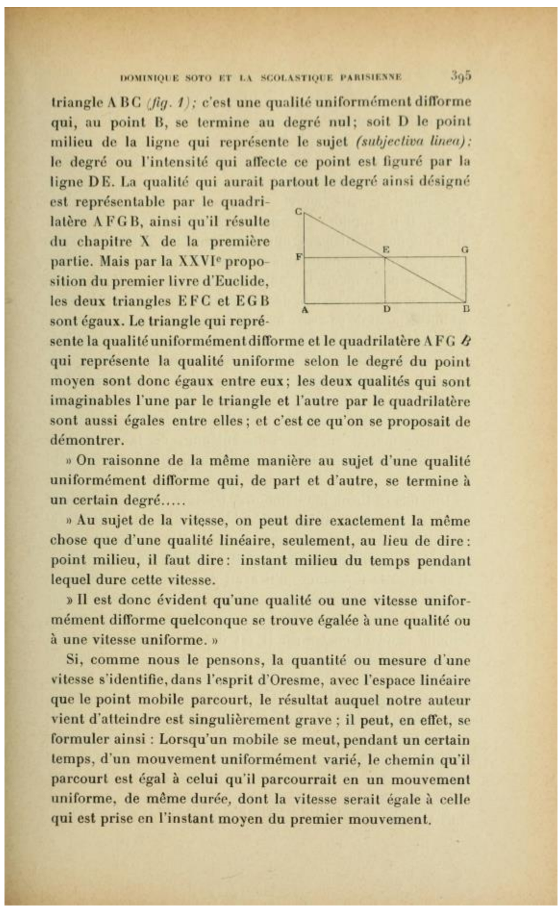
The Oxford school had reached similar results. Indeed, William Heytesbury (c. 1313 – c. 1372), one of its most representative exponents[29] who was also Chancellor of the University of Oxford, in his *Regulee solvendi sophismata* (Heytesbury [1335], 1491), had enunciated the rule, according to which "if any moving body uniformly increases its motion from zero degree up to any degree in an hour, it will traverse in the second half hour precisely three times the distance that it will traverse in the first half of the same [hour]" (Clagett [1959], 1979, p. 285) anticipating what would be the law of odd numbers defined by Galileo. Another important and well-known result of the Oxford school was the mean speed theorem according to which a moving body uniformly acquiring or losing an increment of speed "will traverse in some given time a magnitude completely equal to that which it would traverse if it were moving continuously through the same time with the mean degree [of speed]" (Clagett [1959], 1979, p. 284).

Oresme formulates the mean theorem in a very general way. In the *Tractatus de figuratione potentiarum et mensurarum difformitatum*, he states that "every uniformly dissimilar quality has the same total magnitude as if it uniformed the same subject according to the degree of the midpoint" (Duhem [1913], 1984, p. 394). Oresme's proof is based on the fact that a triangle is equivalent to a rectangle of equal base with a height equal to half that of the triangle. The total magnitude of uniformly dissimilar quality is represented by the area of the triangle, that of uniform quality by the area of the rectangle. At the conclusion of the proof, he then mentions the case of the comparison

29 Another prominent exponent of the Oxford School was Richard Swineshead known as the Calculator (Swineshead 1520). It is difficult to distinguish his contributions to the foundation of kinematics and dynamics from those of Heytesbury (see e.g. Clagett p. 215) where it is emphasised that both anticipated the concept of instantaneous velocity by writing words very similar to those Galileo would use in his *Discorsi su Due Nuove Scienze* (e.g. see "*celeritatis gradus*" in Galileo 1890-1909, vol. 8, p. 198). Even an analysis of his Juvenilia writings, in which he uses the concepts of the qualities of latitudes, shows that Galileo was familiar with what had been written before him (Galilei 1890-1909, vol. 1, pp. 120-121; see Carugo and Geymonat 1958, p. 777).

between motions with uniform and uniformly dissimilar speeds, and then, according to Duhem (Ivi, p. 395), arrives at the mean theorem for speed. The only clarification made by Oresme in this regard is that the uniformly dissimilar mean speed of motion corresponds to the speed of the mean instant of time of the interval during which the motion lasts.

The results achieved on the interpretation of accelerated motion during the fourteenth century were remarkable. Nevertheless, they have not been applied to the motion of the fall of the bodies. Accelerated motion was eventually associated only with the first phase of the fall (during which it was believed that the air resistance made its effect felt more) and possibly with the last phase. The reason is that, according to the Aristotelian school, the gravity of the bodies had to correspond to a certain speed and an increase of this could only be related to an increase in gravity (Koyré [1939], 1978, pp. 31-32).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triangle A B C (*fig. 1*); c'est une qualité uniformément difforme qui, au point B, se termine au degré nul; soit D le point milieu de la ligne qui représente le sujet (*subjectiva linea*); le degré ou l'intensité qui affecte ce point est figuré par la ligne D E. La qualité qui aurait partout le degré ainsi désigné est représentable par le quadrilatère A F G B, ainsi qu'il résulte du chapitre X de la première partie. Mais par la XXVI[e] proposition du premier livre d'Euclide, les deux triangles E F C et E G B sont égaux. Le triangle qui représente la qualité uniformément difforme et le quadrilatère A F G B qui représente la qualité uniforme selon le degré du point moyen sont donc égaux entre eux; les deux qualités qui sont imaginables l'une par le triangle et l'autre par le quadrilatère sont aussi égales entre elles; et c'est ce qu'on se proposait de démontrer.

» On raisonne de la même manière au sujet d'une qualité uniformément difforme qui, de part et d'autre, se termine à un certain degré.....

» Au sujet de la vitesse, on peut dire exactement la même chose que d'une qualité linéaire, seulement, au lieu de dire : point milieu, il faut dire : instant milieu du temps pendant lequel dure cette vitesse.

» Il est donc évident qu'une qualité ou une vitesse uniformément difforme quelconque se trouve égalée à une qualité ou à une vitesse uniforme. »

Si, comme nous le pensons, la quantité ou mesure d'une vitesse s'identifie, dans l'esprit d'Oresme, avec l'espace linéaire que le point mobile parcourt, le résultat auquel notre auteur vient d'atteindre est singulièrement grave ; il peut, en effet, se formuler ainsi : Lorsqu'un mobile se meut, pendant un certain temps, d'un mouvement uniformément varié, le chemin qu'il parcourt est égal à celui qu'il parcourrait en un mouvement uniforme, de même durée, dont la vitesse serait égale à celle qui est prise en l'instant moyen du premier mouvement.

Fig. 1.6. Page reporting Oresme's demonstration of the mean speed theorem. Duhem P (1913) *Etudes sur Léonard de Vinci. Les précurseurs parisiens de Galilée. Troisième série*. Librairie scientifique A. Hermann et fils, p. 395. Source: Public domain, PIMS - University of Toronto. Retrieved via Archive.org.

1.5 Nicolò Tartaglia: the motion of the projectile

Significant progress in the study of projectile motion was made by Nicolò Tartaglia, a mathematician from Brescia, in the sixteenth century, well known for the elaboration of the triangle of binomial coefficients for the development of the power of the binomial presented in *General trattato di numeri e misure* (Tartaglia 1556 –1560) and for the solution of the third degree equation (Tartaglia 1554, Book IX, Qs. XXXIII). Tartaglia achieved other remarkable results in mathematics such as the calculation of the volume of a tetrahedron from the length of its sides and the inscription of three circles tangent to each other in a triangle (Tartaglia 1556–1560).

We find his studies on the motion of the projectile in *Nova Scientia* (Tartaglia 1537) e in *Quesiti et Inventioni Diverse* (Tartaglia 1554). A particularly important scientific achievement is that already in Tartaglia's view the launch angle for the maximum projectile travel distance was 45° for every kind of cannon (see also Pisano and Capecchi 2016, p. 39).

Fig. 1.7. Left: frontispiece with the trajectory of a projectile. Right: cannon tilted at 45° to the horizontal to obtain the maximum range, using the bombardier’s quadrant invented by Tartaglia. Plates from Tartaglia (1537) *Nova Scientia inventa da Nicolo Tartalea.* Stephano da Sabio, Venezia. Book I, ff. 1r, 4r. Source: Public domain. The Dibner Library of the History of Science and Technology. Smithsonian Institution Libraries. Images retrieved via Library.si.edu/digital-library.

The events related to this discovery by Tartaglia are particularly interesting as regards the relationship between science and society, because in the history of physics it was one of the first examples in which a scientist questioned the consequences of his research and acted accordingly. Tartaglia in fact was aware that, in that historical period, due to the continuous wars between Christians, his knowledge could be "harmful to others or even destructive to the human species". Therefore, he decided to postpone his studies on that subject to deal with other issues and not to divulge, either verbally or in writing, his knowledge on the motion of heavy bodies such as cannonballs. He later changed his mind, publishing his works when the "wolf [Turkish emperor Suleiman]" was about to invade Italy (see the introduction to *Nova Scientia* by Tartaglia 1537, ff. 4rv, and its English translation by Pisano and Capecchi 2016, p. 56).

Tartaglia drew the trajectory of the projectile (see fig. 1.8) with, in the first side, a straight line, corresponding to a violent motion, then traced a curved branch and in the end, drew a descending rectilinear trajectory for the natural motion (Tartaglia 1537, Book II, Prop. VI). Later he would revise his vision in order to consider curvilinear trajectories due to the continuous action of gravity (Tartaglia 1554, Book I, Qs. I–II–III–VI) but the idea that a mathematical relationship of the second degree could interpret the trajectory of the cannonball was still premature.[30]

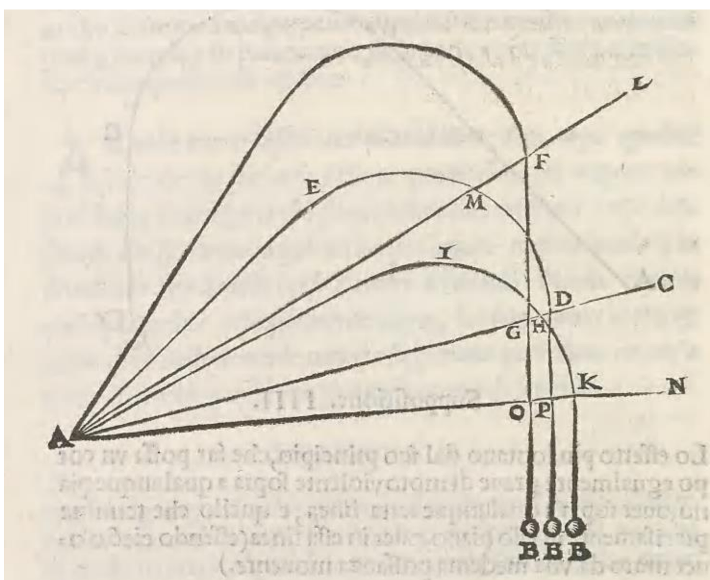

Fig. 1.8. Trajectories of projectiles according to Tartaglia. Plates from Tartaglia (1537) *Nova Scientia inventa da Nicolo Tartalea.* Stephano da Sabio, Venezia. Book I, f. 8v. Source: Public domain. The Dibner Library of the History of Science and Technology. Smithsonian Institution Libraries. Images retrieved via Library.si.edu/digital-library.

Tartaglia played an extraordinary cultural role because he was the first to translate the *Elements* of Euclid (Tartaglia 1543; see also Tartaglia 1565)

[30] For details on Tartaglia's ballistics and related considerations see Pisano and Capecchi 2016, pp. 39-72.

and *Archimedis de Insidentibus Aquae* (Tartaglia 1543c), with dialog commentaries, into Italian (Tartaglia 1551a). In his *History of Free Fall*, Drake (1989, p. 25) hypothesizes that Galileo studied these texts also thanks to Ostilio Ricci who had been his teacher and also a student of Tartaglia. In his edition of the *Elements* (Tartaglia 1543), Tartaglia introduced the corrected version of Book V on proportions, the Eudoxian theory, which would later allow Galileo to deal with the relationships between continuous and non-homogeneous quantities (Pisano and Capecchi 2016, p. 17). In his *Regola Generale da Sulevare con Ragione e Misura nò solaméte ogni affondata Nave* (Tartaglia 1551b; see also fig. 1.9 reproduced from Tartaglia 1562), introducing Archimedes' work, Tartaglia summarizes the fundamental principles relating to the buoyancy of bodies. He remarked that the speed with which a ship sinks increases with the difference between the specific gravity of the ship and that of water, before Galileo found the rate of fall of a body in a viscous fluid is directly proportional to the difference between the specific weights of the body and the fluid, confirming Tartaglia's great influence on the development of modern science.[31]

[31] A further proof of this is that another illustrious pupil of Tartaglia, Giovanni Battista Benedetti, just two years later, published a volume concerning solutions of Euclidean geometry that can be solved with ruler and compass, in which he asserted that bodies of the same material but different weights fall with equal speed through the same medium (Benedetti 1553).


REGOLA GENERALE DI SOLEVARE OGNI fondata Naue & nauilli con Ragione.

Dechiaratione prima.

INANTI che si vegna alla dechiaratione del antedetto modo de recuperare ogni affondata Naue carga, ouero altro Nauiglio conueniente cosa mi pare, à dechiarire prima la causa propinqua del affondar de quelli. Dico adunque esser impossibile, che l'acqua riceua, ouero ingiottisca totalmente dentro di se alcun material corpo che sia piu leggero di essa acqua (in quanto alla specie) anci sempre ne lasciera ouer farà stare vna parte di quello di sopra la superficie di detta acqua, (cioè discoperto da quella) & tal proportione qual hauera tutto quel corpo in acqua posto, à quella sua parte, che sarà accettata, ouer receputa da l'acqua, quella medesima haurà la grauità dell'acqua alla grauità di quel tal corpo materiale (secondo la specie) Ma quelli corpi materiali che sonno poi piu graui dell'acqua posti che siano in acqua subito se fano dar loco alla detta acqua & non solamente intrano totalmente in quella, ma uanno discendendo continuamente per fin al fondo, & tanto piu uelocemente uanno discendendo quanto che sonno piu graui dell'acqua. Et quelli poi, che per sorte sono di quella medesima grauità, che è l'acqua necessariamente posti in essa acqua, sono acettati, ouer receputi totalmente da quella, ma conseruati però nella superficie di essa acqua, cioè che la non lassa in parte alcuna

A

General rule for lifting any sunken ship or vessel by reason

Before dealing with the problem of recovering any cargo ship or any other vessel, it seems to me convenient to illustrate the proximate cause of their sinking. I say therefore it is impossible that water receives – i.e. engulfs totally inside itself - any material body that is lighter than it (depending on their specific gravity). Indeed, water will always leave it out, will make a part of that body stand above the surface – i.e. uncovered from it - that will have with respect to the part of the body immersed in water the same proportion that will have the (specific) gravity of the water compared to that of the material body. But those material bodies that are heavier than water, as soon as they are placed in the water, immediately sink and not only completely enter it, but continuously descend to the bottom, and descend faster the heavier they are than water …[32]

Fig. 1.9. Left: title page of Tartaglia (1562) *Regola Generale di soleuare ogni fondata Naue & Nauilli con Ragione.* Source: Public domain, *Biblioteca Central Militar* / Virtual Library of the Spanish Ministry of Defense. Retrieved via Bibliotecavirtual.defensa.gob.es. Right: my translation of the passages reported on the left, which summarize the fundamental principles relating to the buoyancy of bodies.

[32] The translation of this passage from Italian to English is mine. “Inanti che li vegna alla dechiaratione del antedetto modo de recuperare ogni affondata “Nave carga”, ovvero altro Naviglio conveniente cosa mi pare, à dechiarire prima la causa propinqua del affondar de quelli. Dico adunque esser impossibile, che l’acqua riceva, ovvero inghiottisca totalmente dentro di se alcun material corpo che sia piu leggero di essa acqua (in quanto alla specie) anci sempre ne lascieria over farà stare una parte di quello di sopra la superificie di detta acqua, (cioè discoperto da quella) e tal proportione qual havera tutto quell corpo in acqua posto, à quella sua parte, che sarà accettata, ove receputa dal’acqua, quella medesima havrà la gravità dell’acqua alla gravità di quell corpo material (secondo la specie). Ma quei corpi materiali che sonno poi più gravi dell’acqua, posti che siano in acqua subito se fano dar loco alla detta acquae non solamente intrano totalmente in quella, ma vanno discendendo continuamente per fino al fondo, e tanto più velocemente vanno discendendo quanto che sonno più gravi dell’acqua” (Tartaglia 1562, f. 1r).

Epilogue I

In this chapter I analysed historical sources dealing with the fall of bodies and the motion of projectiles by scholars who in some respects anticipated Galileo, dealing in particular with the issues that remained open in Aristotle's physics and that correspond to the very difficulties encountered by students in dynamics.

I began by illustrating Aristotle's distinction between natural motion and violent motion. Similar to students' common sense conceptions, Aristotle emphasised the dependence of the free-fall velocity of bodies on their weigh. Aristotle's conception of falling bodies in fluids is also similar to that generally assumed by students that velocity is inversely proportional to the density of the medium. This led Aristotle to consider the impossibility of the existence of a vacuum because it corresponds to a zero density, for which velocity would be of infinite magnitude. Was Philoponus, many years before Galileo, in the sixth century AD, who observed two bodies of different weights fall from the same height with very small differences in time, refuting the position of Aristotle.

With regard to violent motion, Aristotle tried to justify how the violent motion of a projectile could continue after the application and the removal of the force that had caused it. The first theory, probably due to Plato, involved the mutual replacement of the air that, pushed by the projectile, in turn pushed more air to take the place left free by the projectile, not allowing the formation of the vacuum, and that - attracted quickly - gives the necessary thrust for motion. The second included the idea that the air in contact with the original motor moves the contiguous air up to the projectile in order to give it continuous motion. I have outlined the main criticisms and additions that have been made to these conceptions by Hipparchus, Philiponus and Buridan to describe violent motion.

With reference to the theorems and mathematical relations on accelerated motion that are generally attributed to Galileo, I identified the contributions of William Heytesbury and Nicole Oresme. Based on these results, in the following chapter, I could recognise Galileo's main and certainly original contribution to the development of kinematics and dynamics was - in addition to having systematised the theory of accelerated motion in an organic manner - that of having associated this type of motion with free-fall motion and demonstrated it experimentally. As a final point, I demonstrated that Nicolò Tartaglia had determined the launch angle to obtain the maximum range of the projectile, a very interesting question even in those days, for the relationships between science and society.

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Tartaglia N (1537) Nova Scientia inventa da Nicolo Tartalea. B. Stephano da Sabio, Venezia.
Tartaglia N (1543a) Euclide Megarense Philosopho solo introduttore delle scientie mathematice: diligentemente rassettato, et alla integrità ridotto, per il degno professore di tal Scientie Nicolo Tartalea, Brisciano, Secondo le due tradottioni: e per commune commodo & utilita di latino in volgar tradotto, con una ampla

esposizione dello istesso tradottore di novo aggionta. Talmente chiara, che ogni mediocre ingegno, senza la notitia, over suffragio di alcun’altra scientia con facilità, serè capace a poterlo intendere. Venturino Rossinelli, Venezia.

Tartaglia N (1543b) Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi per Nicolaum Tartaleam Brixianum (mathematicarum scientiarum cultorem) multis erroribus emendata, expurgata, ac in luce posita, multisque necessariis additis, quae plurimis locis intellectu difficillima erant, commentariolis sane luculentis & eruditissimis aperta, explicata atque illustrata existunt, appositisque manu propria figuris quae graeco exemplari deformatae ac depravatae erant, ad rectissimam symetriam omnia instaurata reducta & reformata elucent. Venturinum Ruffinellum, Venezia.

CHAPTER II

The fall of bodies and the motion of the projectile in Galileo's writings

Prologue II

The replication of historically relevant physics experiments (Heering and Höttecke 2014) can help students to understand how science works (Nature of Science - NoS). In my thesis work, I identified a comprehensive educational path using physics, mathematics and historical sources concerning Galileo’s experiments; it was part of the curriculum education at the *Liceo Scientifico* “F. Sbordone” of Naples.

Indeed, Galileo is a central figure in the history of science on account of his physical discoveries about motion and astronomy, but also for his methodological innovations that made a fundamental contribution to the birth of modern science. The context of the Galilean affair is very interesting for the relationship between science and religion and between science and authority. The mathematics he used (Euclidean geometry and proportions) can be easily understood by the students of the first high school classes. The physics of falling bodies and of the motion of projectiles is close to students’ everyday experience and gets them started to ask questions about the interactions between science, technology and society.

My idea was to bring students closer to Galileo's writings and to start from these for the construction of the experimental educational path, and for correct teaching methodology, naturally contextualising everything with the school order (high school/university). The aim of this second chapter is therefore to examine in detail the Galilean sources pertaining to free fall and the motion of projectiles that will be used for the construction of the educational path, both in terms of Galileo's remarkable manuscripts, which are kept at the *Biblioteca Nazionale Centrale di Firenze*, and Galileo's volumes pertaining to kinematics and the first elements of dynamics. In this thesis work, Galileo's thought has only been examined in relation to the indicated topic due to the vastness of Galileo's contributions to various branches of physics.

The salient point has been the examination of laboratory experiments that constituted an important advance in the history of science for the birth of the experimental method and the verification of the law of falling bodies.

This chapter will examine the evolution of Galileo's thought from his critique of Aristotle's thought and the first results achieved. The analysis of this historical process - together with the development of physical theory from its precursors - will be particularly interesting for the didactic purposes of overcoming students’ common sense conceptions (Halloun and Hestenes 1985; Viennot 2001).

2.1 Manuscript Gal. 71: *De motu antiquiora*

De motu antiquiora is the title of six latin documents on motion written by Galileo between 1589 and 1592 - when he worked at Pisa University - in which he discovered the fundamental law of the inclined plane, the first version of the principle of inertia and the description of the motion in fluids. The texts *De motu* were edited by Favaro in Opere (Galileo 1890-1909, Vol. I, pp. 251-419). Following Camerota (1992), they include:

a) The treatise [Vol. I, pp. 251-340];
b) A reworking of the first two chaps. [Vol. I, pp. 341-343];
c) An essay in ten chaps [Vol. I, pp. 344-366];
d) A dialogue (incomplete) [Vol. I, pp. 367-408];
e) Some Notes [Vol. I, pp. 409-417] (in Gal. 46);
f) The work plan [Vol. I, pp. 418-419]

Great emphasis is placed on the motion in fluids in this work, highlighting the influence of Archimedes over Galileo. In this first phase of his thought, Galileo believed that the falling velocity of a body in natural motion (that he supposed to be uniform) was proportional to the difference between the specific weight of the body and that of the surrounding medium, rejecting the idea of Aristotle who believed that the fall velocity was proportional to the gravity of the body and inversely proportional to the resistance or density of the medium (see par. 1.1). So the Archimedean influence over Galileo's thought had the important consequence that the motion of a body in a vacuum (to which null resistance and density corresponded) could be accepted because its speed would not have been infinite as predicted by Aristotle's theory. An important result, which we will try to reproduce in the experimental didactic path (the demonstration is in the detailed sheet), is that the falling speeds "of a given body in two different media" are proportional to the differences between the specific weight of the body and of the media (Galilei [1589-1592], 1890–1909, vol. I, p. 270; Galilei 1960, p. 35).

The Archimedean approach also helped Galileo to determine the right expression of the upward force necessary to balance the effect of gravity on an inclined plane. In fact, he observed that "the same weight can be drawn up an inclined plane with less force than vertically, in proportion as the vertical ascent is smaller than the oblique" (Galilei [1589-1592], 1890–1909, vol. I, p. 298; Galilei 1960, p. 65). This statement is known as "The *De Motu* Theorem". In the demonstration, Galileo considers bent levers moving along a circumference and assumes the motion of a body in a point of the lower quadrant of the circle to be equivalent to that of a body descending on a sloping plane tangent to the circle in that point.

Galileo formulates an analogous theorem also for velocities, arguing that the velocities along the inclined planes are inversely proportional to the length of the plane, provided that these planes all have the same vertical height.

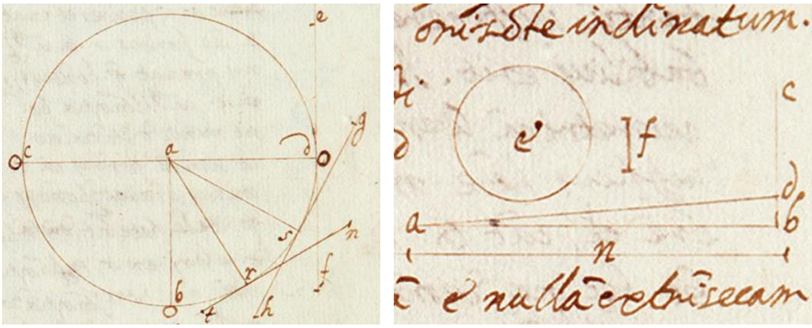
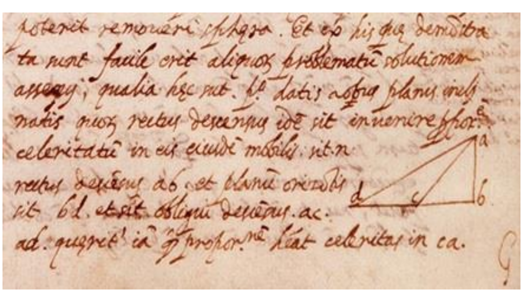


Fig. 2.1. Left: Demonstration of the *De motu* theorem. Galilei 1589-1592. Ms. Gal. 71. Detail of folio 95r. Right: Neutral motion along a horizontal plane. Galilei 1589-1592. Ms. Gal. 71. Detail of folio 97r. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.

It is not possible to affirm the correctness of this proposition according to the modern notion of velocity. However, Souffrin (1993) supported the correctness of this result of Galileo even in an accelerated motion, taking into account Galileo's concept of velocity. Until a few years before his death when Galileo tried to re-establish Euclid's theory of proportions, the scientist could not consider velocity as a ratio between two non-homogeneous quantities as space and time, but as a space travelled in a given interval of time or as the time taken to travel a fixed space. In this hypothesis, the theorem would also be true for accelerated motions because in this case the spaces travelled on planes of different length are proportional to the accelerations multiplied by the squares of the times (and if the times are equal the spaces are directly proportional to the accelerations and therefore inversely proportional to the lengths of the planes).



"And therefore line DA will be to line AC also as the celerity on AC is to celerity on AD. It is clear, then, that the celerity of the same body moving on different inclinations are to each other inversely as the lengths of the oblique paths, if these entail equal vertical descents".

Fig 2.2. The *De Motu* Theorem for velocities. Left: Galilei 1589-1592. Ms. Gal. 71. Detail of f. 97v. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it. Right: Galileo 1890-1909, vol. I, p. 301; Galilei 1960, p. 68.

A consequence of the *De motu* theorem is an earliest formulation of the inertia principle. As we can see in fig. 2.1 (right) the sphere e is subject only to its weight f and it is moving along the plane *ad*. The lower the slope of the plane, the smaller the force necessary to balance the weight. So, a spherical body "that does not resist motion […] on a plane parallel to the horizon will be moved by the very smallest force, indeed, by a force less than any given force" (Galilei [1589-1592], 1890–1909, vol. I, pp. 298-299; Galilei 1960 pp. 65-66). Therefore, while the spontaneous downward motion along an inclined plane must be considered to be natural and the upward movement to be forced, the motion along a horizontal plane is neither one nor the other but what Galileo called "neutral motion" (Galilei 1960, p. 67, ft. 9).

Galileo gave another example of "neutral motion" as that of a body that moves undisturbed with uniform circular motion (circular inertia) around the centre of the Universe as the skies and the stars do - in a still geocentric vision - along a concentric sphere with the Earth without moving away or approaching its centre (Galilei [1589-1592], 1890–1909, vol. I, p 305).

2.2 *Le Mecaniche* and the concept of *momento*

Galileo returned to the concept of circular inertia in *Le Mecaniche* (Galilei 1890-1909, vol. II, pp 179-180), a treatise on simple machines that he never published, but more explicitly in *Seconda Lettera sulle Macchie Solari* (to Marco Velseri) of 1612 (Galilei [1612], 1613; Galilei 1890-1909, vol. V, pp. 134-135) and in *Dialogo sui Due Massimi Sistemi del Mondo* (Galilei [1632], 1890-1909, vol. VII, pp. 56-57). In his maturity, in *Discorsi su due Nuove Scienze*, we find that the formulation of the inertia principle is very similar to the modern one (Galilei [1638], 1890-1909, vol. VIII, p. 243; Galilei 1914, p. 215), but lacking the theory of gravitation, Galileo would never be able to solve the problem of uniform circular motion (Galilei [1638], 1890-1909, vol. VIII, pp. 283-284; Galilei [1638], 1914, pp. 261-262).
In *Le Mecaniche*, he defined the relevant concept of *momento* as:

> the tendency to move downwards, caused not so much by the heaviness of the moving body as by the arrangement of different heavy bodies between them; through which *momento* one will often see a less heavy body counterweight another of greater heaviness: as in the steelyard one sees a small counterweight lift another very heavy weight, not because of excess of heaviness, but because of the distance from the point from which the steelyard is held […] Therefore, the *momento* is that impetus to go down, composed of heaviness, position, and other things from which this tendency may be caused.[33]

From this definition, two different dimensions of moment emerge: the

[33] The English translation is mine (compare with Galilei 1960, p. 151). "Momento è la propensione di andare al basso, cagionata non tanto dalla gravità del mobile, quanto dalla disposizione che abbino tra di loro diversi corpi gravi; mediante il qual momento si vedrà molte volte un corpo men grave contrapesare un altro di maggior gravità: come nella stadera si vede un picciolo contrapeso alzare un altro peso grandissimo, non per eccesso di gravità, ma sì bene per la lontananza dal punto donde viene sostenuta la stadera […] È dunque il momento quell'impeto di andare al basso, composto di gravità, posizione e di altro, dal che possa essere tal propensione cagionata" (Galilei 1890–1909, vol. II, p. 159).

meaning of static moment, understood as the product of weight and distance (e.g. on a scale), and the dynamic implication of the concept that integrates weight and velocity (Abattouy 1996c, p. 17).

Galileo considered this dynamic aspect of moment, especially in the *Discorso intorno alle cose che stanno in su l'acqua* (Galilei 1612):

> *Momento*, according to mechanics, means that virtue, that force, that effectiveness with which the motor moves and the body in motion resists; which virtue depends not only on simple heaviness, but also on the speed of the motion, on the different inclinations of the spaces over which the motion is made, because a heavy body descending on a very sloping space has more impetus than on a less sloping one.[34]

2.3 The law of inertia and the principle of relativity in *Dialogo sui due Massimi Sistemi del Mondo*

There would be much to discuss about his famous work *Dialogo sui due Massimi Sistemi del Mondo Tolemaico e Copernicano Proponendo indeterminatamente le ragioni Filosofiche e Naturali tanto per l'una quanto per l'altra parte* (Galilei [1632], 1890-1909, vol. VII; English translation Galileo [1632], 1953-1967, 2001; hereafter *Dialogo*), starting with the choice of Italian language for his text, and also for its applications for didactic purposes. An important contribution for a better understanding by the students of the NoS (Nature of Science), can be found, for example, in Cioci (2021), regarding the trial that Galileo had to face for having defended the Copernican theory and the fundamental relationship between science and society, and between science / authority / religion. This paragraph will be dedicated only to the presentation of some analyses made by Galileo on the motion of bodies that led to the introduction of the principle of relativity and the law of inertia. Discussing these with students can be particularly important for overcoming their difficulties in learning and concepts of common sense. In fact, we saw in the introduction that the latter are often presented in the same way as they occur in the history of science.

In the *Dialogo sui due Massimi Sistemi del Mondo* we first find the thought experiment that is used by Galileo to justify the law of inertia. He first considers the accelerating effect (of increase in speed) given by an inclined plane to a moving sphere. Then he considers the retarding effect (of decrease in speed) provided by an inclined upward plane. Finally, he observes that along a perfectly smooth horizontal plane, a spherical and

[34] The English translation is mine. "Momento, appresso i meccanici, significa quella virtù, quella forza, quella efficacia, con la quale il motor muove e 'l mobile resiste; la qual virtù depende non solo dalla semplice gravità, ma dalla velocità del moto, dalle diverse inclinazioni degli spazii sopra i quali si fa il moto, perché più fa impeto un grave descendente in uno spazio molto declive che in un meno" (Galilei [1612], 1890–1909, vol. IV, p. 68).

smooth ball will remain stationary if initially stationary or will continue its motion along the entire length of the plane if the ball was initially in motion (Galilei [1632], 1890-1909, vol. VII, pp. 171-173, Galilei [1632], 1953-1967, 2001 pp. 169-173).[35]

The law of inertia will allow Galileo to correctly interpret some phenomena and describe them using the principle of composition of movements, thus overcoming the Aristotelian conception of violent motion. A very instructive typical example that Galileo gives is that of a horseman who drops a heavy ball from a galloping horse. If there are no major irregularities in the ground and the horse proceeds at a constant speed, it is seen that the ball, falling, follows the gait of the horse, so much so that the horseman sees it fall vertically - that is, it maintains its speed in the horizontal direction. Galileo wonders how it is possible that the hand gives impetus to the ball if it is not thrown but is simply dropped (Galilei [1632], 1890-1909, vol. VII, pp. 182-183; Galilei [1632], 1953-1967, 2001, pp. 181-182).

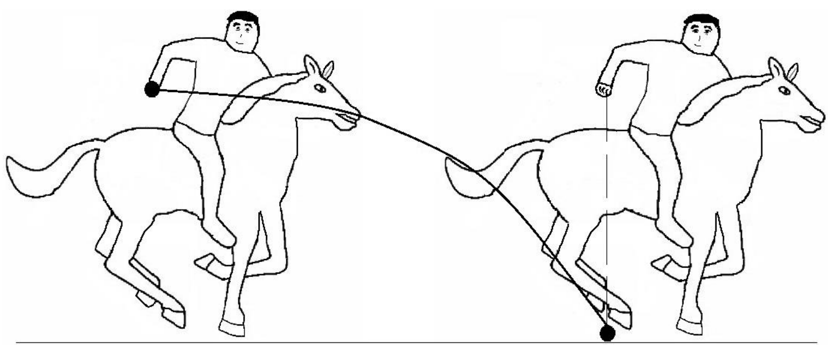


Fig. 2.3. Conservation of its horizontal velocity − equal to that of the galloping horse − when a heavy ball is dropped by the horseman (source: my own production).

Similarly, a stone dropped from the top of a ship's mast would fall to its base, in the same point, whether the ship was moving at a constant speed or whether it was stationary (Galilei [1632], 1890-1909, vol. VII, p. 170; Galilei 1953-1967, 2001 pp. 168). Galileo uses these considerations to defend the plausibility of Earth’s motion. Its opponents disprove the belief that if the Earth were in rotation, a stone, dropped from the top of a tower, should fall along the vertical, but they held that it should remain behind because, while the body falls, the base of the tower would surpass it by a distance that depends on the rotation speed of the Earth.

[35] Compare with the version of the *Discorsi* (Galilei 1890-1909, VIII, p. 243; Galilei 1914, p. 215), between which there are minimal differences, perhaps the most important, is the severity with which Galileo presents the subject, in Latin.

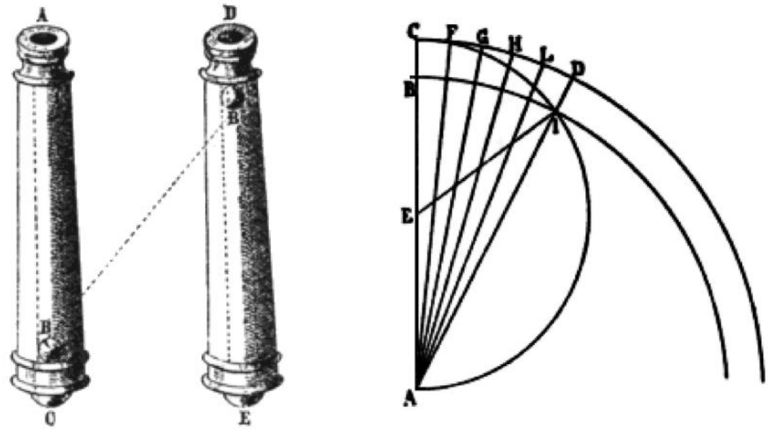


Fig. 2.4. Left: a cannon that fires a shot vertically and is transversely displaced by the motion of the Earth. Galilei 1890-1909, vol. VII, p. 202. Right: Motion of a stone dropped from the top of a tower .[36] Galilei [1632], 1890-1909, vol. VII, p. 191. Source: *Bibliothèque nationale de France* / gallica.bnf.fr.

A similar situation that Galileo considers consists of a cannon shot fired vertically, which would fall on the vertical of the cannon despite the fact that the Earth was in motion and that the cannonball actually travelled, according to Galileo, a trajectory composed of two motions: vertical and uniform circular imparted by the Earth's rotation (Galilei [1632], 1890-1909, vol. VII, pp. 201-202; Galilei [1632], 1953-1967, 2001, pp. 203-204). Despite the results achieved and the modern formulation he gives of the law of inertia in the *Dialogo sui due Massimi Sistemi del Mondo*, Galileo continues to consider uniform circular motion in terms of circular inertia. In fact, even in the interpretation of the motion of a stone falling from a high tower, he hypothesizes that this is given by the composition of a uniformly accelerated motion directed towards the centre of the Earth and a uniform circular motion due to the Earth's rotation (Galilei [1632], 1890-1909, vol. VII, pp. 190-192; Galilei [1632], 1953-1967, 2001, pp. 191-193). In the same work, however, he underlines how inertial motion occurs in the tangent direction to the circumference along which a body moves before its detachment. (Galilei [1632], 1890-1909, vol. VII, pp. 218-220; Galilei [1632], 1953-1967, 2001, pp. 222-225).

A useful exercise - although with more advanced prerequisites - could be to discuss with students the fall of a body to Earth, then considered possible, from the lunar surface. Given the distances, the composition of the uniform rotational motion of the Moon and of the uniformly accelerated fall motion, would produce, according to Galileo, the advancement of the point of fall in the direction of rotation because, falling, the body would advance reaching

[36] To Pierre Fermat's criticism of the correctness of the construction, which in his opinion must have been more like a spiral, Galileo replied that in fact he had not taken his speculations very seriously in this passage and that the motion of the stone would have been of a parabolic type (see Drake ft. 192, in Galileo [1632], 1953-1967, 2001, p. 555-556).

regions of radius always smaller and therefore characterized by lower rotation speed (Galilei [1632], 1890-1909, vol. VII, pp. 259-260; Galilei [1632], 1953-1967, 2001 pp. 270-271).

Generalizing the experiments already discussed, Galileo introduces the principle of relativity, according to which, from the observations of mechanical phenomena it is not possible to determine whether the system in which one is standing or moving with constant speed. In this regard Galileo gives us the beautiful expression that has remained in the history of physics about the experiment performed "in the main cabin below decks on some large ship […] with some flies, butterflies […] a large bowl of water with some fish in it" (Galilei [1632], 1890-1909, vol. VII, pp. 212-214; Galilei [1632], 1953-1967, 2001, pp. 216-218).

2.4 *Discorsi su due Nuove Scienze* and the preliminary study of free fall

The work *Discorsi e Dimostrazioni Matematiche Intorno a Due Nuove Scienze Attenenti alla Meccanica e i Movimenti Locali* (Galilei [1638], 1890-1909, vol. VIII; English translation Galilei [1638], 1914; hereafter *Discorsi*) was published in the Netherlands while Galileo was under house arrest in Arcetri and is mostly written in Italian - in dialogic form - but with definitions and theorems in Latin. The *Discorsi* are divided into four days dedicated to two new sciences: the first treating of the resistance which bodies offer to fracture; the second new science treats of motion, starting from uniform motion; Galileo then builds the theory of uniformly accelerated motion, describing the fundamental experimental experiences that led him, even using the inclined plane, to associate this type of motion with the natural motion of free fall. The last day is dedicated to violent motion - ie the motion of the projectile - which Galileo describes with a second degree equation geometrically represented by a parabola.

The *Discorsi* constitute Galileo's work of maturity where his thought reaches the most correct scientific formulation. For example, there we find the statement of the inertia principle that is very similar to the modern one: "[…] any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of acceleration or retardation are removed, a condition which is found only on horizontal planes" (Galilei [1638], 1890-1909, vol. VIII, p. 243; Galilei [1638], 1914, p. 215). However, lacking the theory of gravitation, the Pisan scientist would never be able to solve the problem of uniform circular motion (Galilei [1638], 1890-1909, vol. VIII, pp. 283-284; Galilei [1638],1914, pp. 261-262).

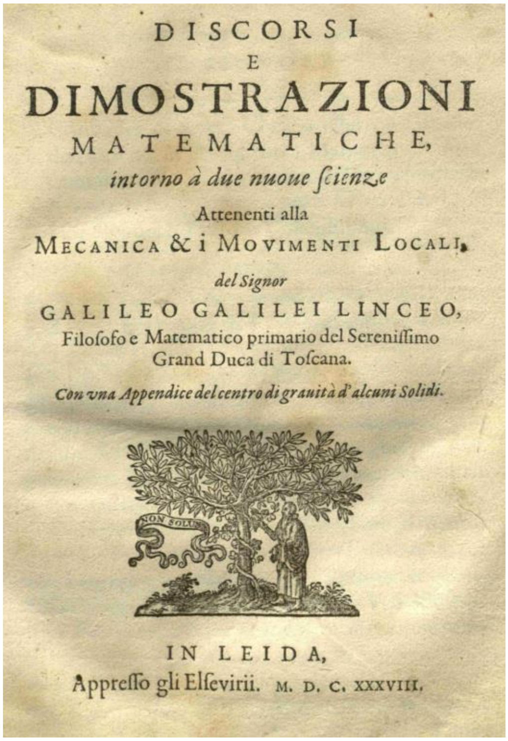


Fig. 2.5. Frontispiece of the *Discorsi e Dimostrazioni Matematiche intorno a due Nuove Scienze Attenenti alla Mecanica & I Movimenti Locali del Signor Galileo Galilei Linceo, Filosofo e Matematico Primario del Serenissimo Gran Duca di Toscana. Con una Appendice del centro di gravità di alcuni Solidi*. In Leida. Appresso gli Elsevirii, MDCXXXVIII (Galilei 1638). Source: Public domain, via Wikimedia Commons.

Galileo renounced any explanation of the nature of gravity but further developed his considerations on the fall of bodies. He gave a correct description (anticipating the conceptual use of the fundamental law of Newton's dynamics) of the motion of falling bodies in viscous media: a first phase of accelerated motion and then - when the resistance of the medium is equal to the excess of the weight force of the body over that of the medium - a second one of uniform motion with terminal velocity of fall that is proportional to the difference between the density of the body and that of the medium (Galilei [1638], 1890-1909, vol. VIII, pp. 118-121; Galilei [1638], 1914, pp. 74-77). He claims to have performed experiments by dropping bodies of the same volume but of different specific gravity (for example glass and steel) in different fluids (for example oil, water and air) and observing that the difference between the times of fall is less and less with the decrease of the resistance that the fluid opposes to the passage of the body (see fig. 2.6). Therefore, he can conclude, extrapolating what occurs in the void, that the free fall of bodies of different density should not show any difference (Galilei [1638], 1890-1909, vol. VIII, pp. 116-117; Galilei [1638], 1914, pp. 70-72). We can say that since Galileo could not perform the measurements in a vacuum, he came to his conclusions through a process of extrapolation by "thinking" about what would happen in a vacuum.

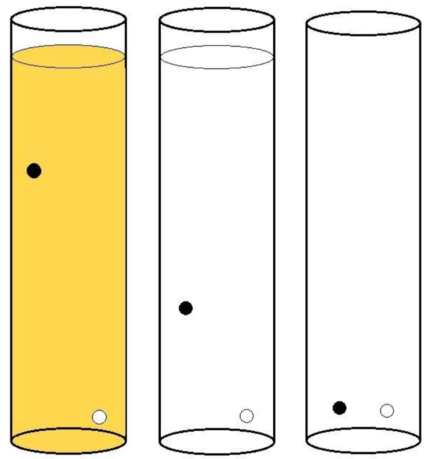


Fig. 2.6. Dropping balls with different specific gravity in media with different resistance (for example, oil, water and air). Source: my graphic representation.

Galileo's thought experiment ends by considering instead two bodies of the same specific weight but of different volume (for example two stones of the same material). If the two bodies had a free fall motion with different speeds, as Aristotle supposed, then they could be joined to each other, obtaining a body with greater mass and therefore should fall even faster. But we also know that the slower body would slow down the faster so the overall speed should be somewhere between the two. There follows a contradiction absurdity originating from having assumed that the fall time depended on the mass (Galilei [1638], 1890-1909, vol. VIII, pp. 107-109; Galilei [1638], 1914, pp. 62-64).

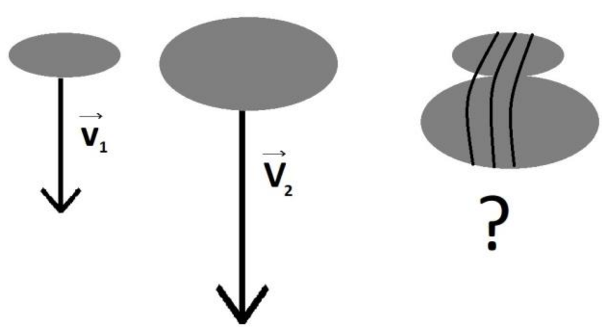


Fig. 2.7. Thought experiment on the fall of two bodies of different weight but of the same material - a larger stone and a smaller one - falling to the ground with the same speed (proof by contradiction). Source: my graphic representation.

2.5 The third day of *Discorsi su due Nuove Scienze*: from the study of uniform motion to that of naturally accelerated motion. The motion along the inclined plane

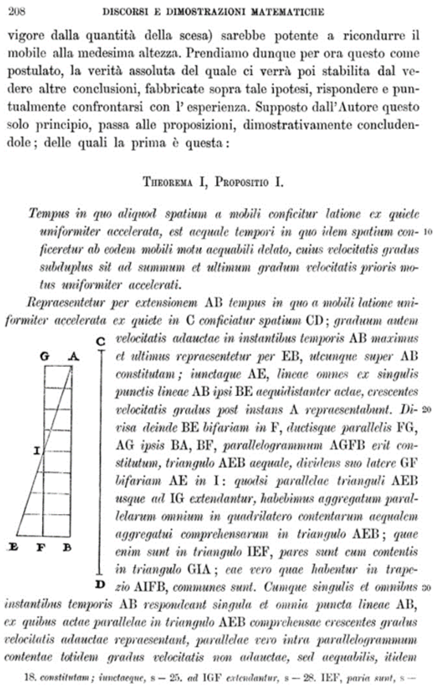
The third day of *Discorsi* constitutes an organic discussion of local motion starting from uniform motion, up to uniformly accelerated motion typical of free fall motion. The discussion includes an accurate definition of uniform motion and the theorems on proportionality laws of uniform motion in the case of two movements with equal speed, with the same distance travelled, with equal time intervals or in the more general case in which all three quantities are unequal for the two motions (Galilei [1638], 1890-1909, vol. VIII, pp. 191-196; Galilei [1638], 1914, pp. 154-160).

Then follows the definition of naturally accelerated motion in which the speed starting from rest acquires equal increments after equal intervals of time (Galilei [1638], 1890-1909, vol VIII, pp. 197-198; Galilei [1638], 1914, 161-162).

As noted above, in *Discorsi* Galileo presents a series of theorems on naturally accelerated motion, typical of the free fall, in which a body starting from rest, acquires, during equal-time intervals, equal increments of speed. By means of purely geometric considerations, Galileo showed that the distance travelled in a naturally accelerated motion is equal to that travelled in a uniform motion having as its speed half of the final one achieved in the accelerated motion. As can be seen from Galileo’s statement, shown in fig. 2.8, the formulation is a little different from the one chosen by Heytesbury but is perfectly equivalent to it because only the order in which the words are enunciated changes.

This − as already pointed out by several authors (see for example the works of Clagett, Duhem, Koyré) − is the most striking confirmation that not everything indicated in the *Discorsi* is Galileo's original work. The fact that he had made use of the terminology coined by the Parisian school (Carugo and Geymonat, n. 208, p. 777)[37] is an important element in favour of the hypothesis that Galileo was already aware of their writings. His demonstration of the theorem is based on the interpretation of space as the area under the speed-time graph, on the equivalence theorem between the triangle and the rectangle of Euclidean geometry, and on the proportionality between speed and time in a naturally accelerated motion. Anticipating the Cartesian plane and integral calculus, Galileo shows that the space travelled in a uniformly accelerated motion starting from rest represented by the area of the triangle AEB (with speeds continuously increased from A to EB) is equal to the space travelled in a uniform motion represented by the area of the rectangle GAFB (with speeds all equal to EB), distances both travelled at the same time CD (Galilei [1638], 1890-1909, vol VIII, pp. 208-209; Galilei [1638], 1914, 173-174).

[37] See the writings Juvenilia (Galilei 1890-1909, vol. I, pp. 119-121).



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vigore dalla quantità della scesa) sarebbe potente a ricondurre il mobile alla medesima altezza. Prendiamo dunque per ora questo come postulato, la verità assoluta del quale ci verrà poi stabilita dal vedere altre conclusioni, fabbricate sopra tale ipotesi, rispondere e puntualmente confrontarsi con l' esperienza. Supposto dall'Autore questo solo principio, passa alle proposizioni, dimostrativamente concludendole; delle quali la prima è questa:

THEOREMA I, PROPOSITIO I.

Tempus in quo aliquod spatium a mobili conficitur latione ex quiete uniformiter accelerata, est aequale tempori in quo idem spatium conficeretur ab eodem mobili motu aequabili delato, cuius velocitatis gradus subduplus sit ad summum et ultimum gradum velocitatis prioris motus uniformiter accelerati.

Repraesentetur per extensionem AB *tempus in quo a mobili latione uniformiter accelerata ex quiete in* C *conficiatur spatium* CD; *graduum autem velocitatis adauctae in instantibus temporis* AB *maximus et ultimus repraesentetur per* EB, *utcunque super* AB *constitutam; iunctaque* AE, *lineae omnes ex singulis punctis lineae* AB *ipsi* BE *aequidistanter actae, crescentes velocitatis gradus post instans* A *repraesentabunt. Divisa deinde* BE *bifariam in* F, *ductisque parallelis* FG, AG *ipsis* BA, BF, *parallelogrammum* AGFB *erit constitutum, triangulo* AEB *aequale, dividens suo latere* GF *bifariam* AE *in* I: *quodsi parallelae trianguli* AEB *usque ad* IG *extendantur, habebimus aggregatum parallelarum omnium in quadrilatero contentarum aequalem aggregatui comprehensarum in triangulo* AEB; *quae enim sunt in triangulo* IEF, *pares sunt cum contentis in triangulo* GIA; *eae vero quae habentur in trapezio* AIFB, *communes sunt. Cumque singulis et omnibus instantibus temporis* AB *respondeant singula et omnia puncta lineae* AB, *ex quibus actae parallelae in triangulo* AEB *comprehensae crescentes gradus velocitatis adauctae repraesentant, parallelae vero intra parallelogrammum contentae totidem gradus velocitatis non adauctae, sed aequabilis, itidem*

18. *constitutam; iunctaeque*, s — 25. *ad* IGF *extendantur*, s — 28. IEF, *paria sunt*, s —

Theorem I

The time in which any distance is traversed by a moving body starting from rest and uniformly accelerated is equal to the time in which that same distance would be traversed by the same body moving with uniform motion whose degree of speed value is one-half the maximum and final degree of the speed of the previous uniformly accelerated motion[38]

Fig. 2.8. Theorem I of naturally accelerated motion. Galileo [1638], 1890-1909, vol. VIII, p. 208. Source: *Bibliothèque nationale de France* / gallica.bnf.fr.

The most important theorem on naturally accelerated motion is Theorem II on the direct proportionality between the space travelled and the squares of the times (the statement and the graphical interpretation of the theorem are represented in fig. 2.9). Its demonstration is based on the mean speed theorem, on the proportionality between speed and time in a naturally accelerated motion and on the proportionality between spaces and the products of speed and time in uniform motion (Galilei [1638], 1890-1909, vol VIII, pp. 209-210; Galilei 1914, pp. 174-175).

[38] The English translation is mine. "Theorema I, Propositio I. Tempus in quo aliqtiod spatium a mobili conficitur latione ex quiete uniformiter accelerata^ est acquale tempori in qtio idem spatium con- io ficeretur ab eodem mobili motti aequabili delato, cuius velocitatis gradus subduplus sit ad stimmum et tdtimum gradum velocitatis prioris motus uniformiter accelerati" (Galileo [1638], 1890-1909, vol. VIII, p. 208). Compare with Galilei [1638], 1914, p. 173.

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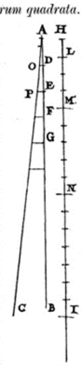
repraesentent; apparet, totidem velocitatis momenta absumpta esse in motu accelerato iuxta crescentes parallelas trianguli AEB, *ac in motu aequabili iuxta parallelas parallelogrammi* GB: *quod enim momentorum deficit in prima motus accelerati medietate (deficiunt enim momenta per parallelas trianguli* AGI *repraesentata), reficitur a momentis per parallelas trianguli* IEF *repraesentatis. Patet igitur, aequalia futura esse spatia tempore eodem a duobus mobilibus peracta, quorum unum motu ex quiete uniformiter accelerato moveatur, alterum vero motu aequabili iuxta momentum subduplum momenti maximi velocitatis accelerati motus: quod erat intentum.*

THEOREMA II, PROPOSITIO II.

Si aliquod mobile motu uniformiter accelerato descendat ex quiete, spatia quibuscunque temporibus ab ipso peracta, sunt inter se in duplicata ratione eorundem temporum, nempe ut eorundem temporum quadrata.



Intelligatur, fluxus temporis ex aliquo primo instanti A *repraesentari per extensionem* AB, *in qua sumantur duo quaelibet tempora* AD, AE; *sitque* HI *linea, in qua mobile ex puncto* H, *tanquam primo motus principio, descendat uniformiter acceleratum; sitque spatium* HL *peractum primo tempore* AD, HM *vero sit spatium per quod descenderit in tempore* AE: *dico, spatium* MH *ad spatium* HL *esse in duplicata ratione eius quam habet tempus* EA *ad tempus* AD; *seu dicamus, spatia* MH, HL *eandem habere rationem quam habent quadrata* EA, AD. *Ponatur linea* AC, *quemcunque angulum cum ipsa* AB *continens; ex punctis vero* D, E *ductae sint parallelae* DO, EP: *quarum* DO *repraesentabit maximum gradum velocitatis acquisitae in instanti* D *temporis* AD; PE *vero, maximum gradum velocitatis acquisitae in instanti* E *temporis* AE. *Quia vero supra demonstratum est, quod attinet ad spatia peracta, aequalia esse inter se illa, quorum alterum conficitur a mobili ex quiete motu uniformiter accelerato, alterum vero quod tempore eodem conficitur a mobili motu aequabili delato, cuius velocitas subdupla sit maximae in motu accelerato acquisitae; constat, spatia* MH, LH *esse eadem quae motibus aequalibus, quorum velocitates essent ut dimidiae* PE, OD, *conficerentur in temporibus* EA, DA. *Si igitur ostensum fuerit, haec spatia* MH, LH *esse in*

VIII. 27

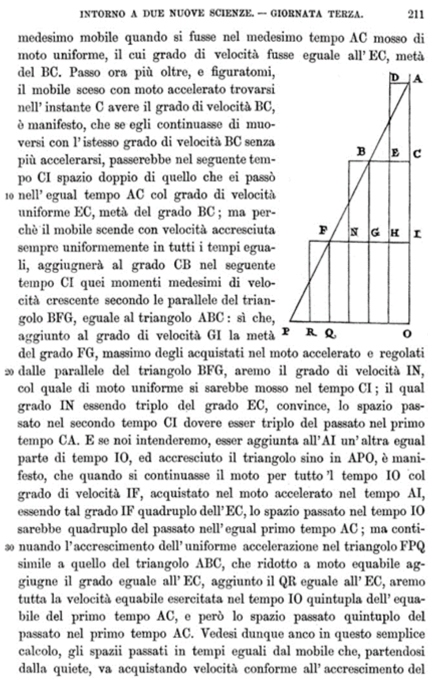
Theorem II

If any moving object descends from rest with a uniformly accelerated motion, the distances traversed by it in whatever times are to each other in the duplicate ratio of the same times, that is, are as the squares of the same times. It is understood that the flow of time from some first instant A is represented by the line AB, in which any two times AD, AE are taken; let HI be the line in which the mobile descends uniformly accelerated from the point H, as if at the first beginning of its motion; and let the distance HL be traversed in the first time AD, and let HM be the distance through which it descends in the time AE: I mean, the space MH is to the space HL in the duplicate ratio of that which the time EA is to the time AD; or let us say that the spaces MH, HL have the same ratio as the squares of EA and AD.[39]

Fig. 2.9. Theorem II of naturally accelerated motion. Galilei [1638], 1890-1909, vol. VIII, p. 209. Source: *Bibliothèque nationale de France* / gallica.bnf.fr.

A consequence of the mean speed theorem on naturally accelerated motion is Corollary I. Galileo, in fact, has shown that a naturally accelerated motion occurs according to the so-called law of odd numbers, which states that *a moving body starting from rest and acquiring velocity at a rate proportional to the time, will, during equal intervals of time, traverse distances which are related to each other as the odd numbers beginning with unity, 1, 3, 5, 7, …* (Galilei [1638], 1890-1909, vol. VIII, pp. 210; Galilei [1638], 1914, pp. 175).

[39] The English traslation is mine. "Theorema II Propositio II. Si aliquod mobile motu uniformiter accelerato descendat ex quiete, spatia quibuscunque temporibus ab ipso peracta, sunt inter se in duplicata ratione eorundem temporum, nempe ut eorundem temporum quadrata. Intelligatur, fluxus temporis ex aliquo primo instanti A repraesentari per extensionem AB, in qua sumantur duo quaelibet tempora AD, AE ; sitque HI linea, in qua mobile ex puncto H, tanquam primo motus principio, descendat uniformiter acceleratum; sitque spatium HL peractum primo tempore AD, HM vero sit spatium per quod descenderit in tempore AE : dico, spatium MH ad spatium HL esse in duplicata ratione eius quam liabet tempus EA ad tempus AD ; seu dicamus, spatia MH, HL eandem habere rationem quam habent quadrata EA, AD" (Galilei [1638], 1890-1909, vol. VIII, p. 209). Compare with Galilei [1638], 1914, p. 174.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medesimo mobile quando si fusse nel medesimo tempo AC mosso di moto uniforme, il cui grado di velocità fusse eguale all' EC, metà del BC. Passo ora più oltre, e figuratomi, il mobile sceso con moto accelerato trovarsi nell' instante C avere il grado di velocità BC, è manifesto, che se egli continuasse di muoversi con l' istesso grado di velocità BC senza più accelerarsi, passerebbe nel seguente tempo CI spazio doppio di quello che ei passò nell' egual tempo AC col grado di velocità uniforme EC, metà del grado BC; ma perchè il mobile scende con velocità accresciuta sempre uniformemente in tutti i tempi eguali, aggiugnerà al grado CB nel seguente tempo CI quei momenti medesimi di velocità crescente secondo le parallele del triangolo BFG, eguale al triangolo ABC: sì che, aggiunto al grado di velocità GI la metà del grado FG, massimo degli acquistati nel moto accelerato e regolati dalle parallele del triangolo BFG, aremo il grado di velocità IN, col quale di moto uniforme si sarebbe mosso nel tempo CI; il qual grado IN essendo triplo del grado EC, convince, lo spazio passato nel secondo tempo CI dovere esser triplo del passato nel primo tempo CA. E se noi intenderemo, esser aggiunta all' AI un' altra egual parte di tempo IO, ed accresciuto il triangolo sino in APO, è manifesto, che quando si continuasse il moto per tutto 'l tempo IO col grado di velocità IF, acquistato nel moto accelerato nel tempo AI, essendo tal grado IF quadruplo dell' EC, lo spazio passato nel tempo IO sarebbe quadruplo del passato nell' egual primo tempo AC; ma continuando l' accrescimento dell' uniforme accelerazione nel triangolo FPQ simile a quello del triangolo ABC, che ridotto a moto equabile aggiugne il grado eguale all' EC, aggiunto il QR eguale all' EC, aremo tutta la velocità equabile esercitata nel tempo IO quintupla dell' equabile del primo tempo AC, e però lo spazio passato quintuplo del passato nel primo tempo AC. Vedesi dunque anco in questo semplice calcolo, gli spazii passati in tempi eguali dal mobile che, partendosi dalla quiete, va acquistando velocità conforme all' accrescimento del

Corollary I

From this, it is evident that if we take successively any number of equal time intervals from the first instant or beginning of the motion, as AD, DE, EF, FG, by which the spaces HL, LM, MN, NI are traversed, the spaces themselves will be to each other as odd numbers from unity, namely, as 1, 3, 5, 7: for these are the ratios of the differences of the squares of lines which exceed one another by equal amounts and whose excess is equal to the least of them, or, let us say, the ratios of the differences of the squares of the natural numbers beginning with unity.[40]

Fig. 2.10. Left: demonstration of Corollary I on naturally accelerated motion. Galileo [1638], 1890-1909, vol. VIII, p. 211. Source: *Bibliothèque nationale de France* / gallica.bnf.fr. Right: its statement and its geometric interpretation.

Galileo's demonstration of Corollary I is based on the proportionality between speed and time in a naturally accelerated motion, on the mean speed theorem and on the proportionality between space and speed in uniform motion, the time taken being fixed (Galilei [1638], 1890-1909, vol. VIII, pp. 210-212. Galilei 1914, pp. 175-177). The law of odd numbers is equivalent to the law of quadratic proportionality between space travelled and time employed. By using modern analytical methods, we can obtain the result of the law directly from the fundamental equation s=s(t) of naturally accelerated motion.

$$\Delta s = ½ g (t_2^2 - t_1^2) = g/2 (t_2 + t_1) (t_2 - t_1)$$

where g is the acceleration of gravity and Δs the space traversed between the instants t_1 and t_2. If now $t_2 - t_1 = 1$, say one second, then $t_2 = t_1 + 1$ and

$$\Delta s = g/2 (t_2 + t_1) = g/2 (2t_1+1)$$

where $2t_1+1$ must always be an odd number if t_1 is a natural number.

40 The English translation is mine. "Hinc manifestum est, quod si fuerint quotcunque tempora aequalia consequenter sumpta a primo instanti seu principio lationis, utputa AD, DE, EF, FG, quibus conficiantur spatia HL, LM, MN, NI, ipsa spatia erunt inter se ut numeri impares ab unitate, scilicet ut 1, 3, 5, 7: haec enim est ratio excessuum quadratorum linearum sese aequaliter excedentium et quarum excessus est aequalis minimae ipsarum, seu dicamus quadratorum sese ab unitate consequentium. Dum igitur gradus velocitatis augentur iuxta seriem simplicem numerorum in temporibus aequalibus, spatia peracta iisdem temporibus incrementa suscipiunt iuxta seriem numerorum imparium ab unitate" (Galileo [1638], 1890-1909, vol. VIII, p. 211). Compare with Galilei [1638], 1914, p. 175.

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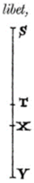
COROLLARIUM II.

Colligitur, secundo, quod si a principio lationis sumantur duo spatia quaelibet, quibuslibet temporibus peracta, tempora ipsorum erunt inter se ut alterum eorum ad spatium medium proportionale inter ipsa. Sumptis enim a principio lationis S *duobus spatiis* ST, SV, *quorum medium sit proportionale* SX, *tempus casus per* ST *ad tempus casus per* SV *erit ut* ST *ad* SX, *seu dicamus, tempus per* SV *ad tempus per* ST *esse ut* VS *ad* SX. *Cum enim demonstratum sit, spatia peracta esse in duplicata ratione temporum, seu (quod idem est) esse ut temporum quadrata; ratio autem spatii* VS *ad spatium* ST *sit dupla rationis* VS *ad* SX, *seu sit eadem quam habent quadrata* VS, SX; *patet, rationem temporum lationum per* SV, ST *esse ut spatiorum, seu linearum,* VS, SX.



SCHOLIUM.

Id autem quod demonstratum est in lationibus peractis in perpendiculis, intelligatur etiam itidem contingere in planis utcunque inclinatis: in iisdem enim assumptum est, accelerationis gradus eadem ratione augeri, nempe secundum temporis incrementum, seu dicas secundum simplicem ac primam numerorum seriem.(1)

(1) Era intenzione di GALILEO (come più particolarmente diciamo nell'Avvertimento) che, quando si stampassero di nuovo i suoi *Discorsi*, dopo questo Scolio della seconda Proposizione fosse inserita la seguente aggiunta alla stampa originale; la quale aggiunta fu distesa in dialogo da VINCENZIO VIVIANI:

SALV. Qui vorrei, Sig. Sagredo, che a me ancora fosse permesso, se ben forsi con troppo tedio del Sig. Simplicio, il differir per un poco la presente lettura, fin ch' io possa esplicare quanto dal detto e dimostrato fin ora, e congiuntamente dalla notizia d' alcune conclusioni mecaniche apprese già dal nostro Academico, sovviemmi adesso di poter soggiugnere per maggior confermazione della verità del principio che sopra con probabili discorsi ed esperienze fu da noi esaminato, anzi, quello più importa, per geometricamente concluderlo, dimostrando prima un sol lemma, elementare nella contemplazione de gl' impeti.

SAGR. Mentre tale deva esser l' acquisto quale V.S. ci promette, non vi è tempo che da me volentierissimo non si spendesse, trattandosi di confermare e interamente stabilire queste scienze del moto: e quanto a me, non solo vi con-

Corollary II

Secondly, if we take, from the beginning of the motion, any two spaces travelled in any time, their respective times will stand between them as one of the two spaces stands to the proportional mean between the two given spaces. In fact, if we take, starting from the beginning of the motion S, the two spaces ST and SV, whose proportional mean is SX, the time of the fall along ST will stand to the time of the fall along SV, as ST stands to SX, or let us say, the time for SV will stand to the time for ST as VS stands to SX.[41]

Fig. 2.11. Corollary II on naturally accelerated motion. Galilei [1638], 1890-1909, vol. VIII, p. 214. Source: *Bibliothèque nationale de France* / gallica.bnf.fr.

Corollary II, in modern mathematical language, states that in a naturally accelerated motion the fall times are directly proportional to the square roots of the distances travelled (Galilei 1890-1909, vol. VIII, pp. 214; Galilei 1914, p.180; see fig. 2.11.) and is essentially the inverse of Theorem II that instead expresses the quadratic proportionality between the space used and the time travelled.

It can be seen from Galileo's formulation that the "time-intervals bear to one another the same ratio as one of the distances to the mean proportional of the two distances" it follows that the times employed are to each other as the square roots of the spaces travelled. Indeed, said t_1 and t_2 the time-intervals taken, s_1 and s_2 the spaces travelled and s_m the proportional mean of the two distances, we can write that

$$\begin{cases} t_1 : t_2 = s_1 : s_m \\ s_1 : s_m = s_m : s_2 \end{cases}$$

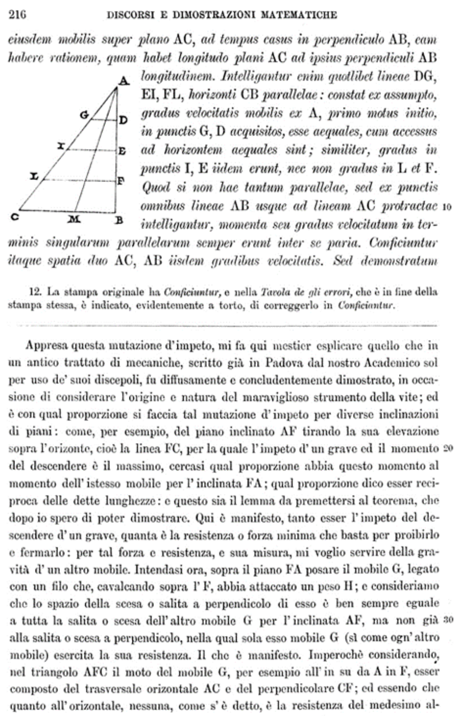
[41] The English translation is mine. "Colligitur, secundo, quod si a principio lationis sumantur duo spatia quaelibet, quibuslibet temporibus peracta, tempora ipsorum erunt inter se ut alterum eorum ad spatium medium proportionale inter ipsa. Sumptis enim a principio lationis S duobus spatiis ST, SV, quorum medium sit proportionale SX, tempus casus per ST ad tempus casus per SV erit ut ST ad SX, seu dicamus, tempus per SV ad tempus per ST esse ut VS ad SX." (Galilei [1638], 1890-1909, vol. VIII, p. 211). Compare with Galilei [1638], 1914 p. 180.

that is, by combining the two equations,

$$t_1 : t_2 = s_1 : \sqrt{s_1 s_2}$$

Bringing s_1 under the square root sign and simplifying, we get

$$t_1 : t_2 = \sqrt{s_1} : \sqrt{s_2}$$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eiusdem mobilis super plano AC, *ad tempus casus in perpendiculo* AB, *eam habere rationem, quam habet longitudo plani* AC *ad ipsius perpendiculi* AB *longitudinem. Intelligantur enim quotlibet lineae* DG, EI, FL, *horizonti* CB *parallelae: constat ex assumpto, gradus velocitatis mobilis ex* A, *primo motus initio, in punctis* G, D *acquisitos, esse aequales, cum accessus ad horizontem aequales sint; similiter, gradus in punctis* I, E *iidem erunt, nec non gradus in* L *et* F. *Quod si non hae tantum parallelae, sed ex punctis omnibus lineae* AB *usque ad lineam* AC *protractae intelligantur, momenta seu gradus velocitatum in terminis singularum parallelarum semper erunt inter se paria. Conficiuntur itaque spatia duo* AC, AB *iisdem gradibus velocitatis. Sed demonstratum*

12. La stampa originale ha *Conficiuntur*, e nella *Tavola de gli errori*, che è in fine della stampa stessa, è indicato, evidentemente a torto, di correggerlo in *Conficiantur*.

Appresa questa mutazione d'impeto, mi fa qui mestier esplicare quello che in un antico trattato di mecaniche, scritto già in Padova dal nostro Academico sol per uso de' suoi discepoli, fu diffusamente e concludentemente dimostrato, in occasione di considerare l'origine e natura del maraviglioso strumento della vite; ed è con qual proporzione si faccia tal mutazione d'impeto per diverse inclinazioni di piani: come, per esempio, del piano inclinato AF tirando la sua elevazione sopra l'orizonte, cioè la linea FC, per la quale l'impeto d'un grave ed il momento del descendere è il massimo, cercasi qual proporzione abbia questo momento al momento dell'istesso mobile per l'inclinata FA; qual proporzione dico esser reciproca delle dette lunghezze: e questo sia il lemma da premettersi al teorema, che dopo io spero di poter dimostrare. Qui è manifesto, tanto esser l'impeto del descendere d'un grave, quanta è la resistenza o forza minima che basta per proibirlo e fermarlo: per tal forza e resistenza, e sua misura, mi voglio servire della gravità d'un altro mobile. Intendasi ora, sopra il piano FA posare il mobile G, legato con un filo che, cavalcando sopra l'F, abbia attaccato un peso H; e consideriamo che lo spazio della scesa o salita a perpendicolo di esso è ben sempre eguale a tutta la salita o scesa dell'altro mobile G per l'inclinata AF, ma non già alla salita o scesa a perpendicolo, nella qual sola esso mobile G (sì come ogn'altro mobile) esercita la sua resistenza. Il che è manifesto. Imperochè considerando, nel triangolo AFC il moto del mobile G, per esempio all'in su da A in F, esser composto del trasversale orizontale AC e del perpendicolare CF; ed essendo che quanto all'orizontale, nessuna, come s'è detto, è la resistenza del medesimo al-

Theorem III

The degrees of speed of a moving body descending with natural motion from the same height along planes inclined in any way, are always equal when it reaches the horizontal, all impediments having been removed […]

If the same body moves from rest t and a perpendicular, which have equal height, the times of the motions will stand to each other as the lengths respectively of the plane and of the perpendicular.

Corollary

Hence it follows that the times taken to descend on differently inclined planes, provided, however, that they have the same height, are to each other as their respective lengths.[42]

Fig. 2.12. Left: Galileo's demonstration of Theorem III. Galilei [1638], 1890-1909, vol. VIII, p. 216. Source: Bibliothèque nationale de France / gallica.bnf.fr. Right: the statement of Theorem III and of its corollary.

An interesting application - from the didactic point of view - of the mean speed theorem is given by Theorem III and its corollary (see fig. 2.12). The first part of the theorem refers to the final speed of a body falling along an inclined plane without friction that is always the same, regardless of the inclination of the plane, considering also the case of the free fall along the vertical direction. A simple way for the students to understand this result is that used by Galileo to consider a pendulum and to observe that although

[42] "I gradi di velocità d'un mobile descendente con moto naturale dalla medesima sublimità per piani in qualsivoglia modo inclinati, all'arrivo all'orizonte son sempre eguali, rimossi gl'impedimenti" (Galilei [1638], 1890-1909, vol. VIII, p. 218). "Si super plano inclinato atque in perpendiculo, quorum eadem sit altitudo, feratur ex quiete idem mobile, tempora lationum erunt inter se ut plani ipsius et perpendiculi longitudines" (Ivi, p. 215). "Hinc colligitur, tempora descensuum super planis diversimode inclinatis, dum tamen eorum eadem sit elevatio, esse inter se ut eorum longitudines" (Ivi, p. 219). Compare with Galilei [1638], 1914, pp.184-187.

other centres of rotation are fixed and the pendulum goes up following arcs of circumference of different radius, the pendulum rises to the same height, highlighting that, in the event that the friction is negligible, the speed assumes the same null value always at the same height.

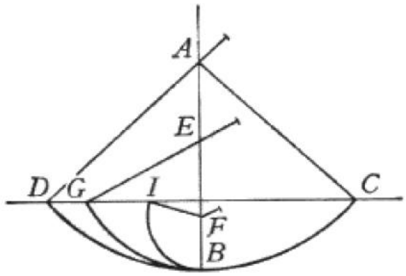

Fig. 2.13. Pendulum with different centres of rotation. Galilei [1638], 1890-1909, vol. VIII, detail of p. 206. Source: *Bibliothèque nationale de France* / gallica.bnf.fr.

The second part of Theorem III and the corollary refer to the time of fall. By the mean speed theorem, the spaces of naturally accelerated motion are equal to the spaces travelled in the same time interval by a uniform motion with speed equal to half the final speed. Since the final speeds are the same, the times taken will be proportional to the distances travelled, that is to say to the lengths of the planes or the vertical height of fall.

Certainly the most important contribution made by Galileo to the study of the free fall motion of bodies was to have assumed that it was a uniformly accelerated motion and to have been able to verify this hypothesis experimentally - as it was possible in his time. This was possible by slowing down the fall motion and using for this purpose an inclined plane about 12 *braccia*[43] long (corresponding to 6.61 m) in which a very smooth channel covered with parchment was obtained to minimize friction; a perfectly spherical bronze ball was rolled and the descent times were measured thanks to the water falling through a thin tube and then weighed with a precision balance. Galileo first measured the descent time relative to the entire length of the plane, then relative to the descent along half the length of the plane, then to a quarter, three quarters and other fractions of length. He confirmed that the time taken for the descent along the half-length of the plane was not half that needed to descend along the whole plane, as would be expected in the case of uniform motion. Galileo verified that the time it took was half when the ball was allowed to roll along a quarter of the length of the plane. All the experiments confirmed the hypothesis that the distances travelled were proportional to the squares of the times employed as was expected if the motion was naturally accelerated (Theorem II). Galileo also discusses the experimental reliability and uncertainties, managing to reproduce the motion along the inclined plane many times so that the time measurements did not differ by even a tenth of the duration of a pulse-beat. (Galilei [1638], 1890-

[43] The Florentine *braccio a terra* measured 0.55063 m (Vergara Caffarelli 2009, p. 17).

1909, vol. VIII, pp. 212-213; Galilei 1914, pp. 178-179).

Further details on this matter will be dealt with in the following paragraphs.

2.6 The laws of the pendulum motion and the measure of time

Galileo made a great contribution to the history of physics with regard to the temporal description of accelerated motion and in particular of free fall motion. Before him there was no systematic mathematical study of motions as a function of time. We find time essentially in the description of motion only in the formulation of Kepler's third law and in the mathematical description of the spiral as a function of time by Archimedes (Abattouy 1992, p 130). In fact, Archimedes defines the spiral as the composition of two motions, a uniform rectilinear motion and a uniform circular one:

> If a straight line of which one extremity remains fixed be made to revolve at a uniform rate in a plane until it returns to the position from which it started, and if, at the same time as the straight line revolves, a point move at a uniform rate along the straight line, starting from the fixed extremity, the point will describe a spiral in the plane (Archimedes 1897, 2009, p. 154).

However, there is a substantial difference between his approach and that of Archimedes which consists in the use of experimentation. While in fact Archimedes, Galileo states in his *Discorsi*, "imagined helices and conchoids as described by certain motions which are not met with in nature, and have very commendably established the properties which these curves possess in virtue of their definitions", Galileo described real motions such as that of free fall, experimentally proving the properties that had previously been demonstrated by him (Galilei [1638]1890–1909, vol. VIII pp. 197; Galilei 1914, p. 160).

A significant contribution to this development is related to the ability to measure time. In the letter to Giovan Battista Baliani dated 1 August 1639, Galileo claims to have obtained the descent time along the inclined plane thanks to the "admirable property of the pendulum, which is to make all its vibrations, large or small, under equal times", although more accurate time measurements could be made by measuring the “flow of water through a thin pipe” in a defined time - for example one minute - and then “during the time of descent along the channel” (Galileo 1890-1909, XVIII [1639], p. 76-77. English translation is mine. See also Vergara Caffarelli 2009, pp. 211-212). The constant-flow chronograph, coupled with a fairly sensitive weight scale, was a particularly useful tool for Galileo because it could measure short intervals of time in a continuous manner in which the uncertainty of the measure is essentially due to the investigator's reflexes (Ivi, p. 212).

Galileo announced his discovery of the pendulum isochronism in the letter to Guidobaldo del Monte[44] dated 29 November 1602 (Galilei 1890-1909, vol. X, pp. 97-100) and described, first in the *Dialogo* (Galilei [1632], 1890-1909, vol. VII, pp. 475-477) but above all in the *Discorsi* (Galilei [1638], 1890-1909, vol. VIII, pp. 128-130, 139-140), some simple experiments that highlight the properties of the pendulum and have great educational value. He carried out three simple experiments that can be repeated with the students: two pendulums – of different masses, with different amplitudes or different lengths – are made to oscillate together. The oscillation time changes only in the last case. For example, Galileo found no difference by swinging two pendulums made one with a lead ball and the other with a cork ball. He also did not notice that the independence of the pendulum swing time from the amplitude of the swing was valid only for small oscillations as was verified by Huygens who established that the perfectly isochronic curve is the cycloid (Huygens [1673], 1986, p. 11). Galileo studied the mathematical relationship between the swing time and the length of the pendulum. By comparing the behaviour of two pendulums, one with a length of four times that of the other, he obtained a swing time that is twice that of the other. If the string of one pendulum is nine times that of the other, the time is 3 times that of the other. This proves that the swing time of the pendulum is directly proportional to the square root of its length.

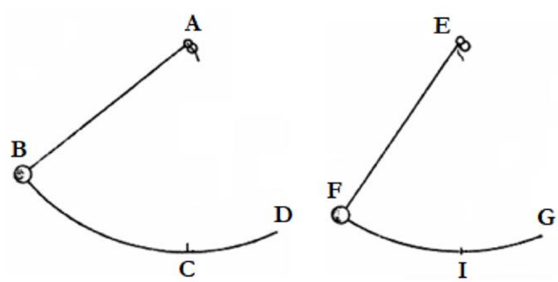

Fig. 2.14. Two pendulums with different amplitudes are made to oscillate together. Galileo, 1890-1909, vol. X, p. 98. Source: *Bibliothèque nationale de France* / gallica.bnf.fr.

Galileo tried to demonstrate the isochronism of a pendulum starting from the observation of the equality of the descent times along the chords of a circumference with its lower extreme at the lowest point of the circle, then analyzing the fall times along the arcs of circumference (Galilei 1890–1909, vol. X, pp. 99; Galilei 1890–1909, vol. VIII p. 139; Galilei 1914, pp. 95-96).

Pendulum motion is particularly significant for Galileo because of its

[44] The cultural relationship with Guidobaldo del Monte was very fruitful for Galileo. Other famous discussions concern the parabolic form of projectile motion. According to Damerow, Freudenthal, McLaughlin and Renn ([1992], 2004) "There is strong evidence that already in the course of an experiment he conducted jointly with Guidobaldo del Monte in late summer 1592, Galileo was confronted with results that showed the symmetry of the trajectory of projectile motion and suggested that it is parabolic" (Ivi, pp. 159-161). The experiment consisted of throwing a coloured ball on an inclined plane and observing the shape of the trajectory traced by the moving ball. Guidobaldo del Monte had also observed that the shape of the projectile's trajectory was similar to that of a chain attached to its ends (catenary).

analogies with free fall motion such as the independence of time from mass or the relationship of proportionality between time and the square root of length (Galilei 1890–1909, vol. VIII pp. 128; Galilei 1914, p. 84). Another important aspect is the impossibility of explaining the pendulum motion by means of the thrust received by the medium in which it is immersed - as predicted by Aristotelian theory - due to the alternating direction of its motion, to the right and then to the left (Galilei 1890–1909, vol. VIII, p. 140-141; Galilei 1914, p. 97).

Confirming the will to use the pendulum to measure time, it should be stressed that Galileo designed a pendulum clock. Its representation (s. fig. 2.15) was published by Favaro in the letter dated 20 August 1659 of Vincenzo Viviani to Prince Leopoldo de' Medici about the application of the pendulum to the clock because Viviani refers in his writing to this scheme made by Galileo (Galilei 1890–1909, vol. XIX, pp. 647-659).

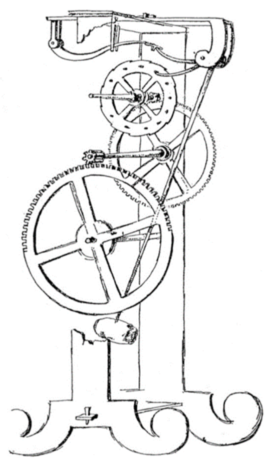

Fig. 2.15. Galileo’s design of a pendulum clock. Galilei 1890–1909, vol. XIX, p. 656. Source*: Bibliothèque nationale de France* / gallica.bnf.fr.

As highlighted by Vergara Caffarelli (2005, p. 78) and then by Pisano and Cioci (2020a, p. 266), Galileo, in particular, would use the pendulum to get time intervals all equal to each other in which to experimentally verify the so-called law of odd numbers.[45] This hypothesis is, as we have seen, based on practical considerations, on the statements of Galileo and on laboratory notes of Galileo and which we find in the remarkable manuscript Gal. 72:

> Therefore, the distances from the beginning of motion are as the squares of the times and, dividing the travelled spaces in equal times, are as odd numbers starting from the unit: that responds to what I have always said and observed with experiments (Galilei 1604-1608, f. 128v; Galilei 1890–1909, vol. VIII, p. 374).[46]

[45] Another way in which Galileo probably managed to obtain equal time intervals was by using the refrain of a song. This hypothesis is supported by the fact that he was the son of a musician (Drake 1975).

[46] English translation is mine. “Le distanze dunque dal principio del moto sono come i quadrati de i tempi et, dividendo gli spazi passati in tempi eguali sono come i numeri

2.7 The Gal. 72 manuscript and the Free Fall Law

Vol. 72 of the *Manoscritti Galileiani* comprises his mature work on motion and some other sheets concerned with other topics. They range from 1602 to the publication of the *Discorsi*. Favaro published most documents in the footnotes to *Discorsi* or in the fragments relating to the manuscript (Galilei 1890–1909, VIII, pp. 363-448). Drake published them entirely in a probable order in 1979 (Galilei [1602-1637], 1979). It includes:

a) A copy of an early work on centres of gravity (ff. 3-6).
b) An incomplete copy of *Le Mecaniche* (ff. 9-26) (Galilei 1890–1909, vol. II, pp. 155-190).
c) Another copy of the same treatise inserted between folios 27bis and 28.
d) A copy of the treatment of the properties of the lever (f. 27r-v) (Galilei 1890–1909, vol. VIII, pp. 366.1-367.14),
e) Two drafts of the original dedication of the *Discorsi* by Galileo to the "Conte de Noailles" (f. 28r-v), and the preface to the *Discorsi* (ff. 31-32).
f) The main part of the manuscript consisting of notes on motion - essentially naturally accelerated motion and parabolic motion - as autograph drafts in the original hand of Galileo. They include almost 30 sheets which are copies written by two of his disciples, Mario Guiducci and Nicolò Arrighetti (f. 33r to f. 194r) (Galilei 1890–1909, vol. VIII, pp. 557-642).

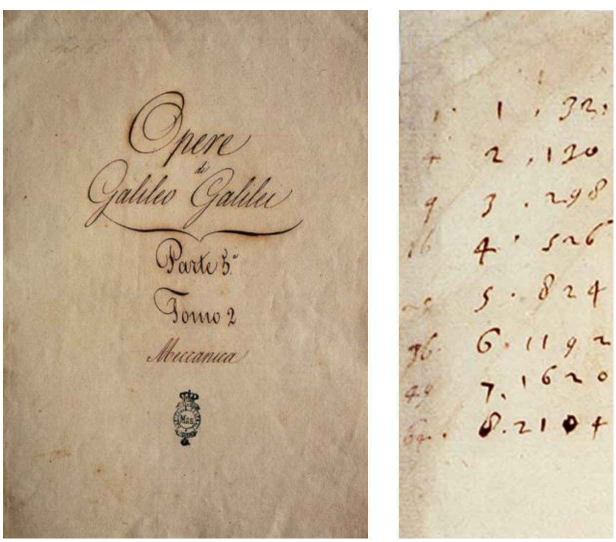

Fig. 2.16. Left: frontispiece of Ms. Gal. 72, f. 1r. Right: Probably data obtained in the course of an experiment performed in order to confirm the law of fall. Ms. Gal. 72, detail of f. 107v. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze.* Retrieved via www.bncf.firenze.sbn.it.

impari ab unitate: che risponde a quello che ho sempre detto et con esperienze osservato" (Galilei 1604-1608, f. 128v; Galilei 1890–1909, vol. VIII, p. 374).

This document, with Galileo's laboratory notes, is particularly important because it provides strong evidence that he actually performed the experiments described in his writings (see the works of Drake, Segre 1980). Descartes had already judged in his letter to Mersenne in April 1634 that Galileo did not perform his experiments set inside a ship or on a running horse or with a cannon shot from the top of a tower, but that they were mostly thought experiments (Descartes 1897-1913, pp. 284-291).

Mersenne, on the other hand, had explicitly disputed Galileo's experiments concerning the descent along the inclined plane:

> I doubt that Mr. Galilée has made the experiments of the falls on the plane, since he does not speak of it at all and that the proportion he gives often contradicts the experience (Mersenne 1636, p. 112).[47]

Alexandre Koyré, likewise, argued that it was impossible to actually perform the experiments with the results that Galileo claimed to have achieved, considering

> the amazing and pitiful poverty of the experimental means at his disposal […] It is obvious that the Galilean experiments are completely worthless: the very perfection of their results is a rigorous proof of their incorrection (Koyré 1953, p. 224).

It was Thomas Settle (1961) who, having carefully repeated the inclined plane experiment as it had been described by Galileo, confirmed the reality of the experiments and Galileo's excellent quality as a physical experimenter, having obtained *relatively precise time measurements*.

One of the most relevant folios of the manuscript Gal. 72 is *107v* in which calculations and diagrams are provided that in Drake's opinion (Drake 1975, pp. 100-101) represent the data obtained in the course of an experiment performed in order to confirm the law of fall. Fig. 2.16 (right) shows a detail of the paper with a table in whose third column "the distances from rest of a ball rolling down an inclined plane at the ends of eight equal times" because they can be obtained "from the products of the first number, 33, by the square numbers 1, 4, 9, ... , 64". These numbers most likely represent experimental results because they differ slightly from the theoretical results expected if distances from the rest position had been measured at intervals all equal to each other. It is also reasonable to assume that Galileo would have used the pendulum to get time intervals all equal to each other in which to experimentally verify the so-called law of odd numbers.

The documents of the period, this manuscript and Galileo's laboratory notes, in particular, also fulfil the important didactic function of showing the very process of research, how scientific theory was constructed.

For example, in the letter to Paolo Sarpi, dated October 16, 1604, Galileo announced the free fall law with the wrong hypothesis of proportionality of speed to the space traversed stating that to prove the experimental law he lacked "a completely indubitable principle to put as an axiom [....] And the

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[47] English translation is mine. "Je doute que le sieur Galilée ayt fait les expériences des cheutes sur le plan, puis qu'il n'en parle nullement et que la proportion qui donne contredit sovent l'expérience" (Mersenne 1636, p. 112).

principle is this: that the natural movable goes increasing in speed with that proportion with which it departs from the beginning of its motion” (Galileo, 1890-1909, vol. X, p. 115; Abattouy 1992, p. 42-43).

Galileo makes this demonstration in folios *128r* and *128v* of the manuscript Gal. 72. It must be emphasised, however, that he was already aware of the experimental law of the squares of times and was looking for an a priori principle from which to derive it. The choice of direct proportionality between speed and distance travelled as an axiom derives from the observation that the effects of the speed-related impact of the incident body with wax are directly proportional to the height of fall (Galilei 1604-1608, f. 128r; Galilei 1890-1909, vol. VIII, p. 373; Galilei 1979, p. 41; Abbatouy 1992, p. 47).

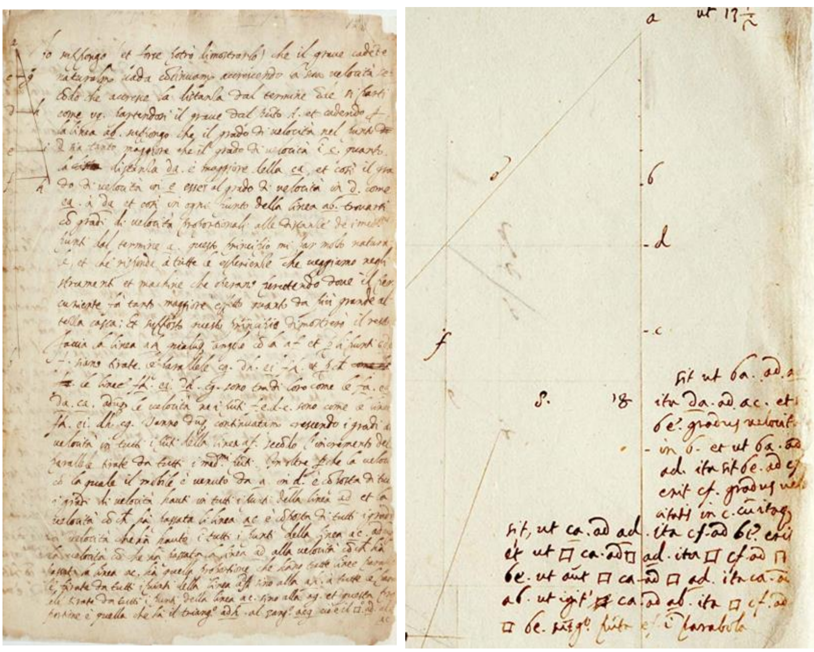


Fig. 2.17. Left: Attempt to demonstrate the law of the squares of times from the incorrect assumption of direct proportionality between velocity and space. Galilei 1602-1637, Ms. Gal. 72, f. 128v. Right: Observation of the contradiction arising from the assumption that speed was proportional to distance travelled. Source: Galilei 1602-1637, Ms. Gal. 72, detail of f. 152r. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.

Galileo, wanting to demonstrate a relation of quadratic proportionality, as a true expert in the Euclidean theory of proportions, opts to consider the areas of two similar triangles that are in proportion to their respective sides (see also Damerow et al. 2004, p 171). His interpretation of geometric representation (in terms of speed-space instead of speed-time) is wrong because it is invalidated by his incorrect hypothesis of proportionality between speed and space travelled.

The conceptual change (Posner et al. 1982) of the passage from the assumption of the wrong law of proportionality between speed and distance travelled in a naturally accelerated motion to the law of proportionality between speed and time is witnessed in folio *152r*. In a geometrical way, Galileo observes that if we start from the quadratic proportionality between the space travelled and the time taken we arrive at a contradiction, that is, that speed is proportional to the quadratic root of the distance.

2.8 Parabolic Motion in the Fourth Day of *Discorsi su Due Nuove Scienze* and in the Gal 72 Manuscript

Accordingly, with Drake and MacLachlan (1975), the most significant of Galileo's experiments described in manuscript Gal. 72 is about parabolic motion.

Galileo discusses parabolic motion in the fourth day of the *Discorsi*, i.e. the motion that Aristotle had defined as violent and that Galileo instead considers as the composition of a naturally accelerated motion in the vertical direction and a uniform rectilinear motion in another direction.

Galileo ([1638], 1890-1909, vol. VIII, pp. 270-271) constructs the parabola according to Apollonius of Perga ([1896], 2013) considering a right cone with the circle ibkc as base and the point l as vertex. Its section with a plane parallel to the side of the cone bc defines the parabola. Galileo demonstrates the important property according to which the part of the axis ae is proportional to the square of the height of the parabola at the corresponding point, i.e.

$$ad : ae = bd^2 : fe^2$$

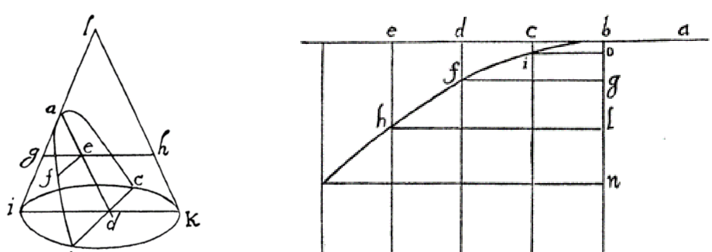


Fig. 2.18. Left: the parabola as a conic section. Galilei [1638], 1890-1909, vol. VIII, detail of p. 270. Right: composition of an equable and a naturally accelerated motion. Ibid, detail of p. 272. Source: *Bibliothèque nationale de France* / gallica.bnf.fr. R

Galileo uses this result to demonstrate that the composition of the motions described above produces an arc of a parabola (see fig. 2.18 right). If we consider a body moving along the plane ab of equable motion, i.e. uniform rectilinear motion, and then dropped in the vertical direction, the uniform motion in the horizontal direction is superimposed by a naturally accelerated motion in the vertical direction. If on the line be we indicate the measure of time - which is in any case proportional to the space travelled in the horizontal direction - we observe that to the equal time intervals bc, cd, de correspond spaces travelled in the vertical direction ci, df, eh which are proportional to the squares of the same time intervals, therefore the points b, i, f, h lie on the same parabolic line. In the same way, given the proportionality of the times with the spaces travelled in the horizontal direction, it is demonstrated that the points travelled by the body during compound motion lie on the same semi-parabola (Galilei [1638], 1890-1909, vol. VIII, p. 272-273).

Perhaps the most complex and most interesting experiment, which can be considered an indirect proof of the principle of inertia, is shown at the top of folio *116v* (fig. 2.19). The diagram drawn by Galileo (reproduced in fig. 2.20 also with graphic elements that were not explicitly represented, for a better understanding) represents the trajectories of a ball that, after falling along an inclined plane, is deflected to move initially in the horizontal direction and then dropped. The horizontal motion, ignoring air resistance, obeys the Galilean inertia principle according to which, in the absence of forces, the ball must continue to move with a straight line motion at constant speed. In the vertical direction, the ball is subject to gravity and therefore moves with a naturally accelerated motion. The result of this composition of movements is a parabolic motion.

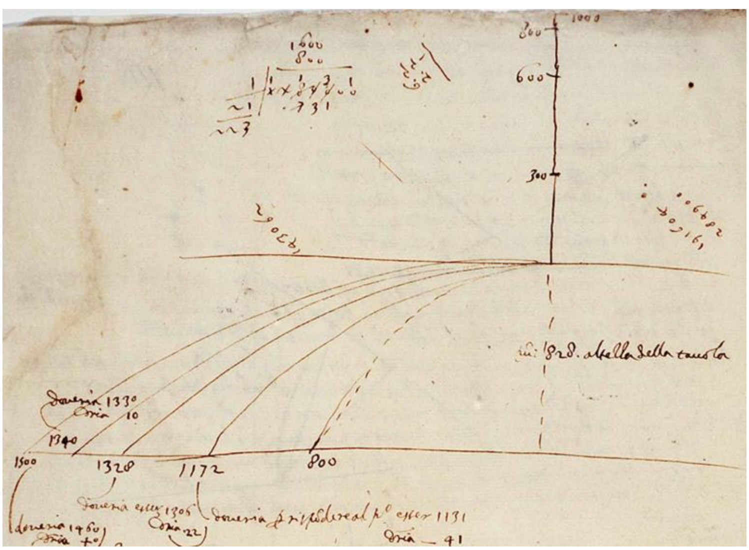


Fig. 2.19. The measures of the ranges of the projectile motion with horizontal initial velocity. Source: Galilei 1602-1637. Ms. Gal. 72, detail of f. 116v. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.

Along a vertical line Galileo recorded the numbers 300, 600, 800, and 1000. They are the heights from which a ball is descending along an inclined plane (that has not been drawn). The unit of measure taken by Galileo is the point, which is equivalent to 0.9 mm. The table is placed at a height of 828 points. Along the horizontal axis are reported the distances in points from the vertical to which the ball touches the ground for the different heights of the inclined plane. Also indicated are the expected values of these distances calculated by Galileo, assuming the conservation of velocity in the horizontal direction equal to the final velocity of the ball's falling motion along the inclined plane and therefore proportional to the square root of its height.

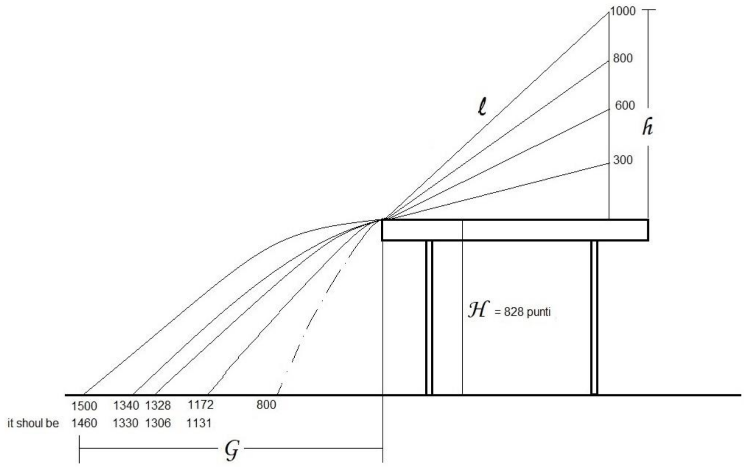
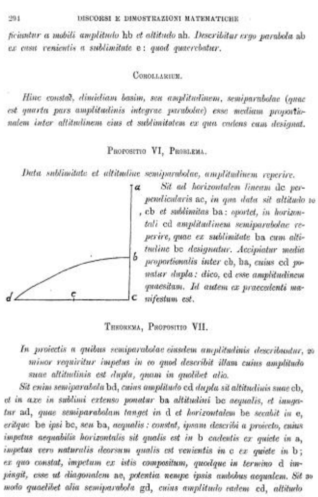


Fig. 2.20. Diagram of the picture on f. 116v of the manuscript Gal. 72 with the implied elements. Source: my graphic representation, similar to fig. 9 in Abbatouy 1996b, p. 43.

In the corollary to Proposition V of the parabolic motion - which answers the question posed by the problem of Proposition VI - given in the fourth day of the *Discorsi*, Galileo enunciates and demonstrates the statement that allows him to establish this relationship, according to which "half the base, or amplitude, of the semiparabola (which is one-fourth of the entire amplitude) is a mean proportional between its altitude and the sublimity" (Galilei [1638], 1890-1909, vol. VIII, p. 294).



294 DISCORSI E DIMOSTRAZIONI MATEMATICHE

ficiuntur a mobili amplitudo hb *et altitudo* ab. *Describitur ergo parabola* ab *ex casu venientis a sublimitate* e : *quod quaerebatur.*

COROLLARIUM.

Hinc constat, dimidium basim, seu amplitudinem, semiparabolae (quae est quarta pars amplitudinis integrae parabolae) esse mediam proportionalem inter altitudinem eius et sublimitatem ex qua cadens eam designat.

PROPOSITIO VI, PROBLEMA.

Data sublimitate et altitudine semiparabolae, amplitudinem reperire.

Sit ad horizontalem lineam dc *perpendicularis* ac, *in qua data sit altitudo* cb *et sublimitas* ba : *oportet, in horizontali* cd *amplitudinem semiparabolae reperire, quae ex sublimitate* ba *cum altitudine* bc *designatur. Accipiatur media proportionalis inter* cb, ba, *cuius* cd *ponatur dupla : dico,* cd *esse amplitudinem quaesitam. Id autem ex praecedenti manifestum est.*

THEOREMA, PROPOSITIO VII.

In proiectis a quibus semiparabolae eiusdem amplitudinis describuntur, minor requiritur impetus in eo quod describit illam cuius amplitudo suae altitudinis est dupla, quam in quolibet alio.

Sit enim semiparabola bd, *cuius amplitudo* cd *dupla sit altitudinis suae* cb, *et in axe in sublimi extenso ponatur* ba *altitudini* bc *aequalis, et iungatur* ad, *quae semiparabolam tanget in* d *et horizontalem* be *secabit in* e, *eritque* be *ipsi* bc, *seu* ba, *aequalis : constat, ipsam describi a proiecto, cuius impetus aequabilis horizontalis sit qualis est in* b *cadentis ex quiete in* a, *impetus vero naturalis deorsum qualis est venientis in* c *ex quiete in* b ; *ex quo constat, impetum ex istis compositum, quodque in termino* d *impingit, esse ut diagonalem* ae, *potentia nempe ipsis ambobus aequalem. Sit modo quaelibet alia semiparabola* gd, *cuius amplitudo eadem* cd, *altitudo*

Corollary

Hence it follows that half the base, i.e. half the amplitude of a semiparabola (which is then the fourth part of the amplitude of the whole parabola) is a mean proportional between its height and that sublimity, falling from which the moving body describes the semiparabola itself.

Proposition VI Problem

Given the sublimity and the height of a semiparabola, find its amplitude[48]

Fig. 2.21. Range of parabolic motion in *Discorsi su Due Nuove Scienze*. Galilei [1638], 1890-1909, vol. VIII, p. 294. Source: Bibliothèque nationale de France / gallica.bnf.fr.

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[48] "Corollarium. Hinc constat, dimidium basim, seu amplitudinem, semiparabolae (quae est quarta pars amplitudinis integrae parabolae) esse mediam proportionalem inter altitudinem eius et sublimitatem ex qua cadens eam designat. Propositio VI. Problema. Data sublimitate et altitudine semiparabolae, amplitudinem reperire" (Galilei [1638], 1890-1909, vol. VIII, p. 294).

The situation referred to in this corollary is that illustrated on folio 116v of the manuscript Gal. 72. By the term sublimity Galileo indicates the height of the inclined plane h from which the ball falls, which determines the initial velocity of the parabolic motion, the altitude of the parabola indicates the height H above the ground of the vertex of the parabola, i.e. the point from which the parabolic motion begins, the base or amplitude G of the parabola is the range of the launch, i.e. the distance in the horizontal direction travelled during the parabolic motion.

It is possible to demonstrate this relationship in a modern way using only the equations of accelerated motion.

By writing the equations of the falling motion along the inclined plane (neglecting the force of friction), it is possible to determine the velocity at the end of the first path:

From $\begin{cases} s = \frac{1}{2}\, g\, \frac{h}{l}\, t^2 \\ v = g \frac{h}{l} t \end{cases}$, in fact, by $s = l$, we obtain $t_f = \sqrt{\frac{2}{gh}}\, l$ which substituted into the second equation gives the value of the final velocity $v_f = \sqrt{2\, g\, h}$.

The range of parabolic motion can be calculated by observing that the initial velocity in the horizontal direction is equal to the final velocity of motion along the inclined plane.

The range of parabolic motion is equal to the product of its initial (horizontal) velocity, which remains constant during motion, and the time of flight, which corresponds to the time of fall in the vertical direction:

$$G = \sqrt{2\, g\, h} \cdot \sqrt{\frac{2\, H}{g}} = 2\, \sqrt{h\, H}$$

That is, half of G is a proportional average between *h* and *H*.

A special case is when $h = H$, for which $G = 2\, H$ results.

Galileo also tried to study the parabolic motion of a projectile launched in the oblique direction. In this case, the relationship between the displacement in the vertical and horizontal direction is more complicated. In this case, Galileo realised that the angle at the base of the inclined pine would determine the relationship between the components of the initial velocity of the motion in the vertical and horizontal directions. The most important result that Galileo obtained in this regard concerned the determination of the launch angle - equal to 45° - corresponding to the maximum range, a result already achieved by Tartaglia who, however, had not considered parabolic trajectories, but straight lines with a circular intermediate phase.

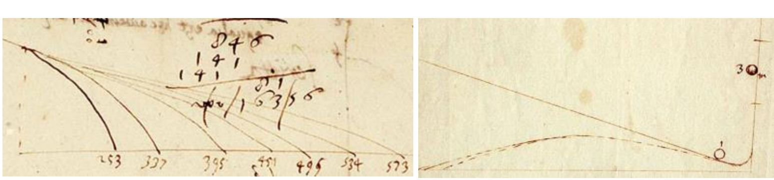

Fig. 2.22. Left: fall with initial velocity in oblique direction. Galilei 1602-1637. Ms. Gal. 72, detail of f. 114v. Right: fall with initial velocity oblique upwards, Galilei 1602-1637. Ms. Gal. 72, detail of f. 175v. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.

The results of an experiment used by Galileo in order to visualise the whole trajectories of a falling ball after descending from an inclined plane are shown in folio *81r* (Hill 1988; see also Drake 1973). The first half of this folio (fig. 2.23) shows Galileo's record of the positions of the points of intersection of the trajectories with multiple parallel planes placed at different heights. This method, together with the condition of quadratic proportionality between the part of the axis located between the vertex and the corresponding point and the height of the parabola, in the case where the initial velocity is in the horizontal direction, could be used from an educational point of view to experimentally verify that the curve obtained is a parabola.

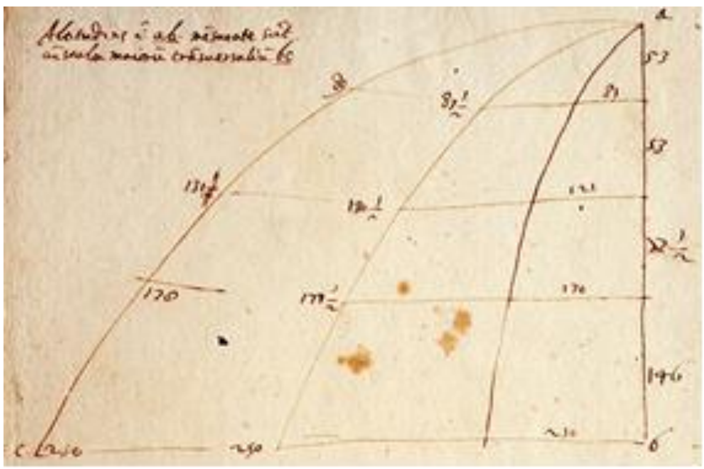

Fig. 2.23. Experimental reconstruction of parabolic trajectories described by a horizontally launched ball. Galilei 1602-1637, Ms. Gal. 72, detail of f. 81r. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.

The application of the law of odd numbers in the vertical direction of a parabolic motion can be very useful for theoretically studying the shape of the trajectory and for estimating its length. The diagram shown in sheet *117r* (fig. 2.24) can be used didactically as a hint for verifying the direct quadratic proportionality between the space travelled in the vertical and horizontal directions.

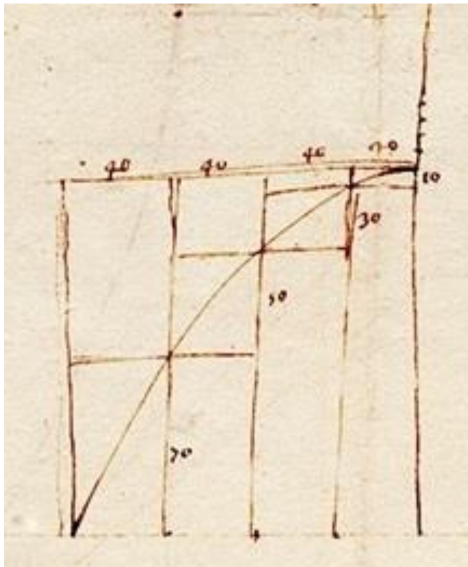

Fig. 2.24. Reconstruction of parabolic trajectories using the law of odd numbers. Galilei 1602-1637, Ms. Gal 72, detail of f. 117r. By concession of Italian Ministry of Culture / *Biblioteca Nazionale Centrale di Firenze*. Retrieved via www.bncf.firenze.sbn.it.

Epilogue II

In this chapter, I analysed Galileo's writings in order to identify significant elements to be included in a historical theoretical and experimental educational path.

First, I developed and examined the evolution of Galileo's thought from his critique of Aristotle's thought and the influence of Archimedes in the determination of the falling velocity of bodies in media and in the determination of the upward force necessary to balance the effect of gravity on an inclined plane.

Then, I discussed the important role played by the *Dialogo sui due Massimi Sistemi del Mondo* in introducing the principle of inertia – and later the principle of relativity – and how Galileo used it to interpret violent motion and introduce the composition of simultaneous motions. I analysed in detail the third and fourth day of Galileo's *Discorsi su due nuove scienze*, reconstructing – even with modern mathematical expressions – his theorems on naturally accelerated motion and parabolic motion.

After that, his manuscripts allowed me to reconstruct experiments he actually performed on the fall along the inclined plane and the pendulum, going so far as to assume that he actually used the pendulum to prove the odd-number law of naturally accelerated motion along the inclined plane.

Finally, I analysed Galileo's laboratory notes in order to reconstruct the experiments on parabolic motion that he actually carried out, understanding the conditions and results achieved, so that they could serve as a starting point for further experiments.

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PART II

Research in physics education. A NoS innovative educational path

An outline

This part of the thesis is dedicated to teaching, to how to bring the knowledge of scholars into schools taking into account the centrality of the student, his needs and his difficulties. Research in physics education, with an in-depth study of the history of physics, can make an important contribution to the effectiveness of teaching that takes into account the pre-conceptions of students. A didactic path is then outlined in which the most recent results in physics education and in the history of physics are considered. The last part of the thesis will analyse the results of the experimentation of this historical educational path in a high school in order to derive insights into the physics education

CHAPTER III

From research in physics education to an innovative historical educational path

Prologue III

Teaching / learning is a complex operation. To be effective, it must take into account the figure of the student, his way of learning, his potential, his learning difficulties. The abilities of the student must be related to the discipline content to be learned through the intervention of the teacher. It is possible to schematize this situation by means of a pedagogical triangle that was introduced by Houssaye ([1988], 1992) that can help the teacher understand the dynamics involved in the relationships between the subjects (teacher and student) and the object of knowledge.
Maintaining the centrality of the student means the upper secondary school teacher must know both how the student learns, with his or her difficulties (Pedagogy), and how best to present the content of the discipline (Content of the Discipline) to the student (*Pedagogical Content Knowledge*).
John Dewey (1897; 1900) had already identified *learning by doing*, i.e. through practical activity, as the best way of educating children and adolescents. For physics education this certainly means putting *hands on* objects in the laboratory but also the reconstruction of real situations, problem-based teaching, inquiry-based learning and the active role of the student. This is the intended outcome of the educational path on the history of Galilean physics presented in this chapter.

3.1 Relationships between the subjects of teaching and knowledge: the pedagogical triangle

The exclusive relationships between the elements that characterise the teaching process can be represented by means of the pedagogical triangle that was introduced by Houssaye in 1988. At its vertices are the student, the teacher and the knowledge. The sides instead represent the relations between these three elements. The relationship between the student and knowledge is learning, with its strategies, conceptions and representations; the relationship between the teacher and the student is training, with the whole psychological and pedagogical dimension; the relationship between the teacher and knowledge is teaching, with the in-depth analysis of the content (and fundamentals) of the disciplines and its didactic transposition. Particularly expressive is the way Houssaye describes how these relationships take place between the subjects, two by two, with the third playing “the dummy”, as in the card game Bridge.

> Not everyone is the subject of everyone else. Many people could be subjects for us, but they do not interest us, they do not enter the field of our desire: we do not consider them as subjects, they have the place of the dummy […] they are the others, the real subjects, who assign him his place, his game and his way of playing (Houssaye [1988], 1992, p. 43. The translation of the passage into English is mine).

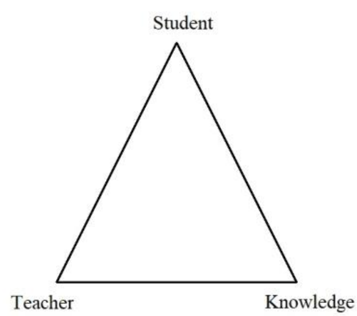

Fig. 3.1 Houssaye's pedagogical triangle (1992, p. 41) modified with the student at the top vertex of the triangle. Source: my own representation.

The didacticians of mathematics instead proposed a didactic triangle to characterize didactic conditions so that the behaviour of the three elements of a teaching system (the student, the teacher and knowledge) only acquires significance as the manifestation of an interaction between them (Develay 1992, p. 64).

Develay emphasises the importance of didactics seen as a new science centred on "the processes of acquiring and transmitting knowledge about a given discipline", taking into account epistemological frameworks, historical implications, the ways in which knowledge is transmitted by teachers and in which students learn (Ivi, p. 67)

3.2 How to teach scholarly knowledge in the classroom? The didactic transposition

The first major didactic problem is how to construct curricula that can be taught to students of the various school levels – adapting the difficulties of the study content to them – and that reveal the basic or underlying principles of various fields of knowledge. These principles, for physics, include the awareness that the final order of nature can be understood or that there are quantities that are conserved and others that are transformed (Bruner [1960], 1977, p. 18 and following). For example, children may have difficulties in studying Euclidean geometry but can easily understand concepts of “intuitive geometry” which can prepare for the subsequent study of postulates and theorems. The same can be said for physics.

> Basic notions in these fields are perfectly accessible to children of seven to ten years of age, provided that they are divorced from their mathematical expression and studied through materials that the child can handle himself (Ivi, p. 43).

Bruner, however, considered it possible to teach any content, as long as it was suitably simplified. Piaget (1975), on the other hand, emphasised the desirability of defining the complexity of the topics to be covered, taking into account the child's psycho-motor and cognitive developmental stages.

With regard to this problem, we can consider the notion of didactic transposition, which, after it had been introduced by Michel Verret (1975), was deepened by Yves Chevallard with the meaning of "work that transforms an object of knowledge to be taught into an object of teaching" (Chevallard 1985, 1991, p. 39. English translation is mine).

The process of didactic transposition begins with the designation of knowledge contents (called “wise” for being the achievements of the scientific and research community) as contents to be taught and continues with a series of transformations that make them the object of teaching.

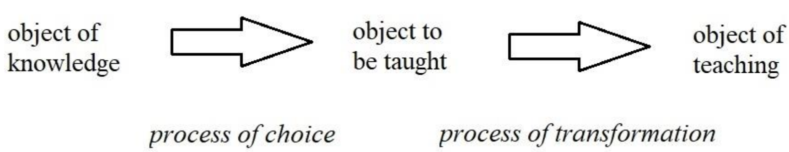

Fig. 3.2 Didactic Transposition according to Chevallard (1985, 1991, p. 39). Source: my own representation.

The didactic transposition is divided into external and internal: the first is that carried out by the commissions that create the ministerial programs and by those who draw up school manuals; the second is that performed by the teachers. Chevallard points out that the objects of teaching are not really objects of knowledge. A real responsibility then arises on the part of teachers towards the object of teaching (the principle of epistemological vigilance) with the task of checking whether it is a correct expression of the object of scholarly knowledge.

Michel Develay deepened the notion of didactic transposition in a text entitled *Savoirs scolaires et didactiques des disciplines* (Develay 1995), specializing it in the context of twelve known disciplines and one new one (Plastic Arts; Physical Education; French Language; Geography; History; English Language; Mathematics; Music Education; Philosophy; Economic and Social Sciences; Physics; Biology; Technology). Develay emphasizes the importance of the social practices of reference that intervene in the choice of knowledge to be taught starting from the knowledge of scholars (Ivi, p. 26). He also considers an extension of the process of transposition to the mediation work of teachers and pupils' learning in which the subjectivity of the teacher takes on an important role that manifests itself precisely in the moment of his action because of the conception that he has of the discipline.

> didactic transposition concerns all the transformations that scholarly knowledge and social practices of reference undergo in order to give rise not only to knowledge to be taught, but to knowledge taught and, ultimately, to knowledge learnt by pupils (Ibid.).

According to Develay, the transposition process incorporates two different aspects*: didactisation* and *axiologisation*. The first concerns the work of choosing the content to be taught and the construction of situations for learning that content; the second is related to the influence that the value system of the teacher and society – but also of the students with their interests and preferences – exerts on the choice of knowledge to be taught and how this is done (Ibid.; Rossi and Pezzimenti 2017, p. 195).

3.3 The particular competences of teachers and the *Pedagogical Content Knowledge*

Great importance has been given to the subjective role of the teacher, to his particular competence, to the specific content involved in his professional growth. Starting from these bases, it has been hypothesized that the renewal based on didactic research may have as a reference Shulman's theoretical assumptions which can be summarized in the definition of *Pedagogical Content Knowledge* (PKC). It is

> that special amalgam of content and pedagogy […] into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction (Shulman 1987 p. 8).

Shulman argued that teachers should know how to structure and represent academic content for direct teaching to students, the misconceptions and difficulties encountered by students in learning a particular disciplinary content, and the specific teaching strategies that can be used to meet the learning needs of students.

According to Shuman's thinking, the teacher should perform a series of operations to transform knowledge content into teaching content. The teacher must understand the content to be taught – but also the educational purposes of developing skills and values necessary to integrate into society – by conveniently transforming it for the students and appropriately identifying alternative methods to represent the fundamental concepts. Through the operation of education, we move on to the actual pedagogical act, followed by a phase of evaluation of the learning of the pupils and the effectiveness of the teacher's action.

Genuine reform can only take place by incentivising a more thorough preparation of teachers, by raising the standards for their admission to teaching careers, by increasing the number of years of their training and by "organising elaborate programmes of new teacher induction and mentoring" in the workplace (Ibid., p. 19).

3.4 The *Model of Educational Reconstruction* and the consideration of students' initial conceptions

The integration of the teacher's profession with educational concepts that take into account the *Model of Educational Reconstruction* (MER) and its latest developments can make a valuable contribution to improved teaching and learning (Duit, Gropengießer and Kattmann 2005; Duit, Gropengießer, Kattmann, Komorek and Parchmann 2012).

According to the educational reconstruction model the attempts to improve didactics must give attention to either problems related to the content of the discipline or to the learning needs of the students. The problems to be considered are closely related: the clarification and analysis of the subject matter (including key ideas of science and principles) and analysis of the dispositions of students and teachers in relation to the choice of contents, including interests, attitudes, skills and pre-didactic conceptions.

Duit et al. analytically identify the phases in which the didactic transposition must occur. According to the MER, the teacher must follow an elementarisation process in order to identify "the elementary features (basic phenomena, basic principles, general laws)", that characterize a certain topic of scholarly knowledge to be taught in accordance with the teaching goals of cultural development of students (Duit et al. 2012, p. 6). The process ends with the "construction of the content structure for instruction" (Ivi, p. 14). This must be complex enough to meet the students' interest and facilitate understanding.

The *Model of Educational Reconstruction* emphasises the importance of research on students' pre-instructional conceptions. Specialists should be aware of the research on students' conceptions in relation to the content to be learned because they need take this into account when designing the curriculum for students.

Since students' initial conceptions are linked to common-sense, everyday knowledge, generally they have been regarded as epistemological stumbling blocks on the way to properly scientific knowledge achieved through a process of conceptual change that first undermines previous knowledge.

The *Model of Educational Reconstruction*, on the other hand, by virtue of its constructivist approach, considers them "as points to start from and mental instruments to work with in further learning" (Ivi, p. 8). The process that characterises learning would also take place in a continuous/evolutionary manner because knowledge would be connected in a new way according to a pattern of growth, of accretion (Strike & Posner, 1992; Vosniadou, 1996), of "conceptual reconstruction" (Kattmann 2007) analogous to the processes of educational reconstruction (Duit et al. 2012, p. 16).

To apply the *Model of Educational Reconstruction*, I have drawn up a planning sheet that takes its approach into account. In fact, I considered the students' pre-conceptions and their interests. The three columns entitled *Structured academic knowledge, Elementary knowledge accessible to students, Logical organisation of content for teaching* facilitate the implementation of the *Model of Educational Reconstruction*.

<table>
<tr><td colspan="3" align="center">TITLE</td></tr>
<tr><td colspan="3">What are the students' pre-instructional conceptions or misconceptions about the content to be learned?</td></tr>
<tr><td colspan="3">What are the students' interests that can be related to the content to be learnt?</td></tr>
<tr><td>Structured academic knowledge
(generally organised in an abstract and concise manner)</td><td>Elementary knowledge accessible to students
(essentially fundamental laws and principles; skills to be achieved, understanding of the nature of science and the way scientists work)</td><td>Logical organisation of teaching content in a way that makes sense to students
(e.g. problematic, historical, in relation to everyday life or society)</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Tab. 3.1. Project sheet outline in application of the Educational Reconstruction Model. Source: my own production.

Duit et al. also emphasise the importance of designing new experimental physics or multimedia learning environments. Scientific teaching is interdisciplinary: science is one of the main disciplines of reference, but other skills are also necessary. The philosophy of science and the history of science provide models of thought to analyse (critically) the nature of science (NoS) and the special contribution that science can give "to understand the world" (Duit et al. 2012, p. 14).

3.5 The contribution of the history of physics to learners' conceptual change. Experiments of particular historical value in physics education

The history of physics can make a remarkable contribution to physics education. The reference to the history of physics in fact makes it possible to argue alternative hypotheses critically by stimulating the students to follow the scholars' reasoning and by motivating them by comparison (for example the comparison between Galileo and the Aristotelians about motion or between Bohr and Einstein on the foundations of quantum mechanics). Some spontaneous ideas are similar to outdated interpretive models, and these can become instruments for overcoming ideas of common sense (Viennot 1979a; Wandersee 1986): studying the theories of historical interest, the students will recognise their own conceptions and will discuss and revise them, by processes of conceptual change (Posner, Strike, Hewson and Gertzog 1982) similar to those that actually occurred in the history of physics. These processes, according to the authors, can occur on an individual basis when there is disaffection with current conceptions and the proposed replacement conception is clear enough to be understood. It must offer solutions to old and new problems, and offer a promising research development, just as happens in science according to the theories of Kuhn ([1962], 1970) and Lakatos (1982). Thus individual cognitive development is supposed to somehow recapitulate the historical development of science (see also Duhem 1906). Of course, it is not said that this process takes place for every single student. Furthermore, the history of the science can contribute to improve the understanding of the Nature of Science in its conceptual, procedural and contextual aspects, and of the relationships among science, technique and society (Teixeira, Greca and Freire, 2012; Maurines and Beaufils 2012).

The History of Science can make Cultural Content Knowledge (Galili 2008; 2012) that it directly related to the narrative component of science. It includes the principal alternative historical theories that have preceded the actual structuring and – in the didactic context – the common conceptions of the students. The inclusion in science teaching of debates and controversies immerses the discipline in a vast context and qualifies it as cultural fact.

In order to include the History of Science in the teaching of science subjects, it is possible - as many high school textbooks are organised - to add references to episodes from the history of science to a non-historical science course; sometimes, however, a historical line is followed in teaching, i.e. history is used to create a narrative development within which the various topics can be organised (think of how introductory quantum mechanics courses are often structured). It is very different to follow an integrated approach, according to which an entire science course is organised on a historical basis (for example, the historically oriented *Project Physics Course* for high schools by Gerald Holton first published in 1970) and scientific topics are "learned in another context, cultural, historical and philosophical. Only such broader perspectives can give point and lasting value to scientific information and experience for the general student" (Conant 1945, p. 115).

However, the laboratory and practical application can also make an important contribution to overcoming learning difficulties. If students not only perform a task of verifying the physical laws but also play an active part in the formulation of research questions, through the design and execution of experiments they can become aware of scientists' way of working (inquiry based learning) (Rutherford 1964). The replication of experiments that were historically relevant for the development of physics, introduced in closest way possible to their real execution in their historical, cultural, ethical and social context, can help students to understand the Nature of Science (NoS) (Heering and Höttecke 2014). It is also possible to replicate these experiments with modern methods and technologies, encouraging the students to take a more active role, and then comparing the results with the original work (Metz and Stinner 2006).

3.6 A NoS historical educational path at the *Liceo Sbordone* in Naples

In this thesis work, an ambitious interdisciplinary educational path is presented, comprising both theoretical and experimental parts, based on the history of physics, in particular on Galileo's writings and laboratory notes. This project was actually carried out within the curriculum in the first three grades of the *Liceo scientifico "F. Sbordone"* in Naples and the educational results achieved were analysed in a rigorous manner by means of quantitative and qualitative surveys using special questionnaires.
In tab. 3.2. the project sheet relating to the educational path is shown, built following the scheme I created in accordance with the Educational Reconstruction Model. Pre-instructional conceptions or misconceptions about the content to be learned were identified during a first year of experimentation (during the following years, a more in-depth investigation was carried out by means of an input questionnaire whose results will be shown in the next chapters). This is followed by further two tables in which the questions posed to students during the lessons are presented alongside

the experimental learning environments that were used to overcome their learning difficulties, and finally the learning objectives to be achieved. Further information on the project is indicated in the last table. They are: the specification of the classes engaged in the study of Galileo's experimental physics, the duration of the teaching activities, the type of evaluation of the pupils and the effectiveness of the project, and the ethical approval of the Headmaster of the *Liceo Sbordone* in Naples. Fig. 3.3 represents the map of concepts that are presented in the paragraph.

TITLE		
Galileian Physics of Motion		
What are the students' pre-instructional conceptions or misconceptions about the content to be learned? Dependence of pendulum swing time on mass and amplitude. Dependence on the mass of the time of fall of a body in the void. Direct proportionality between the oscillation period of the pendulum and the length of the string. Direct proportionality between the speed in a free fall motion and the distance travelled. The force must continue to act to keep a body in motion. Direct proportionality between the space travelled along an inclined plane and the time interval spent. Trajectory of the motion of the projectile consisting of a straight line until the initial thrust is exhausted and an almost vertical line due to the weight effect. Straight trajectory of the projectile falling motion with oblique downward velocity.		
What are the students' interests that can be related to the content to be learnt? High school students are very interested in playing sports such as football and volleyball in which parabolic motions are often observed		
Structured academic knowledge (Core foundation of the discipline)	**Elementary knowledge accessible to students (Subject content)**	**Logical organisation of teaching content in a way that makes sense to students (The context)**
- The motion of the simple pendulum	- Period of small oscillations: independence from mass and amplitude of oscillation - Period of small oscillations: quantitative dependence on the length of the string	- Subjects are presented problematically using the method of enquiry - The questions posed are resolved following the approach taken by Galileo, reproducing a series of experiments performed by him
- Uniform motion	- Characteristics of uniform motion - The principle of inertia	
- Uniformly accelerated motion	- The laws of speed as a function of time. - Speed-time graph - The mean speed theorem - The laws of space as a function of time	- The physics of the falling bodies and of the motion of projectiles is close to the students' every day experience and gets them started asking questions about the interactions between science, technology and society
- Falling motion	- Motion along an inclined plane - Motion of falling bodies in a fluid - Motion of falling bodies in a vacuum	- The mathematics he used (Euclidean geometry and proportions) can be easily understood by the students of the first high school classes
- The parabolic motion of the projectile	- Principle of independence of simultaneous movements - Motion of the projectile with initial horizontal velocity, oblique downwards and upwards velocity - Calculation of range	

Tab. 3.2. Project sheet of knowledge reconstruction relating to the educational historical path.

Questions to the students (Problem posing)	**Activities and Methods (Problem solving)**
a. How could we measure time if we were Galileo?	a. Let us swing simultaneously two pendulum of different masses, with different amplitude or with wires of different lengths. It is observed that the period of oscillation depends only on the length of the wire (Galileo suggests a pendulum with a length of 4 times that of the other one, obtaining an oscillation time that is twice of the other). The fact that it does not depend on the amplitude of the pendulum oscillation during the motion allows us to use the period of oscillation as unit of time measurement (a pendulum length of 25 cm is chosen). To obtain then a continuous measurement of time it is necessary to construct a water clock that is calibrated using the pendulum.
b. Does the free fall of bodies in a vacuum depend on mass?	b. As Galileo did, we drop small balls of different masses first in oil, then in water, then in air by extrapolating their behaviour in a vacuum. We can see that the two balls have different fall times in oil. In water, the difference between them is smaller and in air it is much smaller. Galileo extrapolates that in the void (where in his time he cannot do the experiment) the two balls fall with the same speed. An in-depth analysis is planned: studying the falling motion of the two balls in different fluids using Tracker Video Software for Physics Education.
c. In free fall motion, how does the distance travelled depend on the time employed?	c. We slow the free fall movement by using a 6,66 meters long inclined plane so that we can study motion, just as Galileo did. A moving body starting from rest and falling on an inclined plane travels during equal intervals of time distances, which are related to each other as the odd numbers beginning with unity 1, 3, 5, 7. Considering the total spaces travelled, they are in the ratio of the squares of the times. We do two experiments: we fix the times that are marked by the pendulum and measure the spaces travelled; we fix the spaces that must be travelled and measure the corresponding time intervals using the water clock.
d. Is it possible to obtain a motion at constant speed in one direction when a body is not subject to forces?	d. We drop a little ball on an inclined plane, varying the height and thus its final speed; then the fall motion continues in air with a parabolic trajectory obtaining a uniform movement in the horizontal direction. If the initial velocity of the ball in the parabolic motion is horizontally directed, we can verify the directly proportional relationship between the range and the square root of the launch pad height, just as Galileo did.
e. What are the trajectories of a projectile launched in the horizontal and in the oblique direction?	e. We can measure the range of the ball at different heights of the impact point so as to reconstruct the trajectory and verify quantitatively, in the case that the initial velocity of the parabolic motion is horizontally directed, the quadratic dependence between the vertical distance and the horizontal one. We can attempt to design an experiment to verify that the trajectory is a parabola, even if a small ball is thrown downwards in an oblique direction.

Tab. 3.3. Project sheet about problem posing and problem solving.

Entry requirements	- Concepts of space and time - Scalar and vector quantities - Mass and weight - Friction - The moving material point and the trajectory - Reference systems
General skills	Students should be able to: - Understand the Nature of Science - Design an experiment with the available tools - Draw conclusions about physical laws - Know how to experimentally reject or confirm the mathematically formulated hypotheses - Understand historical physics texts - Know how to distinguish and refute two or more alternative hypotheses - Use modern technology for educational purposes - Work in a group for scientific purposes
Specific skills	- Perform time measurements - Identify the characteristics of the motion - Measure the travelled space as variation of position - Recognize a uniform motion quantitatively - Define average speed and instant speed correctly - Know how to measure the average speed of a motion - Know how to relate space, time and speed in a uniform motion - Know how to correctly define a uniformly accelerated motion - Interpret correctly the speed-time graph relative to a naturally accelerated motion - Know how to deduce the space travelled by the speed-time graph - Know how to demonstrate the equivalence of a naturally accelerated motion with a suitable uniform motion (mean speed theorem) - Know how the speed with which a body falls to the ground along an inclined plane does not depend on the inclination of the plane - Be able to hypothesize laws of linear or quadratic proportionality between the variables that characterize a motion - Know how to apply the principle of the composition of simultaneous movements - Know how to identify a parabolic motion - Know how to use the relations between the variables of motion both in the modern algebraic form and in the form of proportionality used by Galileo - Know how to calculate the range of a projectile - Know how to design and analyze complex experiments about motion

Tab. 3.4. Project sheet about entry requirements and skills

Classes involved	- Two first scientific high school classes - Two second scientific high school classes - Two third scientific high school classes - The overall number of pupils engaged in the educational experimentation is 127; a total of 90 students answered all four questionnaires. - Two other classes of students were engaged in a pre-experimentation year during which their main misconceptions were analysed
Duration	20h per class
Ongoing evaluation	Evaluation of interest levels, of answers to questionnaires on the scientific content and textual comprehension of the historical texts, of participation in group activities, both in the laboratory and inquiry-based learning activities, and in the proposed exercises
Entry/exit evaluation of the students	A skills-and-knowledge questionnaire and one about motivations and interests
Project analysis	By comparing the results of the entry questionnaires on students' skills and motivations and the exit questionnaires, the educational action carried out with the project will be causally analyzed
Methodology	Guided inquiry process is used but pupils can design variations of the experiments to be carried out as long as they are compatible with the aims of the project Group work: pupils participate in person to carry out the experiments Guided discussion for the identification and overcoming of misconceptions Brainstorming for solving problems posed during school lessons (mainly of experimental nature) Learning units organized with a predictive-exploration-comparison activity in which the reading of the historical texts written by Galileo is integrated with the conducting of experiments
Ethical approval	The headmaster of *Liceo Scientifico "F. Sbordone"* in Naples has released a positive ethical evaluation (there is no ethics committee in *Liceo Sbordone*)

Tab. 3.5. Further specification of the historical educational path

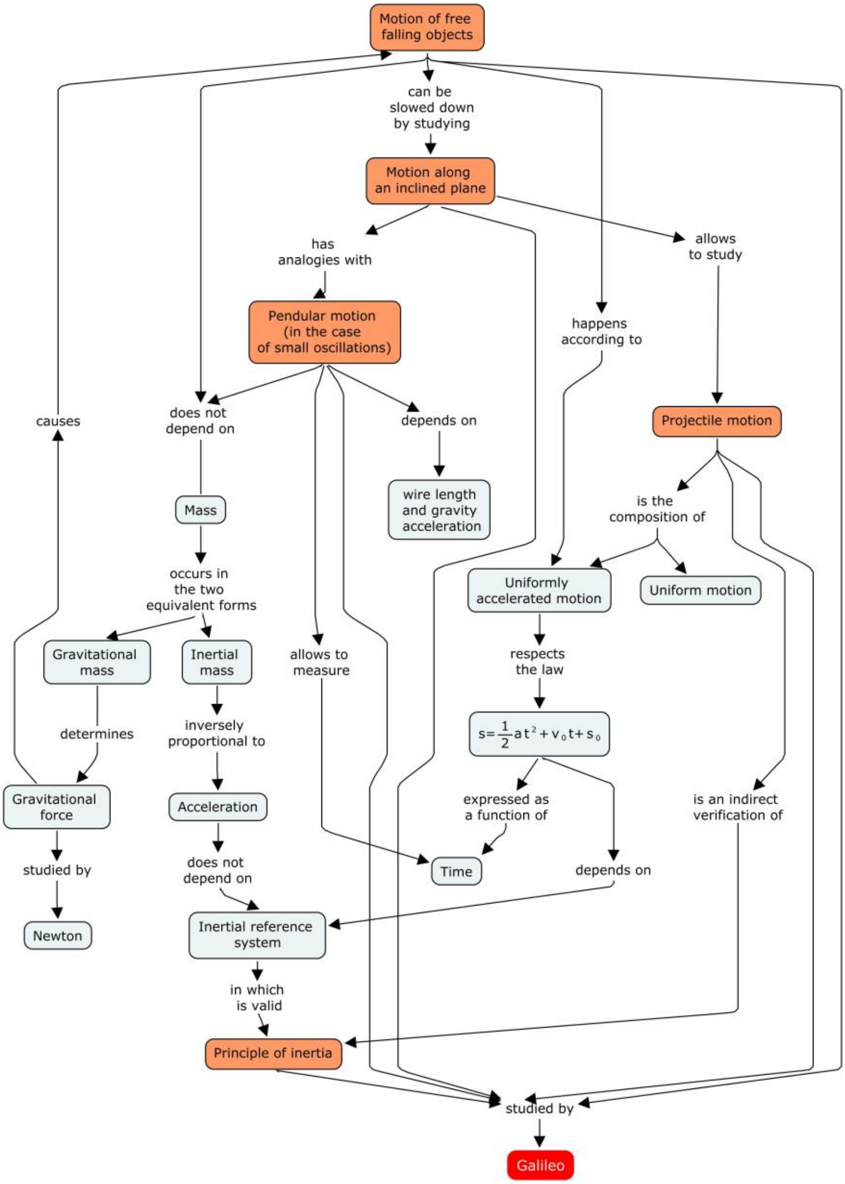

Fig. 3.3. The map of concepts that are presented in the paragraph Source: my own representation.

3.7 The stages of the historical educational path: history-of-physics topics in learning units

In this paragraph, the sequence of historical arguments will be shown in detail with reference to the original writings that make up the theoretical / experimental educational path on Galileo's physics. In fact, it should be emphasized that the study of his theoretical construction will also be addressed, together with his experiments, both in relation to free fall motion (naturally accelerated motion) and to parabolic motion.

The first part of the educational path is dedicated to the study of pendulum motion, from the first analyses of it by analogy with the motion of falling along arcs or chords of a circumference to the determination of the laws governing its motion. Also of interest are Galileo's considerations on the criticism of Aristotelian physics in the interpretation of pendulum motion. The learning module concludes with passages in which Galileo describes the use of the pendulum to measure time and introduces the construction of the water clock.

- Analogies and differences between free fall motion, motion along an inclined plane and pendulum motion (Galilei 1890–1909, vol. VIII pp. 128; Galilei 1914, p. 84).
- Experiment on pendulum's oscillation time independence from its weight: comparison of the oscillation times of two pendulums made with a lead ball and a cork ball (Galilei 1890–1909, vol. VIII, pp. 128-130; Galilei 1914, p. 84-85).
- Demonstration of the isochronism of the oscillations starting from the demonstration of the equality of the times of descent along the chords of a circumference with its inferior extreme at the lowest point of the circle. (Galilei 1890–1909, vol VIII p. 139; Galilei 1914, pp. 95-96).
- Comparison of the time of fall along an arc of a circle and along its chord. (Galilei's letter to Guidobaldo Del Monte dated 29 November 1602 in Galilei 1890–1909, vol. X, pp. 99; Galilei 1890–1909, vol VIII p. 139; Galilei 1914, pp. 95-96).
- Observation of the isochronism in the oscillation of the lamp of a church by Galileo (Galilei 1890–1909, vol VIII, p. 140; Galilei 1914, p. 97).
- Comparison of the oscillation times of two pendulums describing arcs of different amplitudes (Letter from Galileo to Guidobaldo Del Monte dated 29 November 1602, in Galilei 1890–1909, vol. X, pp. 99-100; Galilei 1890–1909, vol. VIII, pp. 129-130; Galilei 1914, p. 85-86).
- Impossibility of air action to push the pendulum (Galilei 1890–1909, vol. VIII, p. 140-141; Galilei 1914, p. 97).
- Experimental verification of the proportionality between the

oscillation times of the pendulum and the square root of the length of the wire (Galilei, 1890–1909, vol VIII p 139-140; Galilei 1914, pp. 96-97).

- Utilization of the pendulum and of the water clock to measure the time of descent along an inclined plane (Letter from Galileo to Giovan Battista Baliani dated August 1st 1639, in Galilei 1890–1909, vol. XIII, p. 76; Galilei 1890–1909, vol. VIII, p. 213; Galilei 1914, pp. 179).

The second part is dedicated to the motion of falling objects in a vacuum and fluids. In particular, we find the theoretical / experimental demonstrations followed by Galileo to prove that this motion in a vacuum does not depend on the mass of the body while in fluids it depends on the difference between the densities of the body and of the medium.

- Explanation for Aristotle's theory of direct proportionality between speed and weight of bodies falling in the same medium and for inverse proportionality between the speed of the body and the density of the medium with the consequence that the void cannot exist according to Aristotle (Galilei 1890-1909, vol. VIII, pp. 105-106; Galilei 1914, pp. 61-62; Aristotle *Physica* 215a.24-216a.21, *De caelo* 301b.)

- Refutation of Aristotle's theory by experimental proof of the fall of two bodies of different weight - a cannonball and a musket - from the top of a tower: they reach the ground with no more than the difference of a palm (Galilei 1890-1909, vol. VIII, pp. 106-107, 109-110; Galilei 1914, pp. 62, 64-65).

- Thought experiment on the fall of two bodies of different weight but of the same material - a larger stone and a smaller one - falling to the ground with the same speed (proof by contradiction) (Galilei 1890-1909, vol. VIII, pp. 107-109; Galilei 1914, pp. 62-64)

- Extrapolation of the experimental comparison of the speeds of two bodies of different specific gravity falling in various less and less resistant media until reaching their (limit) behaviour in the vacuum (Galilei 1890-1909, vol. VIII, pp. 116-117; Galilei 1914, pp. 70-72).

- Correct description (anticipating the conceptual use of the fundamental laws of Newton's dynamics) of the motion of falling bodies in viscous media: a first phase of accelerated motion and then - when the resistance of the medium is equal to the excess of the weight force of the body over that of the medium - a second one of uniform motion with terminal velocity of fall that is proportional to the difference between the density of the body and that of the medium (Galilei 1890-1909, vol. VIII, pp. 118-121; Galilei 1914, pp. 74-77).

The third part deals with uniform motion and uniformly accelerated motion, with demonstrations of the relationships between space and time. Finally, it includes the famous experiment of the inclined plane that proves that the motion of fall is a uniformly accelerated motion.

- ✓ Definition and theorems I, II, III, IV, V and VI (only statements) on proportionality laws of uniform motion in the case of movements with equal speed, with the same distance travelled, with equal time intervals or in the more general case in which all three quantities are unequal (Galilei 1890-1909, vol. VIII, pp. 191-196; Galilei 1914, pp. 154-160).
- ✓ Definition of naturally accelerated motion in which the speed starting from rest acquires equal increments after equal intervals of time (Galilei 1890-1909, vol. VIII, pp. 197-198; Galilei 1914, 161-162).
- ✓ Galileo's decision to give only a description of the acceleration of naturally falling bodies but no reasons, and possible explanations by Galileo's predecessors and contemporaries (Galilei 1890-1909, vol VIII, pp. 202; Galilei 1914, 166).
- ✓ Theorem I of naturally accelerated motion known as the mean speed theorem or as the Merton rule about equivalence between naturally accelerated motion and a particular uniform motion with its demonstration based on space as area under the speed-time graph, equivalence theorem between triangle and rectangle of Euclidean geometry, proportionality between speed and time in a naturally accelerated motion (Galilei 1890-1909, vol VIII, pp. 208-209 Galilei 1914, 173-174)
- ✓ Theorem II of naturally accelerated motion about the direct proportionality between space travelled and the squares of the times with its demonstration based on the mean speed theorem, on the proportionality between speed and time in a naturally accelerated motion and on the proportionality between spaces and the products of speed and time in uniform motion (Galilei 1890-1909, vol VIII, pp. 209-210; Galilei 1914, pp. 174-175).
- ✓ Corollary I known as law of odd numbers and its proof based on the proportionality between speed and time in a naturally accelerated motion, on the mean speed theorem and on the proportionality between space and speed in uniform motion, fixed the time taken (Galilei 1890-1909, vol VIII, pp. 210-212. Galilei 1914, pp. 175-177).
- ✓ Corollary II on the law of proportionality of the fall times with the square roots of the distances travelled. (Galilei 1890-1909, vol. VIII, pp. 214; Galilei 1914, p.180).
- ✓ Theorem III and its corollary on the relationships between final speeds (only the statement) and between the falling times from the

same height along differently inclined planes: the proof of the second part of the theorem and of the corollary is an interesting application of the mean theorem (Galilei 1890-1909, vol. VIII, pp. 215-219; Galilei 1914, pp. 184-187).

- ✓ Experimental study of the fall of a bronze ball along an inclined plane in which a very smooth channel covered with parchment was obtained to minimize friction; time was measured thanks to the water falling through a thin tube and then weighed with a precision balance. Galileo also discusses the experimental uncertainties that are not greater than a tenth of a second (Galileo 1890-1909, vol. VIII, pp. 212-213; Galilei 1914, pp. 178-179).

The fourth part of the educational path is devoted to the motion of the projectile and the principle of inertia (including discussion on circular inertia) with its indirect experimental verification in connection with parabolic motion.

- Galilean relativity principle (Galileo 1890-1909, vol. VII, pp. 212-214; Galileo 1953-1967, 2001 pp. 216-218).
- Galileo's formulation of the inertia principle (Galileo 1890-1909, vol. VII, pp. 171-173, Galileo 1953-1967, 2001 pp. 169-173; Galilei 1890-1909, VIII, p. 243; Galilei 1914, p. 215).
- Conservation of the circular motion of the Earth's rotation of a ball falling from the top of a tower, of an artillery shot fired vertically and of a ball dropped on the Earth from the Moon (Galileo 1890-1909, vol. VII, pp. 170, 190-192, 201-202, 259-260, Galileo 1953-1967, 2001 pp. 168, 191-193, 203-204, 270-271).
- Theorem I (Proposition I) of parabolic motion about the composition of a naturally accelerated vertical motion and of a uniform horizontal motion (Galileo 1890-1909, vol. VIII, pp. 269, 272-273; Galilei 1914, pp. 245, 248-250).
- Problem (Proposition V) of the search of the height from which a particle must fall along a launch pad ("sublimity") determining a specified amplitude of a semi-parabolic trajectory (statement only) (Galileo 1890-1909, vol. VIII, p. 293; Galilei 1914, p. 272)
- Corollary about the relationship (as mean proportionality) between amplitude, maximum height of the semi-parabola and "sublimity" (Galileo 1890-1909, vol. VIII, p. 294; Galilei 1914, p. 273).
- Problem (Proposition VI) of finding the theoretical amplitude of the semi-parabola in terms of its maximum height and of its "sublimity" (Galileo 1890-1909, vol. VIII, p. 294; Galilei 1914, p. 273).
- Experimental verification of the direct proportionality between

the range of a ball thrown horizontally after falling from a launch pad and the square root of the fall height (Galilei 1602-1637, f. 116v) and of the inertia principle in the horizontal direction.

- Experimental reconstruction of the projectile's parabolic trajectory by measuring the projectile's range along planes at different heights (Galilei 1602-1637, ff. 81r, 117r).
- Elevation angle for maximum projectile range (Galileo 1890-1909, vol. VIII, p. 306; Galilei 1914, p. 286).
- Ethical implications for human beings of Tartaglia's discovery about the angle of maximum range of the projectile (Tartaglia 1537, 3rv-4rv; Pisano and Capecchi 2016, pp. 55-56).

3.8 A new educational method based on inquiry and history-of-physics analysis

At *Liceo scientifico "F. Sbordone"* in Naples, a guided inquiry process was used in the implementation of a new educational method that includes the replication of experiments that were relevant for the development of physics.

Lessons are generally organized with a predictive-exploration-comparison activity in which the reading of the historical texts written by Galileo is integrated with the conducting of experiments.

This new teaching method is a combination of inquiry − in which students are engaged in the design of experiments, their critical analysis and development of models (Cfr. Rutherford 1964) − and the analysis of historical sources (Cfr. Pisano, Agassi et al. 2017; Pisano and Capecchi 2016; Pisano and Cioci 2020a) that were submitted to the students in carrying out the experimental activity.[49]

This method consists in four main parts: 1) physics open question brainstorming (usually from daily common sense) were submitted to students, 2) images and historical documents including problem solving, 3) experiments & data, 4) data sheet reporting and analysing, and comparison, with formulation of physics' properties and laws.

There is a form of *open* inquiry-based learning in which students are left free and encouraged to formulate the problems themselves in their research. At *Liceo Sbordone*, a *guided* inquiry based learning method was used

[49] "We define inquiry as engaging students in the intentional process of diagnosing problems, critiquing experiments, distinguishing alternatives, planning investigations, revising views, researching conjectures, searching for information, constructing models, debating with peers, communicating to diverse audiences, and forming coherent arguments" (Linn, Clark and Slotta 2003). Rutherford (1964) considered inquiry as part of the science content to learn. He argued that scientific concepts are to be understood in the context of how they were discovered so that future inquiries could occur analogously. So he recommended that science teachers should have a background in the history and philosophy of science.

instead, in which the questions were formulated by the teacher, but in some cases it was the students themselves who suggested new experiments and posed new problems (e.g. they proposed repeating the fall experiments along the inclined plane with balls of different radius and therefore different mass in order to study the dependence of the fall time on the mass in free-falling motion).

In this paragraph I report the special structure I used during the lessons in order to better clarify the method I followed.

In the first unit of the educational path, dedicated to the study of pendulum motion, before proposing to the students the reading of the passages from Galileo's writing indicated in the previous paragraph, I asked them the following questions in order to then provoke a discussion.

Activity n. 1. Problem posing: How could we measure time if we were Galileo?

Answer the following questions.

If you were Galileo ...

1. How do you expect the pendulum to swing?

....

....

2. How does the period vary with varying swing amplitude?

....

....

3. How does the period vary with the mass?

....

4. How does the period vary with the length of the string?

....

....

5. How could we use the pendulum for time measurements and why?

....

....

6. How could we use the jerry can with tap for time measurements?

....

....

7. Design experiments to refute or confirm your hypotheses

....

....

8. Compare the results obtained with those expected

....

....

Faced with the difficulty of making precise measurements of the period of the pendulum if they were in Galileo's time, the students are invited to read the texts written by Galileo where, among other subjects, it is suggested to swing two pendulums simultaneously in which only one of their characteristics is varied. The following questions follow.

9. What are the experiments that can be deduced from reading the text?

....

....

10. Based on these experiments, what can we deduce about the pendulum swing period?

....

....

11.Why can we say that there is an analogy between pendulum motion and motion along an inclined plane?

....

....

12. What is the main difference in taking time measurements using the pendulum or the water clock?

....

....

In the second unit of the educational path, dedicated to Galileo's proof by contradiction that bodies of different weight but of the same material fall to the ground with the same velocity.

Activity n. 2. Problem posing: Does the free fall time of bodies in a vacuum depend on their mass?

Answer the following questions.

1. What was Aristotle's position about the fall of the bodies?

....

....

2. What does Galileo argues through the words of Salviati?

....

....

3. What was Simplicius's position?

....

....

4. Do you think that Galileo's arguments can be considered a rigorous demonstration?

....

5. If yes, why? What kind of demonstration is it?

....

....

6. If not, why? What do you think could happen?

....

....

These questions are followed by the reading of Galileo's text concerning the extrapolation of the experimental comparison of the speeds of two bodies of different specific gravity falling in various less and less resistant media until reaching their (limit) behaviour in the vacuum

7. What does Galileo argues through the words of Salviati?

....

....

8. What was Simplicius's position?

....

....

9. Do you think that Galileo's arguments can be considered a rigorous demonstration?

....

....

10. If yes, why? What kind of demonstration is it?

....

....

11. If not, why? What do you think could happen?

....

....

12. In addition to the two different types of demonstration, what is the fundamental difference in the assumptions/conditions under which the two *thought experiments* are performed?

....

....

....

In the third unit of the educational path, dedicated to Galileo's theorems about naturally accelerated motion, I proposed reading Galileo's writings to the students and then asked them the following questions to verify their understanding.

Activity n. 3. Problem posing: How does the distance travelled depend on the time employed in the free fall motion?

Answer the following questions.

1. Complete the statement of the following theorem

Theorem I, Proposition I

The space travelled by a body that moves with uniformly accelerated motion starting from rest is equal to the space travelled at the same time by a body that moves with a uniform motion with speed that …………………………..

....

....

2. Repeat the construction indicated by Galileo in the proof of the theorem.
3. What does the AB segment represent?

....

4. What does the EB segment represent?

....

5. What does the area of the AEB triangle represent?

....

6. What does the area of the AGFB rectangle represent?

....

7. Which theorem of Euclidean geometry can we apply to prove the theorem?

....
8. Write the statement of the following theorem
Theorem II, Proposition II

....

....

9. To rebuild the chain of proportions written by Galileo in the demonstration, he used
- the theorem .... which states that

....

....
- the compound proportionality between the spaces travelled and the product between the speeds and the times in a uniform motion (which corresponds to the law s = ...)
- the proportionality between the speeds and .... in a uniformly accelerated motion starting from rest.
10. Write the statement of the following
Corollary I

....

....

11. Repeat the construction indicated by Galileo in the proof of the corollary
12. In this picture, what do the AC, CI, IO segments represent?

....

13. What do the BC, IF, PO segments represent?

....

14. What do the EC, NI, RO segments represent?

....

15. What do the areas of the rectangles with base EC and height AC, with base NI and height CI, with base RO and height IO represent?

....

16. What do the areas of the trapezes CBFI and IFPO represent?

....

17. If we choose the area of the ADEC rectangle as a unit of measure, by what kind of numbers can we express these areas?

....

18. What do AC, AI and AO heights represent?

....

19. What do the areas of the triangles ABC, AFI and APO represent?

....

20. What relationship can you find between the bases BC, FI, PO and the heights AC, AI, AO?

....

21. What relationship can you then identify between the quantities represented by the areas of the triangles ABC, AFI and APO and those represented by the heights AC, AI and AO?

....

22. Using the theory of proportions, express the relation illustrated in the corollary in modern mathematical language.
Corollary II

....

These questions are followed by the reading of Galileo's text concerning the procedure used by Galileo to determine the experimental relationship between the space travelled and the time taken for a ball that rolls along a 6.66 meters long inclined plane.
23. Wanting to repeat the experiments made by Galileo, in your opinion how could we concretely, fixing the lengths to be travelled, measuring the times used?
....

....

24. How could we measure the spaces travelled at fixed times?

....

....

25. What do you think we will obtain by repeating the measurements of the spaces we have travelled after equal time intervals?

....

....

26. How do you think the spaces travelled vary when the times are doubled? And when times triple?
....

27. Compare the results obtained with your forecasts.

....

In the fourth unit of the educational path, dedicated to the parabolic motion, before reading Galileo’s writings, I proposed to students the following questions.

Activity n. 4. Is it possible to obtain a motion at constant speed when a body is not subject to forces? What are the trajectories of a projectile launched in the horizontal and in the oblique direction?

We drop a ball from a height h along an inclined plane at the end of which there is a deflector that changes the direction of velocity from oblique to horizontal. Then we study the motion of the ball falling from height H to ground.

Answer the following questions.

1. What kind of motion do you foresee?

....

....

2. How is the trajectory?

....

....

3. How much is the range D of the ball in the horizontal direction?

....

....

4. How much is the ball's drop time?

....

....

5. What would change if the ball was thrown in an oblique direction downwards or upwards?

....

....

6. Draw the trajectory that you expect in these other two cases.

....

....

7. How would you design experiments to verify your statements?

....

....

After reading passage by Galileo about parabolic motion theorems, answer the following questions.
8. What do you understand from reading the text?

....

....

9. What can you deduce from Galileo's writings on questions 2, 3, 4 above?

....

....

10. How does Galileo demonstrate that the trajectory followed by the projectile is a parabola?

....

....

9. On what simple considerations are his arguments based?

....

....

11. How does the projectile move in the horizontal direction?

....

....

12. What do you think is the experimental verification of the laws that characterize the motion of the projectile for the principle of inertia?

....

....

In order to visualize the whole trajectories (not just the impact point with the ground), Galileo recorded the intersections of these with multiple parallel planes placed at different heights. In his laboratory notes on the mechanics, sheet 114v, we see the ball, which, coming out of the inclined plane, follows a curved trajectory until it strikes with a plane surface. Galileo recorded the impact points with the utmost precision possible for different planes in order

to obtain a set of points in the space whose interpolation assumes the shape of a curve.
13. Repeat the experiment performed by Galileo. If the initial velocity of the projectile were horizontal, how could you make sure that the points are on parabolic trajectories?

....

....

14. If you arrange the horizontal planes according to vertical distances that are in proportion to each other like the odd numbers starting from one, how do you expect the intersections of the ball in the horizontal direction are and why?

....

....

15.What fundamental principle of physics have we verified?

....

16. Galileo also examined the behavior of a small ball which, after falling down an inclined plane, rather than encountering a deflector at the end of its stroke, could continue downward in an oblique direction. Determine the equation of the trajectories followed by the ball in air as the output (oblique) velocities from the inclined plane vary.

....

....

17. In your opinion, how could you prove experimentally, note the inclination of the plane, that these curves are also parables?

....

....

3.9 Summary of topics studied: remedial and in-depth study

The experimental activities illustrated in the previous paragraph are generally concluded with the formulation of physics' properties and laws, that have been summarised in support, remedial or in-depth worksheets. This section illustrates the most important topics as they were theoretically formalised in the classrooms.

1. What does pendulum motion depend on?

Viviani, Galileo's disciple and biographer, recounts in his 'Racconto istorico' (1654) that the scientist discovered the law of isochronism of the (small) oscillations of the pendulum from observing "the motion of a lamp, while it was one day in Pisa Cathedral; and making very exact experiments, he ascertained the equality of its vibrations".

A pendulum is a ball (or other object of negligible size) that is suspended by an inextensible wire (i.e. of length l that does not change during motion) from a fixed point. We represent the ball as a material point moving without friction and assume that the mass of the wire is negligible compared to that of the ball.

If we move the ball from the vertical and drop it, the pendulum swings along an arc of circumference. To study the properties of motion, it is useful to consider the forces acting on the mass m. These are the weight force $\vec{P} = m\,\vec{g}$ and the tension of the wire $\vec{T}$.

We can begin to observe that because the two forces lie in the same plane, the pendulum motion will always occur in the same plane. This is the law of invariance of the plane of pendulum's oscillation that allowed Foucault in 1851 in Paris to verify the motion of the Earth's rotation.

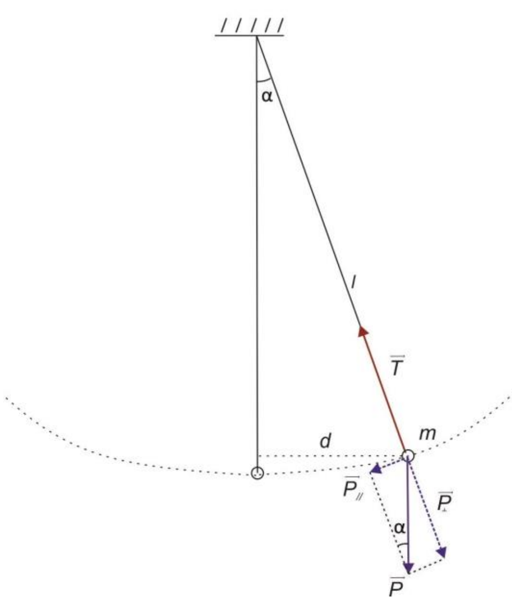

Fig. 3.4 Forces acting on a pendulum. Source: my own representation.

Let us now decompose the weight force into its two component vectors: : $\vec{P}_{//}$ in the direction tangent to the circumference i.e. parallel to the movement, and $\vec{P}_{\perp}$ in the direction tangent to the wire i.e. perpendicular to the movement.

The effect of the latter is balanced by the tension of the thread $\vec{T}$ but the two forces do not cancel each other out: their resultant gives the centripetal force that keeps the ball on its non-rectilinear (circular) trajectory.

The equations of motion in the two directions are

$$\vec{T} + \vec{P}_{\perp} = m\,\vec{a}_c \quad \text{and} \quad \vec{P}_{//} = m\vec{a}_t$$

From the similarity of the two triangles, one of which has as sides the wire of length *l* and the displacement in the horizontal direction *d* and the other the weight force and its parallel component, we can write

$$P_{//} : P = d : l \qquad \text{i.e.} \qquad m\,a_t : mg = d : l$$

and thus

$$a_t = \frac{g}{l}\,d$$

For small oscillations - with respect to the length of the pendulum - i.e. when the angle of oscillation does not exceed 10°, the deflection d can approximate the displacement with respect to the vertical along the arc of the circle and can be written

$$\vec{a}_t = -\frac{g}{l}\vec{s}$$

When this condition is met, the ball undergoes an elastic force, i.e. a force that, like that exerted by a spring, is directly proportional to the displacement $\vec{s}$ from the equilibrium position and is oriented in the opposite direction to this displacement.

As in the case of a body subject to an elastic force, pendulum motion is therefore a harmonic motion whose period is related to the coefficient of proportionality between acceleration and displacement $\omega^2 = \frac{g}{l}$ by the relation

$$T_0 = \frac{2\,\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

In this case, the recall force to which the ball is subjected is the component vector $\vec{P}_{//}$ of the force-weight.

The period of the pendulum, the time required for a complete oscillation, depends on l and g, and not on the angle of oscillation α (provided it is small) or the distance d from the central position. Thus, provided the oscillations of a pendulum are small, its period does not depend on the amplitude of the oscillation.

We have assumed that friction with air is negligible. In reality, due to friction, the oscillations of a pendulum become progressively less ample as time passes. Nevertheless, the duration of each oscillation, which is independent of amplitude, does not change. This property constitutes the isochronism of the pendulum's small oscillations.

Exercises: How long must a pendulum be to have a period of 1 s? How can we estimate the acceleration of gravity by measuring the pendulum's period of oscillation?

How does a falling body move in a viscous fluid?

Galileo wrote:

the medium, although it is a yielding and quiet fluid, opposes itself with resistance that is now less and now greater and greater, according to the fact that it must open slowly or quickly to give transit to the moving body; which, as I have said, because its nature is continually accelerating, consequently encounters continually greater resistance in the medium, and therefore retardation and diminution in the acquisition of new degrees of velocity, so that finally the velocity reaches such a sign, and the resistance of the medium to such a magnitude, that, balancing each other, they lift the most acceleration, and reduce the moving body in an equable and uniform motion, in which it then continues to maintain itself (Galileo's *Discorsi su due nuove scienze*)

When a body falls in a viscous fluid, it is primarily subject to the force of gravity. Unlike in a vacuum, however, in fluids the body is also subject to a velocity-dependent resistance due to the medium.

We assume that the resistance of the medium is proportional to the velocity and in the opposite direction (viscous fluid).

$$\vec{R} = -k\vec{v}$$

Let us assume that the initial velocity is zero. The body begins to fall downwards in accelerated motion

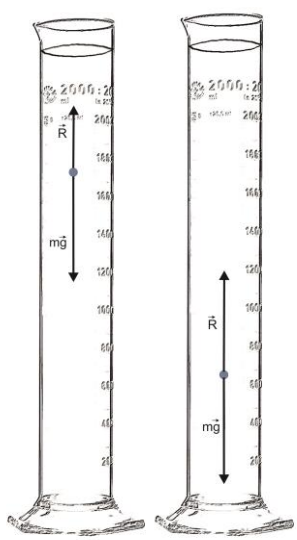

Fig. 3.5 The forces acting on a body falling in a viscous fluid during the two phases of motion. Source: my own representation.

As the speed increases, the resistance of the medium against the force of gravity also increases; the acceleration of the falling motion decreases.

If we write Newton's second law, in fact,

$$m\vec{a} = m\vec{g} - k\vec{v}$$

We can derive acceleration

$$\vec{a} = \frac{m\vec{g} - k\vec{v}}{m}$$

In this first phase, motion is accelerated but not uniformly accelerated.

At a certain point, the resistance of the medium becomes equal to the force of gravity. *In this second phase, the body continues to fall in uniform motion because the resultant of the applied forces is zero.*

Assuming that the two forces are equal, we can calculate the terminal velocity of the fall (limit velocity).

$$m\vec{g} = k\vec{v}$$

from which we obtain

$$\vec{v}_l = \frac{m\,\vec{g}}{k}$$

The constant k depends on the shape of the body and the nature of the medium. For a homogeneous sphere of radius R, k can be written $k = 6\pi R\eta$ where η is the viscosity of the medium.

The mass m of the body can be written as the product of the volume of the body V_c by the density of the body d_c: $m = V_c\,d_c$.

If the density of the body is comparable to that of the medium in which it is immersed, the so-called Archimedes thrust must also be taken into account: every body immersed in a fluid (liquid or gas) receives a upward thrust equal in intensity to the weight of the displaced fluid (of equal volume to that of the immersed part of the body).

$$S_A = V_c\,d_f\,g$$

Where the product of the volume of the body V_c by the density of the fluid d_f is exactly equal to the mass of the fluid displaced.

In this case Newton's second law becomes

$$ma = mg - S_A - kv = V_c(\,d_c - d_f)\,g - kv$$

From which we can derive the terminal speed in the case of zero acceleration:

$$v_l = \frac{V_c(\,d_c - d_f)\,g}{k}$$

Note how the terminal speed is not inversely proportional to the density of the fluid as assumed by Aristotle, but in this model formulated by Galileo it is proportional to the difference between the density of the body and the density of the fluid.

Galileo did not examine the effect due to the shape and size of the body, but if we consider the ratio of the terminal velocities of fall v_1 and v_2 of two bodies of the same shape and size (such as two spheres of equal radius) in the same fluid, the parameter k can be simplified, together with V_c and g, and results in

$$\frac{v_1}{v_2} = \frac{d_{c1} - d_f}{d_{c2} - d_f}$$

Terminal velocities of the balls are proportional to the differences between the density of the body and the density of the medium, in the case that resistance of medium is proportional to velocity. If it is proportional to the square of velocity, we can calculate that the terminal falling velocities result proportional to the squared roots of the differences between densities

$$\frac{v_1}{v_2} = \frac{\sqrt{d_{c1} - d_f}}{\sqrt{d_{c2} - d_f}}$$

i.e. that the terminal velocity is proportional to the square root of the difference between the density of the body and the density of the medium.

Exercise:

What kind of movement do you expect if the body starts to fall with a velocity greater than the terminal velocity in that fluid? Draw a qualitative graph of velocity as a function of time.

How does the speed change along an inclined plane? How does it depend on inclination?

We now calculate the final velocity along an inclined plane of length *l* and verify that it only depends on (i.e. is proportional to) the square root of the height *h* of the fall. Let us write down the equations of this naturally accelerated motion. While space is proportional to the square of time, velocity is proportional to time.

$$\begin{cases} s = \frac{1}{2}\, a\, t^2 \\ v = a\, t \end{cases} \text{ where acceleration } a = g\,\frac{h}{l}$$

We assume that the space travelled along the inclined plane is equal to *l* and derive the time from the first equation and then substitute it into the velocity equation.

$$l = \frac{1}{2}\, a\, t^2 = \frac{1}{2}\, g\,\frac{h}{l}\, t^2 \;\Rightarrow\; t = l\sqrt{\frac{2}{gh}} \text{ and } v = a\, t = g\,\frac{h}{l}.\, l\sqrt{\frac{2}{gh}} = \sqrt{2gh}$$

About the motion along an inclined plane we can also consider this Galileo's theorem*:*

> The final velocities assumed by a body sliding frictionlessly along different inclined planes from a standing position are the same regardless of the inclinations of the planes, provided the heights at which the body falls are equal (Galileo's *Discorsi su due nuove scienze*).

Galileo does not prove this proposition but assumes it to be true on the basis of some of his observations (which today we would say are energy-related): if we drop a ball connected to a string, it rises to the same height after an oscillation (even if we were to follow it around in circular arcs with different centres and radii). Similarly, if we drop (so the initial velocity is zero) a ball along an inclined plane and make it rise again along well-smoothed planes with different inclinations, it always rises to the same height. The result is different if we want to evaluate the *fall time*. In fact, if we apply the mean speed theorem, considering Galileo's hypothesis of the equality of the final velocities to be true, then the mean speeds will also be equal and the travel time along the planes with greater inclination will be longer because the greater the space to be covered at the same speed.

How does the motion of the projectile occur?

Galileo's forerunners believed that the motion of the projectile would take place in two phases: in the first, due to the effect of the momentum (modern momentum equal to mass times velocity) received from the force, it would proceed in the straight direction of launch until it had exhausted its thrust, while in the second phase, gravity would make its effect felt by producing a downward vertical motion. Tartaglia himself, who was the first to identify the launch angle of 45° for maximum throw, considered the two above-mentioned phases, but also hypothesised an intermediate circular phase.

The great innovation Galileo brought was to consider the two effects, accelerated motion in the direction of gravity and uniform motion in the perpendicular direction, compounded throughout the motion.

We can prove Galileo's theorem that *The curve that is described by the body during motion is a parabola* with considerations of proportionality. Since the displacement in the vertical direction - being an accelerated motion from rest - is proportional to the square of time, while the displacement in the horizontal direction - being a uniform motion - is proportional to time, it follows that the displacement in the vertical direction is proportional to the square of the displacement in the horizontal direction.

In formulas, in modern notation, it can be written

$$\begin{cases} x = v_0\, t \\ y = \frac{1}{2}\, g\, t^2 \end{cases} \Rightarrow y = \frac{1}{2}\, \frac{g}{v_0^2} x^2$$

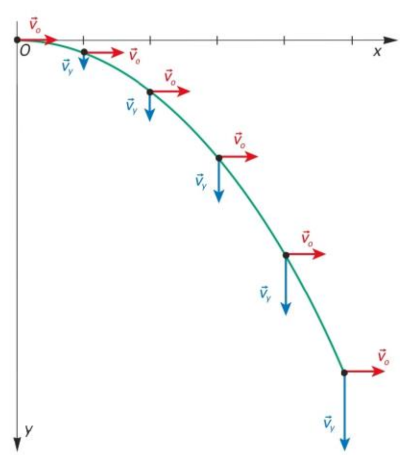

Fig. 3.6 The velocity components in the horizontal and vertical direction of a falling body after being thrown in the horizontal direction. Source: my own representation.

In the representation of the velocities of a free-falling body thrown in the horizontal direction, it can be seen that the velocity components in the horizontal direction remain unchanged while the component in the vertical direction increases.

$$\begin{cases} v_x = v_0 \\ v_y = g\, t \end{cases}$$

Epilogue III

In this chapter, I analysed the problem of teaching scholarly knowledge in the classroom (the didactic transposition) and in particular the way of reconstructing contents to be taught in high school starting with the consideration of the initial conceptions of students (the Model of Educational Reconstruction), taking into account the contribution of the history of physics to learners' conceptual change and the special role played in education by the replication of experiments that were historically relevant for the development of physics.

Following the Educational Reconstruction Model, I created a design sheet outline that takes into account the structured academic knowledge, the elementary knowledge accessible to students and the logical organisation of teaching content in a way that makes sense to students. I used it to design an original interdisciplinary (Physics / Mathematics / History of Physics) educational path for students, with both theoretical and experimental activities, characterised by an organic series of experiments derived from Galileo's writings and laboratory notes. I detailed all of Galileo's writings, which I used to facilitate the repetition of the course by other physics teachers or researchers in the history of physics and physics education. I created a new teaching method – as a combination of inquiry-based learning and the analysis of historical sources. Indeed, a guided inquiry process is used: lessons are generally organized with a predictive-exploration-comparison activity in which the reading and discussion of the historical texts written by Galileo are integrated with the conducting of experiments. I applied this method, preparing guided inquiry sheets for each teaching unit I designed together with support sheets for remedial activity.

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CHAPTER IV

The replication method at the *Liceo Scientifico "F. Sbordone"* in Naples. Results of experiments and discussion

Prologue IV

Several researchers have performed, with different purposes, the experiments based on Galileo's writings and laboratory notes. Settle (1961) in his "An Experiment in the History of Science" proved that the experiment of the inclined plane described in *Discorsi* was technically feasible for Galileo. Riess, Heering and Nawrath (2005) reconstructed Galileo's inclined plane experiments for teaching purposes. Vergara Caffarelli (2009) has rebuilt in university a considerable number of Galilean experiments for almost fifty years. Cerreta (2014) reconstructed some experiments on parabolic motion in the form of exhibits suitable for a science centre.

At the *Liceo Scientifico Sbordone* in Naples, instead, I identified a comprehensive educational path about Galileo's physics and his experiments that was implemented within the curriculum. Lessons were generally conducting of experiments.

Most data reported in the next sections were gathered by students under my supervision. Whenever possible, the experimental work was divided among the students. In some cases, for safety reasons, the teacher made the measurements (for example, to drop the balls into the various viscous fluids). For the measurement of the fall time along the inclined plane, the teacher began to make the measurements using the suggestions obtained from the educational research literature (Riess, Heering and Nawrath 2005), to drop the ball with one hand and start the water clock or the pendulum with the other hand at the same time to reduce measurement uncertainties. Then the pupils continued, several of whom learned the measurement technique so well that they also showed it to the public during the orientation open days. The data were enriched with those acquired in the other classes involved in the project in order to be sufficiently numerous. The physical-mathematical and statistical analysis was started by the pupils under the guidance of the teacher that perfected it, however the pupils had full awareness of the steps taken and the results achieved. The results achieved in all the experiments were remarkable because they confirmed those performed by Galileo and in some cases were original in the way they were performed (procedure and/or instruments used) because although the phenomena had been described by Galileo - think for example of falling bodies in fluids - the methodology used was not explicitly described.

4.1 Galileo’s measure of time: pendulum and water clock

Set-up of the experiment and its analysis. The students were required to analyse pendulum motion as if they were in Galileo's time and therefore unable to use a chronometer. By comparing Galileo's writings, they swung two pendulums simultaneously, changing one at a time the factors that could influence the pendulum swing time: the pendulum mass, the amplitude of the pendulum oscillations (however 'small'), and the length of the wire. It was observed that the period of one oscillation depends only on the length of the wire. The fact that it does not depend on the amplitude of the pendulum's oscillation during movement allows us to use the period of oscillation as a unit of time (a pendulum length of 25 cm is chosen because it corresponds to a period of 1 s).

A final exploration involved analysing the mathematical relationship between the period of the simple pendulum and the length of the wire. As Galileo suggested, students, divided into small groups under my supervision, compared the motion of two pendulums, the first 4 times longer than the second. They obtained an oscillation time of the second that is twice as long as the first. Then, for a pendulum 9 times longer than the other, they measured a period three times greater. Thus, they discovered that the times are proportional to the square roots of the lengths of the wires.

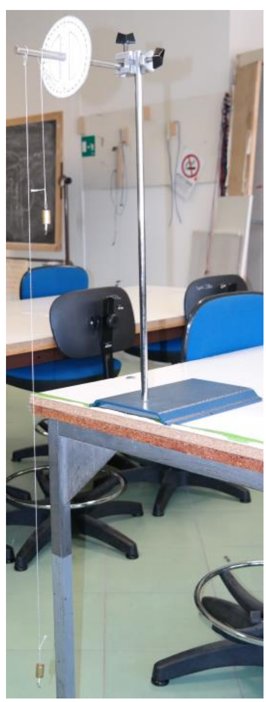

Fig. 4.1. A comparison of the oscillation time of two pendulums, one 4 times longer than the other. Source: my photo of the experiment.

A pendulum can only take measurements at predetermined times. In order to obtain continuous time measurements, we constructed a water clock consisting of a 30 l capacity tank with a tap, combined with a 500 ml measuring cylinder graduated to 5 ml. The water collected in the cylinder was weighed with a balance accurate to 1 g. As the flow rate of the water depends on the height of the liquid in the canister, this height was varied as little as possible by putting water back into the tank after each measurement. In order to calibrate the water clock and verify the maintenance of a constant flow from the tap, the 25 cm long pendulum was used. We carefully opened the tap at maximum flow each time. In this way, we made accurate measurements of the flow rate and found that it was 50 ml (or 50 g) per second.

Fig. 4.2. Our water clock had to keep the height of the liquid in the tank steady and the tap had to be fully open to achieve a constant flow of water over time. Source: my photos of the experiment.

4.2 Free fall motion. The analysis of falling motion in fluids

Set-up of the experiment and its analysis. We dropped small balls of different masses and of the same shape first in glycerine, then in olive oil, then in water, then in air and we extrapolated their behaviour in a vacuum. We noted that the differences between fall times were smaller and smaller as the resistance that fluids oppose to the passage of the body decreased (fig. 4.3). Students could conclude, like Galileo, that the free fall of bodies of different masses in the void should not show any difference.

Fig. 4.3 The fall of two little balls in glycerine, in oil, in water and in air. Source: my experiment images.

As a further study suggested by Galileo's writings,[50] we explored the falling motion of two little balls, one of steel and one of glass, in different fluids. By using Tracker software, a video analysis and modeling tool, and marking the positions of the two little balls (d = 1,5 cm) every 4 cs, we obtained the space-time graphs from which we could calculate the final velocities, after identifying the region where the graph is linear, as the ratio between the space travelled and the time taken (see figures and tables of the next three pages).

[50] In *Discorsi*, Galileo describes the results achieved but not how he actually carries out the experiments. Vergara Caffarelli, on the basis of other writings by Galileo (Galileo 1890-1909, vol. VII, *Dialogo sopra i due massimi sistemi*, p. 275; vol. I, *De motu*, p. 264-265; vol. IV, *Discorso intorno alle cose che stanno in su l'acqua*, p. 83, 89) hypothesizes that these were performed by dropping wax balls mixed with other materials in water and replicates them in his laboratory (Vergara Caffarelli 2009, pp. 35-59).

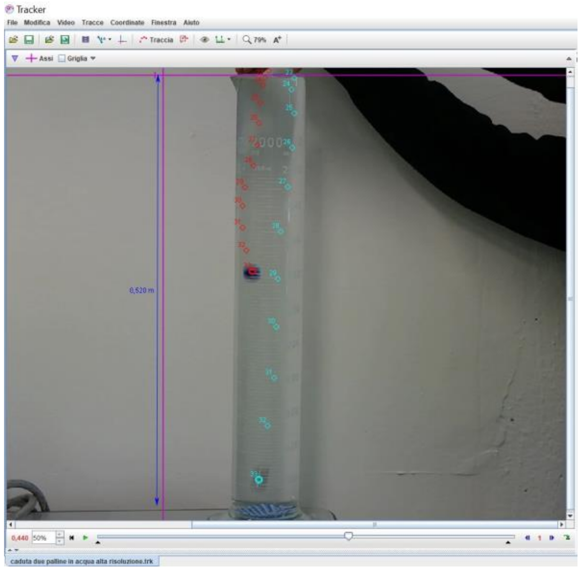

Fig. 4.4. An example of using Tracker software to calculate the terminal speed. The position of the little ball was marked every 4 cs. Source: my experiments image.

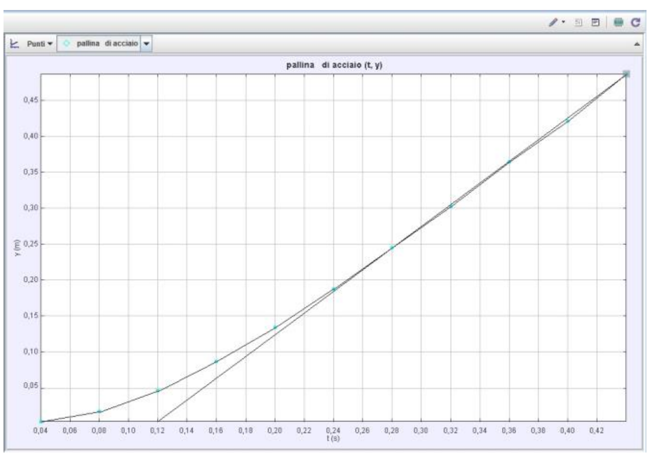

Fig. 4.5 An example of using Tracker software to calculate the terminal speed. Note the final time interval during which the motion is uniform. Source: my experiments image.

Velocity calculation in water	Linearity range		
	Time (s)	Space (m)	Terminal fall velocity (m/s)
Steel ball	0.44	0.510	1.5
	0.28	0.268	
Glass ball	0.80	0.502	0.69
	0.56	0.336	

Tab. 4.1. Calculation of terminal fall velocities in the linearity range of the space-time graph for water. Relative uncertainties have been propagated. The absolute uncertainties affect the second significant digits.

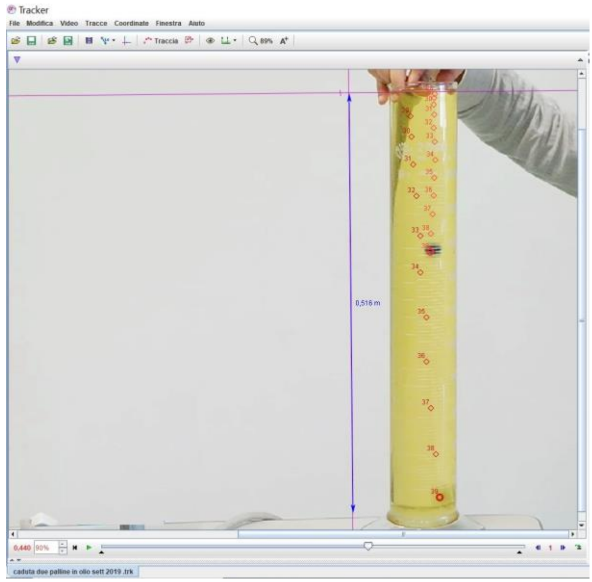

Fig. 4.6. An example of using Tracker software to calculate the terminal speed. The position of the little ball was marked every 4 cs. Source: my experiment image.

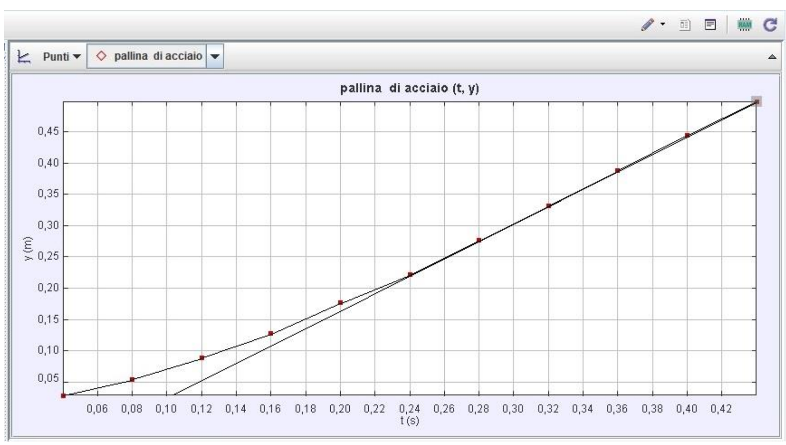

Fig. 4.7. An example of using Tracker software to calculate the terminal speed. Note the final time interval during which the motion is uniform. Source: my experiment image.

Velocity calculation in olive oil	Linearity range		
	Time (s)	Space (m)	Terminal fall velocity (m/s)
Steel ball	0.44	0.498	1.4
	0.24	0.221	
Glass ball	0.92	0.484	0.58
	0.16	0.044	

Tab. 4.2. Calculation of terminal fall velocities in the linearity range of the space-time graph for olive oil. Relative uncertainties have been propagated. The absolute uncertainties affect the second significant digits.

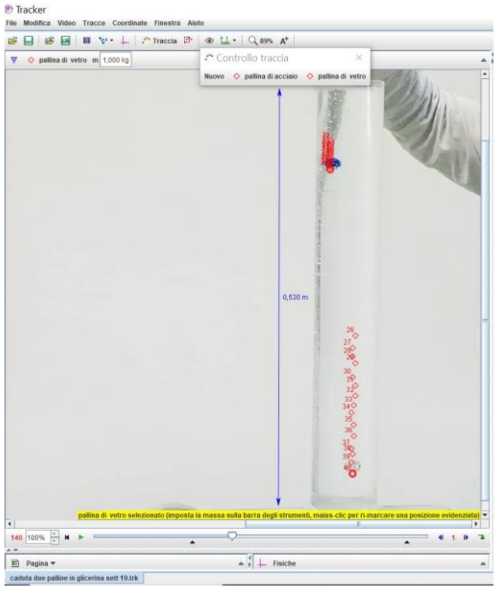

Fig. 4.8. An example of using Tracker software to calculate the terminal speed. The position of the little ball was marked every 4 cs. Source: my experiment image.

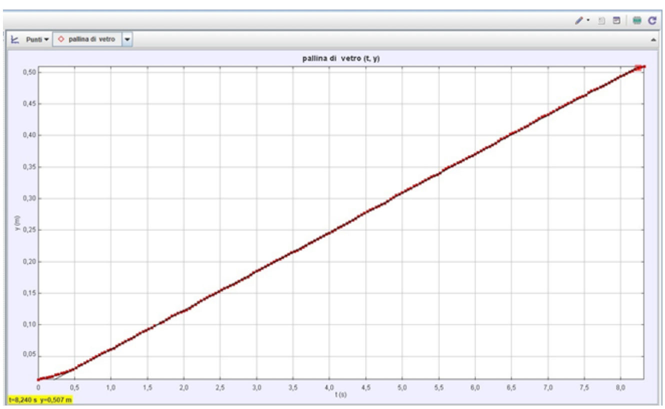

Fig. 4.9. An example of using Tracker software to calculate the terminal speed. Note the final time interval during which the motion is uniform. Source: my experiment image.

Velocity calculation in glycerine	Linearity range		
	Time (s)	Space (m)	Terminal fall velocity (m/s)
Steel ball	1.48	0.445	0.24
	0.12	0.112	
Glass ball	7.60	0,471	0.062
	0.40	0.025	

Tab. 4.3. Calculation of terminal fall velocities in the linearity range of the space-time graph for glycerine. Relative uncertainties have been propagated. The absolute uncertainties affect the second significant digits.

We repeated the measurement in both water and olive oil and in both cases found

$$\frac{v_s}{v_g} = \frac{\sqrt{d_s - d_m}}{\sqrt{d_g - d_m}}$$

i.e. that the terminal velocity is proportional to the square root of the difference between the density of the body and the density of the medium. We deduced that in olive oil and water, the resistance of the medium is proportional to the square of the falling velocity.

An extension of the experiment was to determine the ratio of terminal velocities in a more viscous fluid such as glycerine in order to assess what would be the most appropriate model in this case. We discovered that the terminal velocities of objects as they fall through a fluid are proportional to the differences between the specific weight (or density) of the body and that of the surrounding medium as Galileo supposed.

$$\frac{v_s}{v_g} = \frac{d_s - d_m}{d_g - d_m}$$

	Water	Olive oil	Glycerine
d_m (kg/m^3) at 20°C	998	918	1250
η (Pl) at 20°C*	$1.00 \cdot 10^{-3}$	$8.40 \cdot 10^{-2}$	1.49
$\frac{v_s}{v_g}$	2.2	2.4	4.0
$\frac{d_s - d_m}{d_g - d_m}$	3.9	3.7	4.4
$\sqrt{\frac{d_s - d_m}{d_g - d_m}}$	2.0	1.9	2.1

Tab. 4.4. Ratios between final velocities v_s and v_g of steel and glass balls compared with the ratios of the differences of the steel and glass densities, d_s and d_g, to those of the medium, d_m, and with the ratios of the square roots of their differences. Density values were measured in the laboratory by students: $d_s = 7.86 \cdot 10^3$ kg/m^3, $d_g = 2.57 \cdot 10^3$ kg/m^3. *Viscosity values are taken from technical literature (Liberatore 1992; Fellows [1998], 2022).

4.3 Two experiments on the motion along an inclined plane

Set-up of the experiment and its analysis. The students carried out the experiment in two ways: a) they fixed the spaces and measured the corresponding time intervals using the water clock; b) they fixed the times marked step–by–step by the pendulum in order to measure the corresponding spaces travelled.

Galileo claimed ([1638], 1890-1909, vol. VIII, pp. 212-213; [1638] 1914, pp. 178-179) to have used an inclined plane about 12 *braccia* long (Florentine *braccio a terra* = 0.55063m). We used a wooden beam (6.75 m x 0.15 m x 0.10 m). Our inclined plane was lifted by three cuboids (0.85 m x 0.55 m x 0.30 m; 0.85 m x 0.55 m x 0.20 m; 0.85 m x 0.55 m x 0.11 m). A channel shaped like a circular segment was cut into it: 0.034 m wide and 0.008 m deep. Galileo's notes did not size the balls used. Thus, we used steel balls (0.027 m; 0.030 m; 0.035 m of diameter) so that the balls touched the channel in *one point*. In the following figures, the inclined plane is showed, and then the balls we utilised.

Fig. 4.10. The inclined plane utilized at *Liceo Scintifico "F. Sbordone"* in Naples. Source: my own experiment image.

Fig. 4.11. The groove and the balls used. Source: my own experiment images.

The students fixed spaces. The distances to be covered were set by placing a small wooden block in the canal. The experimenter started the water clock with one hand and let the ball fall by the other one; in order to reduce the inaccuracy of the measurement. The water clock turned off when a sound of the ball impact occurred. The ball was lowered first along the entire length of the canal. The problem of how to fix the distance to get half of the fall time arose; the students required a solution. Many students were surprised to verify that the situation does not happen at half the plane length but at a quarter of it. Thus, they repeated the measures for different values of distance. First, they calculated the ratio between distances travelled and the times taken and found that it varied significantly. Finally, they calculated the ratio between distances travelled and the squares of the times and verified that it was constant (Table 4.5).[51] Hence, they deduced the famous quadratic proportionality relationship between the distance travelled and the time taken that characterises naturally accelerated motion. An important aspect proving the precision of the measure was that the uncertainty on times (the half-range of measurements) was of the order of a few tenths of a second.

d (m)	Water (g)										t (s)	d/t (m/s)	d/t^2 (m/s^2)
	I=1	I=2	I=3	I=4	I=5	I=6	I=7	I=8	I=9	Average			
6,66	397	400	400	404	410	412	419	420	422	408±13	8.2±0.3	0.82	0.100
5.00	356	366	363	368	363	344	354	356	363	359±10	7.2±0.2	0.70	0.097
3.33	296	301	286	293	284	280	282	279	287	289±11	5.8±0.2	0.58	0.100
1,67	198	204	205	206	206	209	211	212	213	206±8	4.1±0.2	0.41	0.098

Tab. 4.5 Fall times t taken to travel the distances d from the rest along an inclined plane measured with the water clock. The d/t^2 ratios differ by a few percent. The uncertainty on times (the half-range of measurements) was of the order of a few tenths of a second.

In the experiment with fixed times, eight small bridges with bells were set along the channel in such positions that a bell rang every time a 25 cm long pendulum described a complete oscillation with a duration of 1 s. The distances Δd travelled in subsequent equal time intervals starting from rest followed Galileo's law of odd numbers (Galilei 1890-1909, vol VIII, pp. 210-212; [1638] 1914, pp. 175-177; see also Pisano and Cioci 2020a): the differences between experimental and theoretical values were less than 3% of their value (tab. 4.6).

Fig. 4.12. The inclined plane with the bells and the pendulum used to verify the law of odd numbers. Source: my own experiment image.

[51] The data in Tables 4.5, 4.6, 4.7 were published in Pisano and Cioci 2020b.

t (s)	OddN.	Δd_{sper} (m)	Δd_{theor} = OddN· Δd_{sper1} (m)	Diff (m)
1.0	1	0.099	0.099	0
2.0	3	0.296	0.297	-0.001
3.0	5	0.494	0.495	-0.001
4.0	7	0.699	0.693	0.006
5.0	9	0.896	0.891	0.005
6.0	11	1.11	1.09	0.02
7.0	13	1.30	1.29	0.01
8.0	15	1.45	1.49	-0.04

Tab. 4.6. Measures of the distances Δd_{sper} travelled along the inclined plane in subsequent equal time intervals Δt = 1s and their theoretical values Δd_{theor}.

4.4 Parabolic motion and the indirect verification of the inertia principle

Set-up of the experiment, modelling & calculus. The students dropped a small ball (diameter = 0.015 cm) along a launch pad/inclined plane (1.000 m x 0.500 m) varying the falling height and thus its final velocity. The initial velocity of the ball in air – in modern terms modelled by a vector – was visibly assumed by the students to be horizontally directed because of the presence of a deflector. Thus, based on Galileo's theorems on parabolic motion and further geometrical and physical assumptions, they calculated the directly proportional relationship between the range G and the square root of the launch pad height H i.e. the module of its initial velocity (see fig. 4.13). In particular, they measured the range G when the height H changed (instead h = 0.810 m was fixed). Then, in order to match experimental and theoretical data, the students after making the first measure calculated the expected theoretical values (based on the historical Galilean proportion):

$$G_1 : G_2 = V_1 : V_2 = \sqrt{H_1 : H_2},$$

that is to say

$$G_{teor} = \sqrt{H_{sper}(m) : 0.300\ m} \cdot 0.798\ m.$$

The differences between the experimental and theoretical data are of the order of magnitude of those calculated by Galileo (1602–1637, BNCF, Ms. Gal72, f. 116v) (tab. 4.7).

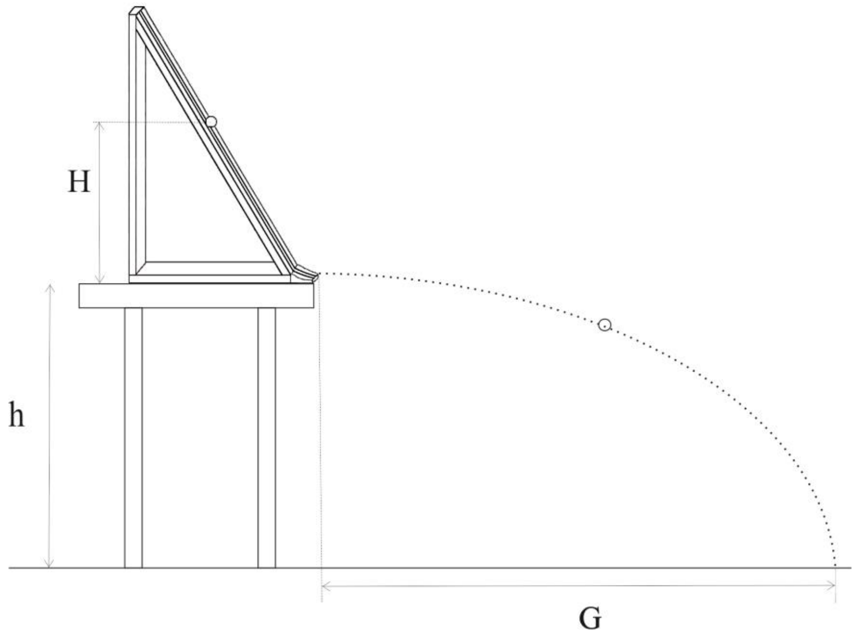

Fig. 4.13. The experimental apparatus for verification of proportionality between range and square root of the height of fall from the launching ramp. Source: my graphic representation.

H (m)	G_{sper} (m)	G_{theor} (m)	Diff (m)
0.300	0.798		
0.400	0.939	0.921	0.018
0.500	1.017	1.030	-0.013
0.600	1.101	1.129	-0.027
0.700	1.195	1.219	-0.024
0.800	1.288	1.303	-0.015
0.900	1.383	1.382	0,001
1.000	1.424	1.457	-0.033

Tab. 4.7. Proportionality between the range G of the projectile and the square root of the falling height H on the launching ramp.

Fig. 4.14. The experimental apparatus for the reconstruction of the parabolic trajectory of the projectile. Source: my own experiment image.

In order to claim the parabolic trajectory of the projectile, the students measured the horizontal range of the ball at different heights (as “x” in tab. 4.8). The position of the ball’s impact point was deduced from the trace left by the ball on a rigid panel placed on an elevator and covered with flour (fig. 4.14). Then they analysed the observed and calculated errors as usual. The “y” values are measured from the highest point of the trajectory. They also verify that the experimental points were arranged along a parabola because the ratios between the values of “y” and the squares of “x” were constant (tab. 4.8).

x (m)	y (m)	x^2 (m^2)	y/x	y/x^2 (m^{-1})
0.72	0.23	0.52	0.32	0.44
1.00	0.44	1.00	0.44	0.44
1.28	0.70	1.64	0.55	0.43
1.46	0.96	2.13	0.66	0.45

Tab. 4.8. The reconstruction of the trajectory of the projectile and the proof that it is a parabola. The y/x differ significantly. The y/x^2 ratios differ by a few percent.

A similar experiment with equally satisfactory results was repeated by choosing the positions (calculated from the launch point) of the horizontal planes advancing according to the rule of odd numbers

- 0.05 m,
- 0.20 m = (0.05 + 3 · 0.05) m,
- 0.45 m = (0.20 + 5 · 0.05) m,
- 0.80 m = (0.45 + 7 · 0.05) m

in order to more easily visualise the verification of the inertia principle in the horizontal direction. While in fact the ball travels through the successive intervals above in the vertical direction in time intervals all equal to each other, the advance in the horizontal direction occurs in spaces equal to each other as would be expected for uniform motion (table 4.9).

N	y (m)	Δy (m)	Δy (m) = N·Δy_1	x (m)	Δx (m)
1	0.05	0.05	0.05	0.24	0.24
3	0.20	0.15	3 · 0.05	0.47	0.23
5	0.45	0.25	5 · 0.05	0.70	0.23
7	0.80	0,35	7 · 0,05	0.91	0.21

Tab. 4.9. The verification of the inertia principle in the horizontal direction for a ball moving in parabolic motion.

As a final experiment, we wanted to reproduce that performed by Guidobaldo del Monte and Galileo in 1592 (see footnote 5 in section 2.6) when a ball dipped in ink was thrown upwards on an inclined plane, leaving a parabola-shaped trace on it.

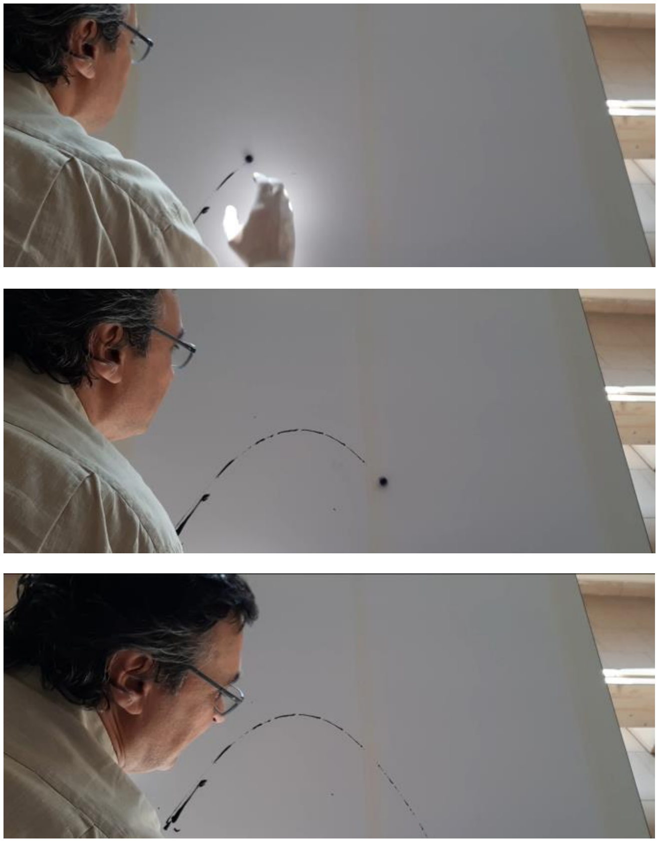

Fig. 4.15. The replication of the experiment performed by Guidobaldo del Monte e Galileo in 1592. A parabola is represented by throwing upwards on an inclined plane a small ball that had previously been dipped in ink. Source: my own experiment image.

Epilogue IV

In this chapter, I analysed the results of the experiments performed by students at the *Liceo Scientifico Sbordone* where, at my suggestion, a Galilean physics laboratory was set up in which in which an attempt was made to reproduce the experimental conditions present in Galileo's time.

For example, to measure the time elapsed during the fall along the inclined plane, a water clock and a pendulum clock were constructed.

To this end, the students explored the phenomenon of the oscillation of two pendulums, observing that the oscillation time does not depend on the mass and amplitude (if small) but only on the length of the wire and determining the law of proportionality between the period of the pendulum and the square root of the length of the wire. They measured the period of oscillation of a pendulum 0.25 m long - corresponding to 1 s. Next, a water clock was constructed using a cylindrical tank with a tap; its calibration was performed using the pendulum. Eventually, the difference between the two instruments was identified in terms of the type of measurement (discrete for the pendulum, continuous for the water clock).

The students were thus able to determine the law of quadratic proportionality between the distance travelled by a steel sphere falling along an inclined plane 6.75 m long and the time taken (measured with a water clock) and to discover the law of odd numbers for the spaces travelled along the inclined plane in equal and successive time intervals (measured with the pendulum and a system of mobile bells).

Finally, the students replicated the famous experiment performed by Galileo and depicted on folio 116v of his manuscript Gal. 72 preserved at the *Biblioteca Nazionale Centrale di Firenze*, verifying the law of direct proportionality between the range of a projectile launched horizontally from a fixed height and its initial velocity, i.e. that the motion is uniform in the horizontal direction, thus confirming the principle of inertia.

An experiment was also performed using modern devices that were easily accessible and usable by the students. An example would be using the acceleration sensor of a smartphone as I did a few years ago. In this more recent educational course, the phenomenon of the fall of two balls was explored. In this recent educational path, students explored the phenomenon of the fall of two balls of the same size but with different masses in different media (glycerine, oil, water, air) with the observation that the difference between the fall times is smaller and smaller as the viscosity of the medium decreases; following Galileo by extrapolation in a vacuum the two bodies arrive together. Students performed a quantitative measurement of the terminal velocities of fall using the Tracker Video Analysis and Modelling Tool for Physics Education software, which made it possible to affirm that these velocities are proportional to the difference between the density of the ball and that of the medium, if the latter is viscous such as glycerine, or to the square root of the differences in densities in the case of oil and water.

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PART III

NoS study case about the kinematics of Galileo

An outline

This part of the thesis is dedicated to this NoS study case about kinematics of Galileo; the results of the entry/exit questionnaires completed by the students are examined in order to understand the effectiveness of the teaching action both in terms of the knowledge and skills acquired by the students and in terms of changes in their interests in the history of physics and physics more generally.

CHAPTER V

The analysis of the questionnaire on students' acquired knowledge and skills and on their interests and motivations

Prologue V

A crucial aspect for didactic effectiveness is the training of teachers and their relationship with didactic research. In addition to disciplinary training, teachers need methodological training so that students, in turn, also receive instruction that is not overly content-related but more aimed at acquiring the research method and the procedures used for problem-solving. Teachers should be made aware that an active, laboratory-based learning procedure for students should be favoured but often, due to the actual conditions of school practice, this does not happen.

In order to overcome these difficulties, it is important to accustom the teacher to distance himself from his daily work and to look at teaching practice from a new perspective, helping him to develop a more scientific approach to teaching and learning processes, e.g. by formulating working hypotheses on specific teaching activities and then evaluating the results achieved. It is important that as far as possible, the teacher also participates in didactic research initiatives, so that he or she is then also more receptive to the results of research, whether carried out personally or by others.

5.1 Constructing of the questionnaire on concepts, skills and knowledge

An original questionnaire on concepts, skills and knowledge was prepared to analyse whether the students had achieved the objectives set out in the educational planning phase. This questionnaire consists of ten disciplinary questions divided into twenty-five sub-questions to assess the students' acquired skills and knowledge. In detail, as to the questionnaire type, it consists of 23 open-ended questions, in order not to influence the students' answers in any way, and only two multiple-choice questions where the possible answers were graphically schematised.

The questions (both open and closed) are predominantly conceptual (cf. Hestenes, Wells and Swackhamer 1992) because they aim to examine the understanding of physics topics starting from the students' difficulties and from their common sense knowledge. The choice of questions therefore had to take into account the students' misconceptions of the topics studied. For example, multiple choice question n. 7 concerned the choice of the trajectory followed by a cannon shot (see Fig. 5.1).

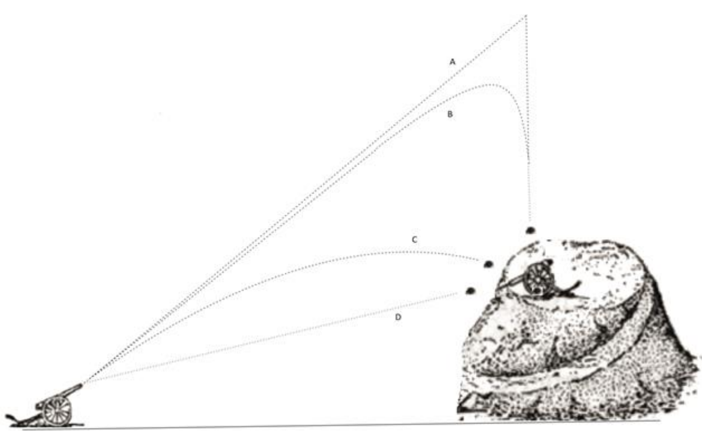


Fig. 5.1. The possible trajectories of a cannon shot. Source: my own representation obtained by modifying an image taken from Galileo's *Trattato di fortificazione* (Galilei 1890-1909, vol. II, p. 93). In public domain.

In the possible answers, in addition to the parabolic trajectory. I have indicated those that have been proposed throughout the history and development of physics: the first movement occurs in a straight direction followed directly by vertical downward movement due to gravity or with an intermediate phase. We find these misconceptions, in particular in Jean Buridan's theory of *impetus* or Tartaglia's projectile trajectory, but they are also frequent among students (see Halloun and Hestenes 1985; McCloskey 1983). A similar question is found in the Force Concept Inventory (Hestenes et al. 1992), but in that case the projectile is fired from above. In my question, the image is taken from Galileo's *Trattato di fortificazione*, with modifications. The other multiple-choice question is about the trajectory followed by a package launched from an airplane in flight. This question is also similar to one from the Concept of Force Inventory, but whereas in that case only the type of trajectory of the particle is studied, in this question, also following McCloskey (1983), knowledge of the principle of inertia is also investigated.

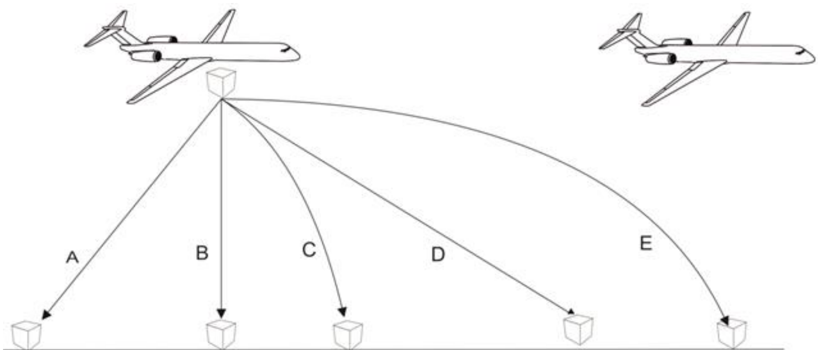


Fig. 5.2. The possible trajectories followed by a parcel launched from an aircraft flying at constant speed. Source: my own representation.

The other questions concern:

a) Galileo's contribution to the development of physics
b) the principle of inertia
c) the fall of a body in a vacuum
d) the fall of a body in a fluid
e) the fall of a body along an inclined plane
f) the motion of the pendulum.

The following section contains the questionnaire on concepts, skills and knowledge that was administered to the students both before and after the history of physics educational path. The entry questionnaire was used to further study of students' pre-instructional conceptions. The exit questionnaire was used for the assessment of the achievement of learning objectives and for the study of students' learning difficulties.

5.2 The questionnaire on concepts, skills and knowledge

1. Illustrate in a few words the main contributions made by Galileo to the development of physics.

....

....

2.1 What conditions must be created to keep a body in motion with constant velocity ?

....

2.2 How is it possible to realize this motion concretely?

....

....

2.3 How did Galileo succeed?

....

....

3.1 What is the relationship between the speed of a body falling in the vacuum and its weight?

....

....

3.2 How can you prove this claim?

....

....

3.3 How did Galileo establish it?

....

....

4.1 What kind of relationship is there between the final speed of a body falling (from rest) in the vacuum and the height of the fall?

....

....

4.2 And between the speed reached at a certain instant starting from rest and the time elapsed from the beginning of the motion?

....

5.1 Describe what happens when a body falls through a fluid. What kind of motion is it?

....

....

5.2 In what ways does the fluid affect the falling-body motion?

....

....

5.3 What does terminal falling speed depend on?

....

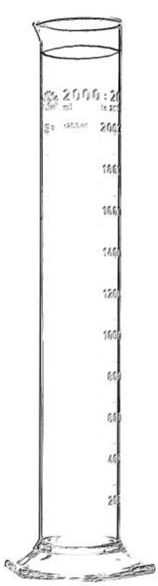
....

5.4 What is the relationship between this speed and the (specific) weight/density of the body and of the medium?

....

....

5.5 Widening of the inquiry: which forces act on the body? Represent the velocity vector and the applied forces in the different phases of the motion.



....

....

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....

....

notions of two equal bodies that both are dropped with zero

initial speed from the same height. one freely falling and the other sliding without friction on an inclined plane.
6.1What is the relationship between the speeds with which the two bodies reach the ground?

....

....

6.2 And what is the relationship between the fall times of the two motions?

....

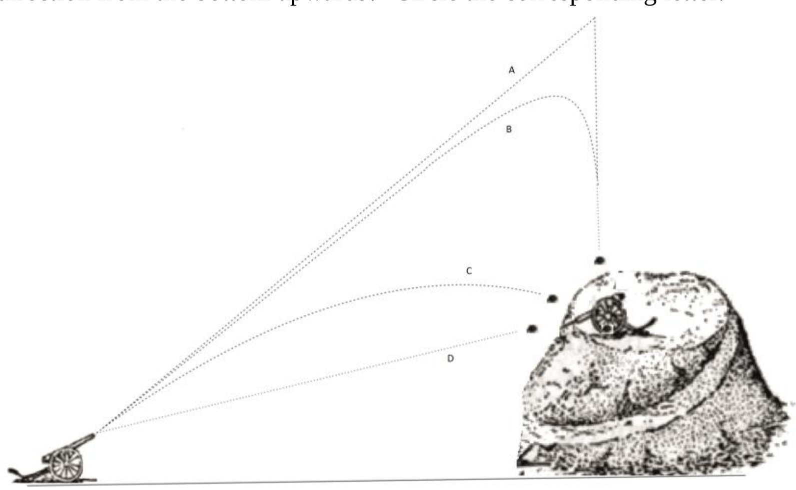
....

6.3 Explain briefly why.

....

....

7.1 In your opinion. what is the trajectory of a cannonball shot in the oblique direction from the bottom upwards? Circle the corresponding letter.



7.2 Briefly analyse the motion.

....

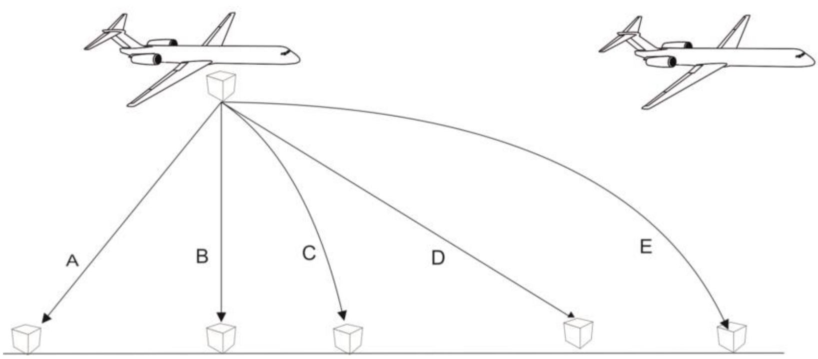
....
8. Only on account of proportionality between quantities, how could you

deduce the type of trajectory followed by a projectile shot horizontally, disregarding the air resistance?

....

....

9.1 A parcel falls from an airplane that moves in a rectilinear uniform motion parallel to the Earth's surface. In the hypothesis that the effect of the winds, the air resistance and the Earth's rotation can be disregarded, which is the trajectory followed by the parcel among those drawn in the figure? (circle the corresponding letter)



9.2 Why?

....

....

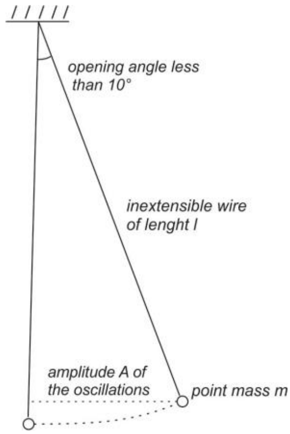
9.3 Widening of the inquiry: represent the velocity and the acceleration of the parcel in three points of the selected trajectory

....

....

10.1 How can the pendulum swing time be increased?
10.2 Widening of the inquiry: draw the forces involved assuming that the mass of the wire and the friction can be disregarded

....



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5.3 The results of the entry questionnaire on concepts, skills and knowledge: students' pre-instructional conceptions

The results of the entry questionnaire were used qualitatively to examine the various kinds of answers given by the pupils, i.e. to understand their pre-educational conceptions and, by comparing them with those already known in the research literature, to discover new ones.

With regard to questions on the principle of inertia, the most common conception of students is related to the Aristotelian view that all motion with constant velocity is generated by a force or, more specifically, to Buridan and Oresme's theory of impetus (Arons 1990; 1997; see also for example Viennot 1979, di Sessa 1982, McDermott 1984). Moreover, students often fail to take friction into account in the dynamic interpretation of motion.

As for the motion of free fall, the most common pre-instructional conception (see e.g. Gunstone and White1981, Whitaker 1983) – which was also historically confirmed in Aristotle's thought – is that the falling velocity of the body depends on the weight of the body. This misconception is due to the fact that we experience a greater downward pull of heavier bodies, but the greater inertial mass that compensates for this effect is an issue that students do not consider.

When they consider the motion of bodies in a vacuum, in the absence of air resistance, students often confuse absence of gravity with absence of air.

Regarding the motion of bodies in fluids, I found that students think that the speed of fall is inversely proportional to the density of the medium, as Aristotle believed. An unexpected discovery was that several students, before studying the subject, believe that the body is accelerated downwards due to the action of the fluid (because of its weight).

If we compare the falling motion along a perfectly smooth inclined plane with the free-falling motion from the same height, students generally think that the final speed of the motion in the vertical direction is greater. Perhaps this is because longer travel times are misleading or because students' conceptions are influenced by concrete experience and the fact that motion along the inclined plane is actually slowed by friction.

With regard to the motion of the pendulum. most students believe that its weight or the amplitude of the oscillation influences the period. Many students do not consider the tension of the wire and only represent a force along the trajectory in the direction of motion (McDermott 1984) because they believe that motion can only occur in the direction in which the force is applied. A common misconception even among teachers is that tension is equal to the radial component of the weight force, not taking into account the centripetal force required to keep the mass in circular motion (García, Ramírez and Rodríguez 2013).

5.4 The results of the exit questionnaire on concepts, skills and knowledge

Table 5.1 shows all the answers given by the pupils to the questionnaire on concepts, skills and knowledge with the relative frequencies for each answer for each of the three grades of the *Liceo scientifico* involved in the project (30 students for the first grade, 28 students for the second grade, 32 students for the third grade, 90 pupils in total who answered all the questionnaires). Answers deemed to be correct were written in italics. Complete answers and slightly inaccurate answers are considered correct. Answers that correspond to misconceptions encountered by the students or answers not given are considered incorrect.

The correct answers were summarised in tab. 5.2. These results are visualised by means of the bar graphs shown in Figures 5.1 and 5.2.

These results help us to analyse the learning difficulties encountered at the end of the educational action, also as a didactic reflection for possible improvements to the project.

In fact, from the observation of the graph in tab. 5.1, it can be observed that the correct answers to question N. 8 are less than 40%. From the graph in fig. 5.2, however, it can be seen that the negative answers only concern students in the first and second class, but not in the third class. The subject of the question is related to the representation of the parabola in the Cartesian plane, a subject that is generally proposed in Italy in the third grade of the *Liceo scientifico*. It can be deduced from this that the topic of parabolic motion would be more suitable for third grade or vice versa that the study of this topic in physics should be accompanied by the study of the parabola in the Cartesian plane in mathematics.

An analysis of the answers to the other questions shows that, on the whole, learning difficulties due to common-sense conceptions have been overcome, especially for those questions in relation to which students were able to question nature themselves by performing the experiment, as in the case of the mass-independence of both pendulum motion and free-fall motion (correct answers with percentages above 90%).

Interesting observations for improving teaching also emerge from the answers to question 6.3 because students do not take into account the role played by the plane's constraining reaction in influencing the motion of the ball on the inclined plane. Finally, answers to question 10.1, on the forces acting on a pendulum, show that there are still some students in the first class who think that motion occurs only in the direction of the force.

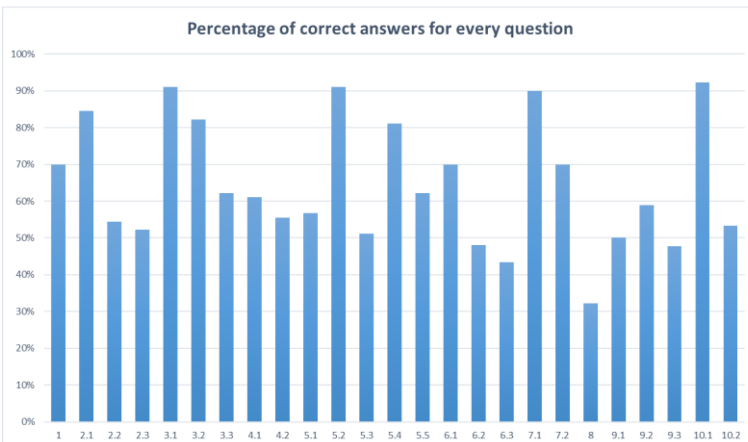

Fig. 5.3. Bar graph of the percentage of correct answers for every question of the exit questionnaire on concepts, skills and knowledge.

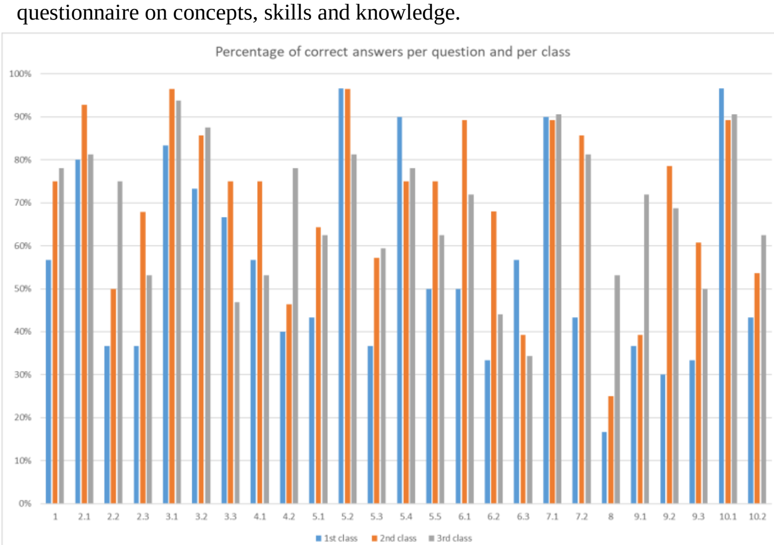

Fig. 5.4. Bar graph of the percentage of correct answers per question and per grade of the exit questionnaire on concepts, skills and knowledge.

Questions	Answers	1st gr.	2nd gr.	3rd gr.	Total	Percentage
1 Illustrate in a few words the main contributions made by Galileo to the development of physics.	*1.1 Four correct answers or more*	13	17	21	51	56.7%
	1.2 Three correct answers	4	4	4	12	13.3%
	1.3 One or two correct answers	7	5	2	14	15.6%
	1.4 A wrong attribution to Galileo	0	2	5	7	7.8%
	1.5 Answer not given	6	0	0	6	6.7%
Contributions made by Galileo to the development of physics.	1a. Introduction of the scientific / experimental method	6	19	16	41	45.6%
	1b. Laws of falling bodies	16	14	10	40	44.4%
	1c. Analysis of the pendulum motion	16	22	21	59	65.6%
	1d. Astronomical studies and support to the Copernican theory	2	14	14	30	33.3%
	1e. Fall motion along an inclined plane	10	14	16	40	44.4%
	1f. Parabolic motion analysis	13	11	18	42	46.7%
	1g. Fall motion into fluids	6	4	11	21	23.3%
	1h. Law of inertia	4	10	13	27	30.0%
	1i. The relativity principle	0	3	10	13	14.4%
	1j. Refinement of the telescope	2	8	7	17	18.9%
	1k. Kepler's laws	0	0	2	2	2.2%
	1l. The three principles of dynamics	0	1	4	5	5.6%
2.1 What conditions must be created to keep a body in motion with constant velocity in one direction?	*2.1.1 A system of forces whose resultant has a zero component in that direction must be applied to a moving body*	7	5	0	12	13.3%
	2.1.2 A system of forces whose resultant is zero must be applied to a moving body	13	18	21	52	57.8%
	2.1.3 No force should be applied.	4	3	5	12	13.3%
	2.1.4 Constant force must be applied.	1	0	1	2	2.2%
	2.1.5 Other non-significant (wrong) answers	5	1	5	11	12.2%

	2.1.6 Answer not given	0	1	0	1	1.1%
2.2 How is it possible to realize this motion concretely?	*2.2.1 By making a motion approximately without friction in the horizontal direction after throwing a ball or firing a projectile (because it is subject only to the weight force that is vertical)*	0	1	0	1	1.1%
	2.2.2 By setting a body in motion without friction on an air cushion rail	0	1	2	3	3.3%
	2.2.3 By reducing friction by using a dry ice disk	1	0	0	1	1.1%
	2.2.4 By reducing friction by sliding an object on ice	0	2	0	2	2.2%
	2.2.5 By applying to a moving body a system of forces whose sum is zero (generic reference to the principle of inertia)	3	0	1	4	4.4%
	2.2.6 By rolling a ball on a smooth surface as much as possible / using roller skates or a cart	2	2	7	11	12.2%
	2.2.7 By reducing friction as much as possible (and increasing body mass)	4	6	8	18	20.0%
	2.2.8 By applying an equal and opposite force to friction (e.g. in order to keep a heavy truck or a car in motion at constant speed)	0	1	6	7	7.8%
	2.2.9 By keeping a car/ a train moving at a constant speed in a straight direction	1	1	0	2	2.2%
	2.2.10 By starting a ball on a horizontal plane (incomplete)	0	5	0	5	5.6%
	2.2.11 By dropping a body from a high altitude	1	0	1	2	2.2%
	2.2.12 By making an object move in an empty space (does not take into account remote interactions)	5	1	0	6	6.7%
	2.2.13 By keeping a car moving at a constant speed	1	1	1	3	3.3%

	2.2.14 By taking into consideration an inertial frame of reference	0	1	0	1	1.1%
	2.2.15 By applying a constant force (e.g. pushing a trolley with a constant force / slightly tilting a plane / by means of the engine of a vehicle / pedalling with constant strength on a bicycle)	1	4	0	5	5.6%
	2.2.16 By rolling a ball on an inclined plane (without friction)	0	1	2	3	3.3%
	2.2.17 By using a pendulum	2	0	0	2	2.2%
	2.2.18 By a uniformly accelerated motion	2	0	0	2	2.2%
	2.2.19 Other non-significant (wrong) answers	2	1	1	4	4.4%
	2.2.20 Answer not given	5	1	3	9	10.0%
2.3 How did Galileo succeed?	*2.3.1 As a horizontal component of the parabolic motion*	1	0	3	4	4.4%
	2.3.2 As a limiting case of motion along a plane inclined upwards or downwards	1	10	3	14	15.6%
	2.3.3 By the motion of a ball launched on a horizontal plane"removed any impediment"	1	3	5	9	10.0%
	2.3.4 By the terminal falling motion in a viscous fluid	1	0	0	1	1.1%
	2.3.5 By using an inclined plane	7	6	6	19	21.1%
	2.3.6 By using a pendulum and related-energy-conservation remarks	2	2	0	4	4.4%
	2.3.7 Only reference to the principle of inertia	2	5	0	7	7.8%
	2.3.8 Other non-significant wrong answers	2	0	8	10	11.1%
	2.3.9 Answer not given	11	2	7	20	22.2%
	3.1.1 No relationship	25	27	30	82	91.1%

3.1 What is the relationship between the speed of a body falling in a vacuum and its weight?	3.1.2 The speed increases with the weight of the body if it falls into the air	1	0	1	2	2.2%
	3.1.3 Direct proportionality	2	1	1	4	4.4%
	3.1.4 Other non-significant (wrong) answers	1	0	0	1	1.1%
3.2 How can you prove this claim?	*3.2.1 In the void of the Newton's tube*	0	1	4	5	5.6%
	3.2.2 Without atmosphere. on the lunar surface	1	0	0	1	1.1%
	3.2.3 By the limit procedure used by Galileo	4	2	5	11	12.2%
	3.2.4 By means of new technologies (e.g. the acceleration sensor of the smartphone)	0	1	0	1	1.1%
	3.2.5 With a generic motion in the void or without resistance of the medium	12	6	14	32	35.6%
	3.2.6 In the first approximation. by the fall of two balls with the same size and different mass in the air	5	14	5	24	26.7%
	3.2.7 Motion without gravity (for example in a lift that moves with acceleration opposite to the acceleration of gravity)	0	0	1	1	1.1%
	3.2.8 Repetition of theory without experimental suggestions	0	3	0	3	3.3%
	3.2.9 Other non-significant wrong answers	3	1	2	6	6.7%
	Answer not given	5	0	1	6	6.7%
3.3 How did Galileo established it?	*3.3.1 By a limit process. comparing the motion of two balls of the same size and different mass in means less and less resistant*	10	0	10	20	22.2%
	3.3.2 By comparing the motion of two balls in free fall from a high tower (e.g. the tower of Pisa)	8	17	4	29	32.2%
	3.3.3 By a thought experiment	2	4	1	7	7.8%

	3.3.4 By studying the falling motion of one ball in the air or in other media	1	0	2	3	3.3%
	3.3.5 By the limit process but with confusion between density and resistance of the medium	1	1	5	7	7.8%
	3.3.6 Other non-significant (wrong) answers	2	5	5	12	13.3%
	3.3.7 Answer not given	7	1	5	13	14.4%
4.1 What kind of relationship is there between the final speed of a body falling (from rest) in the vacuum and the height of fall ?	*4.1.1 The final speed is proportional to the square root of the height*	11	11	12	34	37.8%
	4.1.2 The final speed generally increases with the falling height	6	10	5	21	23.3%
	4.1.3 The final speed is directly proportional to the falling height	4	4	4	12	13.3%
	4.1.4 The final speed is proportional to the square of the falling height	4	2	9	15	16.7%
	4.1.5 The speed does not change during the motion	3	0	2	5	5.6%
	4.1.6 Other non-significant (wrong) answers	1	1	0	2	2.2%
	4.1.7 Answer not given	1	0	0	1	1.1%
4.2 And between the speed reached at a certain instant starting from rest and the time elapsed from the beginning of the motion ?	*4.2.1 Speed is directly proportional to the fall time*	12	12	25	49	54.4%
	4.2.2 Speed (generally) increases as the fall time increases	0	1	0	1	1.1%
	4.2.3 Speed increases linearly with the fall time	1	2	0	3	3.3%
	4.2.4 Speed is inversely proportional to time	4	7	4	15	16.7%
	4.2.5 Speed is directly proportional to the square of time	7	0	1	8	8.9%
	4.2.6 Speed decreases over time	2	6	1	9	10.0%
	4.2.7 The speed does not change during the motion	1	0	1	2	2.2%
	4.2.8 Other non-significant (wrong) answers	1	0	0	1	1.1%
	4.2.9 Answer not given	2	0	0	2	2.2%

5.1 Describe what happens when a body falls through a fluid. What kind of motion is it?	*5.1.1 Firstly. the body moves with an accelerated motion and then - when the resistance force of the medium is equal to the weight of the body -with a uniform motion*	8	13	19	40	44.4%
	5.1.2 When the resultant of the forces becomes zero, there is an uniform motion	5	5	1	11	12.2%
	5.1.3 A uniform motion	0	3	0	3	3.3%
	5.1.4 An accelerated motion / a non-uniform motion	4	2	1	7	7.8%
	5.1.5 A decelerated motion	0	1	1	2	2.2%
	5.1.6 A uniformly accelerated motion (even taking into account the resistance of the medium that slows the motion)	10	1	4	15	16.7%
	5.1.7 Firstly, the body moves with a uniformly accelerated motion and then – when the resistance force of the medium is equal to the weight of the body – with a uniform motion	0	1	4	5	5.6%
	5.1.8 Answer not given	3	2	2	7	7.8%
5.2 In what ways does the fluid affects the body-falling motion?	*5.2.1 The slowdown of the body is due to the drag force and to the upward buoyant push*	9	11	12	32	35.6%
	5.2.2 The slowdown of the body is due to the drag force	18	11	14	43	47.8%
	5.2.3 The slowdown of the body is due to the upward buoyant thrust (governed by Archimedes' principle)	2	5	1	8	8.9%
	5.2.5 Other non-significant wrong answers	1	0	4	5	5.6%
	5.2.6 Answer not given	0	1	1	2	2.2%
5.3 What does terminal falling speed depend on?	*5.3.1 Four exact answers*	0	5	9	14	15.6%
	5.3.2 Three exact answers	2	6	4	12	13.3%
	5.3.3 Two exact answers	9	5	6	20	22.2%

	5.3.4 One exact answer	15	4	5	24	26.7%
	5.3.5 A wrong answer	3	6	6	15	16.7%
	5.3.6 Answer not given	1	1	1	3	3.3%
Quantities on which the terminal speed of fall depends	5.3a. On body mass / density	10	14	18	42	46.7%
	5.3b. On body shape and size	1	9	6	16	17.8%
	5.3c. On medium density	5	13	17	35	38.9%
	5.3d. On the viscosity of the medium	18	10	20	48	53.3%
	5.3e. On gravitational acceleration	9	5	10	24	26.7%
	5.3f. On the resultant of the forces	0	1	0	1	1.1%
	5.3g. On the fall height	3	1	6	10	11.1%
	5.3h. On the initial speed with which the body enters the fluid	1	0	0	1	1.1%
5.4 What is the relationship between this speed and the specific-weights/densities of the body and of the medium?	*5.4.1 The terminal falling speed is proportional to the difference between densities*	10	15	18	43	47.8%
	5.4.2 The terminal speed is proportional to the density of the body	5	2	2	9	10.0%
	5.4.3 The terminal speed decreases with increasing density of the body and increases with the density of the fluid	5	1	3	9	10.0%
	5.4.4 The terminal falling speed is proportional to the square root of the difference between densities	0	2	2	4	4.4%
	5.4.5 The terminal speed depends on the difference between densities	2	1	0	3	3.3%
	5.4.6 The terminal speed is inversely proportional to the density of the fluid	4	5	6	15	16.7%
	5.4.7 Other non-significant (wrong) answers	2	2	0	4	4.4%
	5.4.8 Answer not given	2	0	0	2	2.2%
5.5 Widening of the inquiry: which forces act on the	*5.5.1 The student identifies the two phases of the motion and*	6	16	13	35	38.9%

ball? Represent the velocity vector and the applied forces in the different phases of the motion	*represents the weight of the ball, the drag force and the upward buoyant push correctly*					
	5.5.2 The student identifies the two phases of the motion and represents the weight of the ball and the drag force correctly	9	5	7	21	23.3%
	5.5.3 The student identifies only one phase, correctly representing the forces acting on the ball	6	1	2	9	10.0%
	5.5.4 The student draws the drag-force vector longer than the weight of the ball in the first phase of the motion	1	2	0	3	3.3%
	5.5.5 The student, in the second phase of the motion, considers the drag force greater than the weight of the ball and the velocity smaller and smaller	4	3	4	11	12.2%
	5.5.6 The student represents the drag force smaller than the weight, not identifying the phase at constant speed	1	0	3	4	4.4%
	5.5.7 The student represents the weight force of the overlying fluid acting on the ball and its velocity is greater and greater	0	0	0	0	0.0%
	5.5.8 Other non-significant (wrong) answers	0	0	1	1	1.1%
	5.5.9 Answer not given	3	1	2	6	6.7%
6.1 Compare the motions of two equal bodies that both are dropped with zero initial speed from the same height, one freely falling and the other sliding	*6.1.1 The two speeds are equal*	15	25	23	63	70.0%
	6.1.2 The two speeds are proportional	10	0	1	11	12.2%
	6.1.3 The final speed of the motion along the vertical path is greater	2	2	0	4	4.4%
	6.1.4 The two speeds are different	1	1	6	8	8.9%

without friction on an inclined plane. What is the relationship between the speeds with which the two bodies reach the ground?	6.1.5 Other non-significant (wrong) answers	2	0	0	2	2.2%
	6.1.6 Answer not given	0	0	2	2	2.2%
6.2 And what is the relationship between the fall times of the two motions?	*6.2.1 Once the height is fixed. the ratio between the falling time along the inclined plane and along the vertical is equal to the ratio of the two lengths*	4	8	1	13	14.4%
	6.2.2 The falling time of the motion along the inclined plane is greater	6	10	12	28	31.1%
	6.2.4 The falling times along the inclined plane are directly proportional to the square roots of the spaces travelled	0	1	1	2	2.2%
	6.2.3 The falling times are equal	17	7	15	39	43.3%
	6.2.5 The fall time is directly proportional to the square of the space	0	1	0	1	1.1%
	6.2.6 Other non-significant wrong answers	2	1	1	4	4.4%
	6.2.7 Answer not given	1	0	2	3	3.3%
6.3 Explain briefly why.	*6.3.1 Since speeds depend only on height, for application of the first theorem of Galileo, the travel times are directly proportional to the spaces covered*	9	2	0	11	12.2%
	6.3.2 The falling time of the motion along the inclined plane is greater because the length to be covered is greater	3	6	3	12	13.3%
	6.3.4 The falling time of the motion along the inclined plane is greater because the acceleration/the component of the weight force along the inclined plane is smaller	4	2	1	7	7.8%

	6.3.5 The speeds are equal because in both cases there is no friction	1	1	7	9	10.0%
	6.3.6 The falling times are equal because of similarity between the free fall motion and the motion along the inclined plane: in both cases friction is not considered and the two bodies are subject only to the force of gravity	2	8	8	18	20.0%
	6.3.7 The falling times are equal because the fall time depends only on the height. according to the law $t = \sqrt{2h/g}$	0	2	2	4	4.4%
	6.3.8 Other non-significant (wrong) answers	3	5	6	14	15.6%
	6.3.9 Answer not given	8	2	5	15	16.7%
7.1 In your opinion, what is the trajectory of a cannonball shot in the oblique direction from the bottom upwards? Circle the corresponding letter.	7.1.1 C. The trajectory is parabolic	27	25	29	81	90.0%
	7.1.2 B. First, motion occurs in the straight direction and then a vertical movement downwards (due to the effect of gravity) develops with an intermediate curved path phase	3	2	3	8	8.9%
	7.1.3 A. The two motions occur one after the other	0	0	0	0	0.0%
	7.1.4 D. The motion is rectilinear	0	0	0	0	0.0%
	7.1.5 Answer not given	0	1	0	1	1.1%
7.2 Analyze briefly the motion.	*7.2.1 The projectile motion is the composition of an uniform motion in the horizontal direction and an accelerated one in the vertical direction so that the trajectory is a parabola*	4	18	12	34	37.8%
	7.2.2 The trajectory is parabolic (depending on the initial speed and the orientation of the cannon)	7	6	13	26	28.9%

	7.2.3 The force with which the cannon fires the projectile determines the initial velocity which breaks down into its x and y components	0	0	1	1	1.1%
	7.2.4 Trajectory would change due to air friction	2	0	0	2	2.2%
	7.2.5 First, the projectile continues its rectilinear motion until its thrust is exhausted. Then, it is slowed down by the air resistance. Finally, it falls downwards because of the effect of gravity / In the first phase of the motion, the force impressed by the cannon is greater than the force of gravity. At some point, there is a balance. Finally, the force of gravity predominates causing the cannonball to fall vertically	3	1	0	4	4.4%
	7.2.6 In the first phase the motion is uniform and in the second it is uniformly accelerated by the action of gravity / In the first phase, the projectile accelerates upwards and forwards. Once the maximum height is reached, the projectile begins the falling motion	4	0	1	5	5.6%
	7.2.7 The force of the cannon and the weigh act on the body and determine the parabolic motion / The parabolic motion is composed of a uniformly accelerated motion along the y axis upwards and a uniform motion along the x axis	2	1	4	7	7.8%
	7.2.8 Other non-significant (wrong) answers	6	2	0	8	8.9%
	7.2.9 Answer not given	2	0	1	3	3.3%

8. Only on account of proportionality between quantities, how could you deduce the type of trajectory followed by a projectile shot horizontally, disregarding the resistance of the air?	*8.1 The space travelled in the vertical direction is proportional to the square of the time used and is proportional to the space travelled in the horizontal direction therefore there is a quadratic proportionality between the space travelled in the vertical direction and that travelled in the horizontal direction and therefore the trajectory is parabolic*	0	6	3	9	10.0%
	8.2 The trajectory is a parabola because there is a quadratic proportionality between the space travelled in the vertical direction and that in the horizontal direction	5	1	14	20	22.2%
	8.3 The trajectory is a parabola	8	13	3	24	26.7%
	8.4 Because the motion is the composition of a uniform horizontal motion and a uniformly accelerated vertical motion	5	7	1	13	14.4%
	8.5 There is a quadratic proportionality between the space travelled and the time taken	0	0	5	5	5.6%
	8.6 Other non-significant (wrong) answers	6	0	3	9	10.0%
	8.7 Answer not given	6	1	3	10	11.1%
9.1 A parcel falls from an airplane that moves in a rectilinear uniform motion parallel to the earth's surface. In the hypothesis that the effect of the winds, the resistance of the air and the Earth's rotation can be	*9.1.1 E. The trajectory is parabolic and the parcel keeps the speed in the horizontal direction it had on the plane / there is inertia in the horizontal direction*	11	11	23	45	50.0%
	9.1.2 C. The trajectory is parabolic; the parcel retains part of its motion – but not all its velocity – in the horizontal direction	6	10	7	23	25.6%

disregarded, which is the trajectory followed by the parcel among those drawn in the figure? (circle the corresponding letter)	9.1.3 B. The trajectory is vertical. The parcel does not retain its velocity when it falls from the plane / the student gets confused with the plane's reference system	4	3	1	8	8.9%
	9.1.4 A. The trajectory is straight to the back / the student gets confused with the plane's reference system and does not consider inertial motion	7	4	1	12	13.3%
	9.1.5 D. The trajectory is straight forward / as the composition of two motions of the same type in the horizontal and vertical direction	2	0	0	2	2.2%
9.2 Why?	*9.2.1 The motion of the parcel is the composition of a horizontal uniform motion with the velocity of the airplane and a vertical motion uniformly accelerated*	5	6	13	24	26.7%
	9.2.2 The motion of the parcel is parabolic because it is the composition of a uniform horizontal motion and a uniformly accelerated vertical motion	4	16	9	29	32.2%
	9.2.3 The motion is parabolic	0	2	3	5	5.6%
	9.2.4 The motion is vertical because the parcel is subject only to its weight	4	4	1	9	10.0%
	9.2.5 It's a straight motion in an oblique direction backwards because the parcel is moving in the opposite direction of the airplane	3	0	0	3	3.3%
	9.2.6 The parcel is subject to two forces: the force exerted by the plane and the weight. At first the force of the plane predominates, then the	2	0	0	2	2.2%

	parcel is significantly affected by the gravitational force / For a short interval of time follows the plane and then proceeds with a curved trajectory					
	9.2.7 Other non-significant answers	4	0	1	5	5.6%
	9.2.8 Answer not given	8	0	5	13	14.4%
9.3 Widening of the inquiry: represent the velocity and the acceleration of the parcel in three points of the selected trajectory	*9.3.1 The student draws the two components of which the horizontal one is constant and the vertical one is increasing*	9	16	13	38	42.2%
	9.3.2 The student draws the two components in a slightly inaccurate way	1	1	3	5	5.6%
	9.3.3 The student only draws the vertical component increasing	1	3	0	4	4.4%
	9.3.4 The student draws only the velocity vector tangent to the trajectory	0	1	0	1	1.1%
	9.3.5 The student draws the horizontal component in the opposite direction of the movement of the airplane	3	0	0	3	3.3%
	9.3.6. The student draws the two components of the velocity approximately unchanged at different points of the trajectory	0	2	5	7	7.8%
	9.3.7 The student draws smaller and smaller horizontal component of velocity	0	0	1	1	1.1%
	9.3.8 Other non-significant (wrong) answers	5	1	0	6	6.7%
	9.3.9 Answer not given	11	4	10	25	27.8%
10.1 How can the pendulum swing time be increased?	*10.1.1 By increasing wire length*	29	25	29	83	92.2%
	10.1.2 By increasing the amplitude of the oscillation angle	0	1	2	3	3.3%
	10.1.3 By decreasing the applied mass	0	1	0	1	1.1%

	10.1.4 Other non-significant (wrong) answers	1	0	0	1	1.1%
	10.1.5 Answer not given	0	1	1	2	2.2%
10.2 Widening of the inquiry: draw the forces involved assuming that the mass of the wire and the friction can be disregarded	*10.2.1 The student draws the weight force and the tension greater of the radial component of the weight at an intermediate point of the trajectory*	11	11	15	37	41.1%
	10.2.2 The student graphs the weight force and the tension equal to the radial component of the weight at the starting point of the trajectory	2	4	5	11	12.2%
	10.2.3 The student only draws a force with the direction of the trajectory	2	0	0	2	2.2%
	10.2.4 The student graphs the weight force and the tension equal to the radial component of the weight at an intermediate point of the trajectory	4	4	1	9	10.0%
	10.2.5 The student graphs a radial downward force, equal and opposite to the tension of the wire	0	3	1	4	4.4%
	10.2.6 The student only draws the vertical weight force (and its components: tangent to the trajectory and in the direction of the wire)	7	3	4	14	15.6%
	10.2.7 The student draws an inaccurate representation	1	2	4	7	7.8%
	10.2.8 Answer not given	3	1	2	6	6.7%

Tab. 5.1 Students' answers to the exit questionnaire on concepts. skills and knowledge

Questions	**1st gr.**	**2nd gr.**	**3rd gr.**	**Total**
1. Illustrate in a few words the main contributions made by Galileo to the development of physics.	57%	75%	78%	70%
2.1 What conditions must be created to keep a body in motion with constant velocity in one direction?	80%	93%	81%	84%
2.2 How is it possible to realize this motion concretely?	37%	50%	75%	54%
2.3 How did Galileo succeed?	37%	68%	53%	52%
3.1 What is the relationship between the speed of a body falling in the vacuum and its weight?	83%	96%	94%	91%
3.2 How can you prove this claim?	73%	86%	88%	82%
3.3 How did Galileo established it?	67%	75%	47%	62%
4.1 What kind of relationship is there between the final speed of a body falling (from rest) in the vacuum and the height of fall ?	57%	75%	53%	61%
4.2 And between the speed reached at a certain instant starting from rest and the time elapsed from the beginning of the motion ?	40%	46%	78%	56%
5.1 Describe what happens when a body falls through a fluid. What kind of motion is it?	43%	64%	63%	57%
5.2 In what ways does the fluid affects the body-falling motion?	97%	96%	81%	91%
5.3 What does terminal falling speed depend on?	37%	57%	59%	51%
5.4 What is the relationship between this speed and the specific-weights/densities of the body and of the medium?	90%	75%	78%	81%
5.5 Widening of the inquiry: which forces act on the ball? Represent the velocity vector and the applied forces in the different phases of the motion	50%	75%	63%	62%

6.1 Compare the motions of two equal bodies that both are dropped with zero initial speed from the same height, one freely falling and the other sliding without friction on an inclined plane. What is the relationship between the speeds with which the two bodies reach the ground?	50%	89%	72%	70%
6.2 And what is the relationship between the fall times of the two motions?	33%	68%	44%	48%
6.3 Explain briefly why.	57%	39%	34%	43%
7.1 In your opinion, what is the trajectory of a cannonball shot in the oblique direction from the bottom upwards? Circle the corresponding letter.	90%	89%	91%	90%
7.2 Analyse briefly the motion.	43%	86%	81%	70%
8. Only on account of proportionality between quantities. how could you deduce the type of trajectory followed by a projectile shot horizontally, neglecting the resistance of the air?	17%	25%	53%	32%
9.1 A parcel falls from an airplane that moves in a rectilinear uniform motion parallel to the earth's surface. 9.A In the hypothesis that the effect of the winds. the resistance of the air and the Earth's rotation can be neglected, which is the trajectory followed by the parcel among those drown in the figure? (circle the corresponding letter)	37%	39%	72%	50%
9.2 Why?	30%	79%	69%	59%
9.3 Widening of the inquiry: represent the velocity and the acceleration of the parcel in three points of the selected trajectory	33%	61%	50%	48%
10.1 How can the pendulum swing time be increased?	97%	89%	91%	92%
10.2 Widening of the inquiry: draw the forces involved assuming that the mass of the wire and the friction can be neglected	43%	54%	63%	53%

Tab. 5.2. Percentage of correct answers per grade and per question to the exit questionnaire on concepts, skills and knowledge

5.5 Entry/exit questionnaire on motivational aspects

If you entirely agree with the statement, choose 5. If you entirely disagree, choose 1. If you partially agree, choose a number between 2 and 4 that best expresses your opinion.

1. I am highly intrigued with the study of physics.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
I believe that physics can help me to understand

....

....

2. I find physics obscure and complicated
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
The greatest difficulty I meet concerns

....

....

3. I am very interested in the study of physics as an experimental science.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
I'm really/I'm not interested in experimental science because

....

....

4. I regard the relationship between physics and mathematics as very fruitful for the development of science.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
I think that the study of the relationship between physics and mathematics can help/cannot help us to

....

....

5. I consider solving physics problems to be very useful.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Solving physics problems helps/doesn't help students to

....

....

6. I believe that the history of physics can make a significant contribution to the teaching of physics.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

The history of physics can/ cannot make an important contribution to the comprehension of

....

....

7. I believe that the history of ideas and concepts that have led to the development of physics is very important for students to completely understand basic physics concepts.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

The history of the development of physics concepts can/cannot help students to ....

....

8. I would like to learn more about the biographies of scientists.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Learning the biographies of scientists can/cannot make an important contribution to the comprehension of

....

....

9. I very much appreciate reading original texts (books and scientific papers) during physics lessons.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Reading original texts can/cannot make an important contribution to the comprehension of

....

....

10. I consider it instructive to analyse the original historical documents (notes, letters, etc.) during physics lessons.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
Reading original documents can/cannot make a specific contribution to the understanding of

....

....

11. I find it very useful to read summary texts written by researchers into the history and the teaching of physics
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
Reading texts written by researchers into the history and the teaching of physics helps/doesn't help the students to

....

....

12. I find the narration of historical anecdotes interesting.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
The narration of historical anecdotes can/cannot enrich physics lessons because

....

....

13 I think that history-of-physics books are difficult to understand
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
I find the texts of history of physics difficult/easy to understand because

....

....

14. I believe that the replication of the experiments that led to the development of physics can be very useful for education.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Laboratory experiments of historical interest can/cannot help to better understand

....

....

15. The replication of experiments of historical interest unnecessarily complicates the learning of physics.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Following the reproduction of experiments of historical interest is/isn't difficult because

....

....

16 . I'm very interested in attending the replication of famous physics experiments for educational purposes.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Attending the replication of famous physics experiments can/cannot promote

....

....

17. I am embarrassed to perform physics experiments first-hand.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

I have/haven't difficulty carrying out physics experiments because

....

....

18. I find it interesting to plan an experiment as if I were a real scientist

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Planning an experiment means to me that

....

....

19. I believe that the history of physics can emphasize the relationship between science and society
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
For example, the history of physics can emphasize the relationship between science and society in the case

....

....

20. I think that modern technology can make a valuable contribution to the teaching of physics.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
Modern technology can/cannot help students to

....

....

21. I am very interested in the context in which physics has historically developed.
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
The historical context of physics is really /isn't useful to

....

....

22. I greatly appreciate that educational laboratory experiments are conducted under the same experimental conditions when they were carried out for the first time in the history of the physics
Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)
□1. □2. □3. □4. □5.
The replication of the original experimental conditions that were available at the time of the experiment can/cannot help students to

....
....

23. I find it useful to replicate the experiments that led to the development of physics using modern tools.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

The replication of experiments of historical interest with modern instruments can help students to

....

....

24. I prefer that physics theories are introduced to me in a different mathematical form (eg. more modern or simplified) from the way in which they were originally formulated

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Learning about original mathematical formulation of a physics theory can/cannot help students to

....

....

25. I am interested to learn about the difficulties experienced by scientists in their research and the way in which they overcame them

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

Learning about the difficulties overcome by researchers help/doesn't help to understand

....

....

26. I find it very instructive to compare successive theories in the history of physics and learn which experiments have determined the choice of the most correct one.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

The comparison between two different physics theories can/cannot be very instructive because

....

....

27. I think it unimportant to know the experimental anomalies that led to the rejection of scientific theories that turned out to be not exhaustive

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

The knowledge of the experimental anomalies that led to reject a scientific theory isn't/is helpful to

....

....

28. I would like it if more history-of-physics aspects than other elements were to be introduced in a physics course.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

More history of physics lessons help/doesn't help students to

....

....

29. I consider an educational path on the history of Galileo's physics useful for students.

Give an evaluation from 1 (strongly disagree) to 5 (strongly agree)

□1. □2. □3. □4. □5.

An educational path on the history of Galileo's physics is/isn't useful because

....

....

30. What do you think are the strengths of the educational path about Galileo's physics proposed to you for the understanding of what science is in its context and what the work of scientists is (the Nature of Science)? And the weaknesses?

....

....

....

....

5.6 Statistical analysis of the results of the questionnaire on motivational aspects: the Wilcoxon Signed Rank Test

The key point of statistical analysis of the results for the questionnaire on motivational aspects is to evaluate whether the variations in the score of every question before and after the laboratory historical educational path may be due to chance and therefore to consider the two groups of students as belonging to the same statistical population. On the contrary, we should reject this hypothesis and consider them actually unlike each other because of the teaching action that could, for this reason, be considered effective.

The first step would be to calculate mean, standard deviation and median of the scores of the answers given by the students to each question in the questionnaire and then apply a parametric analysis test. Parametric tests, such as the analysis of the mean or of the variance, are so called because they are applied to samples relating to distributions for which the statistical parameters that characterize them are known (or analyzed). If the type of distribution to which the data belongs is unknown, or if the observations are measured on an ordinal qualitative scale, parametric methods might be unreliable. In these cases, it is possible to use, instead of the observations, the ranks, i.e. the order number of the observations themselves, in order to calculate non-parametric hypothesis tests (i.e. tests free from distribution).

Thus, I performed the Wilcoxon Signed Rank Test (Wilcoxon 1945) on the scores (of each answer) of the entry/exit motivational questionnaire, before and after the laboratory historical educational path. The Wilcoxon Signed Rank Test is generally used in experiments in which the data are paired as the answers of a questionnaire before and after an educational path and in which the conditions of applicability of the parametric tests are not respected. The differences in scores (of each answer of each student) caused by the teacher action were calculated, and a rank was assigned to each difference in relation to the absolute value; then we could attribute the sign of the difference to the rank and, finally, we could calculate the algebraic sum of the ranks in order to obtain the statistical test W.

If the treatment has no effect, the ranks associated with positive variations should be almost equal to the ranks associated with negative variations and the sum of the ranks with sign W should assume a value close to 0. Vice versa, the more the educational activity modifies the scores of the answers, the more the variations will tend to assume the same sign. In this case, the sum of the ranks with the W sign will tend to take on a "large" positive or negative value. So, we can compare the W value obtained with the distribution of the possible W values to decide if it is "high" enough to reject the hypothesis that the teacher action had no effect. Other equivalent statistics tests are sometimes used for convenience. One might calculate the sum of the positive signed ranks, the sum of the negative signed ranks, and W_S, the smaller of both of them.

The closed answers to the motivational questionnaire (before and after the historical laboratory educational activity) were analyzed by the Wilcoxon sign test, in order to assess whether this path has produced a greater interest and penchant for physics and the history of physics. There was no need to compare the results of this questionnaire with those achieved by the students of a control group because all pupils have previously experimented with traditional physics teaching and therefore the comparison between before and after the didactic path is sufficient. Only useful information was drawn from the calculation of the median, the average and the standard deviation of the score given to each question. The Wilcoxon Signed Rank Test was repeated for each answer.

5.7 A first example of Wilcoxon Signed Rank Test

The students participated in the educational activities with a personal involvement by encouraging further exploration with respect to the previously planned educational path. Basing on the observation of students' participation, laboratory reports, didactic forms related to both experimental and theoretical activities, historical research and studies, oral tests, they are shown, in general, to have overcome their misconceptions. The results of the rigorous quantitative—and—qualitative analysis of the students' opinions and concepts they had learned confirms the great interest of the pupils especially for laboratory activities and the replication of experiments of particular historical value. As a critical point, the systematic study of Galilean motion theorems was a bit challenging - some demonstrations were left as optional - but overall the pupils greatly appreciated the study of Galilean physics in its context as a single body bearing the evidence of a renewed interest in physics.

For example, the application of Wilcoxon Signed Rank Test on the scores of the answers of question 17 of the entry/exit motivational questionnaire "I am embarrassed to perform physics experiments first-hand" in a third grade class of high school determined that we can reject the hypothesis that the changes are due to chance with a risk of an experimental error less than 1.9%.

Tab. 5.3 shows for each pupil in the class the score - level of agreement - recorded both before and after the educational historical path, the differences between the two, the division into ranks for the non-zero differences, the sum of the ranks, the number of non-zero differences[52].

[52] I always applied the classic Wilcoxon Signed Rank Test, which discards all zero differences. My choice was due to the fact that for several questions students gave the highest (5) or lowest (1) score in both the entry and exit questionnaires.

Person	Before	After	Diff
1	4	3	-1
2	3	3	0
3	2	1	-1
4	1	1	0
5	4	1	-3
6	1	1	0
7	1	1	0
8	1	1	0
9	4	1	-3
10	4	1	-3
11	3	1	-2
12	2	1	-1
13	2	2	0
14	2	1	-1
15	1	1	0
16	3	2	-1
17	1	2	+1
18	2	1	-1
19	5	2	-3

x_{2i}-x_{1i}	R_i
-1	-4
-1	-4
-1	-4
-1	-4
-1	-4
-1	-4
+1	+4
-2	-8
-3	-10.5
-3	-10.5
-3	-10.5
-3	-10.5

ND
12

Sum Ri	W_S=Min $Sum_{+/-}$ R_i
-70	+4

Tab. 5.3 Application example of Wilcoxon Sign Test on the scores of the entry/exit motivational questionnaire before and after the laboratory historical educational path.

Based on the probability table for Wilcoxon with N=12 e W_S=4 (Mosteller and Rourke 1973, p. 341), it was found that we could reject the hypothesis that the changes were due to chance with a risk of error less than 2.1% and therefore we have strong evidence that the historical didactic path was effective, at least as regards the experimental aspect in that class.

5.8 The general statistical analysis of the results of motivational tests

The following table n. 11 shows the results of the test on the opinions and motivations of the students who followed the historical educational path and who carried out this part of the questionnaire both before and after the didactic action.

In the table, m is the median of the score, μ is its average, σ is the standard deviation, *ND* is the number of different answers between before and after the educational path, W_S is the Wilcoxon statistical parameter corresponding to the minimum of the sums of the ranks of the same sign and p is the probability of making a mistake by rejecting the hypothesis that the changes were due to chance and so assuming that the students have changed their opinion regarding what is stated in the question.

An analysis of the scores assigned to the various answers in both the entry and the exit questionnaires showed that the students considered physics a fairly interesting subject (question N. 1) and not excessively complicated (N. 2). However, the students attached great importance to the replication of the experiments that led to the development of physics (N. 14).

The effects of the educational action were assessed through the variations in the score by which students agree with the statements of the questionnaire. Taking into account the answers, there is a strong evidence that the understanding of physics as an experimental science had improved (N. 3) but also of the interest in the context in which physical theories were developed (N. 21) (with the probability of being wrong of 0.004 and 0.057 respectively): they are both aspects that combine to clarify what the nature of science is. The students also changed their views on their own participation in physics experiments (N. 17). Useful indications were also provided regarding the methodology to be followed in introducing the history of physics to upper secondary school. In fact, the pupils would like to devote themselves less to the direct study of books and scientific articles (N. 9) that are difficult to understand (N. 13) but students prefer to take advantage of the contribution and mediation of researchers in history and physics education (N. 11), i.e. to writings prepared especially for them.

N	*Question*[53]	*Before*			*After*			*ND*	W_S	*p*
		m	*μ*	*σ*	*m*	*μ*	*σ*			
1	I am heavily intrigued with the study of physics	4	4.04	0.79	4	4.00	0.90	37	324	0.655
2	I find physics obscure and complicated	2	2.40	0.84	2	2.33	0.99	50	540	0.317
3	I am very interested in the study of physics as an experimental science	4	4.01	0.85	4	4.31	0.83	50	356	0.004
4	I regard the relationship between physics and mathematics as very fruitful for the development of science	5	4.47	0.73	4.5	4.33	0.78	37	259.5	0.139
5	I consider very useful solving physics problems	4	4.30	0.77	4	4.16	0.78	52	545.5	0.163
6	I believe that the history of physics can make a significant contribution to the teaching of physics	4	3.50	1.10	3	3.46	1.08	57	776	0.678
7	I believe that the history of ideas and concepts that have led to the development of physics is very important for students to completely understand basic physics concepts	4	3.67	1.07	4	3.66	1.12	47	553	0.908
8	I would like to learn more about the biographies of scientists	3	2.91	1.26	3	2.72	1.17	57	655	0.161
9	I appreciate very much reading original texts (books and scientific papers) during physics lessons	3	3.10	1.19	3	2.81	1.16	59	646.5	0.063
10	I consider instructive to analyse the original historical documents (notes, letters, etc.) during physics lessons	3	3.00	1.22	3	2.93	1.21	65	1023.5	0.743

[53] The data analysis was generated using the Real Statistics Resource Pack software (Release 7.6). Copyright (2013 – 2021) Charles Zaiontz. www.real-statistics.com. With kind permission. Date of access to the website 15 March 2022. In the case where the number of differences was greater than 30, I considered the normal distribution for Ws. I also applied the correction for a non-continuous variable (see Appendix).

11	I find it very useful to read summary texts written by researchers into the history of physics and physics education	4	3.58	1.07	4	3.71	1.05	53	603	0.305
12	I find the narration of historical anecdotes interesting	4	3.71	1.15	4	3.69	0.99	57	789	0.760
13	I think the texts of history of physics are difficult to understand	3	2.69	1.03	3	3.20	1.13	64	612	0.003
14	I believe that the replication of the experiments that led to the development of physics can be very useful for education	5	4.61	0.79	5	4.65	0.59	35	311	0.950
15	The replication of experiments of historical interest complicates the learning of physics without reason	1	1.98	1.24	2	1.93	1.14	50	598.5	0.703
16	I'm very interested in attending the replication of famous physics experiments for educational purposes.	5	4.21	0.99	4.5	4.26	0.91	46	512.5	0.755
17	I am embarrassed to perform physics experiments first-hand	2	2.07	1.17	1	1.85	1.26	52	526.5	0.129
18	I find it interesting to plan an experiment as if I were a real scientist	5	4.24	0.99	4	4.15	0.98	39	357	0.642
19	I believe that the history of physics can emphasize the relationship between science and society	4	3.58	1.21	4	3.74	1.20	50	533	0.300
20	I think that modern technology can make a valuable contribution to the teaching of physics	5	4.56	0.72	5	4.69	0.63	31	169.5	0.096
21	I am very interested in the context in which physics has historically developed	3	3.31	1.20	3	3.58	1.18	50	447	0.057
22	I greatly appreciate that educational laboratory experiments are conducted under the same experimental conditions when they	4	3.72	1.08	4	3.78	1.00	63	945	0.659

	were carried out for the first time in the history of the physics								
23	I find it useful to replicate the experiments that led to the development of physics using modern tools	4	4.19	0.90	4	4.08	1.03	51	590.5	0.480
24	I prefer that physics theories are introduced to me in a different mathematical form (e.g. more modern or simplified) from the way in which they were originally formulated	4	3.74	1.08	4	3.94	1.05	55	623.5	0.211
25	I am interested to learn about the difficulties experienced by scientists in their research and the way in which they overcame them	4	3.66	1.14	4	3.76	1.27	51	585.5	0.457
26	I find it very instructive to compare successive theories in the history of physics and learn which experiments have determined the choice of the most correct one	4	3.63	1.15	4	3.78	1.07	58	708.5	0.236
27	I think it unimportant to know the experimental anomalies that led to the rejection of scientific theories that turned out to be not exhaustive	3	2.83	1.26	3	2.71	1.20	61	825.5	0.380
28	28. I would like that more history-of-physics elements than other features should be introduced in a physics course	3	2.57	1.19	2	2.40	1.05	32	613.5	0.173
29	I consider useful for students an educational path on the history of Galileo's physics	4	3.82	1.01	4	3.87	0.94	47	519.5	0.629

Tab. 5.4. General statistical analysis of motivational questionnaire before and after the laboratory historical educational path for 90 students.

5.9 Sharing the project: the teachers training course

Once the historical educational path was defined, in May 2019, I organized a training course at the *Liceo Scientifico "F. Sbordone"* in Naples about Galileo’s contribution to the development of physics and mathematics in the context of basic physics education and experimentation of innovative interdisciplinary paths in order to share with my colleagues the first results achieved and prepare a broadening of experimentation. This in–service training course lasting a total of 12 hours in 3 weeks. It was devoted to twenty physics/mathematics teachers from ten different high schools in the province of Naples.

In order to take into account the training needs of the teachers involved, the course covered an historical deepening on the parabola and Galileo and his reform of the theory of proportions for the study of the physical world. It introduced the experiments listed in the previous paragraphs in which teachers collaborated in the practical implementation and discussion of possible improvements.

The teachers were provided with explanatory sheets of the experiments with an indication of the conceptual issues and possible difficulties of the students.

Fig. 5.5. The training course for teachers (with permission).

Their interest and satisfaction were clearly–positively declared by their answers to questionnaires submitted to them. They especially appreciated the role that the history of physics can play in physics education, particularly when it is linked to laboratory activities, however they also highlighted their need for training in it.

Table 11 shows the strengths and weaknesses of the teacher training course - and therefore of the historical educational path - identified by the teachers who participated in the training course.

We are planning, when the COVID emergency ends, to propose the implementation of the historical educational path in the institutes and classrooms of the trainees, to also extend the experimentation further.

Strengths	**Weaknesses**
✓Physics presented in this course is based on both the theoretical and experimental modules ✓The historical references that made it possible to reproduce the experiments ✓The material viewed and distributed to reproduce the experiments ✓Discussions with colleagues ✓Simplicity with which the experiments were carried out ✓The lessons with historical references were very interesting ✓It is interesting to retrace the historical aspect that brought the evolution of ideas into the reality of those times ✓It is very useful for teaching to reproduce laboratory experiments as they have been conceived and analyzed in the history of physics taking into account the context ✓Experimental activity helps to overcome misconceptions	➢The educational path has little applicability in professional institutes ➢It was not possible to deepen the historical references properly ➢The duration of the course should be longer. also giving more space to the practical laboratory activity ➢At school. it is difficult to find the time to complete the educational path ➢Lack of adequate instrumentation in schools

Tab. 5.5. Strengths/weaknesses of the teachers' training course reported in the exit questionnaire.

Epilogue V

In this chapter, I evaluated the effectiveness of the teaching action. I constructed two different questionnaires, on concepts, knowledge and skills acquired by the students and on their interests and motivations about physics and the work of scientists, which I administered to the students both before and after the teaching action, in order to assess the achievement of the objectives I had set.

By focusing on the questionnaire relating to knowledge and skills, also taking into account what is reported in the research literature in the educational field, I was able to further evaluate the students' initial conceptions in kinematics and dynamics, I analysed all the different final answers given to each question by deriving the percentage of correct answers for each question in order to identify the learning difficulties encountered at the end of the educational action (also as a didactic reflection for possible improvements to the project), I drew indications on the curriculum of the school subjects thanks to the comparative analysis of the answers with reference to the class attended by the students.

Focusing on the second questionnaire, I carefully considered how to evaluate the effectiveness of the project by examining the students' preferences with regard to the various parts of physics at the end of the course and comparing them with the initial situation before the teaching activity was implemented. In particular, I evaluated, pupil by pupil, for each of the questions, the differences between the levels of interest indicated in the final and initial answers; I analysed the results obtained by means of the Wilcoxon Signed Rank Test, not finding parametric tests suitable; I obtained very interesting elements on the teaching of the Nature of Science because I found strong evidence that the students had changed their interest in physics, considered as an experimental science, and in the context in which physics has historically developed.

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PART IV

CONCLUSION

CHAPTER VI

Concluding Remarks

6.1 General Conclusion

This interdisciplinary thesis (History of Physics, Mathematics, Epistemology, Philosophy of Science, Educational Sciences, Nature of Science, Teaching Science) is part of the historical study and teaching of classical physics (Kinematics and Dynamics) and deals with the case study of the history of the discovery of the laws of falling bodies and the motion of the projectile.

The narrative I have reconstructed in my thesis portrays a summary of the main historical developments in the study of the subject from the time of Aristotle to Galileo. My analysis takes into account the most recent critical studies and original manuscripts and has several elements of originality both in the reconstruction of the way in which Galileo's experiments were carried out (e.g. the role assigned to the pendulum for the measurement of time) and in the organic connection between them that I have highlighted.

The synthesis I produced has included both theoretical and experimental elements with the little-known contributions of scholars who in some respects anticipated Galileo and who deserve to be studied more closely. This serves to highlight how scientific progress is always the result of the efforts of a community of scholars in addition to the undeniable contributions of the great personalities who, with their intuition and creativity, have made the history of science. Analysing the works of these scientists, situating their studies and choices in the historical period in which they lived − to form a detailed synoptic picture − has the advantage of helping us understand the choices that scientists made and humanise science by linking it to the ethics and values that inspired them. In general, the study of the context, the experimental conditions present at the time of the experiments, the mathematics that could be used, and the particular relationships between science and society have highlighted the importance of considering the Nature of Science framework as the best candidate for the teaching-learning of science.

The novelty and originality of this work lies precisely in the area of "history of science and teaching" because it constitutes an example in which this richness has been implemented in a physics teaching path, held in curricular hours, whose outcome and impact on the interest of students (from the first three years of the Italian *Liceo scientifico*) have been carefully examined.

Naturally, in designing the educational path I took into account the most modern physics teaching theories (etc., learning by doing, inquiry-based learning), also drawing on my thirty years' experience of teaching in high school.

The experimentation carried out, which involved eight classes in the first three years of the *Liceo scientifico*, was also important in defining the curriculum and the subdivision of the learning topics in the various years of study, based on the results obtained regarding the interest and learning difficulties shown by the students.

An important aspect, however, has been successfully reproducing with the students, in laboratories and school classrooms, the famous experiments performed by Galileo that sanctioned the birth of the experimental method and that have been the subject of criticism and discussion from the time of the publication of Galileo's *Discorsi* to the present day.

6.2 Specific Conclusion

In this thesis work, I tried to address the Nature of Science (NoS)-teaching of science and educational aspects of the teaching-learning of physics in high school and university, placing the student with his learning difficulties at the centre of my action.

The first part of my work, is dedicated to the analysis of historical sources dealing with the fall of bodies and the motion of projectiles that served as a basis for designing an educational path for upper secondary school students. The historical period I have covered with goes from the Greek age to the seventeenth century. In particular, in the first chapter I analysed *The first studies on the fall of bodies*, going to examine in detail the issues that remained open in Aristotle's physics regarding these topics that correspond precisely to the major difficulties encountered by students in dynamics.

1. I began by illustrating Aristotle's thinking about motion by starting with the distinction between natural motion and violent motion. As for the former, Aristotle emphasised the dependence of the free-fall velocity of bodies on their weight. Similar to students' common sense conceptions, their velocities are considered to be the greater the greater their mass. Aristotle's conception of falling bodies in fluids is also similar to that generally assumed by students that velocity is inversely proportional to the density of the medium. This led Aristotle to consider the impossibility of the existence of a vacuum because it corresponds to a zero density, for which velocity would be of infinite magnitude.
2. I investigated the theories reported by Aristotle that justified how the violent motion of a projectile could continue after the application and the removal of the force that had caused it. The first, probably due to Plato, involved the mutual replacement of the air that, pushed by the projectile, in turn pushed more air to take the place left free by the projectile, not allowing the formation of the vacuum, and that - attracted quickly - gives the necessary thrust for motion. The second included the idea that the air in contact with the original motor moves the contiguous air up to the projectile in order to give it continuous motion.

I outlined the main criticisms that were levelled at these conceptions and how they had evolved over time.

3. Hipparchus believed that the motion of the projectile takes place in three phases, the first of which is characterised by the effect of force; during the second phase this effect decreases and that exerted

by its weight begins, which completely determines the motion of the object in the third phase.

4. Philoponus, regarding the mutual replacement theory, considered it unreasonable to think that air could move in a circular way and judged it wrong to consider the action due to the thrust of the air because it meant not taking into due account the action of the force, without which, in fact, motion does not occur.
5. I found that Philoponus, many years before Galileo, observed two bodies of different weights fall from the same height with very small differences in fall times, refuting the position of Aristotle.
6. Jean Buridan to describe violent motion introduced the concept of *impetus* as the force impressed on a body; it would be a permanent quality of the body, responsible for its motion, if not for the resistance suffered or due to other tendencies to movement. I discussed the various interpretations of this quantity by historians of physics as an anticipator of modern momentum.

With reference to other authors' anticipations of the mathematical relations that are generally attributed to Galileo, I obtained the following results, then clarified the originality of his contribution.

7. I found that William Heytesbury, one of the most representative exponents of the Oxford school, had enunciated a property of accelerated motion that anticipated Galileo's rule of odd numbers.
8. Moreover, another important and well-known result of the Oxford school was the mean speed theorem, used by Galileo to demonstrate the famous law of naturally accelerated motion.
9. In a simplified form, this had also been anticipated by Nicole Oresme. Oresme had also proved the mean value theorem, in a more general form of the speed theorem, using the equivalence of plane figures.
10. I thus obtained the result that Galileo's major and certainly original contribution was - in addition to having systematised the theory of accelerated motion in an organic manner - that of having associated this type of motion with the motion of free fall and having proved this experimentally.
11. As a final point, I demonstrated that Nicolò Tartaglia had determined the launch angle to obtain the maximum range of the projectile, a very interesting question even in those days, for the relations between science and society.

In the second chapter, *The fall of bodies and the motion of the projectile in Galileo's writings*, I developed and examined the evolution of Galileo's thought from his critique of Aristotle's thought and up to the analysis of his laboratory experiments that constituted an important advance in the history of science for the birth of the experimental method and in particular the formulation of the law of falling bodies. I studied the Galilean sources in detail: his rich and organic volumes, and his manuscripts with his laboratory notes that allowed me to reconstruct details on the experiments he performed. They are preserved at the *Biblioteca Nazionale Centrale di Firenze*, which has authorized me to reproduce them in this publication. In particular, I obtained the following results.

12. I found that Archimedes influenced Galileo both in the determination of the falling velocity of bodies in media, rendering Aristotle's conclusion about the impossibility of the existence of a vacuum inconsistent, and in the determination of the upward force necessary to balance the effect of gravity on an inclined plane, considering in the proof bent levers moving along a circumference.
13. I discussed the important role played by the *Dialogo sui due Massimi Sistemi del Mondo* in introducing the principle of inertia − and later the principle of relativity − and how Galileo used it to interpret violent motion and introduce the composition of simultaneous motions. Galileo used his arguments to defend his thesis on the motion of the Earth. His arguments, after he was first enjoined by Cardinal Bellarmine not to defend the Copernican system, led to the condemnation he received from the Holy Office. I therefore proved that the *Dialogo* is an important text for understanding the work of scientists and the relationship between science and authority/religion.
14. I analysed in detail the third and fourth day of Galileo's *Discorsi su due nuove scienze*, reconstructing − even with modern mathematical expressions − his theorems on naturally accelerated motion and parabolic motion.
15. I studied in detail the description of Galileo's experiments on the fall along an inclined plane and the pendulum, analysing the relationship between them and arriving at the hypothesis, based on some of Galileo's statements and his laboratory notes, that he actually used the pendulum to prove the odd-number law of naturally accelerated motion along an inclined plane.
16. Finally, I analysed Galileo's laboratory notes in order to reconstruct the experiments on parabolic motion that he carried out, understanding the conditions and results achieved, so that they could serve as a starting point for further experiments.

In the second part of this thesis, I set out the fundamentals of didactics and physics education that I have studied and analysed in order to design and implement an interdisciplinary and innovative educational path − with which I have been acquainted for several years as I have been a high school physics teacher for thirty years, a trainer in science didactics, and an organiser of teacher training courses as head of the Naples section of *AIF (Associazione per l'Insegnamento della Fisica)*.

In the third chapter, *From research in physics education to an innovative historical educational path*, I analysed the following points:

17. the different relationships between the subjects of teaching (the student and the teacher) and knowledge;

18. the problem of teaching scholarly knowledge in the classroom (the didactic transposition);
19. the particular competences of teachers as “amalgam of content and pedagogy […] for instruction” (*Pedagogical Content Knowledge*);
20. the reconstruction of the contents to be taught in high school starting with the consideration of the initial conceptions of students (the Model of Educational Reconstruction);
21. the contribution of the history of physics to learners' conceptual change and the special role played in education by the replication of experiments that were historically relevant for the development of physics.

Following the *Educational Reconstruction Model*, I created a design sheet outline that takes into account the structured academic knowledge, the elementary knowledge accessible to students and the logical organisation of teaching content in a way that makes sense to students. I used it to design an original interdisciplinary (Physics / Mathematics / History of Physics) educational path for students, with both theoretical and experimental activities, characterised by an organic series of experiments derived from Galileo's writings and laboratory notes.

22. I detailed all of Galileo's writings, which I used to facilitate the repetition of the course by other physics teachers or researchers in the history of physics and physics education.
23. I created a new teaching method – as a combination of inquiry-based learning and the analysis of historical sources. Indeed, a guided inquiry process is used: lessons are generally organized with a predictive-exploration-comparison activity in which the reading and discussion of the historical texts written by Galileo are integrated with the conducting of experiments.
24. I applied this method, preparing guided inquiry sheets for each teaching unit I designed together with support sheets for remedial activity.

In the fourth chapter, *The replication method at the Liceo Scientifico 'F. Sbordone' in Naples. Results of experiments and discussion*, I analysed the results of the experiments I held with my students at the *Liceo Scientifico Sbordone* where, at my suggestion, a Galilean physics laboratory was set up in which the experiments performed by Galileo were replicated, attempting to reproduce the experimental conditions present in his time or in some cases with modern devices that were easily accessible and usable by the students. In particular, I obtained:

25. exploration of the phenomenon of the oscillation of two pendulums with the observation that the oscillation time does not depend on the mass and amplitude (small) but only on the length of the wire; determination of the law of proportionality between the period of the pendulum and the square root of the length of the wire;
26. measurement of the period of oscillation of a pendulum 0.25 m long - corresponding to 1 s;

27. construction of a water clock using a cylindrical tank with a tap and its calibration using the pendulum;
28. identification of the difference between the two instruments with regard to the type of measurements (discrete for the pendulum, continuous for the water clock);
29. exploration of the phenomenon of the fall of two balls of the same size but with different masses in different media (glycerine, oil, water, air) with the observation that the difference between fall times is smaller and smaller as the viscosity of the medium decreases; following Galileo, by extrapolation, the two bodies falling into the void reach the ground together;
30. quantitative investigation into the measurement of the terminal velocities of fall using the Tracker Video Analysis and Modelling Tool for Physics Education software, which made it possible to affirm that these velocities are proportional to the difference between the density of the ball and that of the medium, if the latter is viscous such as glycerine, or to the square root of the differences between densities in the case of oil and water (in these cases, according to the model created by applying the first law of dynamics, the resistance of the medium is proportional to the square of the velocity);
31. determination of the law of quadratic proportionality between the distance travelled by a steel ball falling along an inclined plane 6.75 m long and the time taken (measured with a water clock); with a few small tricks - such as dropping the ball with one hand and opening the tap with the other by the same experimenter - I obtained a measurement uncertainty of a few tenths of a second on the times;
32. determination of the law of odd numbers for the spaces travelled along an inclined plane in equal and successive time intervals (measured with the pendulum and a system of bells);
33. verification of the law of proportionality between the range of a projectile launched horizontally from a fixed height and its initial velocity (since the fall time does not change, this means that the motion is uniform in the horizontal direction, thus confirming the principle of inertia).

The third part of the thesis is devoted to this NoS case study on the kinematics of Galileo; the results of the entry/exit questionnaires completed by the students are examined in order to understand the effectiveness of the teaching action both in terms of the knowledge and skills acquired by the students and in terms of changes in their interests in the history of physics and physics more generally.

In chapter five, *The analysis of the questionnaires on students' acquired knowledge and skills and on their interests and motivations*, I outline the tools and methods I developed to evaluate the effectiveness of the educational action. This involved constructing two different questionnaires to assess the attainment of the objectives I had set myself, both with reference to the learning of the concepts of the discipline studied and the variation of interest in the various aspects of science and the understanding of the NoS. Focusing on the first questionnaire, also taking into account that reported in the research literature in didactics, I obtained the following results:

34. the examination of students' pre-instructional conceptions obtained during an analysis phase prior to the actual experimentation;
35. an original questionnaire with 23 open-ended questions, in order not to influence the students' answers in any way, and only two multiple-choice questions where the possible answers were graphically schematised;
36. the examination of the answers given to the entry questionnaire, for a more accurate study of the students' initial conceptions;
37. the analysis of the different answers given to each question with the choice of the correct ones and the attribution of the score i.e. the percentage of correct answers for each question;
38. indications on the school subjects' curriculum thanks to the comparative analysis of the answers according to the class attended by the students;
39. the study of the learning difficulties encountered at the end of the educational action, also as a didactic reflection for possible improvements to the project.

Focusing on the second questionnaire, I carefully considered how to evaluate the effectiveness of the project by examining the students' preferences regarding the various parts of physics at the end of the educational path and comparing them with the initial situation before the implementation of the educational activity. In particular:

40. I created an original entry/exit questionnaire that consists of 29 questions (5-level agreement/disagreement type) with 30 open-ended sub-questions. It concerns the motivational aspects. I used this test in order to evaluate if the educational path was able to increase students' interest in various parts and aspects of physics and in the history of physics;
41. I assessed, pupil by pupil, for all 90 students who had answered all the questionnaires, the differences between the levels of agreement recorded in the pre- and post-questionnaire responses;
42. I analysed the results obtained by means of the Wilcoxon Signed Rank Test, not finding parametric tests suitable;
43. I obtained very interesting elements about the teaching of the Nature of Science because I found strong evidence that students had changed their interest in physics, considered as an experimental science, and in the context in which physics has historically developed;
44. I obtained important methodological indications because I found strong evidence that the students considered it difficult to understand the original texts (books or scientific papers) relevant to

the history of physics, despite the fact that they had appreciated the didactic proposal - organised in an organic manner - that had been made to them and that I had tried to simplify the texts written by Galileo by translating them into a more modern Italian language. Instead, I found almost discrete evidence that the reading of summary texts written by researchers into the history of physics and physics education was very useful to students.

Although I am very satisfied with the work done, I am aware that it can be further extended and improved. It might be appropriate to:

45. compare the levels of knowledge achieved by students who participated in this interdisciplinary Physics / Mathematics / History of Physics activity / educational path with the levels achieved by students who followed a traditional path (control group);
46. monitor the results of activities carried out by other upper secondary school teachers in other schools, which at the moment, although it was planned (see the teacher training course I held at the *Liceo Scientifico Sbordone* in May 2019) due to the COVID pandemic, could not be carried out;
47. extend the experimentation to university students and compare their answers with high school students
48. extend this Physics, Mathematics and History of Physics experimentation to other topics that can be studied experimentally as well as theoretically in both classical and modern physics
49. involve other school disciplines in this study, such as literature (see for example my 2011 paper on *The Life of Galileo* by Bertold Brecht), religion (see my presentation in the Appendix on the proceess to Galileo and Oppenheimer), Latin (for the Italian translation of part of the *Discorsi*) and so on.

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Facendo seguito alla mia richiesta dell'11 settembre 2018, di utilizzare, nella mia tesi di dottorato, immagini o parti d'immagini dei seguenti manoscritti del Fondo Galileiano (immagini scaricate dai siti https://www.bncf.firenze.sbn.it. e http://www.imss.fi.it):
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80131 Napoli
Tel. 3280142719

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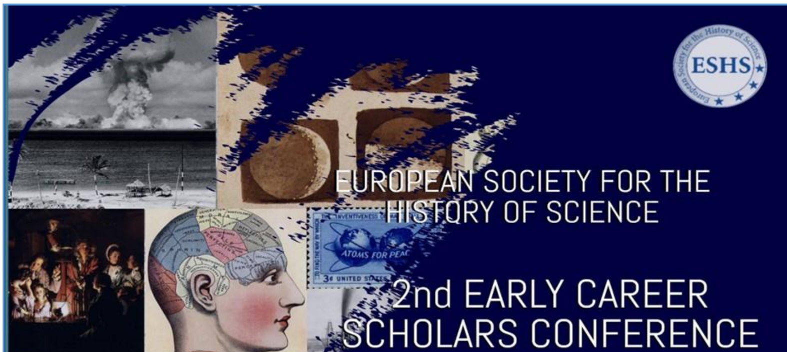
On 18/10/2022 20:31, vincenzo cioci wrote:

> Dear Dr. Zaiontz
> As I already wrote to you a few months ago, I am a PhD student at the University of Lille and I used your site for my experimentation.
> I would be very grateful if you would give me permission to include in my doctoral thesis the Wilcoxon Signed-Ranks Table retrived at https://www.real-statistics.com/statistics-tables/wilcoxon-signed-ranks-table/
> and the Excel spreadsheet representation I used (I enclose 2 examples) and which I obtained by appropriately modifying your examples.
> Thank you for everything
> Vincenzo Cioci
>
> Il giorno mar 28 giu 2022 alle ore 08:38 Charles Zaiontz <czaiontz@gmail.com> ha scritto:
>
>> Hello Vincenzo,
>>
>> You don't need permission to use my website to help you analyze your results. In any case, I am pleased that you have chosen to use Real Statistics. You can cite it as described at
>>
>> https://www.real-statistics.com/appendix/citation-real-statistics-software-website/
>>
>> Charles
>>
>> On 27/06/2022 23:38, vincenzo cioci wrote:
>>
>>> Dear Dr. Zaiontz
>>> First of all, I want to congratulate you on the important work you have done in the edition of the real-statistics.com website
>>> I am a physics teacher in a scientific high school in Naples. I am currently a doctoral student in history and physics education at the University of Lille (France).
>>> I carried out an educational experimentation with high school students where I teach.
>>> To review a questionnaire, I am using the Wilcoxon Signed Ranks Test.
>>> I ask your permission to use the resources of your web-site to analyze the results of the experimentation in my doctoral thesis and in any publications that may come out.
>>> I also take this opportunity to ask you how you wish to be cited and if you have written papers about your website or if there are other scientific works that cite it.
>>> Best regards
>>> Vincenzo Cioci

2. Conference presentation



Conflicts and Alliances in Sciences

Galileo and Oppenheimer: history of two scientists fought but not won

VINCENZO CIOCI

PhD Student, CIREL, University of Lille, France

Introduction

Portrait of Galileo Galilei by Ottavio Leoni, 1624 (public domain)

J. Robert Oppenheimer. Source: Los Alamos National Laboratory, U.S. Department of Energy (use with no charge)

This paper presents a comparison between Galileo and Oppenheimer, focusing on the trials that both scientists had to endure. A common aspect at the origin of the two judicial proceedings is the relationship between science and authority: political and religious. The affairs of the two scientists, with the initial condemnations and their late rehabilitations, help to clarify what the relationship should be between science and faith and between science and ethics.

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Galileo on trial

In 1633 Galileo was tried by the court of the Holy Office

- for having supported in the *Dialogue over the two greatest systems of the world* the Earth's motion and the Sun's immobility (absolutely and not only as a hypothesis),
- for having used the inconclusive argument of the ebb and flow of the sea,
- for having ridiculed the thought of Pope Urban VIII on the question ("mistreatment of contrary authors"),
- for having disobeyed the injunction that had been imposed on him in 1616 by the same Holy Office, that he could not "hold, teach or defend the Copernican system in any way, orally or in writing, otherwise the Holy Office would initiate proceedings against him".

(Favaro 1890-1909, vol. XIX, pp. 326-327. English translations of writings about Galileo trial are mine. Compare with Finocchiaro 2014, pp. 121-122)

How did these charges come about?

Giorgio de Santillana (1955) argued that Galileo's quarrel with the ecclesiastical authorities had first and foremost originated from the insidious machinations of the Aristotelians, supporters of the Ptolemaic system, who, failing to contain his arguments rationally, shifted the terms of the discourse to the theological plane, emphasizing that the Copernican system was contrary to the literal interpretation of the Scriptures.

Galileo and the interpretation of the Scriptures

Galileo expressed his position in the so-called "Copernican Letters" of which the best known are those addressed to Benedetto Castelli (1613) and Madame Christina of Lorraine (1615).
He argued that "in disputes about natural problems, one should not start from the authorities of the texts of Scripture, but from sensible experiences and necessary demonstrations".
Concerning the truthful value of the sacred writings, Galileo cited a famous expression attributed to Baronius according to which "the intention of the Holy Spirit is to teach us how to go to heaven, not how the heavens go". So, "scriptures being unable to lie, theologians should penetrate the true meaning of the sacred texts, that cannot be inconsistent with the certain and proven scientific conclusions".

(Favaro 1890-1909, vol. V, pp. 281-288, 309-348. Finocchiaro 2014 pp. 43-48, 48-77)

Galileo, the Church and the autonomy of scientific research

Prophetically, the scientist argued that:

"it would be very prudent not to allow anyone to force scriptural passages to have to support the truth of any physical conclusion whose contrary could be proven by the senses and necessary demonstrative reasons"

(Favaro 1907, vol. XIX, pp. 300. Finocchiaro 2014 p. 45)

With these words, Galileo described precisely the error that the Catholic Church would have committed, by denying «legitimate autonomy» to scientific research.

The long road of the Catholic Church

The attitude of the Church towards Copernicanism was expressed by Cardinal Roberto Bellarmino, who had employed much of his energy in defending «Catholic tradition» against Protestantism. A letter which he wrote on 12 April 1615 to the Carmelite Foscarini, reiterated that only when a certain demonstration of the Copernican system had been given, would it be justifiable to abandon the literal interpretation of the Scriptures as expounded by the Fathers of the Church (ENOG, vol. XII, pp. 171-172).

Galileo's reasons for this theme were fully accepted by Pope Leo XIII in the encyclical Providentissimus Deus only in 1893.

Meanwhile, after the "evidence" of the motions of revolution and rotation of the Earth - the measurement of the parallax of the star Vega by Giuseppe Calandrelli, in 1806, and the observation of the deviation of the motion of the bodies from the vertical by Giovanni Battista Guglielmini, in 1789 - thanks to the report of the Commissioner of the Holy Office, the Dominican Olivieri, and the relative Resolution of the Congregation in 1820, it was established that «the Copernican opinion is no longer subject to those difficulties in which it was involved in past times, before subsequent observations». Copernicus' Revolutionibus and Galileo's Dialogue did not appear in the following Index of Forbidden Books of 1835. (Brandmüller et al. 1992).

The Second Vatican Council, with the constitution Gaudium et Spes, called for the «legitimate autonomy» of science (n. 36) referring to the Galileo case (note 63).

Oppenheimer on Trial

- In 1954 J. Robert Oppenheimer was forced to submit to a long series of hearings to defend himself from the accusations of the United States Atomic Energy Commission (AEC) of being a potential security risk to his country.
- He had to answer for his leftist friendships - well known already during World War II - and especially for his opposition to the construction of the hydrogen bomb which, influencing the choices of other American scientists, had actually slowed its development.

Oppenheimer on H-bomb

In 1949 the General Advisory Committee of the AEC, of which Oppenheimer was the Chairman, was asked for an opinion on a priority programme for the development of the H-bomb.

The Committee expressed the view that, for the unique power of such a weapon, “its use therefore carries much further than the atomic bomb itself, the policy of exterminating civilian populations”.

The majority of the Committee, together with Oppenheimer, considered that the United States' commitment not to develop this bomb should be “unconditional”.

(Minutes of the EAC General Advisory Committee were published in full in York (1976) pp. 150-159)

Oppenheimer after the «sin»

After Hiroshima, Oppenheimer had become the symbol of the ethical crisis that had affected the physical sciences. He had abandoned war research and devoted himself with all his energy, as an advisor to the Government and the United Nations, to international atomic weapons control. By 1953, he had made numerous enemies opposing the strategic use of the hydrogen bomb by aviation. Instead, he had advocated the tactical use of the atomic bomb, on the battlefield towards military objectives, in order to balance the superiority of the Soviet armies, especially in Europe.

Galileo and Oppenheimer, two humiliated scientists ...

As Giorgio De Santillana has already observed (1955) Galileo and Oppenheimer, despite their considerable oratorical abilities, were both overwhelmed but their ideas, however, resisted.
Galileo was forced to abjure for having been judged "vehementemente sospetto d'heresia", Oppenheimer had his security clearance suspended, authorization for access to secret information, for having shown «fundamental defects of character». Their stories have taken on a character tragic enough to be represented in plays. While *The Life of Galileo* by Brecth (1938-1956) opened to the social responsibility of scientists, Kipphardt's drama was written when the scientist was still alive, showing him as a broken man for what he had done, such that he remained disgusted by the text (draft letter by Oppenheimer to David Bohm - JRO Papers box 20). The prestige of the two scientists did not allow them to be considered fully guilty. The Church and the AEC headed by Strauss achieved their goal: to destroy the two scientists and discourage others from taking similar positions.

The Charge against Oppenheimer

Teller, the father of the hydrogen bomb, with his testimony, while not explicitly arguing Oppenheimer's infidelity to the United States, nailed the scientist by stating:

"I thoroughly disagreed with him in numerous issues and his actions frankly appeared to me confused and complicated. To this extent I feel that I would like to see the vital interests of this country in hands I understand better, and therefore trust more."

(United States Atomic Energy Commission 1954 p. 710)

The questions about the 1616 injunction

The trial against Galileo, instead, beyond the same content of the Dialogue, whose censorship would have been the competence of the Congregation of the Index of Forbidden Books, originated in particular from a document the discovery of which in the archives of the Holy Office is still being discussed today.

This report (drawn up in the presence of witnesses and a notary), not signed, was the warning addressed to Galileo by cardinal Bellarmine to abandon the Copernican opinion and the subsequent injunction by the Commissioner of the S. Uffizio Michelangelo Segizzi «not to hold, defend or teach it in any way, orally or in writing,».

Wohlwill (1909-1926) had argued that it was a forgery.

The thesis of the plot was also supported by Redondi (1983; 1987), after the discovery, in the Archives of the Holy Office, of an anonymous censorship of the *Assayer* of Galileo in which the atomist conception expressed by the scientist was judged irreconcilable with the miracle of eucharistic transubstantiation.

Annibale Fantoli (2003), based on documents found a few years ago in the Vatican Archives, has advanced the hypothesis that, after the warning of Cardinal Bellarmino, Galileo has tried to explain his reasons and Commissioner Segizzi had spoken severely, but the report was not signed by Cardinal Bellarmino. Proof that he did not agree with the injunction is that, after a few months, he issued to Galileo, at his request, a statement stating that the scientist had not been forced to abjure «in his hands» but only that he had been notified of the decision of the Congregation of the Index on the theory of Copernicus, which Galileo could neither «defend nor hold».

Lewis Strauss made President Eisenhower believe that there were new elements about Oppenheimer beyond that which had already emerged during World War II.

The two trials are therefore fundamentally flawed.

They were also unjust in the course of their development since Galileo did not have any lawyer while Oppenheimer's lawyer was not allowed to consult the voluminous material related to more than ten years of FBI investigations on his client since the time of the Manhattan Project.

Neither was Galileo allowed to defend his scientific work nor was there any real discussion of Oppenheimer's views on the hydrogen bomb issue.

The debates focused only on the verification of the obedience of the two scientists to the established authority.

There are obviously differences, beyond the different historical period. The two authorities are different, one essentially political, that judged Oppenheimer, one of spiritual origin that condemned Galileo.
Moreover, while Oppenheimer was virtually destroyed by the process and could no longer participate in decisions regarding atomic policy of his country, Galileo, although in captivity, continued to research and publish.
The Church and the AEC achieved their goal in discouraging other scientists from taking similar positions.
But what damage there was in the relationship between science and faith and in the development of the arms race with the actual possibility of the destruction of human civilization!

As for the defensive line, Galileo stated that his true intention was to "refute the Copernican system" and not to prove it. Although it was clear to his judges that the statement was false, it was undoubtedly a cunning action, because Galileo could not affirm his true intention.

Oppenheimer's behavior was quite different, who, once put in a corner, evidently lied giving different versions of the "Chevalier" affair (a Soviet spy who had approached him).

Finally, it is necessary to evaluate the behaviour of Urban VIII who followed the trial of Galileo with great attention and certainly participated in the sitting of the Holy Office on 16 June 1633 which decreed the sentence of the scientist to prison (then commuted into a kind of isolation at his private residence), the abjuration and prohibition of *Dialogue*.

Of course the pope was also somewhat at the origin of the proceedings (as was Eisenhauer) when he realized that his arguments had been attributed to the foolish Simplicius.

They were not defeated: the legacy of Galileo and Oppenheimer

The two distinguished scientists have been fully rehabilitated. In 1979 Pope John Paul II reopened the Galileo case by establishing a Commission of Studies. In 1992 he recognized the responsibility for a «tragic mutual misunderstanding» between Galileo and the theologians of his time.

In 1963, shortly before his assassination, John F. Kennedy decided to award Oppenheimer the prestigious Fermi Prize. It was to be presented to him by his deputy, Lyndon Johnson, on December 2.

Oppenheimer receives Enrico Fermi Award . Source: Los Alamos National Laboratory, U.S. Department of Energy (use with no charge)

Conclusions

Although there are still requests to close the cases of the two distinguished scientists with even greater clarity (Segre 2011-2012, D'Agostino 2021), conclusions can be drawn on the issues that had been at the basis of the trials.
The problem of the relationship between science and faith, linked to the Galileo process, seems, at least in principle, to have been resolved with a clarification of the disciplines of knowledge, strictly delimiting the respective fields of application, better defining their different methods and the exact scope of the conclusions. The methodologies of each discipline will also allow different aspects of the complexity of reality to be highlight.
The question of the relationship between science and ethics is more relevant than ever. The idea of the social responsibility of scientist is increasingly spreading but it cannot be considered only an exclusively personal affair, linked to the conscience of the individual but it is a broader discourse that concerns the profession as a whole. In this context, specialist societies can also play a key role.

3. Spreadsheets for data analysis

The data analysis was generated using the Real Statistics Resource Pack software (Release 7.6). Copyright (2013 – 2021) Charles Zaiontz. www.realstatistics.com. With kind permission. Date of access to the website 15 March 2022.

	A	B	C	D	E	F	G	H	I
1	1. I am heavily intrigued with the study of physics.								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	4	4	0					
5	2	3	4	1	1	17	17		
6	3	4	5	1	1	17	17		
7	4	3	3	0					
8	5	4	5	1	1	17	17		
9	6	5	5	0					
10	7	5	5	0					
11	8	4	5	1	1	17	17		
12	9	3	3	0					
13	10	4	4	0					
14	11	5	4	-1	1	17		17	
15	12	3	4	1	1	17	17		
16	13	3	3	0					
17	14	4	4	0					
18	15	4	4	0					
19	16	5	4	-1	1	17		17	
20	17	4	4	0					
21	18	5	5	0					
22	19	4	4	0					
23	20	4	5	1	1	17	17		
24	21	4	1	-3	3	37		37	
25	22	5	5	0					
26	23	5	5	0					
27	24	3	4	1	1	17	17		
28	25	4	4	0					
29	26	5	5	0					
30	27	5	5	0					
31	28	4	4	0					
32	29	4	4	0					
33	30	4	5	1	1	17	17		
34	31	5	5	0					
35	32	5	5	0					
36	33	4	5	1	1	17	17		
37	34	4	5	1	1	17	17		
38	35	5	5	0					
39	36	5	4	-1	1	17		17	
40	37	4	4	0					
41	38	4	5	1	1	17	17		
42	39	4	4	0					
43	40	4	3	-1	1	17		17	
44	41	5	4	-1	1	17		17	
45	42	5	5	0					
46	43	5	5	0					
47	44	4	5	1	1	17	17		

	A	B	C	D	E	F	G	H	I
48	45	4	4	0					
49	46	4	4	0					
50	47	3	3	0					
51	48	3	5	2	2	35	35		
52	49	4	4	0					
53	50	4	5	1	1	17	17		
54	51	5	5	0					
55	52	4	4	0					
56	53	4	3	-1	1	17		17	
57	54	3	3	0					
58	55	3	4	1	1	17	17		
59	56	5	4	-1	1	17		17	
60	57	3	3	0					
61	58	5	5	0					
62	59	3	3	0					
63	60	2	1	-1	1	17		17	
64	61	5	4	-1	1	17		17	
65	62	3	3	0					
66	63	4	4	0					
67	64	3	3	0					
68	65	3	3	0					
69	66	5	5	0					
70	67	4	5	1	1	17	17		
71	68	4	3	-1	1	17		17	
72	69	4	2	-2	2	35		35	
73	70	5	5	0					
74	71	5	4	-1	1	17		17	
75	72	5	4	-1	1	17		17	
76	73	4	4	0					
77	74	3	3	0					
78	75	4	4	0					
79	76	2	3	1	1	17	17		
80	77	3	3	0					
81	78	3	4	1	1	17	17		
82	79	4	3	-1	1	17		17	
83	80	4	4	0					
84	81	4	3	-1	1	17		17	
85	82	5	4	-1	1	17		17	
86	83	5	5	0					
87	84	3	3	0					
88	85	5	4	-1	1	17		17	
89	86	3	3	0					
90	87	4	4	0					
91	88	4	4	0					
92	89	4	4	0					
93	90	5	3	-2	2	35		35	
94	Median	4	4	0		Sum	324	379	
95	Mean	4,04	4,00	-0,04					
96	St dev	0,79	0,90	0,78					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	37		=COUNT(E4:E93)			
8		Ws+	324		=G94			
9		Ws-	379		=H94			
10		Ws	324		=MIN(G94:H94)			
11		Ws-crit	221		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	351,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	4393,8		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	66,285		=SQRT(K13)			
15		Z-score	0,415		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	221,5		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,678		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,031		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	324		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,339		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,678		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,344		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,687		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	351,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	60,376		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,447		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,033		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,327		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,655		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,327		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,655		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	2. I find physics obscure and complicated								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	2	2	0					
5	2	3	4	1	1	20,5	20,5		
6	3	2	2	0					
7	4	3	2	-1	1	20,5		20,5	
8	5	2	2	0					
9	6	1	4	3	3	49	49		
10	7	1	5	4	4	50	50		
11	8	3	4	1	1	20,5	20,5		
12	9	4	3	-1	1	20,5		20,5	
13	10	2	4	2	2	44,5	44,5		
14	11	2	2	0					
15	12	3	2	-1	1	20,5		20,5	
16	13	2	2	0					
17	14	3	3	0					
18	15	4	5	1	1	20,5	20,5		
19	16	2	1	-1	1	20,5		20,5	
20	17	4	3	-1	1	20,5		20,5	
21	18	3	4	1	1	20,5	20,5		
22	19	2	2	0					
23	20	3	2	-1	1	20,5		20,5	
24	21	3	3	0					
25	22	2	2	0					
26	23	1	2	1	1	20,5	20,5		
27	24	2	2	0					
28	25	3	2	-1	1	20,5		20,5	
29	26	4	2	-2	2	44,5		44,5	
30	27	2	1	-1	1	20,5		20,5	
31	28	3	2	-1	1	20,5		20,5	
32	29	2	2	0					
33	30	3	2	-1	1	20,5		20,5	
34	31	2	1	-1	1	20,5		20,5	
35	32	2	2	0					
36	33	2	1	-1	1	20,5		20,5	
37	34	2	1	-1	1	20,5		20,5	
38	35	3	2	-1	1	20,5		20,5	
39	36	2	3	1	1	20,5	20,5		
40	37	2	3	1	1	20,5	20,5		
41	38	2	2	0					
42	39	3	2	-1	1	20,5		20,5	
43	40	2	3	1	1	20,5	20,5		
44	41	2	2	0					
45	42	1	3	2	2	44,5	44,5		
46	43	1	1	0					
47	44	3	1	-2	2	44,5		44,5	

	A	B	C	D	E	F	G	H	I
48	45	3	3	0					
49	46	2	2	0					
50	47	3	4	1	1	20,5	20,5		
51	48								
52	49	3	1	-2	2	44,5		44,5	
53	50	3	3	0					
54	51	2	1	-1	1	20,5		20,5	
55	52	3	2	-1	1	20,5		20,5	
56	53	2	2	0					
57	54	2	2	0					
58	55	3	2	-1	1	20,5		20,5	
59	56	1	2	1	1	20,5	20,5		
60	57	4	4	0					
61	58	1	1	0					
62	59	2	2	0					
63	60	5	5	0					
64	61	3	3	0					
65	62	2	2	0					
66	63	2	3	1	1	20,5	20,5		
67	64	2	2	0					
68	65	3	3	0					
69	66	2	4	2	2	44,5	44,5		
70	67	1	1	0					
71	68	1	1	0					
72	69	2	3	1	1	20,5	20,5		
73	70	2	1	-1	1	20,5		20,5	
74	71	2	1	-1	1	20,5		20,5	
75	72	2	1	-1	1	20,5		20,5	
76	73	2	2	0					
77	74	3	3	0					
78	75	2	2	0					
79	76	4	3	-1	1	20,5		20,5	
80	77	4	2	-2	2	44,5		44,5	
81	78	3	1	-2	2	44,5		44,5	
82	79	2	2	0					
83	80	3	2	-1	1	20,5		20,5	
84	81	2	2	0					
85	82	2	2	0					
86	83	1	2	1	1	20,5	20,5		
87	84	2	2	0					
88	85	2	3	1	1	20,5	20,5		
89	86	3	3	0					
90	87	3	2	-1	1	20,5		20,5	
91	88	3	3	0					
92	89	2	3	1	1	20,5	20,5		
93	90	3	2	-1	1	20,5		20,5	
94	Median	2	2	0		Sum	540	735	
95	Mean	2,40	2,33	-0,08					
96	St dev	0,84	0,99	1,05					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	50		=COUNT(E4:E93)			
8		Ws+	540		=G94			
9		Ws-	735		=H94			
10		Ws	540		=MIN(G94:H94)			
11		Ws-crit	434		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	10731,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	103,592		=SQRT(K13)			
15		Z-score	0,941		=ABS(K10-K12)/K14			
16		Ws-crit (normal appressimation)	434,4		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,347		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,071		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	540		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,173		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,347		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,176		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,352		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	96,893		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,001		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,075		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,158		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,317		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,158		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,317		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	3. I am very interested in the study of physics as an experimental science								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	5	2	2	43,5	43,5		
5	2	3	4	1	1	19,5	19,5		
6	3	5	5	0					
7	4	2	4	2	2	43,5	43,5		
8	5	4	5	1	1	19,5	19,5		
9	6	4	5	1	1	19,5	19,5		
10	7	4	5	1	1	19,5	19,5		
11	8	4	4	0					
12	9	3	4	1	1	19,5	19,5		
13	10	5	5	0					
14	11	5	5	0					
15	12	4	5	1	1	19,5	19,5		
16	13	2	2	0					
17	14	4	5	1	1	19,5	19,5		
18	15	3	4	1	1	19,5	19,5		
19	16	5	4	-1	1	19,5		19,5	
20	17	3	4	1	1	19,5	19,5		
21	18	5	4	-1	1	19,5		19,5	
22	19	4	4	0					
23	20	4	4	0					
24	21	4	4	0					
25	22	5	5	0					
26	23	5	1	-4	4	50		50	
27	24	3	5	2	2	43,5	43,5		
28	25	4	4	0					
29	26	5	4	-1	1	19,5		19,5	
30	27	5	5	0					
31	28	5	5	0					
32	29	4	4	0					
33	30	4	4	0					
34	31	5	5	0					
35	32	5	5	0					
36	33	5	5	0					
37	34	4	5	1	1	19,5	19,5		
38	35	5	5	0					
39	36	4	5	1	1	19,5	19,5		
40	37	4	4	0					
41	38	4	5	1	1	19,5	19,5		
42	39	4	5	1	1	19,5	19,5		
43	40	4	4	0					
44	41	4	4	0					
45	42	4	5	1	1	19,5	19,5		
46	43	5	4	-1	1	19,5		19,5	

	A	B	C	D	E	F	G	H	I
47	44	4	5	1	1	19,5	19,5		
48	45	2	3	1	1	19,5	19,5		
49	46	4	5	1	1	19,5	19,5		
50	47	4	2	-2	2	43,5		43,5	
51	48	4	5	1	1	19,5	19,5		
52	49	5	5	0					
53	50	4	5	1	1	19,5	19,5		
54	51	5	5	0					
55	52	4	5	1	1	19,5	19,5		
56	53	5	4	-1	1	19,5		19,5	
57	54	3	4	1	1	19,5	19,5		
58	55	4	5	1	1	19,5	19,5		
59	56	5	4	-1	1	19,5		19,5	
60	57	2	5	3	3	49	49		
61	58	5	5	0					
62	59	4	4	0					
63	60	2	3	1	1	19,5	19,5		
64	61	5	3	-2	2	43,5		43,5	
65	62	3	2	-1	1	19,5		19,5	
66	63	5	5	0					
67	64								
68	65	4	4	0					
69	66	5	5	0					
70	67	4	5	1	1	19,5	19,5		
71	68	4	3	-1	1	19,5		19,5	
72	69	5	5	0					
73	70	3	4	1	1	19,5	19,5		
74	71	4	4	0					
75	72	3	5	2	2	43,5	43,5		
76	73	5	5	0					
77	74	4	5	1	1	19,5	19,5		
78	75	4	4	0					
79	76	3	4	1	1	19,5	19,5		
80	77	4	4	0					
81	78	4	4	0					
82	79	5	3	-2	2	43,5		43,5	
83	80	4	4	0					
84	81	4	3	-1	1	19,5		19,5	
85	82	4	4	0					
86	83	4	4	0					
87	84	4	5	1	1	19,5	19,5		
88	85	3	5	2	2	43,5	43,5		
89	86	2	4	2	2	43,5	43,5		
90	87	4	4	0					
91	88	4	4	0					
92	89	4	5	1	1	19,5	19,5		
93	90	3	5	2	2	43,5	43,5		
94	Median	4	4	0		Sum	919	356	
95	Mean	4,01	4,31	0,30					
96	St dev	0,85	0,83	1,04					

m

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	50		=COUNT(E4:E93)			
8		Ws+	919		=G94			
9		Ws-	356		=H94			
10		Ws	356		=MIN(G94:H94)			
11		Ws-crit	434		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	10731,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	103,592		=SQRT(K13)			
15		Z-score	2,717		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	434,4		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,007		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	yes		=IF(K17<K4,"yes","no")			
19		Effect size r	0,204		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	356		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,003		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,007		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,003		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,006		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	97,817		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	2,873		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,215		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,002		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,004		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	yes		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,002		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,004		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	4. I regard the relationship between physics and mathematics as very fruitful for the development of science								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	5	5	0					
5	2	4	3	-1	1	15,5		15,5	
6	3	5	5	0					
7	4	4	2	-2	2	33		33	
8	5	5	5	0					
9	6	5	3	-2	2	33		33	
10	7	5	5	0					
11	8	3	4	1	1	15,5	15,5		
12	9	4	4	0					
13	10	2	3	1	1	15,5	15,5		
14	11	5	5	0					
15	12	5	5	0					
16	13	3	3	0					
17	14	4	4	0					
18	15	5	5	0					
19	16	5	5	0					
20	17	4	5	1	1	15,5	15,5		
21	18	5	5	0					
22	19	4	4	0					
23	20	4	4	0					
24	21	5	5	0					
25	22	5	5	0					
26	23	5	4	-1	1	15,5		15,5	
27	24	4	4	0					
28	25	5	5	0					
29	26	5	5	0					
30	27	4	4	0					
31	28	5	5	0					
32	29	5	5	0					
33	30	4	4	0					
34	31	5	5	0					
35	32	5	5	0					
36	33	4	5	1	1	15,5	15,5		
37	34	5	5	0					
38	35	5	4	-1	1	15,5		15,5	
39	36	5	5	0					
40	37	3	4	1	1	15,5	15,5		
41	38	3	5	2	2	33	33		
42	39	5	4	-1	1	15,5		15,5	
43	40	4	3	-1	1	15,5		15,5	
44	41	5	5	0					
45	42	5	4	-1	1	15,5		15,5	
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	5	5	0					
48	45	4	3	-1	1	15,5		15,5	
49	46	5	5	0					
50	47	5	5	0					
51	48	3	5	2	2	33	33		
52	49	5	4	-1	1	15,5		15,5	
53	50	5	5	0					
54	51	5	5	0					
55	52	4	3	-1	1	15,5		15,5	
56	53								
57	54	5	5	0					
58	55	4	4	0					
59	56	4	5	1	1	15,5	15,5		
60	57								
61	58	5	4	-1	1	15,5		15,5	
62	59	3	5	2	2	33	33		
63	60	4	4	0					
64	61	5	5	0					
65	62	4	4	0					
66	63	5	4	-1	1	15,5		15,5	
67	64	4	5	1	1	15,5	15,5		
68	65	4	4	0					
69	66	5	5	0					
70	67	5	4	-1	1	15,5		15,5	
71	68	2	5	3	3	36,5	36,5		
72	69	5	5	0					
73	70	5	4	-1	1	15,5		15,5	
74	71	4	4	0					
75	72	5	4	-1	1	15,5		15,5	
76	73	5	4	-1	1	15,5		15,5	
77	74	4	4	0					
78	75	5	5	0					
79	76	5	5	0					
80	77	4	3	-1	1	15,5		15,5	
81	78	4	3	-1	1	15,5		15,5	
82	79	4	4	0					
83	80	4	5	1	1	15,5	15,5		
84	81	5	5	0					
85	82	5	4	-1	1	15,5		15,5	
86	83	5	5	0					
87	84	4	3	-1	1	15,5		15,5	
88	85	5	4	-1	1	15,5		15,5	
89	86	4	3	-1	1	15,5		15,5	
90	87	5	4	-1	1	15,5		15,5	
91	88	5	5	0					
92	89	5	2	-3	3	36,5		36,5	
93	90	4	4	0					
94	Median	5	4,5	0		Sum	259,5	443,5	
95	Mean	4,47	4,33	-0,14					
96	St dev	0,73	0,78	0,87					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	88		=COUNT(C4:C93)			
7		N-diff	37		=COUNT(E4:E93)			
8		Ws+	259,5		=G94			
9		Ws-	443,5		=H94			
10		Ws	259,5		=MIN(G94:H94)			
11		Ws-crit	221		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	351,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	4393,8		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	66,285		=SQRT(K13)			
15		Z-score	1,388		=ABS(K10-K12)/K14			
16		Ws-crit (normal appressimation)	221,5		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,165		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,105		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	259,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,083		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,165		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,084		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,167		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	351,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	61,881		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,479		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,111		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,070		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,139		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,070		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,139		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	5. I consider very useful solving physics problems.								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	5	4	-1	1	21,5		21,5	
5	2	4	4	0					
6	3	5	5	0					
7	4	3	4	1	1	21,5	21,5		
8	5	4	3	-1	1	21,5		21,5	
9	6	5	5	0					
10	7	2	3	1	1	21,5	21,5		
11	8	4	2	-2	2	47		47	
12	9	3	3	0					
13	10	4	5	1	1	21,5	21,5		
14	11	5	4	-1	1	21,5		21,5	
15	12	5	4	-1	1	21,5		21,5	
16	13	4	3	-1	1	21,5		21,5	
17	14	4	5	1	1	21,5	21,5		
18	15	4	5	1	1	21,5	21,5		
19	16	4	4	0					
20	17	5	5	0					
21	18	4	5	1	1	21,5	21,5		
22	19	4	5	1	1	21,5	21,5		
23	20	4	5	1	1	21,5	21,5		
24	21	4	5	1	1	21,5	21,5		
25	22	4	4	0					
26	23	3	4	1	1	21,5	21,5		
27	24	5	3	-2	2	47		47	
28	25	4	4	0					
29	26	4	4	0					
30	27	5	4	-1	1	21,5		21,5	
31	28	2	3	1	1	21,5	21,5		
32	29	4	4	0					
33	30	4	4	0					
34	31	5	5	0					
35	32	5	5	0					
36	33	3	5	2	2	47	47		
37	34	5	5	0					
38	35	5	4	-1	1	21,5		21,5	
39	36	4	3	-1	1	21,5		21,5	
40	37	5	4	-1	1	21,5		21,5	
41	38	4	5	1	1	21,5	21,5		
42	39	5	5	0					
43	40	5	2	-3	3	52		52	
44	41	5	4	-1	1	21,5		21,5	
45	42	5	5	0					
46	43	5	5	0					
47	44	4	4	0					

	A	B	C	D	E	F	G	H	I
48	45	4	4	0					
49	46	5	3	-2	2	47		47	
50	47	4	4	0					
51	48	3	5	2	2	47	47		
52	49	3	4	1	1	21,5	21,5		
53	50	5	5	0					
54	51	4	4	0					
55	52	5	5	0					
56	53	4	5	1	1	21,5	21,5		
57	54	4	4	0					
58	55	5	4	-1	1	21,5		21,5	
59	56	5	4	-1	1	21,5		21,5	
60	57	3	4	1	1	21,5	21,5		
61	58	5	4	-1	1	21,5		21,5	
62	59	5	4	-1	1	21,5		21,5	
63	60	4	3	-1	1	21,5		21,5	
64	61	5	5	0					
65	62	5	4	-1	1	21,5		21,5	
66	63	4	4	0					
67	64	5	5	0					
68	65	5	5	0					
69	66	3	4	1	1	21,5	21,5		
70	67	4	5	1	1	21,5	21,5		
71	68	4	3	-1	1	21,5		21,5	
72	69	5	3	-2	2	47		47	
73	70	4	4	0					
74	71	4	5	1	1	21,5	21,5		
75	72	5	3	-2	2	47		47	
76	73	5	4	-1	1	21,5		21,5	
77	74	5	5	0					
78	75	4	4	0					
79	76	5	4	-1	1	21,5		21,5	
80	77	5	5	0					
81	78	3	4	1	1	21,5	21,5		
82	79	5	3	-2	2	47		47	
83	80	4	4	0					
84	81	5	5	0					
85	82	5	5	0					
86	83	5	5	0					
87	84	3	4	1	1	21,5	21,5		
88	85	5	3	-2	2	47		47	
89	86	3	4	1	1	21,5	21,5		
90	87	5	5	0					
91	88	5	4	-1	1	21,5		21,5	
92	89	4	3	-1	1	21,5		21,5	
93	90	4	4	0					
94	Median	4	4	0		Sum	545,5	832,5	
95	Mean	4,30	4,16	-0,14					
96	St dev	0,77	0,78	0,98					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	52		=COUNT(E4:E93)			
8		Ws+	545,5		=G94			
9		Ws-	832,5		=H94			
10		Ws	545,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	689,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	12057,5		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	109,807		=SQRT(K13)			
15		Z-score	1,307		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	473,7		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,191		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,097		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	545,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,096		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,191		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,096		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,193		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	689		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	102,469		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,396		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,104		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,081		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,163		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,081		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,163		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	6. I believe that the history of physics can make a significant contribution to the teaching of physics								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	3	0					
5	2	4	5	1	1	20,5	20,5		
6	3	2	5	3	3	54,5	54,5		
7	4	2	1	-1	1	20,5		20,5	
8	5	4	4	0					
9	6	4	3	-1	1	20,5		20,5	
10	7	4	4	0					
11	8	1	4	3	3	54,5	54,5		
12	9	4	3	-1	1	20,5		20,5	
13	10	1	1	0					
14	11	4	3	-1	1	20,5		20,5	
15	12	4	2	-2	2	46		46	
16	13	2	2	0					
17	14	1	2	1	1	20,5	20,5		
18	15	4	3	-1	1	20,5		20,5	
19	16	3	3	0					
20	17	4	3	-1	1	20,5		20,5	
21	18	4	3	-1	1	20,5		20,5	
22	19	5	4	-1	1	20,5		20,5	
23	20	5	4	-1	1	20,5		20,5	
24	21	3	5	2	2	46	46		
25	22	2	3	1	1	20,5	20,5		
26	23	2	3	1	1	20,5	20,5		
27	24	4	4	0					
28	25	4	3	-1	1	20,5		20,5	
29	26	5	4	-1	1	20,5		20,5	
30	27	5	4	-1	1	20,5		20,5	
31	28	1	3	2	2	46	46		
32	29	3	2	-1	1	20,5		20,5	
33	30	4	4	0					
34	31	5	4	-1	1	20,5		20,5	
35	32	5	5	0					
36	33	3	4	1	1	20,5	20,5		
37	34	2	5	3	3	54,5	54,5		
38	35	4	5	1	1	20,5	20,5		
39	36	3	4	1	1	20,5	20,5		
40	37	2	2	0					
41	38	4	5	1	1	20,5	20,5		
42	39	4	4	0					
43	40	3	4	1	1	20,5	20,5		
44	41	4	3	-1	1	20,5		20,5	
45	42	4	3	-1	1	20,5		20,5	
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	4	5	1	1	20,5	20,5		
48	45	4	2	-2	2	46		46	
49	46	3	2	-1	1	20,5		20,5	
50	47	2	2	0					
51	48	4	4	0					
52	49	2	4	2	2	46	46		
53	50	5	5	0					
54	51	5	3	-2	2	46		46	
55	52	4	4	0					
56	53	5	5	0					
57	54	3	3	0					
58	55	4	4	0					
59	56	2	3	1	1	20,5	20,5		
60	57	4	2	-2	2	46		46	
61	58	5	5	0					
62	59	3	3	0					
63	60	5	5	0					
64	61	2	4	2	2	46	46		
65	62	2	3	1	1	20,5	20,5		
66	63	3	5	2	2	46	46		
67	64	5	2	-3	3	54,5		54,5	
68	65	4	3	-1	1	20,5		20,5	
69	66	4	5	1	1	20,5	20,5		
70	67	3	3	0					
71	68	3	2	-1	1	20,5		20,5	
72	69	3	3	0					
73	70	4	3	-1	1	20,5		20,5	
74	71	3	4	1	1	20,5	20,5		
75	72	3	3	0					
76	73	4	2	-2	2	46		46	
77	74	3	3	0					
78	75	5	2	-3	3	54,5		54,5	
79	76	4	3	-1	1	20,5		20,5	
80	77	5	5	0					
81	78	5	5	0					
82	79	3	2	-1	1	20,5		20,5	
83	80	2	3	1	1	20,5	20,5		
84	81	5	4	-1	1	20,5		20,5	
85	82	4	4	0					
86	83	4	5	1	1	20,5	20,5		
87	84	4	4	0					
88	85	4	2	-2	2	46		46	
89	86	3	3	0					
90	87	2	5	3	3	54,5	54,5		
91	88	3	3	0					
92	89	3	2	-1	1	20,5		20,5	
93	90	3	3	0					
94	Median	4	3	0		Sum	776	877	
95	Mean	3,50	3,46	-0,04					
96	St dev	1,10	1,08	1,24					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	57		=COUNT(E4:E93)			
8		Ws+	776		=G94			
9		Ws-	877		=H94			
10		Ws	776		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	826,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	15841,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	125,862		=SQRT(K13)			
15		Z-score	0,401		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	579,8		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,688		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,030		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	776		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,344		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,688		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,347		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,693		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	826,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	120,320		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,416		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,031		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,339		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,678		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,339		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,678		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	7. I believe that the history of ideas and concepts that have led to the development of physics is very important to understand them completely								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	4	4	0					
5	2	3	5	2	2	38	38		
6	3	5	5	0					
7	4	3	1	-2	2	38		38	
8	5	3	3	0					
9	6	4	4	0					
10	7	4	4	0					
11	8	1	3	2	2	38	38		
12	9	5	4	-1	1	16,5		16,5	
13	10	2	2	0					
14	11	4	4	0					
15	12	2	2	0					
16	13	3	2	-1	1	16,5		16,5	
17	14	2	3	1	1	16,5	16,5		
18	15	4	4	0					
19	16	3	4	1	1	16,5	16,5		
20	17	4	4	0					
21	18	4	4	0					
22	19	5	4	-1	1	16,5		16,5	
23	20	5	5	0					
24	21	5	5	0					
25	22	3	4	1	1	16,5	16,5		
26	23	3	3	0					
27	24	4	5	1	1	16,5	16,5		
28	25	4	3	-1	1	16,5		16,5	
29	26	4	3	-1	1	16,5		16,5	
30	27	4	4	0					
31	28	4	3	-1	1	16,5		16,5	
32	29	3	3	0					
33	30	5	5	0					
34	31	5	4	-1	1	16,5		16,5	
35	32	5	5	0					
36	33	3	5	2	2	38	38		
37	34	5	5	0					
38	35	5	5	0					
39	36	3	4	1	1	16,5	16,5		
40	37	2	2	0					
41	38	3	5	2	2	38	38		
42	39	4	5	1	1	16,5	16,5		
43	40	3	2	-1	1	16,5		16,5	
44	41	4	4	0					
45	42	4	4	0					
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	3	5	2	2	38	38		
48	45	4	4	0					
49	46	4	5	1	1	16,5	16,5		
50	47	3	3	0					
51	48								
52	49	2	5	3	3	45	45		
53	50	5	5	0					
54	51	2	3	1	1	16,5	16,5		
55	52	3	4	1	1	16,5	16,5		
56	53	3	5	2	2	38	38		
57	54	3	4	1	1	16,5	16,5		
58	55	3	3	0					
59	56	3	2	-1	1	16,5		16,5	
60	57	4	2	-2	2	38		38	
61	58	5	5	0					
62	59	5	2	-3	3	45		45	
63	60	5	5	0					
64	61	1	2	1	1	16,5	16,5		
65	62	4	3	-1	1	16,5		16,5	
66	63	5	1	-4	4	47		47	
67	64	3	4	1	1	16,5	16,5		
68	65	5	4	-1	1	16,5		16,5	
69	66	3	3	0					
70	67	5	3	-2	2	38		38	
71	68	4	4	0					
72	69	4	3	-1	1	16,5		16,5	
73	70	3	4	1	1	16,5	16,5		
74	71								
75	72	4	3	-1	1	16,5		16,5	
76	73	3	4	1	1	16,5	16,5		
77	74	5	2	-3	3	45		45	
78	75	4	3	-1	1	16,5		16,5	
79	76	4	4	0					
80	77	3	4	1	1	16,5	16,5		
81	78	4	4	0					
82	79	3	3	0					
83	80	3	4	1	1	16,5	16,5		
84	81	5	5	0					
85	82	4	4	0					
86	83	5	5	0					
87	84	4	4	0					
88	85	4	3	-1	1	16,5		16,5	
89	86	2	4	2	2	38	38		
90	87	2	1	-1	1	16,5		16,5	
91	88	3	3	0					
92	89	4	2	-2	2	38		38	
93	90	3	3	0					
94	Median	4	4	0		Sum	575	553	
95	Mean	3,67	3,66	-0,01					
96	St dev	1,01	1,09	1,17					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	88		=COUNT(C4:C93)			
7		N-diff	47		=COUNT(E4:E93)			
8		Ws+	575		=G94			
9		Ws-	553		=H94			
10		Ws	553		=MIN(G94:H94)			
11		Ws-crit	378		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	564,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	8930,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	94,499		=SQRT(K13)			
15		Z-score	0,116		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	378,7		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,907		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,009		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	553		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,454		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,907		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,456		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,912		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	564		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	90,664		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,116		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,009		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,454		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,908		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,454		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,908		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	8. I would like to learn more about the biographies of scientists								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	2	3	1	1	18	18		
5	2	3	4	1	1	18	18		
6	3	1	5	4	4	56	56		
7	4	1	1	0					
8	5	3	3	0					
9	6	4	3	-1	1	18		18	
10	7	3	2	-1	1	18		18	
11	8	4	2	-2	2	43		43	
12	9	2	2	0					
13	10	1	1	0					
14	11	2	4	2	2	43	43		
15	12	4	1	-3	3	52,5		52,5	
16	13	1	1	0					
17	14	2	1	-1	1	18		18	
18	15	1	1	0					
19	16	4	3	-1	1	18		18	
20	17	3	2	-1	1	18		18	
21	18	4	2	-2	2	43		43	
22	19	4	3	-1	1	18		18	
23	20	3	3	0					
24	21	4	3	-1	1	18		18	
25	22	1	5	4	4	56	56		
26	23	4	4	0					
27	24	3	2	-1	1	18		18	
28	25	5	2	-3	3	52,5		52,5	
29	26	4	2	-2	2	43		43	
30	27	5	3	-2	2	43		43	
31	28	1	2	1	1	18	18		
32	29	3	2	-1	1	18		18	
33	30	3	2	-1	1	18		18	
34	31	5	4	-1	1	18		18	
35	32	5	5	0					
36	33	3	3	0					
37	34	2	5	3	3	52,5	52,5		
38	35	4	3	-1	1	18		18	
39	36	3	3	0					
40	37	2	1	-1	1	18		18	
41	38	2	3	1	1	18	18		
42	39	2	4	2	2	43	43		
43	40	3	2	-1	1	18		18	
44	41	4	4	0					
45	42	5	5	0					
46	43	5	3	-2	2	43		43	
47	44	2	4	2	2	43	43		

	A	B	C	D	E	F	G	H	I
48	45	2	3	1	1	18	18		
49	46	2	3	1	1	18	18		
50	47	3	1	-2	2	43		43	
51	48	2	4	2	2	43	43		
52	49	2	4	2	2	43	43		
53	50	2	2	0					
54	51	4	3	-1	1	18		18	
55	52	5	5	0					
56	53								
57	54	3	3	0					
58	55	3	1	-2	2	43		43	
59	56	5	1	-4	4	56		56	
60	57	4	4	0					
61	58	1	3	2	2	43	43		
62	59	1	2	1	1	18	18		
63	60	3	3	0					
64	61	2	2	0					
65	62	1	1	0					
66	63	5	4	-1	1	18		18	
67	64	3	3	0					
68	65	1	4	3	3	52,5	52,5		
69	66	3	3	0					
70	67	2	2	0					
71	68	1	1	0					
72	69	2	2	0					
73	70	3	3	0					
74	71	2	3	1	1	18	18		
75	72	3	2	-1	1	18		18	
76	73	3	1	-2	2	43		43	
77	74	2	2	0					
78	75	1	2	1	1	18	18		
79	76	1	1	0					
80	77	5	4	-1	1	18		18	
81	78	5	4	-1	1	18		18	
82	79	2	1	-1	1	18		18	
83	80	3	3	0					
84	81	4	4	0					
85	82	4	3	-1	1	18		18	
86	83	5	4	-1	1	18		18	
87	84	3	4	1	1	18	18		
88	85	3	2	-1	1	18		18	
89	86	3	3	0					
90	87	3	1	-2	2	43		43	
91	88	4	3	-1	1	18		18	
92	89	3	2	-1	1	18		18	
93	90	3	3	0					
94	Median	3	3	0		Sum	655	998	
95	Mean	2,91	2,72	-0,19					
96	St dev	1,26	1,17	1,41					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	57		=COUNT(E4:E93)			
8		Ws+	655		=G94			
9		Ws-	998		=H94			
10		Ws	655		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	826,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	15841,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	125,862		=SQRT(K13)			
15		Z-score	1,363		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	579,8		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,173		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,102		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	655		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,087		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,173		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,088		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,176		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	826,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	121,971		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,402		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,105		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,080		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,161		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,080		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,161		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	9. I appreciate very much reading original texts (books and scientific papers) during physics lessons								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	2	3	1	1	19,5	19,5		
5	2	1	2	1	1	19,5	19,5		
6	3	3	5	2	2	46	46		
7	4	1	1	0					
8	5	5	1	-4	4	58,5		58,5	
9	6	5	3	-2	2	46		46	
10	7	3	3	0					
11	8	3	3	0					
12	9	2	4	2	2	46	46		
13	10	2	2	0					
14	11	4	3	-1	1	19,5		19,5	
15	12	5	2	-3	3	55,5		55,5	
16	13	1	3	2	2	46	46		
17	14	2	2	0					
18	15	4	3	-1	1	19,5		19,5	
19	16	4	4	0					
20	17	4	4	0					
21	18	4	4	0					
22	19	4	3	-1	1	19,5		19,5	
23	20	3	3	0					
24	21	2	2	0					
25	22	3	2	-1	1	19,5		19,5	
26	23	2	3	1	1	19,5	19,5		
27	24	3	5	2	2	46	46		
28	25	3	2	-1	1	19,5		19,5	
29	26	3	3	0					
30	27	5	2	-3	3	55,5		55,5	
31	28	2	2	0					
32	29	3	2	-1	1	19,5		19,5	
33	30	4	3	-1	1	19,5		19,5	
34	31	4	2	-2	2	46		46	
35	32	5	5	0					
36	33	3	4	1	1	19,5	19,5		
37	34	1	5	4	4	58,5	58,5		
38	35	5	4	-1	1	19,5		19,5	
39	36	2	2	0					
40	37	2	3	1	1	19,5	19,5		
41	38	3	5	2	2	46	46		
42	39	3	4	1	1	19,5	19,5		
43	40	3	1	-2	2	46		46	
44	41	3	4	1	1	19,5	19,5		
45	42	3	2	-1	1	19,5		19,5	
46	43	5	5	0					
47	44	3	4	1	1	19,5	19,5		

	A	B	C	D	E	F	G	H	I
48	45	1	2	1	1	19,5	19,5		
49	46	2	1	-1	1	19,5		19,5	
50	47	3	2	-1	1	19,5		19,5	
51	48	3	3	0					
52	49	5	4	-1	1	19,5		19,5	
53	50	3	2	-1	1	19,5		19,5	
54	51	3	4	1	1	19,5	19,5		
55	52	2	2	0					
56	53	3	3	0					
57	54	2	2	0					
58	55	5	2	-3	3	55,5		55,5	
59	56	4	4	0					
60	57	5	5	0					
61	58	1	2	1	1	19,5	19,5		
62	59	3	1	-2	2	46		46	
63	60	5	4	-1	1	19,5		19,5	
64	61	1	2	1	1	19,5	19,5		
65	62	1	1	0					
66	63	3	3	0					
67	64	2	2	0					
68	65	3	3	0					
69	66	4	5	1	1	19,5	19,5		
70	67	3	2	-1	1	19,5		19,5	
71	68	3	1	-2	2	46		46	
72	69	4	3	-1	1	19,5		19,5	
73	70	4	4	0					
74	71	4	3	-1	1	19,5		19,5	
75	72	4	4	0					
76	73	1	1	0					
77	74	3	4	1	1	19,5	19,5		
78	75	1	1	0					
79	76	2	3	1	1	19,5	19,5		
80	77	4	3	-1	1	19,5		19,5	
81	78	4	3	-1	1	19,5		19,5	
82	79	4	1	-3	3	55,5		55,5	
83	80	4	2	-2	2	46		46	
84	81	4	2	-2	2	46		46	
85	82	4	4	0					
86	83	5	3	-2	2	46		46	
87	84	4	2	-2	2	46		46	
88	85	2	1	-1	1	19,5		19,5	
89	86	2	2	0					
90	87	2	4	2	2	46	46		
91	88	3	4	1	1	19,5	19,5		
92	89	4	3	-1	1	19,5		19,5	
93	90	3	2	-1	1	19,5		19,5	
94	Median	3	3	0		Sum	646,5	1123,5	
95	Mean	3,10	2,83	-0,27					
96	St dev	1,19	1,16	1,34					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	59		=COUNT(E4:E93)			
8		Ws+	646,5		=G94			
9		Ws-	1123,5		=H94			
10		Ws	646,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	885,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	17552,5		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	132,486		=SQRT(K13)			
15		Z-score	1,800		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	625,3		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,072		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,134		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	646,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,036		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,072		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,036		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,072		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	885		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	127,823		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,862		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,139		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,031		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,063		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,031		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,063		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	10. I consider instructive to analyse the original historical documents (notes, letters, etc.) during physics lessons								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	2	3	1	1	20,5	20,5		
5	2	3	2	-1	1	20,5		20,5	
6	3	3	5	2	2	50	50		
7	4	1	1	0					
8	5	4	2	-2	2	50		50	
9	6	4	5	1	1	20,5	20,5		
10	7	4	4	0					
11	8	5	2	-3	3	62,5		62,5	
12	9	2	1	-1	1	20,5		20,5	
13	10	3	4	1	1	20,5	20,5		
14	11	3	1	-2	2	50		50	
15	12	2	2	0					
16	13	3	3	0					
17	14	4	1	-3	3	62,5		62,5	
18	15	4	4	0					
19	16	4	4	0					
20	17	3	4	1	1	20,5	20,5		
21	18	4	3	-1	1	20,5		20,5	
22	19	4	3	-1	1	20,5		20,5	
23	20	3	3	0					
24	21	2	3	1	1	20,5	20,5		
25	22	1	1	0					
26	23	1	4	3	3	62,5	62,5		
27	24	4	5	1	1	20,5	20,5		
28	25	3	3	0					
29	26	4	3	-1	1	20,5		20,5	
30	27	5	4	-1	1	20,5		20,5	
31	28	2	1	-1	1	20,5		20,5	
32	29	3	2	-1	1	20,5		20,5	
33	30	4	3	-1	1	20,5		20,5	
34	31	5	2	-3	3	62,5		62,5	
35	32	5	5	0					
36	33	4	4	0					
37	34	3	5	2	2	50	50		
38	35	5	5	0					
39	36	3	2	-1	1	20,5		20,5	
40	37	1	3	2	2	50	50		
41	38	5	4	-1	1	20,5		20,5	
42	39	4	2	-2	2	50		50	
43	40	4	2	-2	2	50		50	
44	41	3	4	1	1	20,5	20,5		
45	42	5	2	-3	3	62,5		62,5	
46	43	5	5	0					
47	44	4	5	1	1	20,5	20,5		

	A	B	C	D	E	F	G	H	I
48	45	1	3	2	2	50	50		
49	46	3	3	0					
50	47	3	3	0					
51	48	5	4	-1	1	20,5		20,5	
52	49	1	3	2	2	50	50		
53	50	4	3	-1	1	20,5		20,5	
54	51	2	3	1	1	20,5	20,5		
55	52	3	3	0					
56	53	3	2	-1	1	20,5		20,5	
57	54	2	2	0					
58	55	3	3	0					
59	56	4	4	0					
60	57	5	3	-2	2	50		50	
61	58	2	1	-1	1	20,5		20,5	
62	59	1	3	2	2	50	50		
63	60	1	1	0					
64	61	4	2	-2	2	50		50	
65	62	2	3	1	1	20,5	20,5		
66	63	1	3	2	2	50	50		
67	64	3	5	2	2	50	50		
68	65	1	2	1	1	20,5	20,5		
69	66	3	1	-2	2	50		50	
70	67	2	4	2	2	50	50		
71	68	4	3	-1	1	20,5		20,5	
72	69	3	3	0					
73	70	3	4	1	1	20,5	20,5		
74	71	1	1	0					
75	72	3	4	1	1	20,5	20,5		
76	73	2	1	-1	1	20,5		20,5	
77	74	2	3	1	1	20,5	20,5		
78	75	3	5	2	2	50	50		
79	76	3	2	-1	1	20,5		20,5	
80	77	1	2	1	1	20,5	20,5		
81	78	3	5	2	2	50	50		
82	79	3	2	-1	1	20,5		20,5	
83	80	3	2	-1	1	20,5		20,5	
84	81	1	2	1	1	20,5	20,5		
85	82	4	3	-1	1	20,5		20,5	
86	83	4	4	0					
87	84	3	2	-1	1	20,5		20,5	
88	85	3	1	-2	2	50		50	
89	86								
90	87	1	4	3	3	62,5	62,5		
91	88	2	3	1	1	20,5	20,5		
92	89	3	2	-1	1	20,5		20,5	
93	90	3	3	0					
94	Median	3	3	0		Sum	1023,5	1121,5	
95	Mean	3,00	2,93	-0,07					
96	St dev	1,22	1,21	1,39					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	65		=COUNT(E4:E93)			
8		Ws+	1023,5		=G94			
9		Ws-	1121,5		=H94			
10		Ws	1023,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	1072,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	23416,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	153,024		=SQRT(K13)			
15		Z-score	0,320		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	772,5		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,749		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,024		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	1023,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,374		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,749		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,375		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,750		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	1072,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	148,111		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,327		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,025		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,372		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,743		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,372		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,743		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	11. I find it very useful to read summary texts written by researchers into the history of physics and physics education								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	2	5	3	3	49,5	49,5		
5	2	4	3	-1	1	17,5		17,5	
6	3	4	5	1	1	17,5	17,5		
7	4	1	1	0					
8	5	4	4	0					
9	6	4	4	0					
10	7	3	4	1	1	17,5	17,5		
11	8	3	5	2	2	40	40		
12	9	5	3	-2	2	40		40	
13	10	1	3	2	2	40	40		
14	11	3	3	0					
15	12	2	2	0					
16	13	2	3	1	1	17,5	17,5		
17	14	2	5	3	3	49,5	49,5		
18	15	4	5	1	1	17,5	17,5		
19	16	5	3	-2	2	40		40	
20	17	4	4	0					
21	18	4	5	1	1	17,5	17,5		
22	19	4	4	0					
23	20	4	4	0					
24	21	4	3	-1	1	17,5		17,5	
25	22	4	4	0					
26	23	4	5	1	1	17,5	17,5		
27	24	4	4	0					
28	25	5	4	-1	1	17,5		17,5	
29	26	4	5	1	1	17,5	17,5		
30	27	4	4	0					
31	28	3	5	2	2	40	40		
32	29	3	2	-1	1	17,5		17,5	
33	30	4	3	-1	1	17,5		17,5	
34	31	5	2	-3	3	49,5		49,5	
35	32	5	5	0					
36	33	4	3	-1	1	17,5		17,5	
37	34	5	5	0					
38	35	5	5	0					
39	36	3	3	0					
40	37	2	3	1	1	17,5	17,5		
41	38	4	5	1	1	17,5	17,5		
42	39	4	4	0					
43	40	3	2	-1	1	17,5		17,5	
44	41	5	5	0					
45	42	4	4	0					
46	43	5	5	0					
47	44	4	5	1	1	17,5	17,5		

	A	B	C	D	E	F	G	H	I
48	45	5	4	-1	1	17,5		17,5	
49	46	4	5	1	1	17,5	17,5		
50	47	4	5	1	1	17,5	17,5		
51	48	4	4	0					
52	49	4	3	-1	1	17,5		17,5	
53	50	5	5	0					
54	51	2	4	2	2	40	40		
55	52	2	3	1	1	17,5	17,5		
56	53	4	1	-3	3	49,5		49,5	
57	54	2	4	2	2	40	40		
58	55	5	5	0					
59	56	3	4	1	1	17,5	17,5		
60	57	3	2	-1	1	17,5		17,5	
61	58	1	3	2	2	40	40		
62	59	3	3	0					
63	60	4	4	0					
64	61	4	4	0					
65	62	5	5	0					
66	63	3	4	1	1	17,5	17,5		
67	64	3	4	1	1	17,5	17,5		
68	65	4	4	0					
69	66	3	3	0					
70	67	4	3	-1	1	17,5		17,5	
71	68	4	4	0					
72	69	5	3	-2	2	40		40	
73	70	4	4	0					
74	71	5	4	-1	1	17,5		17,5	
75	72	2	4	2	2	40	40		
76	73	2	5	3	3	49,5	49,5		
77	74	3	4	1	1	17,5	17,5		
78	75	5	2	-3	3	49,5		49,5	
79	76	3	3	0					
80	77	4	4	0					
81	78	2	5	3	3	49,5	49,5		
82	79	3	3	0					
83	80	3	4	1	1	17,5	17,5		
84	81	5	2	-3	3	49,5		49,5	
85	82	3	3	0					
86	83	4	4	0					
87	84	4	3	-1	1	17,5		17,5	
88	85	3	3	0					
89	86	3	4	1	1	17,5	17,5		
90	87	1	1	0					
91	88	4	3	-1	1	17,5		17,5	
92	89	4	2	-2	2	40		40	
93	90	3	4	1	1	17,5	17,5		
94	Median	4	4	0		Sum	828	603	
95	Mean	3,58	3,71	0,13					
96	St dev	1,07	1,05	1,29					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	53		=COUNT(E4:E93)			
8		Ws+	828		=G94			
9		Ws-	603		=H94			
10		Ws	603		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	715,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	12759,8		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	112,959		=SQRT(K13)			
15		Z-score	0,996		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	494,1		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,319		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,074		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	603		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,160		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,319		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,162		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,324		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	715,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	109,104		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,027		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,077		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,152		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,305		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,152		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,305		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	12. I find the narration of historical anecdotes interesting								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	4	1	1	19	19		
5	2	4	4	0					
6	3	4	5	1	1	19	19		
7	4	1	1	0					
8	5	4	3	-1	1	19		19	
9	6	3	4	1	1	19	19		
10	7	5	4	-1	1	19		19	
11	8	3	3	0					
12	9	4	4	0					
13	10	1	2	1	1	19	19		
14	11	5	5	0					
15	12	2	3	1	1	19	19		
16	13	3	3	0					
17	14	4	5	1	1	19	19		
18	15	4	5	1	1	19	19		
19	16	5	5	0					
20	17	1	2	1	1	19	19		
21	18	3	4	1	1	19	19		
22	19	5	3	-2	2	45		45	
23	20	4	4	0					
24	21	4	2	-2	2	45		45	
25	22	4	4	0					
26	23	4	3	-1	1	19		19	
27	24	5	5	0					
28	25	5	4	-1	1	19		19	
29	26	3	4	1	1	19	19		
30	27	5	2	-3	3	55		55	
31	28	4	4	0					
32	29	3	2	-1	1	19		19	
33	30	4	4	0					
34	31	5	4	-1	1	19		19	
35	32	5	5	0					
36	33	4	4	0					
37	34	5	5	0					
38	35	5	5	0					
39	36	3	3	0					
40	37	3	4	1	1	19	19		
41	38	4	5	1	1	19	19		
42	39	5	4	-1	1	19		19	
43	40	4	4	0					
44	41	3	4	1	1	19	19		
45	42	4	5	1	1	19	19		
46	43	5	4	-1	1	19		19	
47	44	5	5	0					

	A	B	C	D	E	F	G	H	I
48	45	4	4	0					
49	46	2	3	1	1	19	19		
50	47	2	5	3	3	55	55		
51	48	4	5	1	1	19	19		
52	49	2	4	2	2	45	45		
53	50	3	4	1	1	19	19		
54	51	5	5	0					
55	52								
56	53	3	3	0					
57	54	3	4	1	1	19	19		
58	55								
59	56	5	4	-1	1	19		19	
60	57	2	3	1	1	19	19		
61	58	2	3	1	1	19	19		
62	59	4	2	-2	2	45		45	
63	60	5	3	-2	2	45		45	
64	61	5	5	0					
65	62	2	3	1	1	19	19		
66	63	2	4	2	2	45	45		
67	64	5	4	-1	1	19		19	
68	65	2	5	3	3	55	55		
69	66	5	3	-2	2	45		45	
70	67	5	4	-1	1	19		19	
71	68	3	3	0					
72	69	2	4	2	2	45	45		
73	70	5	2	-3	3	55		55	
74	71	5	3	-2	2	45		45	
75	72	4	3	-1	1	19		19	
76	73	4	4	0					
77	74	4	4	0					
78	75	2	4	2	2	45	45		
79	76	5	3	-2	2	45		45	
80	77	4	3	-1	1	19		19	
81	78	4	4	0					
82	79	3	1	-2	2	45		45	
83	80	2	4	2	2	45	45		
84	81	5	3	-2	2	45		45	
85	82	4	3	-1	1	19		19	
86	83	5	5	0					
87	84	4	3	-1	1	19		19	
88	85	4	4	0					
89	86	4	3	-1	1	19		19	
90	87	2	5	3	3	55	55		
91	88	4	2	-2	2	45		45	
92	89	3	3	0					
93	90								
94	Median	4	4	0		Sum	789	864	
95	Mean	3,71	3,69	-0,02					
96	St dev	1,15	0,99	1,28					
97									

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	87		=COUNT(C4:C93)			
7		N-diff	57		=COUNT(E4:E93)			
8		Ws+	789		=G94			
9		Ws-	864		=H94			
10		Ws	789		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	826,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	15841,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	125,862		=SQRT(K13)			
15		Z-score	0,298		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	579,8		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,766		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,023		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	789		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,383		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,766		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,385		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,770		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	826,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	121,302		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,305		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,023		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,380		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,760		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,380		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,760		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	13. I think that history-of-physics books are difficult to understand								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	5	3	-2	2	48		48	
5	2	3	4	1	1	21	21		
6	3	1	5	4	4	62	62		
7	4	1	3	2	2	48	48		
8	5	4	5	1	1	21	21		
9	6	3	4	1	1	21	21		
10	7	3	3	0					
11	8	3	4	1	1	21	21		
12	9	3	3	0					
13	10	2	2	0					
14	11	2	3	1	1	21	21		
15	12	2	3	1	1	21	21		
16	13	4	4	0					
17	14	4	5	1	1	21	21		
18	15	2	3	1	1	21	21		
19	16	2	1	-1	1	21		21	
20	17	4	4	0					
21	18	4	3	-1	1	21		21	
22	19	3	4	1	1	21	21		
23	20	3	3	0					
24	21	3	2	-1	1	21		21	
25	22	3	4	1	1	21	21		
26	23	5	3	-2	2	48		48	
27	24	3	2	-1	1	21		21	
28	25	2	3	1	1	21	21		
29	26	3	3	0					
30	27	3	3	0					
31	28	5	5	0					
32	29	2	5	3	3	57	57		
33	30	3	3	0					
34	31	4	5	1	1	21	21		
35	32	1	3	2	2	48	48		
36	33	1	3	2	2	48	48		
37	34	2	3	1	1	21	21		
38	35	2	2	0					
39	36	2	3	1	1	21	21		
40	37	3	5	2	2	48	48		
41	38	3	4	1	1	21	21		
42	39	3	3	0					
43	40	3	4	1	1	21	21		
44	41	2	2	0					
45	42	2	3	1	1	21	21		
46	43	1	4	3	3	57	57		
47	44	4	1	-3	3	57		57	

	A	B	C	D	E	F	G	H	I
48	45	1	5	4	4	62	62		
49	46	2	4	2	2	48	48		
50	47	2	3	1	1	21	21		
51	48	5	2	-3	3	57		57	
52	49	4	2	-2	2	48		48	
53	50	2	2	0					
54	51	3	3	0					
55	52	3	2	-1	1	21		21	
56	53	3	3	0					
57	54	1	5	4	4	62	62		
58	55	1	4	3	3	57	57		
59	56	1	2	1	1	21	21		
60	57	4	5	1	1	21	21		
61	58	2	4	2	2	48	48		
62	59	1	5	4	4	62	62		
63	60	2	1	-1	1	21		21	
64	61	3	4	1	1	21	21		
65	62	3	3	0					
66	63	2	1	-1	1	21		21	
67	64	3	3	0					
68	65	3	1	-2	2	48		48	
69	66	3	2	-1	1	21		21	
70	67	4	2	-2	2	48		48	
71	68	3	1	-2	2	48		48	
72	69	4	3	-1	1	21		21	
73	70	4	5	1	1	21	21		
74	71	2	2	0					
75	72	3	4	1	1	21	21		
76	73	2	3	1	1	21	21		
77	74	3	3	0					
78	75	1	1	0					
79	76	3	3	0					
80	77	3	3	0					
81	78	3	4	1	1	21	21		
82	79	3	4	1	1	21	21		
83	80	1	1	0					
84	81	3	3	0					
85	82	3	3	0					
86	83	2	4	2	2	48	48		
87	84	3	4	1	1	21	21		
88	85	3	4	1	1	21	21		
89	86	1	5	4	4	62	62		
90	87	4	3	-1	1	21		21	
91	88	2	3	1	1	21	21		
92	89	3	4	1	1	21	21		
93	90	2	3	1	1	21	21		
94	Median	3	3	1		Sum	1468	612	
95	Mean	2,69	3,20	0,51					
96	St dev	1,03	1,13	1,48					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	64		=COUNT(E4:E93)			
8		Ws+	1468		=G94			
9		Ws-	612		=H94			
10		Ws	612		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	1040,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	22360,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	149,533		=SQRT(K13)			
15		Z-score	2,862		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	746,9		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,004		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	yes		=IF(K17<K4,"yes","no")			
19		Effect size r	0,213		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	612		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,002		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,004		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,002		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,004		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	1040		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	144,480		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	2,959		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,221		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,002		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,003		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	yes		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,002		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,003		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	14. I believe that the replication of the experiments that led to the development of physics can be very useful for education.								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	5	2	2	32	32		
5	2	4	4	0					
6	3	5	5	0					
7	4	4	5	1	1	15,5	15,5		
8	5	5	5	0					
9	6	5	5	0					
10	7	5	5	0					
11	8	4	5	1	1	15,5	15,5		
12	9	5	5	0					
13	10	5	5	0					
14	11								
15	12	5	5	0					
16	13	4	3	-1	1	15,5		15,5	
17	14	5	5	0					
18	15	4	5	1	1	15,5	15,5		
19	16	5	5	0					
20	17	5	5	0					
21	18	5	4	-1	1	15,5		15,5	
22	19	5	5	0					
23	20	5	5	0					
24	21	5	4	-1	1	15,5		15,5	
25	22	5	5	0					
26	23	5	3	-2	2	32		32	
27	24	4	5	1	1	15,5	15,5		
28	25	4	4	0					
29	26	5	5	0					
30	27	5	5	0					
31	28	3	3	0					
32	29	5	4	-1	1	15,5		15,5	
33	30	5	5	0					
34	31	5	5	0					
35	32	5	5	0					
36	33	5	5	0					
37	34	5	5	0					
38	35	5	5	0					
39	36	5	4	-1	1	15,5		15,5	
40	37	5	4	-1	1	15,5		15,5	
41	38	5	5	0					
42	39	5	5	0					
43	40	5	4	-1	1	15,5		15,5	
44	41	5	5	0					
45	42	5	5	0					
46	43	5	5	0					
47	44	5	5	0					

	A	B	C	D	E	F	G	H	I
48	45	5	4	-1	1	15,5		15,5	
49	46	5	5	0					
50	47	5	4	-1	1	15,5		15,5	
51	48	5	5	0					
52	49	5	5	0					
53	50	1	5	4	4	35	35		
54	51	5	5	0					
55	52								
56	53	5	4	-1	1	15,5		15,5	
57	54	5	5	0					
58	55	5	5	0					
59	56	5	5	0					
60	57	5	5	0					
61	58	4	5	1	1	15,5	15,5		
62	59	5	4	-1	1	15,5		15,5	
63	60	4	4	0					
64	61	4	5	1	1	15,5	15,5		
65	62	5	4	-1	1	15,5		15,5	
66	63	5	5	0					
67	64	5	4	-1	1	15,5		15,5	
68	65	5	5	0					
69	66	5	5	0					
70	67	5	5	0					
71	68	3	4	1	1	15,5	15,5		
72	69	1	4	3	3	34	34		
73	70	5	5	0					
74	71	5	5	0					
75	72	3	5	2	2	32	32		
76	73	4	5	1	1	15,5	15,5		
77	74	5	5	0					
78	75	5	4	-1	1	15,5		15,5	
79	76	4	3	-1	1	15,5		15,5	
80	77	4	5	1	1	15,5	15,5		
81	78	4	5	1	1	15,5	15,5		
82	79	5	5	0					
83	80	4	5	1	1	15,5	15,5		
84	81	5	5	0					
85	82	5	4	-1	1	15,5		15,5	
86	83	5	5	0					
87	84	5	5	0					
88	85	5	5	0					
89	86	3	3	0					
90	87	5	4	-1	1	15,5		15,5	
91	88	5	4	-1	1	15,5		15,5	
92	89	4	5	1	1	15,5	15,5		
93	90	5	5	0					
94	Median	5	5	0		Sum	319	311	
95	Mean	4,61	4,65	0,03					
96	St dev	0,79	0,59	0,88					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	88		=COUNT(C4:C93)			
7		N-diff	35		=COUNT(E4:E93)			
8		Ws+	319		=G94			
9		Ws-	311		=H94			
10		Ws	311		=MIN(G94:H94)			
11		Ws-crit	195		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	315,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	3727,5		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	61,053		=SQRT(K13)			
15		Z-score	0,066		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	195,3		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,948		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,005		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	311		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,474		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,948		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,477		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,955		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	315		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	56,259		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,062		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,005		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,475		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,950		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,475		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,950		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	15. The replication of experiments of historical interest unnecessarily complicates the learning of physics								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	3	0					
5	2	3	1	-2	2	36,5		36,5	
6	3	5	5	0					
7	4	2	4	2	2	36,5	36,5		
8	5	3	1	-2	2	36,5		36,5	
9	6	2	1	-1	1	14		14	
10	7	1	1	0					
11	8	1	1	0					
12	9	3	4	1	1	14	14		
13	10	4	1	-3	3	47		47	
14	11	1	2	1	1	14	14		
15	12	3	3	0					
16	13	2	1	-1	1	14		14	
17	14	1	1	0					
18	15	2	1	-1	1	14		14	
19	16	1	1	0					
20	17	4	4	0					
21	18	3	2	-1	1	14		14	
22	19	1	1	0					
23	20	3	3	0					
24	21	1	1	0					
25	22	1	5	4	4	49,5	49,5		
26	23	5	3	-2	2	36,5		36,5	
27	24	3	1	-2	2	36,5		36,5	
28	25	1	2	1	1	14	14		
29	26	1	3	2	2	36,5	36,5		
30	27	2	5	3	3	47	47		
31	28	4	3	-1	1	14		14	
32	29	2	3	1	1	14	14		
33	30	1	2	1	1	14	14		
34	31	5	3	-2	2	36,5		36,5	
35	32	1	1	0					
36	33	1	2	1	1	14	14		
37	34	1	1	0					
38	35	1	1	0					
39	36	3	2	-1	1	14		14	
40	37	4	3	-1	1	14		14	
41	38	1	2	1	1	14	14		
42	39	1	2	1	1	14	14		
43	40	2	3	1	1	14	14		
44	41	2	1	-1	1	14		14	
45	42	2	2	0					
46	43	1	1	0					
47	44	1	1	0					

	A	B	C	D	E	F	G	H	I
48	45	4	4	0					
49	46	3	1	-2	2	36,5		36,5	
50	47	1	1	0					
51	48	1	2	1	1	14	14		
52	49	5	1	-4	4	49,5		49,5	
53	50	4	1	-3	3	47		47	
54	51	1	1	0					
55	52	5	5	0					
56	53	1	1	0					
57	54	1	1	0					
58	55								
59	56	1	1	0					
60	57	1	1	0					
61	58	2	2	0					
62	59	1	2	1	1	14	14		
63	60	1	2	1	1	14	14		
64	61	1	1	0					
65	62	4	2	-2	2	36,5		36,5	
66	63	2	1	-1	1	14		14	
67	64	1	1	0					
68	65	1	1	0					
69	66	1	2	1	1	14	14		
70	67	4	2	-2	2	36,5		36,5	
71	68	3	2	-1	1	14		14	
72	69	3	1	-2	2	36,5		36,5	
73	70	3	1	-2	2	36,5		36,5	
74	71	2	1	-1	1	14		14	
75	72	1	1	0					
76	73	1	3	2	2	36,5	36,5		
77	74	1	1	0					
78	75	1	3	2	2	36,5	36,5		
79	76	1	3	2	2	36,5	36,5		
80	77	1	2	1	1	14	14		
81	78	1	1	0					
82	79	2	2	0					
83	80	1	3	2	2	36,5	36,5		
84	81	1	3	2	2	36,5	36,5		
85	82	1	1	0					
86	83	1	1	0					
87	84	1	1	0					
88	85	1	1	0					
89	86	2	4	2	2	36,5	36,5		
90	87	1	2	1	1	14	14		
91	88	2	2	0					
92	89	2	2	0					
93	90	2	1	-1	1	14		14	
94	Median	1	2	0		Sum	598,5	676,5	
95	Mean	1,98	1,93	-0,04					
96	St dev	1,24	1,14	1,34					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	50		=COUNT(E4:E93)			
8		Ws+	598,5		=G94			
9		Ws-	676,5		=H94			
10		Ws	598,5		=MIN(G94:H94)			
11		Ws-crit	434		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	10731,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	103,592		=SQRT(K13)			
15		Z-score	0,376		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	434,4		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,707		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,028		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	598,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,353		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,707		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,354		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,709		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	100,995		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,381		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,029		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,352		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,703		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,352		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,703		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	16. I'm very interested in attending the replication of famous physics experiments for educational purposes								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	4	4	0					
5	2	3	4	1	1	15,5	15,5		
6	3	5	5	0					
7	4	2	3	1	1	15,5	15,5		
8	5	5	4	-1	1	15,5		15,5	
9	6	4	5	1	1	15,5	15,5		
10	7	5	5	0					
11	8	3	4	1	1	15,5	15,5		
12	9	5	5	0					
13	10	3	4	1	1	15,5	15,5		
14	11	3	2	-1	1	15,5		15,5	
15	12	3	3	0					
16	13	5	5	0					
17	14	4	4	0					
18	15	5	4	-1	1	15,5		15,5	
19	16	5	5	0					
20	17	4	4	0					
21	18	4	4	0					
22	19	4	4	0					
23	20	3	5	2	2	36,5	36,5		
24	21	5	5	0					
25	22	3	5	2	2	36,5	36,5		
26	23	5	1	-4	4	45,5		45,5	
27	24	4	5	1	1	15,5	15,5		
28	25	4	3	-1	1	15,5		15,5	
29	26	5	4	-1	1	15,5		15,5	
30	27	5	5	0					
31	28	3	3	0					
32	29	5	5	0					
33	30	4	4	0					
34	31	5	4	-1	1	15,5		15,5	
35	32	5	5	0					
36	33	5	5	0					
37	34	5	5	0					
38	35	5	5	0					
39	36	2	4	2	2	36,5	36,5		
40	37	2	2	0					
41	38	5	5	0					
42	39	5	5	0					
43	40	5	4	-1	1	15,5		15,5	
44	41	4	4	0					
45	42	4	4	0					
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	5	5	0					
48	45	5	4	-1	1	15,5		15,5	
49	46	5	3	-2	2	36,5		36,5	
50	47	5	5	0					
51	48	5	5	0					
52	49	1	5	4	4	45,5	45,5		
53	50	4	5	1	1	15,5	15,5		
54	51	5	5	0					
55	52	4	4	0					
56	53	5	5	0					
57	54	5	4	-1	1	15,5		15,5	
58	55	5	5	0					
59	56	5	5	0					
60	57	5	5	0					
61	58	3	4	1	1	15,5	15,5		
62	59	3	2	-1	1	15,5		15,5	
63	60	3	5	2	2	36,5	36,5		
64	61	5	4	-1	1	15,5		15,5	
65	62	2	4	2	2	36,5	36,5		
66	63	4	5	1	1	15,5	15,5		
67	64	5	4	-1	1	15,5		15,5	
68	65	3	5	2	2	36,5	36,5		
69	66	4	3	-1	1	15,5		15,5	
70	67	2	5	3	3	43,5	43,5		
71	68	4	5	1	1	15,5	15,5		
72	69	4	5	1	1	15,5	15,5		
73	70	4	4	0					
74	71	2	5	3	3	43,5	43,5		
75	72	4	3	-1	1	15,5		15,5	
76	73	4	5	1	1	15,5	15,5		
77	74	5	3	-2	2	36,5		36,5	
78	75	5	5	0					
79	76	5	3	-2	2	36,5		36,5	
80	77	5	5	0					
81	78	4	5	1	1	15,5	15,5		
82	79	4	4	0					
83	80	5	3	-2	2	36,5		36,5	
84	81	5	5	0					
85	82	5	3	-2	2	36,5		36,5	
86	83	5	5	0					
87	84	5	5	0					
88	85	5	5	0					
89	86	4	5	1	1	15,5	15,5		
90	87	5	3	-2	2	36,5		36,5	
91	88	4	3	-1	1	15,5		15,5	
92	89	5	4	-1	1	15,5		15,5	
93	90	4	4	0					
94	Median	5	4,5	0		Sum	568,5	512,5	
95	Mean	4,21	4,26	0,04					
96	St dev	0,99	0,91	1,20					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	46		=COUNT(E4:E93)			
8		Ws+	568,5		=G94			
9		Ws-	512,5		=H94			
10		Ws	512,5		=MIN(G94:H94)			
11		Ws-crit	361		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	540,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	8377,8		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	91,530		=SQRT(K13)			
15		Z-score	0,306		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	361,1		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,760		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,023		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	512,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,380		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,760		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,381		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,762		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	540,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	88,204		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,312		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,023		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,378		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,755		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,378		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,755		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	17. I am embarrassed to perform physics experiments first-hand								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	1	-2	2	36,5		36,5	
5	2	4	1	-3	3	45,5		45,5	
6	3	2	5	3	3	45,5	45,5		
7	4	3	1	-2	2	36,5		36,5	
8	5	1	1	0					
9	6	1	1	0					
10	7	1	1	0					
11	8	3	4	1	1	16	16		
12	9	2	2	0					
13	10	3	5	2	2	36,5	36,5		
14	11			0					
15	12	1	1	0					
16	13	2	1	-1	1	16		16	
17	14	1	1	0					
18	15	2	1	-1	1	16		16	
19	16	1	2	1	1	16	16		
20	17	2	2	0					
21	18	2	3	1	1	16	16		
22	19	1	1	0					
23	20	1	3	2	2	36,5	36,5		
24	21	1	1	0					
25	22	1	1	0					
26	23	4	4	0					
27	24	3	1	-2	2	36,5		36,5	
28	25	1	1	0					
29	26	1	1	0					
30	27	1	1	0					
31	28	1	2	1	1	16	16		
32	29	1	1	0					
33	30	3	1	-2	2	36,5		36,5	
34	31	4	2	-2	2	36,5		36,5	
35	32	1	1	0					
36	33	1	2	1	1	16	16		
37	34	1	1	0					
38	35	1	1	0					
39	36	2	1	-1	1	16		16	
40	37	3	3	0					
41	38	1	1	0					
42	39	1	1	0					
43	40	2	1	-1	1	16		16	
44	41	2	1	-1	1	16		16	
45	42	3	5	2	2	36,5	36,5		
46	43	3	4	1	1	16	16		

	A	B	C	D	E	F	G	H	I
47	44	2	1	-1	1	16		16	
48	45	3	2	-1	1	16		16	
49	46	1	1	0					
50	47	3	1	-2	2	36,5		36,5	
51	48	1	1	0					
52	49	1	1	0					
53	50	5	1	-4	4	51		51	
54	51	1	5	4	4	51	51		
55	52	3	4	1	1	16	16		
56	53	2	3	1	1	16	16		
57	54	1	1	0					
58	55	1	2	1	1	16	16		
59	56	4	5	1	1	16	16		
60	57	1	4	3	3	45,5	45,5		
61	58	1	2	1	1	16	16		
62	59	4	3	-1	1	16		16	
63	60	3	3	0					
64	61	2	1	-1	1	16		16	
65	62	1	1	0					
66	63	4	1	-3	3	45,5		45,5	
67	64	1	1	0					
68	65	1	1	0					
69	66	1	1	0					
70	67	4	1	-3	3	45,5		45,5	
71	68	4	1	-3	3	45,5		45,5	
72	69	3	1	-2	2	36,5		36,5	
73	70	2	1	-1	1	16		16	
74	71	2	2	0					
75	72	2	1	-1	1	16		16	
76	73	1	1	0					
77	74	3	2	-1	1	16		16	
78	75	1	2	1	1	16	16		
79	76	2	1	-1	1	16		16	
80	77	5	2	-3	3	45,5		45,5	
81	78	3	2	-1	1	16		16	
82	79	1	1	0					
83	80	1	2	1	1	16	16		
84	81	2	1	-1	1	16		16	
85	82	5	2	-3	3	45,5		45,5	
86	83	3	4	1	1	16	16		
87	84	1	1	0					
88	85	4	4	0					
89	86	3	3	0					
90	87	2	1	-1	1	16		16	
91	88	1	5	4	4	51	51		
92	89	2	1	-1	1	16		16	
93	90	2	2	0					
94	Median	2	1	0		Sum	526,5	851,5	
95	Mean	2,07	1,85	-0,21					
96	St dev	1,17	1,26	1,45					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	52		=COUNT(E4:E93)			
8		Ws+	526,5		=G94			
9		Ws-	851,5		=H94			
10		Ws	526,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	689,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	12057,5		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	109,807		=SQRT(K13)			
15		Z-score	1,480		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	473,7		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,139		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,111		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	526,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,069		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,139		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,070		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,140		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	689		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	106,798		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,517		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,114		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,065		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,129		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,065		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,129		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	18. I find it interesting to plan an experiment as if I were a real scientist								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	5	5	0					
5	2	5	5	0					
6	3	5	5	0					
7	4	4	4	0					
8	5	5	5	0					
9	6	4	4	0					
10	7	5	2	-3	3	37		37	
11	8	4	4	0					
12	9	4	4	0					
13	10	5	4	-1	1	11		11	
14	11	5	5	0					
15	12	3	2	-1	1	11		11	
16	13	5	5	0					
17	14	3	5	2	2	28,5	28,5		
18	15	2	4	2	2	28,5	28,5		
19	16	5	4	-1	1	11		11	
20	17	5	4	-1	1	11		11	
21	18	4	3	-1	1	11		11	
22	19	4	3	-1	1	11		11	
23	20	3	5	2	2	28,5	28,5		
24	21	5	5	0					
25	22	5	5	0					
26	23	4	3	-1	1	11		11	
27	24	3	5	2	2	28,5	28,5		
28	25	4	4	0					
29	26	4	4	0					
30	27	5	5	0					
31	28	4	3	-1	1	11		11	
32	29	4	5	1	1	11	11		
33	30	4	4	0					
34	31	5	5	0					
35	32	5	5	0					
36	33	5	5	0					
37	34	3	5	2	2	28,5	28,5		
38	35	5	5	0					
39	36	4	4	0					
40	37	2	1	-1	1	11		11	
41	38	5	5	0					
42	39	5	5	0					
43	40	5	3	-2	2	28,5		28,5	
44	41	5	5	0					
45	42	5	4	-1	1	11		11	
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	5	5	0					
48	45	3	2	-1	1	11		11	
49	46	4	4	0					
50	47	4	4	0					
51	48	3	5	2	2	28,5	28,5		
52	49	1	5	4	4	39	39		
53	50	5	5	0					
54	51	5	5	0					
55	52	3	3	0					
56	53	5	4	-1	1	11		11	
57	54	5	3	-2	2	28,5		28,5	
58	55	5	5	0					
59	56	5	5	0					
60	57	2	4	2	2	28,5	28,5		
61	58	5	3	-2	2	28,5		28,5	
62	59	3	3	0					
63	60	3	4	1	1	11	11		
64	61	5	5	0					
65	62	5	5	0					
66	63	5	5	0					
67	64	5	5	0					
68	65	3	3	0					
69	66	4	4	0					
70	67	4	4	0					
71	68	3	4	1	1	11	11		
72	69	5	5	0					
73	70	4	4	0					
74	71	5	3	-2	2	28,5		28,5	
75	72	3	2	-1	1	11		11	
76	73	2	4	2	2	28,5	28,5		
77	74	5	2	-3	3	37		37	
78	75	5	5	0					
79	76	5	4	-1	1	11		11	
80	77	5	5	0					
81	78	2	4	2	2	28,5	28,5		
82	79	4	3	-1	1	11		11	
83	80	5	2	-3	3	37		37	
84	81	5	5	0					
85	82	5	4	-1	1	11		11	
86	83	5	5	0					
87	84	5	5	0					
88	85	5	4	-1	1	11		11	
89	86	4	4	0					
90	87	3	5	2	2	28,5	28,5		
91	88			0					
92	89	5	5	0					
93	90	4	3	-1	1	11		11	
94	Median	5	4	0		Sum	357	423	
95	Mean	4,24	4,15	-0,09					
96	St dev	0,99	0,98	1,16					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	39		=COUNT(E4:E93)			
8		Ws+	357		=G94			
9		Ws-	423		=H94			
10		Ws	357		=MIN(G94:H94)			
11		Ws-crit	249		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	390,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	5135,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	71,659		=SQRT(K13)			
15		Z-score	0,461		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	249,5		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,645		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,035		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	357		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,323		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,645		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,327		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,654		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	390		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	69,894		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,465		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,035		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,321		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,642		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,321		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,642		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	**19. I believe that the history of physics can emphasize the relationship between science and society**								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	3	0					
5	2	4	4	0					
6	3	2	5	3	3	44	44		
7	4	3	2	-1	1	16		16	
8	5	3	3	0					
9	6	4	5	1	1	16	16		
10	7	4	4	0					
11	8	3	5	2	2	36	36		
12	9	4	4	0					
13	10	2	3	1	1	16	16		
14	11	5	5	0					
15	12	1	2	1	1	16	16		
16	13	2	2	0					
17	14	5	5	0					
18	15	2	5	3	3	44	44		
19	16	5	3	-2	2	36		36	
20	17	4	3	-1	1	16		16	
21	18	4	5	1	1	16	16		
22	19	3	4	1	1	16	16		
23	20	4	4	0					
24	21	3	2	-1	1	16		16	
25	22	5	5	0					
26	23	3	4	1	1	16	16		
27	24	4	4	0					
28	25	4	5	1	1	16	16		
29	26	4	5	1	1	16	16		
30	27	4	3	-1	1	16		16	
31	28	4	5	1	1	16	16		
32	29	3	2	-1	1	16		16	
33	30	3	3	0					
34	31	5	3	-2	2	36		36	
35	32	5	5	0					
36	33	4	5	1	1	16	16		
37	34	4	1	-3	3	44		44	
38	35	5	5	0					
39	36	4	3	-1	1	16		16	
40	37	4	3	-1	1	16		16	
41	38	3	5	2	2	36	36		
42	39	2	4	2	2	36	36		
43	40	4	4	0					
44	41	5	5	0					
45	42	4	4	0					
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	1	5	4	4	49	49		
48	45	4	4	0					
49	46	1	4	3	3	44	44		
50	47	3	5	2	2	36	36		
51	48	4	5	1	1	16	16		
52	49	3	3	0					
53	50	5	5	0					
54	51	4	4	0					
55	52	3	3	0					
56	53	3	2	-1	1	16		16	
57	54	3	3	0					
58	55	5	1	-4	4	49		49	
59	56	4	3	-1	1	16		16	
60	57	3	3	0					
61	58	3	3	0					
62	59	2	3	1	1	16	16		
63	60	4	3	-1	1	16		16	
64	61	4	4	0					
65	62	1	3	2	2	36	36		
66	63	1	2	1	1	16	16		
67	64	5	5	0					
68	65	5	4	-1	1	16		16	
69	66	3	5	2	2	36	36		
70	67	5	3	-2	2	36		36	
71	68	4	4	0					
72	69	5	2	-3	3	44		44	
73	70	4	3	-1	1	16		16	
74	71	2	2	0					
75	72	5	4	-1	1	16		16	
76	73	5	5	0					
77	74	5	5	0					
78	75	4	3	-1	1	16		16	
79	76	4	4	0					
80	77	1	4	3	3	44	44		
81	78	4	3	-1	1	16		16	
82	79								
83	80	2	2	0					
84	81	5	4	-1	1	16		16	
85	82	5	5	0					
86	83	5	5	0					
87	84	4	3	-1	1	16		16	
88	85	4	4	0					
89	86	1	4	3	3	44	44		
90	87	1	5	4	4	49	49		
91	88	4	3	-1	1	16		16	
92	89								
93	90	4	4	0					
94	Median	4	4	0		Sum	742	533	
95	Mean	3,58	3,74	0,16					
96	St dev	1,21	1,10	1,42					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	88		=COUNT(C4:C93)			
7		N-diff	50		=COUNT(E4:E93)			
8		Ws+	742		=G94			
9		Ws-	533		=H94			
10		Ws	533		=MIN(G94:H94)			
11		Ws-crit	434		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	10731,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	103,592		=SQRT(K13)			
15		Z-score	1,009		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	434,4		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,313		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,076		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	533		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,157		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,313		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,159		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,318		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	100,443		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,035		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,078		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,150		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,300		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,150		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,300		=SRTEST(B4:C93)			
45								

	A	B	C	D	E	F	G	H	I
1	20. I think that modern technology can make a valuable contribution to the teaching of physics								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	5	2	2	29,5	29,5		
5	2	4	4	0					
6	3	4	5	1	1	14	14		
7	4	4	5	1	1	14	14		
8	5	5	5	0					
9	6	5	5	0					
10	7	5	4	-1	1	14		14	
11	8	4	5	1	1	14	14		
12	9	5	5	0					
13	10	5	5	0					
14	11	5	5	0					
15	12	3	3	0					
16	13	4	5	1	1	14	14		
17	14	5	5	0					
18	15	4	5	1	1	14	14		
19	16	5	5	0					
20	17	4	4	0					
21	18	3	5	2	2	29,5	29,5		
22	19	5	4	-1	1	14		14	
23	20	4	4	0					
24	21	5	5	0					
25	22	5	5	0					
26	23	3	2	-1	1	14		14	
27	24	5	4	-1	1	14		14	
28	25	5	5	0					
29	26	5	5	0					
30	27	5	5	0					
31	28	5	5	0					
32	29	5	5	0					
33	30	4	5	1	1	14	14		
34	31	5	5	0					
35	32	5	5	0					
36	33	5	5	0					
37	34	5	5	0					
38	35	5	5	0					
39	36	4	4	0					
40	37	3	4	1	1	14	14		
41	38	4	5	1	1	14	14		
42	39	4	5	1	1	14	14		
43	40	5	5	0					
44	41	5	5	0					
45	42	5	5	0					
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	5	5	0					
48	45	5	4	-1	1	14		14	
49	46	5	5	0					
50	47	5	5	0					
51	48	5	5	0					
52	49	4	5	1	1	14	14		
53	50	5	5	0					
54	51	4	5	1	1	14	14		
55	52	4	5	1	1	14	14		
56	53	5	5	0					
57	54	4	5	1	1	14	14		
58	55	1	2	1	1	14	14		
59	56	5	5	0					
60	57	3	5	2	2	29,5	29,5		
61	58	5	5	0					
62	59								
63	60	5	3	-2	2	29,5		29,5	
64	61	5	5	0					
65	62	5	5	0					
66	63	4	4	0					
67	64	4	5	1	1	14	14		
68	65	5	4	-1	1	14		14	
69	66	5	5	0					
70	67	4	5	1	1	14	14		
71	68	5	5	0					
72	69	5	5	0					
73	70	4	4	0					
74	71	5	5	0					
75	72	5	5	0					
76	73	5	4	-1	1	14		14	
77	74	5	5	0					
78	75	5	5	0					
79	76	4	4	0					
80	77	5	5	0					
81	78	5	4	-1	1	14		14	
82	79	5	5	0					
83	80	5	5	0					
84	81	5	5	0					
85	82	4	4	0					
86	83	5	5	0					
87	84	5	4	-1	1	14		14	
88	85	5	5	0					
89	86	5	5	0					
90	87	5	5	0					
91	88	5	4	-1	1	14		14	
92	89	4	5	1	1	14	14		
93	90	5	5	0					
94	Median	5	5	0		Sum	326,5	169,5	
95	Mean	4,56	4,69	0,12					
96	St dev	0,72	0,63	0,69					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	31		=COUNT(E4:E93)			
8		Ws+	326,5		=G94			
9		Ws-	169,5		=H94			
10		Ws	169,5		=MIN(G94:H94)			
11		Ws-crit	147		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	248,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	2604,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	51,029		=SQRT(K13)			
15		Z-score	1,538		=ABS(K10-K12)/K14			
16		Ws-crit (normal appressimation)	147,9		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,124		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,115		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	169,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,062		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,124		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,063		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,125		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	248		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	46,832		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,666		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,125		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,048		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,096		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,048		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,096		=SRTEST(B4:C93)			
45								
46								
47								

21. I am very interested in the context in which physics has historically developed.

	A	B	C	D	E	F	G	H	I
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	4	1	1	17	17		
5	2	4	4	0					
6	3	3	5	2	2	40,5	40,5		
7	4	2	3	1	1	17	17		
8	5	3	3	0					
9	6	5	5	0					
10	7	3	3	0					
11	8	3	4	1	1	17	17		
12	9	3	4	1	1	17	17		
13	10	2	3	1	1	17	17		
14	11	4	4	0					
15	12	1	2	1	1	17	17		
16	13	2	3	1	1	17	17		
17	14	1	5	4	4	49,5	49,5		
18	15	4	4	0					
19	16	4	4	0					
20	17	4	4	0					
21	18	3	4	1	1	17	17		
22	19	4	3	-1	1	17		17	
23	20	3	4	1	1	17	17		
24	21	5	4	-1	1	17		17	
25	22	2	2	0					
26	23	2	4	2	2	40,5	40,5		
27	24	4	2	-2	2	40,5		40,5	
28	25	3	3	0					
29	26	5	4	-1	1	17		17	
30	27	5	5	0					
31	28	1	2	1	1	17	17		
32	29	3	2	-1	1	17		17	
33	30	3	3	0					
34	31	5	4	-1	1	17		17	
35	32	5	5	0					
36	33	4	4	0					
37	34	5	5	0					
38	35	5	4	-1	1	17		17	
39	36	3	4	1	1	17	17		
40	37	2	3	1	1	17	17		
41	38	2	4	2	2	40,5	40,5		
42	39	2	4	2	2	40,5	40,5		
43	40	4	4	0					
44	41	4	4	0					
45	42	4	5	1	1	17	17		
46	43	5	4	-1	1	17		17	

	A	B	C	D	E	F	G	H	I
47	44	4	5	1	1	17	17		
48	45	4	3	-1	1	17		17	
49	46	3	3	0					
50	47	5	4	-1	1	17		17	
51	48	3	5	2	2	40,5	40,5		
52	49	2	3	1	1	17	17		
53	50	4	2	-2	2	40,5		40,5	
54	51	5	5	0					
55	52	2	3	1	1	17	17		
56	53	3	3	0					
57	54	1	5	4	4	49,5	49,5		
58	55	4	2	-2	2	40,5		40,5	
59	56	5	5	0					
60	57	2	4	2	2	40,5	40,5		
61	58	1	2	1	1	17	17		
62	59	2	3	1	1	17	17		
63	60	3	3	0					
64	61	5	5	0					
65	62	4	2	-2	2	40,5		40,5	
66	63	4	3	-1	1	17		17	
67	64	4	5	1	1	17	17		
68	65	2	2	0					
69	66	3	3	0					
70	67	2	2	0					
71	68	3	3	0					
72	69	3	4	1	1	17	17		
73	70	2	5	3	3	48	48		
74	71	3	2	-1	1	17		17	
75	72	3	3	0					
76	73	2	2	0					
77	74	3	3	0					
78	75	5	3	-2	2	40,5		40,5	
79	76	4	4	0					
80	77	5	5	0					
81	78	1	2	1	1	17	17		
82	79								
83	80	3	3	0					
84	81	5	3	-2	2	40,5		40,5	
85	82	4	4	0					
86	83	5	5	0					
87	84	4	4	0					
88	85	4	4	0					
89	86	3	5	2	2	40,5	40,5		
90	87	1	3	2	2	40,5	40,5		
91	88	4	3	-1	1	17		17	
92	89	4	4	0					
93	90	3	3	0					
94	Median	3	4	0		Sum	828	447	
95	Mean	3,31	3,58	0,27					
96	St dev	1,20	0,99	1,18					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	50		=COUNT(E4:E93)			
8		Ws+	828		=G94			
9		Ws-	447		=H94			
10		Ws	447		=MIN(G94:H94)			
11		Ws-crit	434		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	10731,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	103,592		=SQRT(K13)			
15		Z-score	1,839		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	434,4		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,066		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,138		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	447		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,033		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,066		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,033		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,066		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	637,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	99,631		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,907		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,143		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,028		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,057		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,028		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,057		=SRTEST(B4:C93)			
45								
46								
47								

22. I greatly appreciate that educational laboratory experiments are conducted under the same experimental conditions when they were carried out for the first time in the history of the physics

Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks
1	5	3	-2	2	48		48
2	4	4	0				
3	4	5	1	1	19,5	19,5	
4	2	2	0				
5	5	3	-2	2	48		48
6	5	4	-1	1	19,5		19,5
7	4	5	1	1	19,5	19,5	
8	5	4	-1	1	19,5		19,5
9	3	2	-1	1	19,5		19,5
10	5	5	0				
11	1	3	2	2	48	48	
12	2	3	1	1	19,5	19,5	
13	2	5	3	3	60	60	
14	5	2	-3	3	60		60
15	5	5	0				
16	4	4	0				
17	4	4	0				
18	3	5	2	2	48	48	
19	4	3	-1	1	19,5		19,5
20	4	5	1	1	19,5	19,5	
21	3	3	0				
22	3	5	2	2	48	48	
23	3	3	0				
24	3	5	2	2	48	48	
25	3	2	-1	1	19,5		19,5
26	5	3	-2	2	48		48
27	4	3	-1	1	19,5		19,5
28	2	4	2	2	48	48	
29	3	2	-1	1	19,5		19,5
30	5	4	-1	1	19,5		19,5
31	4	4	0				
32	5	5	0				
33	5	5	0				
34	5	5	0				
35	5	4	-1	1	19,5		19,5
36	4	4	0				
37	3	2	-1	1	19,5		19,5
38	3	5	2	2	48	48	
39	3	4	1	1	19,5	19,5	
40	3	3	0				
41	4	4	0				
42	5	4	-1	1	19,5		19,5
43	5	5	0				

	A	B	C	D	E	F	G	H	I
47	44	3	5	2	2	48	48		
48	45	4	3	-1	1	19,5		19,5	
49	46	3	4	1	1	19,5	19,5		
50	47	4	5	1	1	19,5	19,5		
51	48	3	3	0					
52	49	4	5	1	1	19,5	19,5		
53	50	3	5	2	2	48	48		
54	51	3	4	1	1	19,5	19,5		
55	52	3	5	2	2	48	48		
56	53	4	3	-1	1	19,5		19,5	
57	54	4	5	1	1	19,5	19,5		
58	55	5	5	0					
59	56	1	2	1	1	19,5	19,5		
60	57	4	2	-2	2	48		48	
61	58	2	4	2	2	48	48		
62	59	1	5	4	4	63	63		
63	60	3	5	2	2	48	48		
64	61	5	3	-2	2	48		48	
65	62	4	3	-1	1	19,5		19,5	
66	63	4	5	1	1	19,5	19,5		
67	64	4	3	-1	1	19,5		19,5	
68	65	4	4	0					
69	66	4	3	-1	1	19,5		19,5	
70	67	4	4	0					
71	68	4	5	1	1	19,5	19,5		
72	69	2	5	3	3	60	60		
73	70	3	2	-1	1	19,5		19,5	
74	71	3	3	0					
75	72	4	3	-1	1	19,5		19,5	
76	73	2	3	1	1	19,5	19,5		
77	74	5	2	-3	3	60		60	
78	75	5	3	-2	2	48		48	
79	76	5	4	-1	1	19,5		19,5	
80	77	4	4	0					
81	78	2	3	1	1	19,5	19,5		
82	79	5	4	-1	1	19,5		19,5	
83	80	5	2	-3	3	60		60	
84	81	5	3	-2	2	48		48	
85	82	5	4	-1	1	19,5		19,5	
86	83	4	4	0					
87	84	4	4	0					
88	85	4	4	0					
89	86	3	4	1	1	19,5	19,5		
90	87	5	4	-1	1	19,5		19,5	
91	88	4	4	0					
92	89	2	4	2	2	48	48		
93	90	3	3	0					
94	Median	4	4	0		Sum	1071	945	
95	Mean	3,72	3,78	0,06					
96	St dev	1,08	1,00	1,40					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	90		=COUNT(C4:C93)			
7		N-diff	63		=COUNT(E4:E93)			
8		Ws+	1071		=G94			
9		Ws-	945		=H94			
10		Ws	945		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	1008,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	21336,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	146,068		=SQRT(K13)			
15		Z-score	0,431		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	721,7		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,666		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,032		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	945		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,333		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,666		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,335		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,671		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	1008		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	141,593		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,441		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,033		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,329		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,659		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,329		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,659		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	23. I find it useful to replicate the experiments that led to the development of physics using modern tools								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	4	3	-1	1	18,5		18,5	
5	2	4	4	0					
6	3	4	5	1	1	18,5	18,5		
7	4	4	5	1	1	18,5	18,5		
8	5	4	5	1	1	18,5	18,5		
9	6	5	4	-1	1	18,5		18,5	
10	7	5	2	-3	3	47,5		47,5	
11	8	3	4	1	1	18,5	18,5		
12	9	4	5	1	1	18,5	18,5		
13	10	4	4	0					
14	11	5	5	0					
15	12	3	3	0					
16	13	3	5	2	2	41,5	41,5		
17	14	1	5	4	4	50	50		
18	15	5	4	-1	1	18,5		18,5	
19	16	5	1	-4	4	50		50	
20	17	4	5	1	1	18,5	18,5		
21	18	5	3	-2	2	41,5		41,5	
22	19	5	3	-2	2	41,5		41,5	
23	20	5	5	0					
24	21	5	5	0					
25	22	4	5	1	1	18,5	18,5		
26	23	4	2	-2	2	41,5		41,5	
27	24	5	4	-1	1	18,5		18,5	
28	25	4	5	1	1	18,5	18,5		
29	26	4	5	1	1	18,5	18,5		
30	27	5	5	0					
31	28	2	2	0					
32	29	4	3	-1	1	18,5		18,5	
33	30	4	4	0					
34	31	5	5	0					
35	32	5	1	-4	4	50		50	
36	33	3	4	1	1	18,5	18,5		
37	34	5	2	-3	3	47,5		47,5	
38	35	4	5	1	1	18,5	18,5		
39	36	4	4	0					
40	37	3	2	-1	1	18,5		18,5	
41	38	5	5	0					
42	39	3	5	2	2	41,5	41,5		
43	40	3	3	0					
44	41	5	5	0					
45	42	5	5	0					
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	5	5	0					
48	45	4	3	-1	1	18,5		18,5	
49	46	5	4	-1	1	18,5		18,5	
50	47	5	5	0					
51	48	4	5	1	1	18,5	18,5		
52	49	4	5	1	1	18,5	18,5		
53	50	4	5	1	1	18,5	18,5		
54	51	5	5	0					
55	52	5	4	-1	1	18,5		18,5	
56	53	3	5	2	2	41,5	41,5		
57	54	5	4	-1	1	18,5		18,5	
58	55	5	5	0					
59	56	5	5	0					
60	57	5	5	0					
61	58	4	4	0					
62	59	3	3	0					
63	60	5	5	0					
64	61	5	4	-1	1	18,5		18,5	
65	62	3	3	0					
66	63	4	4	0					
67	64	3	2	-1	1	18,5		18,5	
68	65	4	4	0					
69	66	5	4	-1	1	18,5		18,5	
70	67	3	4	1	1	18,5	18,5		
71	68	4	4	0					
72	69	5	5	0					
73	70	3	5	2	2	41,5	41,5		
74	71	2	4	2	2	41,5	41,5		
75	72	3	4	1	1	18,5	18,5		
76	73	5	5	0					
77	74	4	4	0					
78	75	5	5	0					
79	76	5	4	-1	1	18,5		18,5	
80	77	4	4	0					
81	78	4	4	0					
82	79	5	3	-2	2	41,5		41,5	
83	80	5	5	0					
84	81	5	5	0					
85	82	4	4	0					
86	83	5	4	-1	1	18,5		18,5	
87	84	4	4	0					
88	85	5	4	-1	1	18,5		18,5	
89	86	2	3	1	1	18,5	18,5		
90	87	4	5	1	1	18,5	18,5		
91	88	4	3	-1	1	18,5		18,5	
92	89	4	3	-1	1	18,5		18,5	
93	90	5	3	-2	2	41,5		41,5	
94	Median	4	4	0		Sum	590,5	735,5	
95	Mean	4,19	4,08	-0,11					
96	St dev	0,90	1,03	1,26					

	I	J	K	L	M	N	O
1							
2		Wilcoxon Signed-Rank Test for Paired Samples					
3							
4		α	0,05				
5		tails	2				
6		N-tot	90		=COUNT(C4:C93)		
7		N-diff	51		=COUNT(E4:E93)		
8		Ws+	590,5		=G94		
9		Ws-	735,5		=H94		
10		Ws	590,5		=MIN(G94:H94)		
11		Ws-crit			table lookup		
12		Mean-Ws statistic (appr. normal distribution)	663,0		=K7*(K7+1)/4		
13		Variance-Ws statistic (appr. normal distribution)	11381,5		=K12*(2*K7+1)/6		
14		Std dev-Ws statistic (appr. normal distribution)	106,684		=SQRT(K13)		
15		Z-score	0,680		=ABS(K10-K12)/K14		
16		Ws-crit (normal appossimation)	453,9		=K12+K14*NORM.S.INV(K4/2)-0.05		
17		p-normal approximation	0,497		=2*(1-NORM.S.DIST(K12,TRUE))		
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")		
19		Effect size r	0,051		=K15/RADQ(2*K6)		
20							
21		Real Statistics functions (no ties, no cont)					
22							
23		Ws	590,5		=SRANK(B4:B93;C4:C93)		
24		p-normal approximation (1 tail)	0,248		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)		
25		p-normal approximation (2 tails)	0,497		=SRTEST(B4:C93,,2,FALSE,FALSE)		
26		p-exact (1 tail)	0,250		=SRDIST(K10;K7;1)		
27		p-exact (2 tails)	0,500		=SRDIST(K10;K7)		
28							
29							
30		Continuity and ties correction					
31							
32		Mean-Ws statistic (appr. normal distribution)	663		=K7*(K7+1)/4		
33		Std dev-Ws statistic with ties correction	101,926		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;		
34		Z-score with Yates continuity correction	0,706		=ABS(ABS(K23-K32)-1/2)/K33		
35		Effect size r with corrections	0,053		=K34/SQRT(2*K6)		
36		p-normal approximation with corrections (1 tail)	0,240		=1-NORM.S.DIST(K34)		
37		p-normal approximation with corrections (2 tails)	0,480		=2*PK36		
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")		
39							
40							
41		Real Statistics functions (default) for corrections					
42							
43		p-normal approximation with corrections (1 tail)	0,240		=SRTEST(B4:B93,C4:C93,1)		
44		p-normal approximation with corrections (2 tails)	0,480		=SRTEST(B4:C93)		
45							
46							
47							

24. I prefer that physics theories are introduced to me in a different mathematical form (eg more modern or simplified) from the way in which they were originally formulated

Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks
1	5	4	-1	1	15		15
2	3	4	1	1	15	15	
3	4	5	1	1	15	15	
4	4	4	0				
5	3	5	2	2	38,5	38,5	
6	5	5	0				
7	4	5	1	1	15	15	
8	4	4	0				
9	4	4	0				
10	5	4	-1	1	15		15
11							
12	3	5	2	2	38,5	38,5	
13	3	3	0				
14	4	4	0				
15	3	3	0				
16	5	2	-3	3	51		51
17	4	5	1	1	15	15	
18	4	5	1	1	15	15	
19	4	4	0				
20	4	4	0				
21	5	3	-2	2	38,5		38,5
22	1	5	4	4	55	55	
23	3	4	1	1	15	15	
24	4	1	-3	3	51		51
25	4	4	0				
26	5	5	0				
27	4	2	-2	2	38,5		38,5
28	5	5	0				
29	4	4	0				
30	3	3	0				
31	1	2	1	1	15	15	
32	1	3	2	2	38,5	38,5	
33	3	2	-1	1	15		15
34	5	5	0				
35	5	4	-1	1	15		15
36	4	4	0				
37	1	3	2	2	38,5	38,5	
38	4	5	1	1	15	15	
39	4	4	0				
40	2	5	3	3	51	51	
41	5	3	-2	2	38,5		38,5
42	4	5	1	1	15	15	
43	3	3	0				

	A	B	C	D	E	F	G	H	I
47	44	5	3	-2	2	38,5		38,5	
48	45	5	4	-1	1	15		15	
49	46	5	5	0					
50	47	5	5	0					
51	48	2	5	3	3	51	51		
52	49	3	4	1	1	15	15		
53	50	4	4	0					
54	51	4	4	0					
55	52	4	3	-1	1	15		15	
56	53	3	4	1	1	15	15		
57	54	4	3	-1	1	15		15	
58	55	4	2	-2	2	38,5		38,5	
59	56	5	5	0					
60	57	3	3	0					
61	58	5	3	-2	2	38,5		38,5	
62	59	5	5	0					
63	60	5	5	0					
64	61	4	3	-1	1	15		15	
65	62	3	5	2	2	38,5	38,5		
66	63	4	5	1	1	15	15		
67	64	3	5	2	2	38,5	38,5		
68	65	3	3	0					
69	66	3	5	2	2	38,5	38,5		
70	67	4	5	1	1	15	15		
71	68	3	4	1	1	15	15		
72	69	4	4	0					
73	70	4	5	1	1	15	15		
74	71	4	5	1	1	15	15		
75	72	4	4	0					
76	73	3	3	0					
77	74	5	3	-2	2	38,5		38,5	
78	75	5	2	-3	3	51		51	
79	76	4	5	1	1	15	15		
80	77	5	5	0					
81	78	5	2	-3	3	51		51	
82	79	3	4	1	1	15	15		
83	80	5	5	0					
84	81	5	5	0					
85	82	3	3	0					
86	83	3	5	2	2	38,5	38,5		
87	84	3	5	2	2	38,5	38,5		
88	85	3	5	2	2	38,5	38,5		
89	86	1	3	2	2	38,5	38,5		
90	87	2	3	1	1	15	15		
91	88	2	5	3	3	51	51		
92	89	3	2	-1	1	15		15	
93	90	4	3	-1	1	15		15	
94	Median	4	4	0		Sum	916,5	623,5	
95	Mean	3,74	3,94	0,20					
96	St dev	1,08	1,05	1,42					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	55		=COUNT(E4:E93)			
8		Ws+	916,5		=G94			
9		Ws-	623,5		=H94			
10		Ws	623,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	770,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	14245,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	119,352		=SQRT(K13)			
15		Z-score	1,227		=ABS(K10-K12)/K14			
16		Ws-crit (normal appressimation)	536,0		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,220		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,092		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	623,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,110		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,220		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,111		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,222		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	770		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	116,659		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,252		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,094		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,105		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,211		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,105		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,211		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	25. I am interested to learn about the difficulties experienced by scientists in their research and the way in which they overcame them								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	2	3	1	1	15,5	15,5		
5	2	2	4	2	2	39	39		
6	3	5	5	0					
7	4	1	1	0					
8	5	4	5	1	1	15,5	15,5		
9	6	2	4	2	2	39	39		
10	7	4	3	-1	1	15,5		15,5	
11	8	4	5	1	1	15,5	15,5		
12	9	1	3	2	2	39	39		
13	10	4	2	-2	2	39		39	
14	11	2	2						
15	12	2	4	2	2	39	39		
16	13	5	5	0					
17	14	4	2	-2	2	39		39	
18	15	5	5	0					
19	16	3	5	2	2	39	39		
20	17	2	5	3	3	49	49		
21	18	3	4	1	1	15,5	15,5		
22	19	4	3	-1	1	15,5		15,5	
23	20	5	5	0					
24	21	5	5	0					
25	22	1	4	3	3	49	49		
26	23	4	2	-2	2	39		39	
27	24	3	5	2	2	39	39		
28	25	4	4	0					
29	26	5	4	-1	1	15,5		15,5	
30	27	5	4	-1	1	15,5		15,5	
31	28	5	5	0					
32	29	3	2	-1	1	15,5		15,5	
33	30	5	5	0					
34	31	5	5	0					
35	32	5	5	0					
36	33	5	4	-1	1	15,5		15,5	
37	34	5	5	0					
38	35	5	5	0					
39	36	3	4	1	1	15,5	15,5		
40	37	1	2	1	1	15,5	15,5		
41	38	4	5	1	1	15,5	15,5		
42	39	4	4	0					
43	40	4	3	-1	1	15,5		15,5	
44	41	4	4	0					
45	42	5	5	0					
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	5	5	0					
48	45	3	3	0					
49	46	4	4	0					
50	47	2	4	2	2	39	39		
51	48	4	4	0					
52	49	5	5	0					
53	50	3	4	1	1	15,5	15,5		
54	51	2	2	0					
55	52	5	3	-2	2	39		39	
56	53	4	4	0					
57	54	5	5	0					
58	55	5	5	0					
59	56	2	1	-1	1	15,5		15,5	
60	57	4	4	0					
61	58	4	3	-1	1	15,5		15,5	
62	59	4	3	-1	1	15,5		15,5	
63	60	5	4	-1	1	15,5		15,5	
64	61	4	4	0					
65	62	4	5	1	1	15,5	15,5		
66	63	3	3	0					
67	64	3	2	-1	1	15,5		15,5	
68	65	4	4	0					
69	66	4	4	0					
70	67	3	4	1	1	15,5	15,5		
71	68	3	3	0					
72	69	4	2	-2	2	39		39	
73	70	4	3	-1	1	15,5		15,5	
74	71	4	1	-3	3	49		49	
75	72								
76	73	3	5	2	2	39	39		
77	74	5	3	-2	2	39		39	
78	75	5	5	0					
79	76	4	3	-1	1	15,5		15,5	
80	77	5	4	-1	1	15,5		15,5	
81	78	3	5	2	2	39	39		
82	79	3	4	1	1	15,5	15,5		
83	80	1	3	2	2	39	39		
84	81	5	5	0					
85	82	4	3	-1	1	15,5		15,5	
86	83	3	4	1	1	15,5	15,5		
87	84	4	3	-1	1	15,5		15,5	
88	85	5	3	-2	2	39		39	
89	86	1	5	4	4	51	51		
90	87	1	2	1	1	15,5	15,5		
91	88	3	3	0					
92	89	4	4	0					
93	90	2	2	0					
94	Median	4	4	0		Sum	740,5	585,5	
95	Mean	3,66	3,76	0,10					
96	St dev	1,24	1,14	1,27					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	51		=COUNT(E4:E93)			
8		Ws+	740,5		=G94			
9		Ws-	585,5		=H94			
10		Ws	585,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	663,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	11381,5		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	106,684		=SQRT(K13)			
15		Z-score	0,726		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	453,9		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,468		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,054		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	585,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,234		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,468		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,235		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,471		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	663		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	103,524		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,744		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,056		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,229		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,457		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,229		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,457		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	26. I find it very instructive to compare successive theories in the history of physics and learn which experiments have determined the choice of the most correct one								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	2	4	2	2	48,5	48,5		
5	2	2	1	-1	1	20,5		20,5	
6	3	2	5	3	3	57,5	57,5		
7	4	1	2	1	1	20,5	20,5		
8	5	3	3	0					
9	6	5	4	-1	1	20,5		20,5	
10	7	5	3	-2	2	48,5		48,5	
11	8								
12	9	4	3	-1	1	20,5		20,5	
13	10	1	4	3	3	57,5	57,5		
14	11								
15	12	2	4	2	2	48,5	48,5		
16	13	2	3	1	1	20,5	20,5		
17	14	2	4	2	2	48,5	48,5		
18	15	4	3	-1	1	20,5		20,5	
19	16	5	4	-1	1	20,5		20,5	
20	17	4	4	0					
21	18	4	5	1	1	20,5	20,5		
22	19	4	4	0					
23	20	5	5	0					
24	21	5	4	-1	1	20,5		20,5	
25	22	2	2	0					
26	23	4	4	0					
27	24	4	4	0					
28	25	3	5	2	2	48,5	48,5		
29	26	4	5	1	1	20,5	20,5		
30	27	4	3	-1	1	20,5		20,5	
31	28	4	4	0					
32	29	4	4	0					
33	30	4	5	1	1	20,5	20,5		
34	31	5	5	0					
35	32	5	5	0					
36	33	5	5	0					
37	34	5	5	0					
38	35	5	4	-1	1	20,5		20,5	
39	36	4	4	0					
40	37	2	3	1	1	20,5	20,5		
41	38	5	5	0					
42	39	4	5	1	1	20,5	20,5		
43	40	3	4	1	1	20,5	20,5		
44	41	5	5	0					
45	42	4	4	0					
46	43	5	4	-1	1	20,5		20,5	

	A	B	C	D	E	F	G	H	I
47	44	4	5	1	1	20,5	20,5		
48	45	4	4	0					
49	46	5	4	-1	1	20,5		20,5	
50	47	4	5	1	1	20,5	20,5		
51	48	4	5	1	1	20,5	20,5		
52	49	3	5	2	2	48,5	48,5		
53	50	2	4	2	2	48,5	48,5		
54	51	3	3	0					
55	52	5	4	-1	1	20,5		20,5	
56	53	3	5	2	2	48,5	48,5		
57	54	3	4	1	1	20,5	20,5		
58	55	5	3	-2	2	48,5		48,5	
59	56	3	1	-2	2	48,5		48,5	
60	57	5	5	0					
61	58	3	5	2	2	48,5	48,5		
62	59	3	1	-2	2	48,5		48,5	
63	60	3	3	0					
64	61	1	3	2	2	48,5	48,5		
65	62	2	3	1	1	20,5	20,5		
66	63	4	4	0					
67	64	5	4	-1	1	20,5		20,5	
68	65	4	4	0					
69	66	3	3	0					
70	67	4	4	0					
71	68	4	5	1	1	20,5	20,5		
72	69	4	4	0					
73	70	3	4	1	1	20,5	20,5		
74	71	2	2	0					
75	72	2	3	1	1	20,5	20,5		
76	73	2	1	-1	1	20,5		20,5	
77	74	3	4	1	1	20,5	20,5		
78	75	5	4	-1	1	20,5		20,5	
79	76	5	5	0					
80	77	5	3	-2	2	48,5		48,5	
81	78	3	4	1	1	20,5	20,5		
82	79	2	3	1	1	20,5	20,5		
83	80	3	4	1	1	20,5	20,5		
84	81	5	5	0					
85	82	5	3	-2	2	48,5		48,5	
86	83	5	5	0					
87	84	4	3	-1	1	20,5		20,5	
88	85	4	3	-1	1	20,5		20,5	
89	86	3	4	1	1	20,5	20,5		
90	87	3	2	-1	1	20,5		20,5	
91	88	4	2	-2	2	48,5		48,5	
92	89	3	4	1	1	20,5	20,5		
93	90	3	2	-1	1	20,5		20,5	
94	Median	4	4	0		Sum	1002,5	708,5	
95	Mean	3,63	3,78	0,16					
96	St dev	1,15	1,07	1,17					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	88		=COUNT(C4:C93)			
7		N-diff	58		=COUNT(E4:E93)			
8		Ws+	1002,5		=G94			
9		Ws-	708,5		=H94			
10		Ws	708,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	855,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	16682,3		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	129,160		=SQRT(K13)			
15		Z-score	1,138		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	602,3		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,255		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,086		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	708,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,128		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,255		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,129		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,257		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	855,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	123,550		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,186		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,089		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,118		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,236		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,118		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,236		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	27. I think it unimportant to know the experimental anomalies that led to the rejection of scientific theories that turned out to be not exhaustive								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	3	0					
5	2	3	3	0					
6	3	3	5	2	2	42	42		
7	4	1	2	1	1	16	16		
8	5	3	1	-2	2	42		42	
9	6	2	4	2	2	42	42		
10	7	2	3	1	1	16	16		
11	8	3	3	0					
12	9	3	3	0					
13	10	3	2	-1	1	16		16	
14	11	2	5	3	3	55,5	55,5		
15	12	2	3	1	1	16	16		
16	13	3	2	-1	1	16		16	
17	14	5	5	0					
18	15	3	1	-2	2	42		42	
19	16	2	1	-1	1	16		16	
20	17	4	2	-2	2	42		42	
21	18	4	2	-2	2	42		42	
22	19	3	3	0					
23	20	4	3	-1	1	16		16	
24	21	3	1	-2	2	42		42	
25	22	2	2	0					
26	23	2	2	0					
27	24	3	1	-2	2	42		42	
28	25	3	2	-1	1	16		16	
29	26	1	3	2	2	42	42		
30	27	5	5	0					
31	28	3	4	1	1	16	16		
32	29	3	3	0					
33	30	2	1	-1	1	16		16	
34	31	5	5	0					
35	32	1	5	4	4	60	60		
36	33	1	1	0					
37	34	1	1	0					
38	35	1	1	0					
39	36	2	2	0					
40	37	2	4	2	2	42	42		
41	38	4	2	-2	2	42		42	
42	39	5	5	0					
43	40	2	3	1	1	16	16		
44	41	3	2	-1	1	16		16	
45	42	2	2	0					
46	43	1	2	1	1	16	16		

	A	B	C	D	E	F	G	H	I
47	44	4	5	1	1	16	16		
48	45	2	3	1	1	16	16		
49	46	2	3	1	1	16	16		
50	47	4	1	-3	3	55,5		55,5	
51	48	1	1	0					
52	49	4	2	-2	2	42		42	
53	50	2	3	1	1	16	16		
54	51	5	2	-3	3	55,5		55,5	
55	52	2	2	0					
56	53	2	2	0					
57	54	5	2	-3	3	55,5		55,5	
58	55	5	1	-4	4	60		60	
59	56	1	5	4	4	60	60		
60	57	3	5	2	2	42	42		
61	58	3	2	-1	1	16		16	
62	59	1	2	1	1	16	16		
63	60	3	1	-2	2	42		42	
64	61	1	3	2	2	42	42		
65	62	3	3	0					
66	63	2	2	0					
67	64	5	3	-2	2	42		42	
68	65	4	3	-1	1	16		16	
69	66	1	1	0					
70	67	2	4	2	2	42	42		
71	68	4	4	0					
72	69	3	4	1	1	16	16		
73	70	4	5	1	1	16	16		
74	71	5	2	-3	3	55,5		55,5	
75	72	4	2	-2	2	42		42	
76	73	1	3	2	2	42	42		
77	74	4	3	-1	1	16		16	
78	75	4	3	-1	1	16		16	
79	76	2	3	1	1	16	16		
80	77	2	2	0					
81	78	5	2	-3	3	55,5		55,5	
82	79	3	3	0					
83	80	1	3	2	2	42	42		
84	81	1	2	1	1	16	16		
85	82	4	3	-1	1	16		16	
86	83								
87	84	4	3	-1	1	16		16	
88	85	2	3	1	1	16	16		
89	86	2	2	0					
90	87	5	3	-2	2	42		42	
91	88	4	3	-1	1	16		16	
92	89	3	3	0					
93	90	3	4	1	1	16	16		
94	Median	3	3	0		Sum	825,5	1065,5	
95	Mean	2,83	2,71	-0,12					
96	St dev	1,26	1,20	1,57					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	61		=COUNT(E4:E93)			
8		Ws+	825,5		=G94			
9		Ws-	1065,5		=H94			
10		Ws	825,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	945,5		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	19382,8		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	139,222		=SQRT(K13)			
15		Z-score	0,862		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	672,6		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,389		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,065		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	825,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,194		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,389		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,196		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,391		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	945,5		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	136,255		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,877		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,066		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,190		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,380		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,190		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,380		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	28. I would like that more history-of-physics aspects than other elements should be introduced in a physics course.								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	3	0					
5	2	3	5	2	2	45	45		
6	3	2	5	3	3	53,5	53,5		
7	4	1	1	0					
8	5	2	2	0					
9	6	3	2	-1	1	19		19	
10	7	2	3	1	1	19	19		
11	8	2	2	0					
12	9	1	1	0					
13	10	1	1	0					
14	11	2	3	1	1	19	19		
15	12	2	2	0					
16	13	1	2	1	1	19	19		
17	14	3	1	-2	2	45		45	
18	15	3	4	1	1	19	19		
19	16	2	3	1	1	19	19		
20	17	2	1	-1	1	19		19	
21	18	3	1	-2	2	45		45	
22	19	3	2	-1	1	19		19	
23	20	2	3	1	1	19	19		
24	21	3	3	0					
25	22	1	2	1	1	19	19		
26	23	2	4	2	2	45	45		
27	24	5	3	-2	2	45		45	
28	25	2	1	-1	1	19		19	
29	26	1	2	1	1	19	19		
30	27	5	3	-2	2	45		45	
31	28	1	2	1	1	19	19		
32	29	2	1	-1	1	19		19	
33	30	3	3	0					
34	31	4	2	-2	2	45		45	
35	32	5	3	-2	2	45		45	
36	33	3	3	0					
37	34	2	1	-1	1	19		19	
38	35	2	3	1	1	19	19		
39	36	3	2	-1	1	19		19	
40	37	2	2	0					
41	38	1	2	1	1	19	19		
42	39	2	2	0					
43	40	3	2	-1	1	19		19	
44	41	2	2	0					
45	42	3	3	0					
46	43	3	3	0					

	A	B	C	D	E	F	G	H	I
47	44	1	2	1	1	19	19		
48	45	5	3	-2	2	45		45	
49	46	1	2	1	1	19	19		
50	47	1	2	1	1	19	19		
51	48	3	2	-1	1	19		19	
52	49	4	4	0					
53	50	1	2	1	1	19	19		
54	51	3	2	-1	1	19		19	
55	52	3	2	-1	1	19		19	
56	53	4	3	-1	1	19		19	
57	54	4	3	-1	1	19		19	
58	55	3	1	-2	2	45		45	
59	56	4	1	-3	3	53,5		53,5	
60	57	1	3	2	2	45	45		
61	58	3	4	1	1	19	19		
62	59	1	1	0					
63	60	2	2	0					
64	61	2	4	2	2	45	45		
65	62	2	2	0					
66	63	3	2	-1	1	19		19	
67	64	1	2	1	1	19	19		
68	65	3	2	-1	1	19		19	
69	66	2	3	1	1	19	19		
70	67	1	1	0					
71	68	2	3	1	1	19	19		
72	69	1	1	0					
73	70	5	5	0					
74	71	1	1	0					
75	72	5	3	-2	2	45		45	
76	73	3	3	0					
77	74	4	4	0					
78	75	3	3	0					
79	76	3	3	0					
80	77								
81	78	5	5	0					
82	79	2	1	-1	1	19		19	
83	80	2	2	0					
84	81	5	1	-4	4	55		55	
85	82	3	3	0					
86	83	4	4	0					
87	84	3	3	0					
88	85	3	3	0					
89	86								
90	87	1	2	1	1	19	19		
91	88	4	2	-2	2	45		45	
92	89	3	1	-2	2	45		45	
93	90	3	2	-1	1	19		19	
94	Median	3	2	0		Sum	613,5	926,5	
95	Mean	2,57	2,40	-0,17					
96	St dev	1,19	1,05	1,22					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	88		=COUNT(C4:C93)			
7		N-diff	55		=COUNT(E4:E93)			
8		Ws+	613,5		=G94			
9		Ws-	926,5		=H94			
10		Ws	613,5		=MIN(G94:H94)			
11		Ws-crit			table lookup			
12		Mean-Ws statistic (appr. normal distribution)	770,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	14245,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	119,352		=SQRT(K13)			
15		Z-score	1,311		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	536,0		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,190		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,099		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	613,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,095		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,190		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,096		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,191		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	770		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	114,544		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	1,362		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,103		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,087		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,173		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,087		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,173		=SRTEST(B4:C93)			
45								
46								
47								

	A	B	C	D	E	F	G	H	I
1	29. I think that an educational path on the history of Galilean physics is useful								
2									
3	Student	Before	After	Diff= Af-Be	Abs Diff	Rank of Abs Diff	Positive Ranks	Negative Ranks	
4	1	3	4	1	1	16	16		
5	2	1	3	2	2	37,5	37,5		
6	3	5	5	0					
7	4	1	3	2	2	37,5	37,5		
8	5	4	4	0					
9	6	5	5	0					
10	7	4	4	0					
11	8	2	4	2	2	37,5	37,5		
12	9	1	3	2	2	37,5	37,5		
13	10	4	3	-1	1	16		16	
14	11	3	3	0					
15	12	3	2	-1	1	16		16	
16	13	3	4	1	1	16	16		
17	14	5	5	0					
18	15	4	5	1	1	16	16		
19	16	4	4	0					
20	17	5	3	-2	2	37,5		37,5	
21	18	5	5	0					
22	19	5	3	-2	2	37,5		37,5	
23	20	4	4	0					
24	21	4	4	0					
25	22	4	4	0					
26	23	3	3	0					
27	24	4	5	1	1	16	16		
28	25	3	4	1	1	16	16		
29	26	4	5	1	1	16	16		
30	27	5	5	0					
31	28	3	3	0					
32	29	3	2	-1	1	16		16	
33	30	5	4	-1	1	16		16	
34	31	5	1	-4	4	47		47	
35	32	5	5	0					
36	33	5	5	0					
37	34	2	5	3	3	45	45		
38	35	5	5	0					
39	36	4	4	0					
40	37	2	2	0					
41	38	4	5	1	1	16	16		
42	39	4	5	1	1	16	16		
43	40	3	4	1	1	16	16		
44	41	4	4	0					
45	42	5	4	-1	1	16		16	
46	43	5	5	0					

	A	B	C	D	E	F	G	H	I
47	44	4	5	1	1	16	16		
48	45	5	4	-1	1	16		16	
49	46	4	4	0					
50	47	2	4	2	2	37,5	37,5		
51	48	3	5	2	2	37,5	37,5		
52	49	4	4	0					
53	50	4	4	0					
54	51	4	5	1	1	16	16		
55	52	4	4	0					
56	53	4	4	0					
57	54	1	4	3	3	45	45		
58	55	4	2	-2	2	37,5		37,5	
59	56	5	5	0					
60	57	3	4	1	1	16	16		
61	58	3	1	-2	2	37,5		37,5	
62	59	3	4	1	1	16	16		
63	60	4	3	-1	1	16		16	
64	61	4	5	1	1	16	16		
65	62	3	4	1	1	16	16		
66	63	4	4	0					
67	64	3	3	0					
68	65	4	3	-1	1	16		16	
69	66	3	4	1	1	16	16		
70	67	4	4	0					
71	68	3	3	0					
72	69	5	3	-2	2	37,5		37,5	
73	70	5	4	-1	1	16		16	
74	71	4	3	-1	1	16		16	
75	72	4	4	0					
76	73	4	4	0					
77	74	4	4	0					
78	75	4	3	-1	1	16		16	
79	76	5	5	0					
80	77	4	4	0					
81	78	5	4	-1	1	16		16	
82	79	5	2	-3	3	45		45	
83	80	4	4	0					
84	81	4	3	-1	1	16		16	
85	82								
86	83	4	4	0					
87	84	4	4	0					
88	85	4	3	-1	1	16		16	
89	86	5	5	0					
90	87	4	4	0					
91	88	3	5	2	2	37,5	37,5		
92	89	4	4	0					
93	90	4	3	-1	1	16		16	
94	Median	4	4	0		Sum	608,5	519,5	
95	Mean	3,82	3,87	0,04					

	I	J	K	L	M	N	O	P
1								
2		Wilcoxon Signed-Rank Test for Paired Samples						
3								
4		α	0,05					
5		tails	2					
6		N-tot	89		=COUNT(C4:C93)			
7		N-diff	47		=COUNT(E4:E93)			
8		Ws+	608,5		=G94			
9		Ws-	519,5		=H94			
10		Ws	519,5		=MIN(G94:H94)			
11		Ws-crit	378		table lookup			
12		Mean-Ws statistic (appr. normal distribution)	564,0		=K7*(K7+1)/4			
13		Variance-Ws statistic (appr. normal distribution)	8930,0		=K12*(2*K7+1)/6			
14		Std dev-Ws statistic (appr. normal distribution)	94,499		=SQRT(K13)			
15		Z-score	0,471		=ABS(K10-K12)/K14			
16		Ws-crit (normal approssimation)	378,7		=K12+K14*NORM.S.INV(K4/2)-0.05			
17		p-normal approximation	0,638		=2*(1-NORM.S.DIST(K12,TRUE))			
18		Reject null hypothesis (differences are due to chance)	no		=IF(K17<K4,"yes","no")			
19		Effect size r	0,035		=K15/RADQ(2*K6)			
20								
21		Real Statistics functions (no ties, no cont)						
22								
23		Ws	519,5		=SRANK(B4:B93;C4:C93)			
24		p-normal approximation (1 tail)	0,319		=SRTEST(B4:B93,C4:C93,1,FALSE,FALSE)			
25		p-normal approximation (2 tails)	0,638		=SRTEST(B4:C93,,2,FALSE,FALSE)			
26		p-exact (1 tail)	0,320		=SRDIST(K10;K7;1)			
27		p-exact (2 tails)	0,641		=SRDIST(K10;K7)			
28								
29								
30		Continuity and ties correction						
31								
32		Mean-Ws statistic (appr. normal distribution)	564		=K7*(K7+1)/4			
33		Std dev-Ws statistic with ties correction	90,960		=SQRT(K32*(2*K7+1)/6-TiesCorrection(B4:B93;C4:C93;1)/48)			
34		Z-score with Yates continuity correction	0,484		=ABS(ABS(K23-K32)-1/2)/K33			
35		Effect size r with corrections	0,036		=K34/SQRT(2*K6)			
36		p-normal approximation with corrections (1 tail)	0,314		=1-NORM.S.DIST(K34)			
37		p-normal approximation with corrections (2 tails)	0,629		=2*PK36			
38		Reject null hypothesis (differences are due to chance)	no		=IF(K37<K4,"yes","no")			
39								
40								
41		Real Statistics functions (default) for corrections						
42								
43		p-normal approximation with corrections (1 tail)	0,314		=SRTEST(B4:B93,C4:C93,1)			
44		p-normal approximation with corrections (2 tails)	0,629		=SRTEST(B4:C93)			
45								

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Index of Subjects and names

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