FACULTY OF PSYCHOLOGY AND
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Spatial numbers: How visual crowding and redundancy masking modulate numerosity perception

Miao Li

Doctoral thesis offered to obtain the degree of

Doctor of Psychology (PhD)

within the framework of a joint doctorate at KU Leuven and University of Lille

Supervisors: Prof. dr. Bilge Sayim (University of Lille) &
Prof. dr. Bert Reynvoet (KU Leuven)

Jury members:

Dr. Muriel Boucart	University of Lille	Examiner
Dr. Ramakrishna Chakravarthi	University of Aberdeen	Examiner
Pr. Michael Herzog	EPFL	Reviewer
Pr. Anke Huckauf	University of Ulm	Reviewer

Public defense on 22/03/2023 at 14:00 (Paris time)

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ABSTRACT

Humans can visually approximate the number of items without the need for counting, a process known as numerosity perception. It has been suggested that numerosity perception is either the result of a dedicated system to estimate the number of items, or is due to the exploitation of various visual properties of the presented items. There are many limitations of our spatial vision that may be related to numerosity perception. For example, visual crowding is the difficulty of identifying targets in cluttered environments, and it is especially pronounced in peripheral vision. Recently, it has been discovered that individuals often fail to detect identical items when they are presented in small clusters in the visual periphery, a phenomenon referred to as redundancy masking. The influence of these and related limits of spatial vision on numerosity perception have yet to be extensively explored. In the current thesis, we aim to investigate how crowding and redundancy masking modulate numerosity perception. In Chapter 1, existing theories of numerosity perception are reviewed, and the potential confounding factors, such as density and convex hull on numerosity perception, are discussed. These factors have been previously shown to impact numerosity perception, and it is important to consider their potential effects in studies on numerosity perception. The concept of crowding and redundancy masking as specific forms of limits of spatial vision, and their potential influence on numerosity perception are introduced. To investigate these effects, a series of experiments were conducted, as discussed in chapters 2 to 4 of the thesis. In Chapter 2, we present the results of the investigation into the impact of the topology of spatial vision on numerosity perception. Specifically, we examined the effects of the radial-tangential anisotropy that radially arranged items were found to interfere more with target perception than tangentially arranged items on numerosity perception. The results demonstrated that numerosity estimates were lower when target discs on displays were predominantly arranged in a radial direction compared to a tangential direction. These findings provide evidence that the radial-tangential anisotropy of spatial vision modulates numerosity perception and highlights the importance of considering visual field asymmetries when studying numerosity perception. Observers reported radial displays as less numerous compared to tangential displays, regardless of how the radial-tangential arrangements of displays were manipulated (Chapter 3). Our results were consistent across experiments, including when manipulation of the radial and tangential arrangement of displays was weak (Experiment 3.1), strong (Experiment 3.2), or modulated with mixed contrast polarity (Experiment 3.3 and Experiment 3.4). We proposed that crowding and redundancy masking modulate numerosity perception. Next, two more experiments were conducted to investigate redundancy masking with faces using a typical redundancy masking paradigm (Chapter 4). Faces are of great social importance and are usually processed quickly. The results showed that observers often failed to detect faces when presented in small groups, with the number of reported items frequently lower than the number of presented items. The results showed that redundancy masking occurs with highly complex stimuli such as faces, and that it is a key mechanism for compressing redundant visual information. In Chapter 5, we discussed and elucidated all experiments in the pervious chapters and the observed results. Overall, our results demonstrated how crowding and redundancy masking possibly modulate numerosity perception.

SAMENVATTING

Mensen kunnen het aantal visueel gepresenteerde objecten schatten zonder te tellen, een proces dat bekend staat als numerositeitsperceptie. Eerder werd gesuggereerd dat numerositeitsperceptie ofwel het resultaat is van een gespecialiseerd numeriek systeem. Een alternatieve visie is dat dit het gevolg is van het gebruik van verschillende visuele, niet-numerieke eigenschappen van de visuele stimulus (bv. totale oppervlakte).

Er zijn echter ook verschillende beperkingen van onze spatiale perceptie die een impact kunnen hebben op numerositeitsperceptie. Bijvoorbeeld, visuele “crowding” is de moeilijkheid om objecten te identificeren in een “crowded” omgeving en treed vooral op in perifere visie. Onlangs werd duidelijk dat mensen vaak identieke objecten niet kunnen detecteren als ze in kleine clusters in de visuele periferie worden gepresenteerd, een fenomeen dat “redundancy making” wordt genoemd. De invloed van deze en gerelateerde beperkingen op de spatiale perceptie van numerositeitsperceptie is vooralsnog een open vraag.

In dit proefschrift willen we onderzoeken hoe crowding en redundancy masking numerositeitsperceptie beïnvloeden. In Hoofdstuk 1 worden bestaande theorieën over numerositeitsperceptie besproken en de mogelijke verstorende factoren, zoals dichtheid en oppervlakte op numerositeitsperceptie. Van deze factoren is eerder aangetoond dat ze invloed hebben op numerositeitsperceptie, en het is belangrijk om rekening te houden met hun mogelijke effecten in studies naar numerositeitsperceptie. Ook worden de concepten crowding en redundancy masking, als specifieke vormen van limitaties van spatiale perceptie, en hun mogelijke invloed op numerositeitsperceptie worden geïntroduceerd. Om deze effecten te onderzoeken is een reeks experimenten uitgevoerd, zoals besproken in de hoofdstukken 2 tot en met 4 van dit proefschrift. In Hoofdstuk 2 onderzochten we de effecten van de radiaal-tangentiële anisotropie: De resultaten toonden aan dat numerositeitsschattingen lager waren wanneer de targets voornamelijk in radiale richting waren geplaatst in vergelijking met tangentiële richting. Deze bevindingen leveren het bewijs dat de radiaal-tangentiële anisotropie in spatiale perceptie de numerositeitsperceptie beïnvloed en benadrukt het belang van van visuele veld asymmetrieën bij van numerositeitsperceptie. Hoofdstuk 3 bouwt verder op de bevindingen van Hoofdstuk 2 en en toont aan dat radiale displays van objecten als minder talrijk worden geschat dan tangentiële displays, ongeacht de specifieke manipulatie (Hoofdstuk 3). De resultaten waren consistent in alle experimenten, inclusief wanneer manipulatie van de radiale en tangentiële rangschikking van displays beperkt was (Experiment 3.1), sterk was (Experiment 3.2), of wanneer er sprake was van contrastpolariteit (Experiment 3.3 en Experiment 3.4). We stelden voor dat crowding en redundantie masking de numerositeitsperceptie moduleren. In Hoofdstuk 4 werden nog twee experimenten uitgevoerd om redundancy masking bij kleine aantallen van gezichten te onderzoeken. Gezichten zijn van groot sociaal belang en worden meestal snel verwerkt. De resultaten toonden aan dat participanten de gezichten vaak niet allemaal detecteerden als ze in kleine groepen werden gepresenteerd, waardoor het aantal gerapporteerde items vaak lager was dan het aantal gepresenteerde items. De resultaten toonden aan dat redundancy masking optreedt bij zeer complexe stimuli zoals gezichten, en dat het een belangrijk mechanisme is voor het comprimeren van redundante visuele informatie. Het afsluitend Hoofdstuk 5 bevat een samenvatting van de resultaten en een integratie ervan in de bestaande literatuur. In het algemeen toonden onze resultaten aan hoe crowding en redundantie masking numerositeitsperceptie beïnvloeden.

RÉSUMÉ

L'être humain est capable d'estimer visuellement le nombre d'objets sans avoir à les compter, c'est un processus connu sous le nom de perception de la numérosité. Il a été suggéré que cette perception relève soit d'un système dédié à l'estimation du nombre d'objets, soit de l'exploitation de diverses propriétés visuelles des objets présentés. Il existe de nombreuses limitations de notre vision spatiale qui peuvent être liées à la perception de la numérosité. Par exemple, l'encombrement visuel est la difficulté d'identifier des cibles dans des environnements encombrés, et il est particulièrement prononcé dans la vision périphérique. Une découverte récente montre que les individus ne parviennent pas à détecter des éléments identiques lorsqu'ils sont regroupés dans la vision périphérique, un phénomène appelé masquage de redondance. L'influence des limites de la vision spatiale sur la perception de la numérosité n'a pas encore été explorée en profondeur. Dans cette thèse, nous cherchons à étudier comment l'encombrement visuel et le masquage de redondance modulent la perception de la numérosité. Dans le chapitre 1, les théories existantes de la perception de la numérosité sont passées en revue, et les facteurs de confusion potentiels tels que la densité et l'enveloppe convexe sur la perception de la numérosité sont discutés. Il a été démontré que ces facteurs ont un impact sur la perception de la numérosité, et il est important de prendre en compte leurs effets potentiels dans les études sur la perception de la numérosité. Les concepts d'encombrement visuel et de masquage de redondance, en tant que limites spécifiques de la vision spatiale, et leur influence potentielle sur la perception de la numérosité sont présentés. Pour étudier ces effets, une série d'expériences a été menée, comme discuté dans les chapitres 2 à 4 de la thèse. Dans le chapitre 2, nous présentons les résultats concernant l'exploration en fonction de la topologie de la vision spatiale sur la perception de la numérosité. Plus précisément, nous avons examiné les effets de l'anisotropie radiale-tangentielle : il a été constaté que les éléments disposés radialement interféraient davantage avec la perception de la cible tandis que éléments disposés tangentiellement interfèrent davantage avec la perception de la numérosité. Les résultats ont démontré que les estimations de la numérosité étaient plus faibles lorsque les disques cibles sur les écrans étaient principalement disposés dans une direction radiale par rapport à une direction tangentielle. Ces résultats prouvent que l'anisotropie radiale-tangentielle de la vision spatiale module la perception de la numérosité et souligne l'importance de prendre en compte les asymétries du champ visuel lors de l'étude de la perception de la numérosité. Les observateurs ont signalé que les affichages radiaux étaient moins nombreux que les affichages tangentiels, quelle que soit la façon dont la disposition radiale-tangentielle des affichages était manipulée (chapitre 3). Nos résultats étaient cohérents d'une expérience à l'autre, y compris lorsque la manipulation de la disposition radiale et tangentielle des affichages était faible (expérience 3.1), forte (expérience 3.2) ou modulée par une polarité de contraste mixte (expérience 3.3 et expérience 3.4).Nous avons proposé que l'encombrement visuel et le masquage redondance modulent la perception de la numérosité. Ensuite, deux autres expériences ont été menées pour étudier le masquage de redondance avec des visages en utilisant un paradigme typique de masquage de redondance (chapitre 4). Les visages ont une grande importance sociale et sont généralement traités rapidement. Les résultats ont montré que les observateurs ne parvenaient pas à détecter les visages lorsqu'ils étaient présentés en petits groupes, le nombre d'éléments rapportés étant souvent inférieur au nombre d'éléments présentés. Les résultats ont montré que le masquage de redondance se produit avec des stimuli très complexes tels que les visages, et qu'il s'agit d'un mécanisme clé pour la compression des informations visuelles redondantes. Enfin, dans le chapitre 5, nous avons discuté et élucidé toutes les expériences des chapitres précédents et les résultats observés. Dans l'ensemble, nos résultats ont démontré comment l'encombrement visuel et le masquage de redondance peuvent moduler la perception de la numérosité.

PUBLICATIONS AND COMMUNICATIONS

Publications

L-Miao, L., Dandan, Y.R., Reynvoet, B., & Sayim, B. (in preparation). Redundancy masking of faces reveals a substantial failure to detect socially relevant information.

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Communications

Li, M., Yu, D., Reynvoet, B., & Sayim, B. (2022). Redundancy masking prevents the detection of faces. 22nd Conference of the European Society for Cognitive Psychology (ESCoP), August 2022, Lille, France. Poster.

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Li, M., Reynvoet, B., & Sayim, B. (2021). More crowded, less numerous: Crowding reduces the number of perceived items in numerosity perception. Virtual Vision Science Society, May 2021. Poster.

Li, M., Reynvoet, B., & Sayim, B. (2020). Perceived numerosity in crowded and uncrowded displays. Virtual Psychonomics 2020 Annual Meeting, November 2020. Poster.

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CHAPTER 1: GENERAL INTRODUCTION

Humans can estimate the number of visually displayed items. For example, when we are in a crowded hall, we can quickly estimate the approximate number of people without having to count them. Although the estimation is not precise, estimating the number of items in a given set is known as numerosity perception. Having the ability to discern numerosity offers an evolutionary advantage, as it allows one to select an area with a greater quantity of sustenance and to determine which group has fewer adversaries (Gómez-Laplaza & Gerlai, 2011; McComb et al., 1994; Nieder, 2018; Wilson et al., 2001).

1.1 The Innate Numerosity

It is frequently suggested that numerosity, as other primary features of objects such as orientation, color, size, etc., is another primary feature of objects (Ross & Burr, 2010). Our ability to process numerosity or to estimate quantities has been suggested to be innate in our visual brain and driven by an approximate number system (ANS, also known as the "number sense" Anobile et al., 2014; Arrighi et al., 2014; Burr et al., 2017; Chen & Verguts, 2013; Dehaene, 1992; Dehaene & Changeux, 1993; Dehaene et al., 1998; Feigenson et al., 2004; Halberda & Feigenson, 2008; Lipton & Spelke, 2003; Stoianov & Zorzi, 2012; Xu et al., 2005). The number sense account suggests that numerosity perception is spontaneously processed and not depending on other physical properties (e.g., size, convex hull, density, etc., Castaldi et al., 2021; Cicchini et al., 2016, 2019). Some evidence was shown to support the idea. For example, Izard et al. (2009) showed that newborn infants spontaneously associated visual displays that contained a different number of items (4 - 12) with auditory events on the basis of numbers, demonstrating that the ability to abstract number information is innate, and occurs at the start of life (see also, de Hevia et al., 2017). In the visual domain, adaptation is evident in color perception (which can differ significantly depending on the previous color seen, Webster, 2011), orientation (which can be altered after viewing tilted lines, Gibson & Radner, 1937), and movement (where the perception of stationary objects can be changed after viewing moving ones, Nashner, 1982). One of the strong indications that the ANS is an innate system is its sensitivity to adaptation (Mollon, 1974; Thompson & Burr, 2009). Burr and Ross (2008) showed that the perceived number of items in displays that were viewed after adaptation shifted drastically in the opposite direction of the adapted displays; that is, after viewing a dense (sparse) display, the subsequent display appeared to be less (more) numerous.

1.2 Numerosity perception Based on Other Physical Properties

However, the ANS view that numerosity perception is innate has been challenged. One of the arguments is that numerosity co-varies with many other nonnumerical physical properties. For example, given a fixed size of each item on display, the total surface area increases as the numerosity increases. The convex hull (the smallest convex shape that contains all items in a set) also correlates positively with the numerosity. More items on display result in a denser display than the display with fewer items given the same item size. It is impossible to create two displays with a different number of items but keep other nonnumerical physical properties unchanged (Leibovich & Ansari, 2016). Therefore, abstracting only the numerosity independent from other co-varied physical properties from a display seems to be impossible (Gebuis & Reynvoet, 2012a, 2012b, 2012c). Several studies showed that numerosity perception is influenced by other physical properties of the displays (Allik & Tuulmets, 1991; Sophian & Chu, 2008). For example, Clearfield and Mix (2001) showed that infants respond to the length of displays' contour instead of numerosity. Ginsburg and Nicholls (1988) demonstrated that perceived numerosity negatively correlated with item size (see also, Tokita & Ishiguchi, 2010, cf., Allik et al., 1991; Hurewitz et al., 2006). It was observed that the occupancy area (overall area occupied by items on displays), which is closely linked to item sizes, and convex hull have an effect on numerosity perception (Binet, 1890; Gilmore et al., 2016; Katzin, 2018; Shilat et al., 2021; Taves, 1941; Vos et al., 1988). For example, Gilmore et al. (2016) asked participants to ignore either the dot area information or the convex hull information while doing a numerosity perception task. They found that participants were able to ignore the dot area, and the ability improved with increasing age. However, the convex hull information was not easy to be ignored while performing dot comparison tasks, suggesting the convex hull's crucial role in numerosity perception. The occupancy model posits that the perceived numerosity in a relatively sparse random pattern of items is related to the area occupied by all items, which is determined by the size of the items and their fixed radius of influence. (Allik & Tuulmets, 1991). Allik and Tuulmets (1991) proposed that the area collectively occupied by items on displays, rather than the number of items per se, determines the perceived numerosity. This model explained the underestimation observed in many numerosity studies well. Particularly, when items were positioned close to each other, the occupied regions overlapped, thus perceived to be less numerous. When other physical properties of displays (e.g., total surface area, size, convex hull) were manipulated to be congruent or incongruent with the number, the numerosity judgment was impacted (Gebuis & Reynvoet, 2012b, 2012c; Hurewitz et al., 2006). For example, Hurewitz et al. (2006) presented displays where numerosity and dot size were manipulated to be congruent or incongruent. In the congruent condition, displays contain more dots composed of large dots or large total surface area, whereas, in the incongruent condition, displays contain more dots composed of small dots or small total surface area. They observed that participants made more errors and performed slower in a numerosity comparison task in the incongruent compared to the congruent condition.

Numerosity perception is also suggested to be influenced by clustering (Bertamini et al., 2018; Bertamini et al., 2016; Chakravarthi & Bertamini, 2020; Frith & Frut, 1972; Sophian, 2007). Frith and Frut (1972) first demonstrated that numerosity perception is impacted by how items are spatially arranged and that a large cluster appeared to be more numerous than several small clusters, known as the solitaire illusion. When items on displays are arranged into clusters, displays appear to be less numerous. An extreme case is the random-regular numerosity illusion (Cousins & Ginsburg, 1983; Ginsburg, 1980): items arranged into a regular pattern (e.g., on intersections of the grid) were judged as more numerous compared to items in a random, clustering pattern. One explanation is that when items on displays are arranged into "good" Gestalt (e.g., the whole central cluster of the solitaire illusion), the perceived numerosity was affected with this higher-order unit and appeared to be more numerous compared with items arranged into "bad" Gestalt (e.g., four corner clusters of the solitaire illusion, Frith & Frut, 1972). Nevertheless, an early study on how clustering impact numerosity perception showed contradictory results (Taves, 1941). Taves (1941) showed that a regularly arranged 20 items were perceived as less numerous than a cluster of 20 irregularly placed items. He suggested that displays with "good" Gestalt have fewer separate effects than the irregular pattern on the perception, and therefore, the regular pattern appeared to be more numerous than the irregular pattern. Inter-item spacing and regularity determine the spatial proximity of displays and results in different level of clustering of displays. Bertamini et al. (2016) first used different measures of the structural configurations of displays (e.g., the distribution, local clustering, overall convex set, etc.) to quantify items on displays that link to numerosity, clustering, and dispersion. Bertamini et al. (2016) presented displays that always contained the same number of items but varied in terms of clustering and dispersion. They concluded that no matter how clustering was quantified, the increase in clustering is linked to the decrease in perceived numerosity (see also, Bertamini et al., 2018). This evidence suggested that clustering could underline numerosity perception (Anobile et al., 2015; Chakravarthi & Bertamini, 2020).

Gebuis et al. (2016) proposed a more comprehensive explanation that there could be a sensory-integration system that evaluates large approximate numerosities by combining the various sensory cues that constitute number stimuli. They suggest that a combination of sensory inputs is used to create a unified representation of numerosity. The model suggested that salient visual cues are typically weighted heavily when doing numerosity perception. The sensory-integration model's predictions are in agreement with a number of earlier results, including the numerical distance effect (a decrease in the difference between two numerosities is associated with an increase in reaction time, Piazza et al., 2004; Sasanguie et al., 2011), the varying congruency effects (congruency effects scaled with the number of manipulated visual cues, Gebuis & Reynvoet, 2012b), and the opposite congruency effect (e.g., trials with a larger number of smaller dots have yielded better performance than those with a smaller number of larger dots, Ginsburg & Nicholls, 1988; Sophian, 2007). Importantly, Gebuis and Reynvoet (2012c) controlled the nonnumerical physical properties of displays so that these visual cues were manipulated to be uncorrelated with numerosity. They found that participants reported the displays to be more numerous when displays had smaller average diameter, aggregate surface or density, but a larger convex hull. They suggested that numerosity perception is performed by weighing and integrating multiple nonnumerical physical properties of displays (see also, Gebuis & Reynvoet, 2012b). The sensory integration account showed the importance of physical properties of displays in numerosity perception and challenged the existence of the ANS. In a recent review, Lourenco and Aulet (2022) proposed that numerosity is more than just a consequence of other magnitudes; it is its own distinct dimension that is not completely separate from the other magnitudes. A new model of numerosity perception was suggested by Lourenco and Aulet (2022), in which the perception of nonnumerical magnitude is integrated with the perception of numerosity throughout the entire perception process.

1.3 Intertwined Density and Numerosity

Numerosity and density are physically indivisible as density is calculated by dividing numerosity by the total area (Tibber et al., 2012). Burr and Ross (2008) demonstrated that numerosity is subject to adaptation and claimed that it is an independent visual property (from other visual properties, including density), further corroborated by Ross and Burr (2010). Anobile et al. (2014) found evidence that discrimination thresholds of high and low-density displays followed two distinct psychophysical functions, suggesting separate mechanisms for numerosity and density. However, Dakin et al. (2011) suggested that numerosity perception and density perception share a similar mechanism, and therefore, they cannot be clearly distinguished by the visual system (see also, Tibber et al., 2012). Many empirical studies support this idea. For example, Durgin (2008) claimed that the "adaptation on numerosity" described by Burr and Ross (2008) was actually based on texture density. Durgin (2008) presented two adapting displays: one contained more items than the other one, and the other's texture was denser, allowing dissociation between numerosity and density during the adaptation. The results showed that greater adaptation was produced by the region of greater density instead of higher numerosity. Similarly, Dakin et al. (2011) showed that both numerosity and density were biased by item size, suggesting a common visual metric between numerosity and density. Numerosity studies sometimes even indicated that although the task was formulated in terms of numerosity, the results and conclusions were *equally* applied to both numerosity and density since they are not dissociable (e.g., Valsecchi et al., 2013). Nevertheless, Ross and Burr (2010) provided further evidence that numerosity perception is not dependent on the densities of displays. They presented three types of displays to participants: a constant numerosity, a constant area, and a constant density, where one of the three parameters was kept constant in the experiment. Participants made comparisons on numerosity and density in separate blocks. Their results showed that density did not play a role in numerosity judgment as the performance of the constant density condition was not worse compared to the other two conditions. In another experiment, Ross and Burr (2010) showed that the perceived numerosity but not the density was modulated by luminance. Hence, it is unclear whether density and numerosity are independent of each other. This poses certain difficulties for future research on numerosity perception, as we must consider whether density plays a role or to what extent density plays a role in perceived numerosity.

1.4 Numerosity Perception in the Periphery

Investigations into numerosity perception generally involve displays that span a significant portion of the visual field, including the fovea, the parafovea, and often the periphery. However, there are substantial differences between different areas of the visual field (Rosenholtz, 2016; Simpson, 2017). For example, visual performance declines with increasing eccentricity; i.e., the performance is usually worse in the peripheral visual field compared to that in the central visual field (Gurnsey et al., 2011; Levi & Waugh, 1994; Livne & Sagi, 2007; Meinecke & Donk, 2002; Wolford & Hollingsworth, 1974; Zahabi & Arguin, 2014). Previous research also investigated numerosity perception in the periphery. For example, Mengal and Matathia (1980) presented small green and red LED lights to participants. Participants needed to discriminate which color (green or red) of the lights was more. Results showed that the performance decreased from the fovea to the periphery, and the reaction time increased with increasing eccentricity. Following a lack of research investigating how eccentricity modulates numerosity perception (at least with relatively large numbers), Valsecchi et al. (2013) conducted an experiment in which participants performed a numerosity comparison task. For this purpose, two displays were simultaneously presented on either side of the monitor, and participants were cued to look at the center of one of the displays, so that the other display appeared in the periphery. The task was to indicate which displays appeared to be more numerous. Results showed that the peripheral presented displays needed to contain a greater number of dots to be judged as the equivalent of the displays that were looked at, indicating numerosity is perceived as less numerous in the periphery compared to in the fovea. Valsecchi et al. (2013) suggested that crowding (see section 1.5) is the key mechanism of the observed underestimation in the periphery.

Visual input from the fovea and the periphery contribute to numerosity perception differently (Cheyette & Piantadosi, 2019). For example, Cheyette and Piantadosi (2019) revealed that an increase in foveation leads to an increase in numerosity estimation. They speculated that items in the foveal vision have twice the influence on numerosity estimation than those in the peripheral vision. Therefore, taking into account the presentation of items in different visual fields is pivotal for the comprehension of numerosity perception, as the peripheral vision is distinct from the fovea. Further exploration into the constraints of the visual periphery is essential.

1.5 Crowding and Redundancy Masking in Numerosity Perception

Spatial vision is strongly limited by crowding: the inability of target perception in cluttered environments (Bouma, 1970; Bouma, 1973; Levi, 2008; Pelli et al., 2004; Pelli & Tillman, 2008; Strasburger, 2020). It was proposed that crowding is a fundamental limit of spatial vision (Levi, 2008), and it is particularly strong in the visual periphery (Bouma, 1970; Bouma, 1973; He et al., 1996; Levi et al., 2002; Levi et al., 1985; Pelli et al., 2004). Crowding is contingent on the spacing between the target and its flankers (e.g., elements that flank the target): a decrease in the spacing between the target and its flankers leads to an increase in crowding (Bouma, 1970; Toet & Levi, 1992). For targets in the periphery, there is an elongated interference region where flankers interfere with the target perception (Toet & Levi, 1992). It has been shown that flankers positioned outside the region do not impede the target's perception (Toet & Levi, 1992). Target-flanker similarity impacts crowding: the more alike they are, the more crowding is experienced (Chakravarthi & Cavanagh, 2007; Chung & Mansfield, 2009; Kooi et al., 1994; Rummens & Sayim, 2019, 2021; Sayim et al., 2008; but see, Rummens & Sayim, 2021).

Crowding was also suggested to be a contributing factor to numerosity perception, resulting in an underestimation (Anobile et al., 2015; Valsecchi et al., 2013). The crowding hypothesis in numerosity perception is corroborated by the fact that both crowding and numerosity perception are eccentricity-modulated. There is increasing crowding and stronger underestimation with increasing eccentricity. (Dakin et al., 2011; Toet & Levi, 1992; Valsecchi et al., 2013).

It is commonly thought that crowding only affects target identification but not target detection (Levi et al., 2002; Pelli et al., 2004; but see, Allard & Cavanagh, 2011). However, the observed underestimation of numerosity perception implies that some detection errors may have taken place. Chakravarthi and Bertamini (2020) manipulated target-letter similarity (similar vs. dissimilar) and the minimal item spacing (near vs. far) that were shown to impact both crowding and numerosity perception and investigated if similarity and spacing had a comparable effect on numerosity perception. The results revealed that item spacing and item similarity showed different effects on the crowding task and the numerosity comparison task, demonstrating that crowding does not modulate numerosity perception.

Recent research has put forward a concept akin to visual crowding: when three or more identical elements, such as lines and letters, are presented in the periphery, individuals report fewer items than presented, which is termed redundancy masking (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021, 2022). Redundancy masking occurred as soon as three items were presented. For example, when three radially aligned lines were presented in the visual periphery, participants usually indicated that they perceived two lines (Yildirim et al., 2020, 2021). Therefore, redundancy masking suggests a detection error, and thus possibly linking to the underestimation in numerosity perception.

1.6 Impact of Visual Field Asymmetries on Numerosity Perception

The visual performance demonstrated a wide range of variation throughout the visual field, revealed by several ubiquitous asymmetries of the visual field, including horizontal-vertical anisotropy (superior performance along the horizontal compared to the vertical meridian at a fixed eccentricity, Barbot et al., 2021; Carrasco et al., 1995; Carrasco et al., 2001; Corbett & Carrasco, 2011; Mackeben, 1999; Rovamo & Virsu, 1979), vertical asymmetry (better performance in the lower compared to the upper visual field, Barbot et al., 2021; Carrasco et al., 2001; Corbett & Carrasco, 2011; Rubin et al., 1996) in a range of vision tasks such as visual acuity, orientation discrimination. In crowding, radially placed flankers have a more pronounced effect on target perception compared to tangentially placed flankers, given an equal target-flanker spacing (Kwon et al., 2014; Toet & Levi, 1992), which refers to as radial-tangential anisotropy. While there are several distinctions between redundancy masking and crowding, both demonstrate an evident radial-tangential anisotropy. In redundancy masking, the reduction of reporting the number of items occurs when they are arranged radially but not tangentially. (Yildirim et al., 2020, 2022).

It is surprising that not many studies have investigated how visual field asymmetries impact numerosity perception. Only in a recent study, Chakravarthi et al. (2022) revealed that a small number of items can produce a variety of visual field asymmetries in numerosity perception by presenting 1-9 small squares to one of four locations (upper, lower, left, or right visual field). They showed that numerosity performance was more effective along the horizontal meridian than the vertical meridian, in the lower visual field than the upper visual field, and on the left horizontal meridian than the right horizontal meridian. The results pointed to the potential influence of visual field asymmetries on numerosity perception.

1.7 The Current Thesis

In the current thesis, we aim to explore how crowding and redundancy masking modulate numerosity perception with a relatively wide range of numerosities. We tested the numerosity estimations (in a range between 21 and 58) with displays whose discs' interference was either strong or weak (Chapter 2). Discs on displays were predominantly arranged in a radial and a tangential direction for the strong and weak interference conditions, respectively (Experiment 2.1). Our results showed that estimates were lower in the strong compared to the weak interference conditions. We suggest that numerosity perception is a radial-tangential anisotropy of numerosity perception. Next, we asked participants to encircle the items perceived as a group (Experiment 2.2). The results indicated that the number of perceived groups was higher in the weak compared to the strong interference condition, showing an opposite trend with the estimation task. Therefore, grouping among discs may not explain the observed numerosity estimation results that radial displays were presented as less numerous compared to tangential displays. Next, crowding, redundancy masking, and radial-tangential anisotropy were further examined with four experiments (Chapter 3). Numerosities between 31 – 99 were tested. We observed that radial displays were reported as less numerous compared to tangential displays, no matter whether the radial-tangential arrangements of displays were weak, strong, or modulated with mixed contrast polarity. Our results demonstrated that the radial-tangential anisotropy of numerosity perception persists in all conditions. We suggest that crowding and redundancy masking modulate the numerosity perception. Then, redundancy masking was particularly tested in a typical redundancy masking paradigm (Chapter 4). We used human faces as stimuli in two experiments. Detection-like errors in redundancy masking in both multi-feature stimuli (faces) and low-level stimuli (luminance- and shape-matched outlines and noise patches) were examined. Faces are of great social significance and are typically processed rapidly. The results showed that redundancy masking not only occurred with simple stimuli (e.g., lines and letters) but also with faces. The occurrences of redundancy masking in faces reveal the stability and strength of redundancy masking across low- and high-level features. In Chapter 5, we discussed all experiments conducted in the preceding chapters, along with the observed results.

CHAPTER 2: ANISOTROPIC REPRESENTATIONS OF VISUAL SPACE MODULATE VISUAL NUMEROSITY ESTIMATION

Abstract

Humans can estimate the number of visually displayed items without counting. This capacity of numerosity perception has often been attributed to a dedicated system to estimate numerosity, or alternatively to the exploitation of various stimulus features, such as density, convex hull, the size of items, and occupancy area. The distribution of the presented items is usually not varied with eccentricity in the visual field. However, our visual fields are highly asymmetric. To date, it is unclear how inhomogeneities of the visual field impact numerosity perception. Besides eccentricity, a pronounced asymmetry is the radial-tangential anisotropy. For example, in crowding, radially placed flankers interfere more strongly with target perception than tangentially placed flankers. Similarly, in redundancy masking, the number of perceived items in repeating patterns is reduced when the items are arranged radially but not when they are arranged tangentially. Here, we investigated whether numerosity perception is subject to the radial-tangential anisotropy of spatial vision to shed light on the underlying topology of numerosity perception. In Experiment 2.1, observers were presented with varying numbers of discs, predominantly arranged radially or tangentially, and asked to report their perceived number. In Experiment 2.2, observers were presented with the same displays as in Experiment 2.1, and were asked to encircle items that were perceived as a group. We found that numerosity estimation depended on the arrangement of discs, suggesting a radial-tangential anisotropy of numerosity perception. Grouping among discs did not seem to explain our results. We suggest that the topology of spatial vision modulates numerosity estimation and that asymmetries of visual space should be taken into account when investigating numerosity estimation.

Keywords: numerosity estimation, spatial vision, crowding, redundancy masking, radial-tangential anisotropy

Introduction

Humans can perform numerosity estimations without counting. When the number of items is small - usually up to 4 items - people apprehend the number of items rapidly and without errors (i.e., subitizing, Atkinson et al., 1976; Kaufman et al., 1949). However, estimating higher numbers of objects is usually imprecise compared with subitizing. Different mechanisms have been proposed to underlie numerosity estimation. A prominent account of numerosity perception suggests that it is accomplished by a dedicated system - the approximate number system (ANS, also known as the "number sense"). The ANS has been suggested to extract the numerosity independently from other physical properties of the stimulus (Barth et al., 2003; Burr et al., 2017; Dehaene, 1992, 2011; Dehaene et al., 1998; Feigenson et al., 2004; Gilmore et al., 2011; Halberda & Feigenson, 2008; Lipton & Spelke, 2003; Xu et al., 2005).

Other accounts suggest that numerosity perception is not performed by independent mechanisms dedicated to numerosity but by exploiting stimulus properties such as item density (Dakin et al., 2011; Durgin, 2008), occupancy area (Allïk & Tuulmets, 1991), or by combining and weighting multiple visual cues (Gebuis & Reynvoet, 2012b, 2012c). Studies investigating the role of density in numerosity perception have shown diverging results. Burr and Ross (2008) demonstrated that numerosity, just like other primary visual properties, is subject to adaptation, and the effect was dependent on the number of items but not on other properties such as size or density. Hence, the authors suggested that numerosity is an independent visual property (see also, Ross & Burr, 2010). Anobile et al. (2014) also suggested separate mechanisms for numerosity and density, supported by evidence that discrimination thresholds of high and low-density displays followed two distinct psychophysical functions (Weber's law and a square root function for low- and high-density displays, respectively). However, density and numerosity are physically indivisible, as density is calculated by dividing numerosity by the total area (Tibber et al., 2012). Dakin et al. (2011) showed that both numerosity and density judgments were biased by the size of the stimulus, which was interpreted to imply that numerosity perception and density perception share a common metric (see also, Tibber et al., 2012).

In addition to density, several other physical properties of displays have been shown to affect numerosity perception. For example, in the occupancy model, Allïk and Tuulmets (1991) proposed that each presented item occupies a given circular region, and the total area collectively occupied by items (instead of the number of items per se) determined the perceived numerosity: When items are positioned too close to each other, the occupied regions overlap, resulting in lower perceived numerosity (see also, Allik & Raidvee, 2021). While proximity according to the occupancy model yields underestimation, varying proximity between subgroups of displayed items can yield more accurate performance. Specifically, when the presented items could be perceptually separated into subgroups, the number of items was enumerated more accurately and quickly ("groupitizing", Anobile et al., 2020; Ciccione & Dehaene, 2020; Maldonado Moscoso et al., 2020; Pan et al., 2021). Hence, the spatial organization and perceptual grouping of items can modulate perceived numerosity. A similar effect of grouping has been shown with uniform versus regular patterns: Uniform patterns are often perceived to be more numerous than patterns that can be grouped into clusters (Frith & Frut, 1972; Ginsburg, 1976; Taves, 1941). Chakravarthi and Bertamini (2020) investigated numerosity estimation and crowded target discrimination using identical stimulus configurations, varying spacing and similarity among items that are both known to affect numerosity perception and crowding (see below). Based on their results that spacing and similarity impacted crowded discrimination and numerosity estimation differently, they suggested that underestimation in numerosity perception was not due to crowding but due to clustering among items, and that grouping may moderate both. Similarly, Im et al. (2016) found that the number of perceived groups predicted perceived numerosity, with smaller numerosity estimates when items were arranged in subgroups (yielding fewer perceived groups), suggesting that grouping between items plays a role in numerosity perception.

Another suggestion for factors modulating or determining numerosity estimates is that observers combine (and weight) information from various visual cues (including item size, aggregate surface, convex hull, and density) to estimate numerosity (Gebuis et al., 2014; Gebuis & Reynvoet, 2012b, 2012c, 2013). What most experiments on numerosity perception have in common is that they usually apply stimulus features homogenously to the entire display, independent of stimulus locations in the visual field. However, our visual field has strong inhomogeneities (Abrams et al., 2012; Carrasco et al., 2001; Greenwood et al., 2017) which are likely to affect numerosity perception. One of the key factors that modulates perception is the eccentricity in the visual field. For example, a decrease in performance with increasing eccentricity has been shown for various tasks, including letter recognition (Gurnsey et al., 2011; Wolford &

Hollingsworth, 1974; Zahabi & Arguin, 2014), conjunction search (Carrasco et al., 1995; Scialfa & Joffe, 1998), target detection (Meinecke & Donk, 2002), and vernier offset discrimination (Harris & Fahle, 1996; Levi & Waugh, 1994). How eccentricity modulates numerosity perception has also been investigated (Mengal & Matathia, 1980; Valsecchi et al., 2013). For example, it was found that the perceived number of items was lower when stimuli were presented in the periphery compared to central vision (Valsecchi et al., 2013). The authors suggested that the underestimation in the periphery could have been due to crowding where targets that are easily identified in isolation become difficult to discern when flanked by other items (Figure 2.1a, b; Bouma, 1970; Levi, 2008; Pelli & Tillman, 2008; Strasburger et al., 1991; Strasburger et al., 2011; Whitney & Levi, 2011). As crowding occurs when multiple objects interact, it is a plausible mechanism that could underlie underestimation in numerosity perception where multiple - often close-by - items are presented. Importantly, while crowding is usually assumed to affect target identification but not detection (Livne & Sagi, 2007; Pelli et al., 2004), recent studies showed that target parts were often unnoticed under crowding (Coates et al., 2017; Sayim & Taylor, 2019; Sayim & Wagemans, 2017). A particularly strong case of such 'omission errors' occurred when flankers and the target were the same. For example, when presenting three identical letters Ts in the periphery, observers frequently reported only 2 letters (see also, Sayim & Taylor, 2019). This effect was termed "redundancy masking": The reduction of the number of perceived items in repeating patterns (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021, 2022). Redundancy masking has been shown to occur when as few as 3 items were presented (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021, 2022). Notably, redundancy masking – as crowding - has a pronounced radial-tangential anisotropy: In crowding, radially placed flankers interfere more strongly with target perception than tangentially placed flankers (see Figure 2.1c, Greenwood et al., 2017; Kwon et al., 2014; Toet & Levi, 1992); redundancy masking is strong with radially arranged lines and absent with tangentially arranged lines (Figure 2.1d, e, f; Yildirim et al., 2020). As performance in most tasks deteriorates with increasing eccentricity (even if no contextual elements are presented), anisotropies such as the radial-tangential anisotropy are better suited to investigate to what extent numerosity perception is determined by similar contextual interactions as crowding and redundancy masking.

Here, we investigated whether numerosity perception is subject to a radial-tangential anisotropy to shed light on the underlying topology of numerosity perception.

Specifically, we created displays that favored or did not favor these effects to occur (in 2 different alignment conditions: *tangential* and *radial*). We presented two types of arrangements of discs to produce weak or strong interference among the presented discs. To obtain a weak interference condition, close-by discs were predominantly arranged tangentially (*tangential* condition; Figure 2.2a); to obtain strong interference, they were predominantly arranged radially (*radial* condition; Figure 2.2b). In the *tangential* condition, elliptical zones around each disc that were expected to yield strong interference from neighboring discs within the zones ("crowding" zones) were "protected" by preventing discs from being positioned in these regions (hence, allowing tangential arrangements of discs, radial "protection zones" were used). In the *radial* condition, "protection zones" were perpendicular to these interference regions (i.e., tangential oriented), allowing discs to fall into other discs' interference regions (Figure 2.2e). We varied the size of the interference and protection zones as a function of eccentricity. Other physical properties (convex hull, occupancy area, density etc.) did not differ in the two conditions. In two experiments, participants viewed *tangential* and *radial* displays and were asked to perform the numerosity estimation task (Experiment 2.1) and the grouping task (Experiment 2.2). In Experiment 2.1, we tested whether the alignment condition (*radial* vs. *tangential*) influenced the perceived numerosity. Observers were asked to indicate the number of discs on each display. We found that the estimates of the number of discs were lower in the *radial* (strong interference) compared to the *tangential* (weak interference) condition. In Experiment 2.2, we tested whether there were any differences in the perceived number of groups in the two conditions, and thereby whether grouping could underlie the observed results in Experiment 2 1. For that aim, we asked participants to encircle the discs that they perceived to form groups. Interestingly, the results of Experiment 2.2 showed the opposite effect of the alignment condition on the perceived number of groups than Experiment 2.1: The average number of groups reported by observers was larger in the *radial* compared to the *tangential* condition. This result suggests that the relatively lower estimates in the *radial* condition compared to the *tangential* condition (Experiment 2.1) was not likely caused by factors related to perceptual grouping as tested in Experiment 2.2. Overall, our results showed a pronounced radial-tangential anisotropy of numerosity perception, suggesting a similar underlying topology of spatial vision as in other types of contextual interactions.

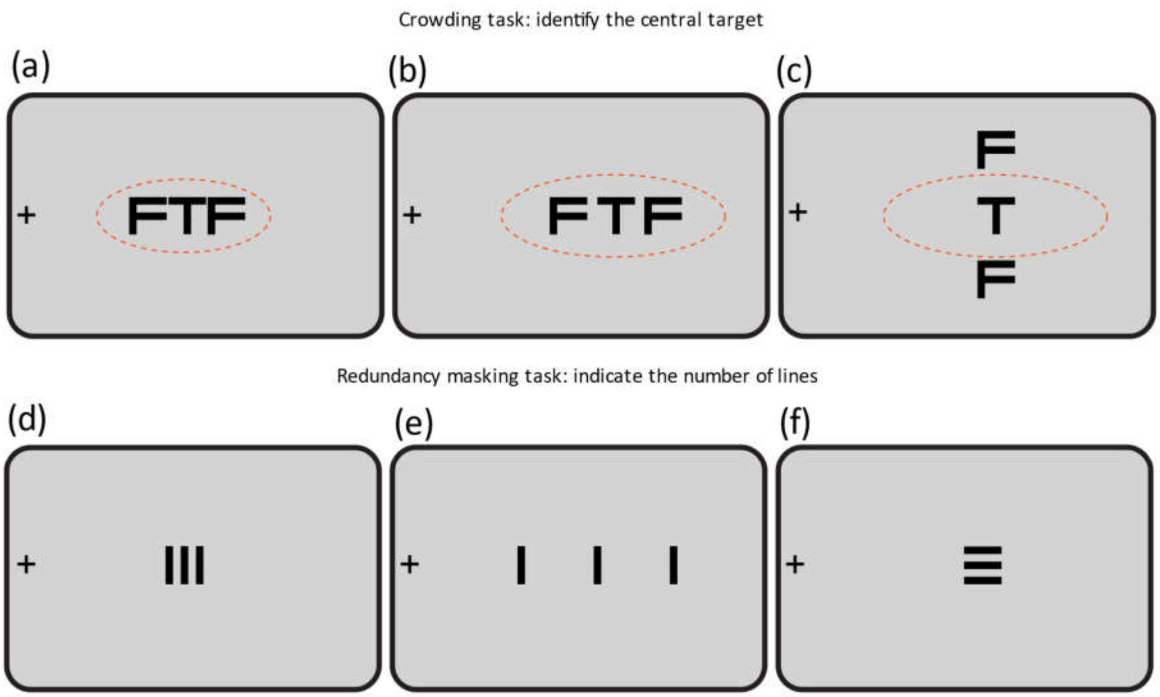

Figure 2.1. Illustration of crowding and redundancy masking. (a) When fixating the cross, identifying the target "T" that is surrounded by 2 flankers "F", is usually difficult when flankers are positioned inside the interference ("crowding") region (indicated by the dashed ellipse). (b) The interference region is eccentricity-dependent: increasing target eccentricity increases the size of the interference region. (c) The interference region is anisotropic: Flankers cease to interfere at smaller distances in tangential (c) compared to radial (b) directions. (d) Redundancy masking is the reduction of the number of perceived items in repeating patterns. When presenting 3 close-by aligned vertical lines in the periphery, most observers reported only 2 lines. (e) Redundancy masking was weaker with large compared to small (d) spacings (Yildirim et al., 2020). (f) There was no redundancy masking when lines were arranged tangentially.

Method

Experiment 2.1: Numerosity estimation

In Experiment 2.1, we tested whether the radial-tangential anisotropy of visual space impacted perceived numerosity.

Participants

Twenty-one healthy participants (7 males, 14 females; mean age: 24.1 years, ranging from 19 to 31) participated in the experiment. All participants were naïve as to the purpose of the study. Participants either received monetary compensation or participated without compensation. All participants reported normal or corrected-to-normal visual acuity and signed informed consent prior to the experiment. The experiment was approved by the ethics committee of the Ulille SHS, University of Lille.

Apparatus

The experiment was programmed in PsychoPy coder v3.1.2 (Peirce, 2009) and ran on a desktop PC. All stimuli were presented on a Vision Master Flat Square CRT monitor (Iiyama MS103DT), with a resolution of 1280 × 1024 pixels (refresh rate was set at 100 Hz). During the experiment, participants sat in front of the monitor with a chin rest at a distance of 57 cm from the monitor. All experiments were conducted in a dim experimental room.

Stimuli

Stimuli consisted of black discs (0.9 cd/m^2; radius: 0.25°) presented on a gray background (25 cd/m^2). In five numerosity range conditions, discs were presented within rectangular regions of different sizes (width × height: 19.5 × 11.5; 21.5 × 13.5; 25 × 16.5; 27 × 18.5; 30 × 21 degrees of visual angle that occupy 30%, 40%, 50%, 60%, and 70% of the screen, respectively), each corresponding to one of the 5 different numerosity ranges (21 –25; 31 – 35; 41 – 45; 49 – 53; 54 - 58). No discs were presented within a circular region (radius: 3.8°) around fixation. There were two types of disc arrangements: tangential and radial, illustrated in Figure 2.2. We surrounded each disc with a virtual "protection zone" free of any other disc. The size of the "protection zone" was based on common estimates of the size of the interference region in crowding (e.g., Bouma, 1970; Toet & Levi, 1992). Both the major axis and the minor axis of the "protection zone" were determined by target eccentricity: the major axis was set to 0.25 × eccentricity and the minor axis to 0.1 × eccentricity (corresponding to a minimum distance of 0.2 and 0.5 × eccentricity when two discs were tangentially or radially aligned). To generate a *tangential* display (Figure 2.2a, c and f), a random position was chosen to place the first disc with its corresponding (radially extended) "protection zone." All the other discs were added with their "protection zones" iteratively on the displays with the constraint not to overlap with any of the "protection zones" of other discs, until no disc could be positioned onto the display without overlapping "protection zones." In the *radial* condition (Figure 2.2b, d, and g), displays were generated the same way as the *tangential* displays, except that the "protection zones" were rotated by 90° compared to the *tangential* condition. Therefore, in the *radial* condition, "protection zones" were orthogonal to the major axis of the interference region (Figure 2.2d, e). For each numerosity range, we generated 5000 displays for each condition (*tangential* and *radial*). We calculated convex hull, occupancy area, average spacing, average eccentricity, and density for each generated display and selected displays from the *tangential* and *radial* conditions that matched their physical properties (see Supplementary Table S2.1). The density was measured by dividing numerosity by occupancy area, excluding the central region where no discs were presented. As an insufficient number of displays in the smallest numerosity range could be matched, we generated an additional 5700 *radial* displays to obtain the required matches.

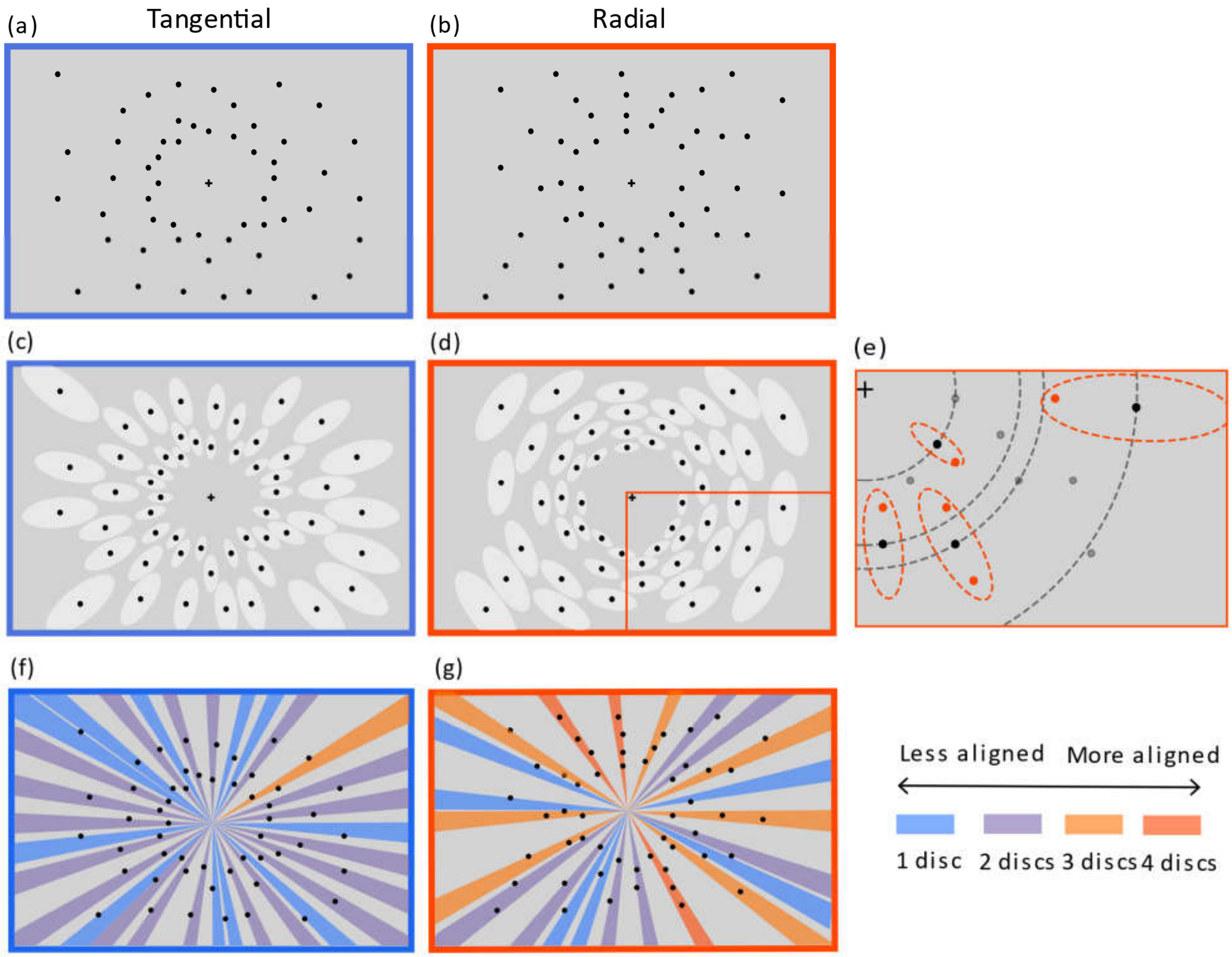

Figure 2.2. Illustration of displays in the (a) *tangential* and (b) *radial* conditions. (c) and (d): Illustration of the geometric principles of the *tangential* and *radial* conditions. (c) In the *tangential* condition, each disc is surrounded by a "protection zone" (indicated by the ellipses), allowing predominantly tangential alignments of discs. No discs were positioned into any other disc's interference region zones. (d) Rotated protection zones in the radial condition, favoring stronger interference. Here, a certain number of discs was positioned inside other discs' interference regions. (e): Detail of the *radial* display, illustrating discs (shown in red for illustration) in the interference region of other discs. (f) and (g) illustrate radial-tangential alignment scores for the *tangential* and *radial* conditions, respectively.

Design and Procedure

At the start of each trial, a black fixation cross (0.75° × 0.75°) was presented at the center of the screen. Observers initiated each trial by pressing the spacebar. The stimulus display was presented for 150 ms. Participants were instructed to respond by entering their best estimation of the number of presented discs with the numeric keypad. No feedback was provided. There was no time limit for participants to respond. Participants were not informed about the numerosity ranges prior to the experiment. Prior to each experimental block, participants viewed 5 reference displays in random order. The numerosities of the 5 reference displays were equally distributed around the averaged numerosity of the block (± 0.125 and ± 0.25 times of the mean numerosity of the block). Each reference display was presented for 150 ms, and participants were informed about the actual numerosity of the display after the reference display offset.

There were two factors: Alignment condition (*tangential* vs. *radial*) and numerosity range (5 levels: 21-25, 31-35, 41- 45, 49-53, and 54-58; for convenience, we use the first numerosity of each numerosity range to denote the actual numerosity range, i.e., N21, N31, N41, N49 and N54 denote numerosity range 21-25, 31-35, 41-45, 49-53, and 54-58, respectively). Each participant performed 10 blocks of 50 trials each. Within each block, each numerosity was presented 10 times (5 different displays, each repeated twice). Participants first completed each of the 5 numerosity ranges (in random order), followed by 5 blocks in the opposite order. The dependent variable was the deviation score (DV) of participants, calculated by subtracting the actual numerosity from participants' estimation for each trial. Hence, positive DVs represent overestimation; negative DVs represent underestimation. We also calculated the relative estimation error by dividing the DV by the numerosity of the display.

Data Analysis

We conducted a within-subject ANOVA on DV scores with alignment condition and numerosity range as within-subject factors. We expected lower DVs in the radial compared to the tangential condition. The ANOVA and pairwise analysis were performed with an open-source Python package, Pingouin version 0.5.1 (Vallat, 2018). Estimates outside of 3 standard deviations around the mean were discarded independently for each numerosity range (0.4% of all trials). The same analyses were conducted on relative estimation error.

Radial alignment scores (RAs).

We calculated RAs as measures of how well discs were radially aligned in a display. RAs were calculated individually for each display by rotating a circle sector with an angle of 6° (half the angle of the minor axis of the protection zones) around fixation for a complete rotation and counting the number of discs falling in the sector at each location a new disc fell into the trailing edge of the sector (i.e., when the edge of the circle sector aligned with a disc center; Figures 2.2f, g). Neighboring circle sectors ("alignment regions") did not overlap. The procedure was repeated with each disc in the display as starting disc, always performing a complete rotation. For each rotation, the proportion of the circle sectors that contained 3 (the minimum number of items to obtain redundancy masking) or more discs was calculated. For example, if there were 20 circle sectors in one rotation and 10 of them contained 3 (or more) discs, the proportion would be 0.5. The RA of that display was the averaged proportion across all rotations for that display.

Crowding strength.

The number of discs that was positioned in other discs' interference regions varied in the *radial* condition but not in the *tangential* condition since no discs could be positioned into the interference region of other discs (Figure 2.2c; by definition, what we denote as the "crowding strength" was 0 in all tangential displays). To quantify "crowding strength" in the *radial* condition, we calculated the number of discs per display that were positioned in other discs' interference regions. The average crowding strength was 1.3 ± 1.1, 2.6 ± 1.3, 4.8 ± 2.4, 6.6 ± 2.9, and 7.1 ± 3.1 for N21, N31, N41, N49, and N54, respectively.

Partial correlations.

We calculated partial correlations between (1) RAs and DVs and (2) crowding strength and DVs, controlling for numerosity. To ensure that RAs, crowding strength, and DVs were comparable across numerosity ranges, they were normalized in the linear regression to predict numerosity.

Experiment 2.2: Grouping into clusters

In Experiment 2.2, we tested whether the number of perceived groups in the *radial* and *tangential* conditions differed. If the number of perceived groups was lower in the *radial* than in the *tangential* condition, grouping among discs could be a factor contributing to the effect found in Experiment 2.1. If the number of perceived groups was similar in the *radial* and the *tangential* displays, the results would suggest that grouping is an unlikely factor underlying the effect observed in Experiment 2.1.

Participants

Thirty healthy participants (4 males, 26 females; mean age: 19.7 years, ranging from 18 to 24) participated in Experiment 2.2. All participants were students at the University of Lille or the KU Leuven, and naïve as to the purpose of the study. All participants received course credits for their participation. All participants reported normal or corrected-to-normal visual acuity and signed informed consent prior to the experiment.

Apparatus

The experiment was programmed in PsychoPy coder v2.1.0 (Peirce, 2009) and ran on a desktop PC. All stimuli were presented on an LCD display with a resolution of 1960 × 1080 pixels. During the experiment, participants sat in front of the monitor with a chin rest at a distance of 57 cm from the monitor.

Stimuli

The stimuli were identical to the stimuli in Experiment 2.1.

Design and Procedure

The design and procedure were identical to Experiment 2.1 except for the following changes: Participants were asked to encircle the discs that they perceived as a group, using the mouse (as a 'pen'). Each display was presented until participants had finished the trial (unlimited viewing time). Participants were presented with the same displays that were used in Experiment 2.1. Each participant was presented with one-third of the total number of displays (250 displays) of Experiment 2.1 to limit the duration of the experiment (about 100 minutes per participant). There were 30 participants (hence, 10 responses per display).

Data Analysis

The analyses were identical to the ANOVA analysis in Experiment 2.1, except that the dependent variable was the number of perceived groups. The number of groups that participants encircled for each display corresponded to the number of perceived groups in the analysis.

Results

Experiment 2.1: Numerosity estimation

Figure 2.3a shows the average deviation scores (DVs) for the *tangential* and the *radial* condition separately for each numerosity range. A repeated measures ANOVA with alignment condition (*tangential* and *radial*) and numerosity range (N21, N31, N41, N49, and N54) as factors showed a main effect of alignment condition ($F(1, 20) = 13.45$, $p < .005$, $\eta_p^2 = .40$) on DVs. Participants reported fewer discs in the *radial* (DV = 1.64 ± 8.65) compared to the *tangential* condition (DV = 2.66 ± 8.78). Pairwise comparisons with Hochberg FDR correction showed significant differences between the *tangential* and the *radial* conditions in all numerosity ranges (N31: t(20) = 2.66, $p < .05$, Cohen's $d = 0.12$; N41: t(20) = 2.32, $p < .05$, Cohen's $d = 0.10$; N49: t(20) = 3.43, $p < .005$, Cohen's $d = 0.15$; N54: t(20) = 3.55, $p = .005$, Cohen's $d = 0.16$), except for the smallest one (N21: (t(20) = 0.85, $p = .40$, Cohen's $d = 0.04$). We also found a main effect of numerosity range with lower DVs for small numerosities. ($F(4, 80) = 3.96$, $p < .05$, $\eta_p^2 = .17$). A significant interaction between alignment condition and numerosity range ($F(4, 80) = 2.68$, $p < .05$, $\eta_p^2 = .12$) indicated that the difference between the *tangential* and the *radial* conditions increased with larger numerosities. Figure 2.3b shows the average relative estimation error for each condition. We also conducted a repeated measures ANOVA on average relative estimation error with alignment condition and numerosity range as within-subject factors. We observed a main effect of alignment condition ($F(1, 20) = 8.79$, $p < .01$, $\eta_p^2 = .31$) on relative estimation errors. No other significant effect was observed (*p*s > .05).

To test whether radial alignment predicted DVs, we correlated radial-alignment scores (RAs) and DVs while controlling for numerosity (partial correlation, Figure 2.3b). For all numerosity ranges combined, the partial correlation was $r = -0.40$ ($p < .0001$, CI 95% [-0.50 -0.29]), showing higher deviation scores with increasing RAs. Except for N21, the partial correlation between DVs and RAs showed a clear negative correlation when controlling for the effect of numerosity. These results showed that estimates were smaller when discs were more strongly radially aligned, at least for larger numerosities (N31 and above). The averaged RAs for separate numerosity ranges for both *tangential* and *radial* displays are shown in Supplementary Table S2.2. The partial correlations for the separate numerosity ranges are shown in Supplementary Table S2.3.

To test whether "crowding strength" predicted DVs, we correlated crowding strength and DVs while controlling for numerosity (partial correlations, Figure 2.3c). Results showed that the overall partial correlation coefficient was $r = -0.40$ ($p < .0001$, CI 95% [-0.50 -0.29]). Hence, there was a clear negative correlation between the number of discs falling into the interference zone of other discs and numerosity judgments: The more discs were presented in other discs' interference zones, the lower the numerosity judgments. Supplementary Table S2.3 shows the partial correlations analysis of each numerosity range separately.

Experiment 2.2: Grouping into clusters

Figure 2.4 illustrates the task and response format in the grouping task for tangential and radial displays, respectively. A repeated measures ANOVA with alignment condition and numerosity range as factors showed a main effect of alignment condition ($F(1, 9) = 6.91$, $p < .005$, $\eta_p^2 = .43$) on the perceived number of groups. Participants reported more groups in the *radial* (13.0 ± 4.25) compared to the *tangential* condition (11.4 ± 3.78). Pairwise comparisons with Hochberg FDR correction showed significant differences between the *tangential* and the *radial* conditions in N21 ($t(9) = 4.11$, $p < .01$, Cohen's $d = 1.10$), but not in the other numerosity ranges (N31: $t(9) = 2.08$, $p = .09$, Cohen's $d = 0.70$; N41: $t(9) = 2.08$, $p =.09$, Cohen's $d = 0.67$; N49: $t(9) = 1.58$, $p = .15$, Cohen's $d = 0.40$, N54: $t(9) = 2.07$, $p = .09$, Cohen's $d = 0.58$). Unsurprisingly, there was also a main effect of numerosity range on the perceived number of groups ($F(4, 36) = 101.94$, $p < .001$, $\eta_p^2 = .92$), showing that more groups were perceived with larger numerosities. No interaction between alignment condition and numerosity range was observed ($F(4, 36) = 0.58$, $p = .68$, $\eta_p^2 = .06$). Supplementary Table S2.4 summarizes the average perceived number of groups for each numerosity range in the *tangential* and the *radial* condition. Importantly, the two alignment conditions affected numerosity estimations (Experiment 2.1) and the perceived number of groups (Experiment 2.2) differently: numerosity estimation was lower and the perceived number of groups higher in the *radial* compared to the *tangential* condition.

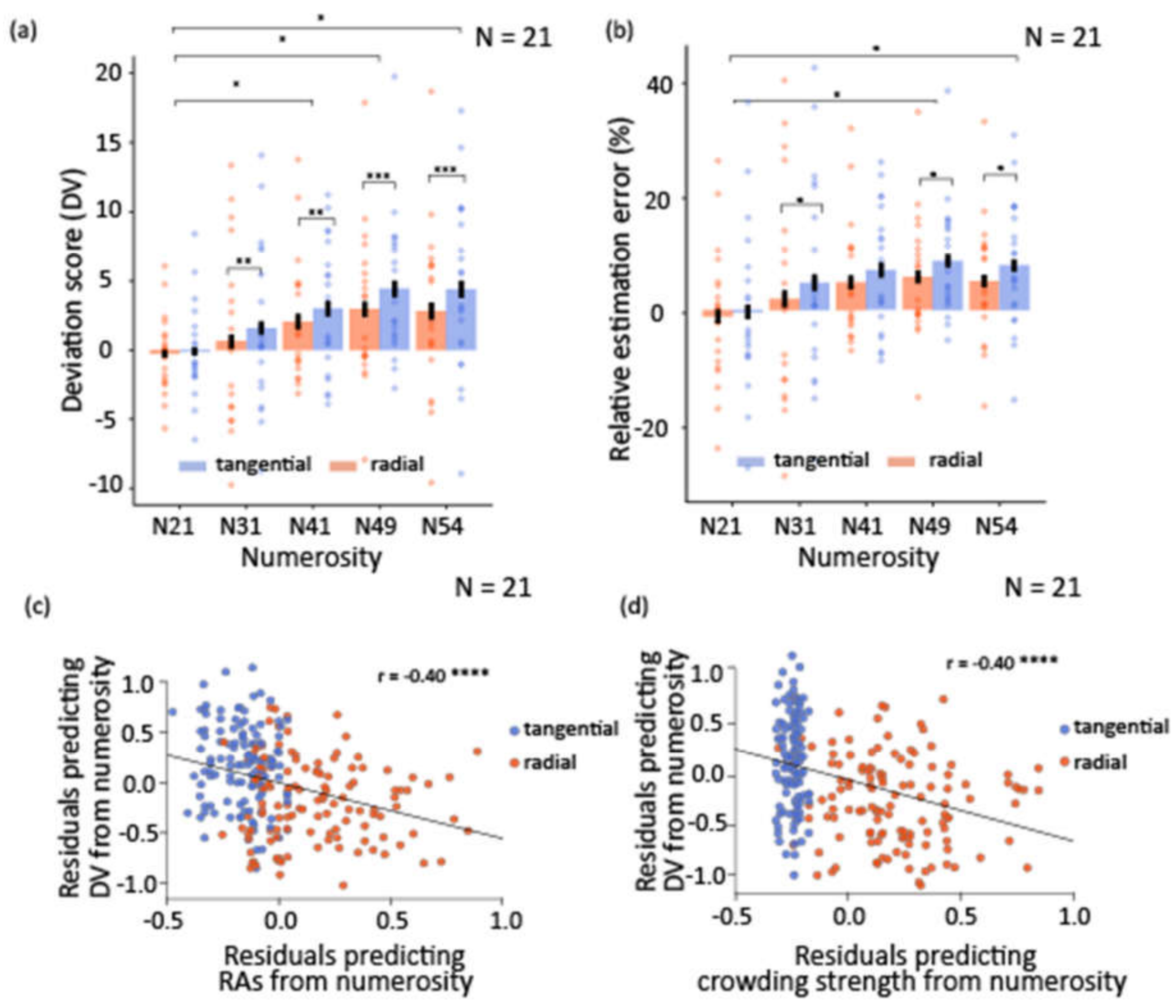

Figure 2.3. Results of Experiment 2.1. (a): Deviation score (DV) as a function of numerosity. DVs of 0 represent no deviation from correct responses, negative DVs represent underestimations, and positive DVs represent overestimations. Error bars indicate (+/-1) standard errors of the mean. Significant pairwise comparisons are indicated with asterisks. Each data point shows the average scores for one observer. (b): Relative estimation error as a function of numerosity. Error bars indicate (+/-1) standard errors of the mean. Each data point represents the average percent changes of one observer. (c): Partial correlation between DVs and radial alignment scores (RAs). when controlling for the effect of numerosity. (d): Partial correlation between DVs and crowding strength when controlling for the effect of numerosity. (*p < .05. **p < .005. ***p < .001. ****p < .0001.)

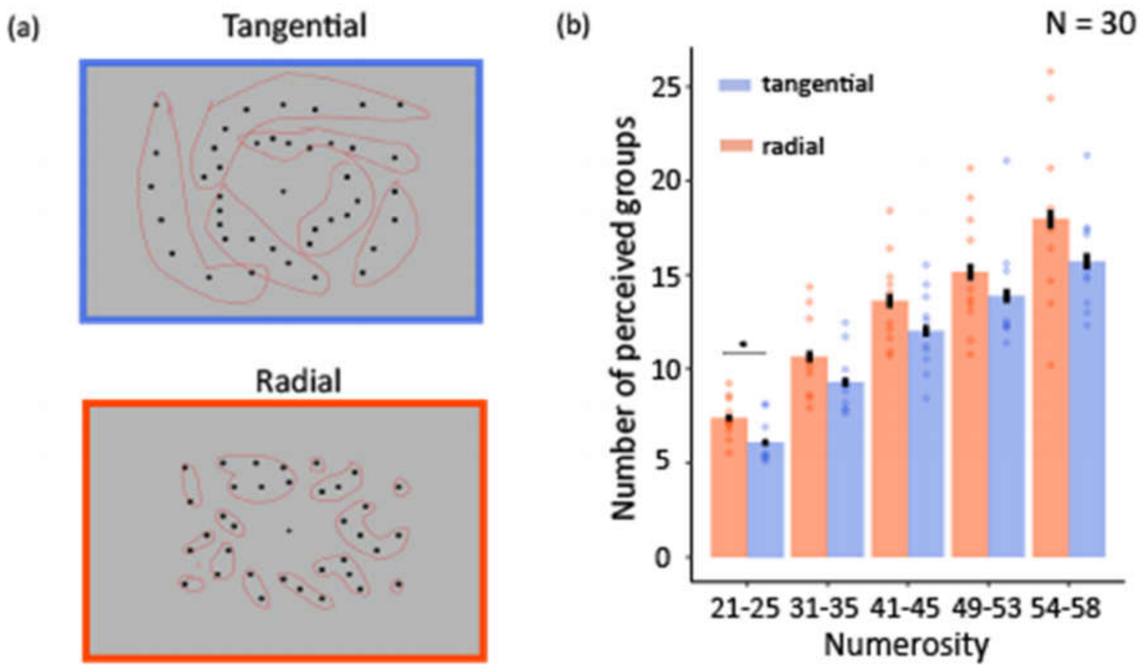

Figure 2.4. Results of Experiment 2.2. (a) Illustration of Experiment 2.2 with possible responses for tangential and radial displays. Each closed red shape was counted as one group of items. (b) The number of perceived groups as a function of numerosity separated for the radial and tangential conditions. Error bars indicate (+/-1) standard errors of the mean. Each data point shows the average scores for one observer. (*p < .05).

Discussion

We investigated to what extent the topology of spatial vision determined numerosity estimation. In particular, based on the radial-tangential anisotropy of spatial interactions in the peripheral visual field, we sought to investigate if numerosity estimation was subject to a similar radial-tangential anisotropy as crowding and redundancy masking. For that aim, we created displays in which neighboring items were predominantly arranged in either tangential or radial directions while keeping other features of the two types of displays, such as inter-item spacing, average eccentricity, convex hull, and density as similar as possible. In Experiment 2.1, we asked participants to report the number of discs they perceived. We found that numerosity estimates were lower in the *radial* compared to the *tangential* condition. The analysis of radial alignment scores (RAs) showed that higher RAs yielded lower numerosity estimates. In the *radial* condition, the number of items falling into the interference regions of other items was taken as a measure of "crowding strength." We found that crowding strength predicted deviation scores (DVs): high crowding strength was associated with smaller numerosity estimates and vice versa. Grouping among items is a good predictor of crowding strength (Livne & Sagi, 2007; Manassi et al., 2012; Sayim et al., 2010; Sayim et al., 2011; but see Melnik et al., 2018; Rummens & Sayim, 2019a). Grouping has also been shown to modulate numerosity perception (Chakravarthi & Bertamini, 2020; Ciccione & Dehaene, 2020; Im et al., 2016; Pan et al., 2021). To test whether the number of perceived groups was related to the relative underestimation in the *radial* compared to the *tangential* condition, we asked observers in Experiment 2.2 to encircle the discs they perceived as a group. We used the same displays in the grouping task as in Experiment 2.1. The results showed that the number of perceived groups in the *radial* condition was higher than in the *tangential* condition, i.e., the opposite pattern of results compared to Experiment 2.1: lower estimations (Experiment 2.1) and higher number of groups (Experiment 2.2) in the *radial* compared to the *tangential* condition. Hence, the perceived number of groups and the perceived numerosity were affected by alignment conditions differently. These results indicate that grouping is unlikely the cause for the different numerosity estimates in the *radial* and the *tangential* condition.

Crowding strongly limits peripheral vision (Bouma, 1970; He et al., 1996; Levi et al., 2002; Pelli et al., 2004), and was proposed to play a role in numerosity estimates

(Anobile et al., 2015; Valsecchi et al., 2013). In particular, the relative underestimation of numerosities in dot displays presented in the fovea compared to the periphery suggested that mechanisms related to crowding might be an important factor in numerosity perception (Valsecchi et al., 2013). A potential role of crowding was also shown when varying eccentricity: Numerosity estimates varied with eccentricity similar to crowding, with stronger interference (lower estimates) farther in the periphery (Valsecchi et al., 2013). However, performance in most tasks deteriorates with increasing eccentricity. For example, besides crowding (Levi, 2008; Pelli et al., 2004; Strasburger, 2020; Toet & Levi, 1992), performance in other tasks, including letter recognition (Gurnsey et al., 2011; Wolford & Hollingsworth, 1974; Zahabi & Arguin, 2014), conjunction search (Carrasco et al., 1995; Scialfa & Joffe, 1998), target detection (Gruber et al., 2014; Meinecke & Donk, 2002), visual search (Carrasco & Frieder, 1997; Carrasco et al., 1998) and vernier offset discrimination (Harris & Fahle, 1996; Levi & Waugh, 1994) deteriorates with increasing eccentricity. Hence, eccentricity dependence is not sufficient to conclude that crowding-like mechanisms underlie numerosity estimation. In a recent study, crowding and numerosity perception were directly compared using identical stimulus configurations (Chakravarthi & Bertamini, 2020). Inter-item spacing and item similarity (same or opposite contrast polarity), both known to modulate crowding as well as numerosity estimates were varied. The results showed that spacing and similarity affected numerosity perception (in a 2AFC numerosity comparison task) and crowding (in an identification task) differently, suggesting a dissociation between numerosity perception and crowding. However, the different tasks and different task-relevancy of the presented items – a single relevant target or many relevant targets – render definite conclusions about the dissociation of crowding and numerosity perception difficult. For example, whether items are task-relevant or not has recently been shown to strongly modulate crowding, inverting the similarity rule of crowding (Rummens & Sayim, 2019b): When all items were task-relevant, performance was superior with target and flankers of the same compared to opposite contrast polarity. Similarly, small spacing between target and flankers does not always yield stronger crowding: Emergent features between the target and a flanker improved performance at small compared to larger distances in a crowding task (Melnik et al., 2020).

Importantly, crowding is usually assumed to impair target identification but not target detection (Andriessen & Bouma, 1976; Levi, 2008; Pelli et al., 2004; but see

Allard & Cavanagh, 2011; Sayim & Wagemans, 2017). As underestimation in numerosity perception implies failures of detection, not discrimination, it might be suggested that crowding is an unlikely candidate to play a role in numerosity perception in general. However, recently it was shown that parts of the targets are often lost in crowding (Sayim & Wagemans, 2017). Such "omission errors" may well be due to the recently discovered phenomenon of redundancy masking, the reduction of the number of perceived items in repeating patterns (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021). Although related to crowding, a key difference is that redundancy masking, unlike crowding, impairs the perception of the number of items (not their identity). As in numerosity estimation, a typical task to investigate redundancy masking is to ask participants to report the number of perceived items (however, see Sayim & Taylor, 2019, for a free verbal report and drawing task). Hence, there are obvious parallels between redundancy masking and numerosity perception, and redundancy masking could underlie underestimation in numerosity perception. Importantly, redundancy masking occurs for as few as three presented items, i.e., in the subitizing range (Yildirim et al., 2020) where reports are usually accurate (Atkinson et al., 1976; Jensen et al., 1950; Kaufman et al., 1949). Although clearly present for larger numbers of items, redundancy masking does not scale linearly with the number of items. For example, with three presented items of which only two are reported, one-third of all items are lost due to redundancy masking. While the absolute number of items lost due to redundancy masking increases with the number of presented items, the ratio decreases (Yildirim et al., 2020). Hence, the exact relation between redundancy masking and numerosity estimation still needs to be investigated, with future studies closing the gap between the paradigms typically used in numerosity perception and in redundancy masking, and shedding light on the extent of their similarities. Importantly, redundancy masking – as crowding – has a pronounced radial-tangential anisotropy: When peripherally presented lines were arranged radially, redundancy masking was strong; when they were arranged tangentially, there was no redundancy masking (Yildirim et al., 2020). Here, we used this radial-tangential anisotropy to manipulate displays where discs were predominantly arranged tangentially or radially to test if radial arrangements would yield lower estimates than tangential arrangements. As expected, radial arrangements yielded lower estimates than tangential arrangements. Taken together, contextual interactions subject to radial-tangential anisotropy, and in particular redundancy masking, are promising phenomena that share characteristics with numerosity perception beyond eccentricity dependence.

Many physical characteristics of displays used in experiments on numerosity perception are potentially confounded with numerosity per se (Gebuis & Reynvoet, 2012c). Importantly, in our *tangential* and *radial* arrangements, we kept physical properties of the displays that have been shown to play a role in numerosity estimation as similar as possible, matching them in regard to items size (Allik et al., 1991; Ginsburg & Nicholls, 1988), occupancy area (Allïk & Tuulmets, 1991), convex hull (Gilmore et al., 2016; Katzin, 2018), regularity (Franconeri et al., 2009; Ginsburg, 1976; Liu et al., 2018; Zhao & Yu, 2016), spatial clustering (Bertamini et al., 2018; Bertamini et al., 2016; Chakravarthi & Bertamini, 2020; Koesling et al., 2004), and texture density (Dakin et al., 2011). Controlling for these possibly confounding physical properties in the two conditions minimized the probability of factors related to these properties to account for the effect of our manipulation. Given the predominantly *tangential* or *radial* arrangements in the two conditions, some systematic structural differences are unavoidable. In particular, the discs in the *tangential* displays tend to be arranged into concentric patterns around fixation and in the *radial* displays into ray patterns. Importantly, while these structural differences between the displays may be a variable that modulates numerosity estimates, the findings in redundancy masking show strong differences between *tangential* and *radial* arrangements without any global, structural differences between *tangential* and *radial* arrangements. Moreover, redundancy masking has been shown to increase – not decrease – with diffused compared to focused attention (Yildirim et al., in preparation). As focused spatial attention is considered not required in numerosity estimation (at least with relatively sparse displays; Anobile et al., 2020; Burr et al., 2010), redundancy masking would not be expected to cease in displays with larger numerosities.

While the number of discs, average eccentricity, average spacing, convex hull, and density were matched in the *tangential and radial* conditions, all displays contained density gradients with higher density in more central regions and decreasing density with increasing eccentricity. Hence, differences of the spatial distributions of the discs as a function of eccentricity in the two conditions were possible. For example, relatively more discs could be close to the center in one display, forming a higher local density region, compared to fewer discs close to the center in another display (with the same number of discs). The local density as a function of eccentricity (Supplementary Figure S2.1) captures such variations of display density. Differences in local densities could be a factor influencing numerosity estimates, for example, by yielding higher numerosity estimates for displays with high local densities compared to displays with low local densities. Such an effect would be expected if central regions were weighted more strongly than peripheral regions (Cheyette & Piantadosi, 2019; see also, Dandan et al., 2022). A small subset of displays in the *tangential* condition had relatively high local densities compared to the average (Supplementary Figure S2.1). However, the majority of these displays were not judged as more numerous than displays with lower local density, suggesting that local density differences between the *tangential* ('concentric') and the *radial* ('ray') conditions did not underlie differences of numerosity estimates. Note that relatively low density (due to relatively larger item size or smaller convex hull) has also been reported to yield higher numerosity estimates compared to displays with relatively high densities (Gebuis & Reynvoet, 2012c), however, in relatively uniform displays, without any systematic density variation with eccentricity as in our displays. If the structural differences per se irrespective of other variables (e.g., local density, overall density, convex hull, etc.) modulated numerosity perception, with generally lower estimates in ray compared to concentric patterns, radial-tangential anisotropies may well underlie such a difference. Systematic investigations to explore if – and how – such structural differences and local density differences modulate numerosity estimations will shed light on their role in numerosity perception.

Our results showed that the relative underestimation in the *radial* compared to the *tangential* condition was primarily driven by larger numerosities, with significant differences observed in N31 to N54 but not for N21. Consistently, in the partial correlation analysis, we found that both RAs and crowding strength negatively correlated with estimations with large numerosities but not small numerosities (see Supplementary Table S2.3). The pronounced effect on large but not small numerosity ranges is not surprising as the radial-tangential manipulation of displays did not yield strong differences in the smallest numerosity (N21, see RAs, Supplementary Table S2.2). While density did not differ between the *radial* and *tangential* conditions within each numerosity range, densities did vary between numerosity ranges: Relative higher density in N21 compared to the other numerosity (see Supplementary Table S2.1). Anobile et al. (2014) suggested that numerosity discrimination and judgments based on density depend on the density of the displayed items, with numerosity discrimination occurring when display densities are less than 0.25 items/deg^2 and judgments based on density with larger densities of the displays. In our displays, the densities in the large numerosity ranges (N41, N49, and N54) where we found differences between the *radial* and *tangential* displays fell into the 'numerosity judgment' range suggested by Anobile et al. (2014). Hence, it is unlikely that judgments in these conditions were based on density (but see Dakin et al., 2011; Durgin, 2008).

In contrast to smaller numerosities (N21) where the number of discs was rather accurately estimated, it was overestimated with larger numerosities (N31 and more). The overestimation with larger numerosities diverged from the general underestimation found in most numerosity studies (Anobile et al., 2020; Au & Watanabe, 2013; Chakravarthi & Bertamini, 2020; Krueger, 1982, 1984; Liu et al., 2017; Liu et al., 2018). The direct estimation task, in contrast to the typical discrimination task, could be one reason for the overestimation in our study. Similar overestimations were found when presenting regular and irregular dots array (28 – 46 dots), asking observers to estimate the number of dots (Alam et al., 1986). Also, when asking participants to report the number of items, Gebuis and Reynvoet (2012c) found that half of the participants overestimated and the other half underestimated the numerosities. We can exclude that the overestimation was due to the overall distribution of numerosities in different blocks as the same pattern of results also occurred in the first block that observers completed. Importantly, irrespective of the overall overestimation, which suggests a general bias, it is the relative underestimation in the *radial* compared to the *tangential* condition that shows the key estimation difference between the two conditions.

Perceptual grouping has been shown to modulate perceived numerosity (Chakravarthi & Bertamini, 2020; Im et al., 2016; Mazza & Caramazza, 2012). When items were arranged into clusters (Chakravarthi & Bertamini, 2020; Frith & Frut, 1972), perceived to contain a larger number of groups (Im et al., 2016), were grouped by connectedness (Franconeri et al., 2009) or by similarity grouping (connectedness, shape, proximity, and common region (Yu et al., 2019), observers tended to underestimate the numerosity compared to similar displays with weaker grouping. Hence, grouping among items may have modulated the perceived numerosity in the present study as well. For example, the relative underestimation in the *radial* compared to the *tangential* condition could have been driven by more grouping (and therefore fewer groups) in the *radial* compared to the *tangential* displays. In Experiment 2.2, we investigated how the discs in our displays were perceived to groups and whether grouping differences between the conditions could underlie the pattern of results in Experiment 2.1. Interestingly, the average number of perceived groups was higher in the *radial* than in the *tangential* condition, in contrast to number estimates which were lower in the *radial* compared to the *tangential* condition. Hence, this result shows that displays with low (high) numbers of perceived groups did not yield low (high) numerosity estimates. These results suggest that the relative underestimation in the *radial* compared to the *tangential* displays was not due to a smaller number of groups in the *radial* compared to the *tangential* condition: Grouping into clusters seems unlikely to play an important role in our results. However, while the same stimuli were used in the estimation (Experiment 2.1) and the grouping task (Experiment 2.2), viewing conditions were different: peripheral viewing with limited presentation time (150 ms) in the estimation task and free viewing with unlimited presentation time in the grouping task. Hence, retinal stimulus locations and presentation time could have influenced the results in the two experiments. For example, different sets of discs could have appeared to group when viewed peripherally compared to when viewed freely. However, as proximity was the principal grouping factor, differences that would systematically reverse grouping strength of the same displays in the two experiments are implausible. Rather, proximity as a grouping factor should be stable and maintain the ordinal relationships among displays across eccentricities. Importantly, in the realm of contextual interactions, i.e., crowding, the very same effects of grouping (and ungrouping) have been observed in the fovea (Sayim et al., 2008; Sayim et al., 2010) and in the periphery (Manassi et al., 2012; Rosen & Pelli, 2015). Similarly, variations of presentation time should maintain the order of grouping strengths across displays (Haladjian & Mathy, 2015). Interestingly, investigations of grouping and ungrouping in a backward masking paradigm showed that complex Gestalts needed more time to yield ungrouping compared to basic features; however, presentation times were very short (20ms), and no modulation occurred beyond the presentation time in our Experiment 2.1 (150 ms, Sayim et al., 2014; see also, Feldman, 2007; Kimchi, 1998). One possible explanation for the divergent numerosity estimation results of Experiment 2.1 and grouping results of Experiment 2.2 is that only single - or subsets of – grouped discs were sampled in a given trial in Experiment 2.1. As the number of (perceived) groups was larger in the *radial* compared to the *tangential* condition (Experiment 2.2), and therefore the average number of discs per group was smaller, numerosity estimates based on single (or a few) groups would be lower. However, given the frequent overestimation in the current study, it is unlikely that such sub-sampling (without overcompensation) has occurred. Another factor that could underlie the diverging results in Experiments 2.1 and 2.2 is that different groups of observers participated in the two experiments. In recent experiments with similar stimuli (including the radial-tangential manipulation), we found similar results with a different group of observers (66 participants), providing further evidence that numerosity estimates depend on the (*radial* or *tangential*) arrangement of items. In Experiment 2.2 of the current study, 87% of the observers indicated more groups in the radial than in the *tangential* condition (on average for all numerosities), while only 13% showed the opposite pattern, indicating a robust pattern of results across participants. Hence, it is unlikely that a different group of observers would show the opposite pattern of results, i.e., higher numerosity estimates and a larger number of perceived groups in the radial condition compared to the tangential condition.

Overall, we demonstrated that numerosity perception was anisotropic in regard to *radial* versus *tangential* arrangements. We suggest that redundancy masking is one of the potential determining factors in numerosity estimation. Going beyond purely physical stimulus descriptions by taking into account asymmetries of the visual field in spatial vision will help to shed light on the underlying mechanisms of numerosity perception.

CHAPTER 3: CROWDING, REDUNDANCY MASKING, AND THE RADIAL-TANGENTIAL ANISOTROPY OF NUMEROSITY ESTIMATION

Abstract

Humans can estimate the number of visually presented items without counting. This ability refers as to numerosity perception. In most numerosity studies, items are uniformly distributed across displays, with identical distributions in central and eccentric parts. However, our visual performance differs between the fovea and the periphery, and the visual field is highly asymmetric in regard to interferences between items. One of such asymmetries is the radial-tangential anisotropy: items arranged radially interfere more strongly with each other than those arranged tangentially. This has been shown for crowding (the deleterious effect when identifying targets in clutter) and redundancy masking where items in repeating patterns are not detected. In the present studies, we tested how the radial-tangential anisotropy of spatial vision impacts numerosity perception. In four experiments, we presented participants with displays containing 34-99 discs, predominantly arranged radially or tangentially, forming strong and weak interference conditions, respectively. Participants were required to report the number of discs. We found that observers reported the radial displays as less numerous than the tangential displays for all radial-tangential manipulations: weak (Experiment 3.1), strong (Experiment 3.2), and modulated with mixed contrast polarity (Experiments 3.3 and 3.4). Our results showed a radial-tangential anisotropy of numerosity perception. We suggested that crowding and redundancy masking modulate the numerosity perception.

Keywords: numerosity, contrast polarity, spatial vision, crowding, redundancy masking, radial-tangential anisotropy

Introduction

Humans are endowed with the competence to estimate the number of visually presented items without counting. It has been proposed that a dedicated system, the approximate number system (ANS, also known as "number sense") underlies such numerosity perception (Burr & Ross, 2008; Castaldi et al., 2021; Dehaene & Cohen, 1995; Dehaene et al., 1998; Feigenson et al., 2004). The ANS was proposed to be independent of other visual properties (Burr & Ross, 2008). However, it was also suggested that visual properties of the displays (e.g., item size, density, convex hull length: the smallest convex set that contains all items, and occupancy area: the encloser area of convex hull length, etc.) determine numerosity estimates (Allik et al., 1991; Aulet & Lourenco, 2021; Dakin et al., 2011; Gilmore et al., 2016; Hurewitz et al., 2006; Shilat et al., 2021). Stimulus properties that are correlated with numerosity (Leibovich & Henik, 2013) are important factors in numerosity estimates. For example, varying the number of items often goes hand in hand with changes of other visual properties of the display, such as the occupancy area (Allik & Tuulmets, 1991), and the convex hull (Gilmore et al., 2016; Katzin et al., 2020; Katzin et al., 2019; Shilat et al., 2021), both of which have been shown to play an important role in numerosity perception. When multiple visual cues that may contain information about the number of items in a display were present, observers weighed the cues to perform a numerosity comparison task (Gebuis & Reynvoet, 2012b), suggesting that multiple visual cues were integrated during numerosity perception (Gebuis & Reynvoet, 2012a, 2012b). In particular, Gebuis and Reynvoet (2012a) presented two displays sequentially in each trial and asked participants to judge which display contained more discs. They manipulated displays in a way that visual properties were fully or partially (un-)correlated with numerosity. For example, in the partial congruent condition, a larger numerosity display has a larger density but a smaller convex hull. In fully congruent or incongruent conditions, all visual properties positively or negatively correlated with numerosity, respectively. Importantly, in the correlated and uncorrelated displays, visual cues were informative and not informative about the numerosity of the displays, respectively. In partial congruent displays, some visual cues were informative, and others were not informative about the numerosity of the displays. They observed that performance was worst in the partial congruent condition. They suggested that participants integrated both the informative and the uninformative visual cues on partial congruent conditions so that the performance declined.

Recently, it was proposed that the topology of spatial vision, especially asymmetries of visual space and the radial-tangential anisotropy of contextual interferences, should be considered when investigating numerosity perception (L-Miao et al., 2022). In particular, it was shown that the arrangement of items predominantly radially or tangentially modulated numerosity estimates, with lower estimates in radial than tangential arrangements (L-Miao et al., 2022). This result has been attributed to the radial-tangential asymmetry of spatial vision, reported for crowding (Kwon et al., 2014; L-Miao et al., 2022; Toet & Levi, 1992) and redundancy masking (Yildirim et al., 2020; 2022; see below).

Studies investigating numerosity perception usually use displays that consist of multiple items that cover a relatively large area of the visual field, including the fovea, parafovea, and often the periphery (e.g., Anobile et al., 2015; Chakravarthi & Bertamini, 2020; Mengal & Matathia, 1980; Valsecchi et al., 2013). However, there are important differences between foveal, parafoveal, and peripheral vision (Rosenholtz, 2016; Simpson, 2017). For example, crowding, the interference of neighboring objects on target perception (Bouma, 1970) occurs over much larger distances in the periphery than in the fovea (Andriessen & Bouma, 1976; Bouma, 1970; He et al., 1996; Levi, 2008; Levi et al., 1985; Pelli et al., 2004; Pelli & Tillman, 2008; Sayim et al., 2014; Sayim & Wagemans, 2017; Strasburger et al., 2011; Whitney & Levi, 2011). Recently, it was suggested that crowding plays a role in numerosity perception (Anobile et al., 2015; L-Miao et al., 2022; Valsecchi et al., 2013). For example, Valsecchi et al. (2013) presented two adjacent dot arrays. Participants were asked to fixate the center of one of the dot arrays so that the other dot array appeared in participants' periphery. In a two-alternative forced choice task (where participants needed to indicate which of the arrays contains more dots), they found that the perceived numerosity of peripherally presented arrays was lower compared to foveally presented arrays. The underestimation in the periphery increased with increasing eccentricity. Based on these results, Valsecchi et al. (2013) suggested that the underestimation in peripherally presented displays was due to crowding (see also, Anobile et al., 2015).

However, the role of crowding in numerosity perception has been questioned. Recently, Chakravarthi and Bertamini (2020) tested whether crowding modulated numerosity perception. They used displays with configurations that affect both crowding and numerosity perception. They found that item spacing and item similarity affected the performance in crowding and numerosity estimation differently. Based on these results, they suggested that crowding does not modulate numerosity perception. Instead, it was proposed that clustering among items, independent of crowding, modulates numerosity perception and contributes to the underestimation of peripherally presented items (see also, Bertamini et al., 2018; Bertamini et al., 2016). How crowding or mechanisms related to crowding, such as redundancy masking (see below), would yield systematic underestimation of peripherally presented items is still unclear. One of the earliest models to explain underestimation is the occupancy model. In the occupancy model, each item occupies a circular area, and people estimate the numerosity base on the total occupied area. If items are close to each other, the occupied areas overlap, therefore resulting in underestimations (Allik & Tuulmets, 1991). Studies also suggested that the capacity of object individuation is limited, thus forming a bottleneck that restricts the number of items that can be encoded in numerosity perception (Mazza, 2017). Additionally, research showed that when the target and flankers are grouped together, crowding is stronger, whereas when they are not grouped together, crowding is weaker (Herzog et al., 2015; Manassi et al., 2013). Therefore, the lack of segmentation between items (due to crowding) may lead to an underestimation of numerosity perception.

While the typically found underestimation in numerosity studies suggests a detection-like error –some items are not detected and are missing from the estimate--crowding is usually assumed to interfere with target identification but not target detection (Levi et al., 2002; Pelli et al., 2004). Alternatively, the crowded items could be missing from numerosity estimates because they were merged, i.e., not segmented from neighboring items (Balas et al., 2009; Levi et al., 2002; Parkes et al., 2001; Pelli et al., 2004). Recently, a new phenomenon named ‘redundancy masking’ that could underlie underestimation in numerosity perception was discovered (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021, 2022). In redundancy masking, the number of perceived items in repeating patterns is lower than the number of presented items: For example, when presenting an array of identical, radially arranged lines in the visual periphery, observers usually reported fewer lines than were presented, even with as few as three lines (Yildirim et al., 2020, 2021). Redundancy masking, in contrast to crowding, is characterized by detection-like errors (see also, Coates et al., 2017; Sayim & Wagemans, 2017 for 'diminishment' or detection-like errors in crowding). With frequent reports of only two items when three items are presented, the error in redundancy masking is profound: one-third of the presented items are not reported. Redundancy masking has recently been suggested to underlie underestimation in numerosity perception (L-Miao et al., 2022).

Both crowding and redundancy masking are subject to a strong radial-tangential anisotropy (Kwon et al., 2014; Toet & Levi, 1992; Yildirim et al., 2020, 2022). In crowding, radially placed flankers interfere more strongly with target perception than tangentially placed flankers (Figure 3.1a; Feng et al., 2007; Greenwood et al., 2017; Kwon et al., 2014; Toet & Levi, 1992). Similarly, redundancy masking has been shown for radially arranged but not tangentially arranged items (Figure 3.1b; Yildirim et al., 2022). Recently, testing several large numerosities (from 21 to 58), we used displays whose discs were predominantly arranged in radial and tangential directions and demonstrated that numerosity estimation was subject to a radial-tangential anisotropy: Estimates were systematically lower when items were arranged radially compared to tangentially (L-Miao et al., 2022). These visual field asymmetries in numerosity estimation suggest that numerosity perception is modulated by the asymmetries of spatial vision.

Crowding has been shown to depend on target-flanker similarity, with higher target-flankers similarity usually yielding stronger crowding (and vice versa; Chakravarthi & Cavanagh, 2007; Chung et al., 2002; Chung et al., 2001; Chung & Mansfield, 2009; Kooi et al., 1994; Rummens & Sayim, 2019b, 2021). For example, crowding was stronger when the target and the flankers shared the same contrast polarity (e.g., both black or both white on a gray background) compared to opposite contrast polarity, crowding (Chakravarthi & Cavanagh, 2007; Chung & Mansfield, 2009; Sayim et al., 2008; Figure 3.1c). Interestingly, there was no such 'polarity advantage' in crowding when reporting all three displayed items, suggesting that attentional selection can modulate similarity effects in crowding (Rummens & Sayim, 2022). Importantly, a similar polarity advantage as in crowding has been found in RM (Hansmann-Roth & Sayim, 2022). They presented 3-5 radially arranged lines in peripheral vision. The lines were either uniform (all black or all white) or alternating in contrast polarity (e.g., black, white, black). Participants reported the number of lines they perceived and indicated the perceived color of each line afterward. Mixed contrast polarity did not prevent RM: two lines were frequently reported compared to three lines, and even the triplets were in mixed contrast polarity. However, the features of mixed contrast polarity were well preserved when RM occurred: participants usually indicated that the perceived two lines were of opposite contrast polarity. Contrast polarity seems to be another common factor that modulates both crowding and RM.

In the realm of standard numerosity studies, divergent results on how contrast polarity impacts numerosity perception have been found. In particular, some studies showed that contrast polarity has no impact on numerosity perception, while others revealed the opposite. For example, in a two-alternative forced-choice task, Tibber et al. (2012) presented test (64-265 items) and reference (128 items) displays to the left and the right of the central fixation. There were three contrast polarity conditions: both test and reference displays were of mixed contrast polarity (mixed with black and white), both test and reference displays were of uniform contrast polarity (all items were either white or black), and either test or reference displays were of mixed contrast polarity (and the other display was uniform). They did not find any advantage of mixed contrast polarity in numerosity perception: all three contrast polarity conditions showed similar patterns of results (see also, Dakin et al., 2011). However, Chakravarthi and Bertamini (2020) found that contrast polarity modulated numerosity perception in low- but not high-density displays: When the displays were of low density (0.08 – 0.24 items/deg2), mixed contrast polarity increased underestimation compared to uniform displays; however, this effect was not observed when the displays were of high density (approximately seven times higher than the low density).

In the presented study, we investigated how limits of spatial vision, i.e., crowding and RM, impact numerosity perception. First, we systematically varied the degree of radial-tangential arrangements, including displays that maximized the probability of being affected by RM. Second, we varied the contrast polarity of the items to (1) break visual configurations that emerge when grouping items of the same contrast polarity, and (2) investigate if mixed contrast polarity displays modulate numerosity estimation. The radial-tangential arrangements of displays were similar to our previous study, where we used the radial-tangential anisotropy of contextual interferences to create displays with different levels of interference by presenting – or not presenting any -- discs inside the interference zones of other discs (L-Miao et al., 2022). The shape of the interference zone can be approximated by an ellipsis with its long axis along the radial direction; its size increases with increasing eccentricity (Figure 3.1d). Items that fall into the interference zone are expected to yield interference with target perception; items that fall outside the interference zone are not expected to yield interference with target perception. We created two conditions: weak and strong interference. In the weak interference condition, no discs were placed in the interference zones of any other discs, and in the strong interference condition, around 10% of discs (range 1.8% - 26.8%, average 10.5%) were placed in other discs' interference zones (L-Miao et al., 2022). We found the numerosity estimations were systematically lower in the strong than in the weak interference condition. However, even in the strong interference condition, the majority of discs did not contain any discs in their interference region, and hence, underestimation driven by these configurations was not expected to be substantial.

Here, we sought to maximize interference (and underestimation) by increasing the number of discs in the interference zones of other discs. In particular, we maximized the potential of interference among discs by creating displays with at least 50% of the discs falling into other discs' interference zones. Our displays were composed of 'base' discs and 'flanking' discs. In radial displays, flanking discs were added into the interference zone of the base discs, and in tangential displays, flanking discs were added at (on average) the same distance to the base discs as in the tangential direction; however, *outside* of the interference zone of the base discs (Figure 3.2a). In Experiment 3.1 and Experiment 3.2, we varied the radial-tangential arrangements of displays (weak and strong): In weak radial-tangential arrangements, we ensured that there were at least 50% of base discs that were paired with one flanking disc, while at the same time avoiding strong structural differences between radial and tangential displays due to grouping among close-by items. In strong radial-tangential arrangements, each base disc was paired with two flanking discs, forming a disc triplet. The strong radial-tangential arrangement of displays were expected to have the highest probability of being affected by RM. To reduce perceived grouping among discs, we used mixed contrast polarity displays, i.e., the base and flanking discs were black and white, respectively (Experiments 3.3 and 3.4). As crowding is reduced when the target and flankers are with mixed contrast polarity, the flanking discs that were of the opposite contrast polarity of the base discs were expected to result in less interference with the base discs.

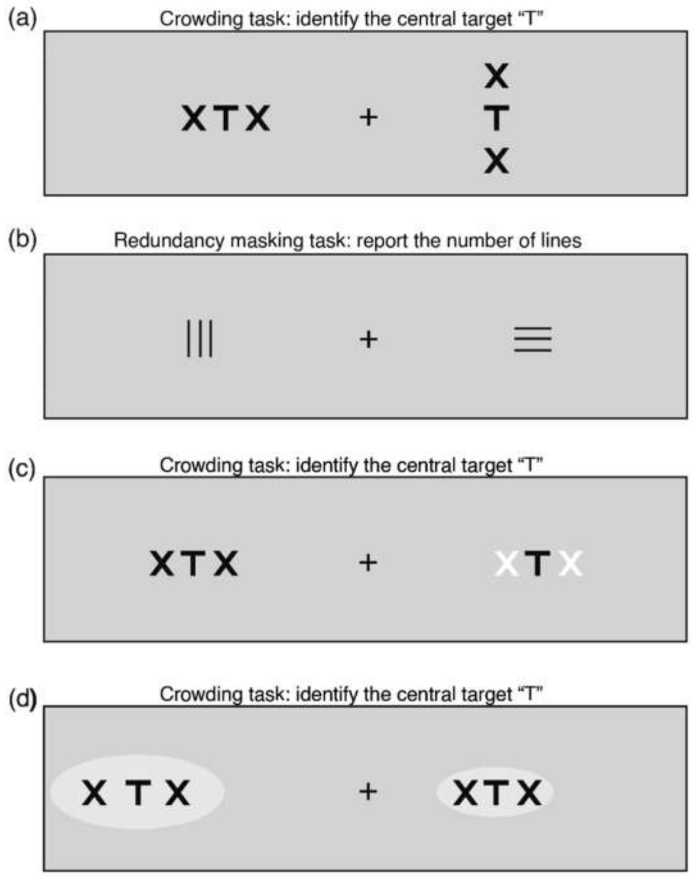

Figure 3.1. Illustration of the radial-tangential anisotropy of crowding (a) and redundancy masking (b), the effect of same vs. opposite contrast polarity in crowding (c), and interference zones of crowding (d). (a): When fixating the fixation cross, the identification of the target "T" (left) in the visual periphery is usually strongly impaired by flankers that are radially positioned in the interference zone (indicated by the shaded ellipse). Flankers cease to interfere with target perception at smaller target-flanker spacing when placed tangentially (outside of the interference zone; right). (b): When a line triplet is arranged radially (left), most observers report a line pair. When the line triplet is arranged tangentially (right), participants usually do not report lower numbers of lines (Yildirim et al., 2020). (c): The identification of the target "T" (left) is strongly impaired by flankers of the same contrast polarity compared to flankers of opposite contrast polarity (right). (d): The size of the interference zone increases with the eccentricity, often estimated to be around 0.5 x eccentricity in radial directions, and significantly less (e.g., around 0.2 × eccentricity; Toet & Levi, 1992) in tangential directions.

Experiment 3.1: Weak radial-tangential arrangements

In Experiment 3.1, we tested how the radial-tangential anisotropy modulated numerosity estimation by presenting displays to participants that were arranged predominantly radially or tangentially. We refer the manipulation in Experiment 3.1 weak (in compared to the manipulation in Experiment 3.2: strong). In Experiment 3.1a, we tested numerosity 34-44 and in Experiment 3.1b, we tested numerosities 54-64.

Method

Participants

Participants were recruited online using Prolific (www.prolific.co). Experiment 3.1a was completed by 34 participants (16 males, 18 females; mean age: 25.3 years, ranging from 18 to 38). Six of the forty recruited participants were removed: 4 participants did not complete the study, 1 participant had more than 5% of invalid responses (e.g., meaningless numbers such as '000', and 1 participant failed the attentional check (performance in subitizing trials were lower than 90%, details see below). Experiment 3.1b was completed by 32 participants (11 males, 21 females, mean age 25.7 years, ranging from 19 to 39). Forty participants who did not participate in Experiment 3.1a were recruited online using Prolific in Experiment 3.1b. We removed 8 participants: 2 participants did not complete the study, and the other 6 participants failed the subitizing attention check. All participants were naïve as to the purpose of the study. All participants received monetary compensation (7.5 £/hour). All participants reported normal or corrected-to-normal visual acuity, and the informed consent was solicited prior to the experiment. All experiments were approved by the ethics committee of the Ulille SHS, University of Lille.

Apparatus and stimuli

The experiment was created using Psychopy coder v3.1.2 (Peirce, 2009) and hosted by Pavlovia (www.pavlovia.org). In the online experiments (Experiments 1a, 1b, Experiment 3.3a, 3.3b), participants were instructed to do the experiment with a 24-inch monitor with a vertical resolution of 1080 pixels, and to sit at a distance of about 45 cm in front of the monitor (1 pixel = 0.04 visual degree).

Stimuli consisted of black (Hex Code #000000) discs presented on a gray (Hex

Code #B6B6B6) background. Discs (radius: 9 pixels) were presented within an imaginary rectangular region that occupied 40% (Experiment 3.1a) or 60% (Experiment 3.1b) of the screen. No disc was presented within a circular region (radius: 100 pixels, around 4 degrees of visual angle) around fixation (see Figure 3.3). Discs were either base discs or flanking discs (Figure 3.2a). To create predominantly radially or tangentially arranged displays, each base disc was surrounded by a radially orientated and a tangentially oriented elliptical interference zone (Figure 3.2a; these zones were only used to construct the displays and were never shown to participants). The major and the minor axis of the elliptical interference zone were $0.25 \times$ eccentricity and $0.1 \times$ eccentricity, respectively (see also, L-Miao et al., 2022). The size of the zones was determined based on common estimates of the size of the interference zone in crowding (Toet & Levi, 1992), and used to control for the distance among discs in the displays. The two zones were free from other base discs. The flanking discs were placed into the radially or tangentially orientated zones to form radial (strong interference) or tangential (weak interference) displays, respectively. No flanking disc was added to the overlap area of the radial and rotated (tangentially elongated) interference zone. In Experiment 3.1, the radial-tangential display manipulation was weak. Each base disc was either presented without any flanking disc (remaining a single disc), paired with one flanking disc (forming a disc pair), or paired with two flanking discs (forming a disc triplet, Figure 3.2b). We varied the percentage of single discs, disc pairs, and disc triplets to reduce the probability that participants estimated the number of discs by multiplication of the number of estimated disc pairs and/or triplets. The percentage of disc pairs varied between 0 and 100% in steps of 25%. For example, the percentage of disc pairs in a display was 50% when 50% of the base discs (randomly selected) were paired with one extra disc. The remaining base discs were presented with two flanking discs forming disc triplets (25%) and without flanking discs (single base discs; 25%).

To generate a display, a random position was selected to place the first base disc with its corresponding interference zones (Figure 3.2c). Additional discs were added iteratively on the displays with the constraint that no interference zones overlapped with the interference zones of any other base disc. Base discs were positioned on the display until no disc without overlapping interference zones could be added anymore. Flanking discs were added into the interference zones or the rotated interference zones (excluding the central, overlapping zone) to form radial and tangential displays, respectively. All discs on the displays were presented within a rectangular region. The size of the rectangular was either small (21.5˚ width × 13.5˚ height, occupying 40% of the entire screen, Experiment 3.1a, 3.2a, 3.3a, and Experiment 3.4: small numerosities) or large (27.0 ˚ width × 18.5˚, occupying 60% of the entire screen, Experiment 3.1b, 2b, 3b, and Experiment 3.4: large numerosities). The size of the rectangular region determined the maximum number of base discs that could be presented. For each percent of disc pairs condition, we generated 10000 displays (5000 radial and tangential displays each). We selected displays with the same numerosity so that radial and tangential displays matched in regard to average eccentricity, average spacing, convex hull length, occupancy area, and density (see Supplementary Table S2.1). The possible numbers of base discs was 17 -22 and 27-32 for small and large displays, respectively. The numerosities were 34, 36, 38, 40, 42, and 44 for Experiment 3.1a (small numerosities), and 54, 56, 58, 60, 62, and 64 for Experiment 3.1b (large numerosities).

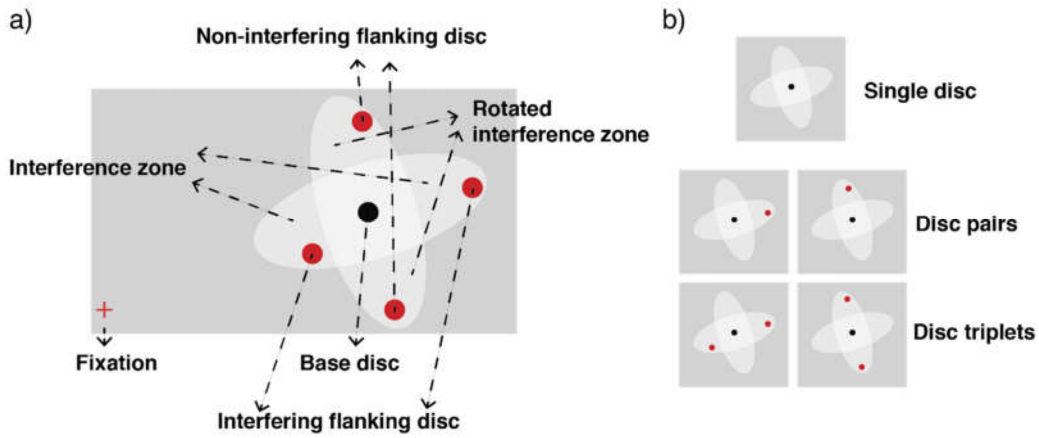

Figure 3.2. a) Illustration of display construction. In radial displays, flanking discs were added into the interference zone of the base discs. In tangential displays, flanking discs were added in the rotated interference zone of the base discs. b) Possible disc configurations.

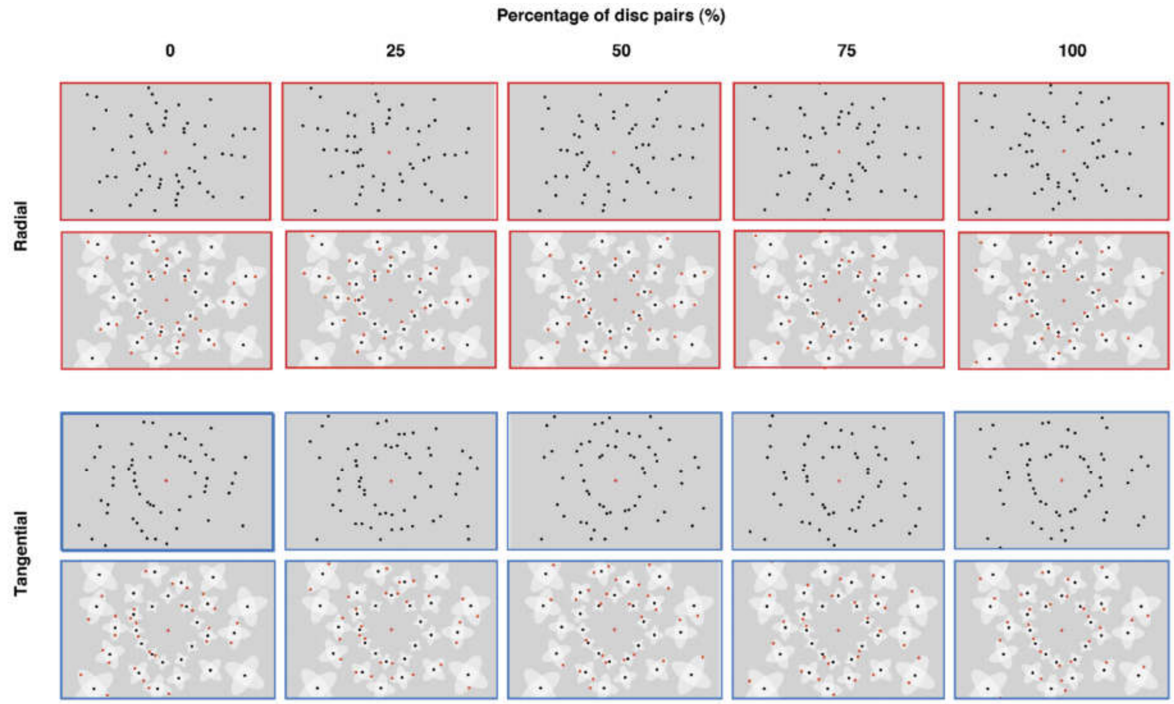

Figure 3.3. Illustration of displays in the radial and tangential conditions (the first and the third row) and their geometric principles (the second and the last row) in Experiment 3.1. Note that all displays in the figure share the same base discs for illustration purposes (in the experiments, no display shared the same base discs as each display was generated independently).

Design and Procedure

Each trial started with a red fixation cross (5 pixels × 5 pixels) presented at the center of the screen. Observers initiated each trial by pressing the spacebar. The display was presented for 150 ms. Participants were required to enter their best estimation of the number of discs on the presented display using the number keys on the keyboard. The estimates entered by participants were displayed on the screen for each trial. There was no feedback in the experiment, and there was no time limit for participants to respond. Participants were not informed about the numerosity ranges prior to the experiment. Prior to the experiment, participants viewed 5 reference displays. The numerosities of the 5 reference displays were equally distributed around the mean numerosity of the experiment (0.125 and 0.25 times the mean numerosity of all displays). Each reference display was presented for 150 ms, and participants were informed about the actual number of the reference display after the offset of the reference display. Each participant performed 300 trials (50 trials for each numerosity in random order). The experiment was interspersed with 30 trials with numerosities in the subitizing range (2-4 discs) for attentional control (participants with incorrect responses in these trials of 10 percent or more were to be excluded from the experiment).

Data analysis

We calculated the deviation score (DV) by subtracting the actual numerosity of the display from the reported numerosity. The raw data were tidied up (including combining all raw data into an intact file, and removing extraneous information) with the tidyverse library (Wickham, 2017) in R 3.6.3 (R Core Team, 2020) and RStudio (RStudio Team, 2020). Linear mixed-effect analyses were conducted using the lme4 package (Bates et al., 2014). The Emmeans package (Lenth et al., 2021) was used for estimation statistics and post-hoc comparisons on the full model. The analysis codes and data are available at

https://github.com/miaoli-psy/numerosity_exps/tree/master/src/stat_tests.

In the models, we standardized the dependent variable deviation score (DV) so that DV has a mean of zero and a standard deviation of one, ensuring that the estimated coefficients were of the same scale in all analyses. To examine the DV differences between the radial and tangential conditions, we entered the alignment condition as a fixed factor. Numerosity was submitted as a random factor (DV differences between numerosities, for example, between displays with 34 and 36 were not analyzed). Using the model comparison method, first, we constructed a full model (that successfully converged) with the alignment condition as a fixed factor and a reduced model without the alignment condition as a fixed factor. We used the random slope model, assuming that the effect of the alignment condition differed among participants, and the difference between numerosities (participants and numerosity had different intercepts and different slopes for the effect of DV in the model). As random effects, we had intercepts for participants and intercepts for numerosity. P-values were obtained by likelihood ratio tests between the full model (with alignment condition as a fixed factor) and the reduced model (without alignment condition as a fixed factor). Visual inspection of all residual plots did not reveal any obvious deviations from homoscedasticity or normality.

Results

Experiment 3.1a: small numerosities (34 - 44)

Figure 3.4a shows the deviation scores (DV) for the radial and tangential alignment conditions in Experiment 3.1a (see Supplementary Figure S3.1 for deviation scores as a function of numerosity). Comparing the full model (including alignment condition as a fixed factor) with the reduced model (excluding alignment condition as a fixed factor) revealed no difference between the radial and the tangential displays (DV; $\chi2(1) = 1.98$, $p = .16$). There was a trend for lower estimates in the radial compared to the tangential condition: $\beta = -0.06 \pm 0.14$.

Experiment 3.1b: large numerosity range (54 – 64)

Figure 3.4b shows the DV for the radial and tangential alignment conditions in Experiment 3.1b (see Supplementary Figure S3.1 for deviation scores as a function of numerosity). Comparing the full model (including alignment condition as a fixed factor) and the reduced model (excluding alignment condition as a fixed factor) revealed significant differences between the radial and the tangential displays(DV; $\chi2(1) = 12.20$, $p < .001$). Estimates in the radial condition were lower compared to the tangential condition: $\beta = -0.08 \pm 0.02$, $p < .0001$.

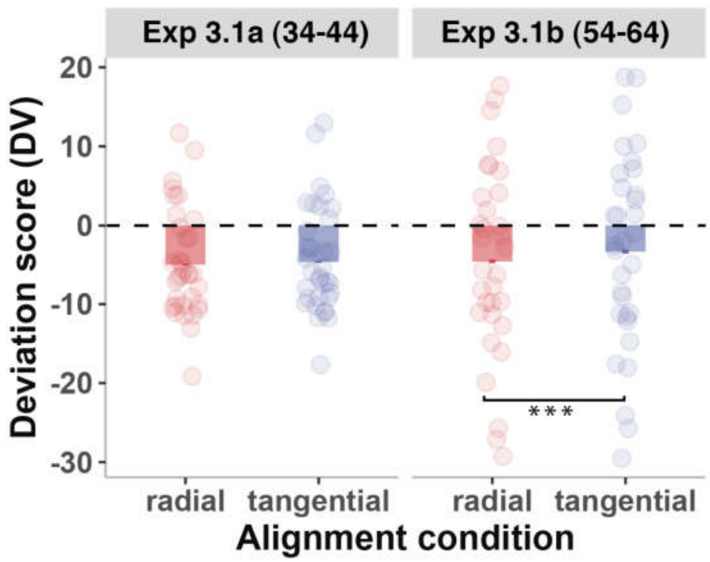

Figure 3.4. Results for Experiment 3.1a (a) and Experiment 3.1b (b). Deviation score (DV) for the radial and tangential conditions. DVs of 0 represent correct estimates, negative DVs underestimations, and positive DVs overestimation. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks. *$p < .05$, **$p < .01$, ***$p < .001$

Experiment 3.2: Strong radial-tangential arrangements

In Experiment 3.2, we created displays to maximize the probability of interference among discs (in the radial condition). Unlike in Experiment 3.1 where (on average) one flanking disc was placed in the (rotated) interference zone of the base disc, in Experiment 3.2, two flanking discs were placed around each base disc, forming “disc triplets” (Figure 3.5).

Method

Participants

Sixteen participants (13 females, 3 males, mean age = 20.4 years, range from 19 to 23 years). All participants were undergraduate psychology students at KU Leuven. They received course credits for their participation. All participants had more than 95% correct in performing the subitizing trials; therefore, no data was removed from the analysis.

Apparatus and stimuli

The experiment was created using Psychopy coder v3.1.2 (Peirce, 2009) and run on a Desktop PC. All stimuli were presented on an LED 24-inch display, with a resolution of 1920 × 1080 pixels and a refresh rate of 60 Hz. During the experiment, participants sat in front of the screen at approximately 45 cm (1 pixel corresponds to 0.04 visual degree angles). The experiments were conducted in a dim experiment room.

Figure 3.5 illustrates the display used in Experiment 3.2. As in Experiment 3.1, the base discs were surrounded by a radially arranged interference zone and a tangentially arranged rotated interference zone for radial and tangential displays, respectively. The stimuli were identical to the displays used in Experiment 3.1, except that all base discs were paired with two flanking discs, forming “disc triplets” (see Figure 3.2b). The numerosities were 51, 54, 57, 60, 63, 69, and 72 in the small displays, and 78, 81, 84, 87, 90, 93, 96, and 99 in the large displays.

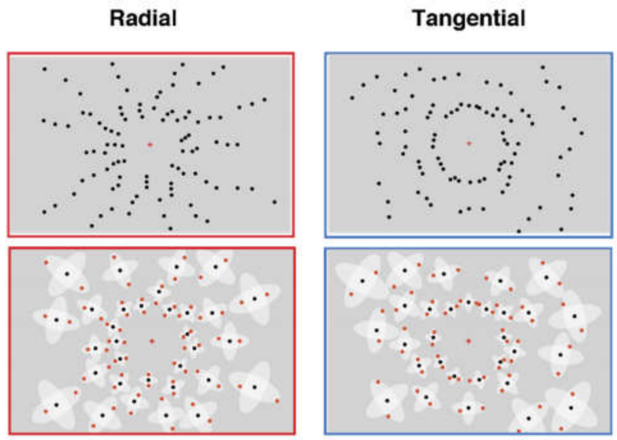

Figure 3.5. Illustration of displays in the radial and tangential conditions (upper-left corner and upper-right corner) and their geometric principles (lower-left corner and lower-right corner) in Experiment 3.2. Note that all displays in the figure share the same base discs for illustration purposes (in the experiments, no display shared the same base discs as each display was generated independently).

Design and Procedure

The design and procedure were identical to Experiment 3.1 except that participants performed a within-subject design experiment where each participant viewed both small (51-72) and large (87-99) numerosity displays. Each participant performed 6 blocks (3 large numerosities and 3 small numerosities) of 80 trials, each in random order.

Data analysis

Data analysis was identical to Experiment 3.1a, except for the following changes. We submitted numerosity (small vs. large) as a fixed factor in the model to compare DV between small and large numerosities. Using the model comparison method, first, we examined whether there was an interference effect between the alignment condition and the numerosity on deviation scores (DV). For that aim, we constructed a full model with the interference effect between the alignment condition and the numerosity as a fixed factor and a reduced model without the interference effect as a fixed factor. We used the random slope model, assuming that the effect of the alignment condition differed between participants (participants had different intercepts and different slopes for the effect of DV in the model). As fixed factors, we entered alignment condition, numerosity, and the interaction between them. As random effects, we had intercepts for participants. We constructed a reduced model without interaction between the alignment condition and the numerosity as a fixed factor. P-values were obtained by likelihood ratio tests of the full model with the interaction against the model without the interaction. In case of a significant interaction effect, the DV differences under each numerosity (small or large) with a contrast comparison would be examined. In case of a non-significant interaction effect, the interaction factor would be removed from the full model, examining the effect of alignment condition and numerosity on DV separately (with the same model comparison method). Visual inspection of all residual plots did not reveal any obvious deviations from homoscedasticity or normality.

Results

Figure 3.6 shows the deviation score (DV) for the radial and the tangential alignment conditions for Experiment 3.2 (see Supplementary Figure S3.2 for deviation scores as a function of numerosity). The model comparison between the full model (including the interaction between alignment condition and numerosity as a fixed factor) and the reduced model (excluding the interaction as a fixed factor) revealed no difference $\chi^2(1) = 0.96$, $p = .33$. There was no significant interaction effect between alignment condition and numerosity on DV. There was a significant main effect of the alignment condition: $\chi^2(1) = 8.48$, $p < .005$. The main effect of numerosity was significant, revealed by the model comparison result: $\chi^2(1) = 75.20$, $p < .0001$. For small numerosities: the DV in the radial condition (β = --0.15 ± 0.12) was lower compared to the tangential condition ($\beta = -0.02 \pm 0.13$), $p < .005$. For large numerosities: the DV in the radial condition ($\beta = 0.02 \pm 0.12$) was also lower compared to the tangential condition ($\beta = 0.19 \pm 0.13$), $p < .005$.

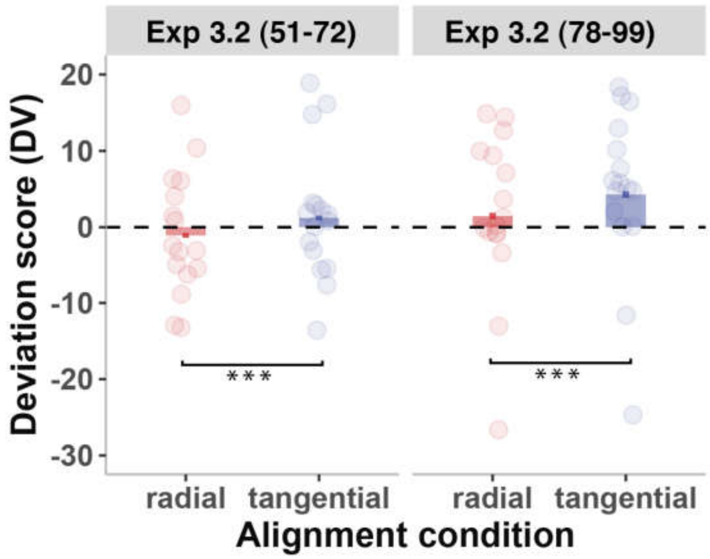

Figure 3.6. Results for Experiment 3.2. Deviation score (DV) as a function of numerosity. DVs of 0 represent correct responses, negative DVs underestimations, and positive DVs overestimation. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks. *$p < .05$, **$p < .01$, ***$p < .001$

Experiment 3.3: Radial-tangential arrangements with mixed contrast polarity

In Experiment 3.3, we investigated if differences between the radial and the tangential alignment conditions were impacted when using mixed contrast polarity displays.

Method

Participants

Data from 29 participants (20 females, 8 males, and 1 participant who did not indicate any sex, mean age = 25.4 years, ranging from 19 to 38 years) were submitted for analysis for Experiment 3.3a. Forty participants who were naïve as to the study were recruited. We removed 11 participants: 10 did not complete the study, and 1 failed the subitizing attention check (performance in subitizing trials was lower than 90%). For Experiment 3.3b, data from 28 participants (19 females and 9 males, mean age: 27.3 years, ranging from 19 to 40 years) were submitted for analysis. Forty naïve participants were recruited. We removed 12 participants: 2 did not complete the study, and 10 failed the subitizing attention check.

Apparatus and stimuli

The apparatus was identical to Experiment 3.1a. The stimuli used in Experiment 3.3a and Experiment 3.3b were identical to Experiment 3.1a and Experiment 3.1b, respectively, except that the displays were mixed contrast polarity (black and white discs) (Figure 3.7; all base discs were black, all flanking discs were white).

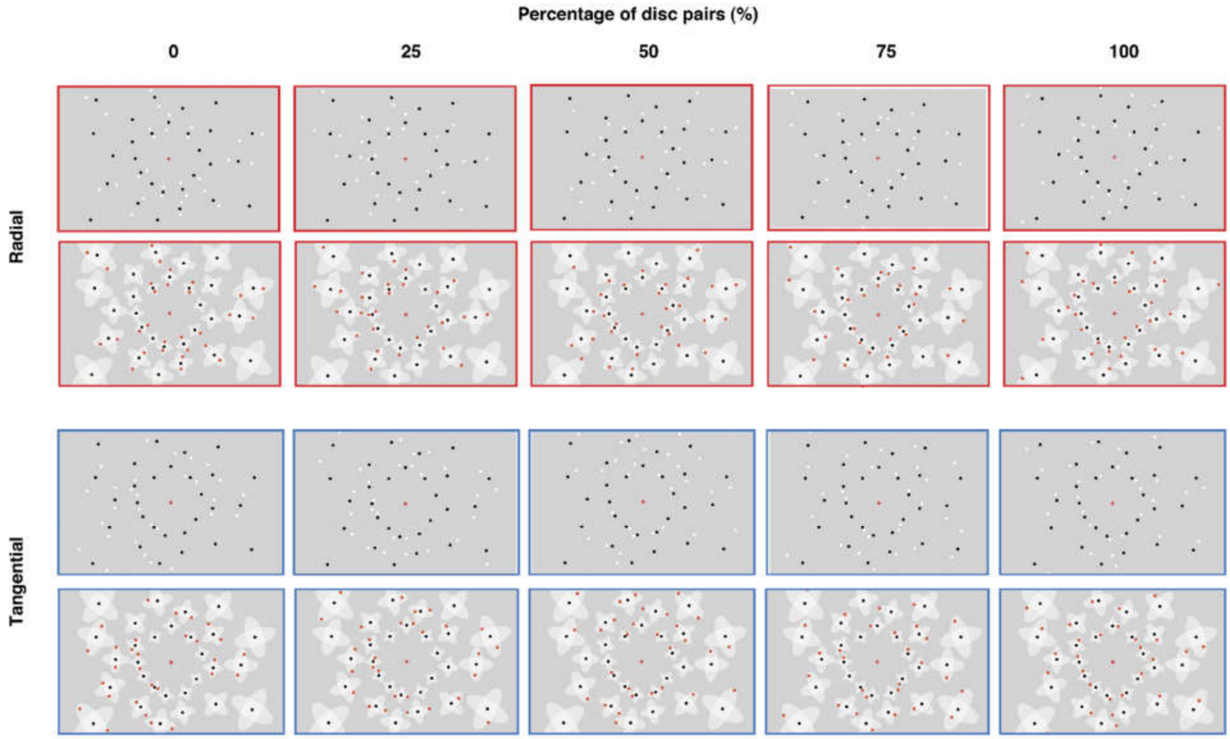

Figure 3.7. Illustration of displays in the radial and tangential conditions (the first and the third row, respectively) and their geometric principles (the second and the last row) in Experiment 3.3. Note that all displays in the figure share the same base discs for illustration purposes (in the experiments, no display shared the same base discs as each display was generated independently).

Design and Procedure

Identical to Experiment 3.1.

Data analysis

Identical to Experiment 3.1.

Results

Experiment 3.3a: small numerosities (34 - 44)

Figure 3.8a shows the DV for the radial and tangential alignment conditions in Experiment 3.1a (see Supplementary Figure S3.3 for deviation scores as a function of numerosity). We found that the alignment condition impacts DV, revealed by the comparison between the full model (including alignment condition as a fixed factor) and the reduced model (excluding alignment condition as a fixed factor): $\chi2(1) = 8.85$, $p < .005$. Estimation in the radial condition is lower compared to the tangential condition: $\beta = -0.08 \pm 0.02$, $p < .0001$.

Experiment 3.3b: large numerosities (54 - 64)

Figure 3.8b shows the DV for the radial and tangential alignment conditions in Experiment 3.1b (see Supplementary Figure S3.3 for deviation scores as a function of numerosity). The results for Experiment 3.3b were very similar to Experiment 3.3a that we observed a significant effect of alignment condition on DV, revealed by the comparison between the full model (including alignment condition as a fixed factor) and the reduced model (excluding alignment condition as a fixed factor): $\chi2(1) = 9.85$, $p < .005$. Estimation in the radial condition is lower compared to the tangential condition: $\beta = -0.1 \pm 0.03$, $p < .0001$.

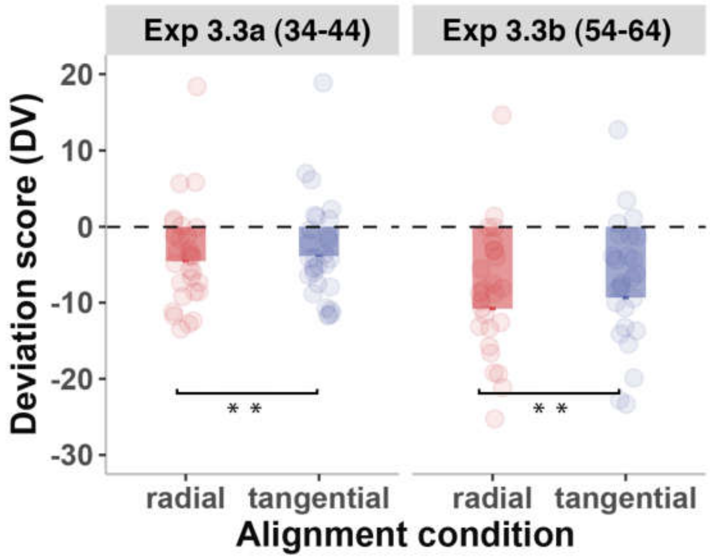

Figure 3.8. Results for Experiment 3.3a (a) and Experiment 3.3b (b). Deviation scores (DV) for the radial and tangential conditions. DVs of 0 represent correct estimates, negative DVs underestimations, and positive DVs overestimations. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks. *$p < .05$, **$p < .01$, ***$p < .001$

Experiment 3.4: Radial-tangential arrangements with uniform and mixed contrast polarity

In Experiment 3.4, we compared the uniform and mixed contrast polarity displays with a within-subject design where participants viewed both types of displays.

Method

Participants

Nineteen participants (14 females, 5 males, mean age = 20.1 years, range from 18 to 24 years). All participants were undergraduate psychology students at KU Leuven. They received course credits after their participation. All participants had more than 95% correct in performing subitizing trials. Therefore, no data was removed from the analysis.

Apparatus and stimuli

The apparatus was identical to Experiment 3.2. The stimuli in Experiment 3.4 included displays used in all the previous experiments, excluding the percentage of disc pairs of 25% and 75% (Figure 3.9). This is to reduce the number of presented displays in each block to control for the experiment duration (less than 120 min). The mixed contrast polarity displays used in Experiment 3.4 were identical in Experiment 3.3. Half of the uniform contrast polarity displays in Experiment 3.3 were identical to displays in Experiment 3.1. To balance the overall luminance between uniform displays and mixed contrast polarity displays, the other half of the uniform displays were identical to displays in Experiment 3.1 but contained white discs.

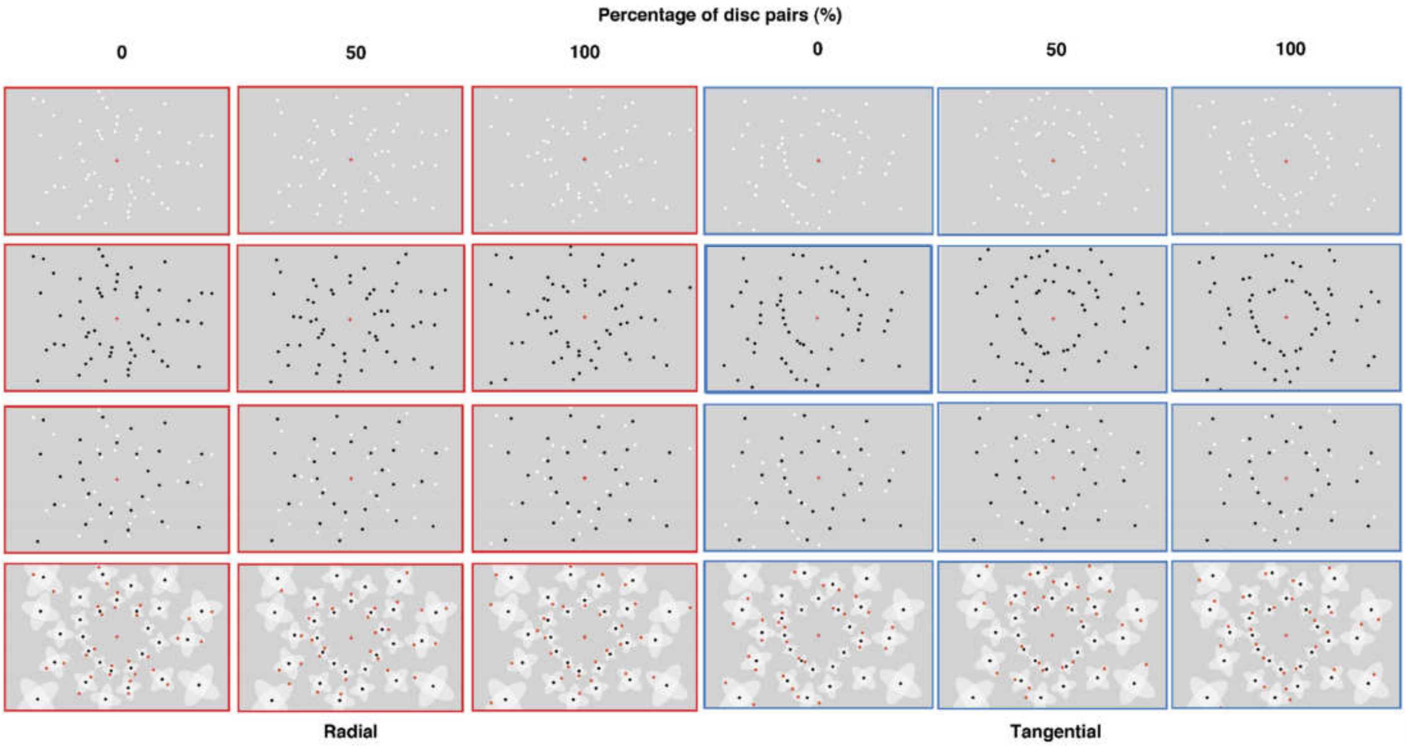

Figure 3.9. Illustration of displays in the radial and tangential conditions (the first and the third row) and their geometric principles (the second and the last row) in Experiment 3.4. Note that all displays in the figure share the same base discs for illustration purposes (in the experiments, no display shared the same base discs as each display was generated independently).

Design and Procedure

The design and procedure were identical to Experiment 3.1 except for the following changes: 1) participants viewed both small (34-44) and large (54-64) numerosities displays, as well as both uniform display and mixed displays in separate blocks in random orders (within-subject design). 2) Each participant completed 8 blocks (4 large numerosities blocks and 4 small numerosities blocks) of 144 trials in random orders.

Data analysis

We performed a linear mixed effects analysis on the obtained data. As in Experiment 3.2, we submitted numerosity (small vs. large) as a fixed factor in the model. First, we examined whether there was a three-way interaction effect among alignment condition, numerosity, and contrast polarity on DV. For that aim, we constructed a full model including the three-way interaction among alignment condition, numerosity, and contrast as a fixed factor, as well as a reduced model without the three-way interaction as the fixed factor. We constructed a random slope model as we assumed that the effect of the alignment condition was different for different participants. Therefore, participants had different intercepts and slopes for the effect of DV in the model. As fixed factors, we entered alignment condition, numerosity, contrast polarity, the three two-way interactions, and the three-way interaction. As random effects, we had intercepts for participants. Then, we constructed a reduced model without three-way interaction as a fixed factor. P-values were obtained by likelihood ratio tests of the full model and the reduced model. If a three-way interaction effect is observed, the contrast will be applied to the model to examine whether there is a significant two-way interaction, and a simple contrast will be applied to examine the (simple) main effect of alignment condition, contrast polarity, and numerosity if any significant two-way interaction is observed. If no significant three-way interaction is observed, the same model comparison method as in Experiment 3.2 will be used to examine two-way interactions and main effects: including the factor that we examined in the full model and excluding it in the reduced model. Visual inspection of all residual plots did not reveal any obvious deviations from homoscedasticity or normality.

Results

Figure 3.10 shows the deviation scores (DV) as a function of tested numerosity for the radial and tangential alignment conditions for Experiment 3.4 (see Supplementary Figure S3.4 for deviation scores as a function of numerosity). Mixed and uniform contrast polarity conditions were plotted separately in the subplots. The model comparison between the full model (including the three-way interaction as a fixed factor) and the reduced model (excluding the three-way interaction as a fixed factor) showed that there was no significant three-way interaction among numerosity, alignment condition, and contrast polarity $\chi2(1) = 0.05$, $p = .83$. The interaction between numerosity and contrast polarity was significant: $\chi2(1) = 9.20$, $p < .01$. No other significant two-way interaction was observed (interaction between numerosity and alignment condition: $\chi2(1) = 2.63$, $p = .11$; interaction between contrast polarity and alignment condition: $\chi2(1) = 0.25$, $p = .62$). Post-hoc pairwise analysis on the full model with Tukey adjustments showed the DV differences between mixed contrast polarity and uniform contrast polarity conditions were significant: ($\beta = -0.21 \pm 0.02$, $p < .0001$ and $\beta = -0.14 \pm 0.02$, $p < .0001$ for small and large numerosities). The existence of an interaction effect between numerosity and contrast polarity prevents us from examining the main effect with the model comparison method. Therefore, we report the pairwise differences of the alignment condition (radial - tangential): $\beta = -0.12 \pm 0.03$, $p < .0005$ and $\beta = -0.15 \pm 0.02$, $p < .0001$.

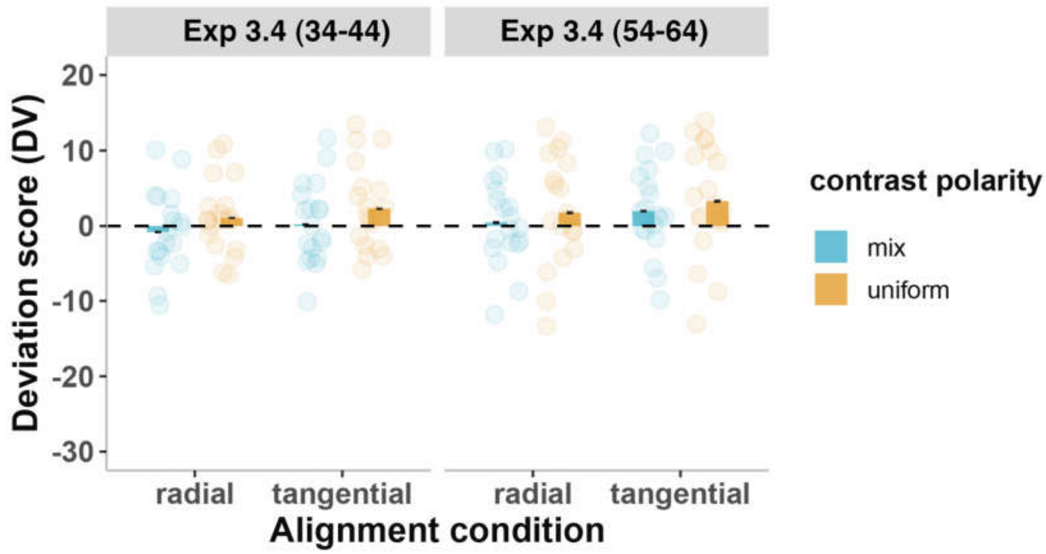

Figure 3.10. Results for Experiment 3.4. Deviation scores (DV) for mixed and uniform displays in the radial and the tangential conditions. DVs of 0 represent correct estimates, negative DVs underestimations, and positive DVs overestimations. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks. *$p < .05$, **$p < .01$, ***$p < .001$

Discussion

In the current study, we sought to understand the role of crowding and redundancy masking in numerosity perception and how they contribute to the radial-tangential anisotropy of numerosity perception. We investigated how the topology of our spatial vision, especially how the radial-tangential anisotropy modulates numerosity perception. We varied displays, arranging discs predominantly radially or tangentially. Other physical properties (e.g., convex hull, occupancy area, density etc.) of the displays were kept as similar as possible between the radial and the tangential displays (Supplementary Table S3.1). In four experiments, we asked participants to report the number of presented discs. We found that the estimates were lower when discs were arranged radially compared to when discs were arranged tangentially (Experiments 3.1–3.4). The relative underestimation in the radial compared to tangential condition occurred when the radial-tangential manipulation of displays was weak (Experiment 3.1 with large numerosities) and strong (Experiment 3.2 with both small and large numerosities). Importantly, also when using mixed contrast polarity displays, radial displays were reported as less numerous than tangential displays (Experiments 3.3 and 3.4).

In Experiment 3.1, numerosity estimates were lower in the radially compared to tangentially arranged displays when the number of presented discs was high (54-64), but not when it was low (34-44). The results seem to indicate that the radial-tangential arrangements of displays impact large but not small numerosities. However, there are other potential differences between the large and small numerosities in the current study. In particular, the small numerosity displays have an overall shorter convex hull and higher density compared to large numerosity displays (see Supplementary Table S3.1). We cannot rule out that the absence of DV differences between the radial and tangential conditions in the small numerosity condition in Experiment 3.1 was a result of possible variations due to the online, unsupervised data collections. In both numerosity ranges in Experiment 3.1, there were either 0, 1, or 2 flanking discs in the interference regions of the base discs, resulting in displays with at least 50% (50 % to 100 %) of base discs flanked by discs within the interference zone (radial condition) or the rotated interference zone (tangential condition). This manipulation was implemented to raise the probability of interference among discs in the radial condition, thus increasing the probability of observing lower estimates in the radial compared to the tangential condition. Compared to an earlier study in which we observed a radial-tangential anisotropy of numerosity estimation (L-Miao et al., 2022), the proportion of flanking discs in the interference zones was more than doubled. We expected more interference among discs in the current radial displays compared to those of L-Miao et al. (2022)'s study.

In Experiment 3.2, we investigated how numerosity estimation was impacted if discs' mutual influences on radial displays were enlarged. To this end, two flanking discs (instead of one) were placed into the interference zone of the base discs in radial displays, which resulted in an increasing probability of displays being affected by redundancy masking (RM) in radial displays. Importantly, RM does usually not occur with two items; for example, participants were able to accurately report two items even when items were aligned radially (Yildirim et al., 2021). RM was strong when three items (e.g., letters and lines) were presented radially (one-third of the items are missing in the enumeration task; Sayim & Taylor, 2019; Yildirim et al., 2020, 2021), and the report number of items were accurate when items were arranged tangentially (Yildirim et al., 2020). Several studies show that redundancy masking is different from crowding (Sayim & Taylor, 2019; Yildirim et al., 2020). One of the major differences is that crowding is usually assumed to not impact target detection (Levi et al., 2002; Pelli et al., 2004), whereas in RM, entire items go unnoticed, that is, they are not detected. Here, when displays were of high probability to be affected by RM, we observed similar results as in Experiment 3.1, that radial displays were estimated as less numerous compared to tangential displays. The results of the study indicated that displays with a high likelihood of RM were perceived as having a lower number compared to displays with a low likelihood of RM. We suggest that RM plays an important role in numerosity perception.

Interestingly, our results did not show consistent underestimation. We previously suggested that the occurrence of the overestimation may be driven by the task that requires participants to report the number of items instead of making comparisons between two or more displays (L-Miao et al., 2022). In the current experiments, the overestimations or underestimations were inconsistent across the different experiments. In Experiments 3.1 and 3.3, the estimates for both small and large numerosities were underestimations (see Figure 3.4, Figure 3.8, Supplementary Figure S3.1, and Supplementary Figure S3.3). However, in Experiments 3.2 and 3.4, the estimates within blocks (small numerosities or large numerosities) showed both under- and overestimations, and were centered approximately on the median numerosity within each block. One possible explanation is that the direct estimation triggered a central response tendency where stimuli tend to be misperceived and biased towards the mean of the distribution (Hollingworth, 1910). Anobile et al. (2019) further suggested a Bayesian model of the central tendency of numerosity perception that observers based on the performance on a distribution that consists of both sensory estimates and a priori hypothesis about the stimuli. Here, our results of numerosity estimation captured the central tendency trend within a block (see Supplementary Figure S3.1-S3.4).

In Experiment 3.3, we used mixed contrast polarity displays with the same radial-tangential arrangement displays as in Experiment 3.1. The mixed contrast polarity displays, compared to the uniform contrast polarity displays, were expected to break visual structures (see L-Miao et al., 2022) that emerge when items are perceived to be grouped together (see L-Miao et al., 2022). Moreover, flanking discs with opposite contrast polarity were expected to interfere less with the base discs compared to the same contrast polarity discs in the radial condition. In crowding, when the target and flankers are distinct from each other (e.g., with opposite contrast polarity), the target identification performance is better than when the target and flankers are similar (Chung & Mansfield, 2009; Kooi et al., 1994; Rosen & Pelli, 2015; Rummens & Sayim, 2019a). In this regard, adding opposite contrast polarity flanking discs into the interference zone of the base disc in radial displays should result in less interference compared to when adding the same contrast polarity flanking discs into the interference zone of the base disc. Therefore, we speculated that the radial mixed contrast polarity displays reduced the overall interference level among discs. Hence, a reduction of the difference between the radial and the tangential conditions was expected. However, we found that the estimates for radial displays were still lower than for tangential displays, as in Experiment 3.1, showing the radial-tangential anisotropy persisted with mixed contrast polarity displays. Critically, it has been shown that opposite contrast polarity did not result in improved performance in full report tasks of peripherally presented targets (Rummens & Sayim, 2021). Specifically, Rummens and Sayim (2021) conducted an identification task on crowding. They presented three orientated letters T and asked participants to either report the orientation of one of the letters or the orientation of all the letters. The stimuli were either of the mixed contrast polarity or of the uniform contrast polarity. They found that mixed contrast polarity improved the orientation identification performance when participants reported the single letter orientation and reduced the performance when participants reported all letters' orientations. The findings of our study are in line with these results, as the numerosity estimation employed in our study bears a resemblance to full report tasks. Therefore, we suggest that the effect of mixed contrast polarity displays on numerosity estimation performance could be similar to Rummens and Sayim (2021). Moreover, research showed that when the target was surrounded by multiple equidistant flankers that were in alternating contrast polarity, crowding increased (Rosen & Pelli, 2015; Sayim et al., 2008). The authors suggested that multiple mixed contrast polarity flankers override the local dissimilarity (e.g., one target with neighboring opposite contrast polarity flankers) by grouping with the target and producing more crowding.

In Experiment 3.4, participants completed the numerosity estimation task with both the uniform and mixed contrast polarity displays, which allowed us to disentangle DV differences between these two types of displays. We replicated the results of Experiment 3.3 that radial displays were estimated to be less numerous compared to tangential displays. Interestingly, in Experiment 3.4 (within-subject design), we found an advantage of mixed contrast polarity: the DV of mixed contrast polarity displays was lower (and closer to DV of 0, the correct estimates) than uniform contrast polarity displays.

Our findings indicated a radial-tangential anisotropy of numerosity perception, with radially arranged items being perceived as less numerous compared to tangentially arranged items. Mixed contrast polarity has been demonstrated to modulate crowding and redundancy, but it does not seem to reduce the difference between the radial and tangential displays. We suggest that the radial-tangential anisotropy of contextual interference plays a role in numerosity perception, possibly mediated by crowding and RM.

CHAPTER 4: REDUNDANCY MASKING OF FACES REVEALS A SUBSTANTIAL FAILURE TO DETECT SOCIALLY RELEVANT INFORMATION

Abstract

Faces are socially highly relevant stimuli that are usually detected rapidly and accurately. For example, it was shown that faces were accurately detected with presentation times as short as 100 ms, and when embedded in highly noisy contexts. Strong performance in face detection tasks highlights the importance of faces as an important stimulus for human observers. Here we show that face detection frequently failed when faces were presented in small groups. In Experiment 4.1, 3-6 identical upright faces, shape-matched outlines of the faces, and luminance-matched noise patches were presented at 10° eccentricity, randomly to the left or right of fixation. In Experiment 4.2, three to six identical upright or upside-down faces were presented. Participants were asked to indicate the number of items (1-9) in Experiment 4.1 and to indicate both the number of faces and their orientation (upright or upside-down) in Experiment 4.2. In both experiments, we found that the number of reported items was frequently lower than the number of presented items. Importantly, people showed substantial failures to report all presented faces, even with only three presented faces. Face orientations were reported highly accurately in Experiment 4.2. We suggest that redundancy masking, the reduction of the number of perceived items in repeating patterns, occurs with highly complex, socially relevant stimuli and that RM is a key mechanism for compressing redundant visual information.

Keywords: redundancy masking, crowding, spatial vision

Introduction

Humans are capable of swiftly processing faces (Reddy et al., 2006; Ro et al., 2001) and facial information, including gaze direction (Chen & Yeh, 2012), gender (Chen & Yeh, 2012), and emotion (Yang et al., 2007). It is generally accepted that humans possess a remarkable ability to identify faces (Besson et al., 2017; Boucart et al., 2016; Carey et al., 1992; Carey & Diamond, 1977; Crouzet et al., 2010; Tanaka & Gauthier, 1997). For example, Besson et al. (2017) found that accurately reporting whether a shortly presented face (100ms) belonged to a target person was remarkably fast, with reaction times as fast as 260 ms. In another face categorization task, participants needed to find a human face among other objects. Besson et al. (2017) showed that participants could perform the task correctly and with reaction times as fast as 240 ms. Recent research conducted by Crouzet et al. (2010) revealed that saccades towards faces could be executed within 100 ms (on average 147 ms) in a two-alternative forced-choice task. Research showed that making a decision and a motor response (e.g., bringing the hand close to a target object) took approximately 110 ms (Kalaska & Crammond, 1992), while processing a complex natural image (e.g., deciding whether a natural image was previously seen) required 150 ms (Thorpe et al., 1996). Comparatively, the processing of faces by human observers is highly remarkable.

Studies that investigate face detection are usually focused on the perception of single faces, although faces are often perceived in groups. How the perception of faces is modulated when they are presented in groups has been investigated, for example, in experiments using visual crowding paradigms (Fischer & Whitney, 2011; Louie et al., 2007; Westheimer, 1975). In crowding, visual items are usually presented in the periphery, and performance on a target surrounded by flankers is measured. Crowding has been shown for a large range of stimuli, such as letters (Bouma, 1970; Bouma, 1973; Grainger et al., 2016; Pelli et al., 2004; Winsler et al., 2022), Gabor patches (Livne & Sagi, 2007, 2011; Parkes et al., 2001) and verniers (Levi et al., 1985; Manassi et al., 2012, 2013; Sayim et al., 2008). Importantly, there is some evidence that crowding occurs on multiple processing levels (Whitney & Levi, 2011), including between complex stimuli such as faces. For example, Fischer and Whitney (2011) presented two groups of faces, each with one central face and six flanking faces to the right and left visual fields of participants, and asked them to compare the facial expressions of the two central faces. Results showed that the performance significantly deteriorated when the two target faces were surrounded by flankers compared to the performance without any flankers. This is a typical crowding effect: Flankers deteriorate performance on a target. Crowding of faces has also been shown when discriminating emotion (To et al., 2019), identity (Louie et al., 2007), when categorizing (Sun & Balas, 2015), and with Mooney faces (Farzin et al., 2009).

Importantly, crowding is usually assumed not to influence detection (Levi et al., 2002; Livne & Sagi, 2007; Pelli et al., 2004; but see, Allard & Cavanagh, 2011). However, recent studies demonstrated that there is a loss of information in crowding (Coates et al., 2017; Sayim & Wagemans, 2017) akin to failures of detection. For example, when asking participants to draw peripherally presented crowded letters, observes often underreport the number of elements (‘omission’ errors) or their size (‘truncation’ errors; Sayim & Wagemans, 2017). Such errors in crowding may well be due to the recently discovered phenomenon of redundancy masking (RM), which has been proposed to underlie detection-like errors in peripheral vision (Sayim & Taylor, 2019; Yildirim et al., 2020). In RM, the number of perceived items in repeating patterns is lower than the presented number for as few as three presented items (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021, 2022). For example, when three radially arranged letters T were presented in the visual periphery, observers often reported only two letters, using a free verbal report and a drawing task (Sayim & Taylor, 2019). RM has been shown for simple stimuli, such as lines (Yildirim et al., 2020, 2021, 2022) and dots (Hansmann-Roth et al., 2021), and more complex stimuli, such as letters (Sayim & Taylor, 2019). Recent findings suggest that RM goes hand in hand with the compression of visual space (Yildirim et al., 2022). Further characteristics of RM include its dependence on stimulus regularity (with high regularity yielding strong RM, Rummens & Sayim, 2022; Yildirim et al., 2020) and spacing (decreasing RM with increasing spacing; Yildirim et al., 2020). RM was also shown to be dependent on the spatial layout of the stimuli (Yildirim et al., 2020, 2022). For example, RM occurred more often when items were arranged radially compared to tangentially, i.e., RM is subject to a radial-tangential anisotropy. RM was found to have atypical visual field asymmetries; e.g., it was stronger on the horizontal meridian compared to the vertical meridian (Yildirim et al., 2020, 2022). Taken together, RM demonstrates a substantial failure to detect parts of a stimulus, often failing to detect one-third of the presented items when observers report only two out of the three presented items.

Here, we investigated if the detection of faces was impacted when they were presented in small groups of 3-6 faces. In Experiment 4.1, we presented multiple identical faces, luminance-matched noise patches, and shape-matched outlines in the visual periphery. Participants reported the number of perceived items. In Experiment 4.2, both upright and upside-down faces were presented. Participants reported the face orientation and the number of faces they perceived on a trial basis. Given the usual high performance for detecting faces, it is expected that face detection in small groups is intact and overcomes RM. However, we found that the detection of faces was impaired: participants reported fewer faces than the actual number of faces for both upright and upside-down faces, although the effect was less strong compared to the low-level matching noise patches and outlines. Importantly, performance was good in the face orientation discrimination task, both when the correct and erroneous numbers of faces were reported, indicating that sufficient features of faces (at least for orientation discrimination) were preserved in RM. Our results showed a substantial failure to detect faces presented in groups. It seems that the visual system's sensitivity to detect the presence of highly relevant stimuli does not hold for the number of exemplars. Instead, the visual system appears to be insensitive when detecting the number of faces in small groups.

Experiment 4.1: Enumeration task

In Experiment 4.1, we tested RM with multi-featured, complex stimuli: human faces. Two control stimuli: luminance-matched noise patches and shape-matched outlines, were also presented.

Method

Participants

Thirteen participants (1 male, 12 females; mean age: 19.5 years, ranging from 18 to 21) participated in Experiment 4.1. All participants were naive as to the purpose of the study. All participants were undergraduate psychology students at KU Leuven. They received course credits for their participation. All participants reported normal or corrected-to-normal visual acuity and signed informed consent prior to the experiment.

Apparatus and stimuli

The experiment was programmed in PsychoPy builder (Peirce et al., 2019) and ran on a Desktop PC (refresh rate: 60hz). The experiment was conducted in a dimly lit room. Participants viewed the monitor from a distance of 57 cm with a chinrest. A fixation that was comprised of a black fixation dot (diameter = 0.14°, luminance: < 0.5 cd/m2) and two concentric circles (diameter = 0.32° and 0.40°) was presented at the center of the monitor throughout the experiment. Three types of stimuli were used: faces, noise patches, and face outlines. All stimuli were gray-scaled (luminance: ~34 cd/m^2) and were presented on a white background (luminance: ~104 cd/m2). There were three sizes of each type of stimulus (small: 0.7° width × 1.0° height; medium: 0.9° width × 1.3° height and large: 1.1° width × 1.6° height, as demonstrated in Figure 3.1a). The edge-to-edge spacing of adjacent items was uniform and varied across trials (small stimuli: 0, 0.2°, 0.4°; medium stimuli: 0, 0.2°, 0.5°; large stimuli: 0, 0.2°, 0.6°). We also included a spacing for each stimulus size to match the width of the stimuli arrays (to match the width for set sizes 3-5, another set of stimuli with varying spacing, set sizes 3-6, were used, illustrated in Figure 3.1b). Hence, the matched spacing varied across stimulus sizes and set sizes. This is to prevent participants from taking the width of the array of items as a cue to estimate the number of items in the enumeration task (see below). The stimuli were randomly presented in the right or the left visual field, centered at an eccentricity of 10°. The first three panels of Figure 3.1c show sample trials with faces, outlines, and noise patches, respectively.

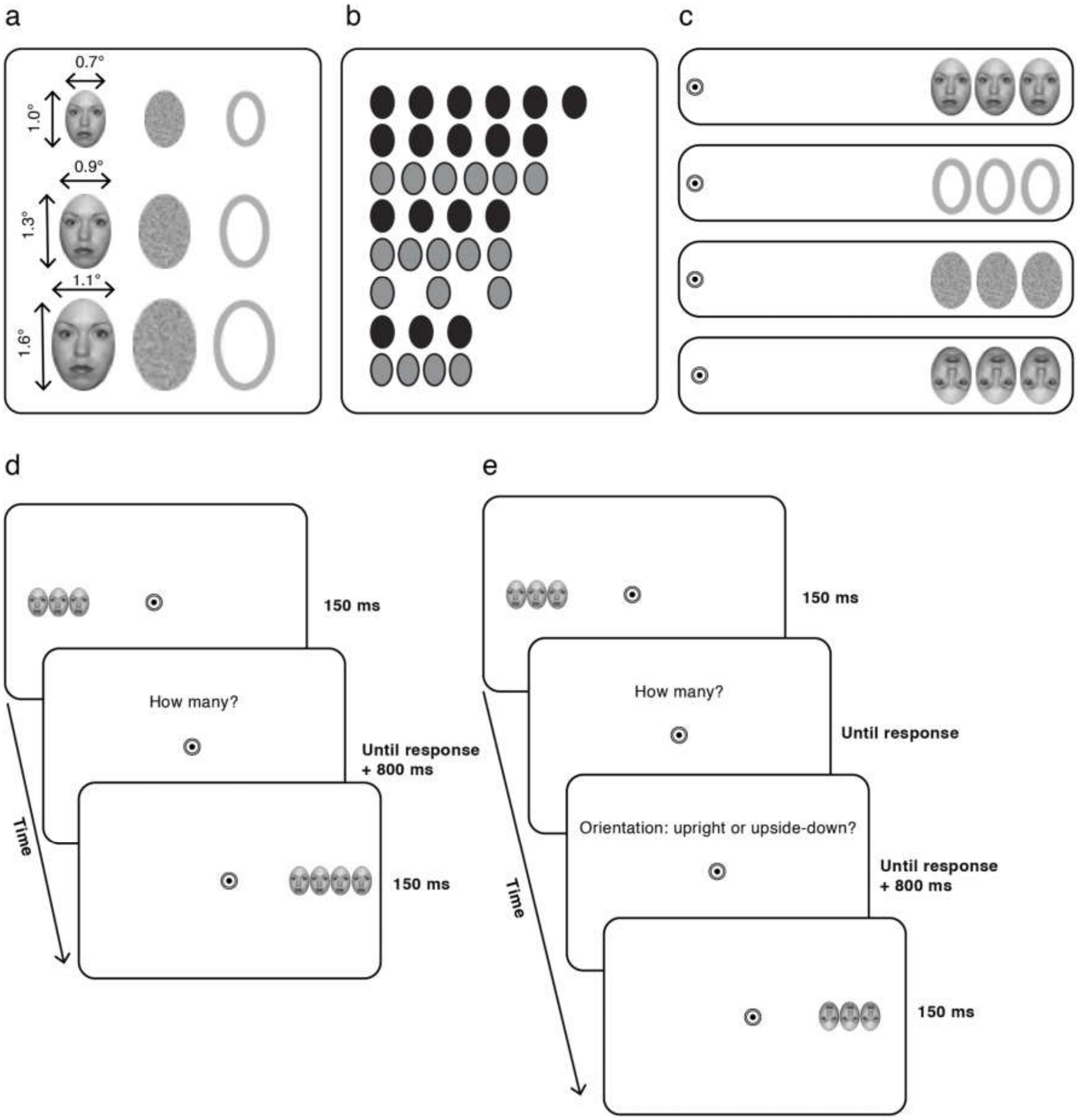

Figure 4.1. (a) Illustration of the stimuli. Each stimulus type had 3 sizes (small: 0.7° width × 1.0° height; medium: 0.9° width × 1.3° height and large: 1.1° width × 1.6° height). (b) Illustration of matching the spacing. Black placeholders indicate the stimuli with 'original' spacings, and gray placeholders indicate the matching stimuli. (c) Illustration of RM with faces, outlines, and noise patches (Experiment 4.1) and with upside-down faces (Experiment 4.2). (d) Schematic depiction of the experiment procedure for Experiment 4.1. (e) Schematic depiction of the experiment procedure for Experiment 4.2 (stimuli and background are not scaled to the actual sizes in the experiments).

Design and procedure

Each trial started with a fixation dot presented at the center of the screen. Three to six items were presented for 150 ms to the left or the right of fixation. Participants were required to indicate the number of items they perceived with a key press on the numeral keypad. Responses from 0-9 were allowed. The stimuli location (left or right visual field), the number of stimuli (3 - 6), and the four edge-to-edge spacings were randomized within each block. Stimulus types (faces, noise patches, and outlines) varied across blocks. Participants performed 9 blocks (3 blocks of each stimulus) in total. There were 288 trials per block. Experiment 4.1 is a 3 (set size: 3-6) × 3 (stimulus type: face, noise patch, outline) × 3 (stimulus size: small, medium, small) within-subject design. A schematic depiction of Experiment 4.1 is shown in Figure 4.1d. We calculated the deviation score (DV) using the reported number minus the actual number of presented stimuli (Yildirim et al., 2020). Therefore, negative DVs represent underestimation where redundancy masking occurs. DV magnitude represents the strength of RM: the more negative the DV, the stronger RM. The precision of the response was measured by the coefficient of variation (CV) using the standard deviation of the responses divided by the actual set size. CV is a classical psychophysical parameter in numerosity perception that allows cross-numerical comparison of average performance, and higher CV represents more sensory noise, therefore, less precise estimation.

After the experiment, a two-stimulus discrimination task for each stimulus type was performed. This was to ensure that the stimuli were above the observers' visual resolution limit. In the discrimination task, either 1 item or 2 items were presented at the farthest eccentricities of the main experiment (13.2˚, 14.1˚, and 14.9˚ for small, medium, and large stimuli, respectively). Stimulus types were blocked as in the main experiment. Participants were asked to indicate whether they perceived 1 or 2 items. Each participant performed 36 trials in a block that contained 6 repetitions per condition (108 trials in total). Performance was equal to or above 93.2% correct in all conditions.

Data analysis

All analyses were conducted in R (v 3.6.3), RStudio (R Core Team, 2020) and Python on a local laptop. The data was preprocessed with the Python Pandas tool (https://pandas.pydata.org/) and tidied up with "tidyverse" package (Wickham, 2017) in R. We conducted a 3-way within-subjects repeated measures ANOVA on both DVs and CVs with set size (3 – 6), stimulus size (small, medium and large), and stimulus type (face, noise patch, and outline) as within-subject factors. In the analysis, as planned, we collapsed over spacing as spacing showed no effect of either DV or CV. The ANOVA and the follow-up comparisons were performed using "rstatix" package (Alboukadel Kassambara, 2020). Normality assumption was checked with Shapiro-Wilk tests for each combination of factor levels. The DV was normally distributed in all factor levels except for the factor combination of outline, set size 3 and large stimuli (p = .02). Therefore, we assumed the normal distribution of the DV for all the other factor combinations (except for the combination of outline, set size 3 and large stimuli) as assessed by Shapiro-Wilk test of normality (ps > .05). The CV was normally distributed in all factor levels except for the factor combination of noise patch, set size 3 and small stimuli (p = .03) and the factor combination of face, set size 3 and middle stimuli (p = .04). Thus, we assume that the distributions of the CV for all the other factor combinations were normally distributed (Shapiro-Wilk test showed ps > .05). The data violated the sphericity assumption, and thus we reported the Greenhouse-Geisser sphericity corrections. The code of data analysis is available at https://github.com/miaoli-psy/RM_face/tree/main/src.

Results

Deviation score (DV)

Figure 4.2a shows deviation scores (DVs) for each condition. The average DV (mean ± SD) for outlines, noise patches, and faces were -0.84 ± 0.47, -0.78 ± 0.49, and -0.60 ± 0.56, respectively. A three-way within-subjects repeat measures ANOVA on DVs revealed a significant main effect of stimulus type (F(1.29, 14.08) = 7.55, p < .05, η^2 =.41). Pairwise comparison for stimulus type with Holm corrections showed that the DVs for noise patches were significantly lower than the DVs for faces (t(429) = 3.05, p < .001). DVs for outlines were significantly lower than the DVs for faces (t(429) = 3.98, p < .0001). There was no significant DV difference between noise patches and outlines (t(429) =0.93, p = .35). These results showed that RM for faces was less strong compared to noise patches and outlines. We observed a significant two-way interaction between set size and stimulus size (F(6, 66) = 2.41, p < .05, η^2 = .18). No other significant main, two-way interaction or three-way interaction effect was observed

(ps > .05).

Coefficient of variation (CV)

Figure 4.2b shows the coefficient of variation (CV) for each condition. The average CV (means ± SD) for outlines, noise patches, and faces were 0.15 ± 0.07, 0.15 ± 0.07, and 0.17 ± 0.08, respectively. A 3 -way within-subject repeat measure ANOVA on CV revealed that all three main effects were significant: stimulus type: $F(2, 22) = 5.31$, $p < .05$, $\eta^2 = .31$; stimulus size: $F(2, 22) = 5.73$, $p < .05$, $\eta^2 = .34$; set size: $F(1.51, 16.66) = 4.04$, $p < .05$, $\eta^2 = .27$. Pairwise comparison with Holm corrections for stimulus type showed that the CVs for noise patches, outlines, and faces were comparable (ps > .05). Pairwise comparisons with Holm corrections for set size showed that the CVs of set size 3 were significantly higher compared to set size 5 ($t(428) = 2.78$, $p < .05$), and set size 6 ($t(428) = 4.49$, $p < .0001$), and the CV of set size 4 was significantly higher than set size 6 ($t(428) = 3.08$, $p < .05$). We did not observe any significant two-way or three-way interaction effect (ps > .05).

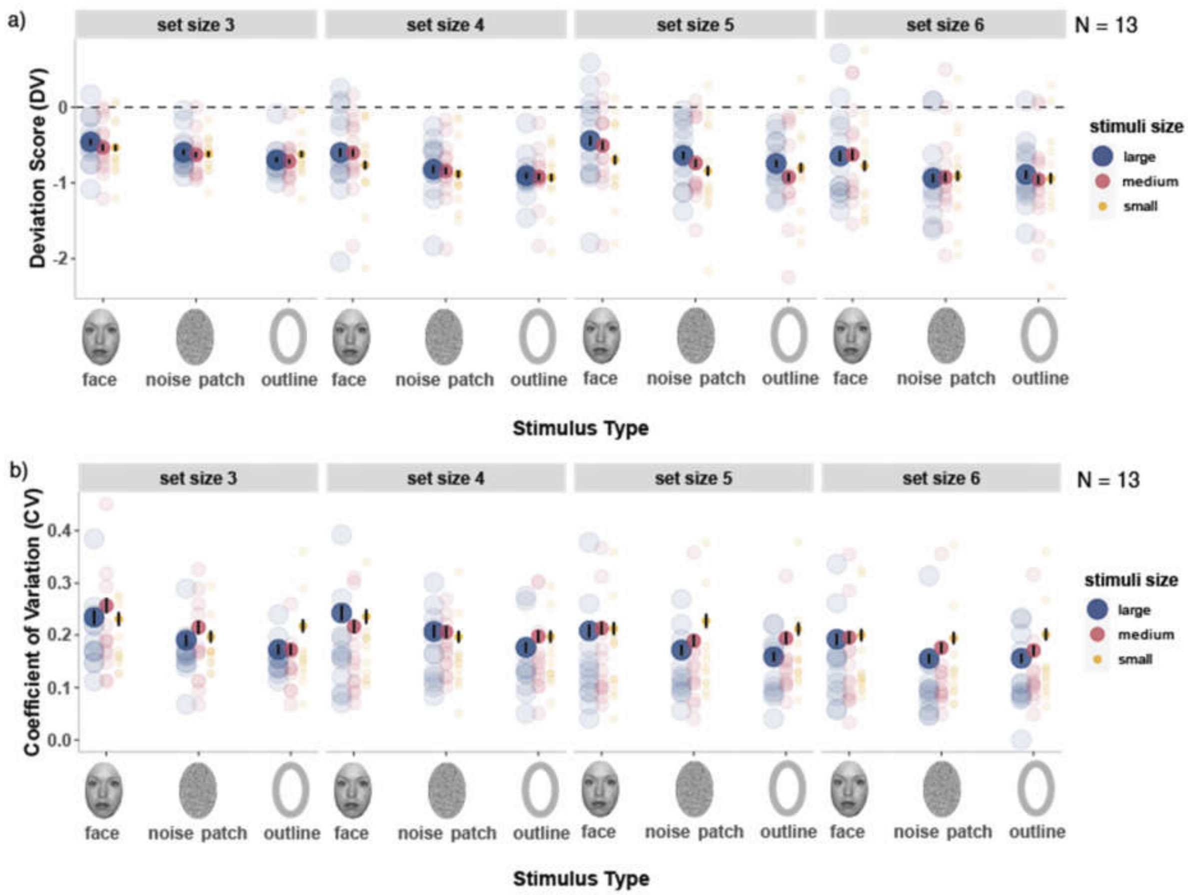

Figure 4.2. Results of Experiment 4.1. Deviation scores (a) and Coefficients of variation (b) for large, medium, and small stimuli, separated for each set size (3-6). Dark data points represent the group average, and light data points represent individual data. Error bars represent standard errors of the mean.

Experiment 4.2: Enumeration and orientation task

In Experiment 4.1, we found redundancy masking for faces, luminance-matched noise patches, and shape-matched outlines. However, what information was extracted from the faces is unclear. In Experiment 4.2, we presented upright and upside-down faces. There were two tasks: Reporting the number of faces as in Experiment 4.1, and additionally, the orientation of the faces (upright or upside-down). The orientation task was used to investigate whether sufficient information was extracted from the faces to perform this task.

Method

Participants

Twelve participants (3 males, 9 females; mean age: 19.6 years, ranging from 18 to 23) who did not participate in Experiment 4.1 participated in Experiment 4.2. All participants were naive as to the purpose of the study. All participants were undergraduate psychology students at KU Leuven. They received course credits after their participation. All participants reported normal or corrected-to-normal visual acuity and signed informed consent prior to the experiment.

Apparatus and stimuli

The apparatus was identical to Experiment 4.1. In Experiment 4.2, only the face stimuli from Experiment 4.1 were used (no noise patch or outline). The first and last panels of Figure 4.1c show the upright and upside-down faces. The face size and spacing manipulations were identical to those in Experiment 4.1.

Design and procedure

The procedure of Experiment 4.2 was identical to Experiment 4.1 except for the following changes: (1) only face stimuli were presented, (2) faces were presented in two possible orientations (upright or upside-down), and (3) participants performed two tasks (indicate the number of faces and the orientation of faces). The order of the two tasks was counterbalanced between subjects. Participants used the numerical keypad (0 - 9) to perform the number task and used "z" and "x" (on a Dutch keyboard layout) to perform the orientation task. Participants performed 6 blocks (3 blocks with each stimulus type 3) of 192 randomly presented trials. Experiment 4.2 is a 4 (set size: 3-6) × 3 (stimulus size: small, medium, small) × 2 (face orientation: upright, upside-down) within-subject design. A schematic depiction of Experiment 4.2 is shown in Figure 4.1e.

After the experiment, a two-stimulus discrimination task for both upright faces and upside-down faces was performed. The procedure of the discrimination task was identical to the discrimination task in Experiment 4.1, except for that stimuli were either upright or upside-down faces. Participants were asked to indicate whether they perceived 1 or 2 items. Each participant performed 144 trials in total with 12 repetitions per condition (2 set sizes × 2 orientations × 3 face sizes). Performance was equal to or above 93.4% correct in all conditions.

Data analysis

Number task

The analysis was identical to Experiment 4.1 except for the following changes: the three within-subject factors are face orientation, set size, and face size. Normality assumption was checked with Shapiro-Wilk tests for each combination of factor levels. The DV was normally distributed for all factor levels except for the factor combinations, including set size 3, the combination of upright face, set size 4, small size, the combination of upside-down face, set size 4, small size, and the combination of upright face, set size 5, medium size (ps < .05). All the other 18 factor combinations were normally distributed. The CV was normally distributed in all factor levels except for the following factor combinations: upright face, set size 4, large size, the combination of upside-down face, set size 5, large size, the combination of upright face, set size 5, medium size, the combination upright face, set size 6, medium size, and upside-down face, set size 6, small size (ps < .05). Thus, we assume that the distributions of the CV for all the other 19 factor combinations were normally distributed (Shapiro-Wilk test showed ps > .05). The sphericity assumption of the data is met.

Orientation task.

We used signal detection theory to determine the sensitivity of orientation discrimination. We defined upright faces reported as upright as “hits”, upright faces reported as upside-down as “misses”, upside-down faces reported as upside-down as “correct rejections”, and upside-down faces reported as upright as “false alarms”.

Sensitivity (d') was calculated using the z-transforms of the hit rate minus the z-transforms of the false alarm rate. We compared trials when redundancy masking occurred (reported a smaller number of faces than presented), with trials when the number of faces was correctly reported. After excluding trials that were overestimated (7.4%), a three-way within-subject repeat measure ANOVA on sensitivity with set size (3 – 6), face size (small, medium, and large), and trial type (RM or correct) as within-subject factors. Normality assumption was checked with Shapiro- Wilk tests for each combination of factor levels. All the factor combinations were normally distributed (ps > .05). The sphericity assumption of the data is satisfied.

Results

Number task

Figure 5a shows the deviation score (DV) as a function of the set size separated for each stimulus size. A 3-way within-subjects repeated measures ANOVA on DV revealed a significant main effect of face size ($F(2, 22) = 12.40$, $p < .05$, $\eta^2 = .53$). The DVs (± SD) for large, medium, and small faces are -0.54 ± 0.47, -0.59 ± 0.48 and -0.69 ± 0.49. We observed a significant interaction between set size and stimulus size ($F(6, 66) = 2.91$, $p < .05$, $\eta^2 = .21$). Pairwise comparison for stimulus size with Holm corrections showed that the DV for set size 3 was significantly lower than set size 4 when stimulus size was small ($p < .05$) but not when stimulus size was medium or large. The results showed that RM for faces was strong when three faces were presented, particularly with small face sizes. We did not observe a significant main effect on face orientation ($F(1, 11) = 0.22$, $p = .65$, $\eta^2 = .02$), showing the DV for upright and upside-down faces were comparable. The ANOVA did not reveal other significant two-way interactions or three-way interactions (ps > .05).

Coefficient of variation (CV)

Figure 4.3b shows the coefficient of variation (CV) as a function of set size, plotting separately for each stimulus size. A 3-way within-subject repeat measure ANOVA on CV revealed that there was a significant main effect of set size ($F(3, 33) = 33.25$, $p < .0001$, $\eta^2 = .75$). The CVs (± SD) were 0.19 ± 0.05, 0.16 ± 0.05, 0.14 ± 0.03 and 0.12 ± 0.04 for set size 3, 4, 5, and 6, respectively. We observed a significant main effect of face size ($F(2, 22) = 4.63$, $p < .05$, $\eta^2 = .30$). The CVs (± SD) were 0.16 ± 0.05, 0.15 ± 0.05 and 0.15 ± 0.05 for the small, medium and large face size, respectively. The CV results revealed that when three faces were presented, the detection of all faces was weakened, particularly when the face size was small. There was no significant main effect of face orientation ($F(1, 11) = 0.01$, $p = .94$, $\eta^2 < .01$). No other significant two-way or three-way interactions as observed.

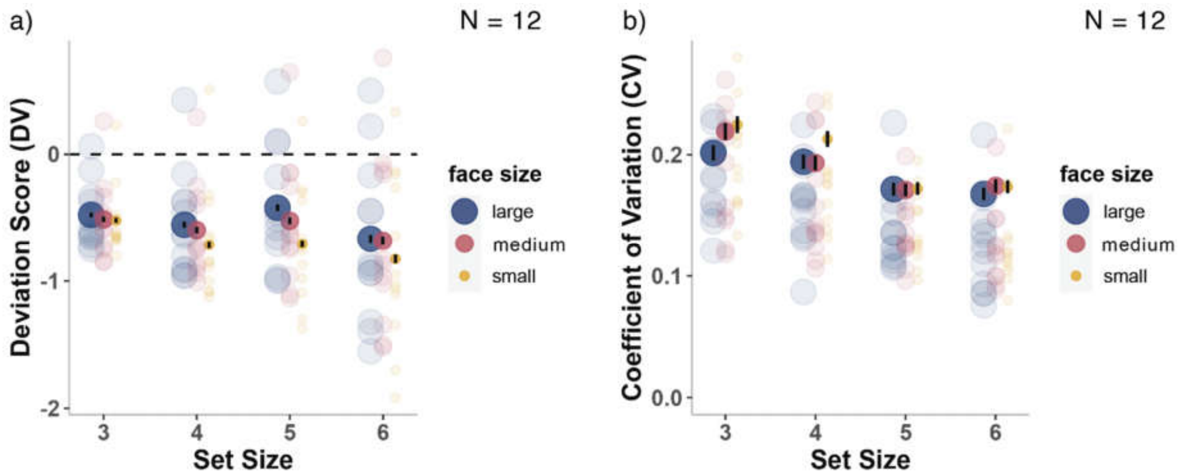

Figure 4.3. Results of Experiment 4.2 number task. Deviation score (a) and coefficient of variation (b) as a function of set size. Dark data points represent the group average, and light data points represent individual data. Error bars represent the standard error of the means.

Orientation task

Figure 4.4 shows the sensitivity (d') against the set size for large, medium, and small faces for both RM trials (59.8%) and correct trials (32.8%). A three-way within-subjects repeated measures ANOVA on sensitivity revealed a significant interaction effect between set size and trial type ($F(3, 21) = 3.56$, $p < .05$, $\eta^2 = .33$). We did not observe any other main effects, two-way interactions, or three-way interactions. Pairwise comparisons with Holm correction showed that the sensitivity for small faces in correct response trials (1.68 ± 1.03) was slightly higher compared to RM trials (1.29 ± 0.87; $p = .0495$). The sensitivity for large and medium faces in correct response trials and RM trials was comparable (ps > .05).

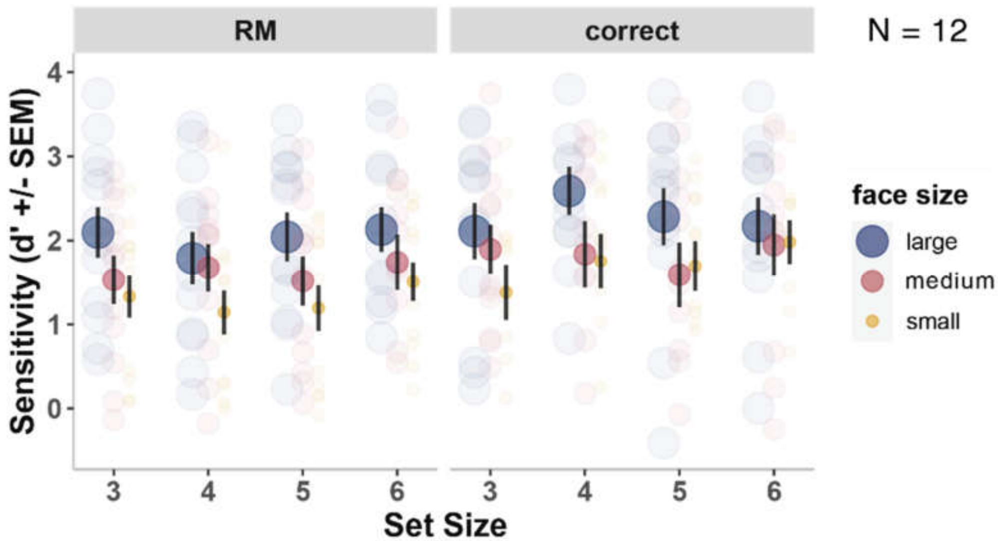

Figure 4.4. Results of Experiment 4.2 orientation task. Sensitivity as a function of set size. Dark data points represent the group average, and light data points represent individual data. Error bars represent the standard error of the mean.

Discussion

Faces are of great importance in social contexts and are usually detected quickly (Reddy et al., 2006; Ro et al., 2001). They are essential for identifying individuals, perceiving emotional states, and evaluating social situations. Despite the high importance of faces, in the current study, we report a substantial inability to detect faces. We examined the ability to detect the presence of faces in small groups in the visual periphery. Usually, very few features are necessary to report the presence of a target face. Here, when reporting fewer faces, in particular, two instead of three presented faces, it seems that none of these features was perceived to a sufficient extent to make participants report the additional face(s). Previous studies showed that human adults have a remarkable ability to detect faces even when some of the facial features were absent (Moscovitch et al., 1997), and when the perception of individual features of faces were transformed to Mooney face (Kanwisher et al., 1998). In peripheral vision, results demonstrated that sparse depictions of a few facial features (eyes, nose, and mouth) were recognized as a face (Brown et al., 1997): Simple features on faces, without a face outline, were sufficient for observers to correctly report the presence of a face. In general, the criterion to report the presence of a face is liberal, as minimal information is required to report that a face is “present”. However, it remains unclear how faces in groups of faces are detected, particularly in the visual periphery. In Experiment 4.1, identical faces and two types of control stimuli (noise patches and outlines) were used. We found that, when performing an enumeration task, observers reported fewer faces than the actual number of faces presented, showing that redundancy masking (RM) occurred with faces. Experiment 4.2 replicated the main finding of Experiment 4.1. Participants were required to report both the number and the orientation of faces. Our results showed that the detection of faces was inaccurate: RM with faces occurred as soon as there were three faces presented in the periphery, and observers frequently failed to detect at least one of the presented faces regardless of the face orientations. Our results demonstrated a massive failure in detecting faces in the visual periphery. As faces convey a wide range of socially relevant characteristics, it is surprising that there is a significant failure in detecting faces in small groups.

Many visual tasks showed a deterioration of performance with increasing eccentricity (Gurnsey et al., 2011; Valsecchi et al., 2013; Wolford & Hollingsworth, 1974). A prominent example of this decline in performance is crowding, the deleterious effect when presenting items in clutter (Bouma, 1970; Herzog et al., 2015; Levi, 2008; Pelli et al., 2004; Pelli & Tillman, 2008; Strasburger, 2020). Crowding has been suggested to be a fundamental limit of our periphery vision (Levi, 2008). Crowding is usually assumed to affect target identification but not detection (Livne & Sagi, 2007; Pelli et al., 2004). In RM, by contrast, items are lost, which points to detection errors. RM also differs from crowding in several other characteristics. For example, evidence showed that the upper-lower visual field asymmetry that was found for crowding (Fortenbaugh et al., 2015; Greenwood et al., 2017; He et al., 1996) did not occur with RM (Yildirim et al., 2022). And the horizontal-vertical asymmetry occurs for crowding but not for RM (Greenwood et al., 2017; Yildirim et al., 2022). The failure to detect faces in the present study seems to be due to RM (Yildirim et al., 2020, 2022).

While RM was strong with faces, it was even stronger with the control stimuli (luminance-matched noise patches and shape-matched outlines), and – in other studies with different observers – simple stimuli such as lines (Yildirim et al., 2020). Previous studies have found that RM is more likely to occur with simple stimuli such as lines or the letter "I" and "T" which have relatively low stimulus complexly (Yildirim et al., in preparation). This suggests that the complexity of the stimuli may be an important factor in determining the likelihood of RM and may contribute to the relatively weaker RM in faces compared to noise patches and outlines.

The observed detection errors with faces are different from the established understanding of the visual system's proficiency in processing small numerical information. Particularly, people are fast and accurate in enumerating small sets of visual items (1-4), known as subitizing (Jensen et al., 1950; Trick & Pylyshyn, 1994). When the number of items exceeds four, the performance of enumeration drops dramatically (Jensen et al., 1950). Importantly, studies tested the ability to subitize usually presented items in central vision, where the subitizing range is more accurate than in peripheral vision. Recently, the subitizing capacity in the periphery was estimated to be significantly lower: Presenting 1-6 tiny lines (1˚ height and 0.25˚ width) concentrically in the visual periphery showed that the capacity of fast and errorless enumerating the number of items can be limited to just two items (Chakravarthi et al., 2022, see also, Chakravarthi & Herbert, 2019; Chakravarthi et al., 2022). One hypothesis of subitizing is that the spatial arrangement of items forms familiar shapes (e.g., two dots as a line and three dots as a triangle), and the process of these shapes involves pattern recognition (Mandler & Shebo, 1982). This is further supported by evidence that participants enumerate the dots that formed a regular shape faster than the ones that formed an irregular shape (Wender & Rothkegel, 2000). RM seems to be a failure of object individuation with a small number of items where additional shape information is not helpful.

In Experiment 4.2, participants were able to correctly discriminate the face orientations, both when RM occurred and when it did not occur. This suggests that the extraction of facial features was sufficient to at least discriminate between the orientations of faces. It could be argued that the high performance in the face orientation task would also be obtained if observers only attended to the innermost face, and attention to the innermost face with an increase of RM (as not the entire array was attended). In this case, better performance in orientation discrimination would have been expected for RM than non-RM trials. However, the observed results show that this was not the case.

Overall, we found RM with complex, multi-features objects: human faces. The results of the current study indicate a substantial failure in detecting faces in groups, with observers frequently failing to detect at least one of the presented faces. We provided new insights into the limitation of humans' ability to detect faces in small groups. Our results underscore the need for future research to investigate the mechanisms of RM and its potential implications in situations where the capability to accurately perceive and process faces is critical.

CHAPTER 5: GENERAL DISCUSSION

In the current thesis, we investigated numerosity perception in the visual periphery by conducting multiple psychophysical experiments. The research objectives were to determine the role of crowding and redundancy masking in numerosity perception by manipulating the spatial arrangement of items on displays. Crowding is the deterioration effect on target identification when the target is flanked by other objects (Bouma, 1970; Manassi et al., 2012; Pelli et al., 2004; Strasburger, 2020; Toet & Levi, 1992). Recently, a crowding-like phenomenon r, termed redundancy masking -- the perceived number of multiple identical items is largely reduced -- has been reported (Sayim & Taylor, 2019; Yildirim et al., 2020, 2021, 2022). For example, when three identical letters T were aligned and presented in the visual periphery, participants often reported that only two Ts were perceived (Sayim & Taylor, 2019). The set of experiments was designed to provide a deeper understanding of the processes involved in peripheral numerosity perception and the factors that influence it. The findings of these studies contribute to the current understanding of the mechanisms of numerosity perception.

In Experiment 2.1, we employed the radial-tangential anisotropy observed in crowding and redundancy masking. Specifically, in crowding, radially placed flankers interfere with the target perception more than tangentially arranged flankers (Kooi et al., 1994; Toet & Levi, 1992). In redundancy masking, the reduction of reporting the number of items occurred when items were arranged radially but not tangentially. We created displays consisting of discs that were arranged either predominantly radially or tangentially. Through this manipulation, we aimed to induce interference among discs in the radial displays and to induce no (or at least reduced) interference in the tangential displays. Participants were presented with displays of discs (between 21 – 58) and asked to report the estimates of the number of discs presented (Experiment 2.1). The results indicated that numerosity estimates were lower for the radial displays compared to the tangential displays. These results showed that numerosity perception is subject to a radial-tangential anisotropy. In the next experiment (Experiment 2.2), we aim to investigate whether the numerosity estimation differences between the radial and tangential conditions were due to perceived grouping differences. Participants who did not perform Experiment 2.1 were presented with the same set of displays as in

Experiment 2.1. They were asked to encircle discs that were perceived as a group. If the number of perceived groups in radial displays is fewer than in tangential displays, we can anticipate that the perceived number of groups has an impact on the numerosity estimation. The results from Experiment 2.2 provided insight into whether grouping among the discs plays a role in the effect, i.e., the relative underestimation in the radial compared to the tangential condition observed in Experiment 2.1. However, the number of perceived groups in the radial displays was higher compared to the tangential displays, showing an opposite pattern from the numerosity estimation task. This result indicates that grouping is not the factor that confounds the numerosity estimation results. Our results from the two experiments indicated that numerosity perception may vary depending on the spatial arrangements of the items (either radial or tangential). Crowding and redundancy masking, the limits of our spatial vision that are subject to a radial-tangential anisotropy, may impact numerosity perception.

Results from a new set of four experiments provided further evidence that crowding and redundancy masking may be related to numerosity perception (Chapter 3). A new set of displays were created into varied degrees of radial-tangential arrangements, both weak and strong interference conditions. Displays contained base and flanking discs. Specifically, displays used in Experiment 3.1 and Experiment 3.2 were manipulated in a weak and strong manner, respectively. In the weak manipulation (Experiment 3.1), base discs, on average, had one flanking disc that was placed to either interfere (radial condition) or not interfere (tangential condition) with the base disc. In the strong manipulation (Experiment 3.3.), two flanking discs (instead of one) were again placed to interfere or to not interfere with the base disc to form radial and tangential conditions, respectively. Three close-by discs (one base and two flanking discs) in radial arrangements are highly similar to typical stimuli that yield redundancy masking. Importantly, the phenomenon of redundancy masking is generally not observed when only one or two items are presented - three items seem to be the minimum to obtain redundancy masking (Yildirim et al., 2021). The results were consistent with those reported in Chapter 2, where participants perceived the displays comprising of radially arranged discs to be fewer in number compared to displays comprising of tangentially arranged discs. Observers frequently reported perceiving three presented lines as two in redundancy masking paradigms (Yildirim et al., 2020, 2021). Previous research provided clear evidence that the spatial arrangement of items impacts redundancy masking: redundancy masking was strong when items were arranged in the radial direction and weak when items were arranged in the tangential direction (Yildirim et al., 2020). Therefore, the observed relative underestimation in radial displays compared to tangential displays in the strong manipulation in Experiment 2 may well be due to redundancy masking.

Next, the contrast polarity of items on displays was varied (Experiments 3.3 and 3.4). Mixed contrast polarity has been shown to reduce crowding (Chung & Mansfield, 2009; Sayim et al., 2008). For example, in crowding, a higher similarity between the target and flankers usually results in a stronger crowding effect (Chakravarthi & Cavanagh, 2007; Chung et al., 1998; Kooi et al., 1994; Rosen & Pelli, 2015; Rummens & Sayim, 2019b, 2021; Sayim et al., 2008). Therefore, the mixed contrast polarity of the items was thought to disrupt the visual structures that could induce by grouping among the items on uniform contrast polarity displays. Thus, the perceived structural difference between the radial and the tangential displays with mixed contrast polarity was small compared to uniform contrast polarity displays. However, estimation results showed that radial displays with mixed contrast polarity were still perceived as fewer compared to tangential displays with mixed contrast polarity. The results with mixed contrast polarity displays mirrored those of the uniform contrast polarity displays, indicating that the mixed contrast polarity had no effect on the radial-tangential differences.

Experiments described in Chapters 2 and 3 demonstrated that numerosity estimation is subject to a radial-tangential anisotropy. Therefore, the topology of our spatial vision seems to be relevant in numerosity perception. The relative estimation difference between radial and tangential displays was consistent and stable (e.g., not impacted by how strong the radial-tangential arrangement was nor by uniform/mixed contrast polarity). Nevertheless, mixed contrast polarity may not always reduce crowding, especially when asking observers to make a full report of the peripherally presented stimuli (Rummens & Sayim, 2021). In their study, Rummens and Sayim (2021) presented three letters to participants' visual periphery and asked them to either report the central letter or all presented letters. The letters' contrast polarity was either uniform or mixed. They observed that performance in the mixed contrast polarity condition was better in the single report task than in the full report task. They concluded that the uniformity of stimuli enhances the perception of crowded objects. Our numerosity estimation tasks (Experiment 2.1, Experiments 3.1-3.4) required participants to attend to and report all discs. Determined by the characteristics of the task instructions, our tasks are similar to a full report task, as participants need to attend to the entire display to report the number of items.

Displays of composite discs were generally used in our numerosity estimation experiments (Experiments 2.1, 3.1-3.4). These displays were all meaningless patches. Two additional experiments were conducted to investigate redundancy masking with a typical redundancy masking paradigm (Chapter 4). Here, we presented grey-scaled human face stimuli instead of discs in the visual periphery. Humans have the capability to rapidly detect faces (Reddy et al., 2006; Ro et al., 2001), even when a single face is presented in the visual periphery, although the detection is deleterious with increasing eccentricity (Farzin et al., 2009). Correctly identifying faces is crucial to us. However, in a typical redundancy masking paradigm, we observed a considerable failure to detect faces appearing in groups. We reported that redundancy masking occurred with multi-featured faces: people often miss one or more faces with identical faces that were aligned and presented in the periphery (Experiment 4.1). Despite redundancy masking of faces, observers were still able to accurately discern the orientation information from them (Experiment 4.2), demonstrating that despite the omitted face(s) in redundancy masking, feature extraction (at least to the degree that allowed discriminating between upright and upside-down faces) was intact.

In the current thesis, we examined the effects of crowding and redundancy masking on numerosity perception. One of the major differences between crowding and redundancy masking is that crowding is assumed to deteriorate target identification but not target detection, whereas redundancy masking affects target detection as one or more items were missed (Levi et al., 2002; Pelli et al., 2004; Taylor & Sayim, 2020) but see (Allard & Cavanagh, 2011). Our results indicated that crowding and redundancy masking could significantly modulate numerosity perception. The results also provided insights into the underlying mechanisms of numerosity perception. Importantly, we showed that radial-tangential anisotropy has a significant impact on numerosity perception. These findings provided evidence for the role of visual field asymmetries in shaping numerosity perception and suggested that the topology of spatial vision plays a crucial role in numerosity perception. In order to further comprehend how crowding and redundancy masking modulate numerosity perception, it is crucial not to solely focus on the physical attributes of the stimulus (e.g., the displays used in numerosity studies) but also to incorporate characteristics of the visual system, e.g., visual field asymmetries, within the context of spatial vision.

Several limitations exist in our studies. One such limitation is the use of a similar algorithm for display generation in Experiment 2.1 and Experiments 3.1-3.4. Although the algorithm ensures the radial and tangential displays differ in terms of the radial-tangential arrangement, it results in displays that were only available in limited numerosity ranges. In order to preserve the minimum differences in physical properties between the radial and tangential displays, the selecting displays required limiting the numerosities available, leading to a trade-off where both too-small and too-large numerosities were sacrificed and could not be tested. A potential solution to address the limitations of our study is to split the existing displays. Particularly, the condition where redundancy masking is expected to have a high probability of occurring (Experiment 3.2) could be split into several equal parts while preserving the locations of the discs. In this way, displays are retained with the original radial-tangential manipulation while exploring the effects of redundancy masking on numerosity estimation over a smaller range of numerosities. Another limitation of our experiments is the use of only one type of face in Experiments 4.1 and 4.2, where face redundancy masking was investigated. This limited our ability to determine the extraction of other local features, such as eyes and mouth, as well as global information, such as emotion and identity, during redundancy masking. Thus, our results can only provide insight into the extent to which the orientation discrimination of faces is preserved in redundancy masking. Future studies can examine the retention or suppression of other features and information in redundancy masking.

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APPENDIX

Supplementary Table S2.1

A summary of physical properties for the radial and the tangential displays across all numerosity ranges.

	Numerosity range 21-25		Numerosity range 31-35		Numerosity range 41-45		Numerosity range 49-53		Numerosity range 54-58	
	Tan*(SD)*	Rad*(SD)*	Tan*(SD)*	Rad*(SD)*	Tan*(SD)*	Rad*(SD)*	Tan*(SD)*	Rad*(SD)*	Tan*(SD)*	Rad*(SD)*
Average spacing (°)	6.80*(0.07)*	6.74*(0.06)*	7.93*(0.07)*	7.91*(0.05)*	9.16*(0.09)*	9.16*(0.08)*	10.41*(0.09)*	10.37*(0.09)*	11.44*(0.09)*	11.38*(0.09)*
Convex hull (°)	35.47(*1.04)*	36.57(*0.98)*	48.28(*0.78)*	48.49*(0.67)*	60.93*(0.98)*	61.49*(0.96)*	73.78*(1.12)*	74.29*(0.96)*	84.43*(1.39)*	85.38*(0.98)*
Average eccentricity (°)	5.07*(0.05)*	5.03(0.05)	5.88*(0.05)*	5.86*(0.04)*	6.71*(0.07)*	6.72*(0.06)*	7.54*(0.07)*	7.54*(0.06)*	8.21*(0.06)*	8.20*(0.07)*
Occupancy area (Convex hull 2D volume)	88.83*(3.60)*	90.81*(3.22)*	157.45*(4.04)*	156.30*(4.05)*	249.79*(4.69)*	251.38*(5.06)*	367.54*(6.48)*	368.95*(6.79)*	482.35*(10.79)*	485.45*(8.17)*
Density (item/deg^2)	0.54*(0.02)*	0.52*(0.01)*	0.31*(0.01)*	0.30*(0.01)*	0.21*(0.01)*	0.21*(0.01)*	0.16*(<0.01)*	0.16*(<0.01)*	0.13*(<0.01)*	0.13*(<0.01)*

Note. Tan: Tangential displays; Rad: Radial displays. SD: Standard deviation. Convex hull and occupancy area were computed using the Qhull library (Barber et al., 1996) with Python. Density was calculated using the numerosity divided by occupancy area, excluding the empty central region (46.28 deg^2).

Supplementary Table S2.2

Averaged radial alignment scores (RAs) for each numerosity range

Numerosity range	Radial (*SD*)	Tangential (*SD*)
21-25	0.075(*0.255*)	0 (*0*)
31-35	0.466 (*0.532*)	0 (*0*)
41-45	1.378 (*0.808*)	0.049 (*0.165*)
49-53	2.939 (*1.080*)	0.525 (*0.461*)
54-58	3.378 (*1.207*)	1.447 (*0.914*)

Supplementary Table S2.3

Partial correlations (partial r_1 and $CI_1$95%) between deviation scores (DVs)and radial alignment scores (RAs) controlling for numerosity and partial correlations (partial r_2 and $CI_1$95%) between DVs and crowding strength controlling for numerosity

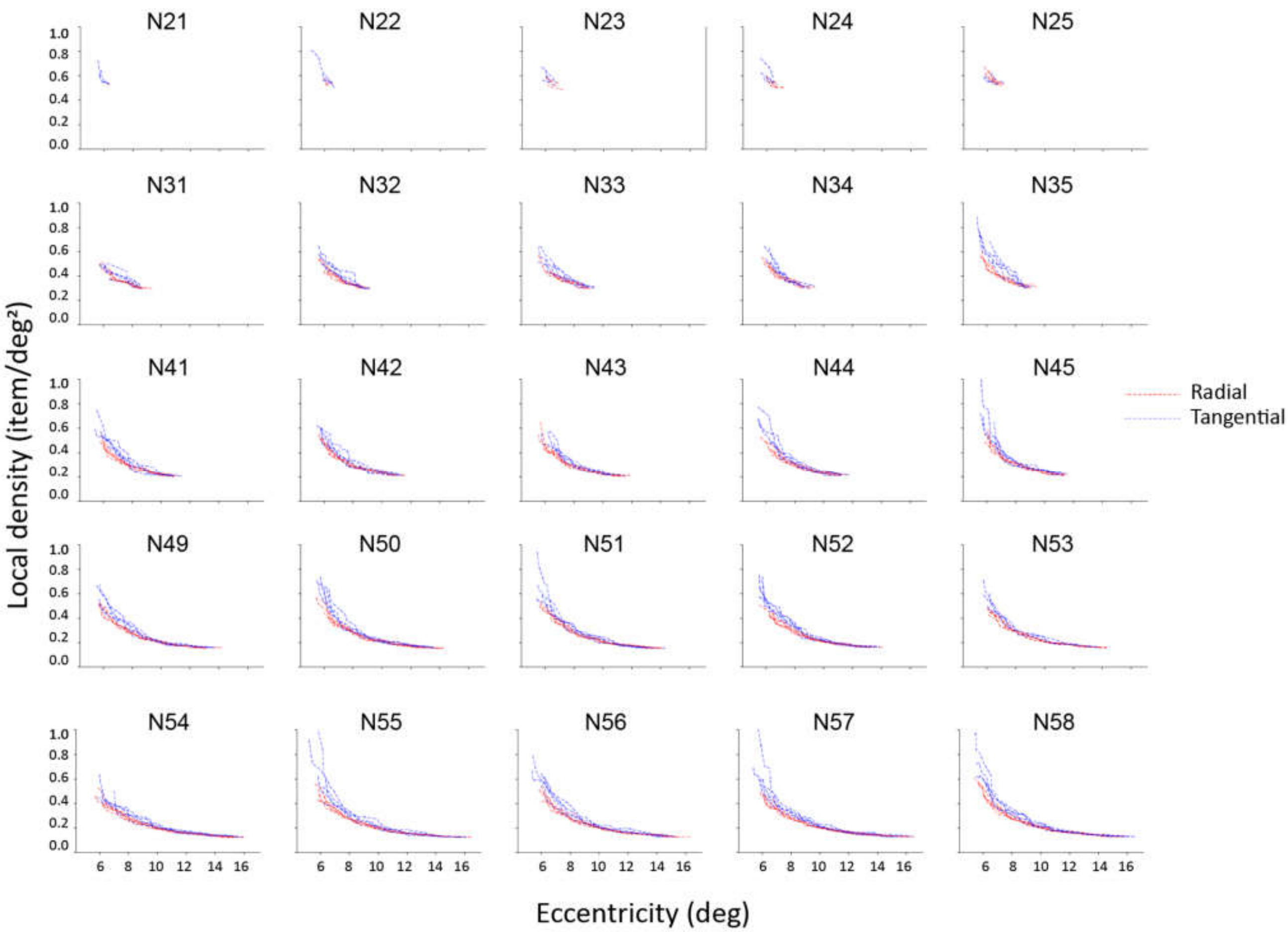
Numerosity range	partial r_1	$CI_1$95%	partial r_2	$CI_2$95%
21-25	0.10	[-0.19 - 0.36]	-0.17	[-0.43 - 0.12]
31-35	-0.23	[-0.48 - 0.05]	-0.49***	[-0.68 - 0.25]
41-45	-0.31*	[-0.54 - 0.03]	-0.31*	[-0.54 - 0.03]
49-53	-0.52***	[-0.7 - 0.28]	-0.44**	[-0.64 - 0.18]
54-58	-0.50***	[-0.68 - 0.25]	-0.52***	[-0.7 - 0.28]
all	-0.40****	[-0.5 - 0.29]	-0.40****	[-0.5 - 0.29]

Note. *$p < .05$. **$p < .005$. ***$p < .001$. ****$p < .0001$. In addition to circle sectors of 6°, we varied the size of the sectors from 1° to 12°, following the same method as described above in Method. Too small and too large angles were expected to yield weaker (or no) correlations with RAs as alignments would be rare (when angles were very small) or counted when far beyond plausible interference zones (when angles were large). The results showed that this was the case, with overall higher correlations for medium angle sizes (from about 5° to 9°).

Supplementary Table S2.4

Descriptive Statistics: means and standard deviations of perceived groups in the tangential and the radial condition for each numerosity range

Numerosity range	Alignment condition	Mean (*SD*)
21-25	Tangential	6.13(*2.50*)
	Radial	7.37(*2.66*)
31-35	Tangential	9.3(*3.66*)
	Radial	10.7(*4.72*)
41-45	Tangential	12.0(*4.74*)
	Radial	13.6(*5.72*)
49-53	Tangential	13.9(*5.95*)
	Radial	15.1(*6.91*)
54-58	Tangential	15.7(*7.80*)
	Radial	18.0(*6.48*)



Supplementary Figure S2.1. Local density as a function of eccentricity. Local density was measured using the number of discs of displays (that fall into the local convex hull region) divided by occupancy area, excluding the empty central region. Each curve represents the local density for a single display.

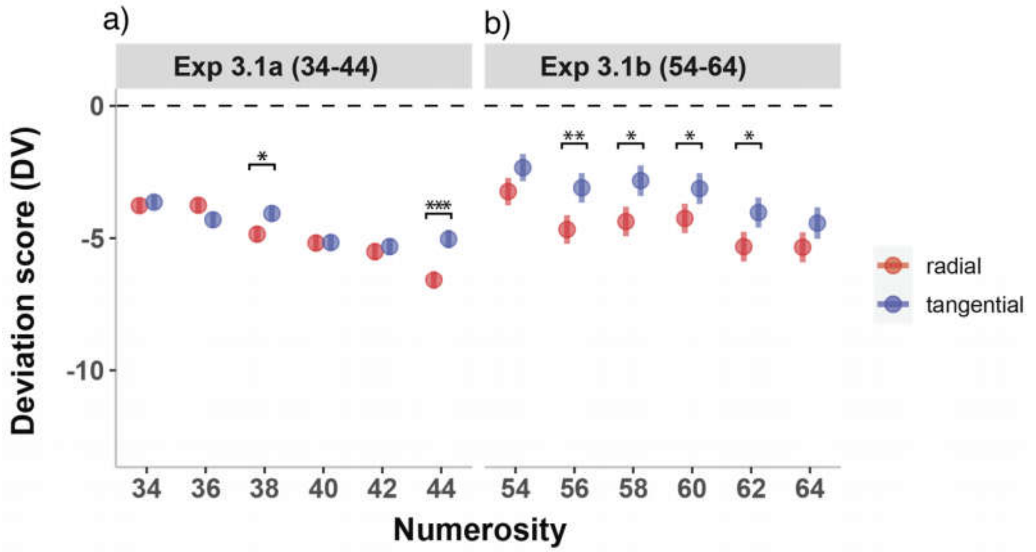
Supplementary Table S3.1

A summary of physical properties for the radial and tangential displays across tested numerosity ranges.

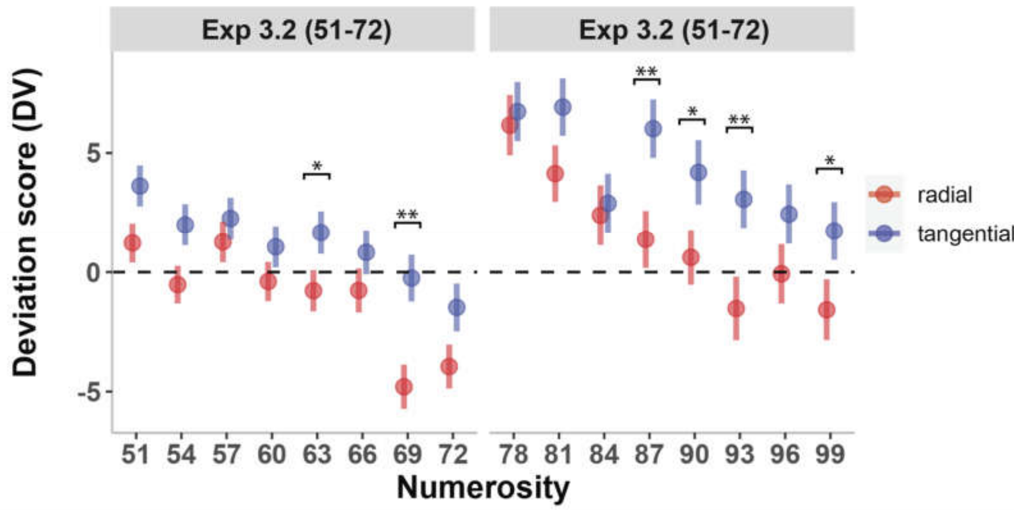
Note. Tan: Tangential displays; Rad: Radial displays. SD: Standard deviation. Convex hull and occupancy area were computed using the Qhull library (Barber et al., 1996) with Python. Density was calculated using the numerosity divided by occupancy area, excluding thee fovea zone

	Experiment 2 displays				Experiment 4 displays			
	Numerosity (51-72)		Numerosity (78-99)		Numerosity (34-44)		Numerosity (54-64)	
	Tan*(SD)*	Rad*(SD)*	Tan*(SD)*	Rad*(SD)*	Tan*(SD)*	Rad*(SD)*	Tan*(SD)*	Rad*(SD)*
Convex hull (°))	51.77 *(1.84)*	52.95*(2.11)*	78.27*(2.73)*	79.74*(2.17)*	49.95*(2.35)*	50.66*(2.57)*	76.29*(2.43)*	77.33*(2.94)*
Occupancy area(Convex hull 2D volume)	186.16*(12.06)*	186.28*(12.70)*	431.60*(19.01)*	430.34*(25.81)*	170.38*(15.03)*	170.46*(15.04)*	400.72*(23.33)*	400.20*(23.51)*
Density (item/deg^2)	0.38*(0.02)*	0.38*(0.02)*	0.22*(0.01)*	0.22*(0.01)*	0.27*(<0.01)*	0.27*(<0.01)*	0.16*(<0.01)*	0.16*(<0.01)*

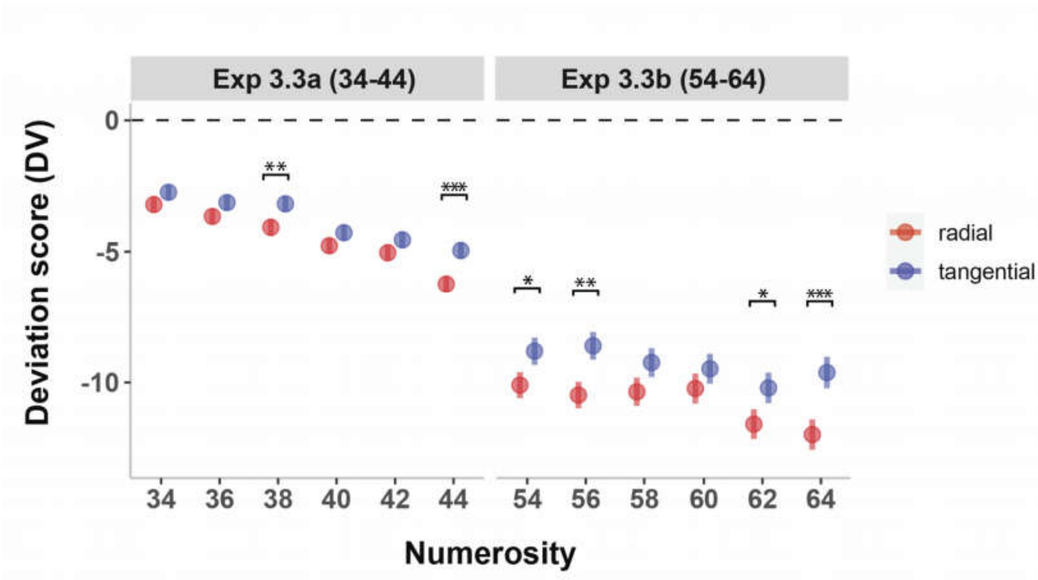
where no disc were presented(24.3 deg^2). Note that display properties of Experiment 4 displays also account for Experiment 1 and 3 (online) if participants correctly follow the experiment instructions (i.e., using a 24-inch monitor and sitting 45 cm away from the screen.)



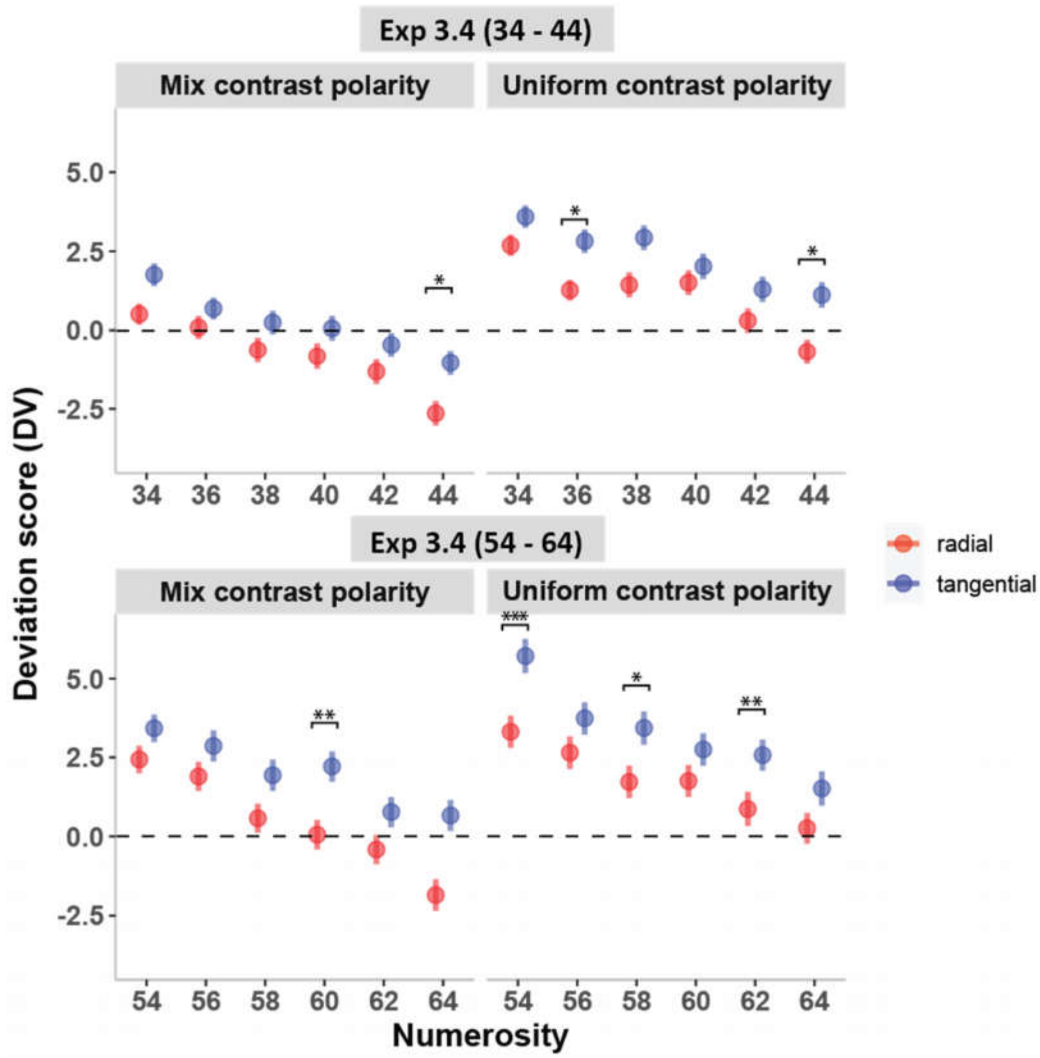
Supplementary Figure S3.1. (a) Results for Experiment 3.1a. (b) Results of Experiment 3.1b. Deviation score (DV) as a function of numerosity for the radial and the tangential conditions. DVs of 0 represent correct estimates, negative DVs underestimations, and positive DVs overestimations. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks. *$p < .05$, **$p < .01$, ***$p < .001$



Supplementary Figure S3.2. Results for Experiment 3.2. Deviation score (DV) as a function of numerosity. DVs of 0 represent correct estimates, negative DVs underestimations, and positive DVs overestimations. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks.*p < .05, **p < .01, ***p < .001



Supplementary Figure S3.3. (a) Results for Experiment 3.3a. (b) Results of Experiment 3.3b. Deviation score (DV) as a function of numerosity for the radial and the tangential conditions. DVs of 0 represent correct estimates, negative DVs underestimations, and positive DVs overestimations. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks. .*$p < .05$, **$p < .01$, ***$p < .001$



Supplementary Figure S3.4. Results for Experiment 3.4. Deviation score (DV) as a function of numerosity. DVs of 0 represent correct estimates, negative DVs underestimations, and positive DVs overestimations. Error bars indicate (+/- 1) standard errors of the mean. Significant post-hoc pairwise comparisons on the full model are indicated with asterisks. **p* < .05, ***p* < .01, ****p* < .001

RÉSUMÉ SUBSTANTIEL

L'être humain est capable d'estimer visuellement le nombre d'objets sans avoir à les compter, c'est un processus connu sous le nom de perception de la numérosité. Par exemple, lorsque nous nous trouvons dans une salle bondée, nous pouvons rapidement estimer le nombre approximatif de personnes sans avoir à les compter. Bien que l'estimation ne soit pas précise, l'estimation du nombre d'éléments dans un ensemble donné est connue sous le nom de perception de la numérosité. La capacité à discerner la numérosité présente un avantage évolutif, car elle permet de choisir une zone où la quantité de nourriture est plus importante et de déterminer quel groupe a moins d'adversaires (Gómez-Laplaza & Gerlai, 2011 ; McComb et al., 1994 ; Nieder, 2018 ; Wilson et al., 2001).

Il est souvent suggéré que la numérosité, comme d'autres caractéristiques primaires des objets telles que l'orientation, la couleur, la taille, etc., est une autre caractéristique primaire des objets (Ross & Burr, 2010). Il a été suggéré que notre capacité à traiter la numérosité ou à estimer des quantités était innée dans notre cerveau visuel et qu'elle était pilotée par un système de numérosité approximative (également connu sous le nom de "sens du nombre", Anobile et al, 2014 ; Burr et al., 2017 ; Chen & Verguts, 2013 ; Dehaene, 1992 ; Dehaene & Changeux, 1993 ; Dehaene et al., 1998 ; Feigenson et al., 2004 ; Halberda & Feigenson, 2008 ; Lipton & Spelke, 2003 ; Stoianov & Zorzi, 2012 ; Xu et al., 2005). La notion de sens du nombre suggère que la perception de la numérosité est traitée spontanément et ne dépend pas d'autres propriétés physiques (par exemple, la taille, la coque convexe, la densité, etc., Castaldi et al., 2021 ; Cicchini et al., 2016, 2019). Certains éléments sont venus étayer cette idée. Par exemple, Izard et al. (2009) ont montré que les nouveau-nés associaient spontanément des affichages visuels contenant un nombre différent d'éléments (4 - 12) à des événements auditifs sur la base des nombres, ce qui démontre que la capacité d'abstraire des informations sur les nombres est innée et apparaît dès le début de la vie (voir également de Hevia et al., 2017). Dans le domaine visuel, l'adaptation est évidente dans la perception des couleurs (qui peut différer de manière significative en fonction de la couleur vue précédemment, Webster, 2011), de l'orientation (qui peut être modifiée après avoir vu des lignes inclinées, Gibson & Radner, 1937), et du mouvement (où la perception d'objets stationnaires peut être modifiée après avoir vu des objets en mouvement, Nashner, 1982). L'une des principales indications que le SNA est un système inné est sa

sensibilité à l'adaptation (Mollon, 1974 ; Thompson & Burr, 2009). Burr et Ross (2008) ont montré que le nombre perçu d'éléments dans des affichages visualisés après adaptation changeait radicalement dans la direction opposée des affichages adaptés ; c'est-à-dire qu'après avoir visualisé un affichage dense (peu dense), l'affichage suivant semblait être moins (plus) nombreux.

Cependant, le point de vue de l'ANS selon lequel la perception de la numérosité est innée a été remis en question. L'un des arguments avancés est que la numérosité covarie avec de nombreuses autres propriétés physiques non numériques. Par exemple, pour une taille fixe de chaque objet exposé, la surface totale augmente à mesure que la numérosité augmente. La coque convexe (la plus petite forme convexe qui contient tous les éléments d'un ensemble) présente également une corrélation positive avec la numérosité. À taille égale, un plus grand nombre d'objets exposés donne un étalage plus dense qu'un étalage comportant moins d'objets. Il est impossible de créer deux présentoirs avec un nombre différent d'objets tout en conservant les autres propriétés physiques non numériques (Leibovich & Ansari, 2016). Par conséquent, il semble impossible de faire abstraction de la seule numérosité, indépendamment des autres propriétés physiques co-variées d'un affichage (Gebuis & Reynvoet, 2012a, 2012b, 2012c). Plusieurs études ont montré que la perception de la numérosité est influencée par d'autres propriétés physiques des écrans (Allik & Tuulmets, 1991 ; Sophian & Chu, 2008). Par exemple, Clearfield et Mix (2001) ont montré que les enfants réagissaient à la longueur du contour des écrans plutôt qu'à la numérosité. Ginsburg et Nicholls (1988) ont démontré que la numérosité perçue est en corrélation négative avec la taille de l'objet (voir également Tokita & Ishiguchi, 2010, cf. Allik et al., 1991 ; Hurewitz et al., 2006). Il a été observé que la zone d'occupation (zone globale occupée par les éléments sur les présentoirs), qui est étroitement liée à la taille des éléments, et la coque convexe ont un effet sur la perception de la numérosité (Binet, 1890 ; Gilmore et al., 2016 ; Katzin, 2018 ; Shilat et al., 2021 ; Taves, 1941 ; Vos et al., 1988). Par exemple, Gilmore et al. (2016) ont demandé à des participants d'ignorer soit l'information sur la surface du point, soit l'information sur la coque convexe lors d'une tâche de perception de la numérosité. Ils ont constaté que les participants étaient capables d'ignorer la surface du point, et que cette capacité s'améliorait avec l'âge. Cependant, il n'était pas facile d'ignorer les informations relatives à la coque convexe lors de tâches de comparaison de points, ce qui suggère le rôle crucial de la coque convexe dans la perception de la numérosité. Le modèle d'occupation part du principe que la numérosité perçue dans un ensemble aléatoire d'éléments relativement clairsemés est liée à la zone occupée par tous les éléments, qui est déterminée par la taille des éléments et leur rayon d'influence fixe. (Allik & Tuulmets, 1991). Allik et Tuulmets (1991) ont proposé que la zone occupée collectivement par les éléments sur les présentoirs, plutôt que le nombre d'éléments en soi, détermine la numérosité perçue. Ce modèle explique bien la sous-estimation observée dans de nombreuses études sur la numérosité. En particulier, lorsque les éléments sont placés à proximité les uns des autres, les zones occupées se chevauchent et sont donc perçues comme moins nombreuses. Lorsque d'autres propriétés physiques des présentoirs (par exemple, la surface totale, la taille, la coque convexe) étaient manipulées pour être congruentes ou incongrues avec le nombre, le jugement de la numérosité était affecté (Gebuis & Reynvoet, 2012b, 2012c ; Hurewitz et al., 2006). Par exemple, Hurewitz et al. (2006) ont présenté des affichages où la numérosité et la taille des points étaient manipulées pour être congruentes ou incongrues. Dans la condition congruente, les affichages contiennent plus de points composés de gros points ou d'une grande surface totale, tandis que dans la condition incongrue, les affichages contiennent plus de points composés de petits points ou d'une petite surface totale. Ils ont observé que les participants commettaient plus d'erreurs et étaient plus lents dans une tâche de comparaison de la numérosité dans la condition incongrue que dans la condition congruente.

Il est également suggéré que la perception de la numérosité est influencée par le regroupement (Bertamini et al., 2018 ; Bertamini et al., 2016 ; Chakravarthi & Bertamini, 2020 ; Frith & Frut, 1972 ; Sophian, 2007). Frith et Frut (1972) ont démontré pour la première fois que la perception de la numérosité est influencée par la façon dont les éléments sont disposés dans l'espace et qu'un grand groupe semble être plus nombreux que plusieurs petits groupes, ce que l'on appelle l'illusion du solitaire. Lorsque les éléments d'un affichage sont disposés en grappes, les affichages semblent moins nombreux. Un cas extrême est l'illusion de la numérosité aléatoire-régulière (Cousins & Ginsburg, 1983 ; Ginsburg, 1980) : les éléments disposés selon un schéma régulier (par exemple, aux intersections de la grille) sont jugés plus nombreux que les éléments disposés selon un schéma aléatoire, en grappes. L'une des explications est que, lorsque les éléments sont disposés dans une "bonne" Gestalt (par exemple, l'ensemble de la grappe centrale de l'illusion du solitaire), la numérosité perçue est affectée par cette unité d'ordre supérieur et semble plus importante que celle des éléments disposés dans une "mauvaise" Gestalt (par exemple, les quatre grappes de coins de l'illusion du solitaire, Frith & Frut, 1972). Néanmoins, une première étude sur l'impact du regroupement sur la perception de la numérosité a donné des résultats contradictoires (Taves, 1941). Taves (1941) a montré qu'un groupe de 20 objets disposés régulièrement était perçu comme moins nombreux qu'un groupe de 20 objets placés de manière irrégulière. Il a suggéré que les affichages présentant une "bonne" Gestalt ont moins d'effets distincts que le motif irrégulier sur la perception et que, par conséquent, le motif régulier semble être plus nombreux que le motif irrégulier. L'espacement entre les éléments et la régularité déterminent la proximité spatiale des affichages et se traduisent par différents niveaux de regroupement des affichages. Bertamini et al. (2016) ont d'abord utilisé différentes mesures des configurations structurelles des affichages (par exemple, la distribution, le regroupement local, l'ensemble convexe global, etc.) pour quantifier les éléments sur les affichages qui sont liés à la numérosité, au regroupement et à la dispersion. Bertamini et al. (2016) ont présenté des affichages qui contenaient toujours le même nombre d'éléments, mais qui variaient en termes de regroupement et de dispersion. Ils ont conclu que, quelle que soit la manière dont le regroupement était quantifié, l'augmentation du regroupement était liée à la diminution de la numérosité perçue (voir également Bertamini et al., 2018). Ces données suggèrent que le regroupement pourrait souligner la perception de la numérosité (Anobile et al., 2015 ; Chakravarthi & Bertamini, 2020).

Gebuis et al. (2016) ont proposé une explication plus complète selon laquelle il pourrait y avoir un système d'intégration sensorielle qui évalue les grandes numérosités approximatives en combinant les différents indices sensoriels qui constituent les stimuli des nombres. Ils suggèrent qu'une combinaison d'entrées sensorielles est utilisée pour créer une représentation unifiée de la numérosité. Le modèle suggère que les indices visuels saillants sont généralement fortement pondérés lors de la perception de la numérosité. Les prédictions du modèle d'intégration sensorielle sont en accord avec un certain nombre de résultats antérieurs, y compris l'effet de distance numérique (une diminution de la différence entre deux nombres est associée à une augmentation du temps de réaction, Piazza et al, 2004 ; Sasanguie et al, 2011), les effets de congruence variables (effets de congruence mis à l'échelle avec le nombre de repères visuels manipulés, Gebuis & Reynvoet, 2012b), et l'effet de congruence opposé (par exemple, les essais avec un plus grand nombre de petits points ont donné de meilleures performances que ceux avec un plus petit nombre de gros points, Ginsburg & Nicholls, 1988 ; Sophian, 2007). Fait important, Gebuis et Reynvoet (2012c) ont contrôlé les propriétés physiques non numériques des affichages de manière à ce que ces indices visuels soient manipulés de manière à ne pas être corrélés avec la numérosité. Ils ont constaté que les participants considéraient que les présentoirs étaient plus nombreux lorsque leur diamètre moyen, leur surface agrégée ou leur densité étaient plus petits, mais que leur coque convexe était plus grande. Ils ont suggéré que la perception de la numérosité s'effectue en pesant et en intégrant de multiples propriétés physiques non numériques des affichages (voir également Gebuis & Reynvoet, 2012b). La théorie de l'intégration sensorielle a montré l'importance des propriétés physiques des écrans dans la perception de la numérité et a remis en question l'existence du SNA. Dans une revue récente, Lourenco et Aulet (2022) ont proposé que la numérosité soit plus qu'une simple conséquence des autres grandeurs ; elle constitue sa propre dimension distincte qui n'est pas complètement séparée des autres grandeurs. Lourenco et Aulet (2022) ont suggéré un nouveau modèle de perception de la numérosité, dans lequel la perception de la magnitude non numérique est intégrée à la perception de la numérosité tout au long du processus de perception.

Numerosity and density are physically indivisible as density is calculated by dividing numerosity by the total area (Tibber et al., 2012). Burr and Ross (2008) demonstrated that numerosity is subject to adaptation and claimed that it is an independent visual property (from other visual properties, including density), further corroborated by Ross and Burr (2010). Anobile et al. (2014) found evidence that discrimination thresholds of high and low-density displays followed two distinct psychophysical functions, suggesting separate mechanisms for numerosity and density. However, Dakin et al. (2011) suggested that numerosity perception and density perception share a similar mechanism, and therefore, they cannot be clearly distinguished by the visual system (see also, Tibber et al., 2012). Many empirical studies support this idea. For example, Durgin (2008) claimed that the “adaptation on numerosity” described by Burr and Ross (2008) was actually based on texture density. Durgin (2008) presented two adapting displays: one contained more items than the other one, and the other’s texture was denser, allowing dissociation between numerosity and density during the adaptation. The results showed that greater adaptation was produced by the region of greater density instead of higher numerosity. Similarly, Dakin et al. (2011) showed that both numerosity and density were biased by item size, suggesting a common visual metric between numerosity and density. Numerosity studies sometimes even indicated that although the task was formulated in terms of numerosity, the results and conclusions were *equally* applied to both numerosity and density since they are not dissociable (e.g., Valsecchi et al., 2013). Nevertheless, Ross and Burr (2010) provided further evidence that numerosity perception is not dependent on the densities of displays. They presented three types of displays to participants: a constant numerosity, a constant area, and a constant density, where one of the three parameters was kept constant in the experiment. Participants made comparisons on numerosity and density in separate blocks. Their results showed that density did not play a role in numerosity judgment as the performance of the constant density condition was not worse compared to the other two conditions. In another experiment, Ross and Burr (2010) showed that the perceived numerosity but not the density was modulated by luminance. Hence, it is unclear whether density and numerosity are independent of each other. This poses certain difficulties for future research on numerosity perception, as we must consider whether density plays a role or to what extent density plays a role in perceived numerosity.

La numérologie et la densité sont physiquement indivisibles car la densité est calculée en divisant la numérologie par la surface totale (Tibber et al., 2012). Burr et Ross (2008) ont démontré que la numérosité est sujette à adaptation et ont affirmé qu'il s'agit d'une propriété visuelle indépendante (d'autres propriétés visuelles, y compris la densité), ce qui a été corroboré par Ross et Burr (2010). Anobile et al. (2014) ont trouvé des preuves que les seuils de discrimination des affichages à haute et à faible densité suivaient deux fonctions psychophysiques distinctes, suggérant des mécanismes séparés pour la numérologie et la densité. Cependant, Dakin et al. (2011) ont suggéré que la perception de la numérosité et la perception de la densité partagent un mécanisme similaire et que, par conséquent, elles ne peuvent pas être clairement distinguées par le système visuel (voir également Tibber et al., 2012). De nombreuses études empiriques soutiennent cette idée. Par exemple, Durgin (2008) a affirmé que l'"adaptation sur la numérologie" décrite par Burr et Ross (2008) était en fait basée sur la densité de la texture. Durgin (2008) a présenté deux écrans d'adaptation : l'un contenait plus d'éléments que l'autre, et la texture de l'autre était plus dense, ce qui permettait de dissocier la numérosité de la densité pendant l'adaptation. Les résultats ont montré qu'une plus grande adaptation était produite par la région de plus grande densité plutôt que par celle de plus grande numérosité. De même, Dakin et al. (2011) ont montré que la numérosité et la densité étaient toutes deux biaisées par la taille de l'objet, ce qui suggère l'existence d'une métrique visuelle commune entre la numérosité et la densité.

Les études sur la numérosité ont même parfois indiqué que, bien que la tâche ait été formulée en termes de numérosité, les résultats et les conclusions s'appliquaient également à la numérosité et à la densité, puisqu'elles ne sont pas dissociables (par exemple, Valsecchi et al., 2013). Néanmoins, Ross et Burr (2010) ont fourni d'autres preuves que la perception de la numérosité ne dépend pas de la densité des affichages. Ils ont présenté trois types d'affichage aux participants : une numérosité constante, une surface constante et une densité constante, l'un des trois paramètres étant maintenu constant au cours de l'expérience. Les participants ont effectué des comparaisons sur la numérosité et la densité dans des blocs séparés. Leurs résultats ont montré que la densité ne jouait pas un rôle dans le jugement de la numérosité, car les performances de la condition de densité constante n'étaient pas plus mauvaises que celles des deux autres conditions. Dans une autre expérience, Ross et Burr (2010) ont montré que la numerosité perçue, mais pas la densité, était modulée par la luminance. Il n'est donc pas certain que la densité et la numérologie soient indépendantes l'une de l'autre. Cela pose certaines difficultés pour les futures recherches sur la perception de la numérosité, car nous devons déterminer si la densité joue un rôle ou dans quelle mesure la densité joue un rôle dans la numérosité perçue.

Les études sur la perception de la numérosité impliquent généralement des affichages qui couvrent une partie importante du champ visuel, y compris la fovéa, la parafovéa et souvent la périphérie. Cependant, il existe des différences substantielles entre les différentes zones du champ visuel (Rosenholtz, 2016 ; Simpson, 2017). Par exemple, les performances visuelles diminuent avec l'augmentation de l'excentricité, c'est-à-dire que les performances sont généralement moins bonnes dans le champ visuel périphérique que dans le champ visuel central (Gurnsey et al., 2011 ; Levi & Waugh, 1994 ; Livne & Sagi, 2007 ; Meinecke & Donk, 2002 ; Wolford & Hollingsworth, 1974 ; Zahabi & Arguin, 2014). Des recherches antérieures ont également porté sur la perception de la numérosité en périphérie. Par exemple, Mengal et Matathia (1980) ont présenté de petites lumières LED vertes et rouges à des participants. Les participants devaient déterminer quelle couleur (verte ou rouge) était la plus importante. Les résultats ont montré que les performances diminuaient de la fovéa à la périphérie et que le temps de réaction augmentait avec l'excentricité. En raison du manque de recherches sur la façon dont l'excentricité module la perception de la numérosité (du moins avec des nombres relativement importants), Valsecchi et al. (2013) ont mené une expérience dans laquelle les participants ont effectué une tâche de comparaison de la numérosité.

Pour ce faire, deux écrans étaient présentés simultanément de part et d'autre du moniteur, et les participants étaient invités à regarder le centre de l'un des écrans, de sorte que l'autre écran apparaissait en périphérie. La tâche consistait à indiquer quels affichages semblaient être les plus nombreux. Les résultats ont montré que les affichages présentés en périphérie devaient contenir un plus grand nombre de points pour être jugés équivalents aux affichages regardés, ce qui indique que la numérosité est perçue comme moins importante en périphérie que dans la fovéa. Valsecchi et al. (2013) ont suggéré que l'encombrement visuel (voir section 1.5) est le mécanisme clé de la sous-estimation observée en périphérie.

Les entrées visuelles provenant de la fovéa et de la périphérie contribuent différemment à la perception de la numérosité (Cheyette & Piantadosi, 2019). Par exemple, Cheyette et Piantadosi (2019) ont révélé qu'une augmentation de la fovéation entraîne une augmentation de l'estimation de la numérosité. Ils ont émis l'hypothèse que les éléments de la vision fovéale ont deux fois plus d'influence sur l'estimation de la numérosité que ceux de la vision périphérique. Par conséquent, la prise en compte de la présentation des éléments dans différents champs visuels est essentielle pour la compréhension de la perception de la numérosité, car la vision périphérique est distincte de la fovéa. Il est essentiel d'explorer davantage les contraintes de la périphérie visuelle.

La vision spatiale est fortement limitée par l'encombrement visuel : l'incapacité de percevoir une cible dans des environnements encombrés (Bouma, 1970 ; Bouma, 1973 ; Levi, 2008 ; Pelli et al., 2004 ; Pelli & Tillman, 2008 ; Strasburger, 2020). Il a été proposé que l'encombrement visuel soit une limite fondamentale de la vision spatiale (Levi, 2008), et qu'il soit particulièrement fort dans la périphérie visuelle (Bouma, 1970 ; Bouma, 1973 ; He et al., 1996 ; Levi et al., 2002 ; Levi et al., 1985 ; Pelli et al., 2004). L'encombrement visuel dépend de l'espacement entre la cible et ses flancs (par exemple, les éléments qui entourent la cible) : une diminution de l'espacement entre la cible et ses flancs entraîne une augmentation de l'encombrement visuel (Bouma, 1970 ; Toet & Levi, 1992). Pour les cibles situées en périphérie, il existe une région d'interférence allongée où les flancs interfèrent avec la perception de la cible (Toet & Levi, 1992). Il a été démontré que les flankers placés en dehors de cette région ne gênent pas la perception de la cible (Toet & Levi, 1992). La similarité entre la cible et le flanker a un impact sur l'encombrement visuel : plus ils se ressemblent, plus l'encombrement visuel est important (Chakravarthi & Cavanagh, 2007 ; Chung & Mansfield, 2009 ; Kooi et al., 1994 ; Rummens & Sayim, 2019, 2021 ; Sayim et al., 2008 ; mais voir Rummens & Sayim, 2021).

Il a également été suggéré que l'encombrement visuel était un facteur contribuant à la perception de la numérosité, entraînant une sous-estimation (Anobile et al., 2015 ; Valsecchi et al., 2013). L'hypothèse de l'encombrement visuel dans la perception de la numérosité est corroborée par le fait que l'encombrement visuel et la perception de la numérosité sont tous deux modulés par l'excentricité. On observe une augmentation de l'encombrement visuel et une sous-estimation plus forte avec l'augmentation de l'excentricité. (Dakin et al., 2011 ; Toet & Levi, 1992 ; Valsecchi et al., 2013).

On pense généralement que l'encombrement visuel n'affecte que l'identification de la cible et non sa détection (Levi et al., 2002 ; Pelli et al., 2004 ; mais voir Allard & Cavanagh, 2011). Cependant, la sous-estimation observée de la perception de la numérosité implique que certaines erreurs de détection ont pu se produire. Chakravarthi et Bertamini (2020) ont manipulé la similarité des lettres cibles (similaires ou dissemblables) et l'espacement minimal entre les éléments (proches ou éloignés), dont il a été démontré qu'ils avaient un impact sur la perception de l'encombrement visuel et de la numérosité, et ont cherché à savoir si la similarité et l'espacement avaient un effet comparable sur la perception de la numérosité. Les résultats ont révélé que l'espacement et la similarité des éléments avaient des effets différents sur la tâche d'encombrement et sur la tâche de comparaison de la numérosité, ce qui démontre que l'encombrement visuel ne module pas la perception de la numérosité.

Des recherches récentes ont mis en avant un concept proche du crowding visuel : lorsque trois éléments identiques ou plus, tels que des lignes et des lettres, sont présentés en périphérie, les individus rapportent moins d'éléments que ceux présentés, ce qui est appelé le masquage de redondance (Sayim & Taylor, 2019 ; Yildirim et al., 2020, 2021, 2022). Le masquage de la redondance se produit dès que trois éléments sont présentés. Par exemple, lorsque trois lignes alignées radialement étaient présentées dans la périphérie visuelle, les participants indiquaient généralement qu'ils percevaient deux lignes (Yildirim et al., 2020, 2021). Par conséquent, le masquage de la redondance suggère une erreur de détection et peut donc être lié à la sous-estimation de la perception de la numérosité.

Les performances visuelles ont montré un large éventail de variations dans l'ensemble du champ visuel, révélé par plusieurs asymétries omniprésentes dans le champ visuel, notamment l'anisotropie horizontale-verticale (performances supérieures le long du méridien horizontal par rapport au méridien vertical à une excentricité fixe, Barbot et al, 2021 ; Carrasco et al, 1995 ; Carrasco et al., 2001 ; Corbett & Carrasco, 2011 ; Mackeben, 1999 ; Rovamo & Virsu, 1979), l'asymétrie verticale (meilleure performance dans le champ visuel inférieur par rapport au champ visuel supérieur, Barbot et al., 2021 ; Carrasco et al., 2001 ; Corbett & Carrasco, 2011 ; Rubin et al., 1996) dans une série de tâches visuelles telles que l'acuité visuelle, la discrimination de l'orientation. Dans les foules, les flankers placés radialement ont un effet plus prononcé sur la perception de la cible que les flankers placés tangentiellement, à distance égale entre la cible et le flanker (Kwon et al., 2014 ; Toet & Levi, 1992), ce que l'on appelle l'anisotropie radiale-tangentielle. Bien qu'il existe plusieurs distinctions entre le masquage de redondance et le crowding, tous deux présentent une anisotropie radiale-tangentielle évidente. Dans le cas du masquage de la redondance, la réduction du nombre d'éléments rapportés se produit lorsqu'ils sont disposés radialement mais pas tangentiellement. (Yildirim et al., 2020, 2022).

Il est surprenant que peu d'études aient étudié l'impact des asymétries du champ visuel sur la perception de la numérosité. Ce n'est que dans une étude récente, Chakravarthi et al. (2022) ont révélé qu'un petit nombre d'éléments peut produire une variété d'asymétries du champ visuel dans la perception de la numérosité en présentant 1 à 9 petits carrés à l'un des quatre emplacements (champ visuel supérieur, inférieur, gauche ou droit). Ils ont montré que les performances de numérosité étaient plus efficaces le long du méridien horizontal que le méridien vertical, dans le champ visuel inférieur que dans le champ visuel supérieur et sur le méridien horizontal gauche que le méridien horizontal droit. Les résultats ont mis en évidence l'influence potentielle des asymétries du champ visuel sur la perception de la numéroté.

Dans la présente thèse, nous visons à explorer comment l'encombrement visuel et le masquage de redondance modulent la perception de la numérosité avec une gamme relativement large de numérosités. Nous avons testé les estimations de numéroté (dans une fourchette comprise entre 21 et 58) avec des écrans dont l'interférence des disques était forte ou faible (Chapitre 2). Les disques sur les écrans étaient principalement disposés dans une direction radiale et tangentielle pour les conditions d'interférence forte et faible, respectivement (expérience 2.1). Nos résultats ont montré que les estimations étaient plus faibles dans les conditions d'interférence forte que dans les conditions d'interférence faible. Nous suggérons que la perception de la numérorité est une anisotropie radiale-tangentielle de la perception de la numéroté. Ensuite, nous avons demandé aux participants d'encercler les items perçus comme un groupe (Expérience 2.2). Les résultats ont indiqué que le nombre de groupes perçus était plus élevé dans la condition d'interférence faible par rapport à la condition d'interférence forte, montrant une tendance opposée avec la tâche d'estimation. Par conséquent, le regroupement des disques peut ne pas expliquer les résultats d'estimation de la numérosité observés selon lesquels les affichages radiaux étaient présentés comme moins nombreux que les affichages tangentiels. Ensuite, l'encombrement visuel, le masquage de redondance et l'anisotropie radiale-tangentielle ont été examinés plus en détail avec quatre expériences (chapitre 3). Des numéros compris entre 31 et 99 ont été testés. Nous avons observé que les affichages radiaux étaient signalés comme moins nombreux que les affichages tangentiels, que les arrangements radiaux-tangentiels des affichages soient faibles, forts ou modulés avec une polarité de contraste mixte. Nos résultats ont démontré que l'anisotropie radiale-tangentielle de la perception de la numéroté persiste dans toutes les conditions. Nous suggérons que l'encombrement visuel et le masquage de redondance modulent la perception de la numéroté. Ensuite, le masquage de redondance a été particulièrement testé dans un paradigme typique de masquage de redondance (Chapitre 4). Nous avons utilisé des visages humains comme stimuli dans deux expériences. Les erreurs de type détection dans le masquage de redondance dans les stimuli multi-fonctions (visages) et les stimuli de bas niveau (contours et patchs de bruit assortis à la luminance et à la forme) ont été examinées. Les visages ont une grande importance sociale et sont généralement traités rapidement. Les résultats ont montré que le masquage de redondance se produisait non seulement avec des stimuli simples (par exemple, des lignes et des lettres), mais également avec des visages. Les occurrences de masquage de redondance dans les visages révèlent la stabilité et la force du masquage de redondance sur les fonctionnalités de bas et de haut niveau. Dans le chapitre 5, nous avons discuté de toutes les expériences menées dans les chapitres précédents, ainsi que des résultats observés.