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Doctoral School **Science de la Matière, du Rayonnement et de l'Environnement (SMRE)**
University Department **Physique des Lasers, Atomes et Molécules (PhLAM) UMR 8523**

Thesis defended by **Quentin DEMAZEUX**

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Academic Field **Diluted media and fundamental optics**

Speciality **Physics**

High-Repetition-Rate Electro-Optic Diagnostics for Accelerator-Based Light Sources : Broadband and Narrowband THz Characterization

Thesis supervised by Pr. Serge BIELAWSKI Supervisor
Pr. Christophe SZWAJ Co-Supervisor

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COLOPHON

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Diagnostiques électro-optiques à haut taux de répétition pour les sources de lumière basées sur des accélérateurs : Caractérisation THz à large bande et à bande étroite

Thèse dirigée par Pr. Serge BIELAWSKI directeur
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HIGH-REPETITION-RATE ELECTRO-OPTIC DIAGNOSTICS FOR ACCELERATOR-BASED LIGHT SOURCES : BROADBAND AND NARROWBAND THz CHARACTERIZATION**Abstract**

Accelerator-based light sources are indispensable research tools, delivering intense, short light pulses ranging from the infrared to X-ray spectral regions. The routine operation and investigation of these sources relies on the real-time characterization of relativistic electron bunches and the resulting photon beams. To address this, the objective of this thesis is the development of detection setups that deliver sub-picosecond temporal resolution at MHz repetition rates, capable of characterizing in single-shot both broadband and narrowband THz signals.

For broadband THz characterization, although Diversity Electro-Optic Sampling (DEOS) combined with the use of a THz reconstruction algorithm, has improved temporal resolution, existing reconstruction algorithms based on linear dispersion models remain unreliable due to non-linear dispersion in chirped probe pulses. This work introduces a novel Self-Adaptive Dispersion (SAD) algorithm for DEOS. This generalized framework transcends previous linear-time-invariant assumptions, providing a high-fidelity reconstruction solution for any arbitrary dispersion profile while simultaneously characterizing the complex amplitude (amplitude, phase, and dispersion) of the chirped probe laser. The DEOS-SAD system was successfully implemented at the European XFEL and FLASH facilities enabling routine, single-shot MHz-rate longitudinal electron bunch characterization.

For narrowband THz characterization, capturing complete multicycle waveforms in high-repetition-rate environments typically requires prohibitively high electronic bandwidths. This thesis proposes and demonstrates a novel electro-optic approach combining all-optical heterodyning with the Photonic Time-Stretch technique. By down-converting THz signals to an intermediate frequency and applying temporal stretching, this method records complete multicycle electric fields while preserving signal integrity. A proof-of-principle demonstration was conducted at the FELBE facility, where a 3.44 THz signal was successfully down-converted to 1.72 GHz, directly enabling the single-shot diagnosis of carrier-envelope phase (CEP) drift using lower electronic readout bandwidths.

By integrating these innovations, this thesis advances the state of the art in THz diagnostics, providing new possibilities for the real-time monitoring of high-repetition-rate accelerator-based light sources.

Keywords: accelerator, photonic, ultrafast measurement, free electron laser

DIAGNOSTICS ÉLECTRO-OPTIQUES À HAUT TAUX DE RÉPÉTITION POUR LES SOURCES DE LUMIÈRE BASÉES SUR DES ACCÉLÉRATEURS : CARACTÉRISATION THz À LARGE BANDE ET À BANDE ÉTROITE**Résumé**

Les sources de lumière basées sur des accélérateurs sont des outils de recherche indispensables, délivrant des impulsions lumineuses intenses et brèves allant de l'infrarouge aux rayons X. L'opération courante et l'étude de ces sources reposent sur la caractérisation en temps réel des paquets d'électrons relativistes et du faisceau de photons qui en résulte. Pour répondre à ce besoin, l'objectif de cette thèse est le développement de dispositifs de détection offrant une résolution temporelle sub-picoseconde à des taux de répétition de l'ordre du MHz, capables de caractériser en monocoup des signaux térahertz (THz) à la fois à large bande et à bande étroite.

Pour la caractérisation THz à large bande, bien que l'échantillonnage électro-optique de diversité (DEOS) combiné à un algorithme de reconstruction ait amélioré la résolution temporelle, les algorithmes existants basés sur des modèles de dispersion linéaire restent peu fiables en raison de la dispersion non linéaire des impulsions de sonde étirées. Ce travail introduit un nouvel algorithme de dispersion auto-adaptative (SAD) pour le DEOS. Ce cadre généralisé dépasse les hypothèses précédentes d'invariance temporelle linéaire, fournissant une solution de reconstruction fidèle pour n'importe quel profil de dispersion arbitraire, tout en caractérisant simultanément l'amplitude complexe (amplitude, phase et dispersion) du laser de sonde. Le système DEOS-SAD a été implémenté avec succès sur les installations European XFEL et FLASH, permettant une caractérisation monocoup longitudinale des paquets d'électrons à des taux MHz.

Concernant la caractérisation THz à bande étroite, la capture de formes d'onde multicycles complètes dans des environnements à haut taux de répétition nécessite généralement des bandes passantes électroniques prohibitives. Cette thèse propose et démontre une nouvelle approche électro-optique combinant l'hétérodynage tout-optique avec la technique d'étirement temporel photonique (Photonic Time-Stretch). En convertissant les signaux THz vers une fréquence intermédiaire et en appliquant un étirement temporel, cette méthode enregistre des champs électriques multicycles complets tout en préservant l'intégrité du signal. Une démonstration de principe a été réalisée à l'installation FELBE, où un signal de 3,44 THz a été converti avec succès en un signal de 1,72 GHz, permettant directement le diagnostic monocoup de la dérive de la phase enveloppe-porteuse (CEP) avec une bande passante électronique réduite.

En intégrant ces innovations, cette thèse fait progresser l'état de l'art des diagnostics THz, offrant de nouvelles possibilités pour la surveillance en temps réel des sources de lumière de nouvelle génération.

Mots clés : accélérateur, photonique, mesure ultra-rapide, laser à électrons libres

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General Introduction

The most exciting phrase to hear in science, the one that heralds new discoveries, is not 'Eureka!' but 'That's funny...'

– Isaac Asimov

Accelerator-based light sources are indispensable research tools. They deliver intense, short light pulses ranging from the infrared to X-ray spectral regions [1–4], making them essential for major scientific breakthroughs across various disciplines [5]. The continuous effort to develop and optimize these light sources relies on the control and real-time characterization of the relativistic electron bunches and the resulting photon beam [6–9]. To address the need for the characterization of both the electron bunches and the associated light, this thesis is dedicated to the single-shot measurement of terahertz (THz) electric fields¹. Measuring these THz fields from the electron bunch's Coulomb field or the THz light radiation from the accelerator provides direct insight into the stability and quality of the generated photon beam [10, 11]. Therefore, the overarching objective of this thesis is to develop single-shot detection setups that meet the requirements for accelerator diagnostics. These tools aim to deliver sub-picosecond temporal resolution at MHz repetition rates, enabling the characterization of both broadband and narrowband THz signals.

To achieve single-shot recording of THz signals at MHz rates, diagnostics evolved from the conventional Electro-Optic Sampling (EOS) method [12–15], which relies on a femtosecond probe pulse to scan the THz field temporally within an electro-optic crystal. One approach to transition from EOS to single-shot measurement was enabled by employing a chirped probe pulse, a technique that establishes a direct temporal-to-wavelength mapping. This innovation led to two main MHz-rate compatible methods: Electro-Optic Spectral Decoding (EOSD) [10, 16] and Photonic Time-Stretch (PTS) [17]. While these techniques enabled single-shot, MHz-rate acquisition, they were inherently limited in temporal resolution for decades. This bottleneck was overcome by implementing the Diversity Electro-Optic Sampling (DEOS) configuration [18], which yields a dual-output containing diversity and complementary information. These outputs are then combined with a reconstruction algorithm to retrieve the THz field waveform. Using the DEOS method with a reconstruction algorithm, the temporal resolution is now solely limited by the hardware's inherent bandwidth (i.e.,

¹While the immediate application of this thesis is accelerator diagnostics, the single-shot technique developed herein offers a versatile tool for measuring any THz electric field.

the laser's Fourier-limited duration and the electro-optic crystal's bandwidth). However, the reliance on a chirped probe pulse still introduces a limitation. Non-linear dispersion compromises the fidelity of broadband measurements by distorting the time-to-wavelength mapping, thus rendering existing reconstruction algorithms based on a linear dispersion model unreliable.

To resolve this barrier, the second part of this thesis aims to develop a novel Self-Adaptive Dispersion (SAD) algorithm for Diversity Electro-Optic Sampling (DEOS), providing a generalized framework for the high-fidelity reconstruction of single-cycle, broadband THz pulses. This research redefines the signal reconstruction problem by introducing a unifying conceptual framework that transcends the limitations of existing methods. Unlike previous approaches, such as the Maximum Ratio Combining algorithm which relies on restrictive linear-time-invariant assumptions, this new framework aims to generalize and encompass previous models by providing a solution for any arbitrary dispersion profile inherent in chirped-probe systems. The development path involves initial numerical validation and a tabletop proof-of-principle demonstration to verify the SAD algorithm's ability to reconstruct THz signal under nonlinear dispersion. Following this, the ultimate objective is the practical implementation of the SAD-integrated DEOS system at the European X-ray Free Electron Laser (EuXFEL) and the Free-electron LASer in Hamburg (FLASH). By deploying this tool at these high-repetition-rate facilities, the goal is to provide routine, MHz-rate longitudinal electron bunch characterization, ensuring high-fidelity monitoring of the bunch properties across entire bunch trains.

Beyond broadband THz characterization, the development of narrowband diagnostics is motivated by the requirements of THz accelerator-based sources such as FELBE, STERN, PITZ, and the future DALI upgrade [19–25]. In these environments, there is a need to characterize multicycle THz pulses in a single shot, particularly to provide Carrier-Envelope Phase (CEP) diagnostics [26]. This parameter is essential for pump-probe users, as it allows them to accurately sort and analyze their data based on the phase of the THz field. However, capturing these complete multicycle waveforms is difficult, as it requires the sampling of at least two points per THz cycle. In high-repetition-rate environments, conventional techniques like Photonic Time-Stretch demand prohibitively high electronic bandwidths (e.g., >10 GHz) to record the full multicycle pulse, severely impeding the realization of cost-effective, real-time diagnostic systems. To address this, the third part of this thesis proposes a novel electro-optic approach that combines all-optical heterodyning with the Photonic Time-Stretch technique. By down-converting THz signals to an intermediate frequency and applying temporal stretching, this novel method aims to record the complete multicycle electric field using lower electronic bandwidths. The ultimate goal is to provide a proof-of-principle demonstration at the FELBE facility, establishing this all-optical system as a viable tool for the real-time monitoring and characterization of narrowband THz sources.

In summary, this thesis pushes the state of the art in THz electro-optic diagnostics on two fronts. It develops a new reconstruction algorithm which, combined with Diversity Electro-Optic Sampling enables high-fidelity broadband THz pulse reconstruction despite nonlinear dispersion within the chirped

probe laser pulse.

Furthermore, it introduces a novel electro-optic technique which, by combining the heterodyne and photonic time stretch techniques, enables the single-shot characterization of narrowband THz fields with a low electronic readout bandwidth. The subsequent chapters of this thesis are organized into three parts:

- Part I establishes the foundational principles of the work. Chapter 1 provides the background on accelerator-based light sources, while Chapter 2 reviews the electro-optics detection systems.
- Part II focuses on achieving high-fidelity broadband THz characterization. Chapter 3 presents the theoretical and numerical development of a new reconstruction algorithm for the Diversity Electro-Optic techniques. This novel algorithm introduces a unifying conceptual framework that encompasses existing methods while transcending their limitations by providing a solution for any arbitrary dispersion profile inherent in chirped-probe systems. The effectiveness of this approach is validated through both numerical simulations and tabletop proof-of-principle demonstrations. Chapter 4 details the implementation of this algorithm at two major facilities. It describes single-shot, MHz-rate longitudinal bunch characterization at the European X-ray Free Electron Laser (EuXFEL) and the Free-electron LASer (FLASH) facilities in Hamburg, providing statistics on bunch length and arrival time stability across full bunch trains.
- Part III introduces a new strategy for narrowband signals. Chapter 5 outlines the theoretical framework and design of a detection system combining all-optical heterodyning with Photonic Time Stretch technique. Finally, Chapter 6 describes the experimental implementation of this novel electro-optic strategy at the Dresden-Rossendorf free-electron laser (FELBE) facility. This includes the single-shot down-conversion of a 3.44 THz narrowband signal to a 1.72 GHz intermediate frequency signal on the oscilloscope. Furthermore it includes the measurement of carrier-envelope phase drift as a function of the cavity detuning.

Part I

Background Knowledge of the Accelerator-Based Light Sources and the Electro-Optic Diagnostics

Accelerator-based light sources

If you want to find the secrets of the universe, think in terms of energy, frequency and vibration.

– Nikola Tesla

1.1 Introduction

Accelerator-based light sources, including electron storage rings and Free-Electron Lasers (FELs), are powerful research tools used by a vast community of scientists working across various technological frontiers. Their capability to generate intense, short light pulses, ranging from the infrared to X-ray spectral regions, makes them indispensable for major scientific breakthroughs across physics, chemistry, biology, materials science, and numerous other disciplines.

However, the continued optimization and improvement of these light sources depend on the control and real-time characterization of the relativistic electron bunches responsible for light generation. This thesis addresses this requirement by focusing on accelerator diagnostics, and more specifically on the measurement of electric fields in the terahertz (THz) range. Direct insight into the stability and quality of the photon beam delivered to users is gained by characterizing either the Coulomb electric field of the electron bunch within the accelerator vacuum chamber or the THz radiation emitted by the bunches.

To establish the necessary context for this accelerator diagnostic work, this chapter provides a general introduction to these light sources and their underlying physics. The chapter begins with Section. 1.2, detailing the historical evolution of accelerator-based light sources. This provides an overview of the technological progress that necessitates the adaptation of diagnostic tools. The

principle of light generation is detailed in Section. 1.3, which covers the fundamental mechanism of charged particle radiation from accelerated electrons and the role of insertion devices generating and shaping these intense, short, flashes of light. Given the experimental focus of this thesis at various FELs facilities, Section. 1.4 examines the operating principles of Free-Electron Lasers, differentiating between the single-pass SASE and FEL oscillator configurations, both of which are investigated using EO diagnostics in Chapter. 4 and Chapter. 6, respectively. Finally, Section. 1.5 establishes the challenging need for robust, single-shot electro-optic accelerator diagnostics operating at MHz acquisition rates, the principles of which are detailed in Chapter. 2.

1.2 Historical overview of accelerator-based light sources

The history of accelerator radiation sources began with the first observations of synchrotron light in 1947 [27, 28]. Though initially viewed as a mere byproduct of particle accelerators dedicated to high-energy physics [29], these emissions quickly revealed a profound scientific potential that warranted dedicated study.

The first generation of synchrotron radiation sources (early 1970s) consisted of electron synchrotrons or storage rings repurposed from high-energy physics programs. While the overall utility of the resulting Synchrotron Radiation (SR) was intrinsically constrained by this arrangement, the compelling scientific results obtained gradually cultivated a dedicated user community. This growing interest soon mounted pressure for the construction of facilities designed specifically for light generation.

This demand marked the transition to the second generation, characterized by storage rings engineered exclusively for the production of synchrotron radiation. The defining technological feature of this era was the ability to accommodate insertion devices, such as wigglers and undulators (for more information, see subsection 1.3.2). These devices, which require long, straight sections within the ring lattice, dramatically improved the intensity of the outgoing photon beam.

Building upon this technological leap, the design and need for the third generation arose from a growing enthusiasm for accessing even shorter wavelengths, which required the implementation of high-energy storage rings. By integrating the latest advancements in electron optics and insertion devices, these installations successfully achieved and increased the brightness of X-rays.

While the third generation optimized brilliance, the fourth generation marked a paradigm shift driven by the pursuit of ultra-high brilliance, higher photon energy, and extremely short pulses (in the sub-picosecond to attosecond range). This era was defined by the emergence of various approaches to push the boundaries of peak brilliance, temporal coherence, and pulse duration. Continued refinement of storage rings (e.g., through low-emittance lattice designs) and recirculating energy-recovery linacs (ERLs) were developed and realized to satisfy various user groups. However, the emergence of Free-Electron Lasers (FELs) provided peak powers several orders of magnitude higher and shorter pulse lengths. This superior performance is essential for femtosecond ultrafast techniques, such as "pump-probe" experiments¹ and non-linear science with photons. Notable examples include FERMI [30], FELBE [31], EuXFEL [4, 32, 33], FLASH [34], and

¹Note that storage rings also render some ultrafast studies (like low-alpha pump-probe experiments) possible

LCLS [2], among others.

Currently, research is defining the potential fifth generation, which focuses on reducing the extensive scale and high cost associated with these facilities. Key advancements include the use of laser plasma acceleration [35–39] to accelerate electron bunches and the exploration of dielectric waveguides to replace conventional undulators, particularly for specific radiation ranges like the THz region [23, 25, 40].

Collectively, these advanced light sources have become indispensable tools, driving significant advancements across a multitude of research fields. Their broad spectrum of applications extends from fundamental sciences, including biology, physics, medicine, materials science, and spectroscopy [41–43], among others.

For further details on the history of accelerator-based radiation sources, readers are referred to ref [44].

1.3 Charged particle radiation principles

This section details the fundamental physical principles governing the generation of light from relativistic electron bunches in accelerator-driven light sources. A detailed overview is presented in the literature [45, 46].

1.3.1 Synchrotron radiation

According to the principles of classical electrodynamics, any accelerated charged particle emits electromagnetic radiation [46]. In the context of accelerator physics, when a relativistic electron bunch is constrained to follow a curved trajectory by magnetic fields, it generates what is termed synchrotron radiation (SR). This light is characterized by its emission in a narrow cone in the direction of the electron's instantaneous motion and a broad spectral distribution spanning from the infrared to hard X-rays.

The total intensity of this radiation is governed by its coherence and the number of electrons, as illustrated in Figure. 1.1. For the incoherent (spontaneous) SR, the radiation wavelength is shorter than the bunch length, this means that the electrons are randomly spaced relative to the wavelength. Thus the radiation fields add with arbitrary phase, and the total intensity scales linearly with the number of electrons, $P \propto N$ (Figure. 1.1.(a)). Coherent synchrotron radiation (CSR) occurs when the radiation wavelength is comparable to or longer than the bunch length, or when the bunch contains micro-structures on the scale of the radiation wavelength. In this case, the emitted fields add constructively, or

coherently, enhancing the intensity such that it scales with the square of the number of electrons, $P \propto N^2$ (Figure. 1.1.(b)).

The distinct intensity profile of the emitted SR serves as an important tool for longitudinal beam diagnostics, as it directly corresponds to the longitudinal profile of the electron bunch. Synchrotron radiation was first observed in 1947 [27, 28] and an historical photo of this radiations is depicted in Figure. 1.1.(c).

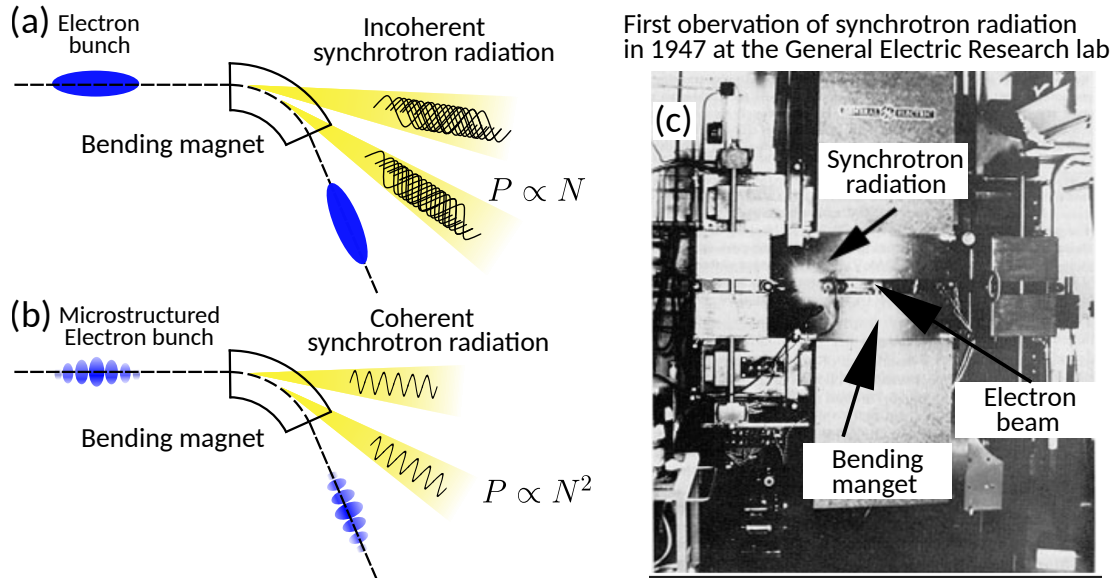


Figure 1.1: Principle of synchrotron radiation emission. (a) Incoherent SR : Radiation from a relativistic electron is emitted into a narrow cone in the direction of motion. When the SR wavelength is much shorter than the bunch length, the radiation fields from the randomly spaced electrons sum incoherently, resulting in a total intensity $P \propto N$. (b) Coherent SR (CSR) : When the radiation wavelength is comparable to or longer than the bunch length, or a density modulation exists, the fields add constructively. This results in highly intense coherent emission, $P \propto N^2$. (c) Adapted picture from [28].

1.3.2 Undulator/Wiggler radiation

To enhance the properties of the synchrotron radiation, insertion devices called undulators and wigglers are employed. These devices consist of periodic arrangements of magnets designed to force relativistic electron bunches to undergo transverse oscillations. A key distinction exists between wigglers and undulators based on the amplitude of electron oscillations. In a wiggler, the oscillations are large, leading to the emission of incoherent but highly intense radiation. Conversely, in an undulator, the oscillations are weaker, and the spontaneous emissions radiation emitted at each period of the undulator interferes coherently, producing quasi-monochromatic and very brilliant radiation [47].

The underlying principle of undulator radiation is depicted in Fig. 1.2(a). A

picture of an undulator is provided in Fig. 1.2(b).

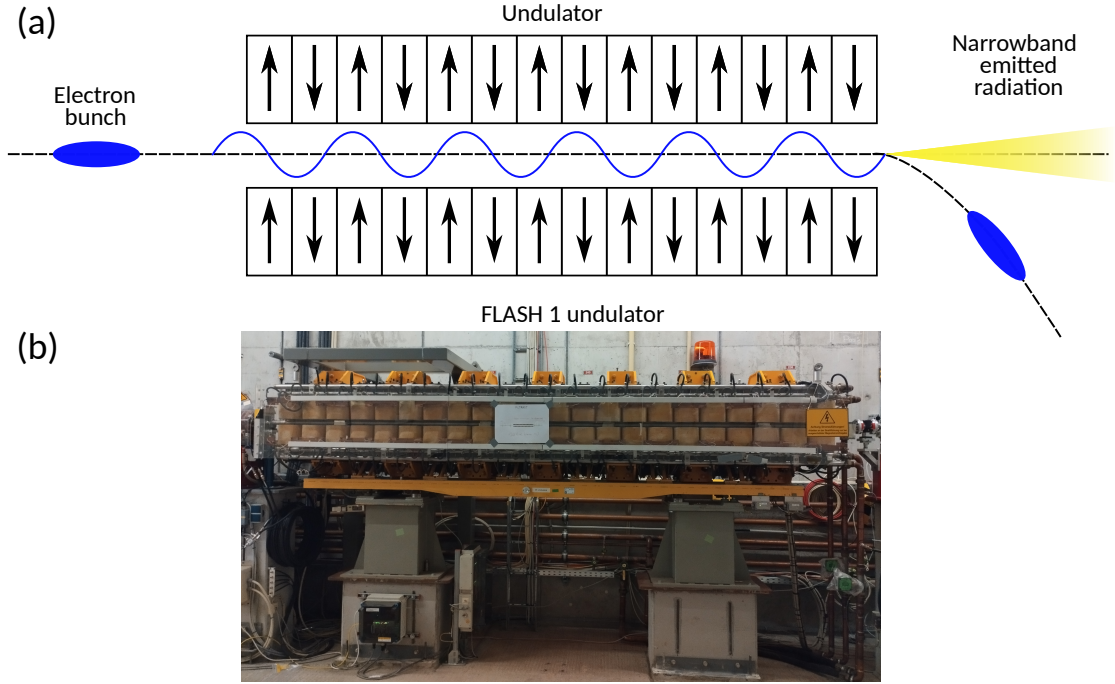


Figure 1.2: **Principle of undulator radiation.** (a) A relativistic electron bunch passes through a periodic magnetic field, causing it to oscillate and emit coherent electromagnetic radiation. Note that the electron oscillation is perpendicular to the plane of the figure. The motion is represented here and in subsequent figures as a simplified 2D projection for clarity. (b) Photograph of the THz undulator of FLASH 1. This undulator is shown because its much longer period (compared to X-ray undulators) allows the individual magnetic periods to be clearly visible for illustrative purposes.

The wavelength of the emitted light, λ , is primarily determined by the undulator period λ_u , the electron energy $E_e = \gamma m_e c^2$, the undulator parameter K_u and θ the observation angle with respect to the electron beam², as defined by the following equation:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K_u^2}{2} + \gamma^2 \theta^2 \right) \quad (1.1)$$

The undulator parameter K_u is given by:

$$K_u = \frac{eB\lambda_u}{2\pi m_e c} \quad (1.2)$$

Here, B denotes the magnetic field strength, γ is the relativistic Lorentz factor and e and m_e are the electron charge and mass, respectively. The final emitted wavelength λ can be tuned by varying the electron beam energy E_e or by adjusting the undulator gap to change the undulator parameter K_u .

²The angle θ is generally small in the undulator

1.4 Free-Electron Lasers (FELs)

Free-electron lasers represent the fourth generation of accelerator-based radiation sources, overcoming some of the limitations of synchrotrons to offer photon pulses of unmatched peak power and coherence. Unlike synchrotrons that use storage rings, most FELs use a linear particle accelerator (linac) to produce ultrashort and high-quality electron bunches. This section will detail the operating principle of an FEL, highlighting the fundamental differences between single-pass FELs and FELs oscillator, both of which are investigated using EO diagnostics in this thesis. A detailed overview is presented in the literature [45, 46].

1.4.1 FEL operating principle

The operating principle of FELs relies on the collective interaction between a relativistic electron bunch and the electromagnetic field it generates within an undulator. In the Self-Amplified Spontaneous Emission (SASE) mode, electrons traversing the undulator initially emit spontaneous radiation. This radiation then interacts with the electron bunch itself, inducing a self-organization process where electrons group into micro-bunches, typically separated by an optical wavelength. This induced micro-bunching intensifies the coherently emitted radiation. This leads to a positive feedback loop, where the amplified radiation further strengthens the micro-bunching, and this iterative process continues along the undulator. The amplification continues until saturation, producing a laser beam of extreme brilliance and coherence, characterized by ultrashort pulses and gigantic peak power [48].

The FEL operation can be categorized into two configurations, a single passage of the electron bunch through a long undulator generating EUV and X-ray light or the use of an undulator integrated within a resonant optical cavity. While the latter generates IR-visible light, modern cavity-based systems can extend this range into the VUV, XUV, and even soft X-ray regimes [49, 50].

1.4.2 Single-pass FELs

1. Principle of single-pass FELs :

Single-pass free-electron lasers are characterized by the interaction of an electron bunch with a long undulator in a single traversal, enabling the self-amplification of its own spontaneously emitted radiation. This configuration is typically associated with the Self-Amplified Spontaneous Emission (SASE) mode, where initial spontaneous emission is amplified as the electron bunch propagates, leading to the generation of highly coherent radiation. Figure. 1.3 shows the principle of a single-pass FEL.

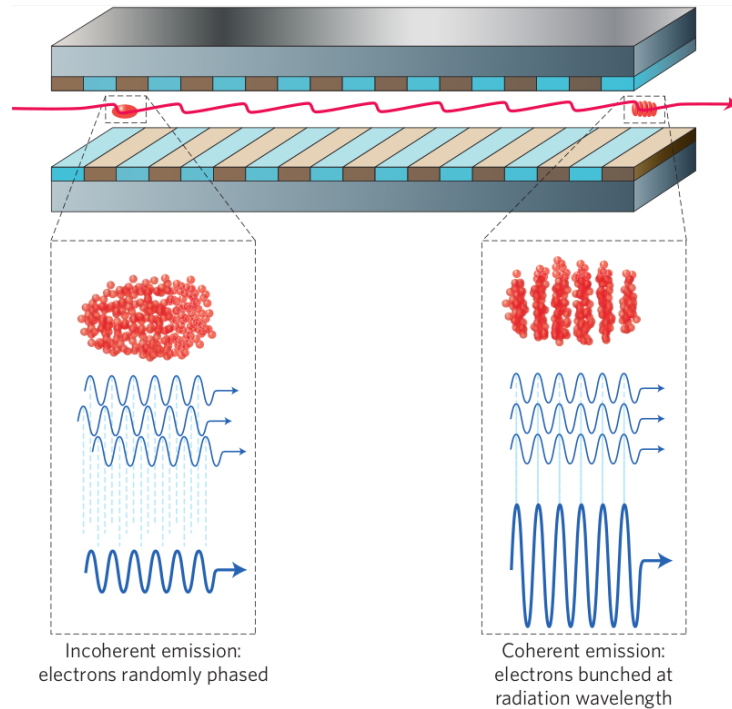


Figure 1.3: **Principle of single-pass free-electron laser operating in self-amplified spontaneous emission mode.** An electron bunch, after being accelerated by a linear accelerator, enters a long undulator. Initial spontaneous radiation from the electron bunch interacts with the electrons, leading to micro-bunching and subsequent amplification of the emitted coherent radiation. The figure comes from ref [48].

1.4.3 FELs oscillators

1. Principle of FELs oscillators :

Free-electron lasers oscillators represent another class of FELs that are based on a resonant optical cavity, similar in principle to conventional lasers. In this configuration, the undulator is integrated within an optical resonator, typically formed by a pair of mirrors. As the relativistic electron bunch traverses the undulator, it generates radiation. This generated light is stored within the optical cavity, reflecting back and forth between the mirrors, and is synchronized to interact repeatedly with subsequent electron bunches. During each pass through the undulator, the light interacts with the electron bunch, which continuously transfers energy to the electromagnetic wave through stimulated emission. One of the cavity mirrors often includes an outcoupling mechanism to extract the amplified coherent light pulses. Precise synchronization of the optical pulse's round-trip time with the arrival of successive electron bunches at the undulator entrance is paramount for sustained amplification and the production of intense, coherent radiation. Figure. 1.4 shows the principle of an FEL oscillator.

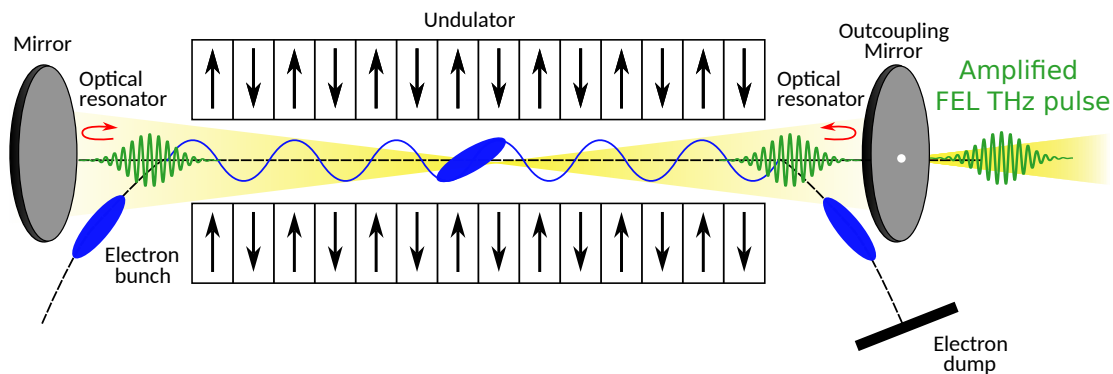


Figure 1.4: **Operational principle of a free-electron laser oscillator.**

An electron bunch passes through an undulator located within an optical cavity, causing light emission. This light is then reflected back by the cavity mirrors to interact with subsequent electron bunches, leading to stimulated emission and amplification of the radiation. A portion of the amplified light is extracted as the output laser pulse.

1.5 The need for single-shot electro-optic accelerator diagnostics

The optimization and routine operation of accelerator-based light sources depend on the real-time characterization of their electron bunches. The quality and specific properties of the generated photon beam, including its intensity, duration, spectrum, and coherence, are directly linked to fundamental characteristics of the electron bunches, such as charge, longitudinal profile, energy, arrival time, and stability. Therefore, any variations in these electron parameters can impact the quality and utility of the produced radiation.

Thus, accelerator diagnostics are indispensable tools for several key reasons. They are used to optimize accelerator parameters in order to maximize source performance and to gain a deeper understanding of the complex physical phenomena occurring during electron acceleration and the subsequent electron-radiation interaction. This facilitates the development of new acceleration and radiation generation techniques. Furthermore, these diagnostics provide crucial information to facility users, enabling them to conduct their experiments effectively.

A diverse and extensive suite of diagnostic techniques is routinely employed to characterize every aspect of either the electron beam or the radiation itself (e.g., see ref [51] for electron beam diagnostics overview and ref [9] for X-ray³ photon diagnostics, both at EuXFEL). However, the central challenge that this thesis addresses is the need for robust, single-shot, and high-repetition-rate measurements of the longitudinal electron bunch shape or its related THz emitted radiation.

A widely employed technique for this purpose is electro-optic sampling (EOS), which involves probing an electric field by using a laser probe pulse and an electro-optic crystal (this method will be detailed in Section 2.2). This technique can be applied to probe the Coulomb electric field of the electron bunch by placing an electro-optic crystal inside the vacuum accelerator chamber [52–54]. Or alternatively, by placing the EO crystal at the THz beamline output to measure the THz radiation emitted by these relativistic electrons, as illustrated in Figure 1.5(b) [55].

³Note that X-ray radiation is chosen here, but there are radiation diagnostics at any wavelength employing specific techniques

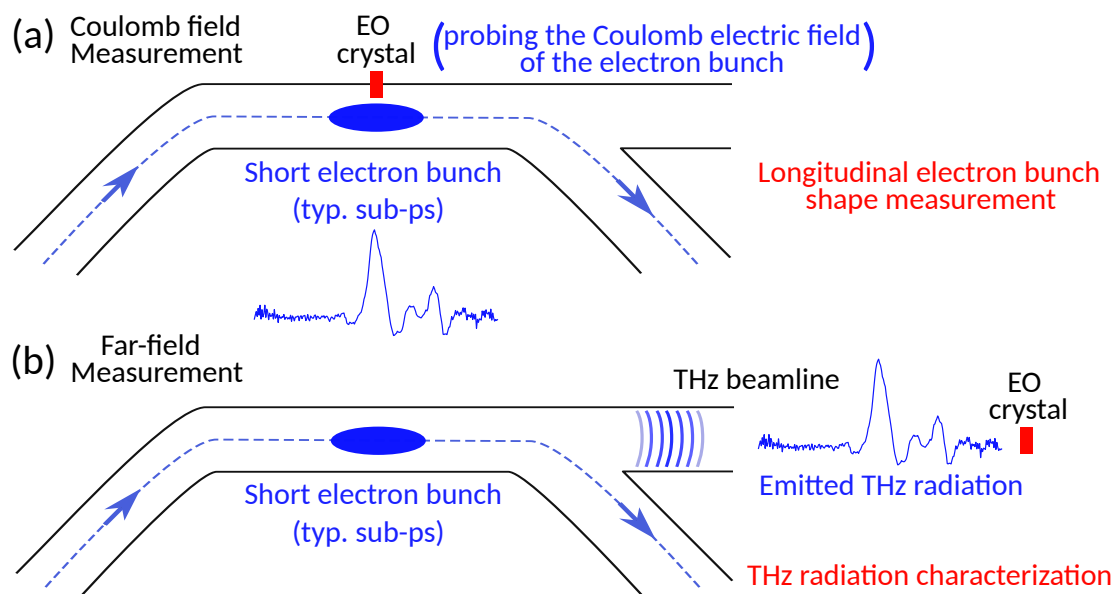


Figure 1.5: Comparison of Coulomb field and far-field electro-optic detection methods for characterizing electron bunches and their emitted radiation in accelerator facilities. (a) In the Coulomb field detection, an electro-optic crystal is placed within the vacuum accelerator chamber, directly probing the Coulomb field of the electron bunch. (b) In the far-field configuration, the THz radiation emitted by the electron bunch is collected and measured remotely using an EO detection system.

Nevertheless, existing EO detectors often present limitations and involve trade-offs between single-shot operation, acquisition rates, sufficient temporal resolution, and sensitivity. The specific challenges and intricacies of these measurements will be described in detail throughout Chapter 2.

The main objective of this thesis, therefore, is to develop a single-shot diagnostic tools based on electro-optic detection. These tools aim to achieve repetition rates from 2 to 13 MHz, provide non-destructive recordings of both broadband and narrowband THz signals with high fidelity, and push the bandwidth limitations of EO detection towards higher frequencies.

The detailed development and experimental results for broadband detection and high fidelity recordings are presented in Part II, while Part III focuses on narrowband detection and the exploration of enhanced bandwidth limitations.

Note on the versatility of EO THz detection: From accelerators to tabletop sources

It is important to emphasize that while this thesis focuses on applying the developed electro-optic detection method to accelerator diagnostics, the underlying principles and advanced capabilities of this tool are equally valid and highly valuable for the characterization of THz fields generated through other means, such as tabletop conventional laser-based THz sources utilizing optical rectification and for exploring diverse non-equilibrium phenomena beyond the scope of accelerator-based studies [56–59].

State of the art of single-shot electro-optic methods for THz detection

*Measure what is measurable, and
make measurable what is not so.*

– Galileo Galilei

2.1 Introduction

Single-shot electric field recording is essential in THz science for investigating fast, non-repetitive phenomena or low-repetition-rate experiments where classical time-domain spectroscopy is impractical.

Furthermore, single-shot THz diagnostics are needed for the investigation and optimization of novel THz sources, including quantum cascade lasers, ultra-wide-bandwidth laser-plasma sources and, specifically for this thesis, Free-Electron Lasers and accelerators. THz diagnostics in accelerator facilities have varying demands. While low-repetition-rate FELs (such as FERMI, SACLA, and LCLS) only require single-shot measurement, high-repetition-rate facilities demand operation at MHz repetition rates. These high rates are essential for electron bunch-to-bunch characterization, along with the required sub-picosecond temporal resolution.

This chapter details the underlying principle of a non-invasive, single-shot THz recorder operating at MHz repetition rates with hundreds of femtosecond temporal resolution.

The first Section.2.2 introduces the Electro-Optic Sampling (EOS) method,

which uses an electro-optic crystal and a femtosecond probe laser to perform a conventional THz waveform scanning measurement.

To meet the specific requirements of accelerator THz diagnostics, the setups necessary for converting the EOS technique into a high-repetition-rate, single-shot method are detailed in Section.2.3. This is achieved using a chirped probe laser pulse instead of a femtosecond pulse, a technique known as spectral encoding. This process encodes the entire THz signal onto the laser spectrum using a time-to-wavelength mapping, thus enabling a single-shot THz measurement. Regarding the MHz-readouts, two options are possible, an electronic approach and a photonic method. The electronic approach uses a spectrometer comprising a grating and an ultrafast camera, allowing acquisition rates of 2.7 MHz [60]. The photonic method employs long optical fibers introducing dispersion to slow down the THz-encoded signal so it can be captured by commercially available photodiodes and oscilloscopes, reaching acquisition rates up to 88 MHz [61]. Finally, the intrinsic temporal resolution limitations of the spectral encoding technique are rigorously examined in Section.2.4, particularly those imposed by the use of a chirped probe pulse. The solution to overcome these limitations and achieve sub-picosecond temporal resolution is the phase diversity method, which is introduced and detailed, along with its associated Maximum Ratio Combining (MRC) reconstruction algorithm. However, it will be revealed in section. 2.5 that even these advanced methods face limitations under conditions of non-linear dispersion (such as higher-order dispersion in pre-chirp fibers).

2.2 Principle of the Electro-Optic Sampling (EOS)

1. Probe laser and THz interaction within an electro-optic crystal:

A linearly polarized femtosecond laser pulse, referred as the probe pulse, and a THz electric field under interest are interacting colinearly in the electro-optic crystal. When the THz electric field is applied, the EO crystal becomes optically anisotropic. This anisotropy creates two distinct refractive indices, n_1 and n_2 along the fast and slow axes [62], respectively, as illustrated in Figure 2.2.(a).

As the probe pulse propagates through the EO crystal, its polarization state is altered by the THz electric field induced birefringence. This interaction leads to a phase retardation $\Delta\phi$ between the ordinary and extraordinary components of the probe light. This phase retardation can be expressed as [63, 64]:

$$\Delta\phi(\alpha, t) = \frac{2\pi d}{\lambda} (n_1(\alpha) - n_2(\alpha)) = \frac{\pi d}{\lambda} n_0^3 r_{41} \sqrt{1 + 3 \cos^2(\alpha)} E_{THz}(t) \quad (2.1)$$

where λ is the laser wavelength, d is the crystal thickness, n_0 is the refractive index of the crystal at the laser wavelength λ , r_{41} is the electro-optic coefficient, α is the angle of the THz field with respect to the $[-110]$ direction of the EO crystal, and E_{THz} is the THz electric field.

In the standard EOS configuration, the THz electric field and the femtosecond probe laser are aligned parallel to the $[-110]$ crystallographic axis, meaning that $\alpha = 0$, as depicted in Figure 2.2.(b).

In this specific alignment, Equation 2.1 simplifies to:

$$\Delta\phi^{std}(\alpha = 0, t) = \frac{2\pi d}{\lambda} n_0^3 r_{41} E_{THz}(t) \quad (2.2)$$

In the absence of a THz electric field, the probe laser's polarization remains unchanged. The electro-optic crystal acts as an electro-optic modulator, where the rapidly varying THz field modulates the polarization state of the probe pulse.

Example : Order of magnitude of phase retardation

The THz electric fields encountered in accelerators are typically on the order of several hundreds of kV/m. Considering a GaP crystal with $n_0 = 3.1$ at a wavelength of $1.03 \mu\text{m}$, an electro-optic coefficient $r_{41} = 0.97 \text{ pm/V}$ [65, 66] and a thickness $d = 5 \text{ mm}$.

Using Equation 2.2, the calculated phase retardation $\Delta\phi$ is typically in the range of tens to hundreds of milliradians. This relatively small phase retardation necessitates a special design in order to reach the required sensitivity to extract the THz signal with a good signal-to-noise ratio (SNR).

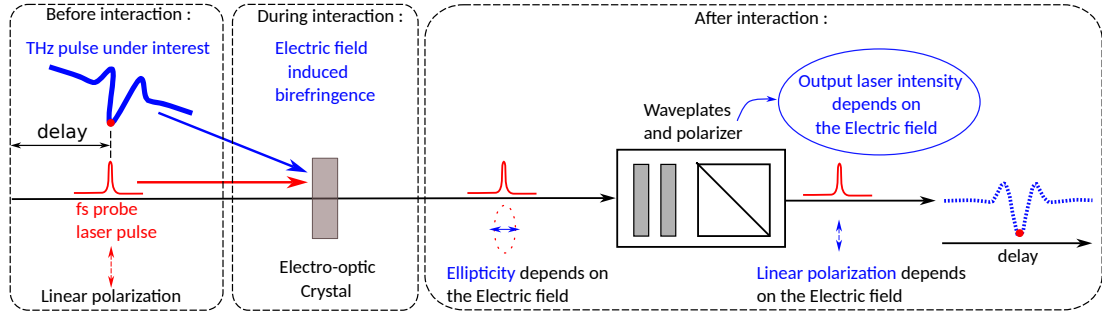


Figure 2.1: **Principle of a conventional electro-optic sampling method.** A variable delay stage is used to scan the relative arrival time between the femtosecond laser probe pulse and the THz pulse.

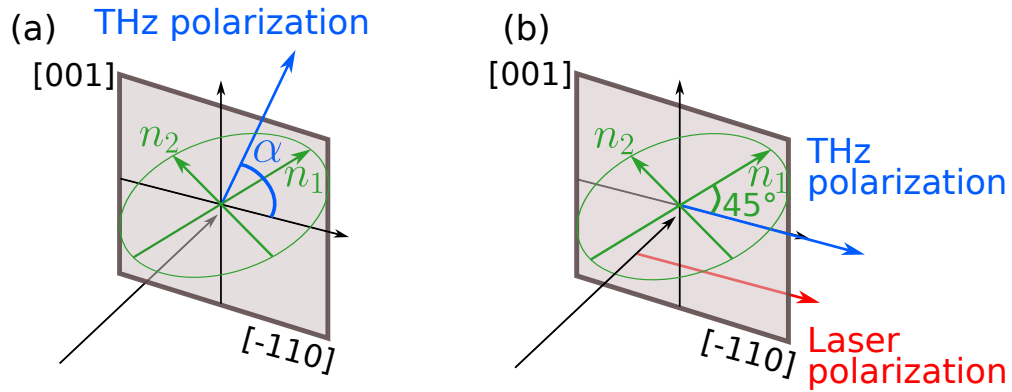


Figure 2.2: **Illustration of an electro-optic crystal under the influence of a THz electric field :** (a) Representation of the anisotropy induced by the THz electric field resulting in a fast and slow axes with a refractive indices n_1 and n_2 . (b) Standard configuration for electro-optic sampling, showing the alignment of the probe laser and THz field with respect to the crystallographic axes.

2. Recording Process:

The induced phase retardation transforms the initially linear polarization of the probe laser into an elliptical polarization. This ellipticity is then converted into an intensity modulation by passing the probe beam through a judiciously chosen combination of waveplates and a polarizer. By measuring this intensity change, the instantaneous value of the THz electric field can be precisely deduced.

Since each measurement yields only a single temporal point of the THz electric field evolution, retrieving the entire temporal profile of the THz pulse requires scanning the relative delay between the femtosecond laser probe pulse and the THz signal. This time-domain sampling process is schematically illustrated in Figure 2.1.

While effective for characterizing stable or repetitive phenomena, this scanning approach is inherently time-consuming and precludes single-shot measurements of THz waveforms.

3. Temporal Resolution:

The temporal resolution of EOS is limited by the duration of the probe pulse T_0 and the inherent properties of the electro-optic crystal, particularly its phonon absorption resonance frequencies f_0 and phase matching condition between the laser and the THz signal [63, 64].

Table 2.1 provides an overview of common EO crystals, detailing their respective first phonon resonance frequencies f_0 and their corresponding probe laser wavelengths used to achieve the optimal phase-matching condition for each material.

EO crystal	f_0 THz	Probe laser wavelength (nm)
ZnTe	5.3	800
GaP	10.98	1060
GaAs	8.025	1550

Table 2.1: Suitable electro-optic crystals for THz detection with their respective first phonon resonance frequencies f_0 , at which absorption occurs, from ref [65]. The selection of the probe laser wavelength correspond to the optimal phase matching conditions for each EO crystal.

With careful selection of the crystal and probe pulse duration, sub-picosecond temporal resolution has been achieved using this technique [14, 67, 68].

4. Applications :

EOS finds widespread applications in characterizing repetitive electric fields [12–15], THz spectroscopy [69], tomography [70], and imaging [71]. In accelerator physics, notable early demonstrations of electro-optic detection include applications in Free Electron Lasers (FELs) like FELIX [68] and in storage ring laser slicing experiments [72].

5. Need for single-shot electro-optic techniques :

Conventional EOS offers excellent temporal resolution, but its reliance on a scanning delay makes single-shot THz waveform measurements impossible. This limitation precludes its use in studying irreversible or non-repeatable processes, and makes it impractical for low repetition rate phenomena. More importantly, in the context of this thesis, non single-shot EOS cannot meet the real-time diagnostics requirements of accelerators, which necessitate the study of electron bunch shape dynamics. To enable bunch-to-bunch characterization, the conventional EOS scheme must be adapted for single-shot operation. The following section will detail how this adaptation was made to allow a single-shot operation working at MHz repetition rates.

2.3 High repetition rate single shot electro-optic detection

There are various single-shot electro-optic techniques, including transverse encoding (time-to-space mapping), angular encoding (time-to-angle mapping) and spectral encoding (time-to-wavelength mapping).

So far, to achieve the high repetition rates needed in accelerator facilities, the most viable technique is spectral encoding using a chirped probe laser pulse. This technique will be detailed in subsection 2.3.1. For more information on the transverse and angular encoding, the reader is referred to the following reference [73].

Once the signal is encoded, there are different ways to read it out. One option is spectral decoding [16], which uses a spectrometer, i.e, a grating combined with an ultrafast camera, as explained in subsection 2.3.2.1. The other is photonic time-stretch (PTS) [17, 61], which uses long optical fibers to slow the signal down, enabling its capture by a photodiode and oscilloscope, as explained in subsection 2.3.2.2. The advantages and disadvantages of choosing between PTS and the ultrafast spectrometer will be discussed in the summary. 2.3.2.2.

2.3.1 Single-shot electro-optic detection using chirped probe laser pulses (Time-to-wavelength conversion)

The electro-optic detection technique using chirped probe laser pulse [16, 17], also known as spectral encoding, is an advanced EO technique, enabling single-shot waveform acquisition, by encoding temporal information of the THz field onto the chirped laser probe spectrum. The principle of the spectral encoding technique is displayed in Figure 2.3.

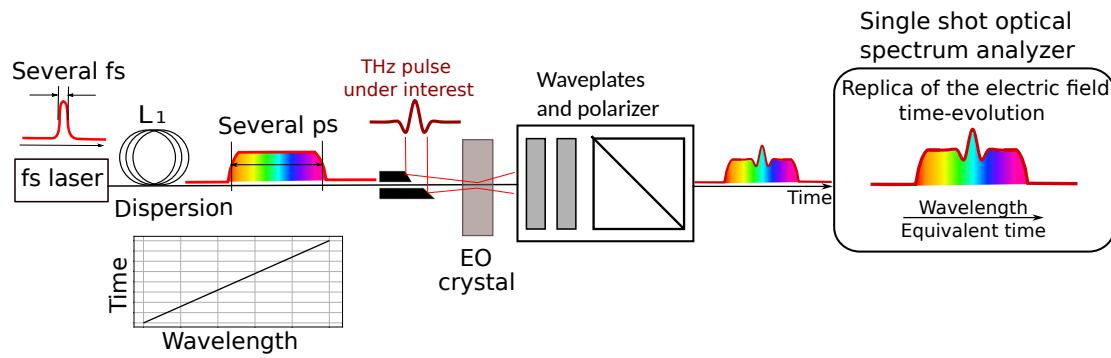


Figure 2.3: Illustration of the single-shot electro-optic spectral encoding : A chirped probe laser pulse interacts with the THz electric field within an electro-optic crystal. The temporal profile of the THz field is encoded onto the chirped laser probe spectrum.

The chirped probe laser is obtained from a femtosecond laser pulse with a spectral bandwidth $\Delta\lambda$ that is temporally stretched to a duration T_1 , i.e. chirped, before interacting with the THz field inside the electro-optic crystal.

$$T_1 = \Delta\lambda \mathcal{D}_1 \quad (2.3)$$

This chirping process, is achieved by propagating the laser pulse through a dispersive medium with a total dispersion $\mathcal{D}_1$ ¹, establishing an approximately linear² relationship between the instantaneous frequency and the temporal position within the probe pulse. A modulation of the chirped pulse in time will therefore cause an identical³ modulation in the spectrum. When this chirped probe pulse interacts with the co-propagating THz electric field in the electro-optic crystal, its polarization state is modulated. Using waveplates and a polarizer, this polarization modulation is converted into an intensity modulation. Therefore, the entire spectral encoding process maps the time-evolution of the THz electric field directly onto the chirped probe laser's spectrum.

¹As example $\mathcal{D}_1 = D_1 L_1$ for an optical fiber having a dispersion D_1 and a length L_1

²The general case in which the relation between the instantaneous frequency and time is not linear will be further discuss in section. 2.5 and chapter. 3

³In section.2.4, we will see that it not always the case.

2.3.2 High-Speed THz Readouts: ultrafast camera-based optical spectrum analyzer vs photonic time stretch

To record the chirped probe laser's spectrum and thus the THz temporal profile, a standard spectrum analyzer can be used. However, bunch-to-bunch characterization in accelerator environments requires a MHz acquisition readout. This high repetition rate acquisition is currently achieved through two main approaches. One approach uses a spectrometer combining a grating and an ultrafast camera able to record up to 2.7 MHz [60], as detailed in section.2.3.2.1. The other approach employs the photonic time stretch technique, which enable an acquisition rate up to 88 MHz[74], as detailed in section2.3.2.2. Pro and cons of these two different readouts are detailed in summary.2.3.2.2.

2.3.2.1 Electro-optic spectral decoding using an ultrafast camera-based optical spectrum analyzer

Electro-optic spectral decoding (EOSD) uses a spectrometer to spatially disperse the chirped probe pulse's frequency components, enabling single-shot recording by a camera or a line array detector. However standard commercial cameras are typically limited to readout rates of a few hundreds of kHz⁴. This falls significantly short of the MHz repetition rates required to record every electron bunch produced by accelerators. To overcome this fundamental limitation, a notable solution designed to address this challenge is KALYPSO (**K**Arlsruhe **L**inear arra**Y** detector for MHz-re**P**etition rate **S**pectr**O**scopy), an ultrafast linear array detector [60]. KALYPSO enables a continuous readout frame rate of 2.7 MHz over 1024 pixels.

The integration of KALYPSO with EOSD schemes allows for high-repetition rate diagnostics of the longitudinal electron bunch.

For instance, at the KARA storage ring, KALYPSO has been instrumental in measuring longitudinal charge density profiles of electron bunches in both Coulomb field and far-field configurations [75]. Coulomb field measurements, where the EO crystal is placed within the vacuum chamber to probe the electron bunch's Coulomb electric field, have been reported in [52–54]. Far-field measurements, in contrast, involve positioning the EO crystal outside the vacuum chamber to probe the coherent synchrotron radiation or coherent transition radiation [55]. A typical setup for KALYPSO within an EOSD system is shown in Figure 2.4.(a). Furthermore, KALYPSO has been successfully deployed within EOSD systems at the EuXFEL [11] and FLASH [76], enabling real-time, single-shot monitoring of electron bunch profiles⁵.

⁴For real-time operation and line scan camera

⁵As it will be presented in chapter.4

The development and deployment of such high-speed camera systems are essential for single-shot spectral encoding techniques, enabling bunch-to-bunch characterization in accelerator environments. However, certain accelerator facilities demand even higher repetition rate performance. For example, the FEL oscillator FELBE operates at a 13 MHz rate. Therefore an alternative option is to use photonics methods for high-repetition rate readout.

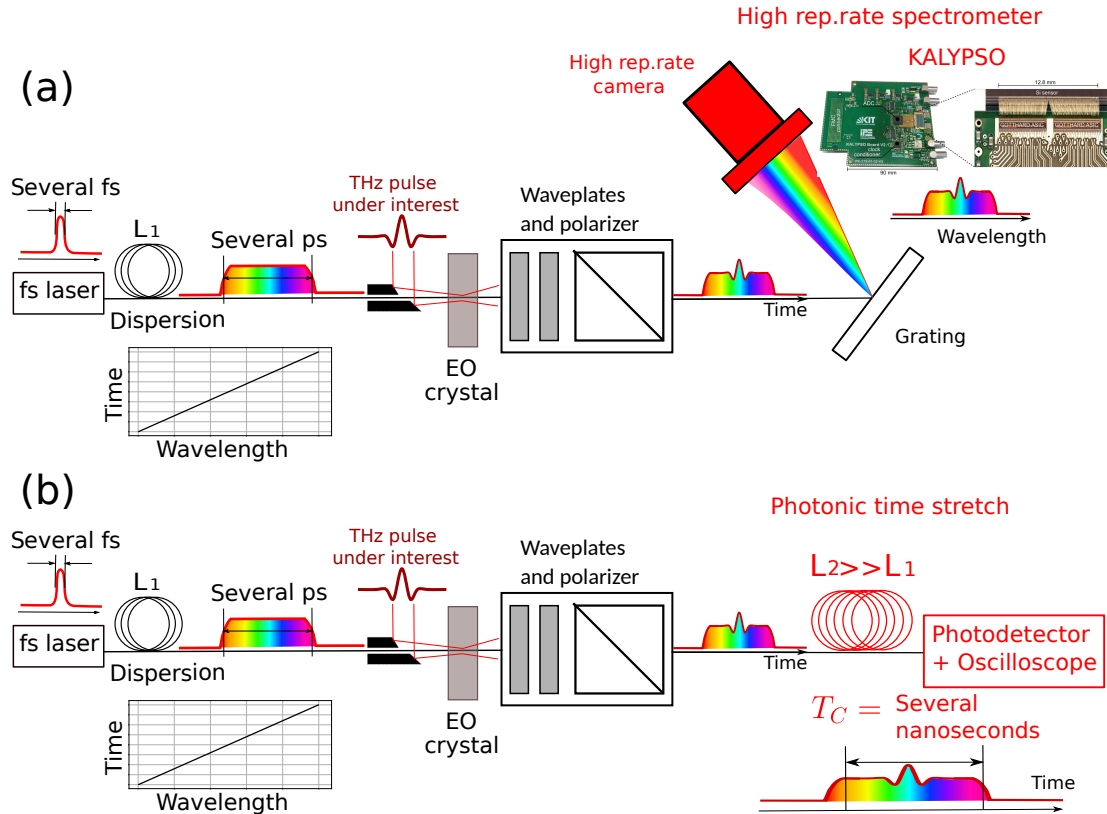


Figure 2.4: **Readouts methods for high-repetition rate electro-optic spectral encoding** (a) Schematic illustration of electro-optic spectral decoding utilizing an ultrafast camera, such as KALYPSO [60], to capture high-repetition-rate THz-encoded signals. (b) Schematic illustration of the photonic time stretch process integrated into a spectral encoding setup. The pre-chirped probe pulse interacts with the THz field in the EO crystal, and the modulated pulse then undergoes additional dispersion in a long fiber, stretching its temporal profile for detection by slower electronics.

2.3.2.2 Electro-optic spectral encoding using photonic time stretch

While ultrafast cameras like KALYPSO offer a compelling readout solution for high-repetition-rate spectral decoding technique, another approach to overcoming detector speed limitations is Photonic Time Stretch (PTS). This technique is based on optical dispersion to effectively slow down a fast optical signal, enabling its capture by slower electronic devices such as photodiodes and oscilloscopes with GHz bandwidth.

1. Photonic Time Stretch technique:

In the spectral encoding scheme, PTS is implemented after interaction between the THz field and the laser probe in the EO crystal, the modulated chirped probe pulse, which already carries the THz waveform encoded in its spectrum, is sent into a long fiber and undergoes a significant additional total dispersion $\mathcal{D}_2 \gg \mathcal{D}_1$. This process further slows down the modulated laser pulse, including the THz waveform encoded on it. Figure 2.4.(b) illustrates the photonic time stretch process within the spectral encoding setup.

The degree of temporal stretching is quantified by the magnification factor M [77], defined as the ratio of the stretched pulse duration T_2 to the original modulated pulse duration T_1 . This factor can be expressed as:

$$M = \frac{T_2}{T_1} = \frac{f_1}{f_2} = 1 + \frac{\mathcal{D}_2}{\mathcal{D}_1} \quad (2.4)$$

Where f_1 and f_2 refer to the bandwidth of the original and stretched signals, respectively. If the same type of optical fiber is used for the pre-chirp (i.e temporal window T_1) and the photonic time stretch, this simplified to :

$$M = 1 + \frac{L_2}{L_1} \quad (2.5)$$

Where L_1 and L_2 are the lengths of the respective fibers. Typically, $L_2 \gg L_1$ to achieve significant stretching⁶.

The total duration T_2 of the modulated laser signal after the stretching fiber, can be calculated by considering the initial spectral bandwidth $\Delta\lambda$ of the probe pulse and the total accumulated dispersion:

$$T_2 = \Delta\lambda(\mathcal{D}_1 + \mathcal{D}_2) \quad (2.6)$$

A key benefit of stretching the fast optical pulse by a factor M , is that the electronic bandwidth required to capture it is reduced by the same factor M .

Practical case : The impact of stretch factor M on THz detection

For instance, if a THz signal encoded on the probe pulse results in an optical modulation with a 3 THz bandwidth, and the pulse is stretched by $M = 300$, the equivalent electronic bandwidth required for detection would be reduced to a manageable 10 GHz.

This reduction enables the use of readily available and cost-effective GHz photodetectors and oscilloscopes, thereby effectively circumventing the speed limitations of commercial cameras.

⁶Note that PTS is a general technique, originally employed for GHz signal processing with relatively low stretch factors ($M < 10$) for GHz signal. In the context of THz diagnostics, however, the need to reduce the multi-terahertz bandwidth to a manageable electronic bandwidth necessitates a large stretch factor, requiring the condition $\mathcal{D}_2 \gg \mathcal{D}_1$

2. PTS combined with balanced detection:

To significantly enhance the sensitivity, defined as the minimum measurable electric field, of spectral decoding when employing PTS, a balanced detection can be performed. The EO signal $I_{EO}(t)$ which contains the THz signal information, manifests as a small alteration, typically only a few percent modulation, encoded upon the pre-chirped probe laser pulse.

The one-output time-stretch detection, as depicted in Figure 2.5.(a), recording this signal means that this essential information occupies a limited number of bins on the digitizer. This inherently constricts the detection's dynamic range and consequently degrades the signal-to-noise ratio.

In an analog balanced detection configuration, two identical photodiodes concurrently detect the two optical signals. Their resulting photocurrents are then differentially amplified by a gain G , as depicted Fig. 2.5.(b). This differential amplification effectively cancels common-mode fluctuations, including inherent laser shape variations, allowing the oscilloscope to record the difference signal with a substantially higher effective dynamic range. This approach has proven particularly valuable in THz time-domain spectroscopy (THz-TDS) enabling high-repetition-rate measurements at elevated sensitivities [61, 78, 79].

A notable trade-off is inherent to this process, while it effectively suppresses common-mode laser noise, it only provides the difference between the two polarization states. This precludes separate, simultaneous measurements of each individual polarization, a capability that will become mandatory in Section.2.4.2.

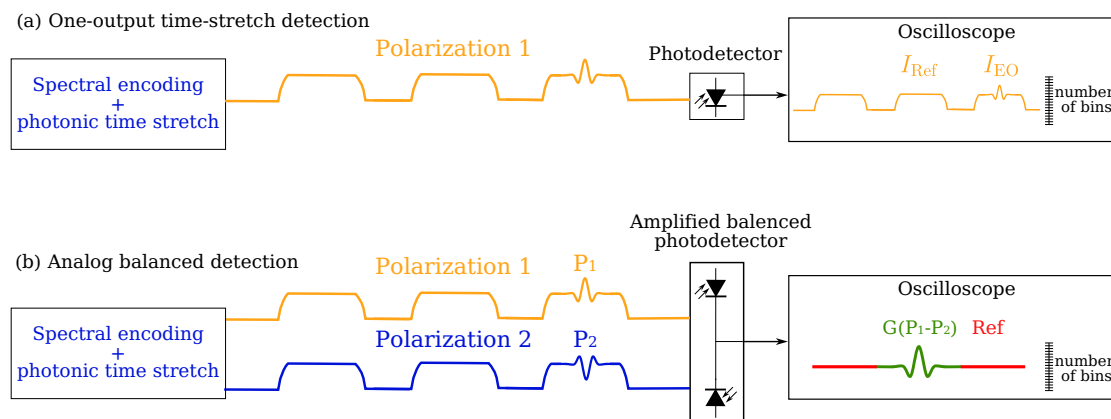


Figure 2.5: Classical vs. balanced detection in spectral encoding technique combined with photonic time stretch. (a) One-output time-stretch detection, where the small EO signal is recorded on top of a large background. (b) Balanced detection configuration, employing two photodiodes and differential amplification (gain G) to subtract the common-mode background, thus maximizing the effective dynamic range for the THz-encoded signal.

3. Historical context and applications :

The concept of photonic time stretch was initially developed to enhance the bandwidth of analog-to-digital converters (ADCs) for electronic signals, allowing for the digitization of ultra-broadband signals that would otherwise exceed the capabilities of conventional electronics [17, 77, 80, 81]. Its application was later introduced to electro-optic detection in THz [61, 78, 82–84], where the need for high-speed, single-shot THz measurements is paramount. Recently, PTS has seen increasing adoption in table-top THz experiments [85, 86], pushing the boundaries of repetition rates in laboratory settings, with demonstrated systems reaching up to 1.1 MHz acquisition speeds [56, 57].

4. Limitations of photonic time stretch technique

While PTS offers a powerful solution for high-repetition-rate measurements, it is not without its own set of limitations that must be carefully considered in system design:

- **Pulse overlap and repetition rate:** A critical constraint is that the stretched pulses must not overlap in the time domain. This implies that the total stretched pulse duration T_2 must be less than or equal to the inverse of the repetition rate $\frac{1}{f_{rep}}$.

$$T_2 < \frac{1}{f_{rep}} \quad (2.7)$$

Excessive stretching beyond this limit will lead to consecutive pulses overlapping, resulting in a loss of distinct signal information and an inability to resolve individual events. This fundamental trade-off exists between the achievable stretch factor and the maximum operational repetition rate.

- **Fiber losses:** Fiber losses are an essential concern in the long stretching fiber, as they limit the maximum achievable stretch factor and thus the bandwidth reduction. The figure of merit for selecting the optical fiber is the ratio of dispersion to loss. For comparison, at 1550 nm wavelength, the SMF28 fiber provide a loss of 0.17 dB/km for a dispersion of 18 ps/nm/km, while at 1030 nm, the HI1060 fiber exhibits a much higher loss of 1.5 dB/km for a dispersion of -53 ps/nm/km. Thus the lower loss at 1550 nm provides more advantageous dispersion/loss ratio, enabling a greater temporal stretch and a more effective electronic bandwidth reduction [84].
- **Higher order dispersion in the stretch fiber L_2 :** Accurate photonic time stretch requires perfectly linear dispersion in the stretch fiber, therefore, any higher-order dispersion creates a nonlinear time calibration that limits measurement accuracy.

An example can be found in appendix.B related to chapter.6.

Note on photonic time stretch and analogy with paraxial diffraction

The final step of the Photonic Time Stretch, in the case of a THz signal (i.e., $\mathcal{D}_2 \gg \mathcal{D}_1$) is based on the Dispersive Fourier Transform (DFT) [87]. This temporal mapping bears a strong analogy to paraxial diffraction in the spatial domain, which is governed by the Fresnel approximation [88]. Under the condition of large dispersion (the "far-field" condition, or Fraunhofer diffraction), the DFT simplifies, making the stretched pulse's temporal waveform a direct representation of the original pulse's optical spectrum. Experimental evidence of DFT is presented in Chapter.6, where Figure 6.4 shows the optical spectrum recorded with an Optical Spectrum Analyzer (OSA), and Figure 6.6(a) displays the corresponding temporal laser profile obtained using PTS, demonstrating that both measurements represent the same underlying optical spectrum.

Summary: Electronic vs. Photonic Readout for High-Speed THz Diagnostics

In summary, both spectral decoding using ultrafast cameras like KALYPSO and photonic time stretch offer compelling solutions for high-repetition-rate, single-shot electro-optic detection, with each presenting trade-offs. The KALYPSO camera provides a direct, high-speed spectral readout, currently operating up to 2.7 MHz. In contrast, PTS offers the advantage of pushing the acquisition rate into the hundreds of MHz range by using readily available commercial devices. This high speed comes with a fiber loss limitation, however, requiring the use of a 1550 nm probe laser. Thus, the choice is determined by the required repetition rate and probe laser wavelength, one would typically use KALYPSO for 1030 nm up to 2.7 MHz, but if the repetition rate is even higher, then PTS becomes mandatory. Experiment measurements performed at DESY (2.26 MHz) in chapter. 4 are made using spectral decoding with the 2.7 MHz KALYPSO at 1030 nm. While experiment measurements performed at FELBE (13 MHz) in chapter.6 are made using PTS combined with balanced detection at 1550 nm.

Building upon our capability to record single-shot THz electric fields at MHz repetition rates, the following section will explore the ultimate temporal resolution limits achievable in our experimental setup.

2.4 Time resolution limitation and phase diversity solution

While electro-optic spectral decoding and photonic time-stretch techniques enable single-shot THz waveform acquisition at MHz repetition rate, this advantage comes with an inherent trade-off in temporal resolution. This limitation arises from the use of a chirped probe pulse, as detailed in section.2.4.1. Fortunately, a recent novel approach, Diversity Electro-Optic Sampling (DEOS), has been developed to overcome this resolution limit, as explained in section.2.4.2. More details about the time resolution limitation and the DEOS solution can be found in this paper [18].

2.4.1 Spectral encoding time resolution limitation

1. Time resolution limitation due to chirped probe pulse

A clear demonstration of the time resolution limitation can be seen in numerical simulations where, even under ideal conditions (i.e., without considering electro-optic crystal bandwidth limitations), the fidelity of the reconstructed THz waveform is compromised for shorter pulses. Figure 2.6 illustrates this point. A $T_{THz} = 2$ ps long Gaussian THz pulse is faithfully encoded onto the chirped laser pulse spectrum, while a shorter $T_{THz} = 200$ fs pulse exhibits significant distortions. This phenomenon is a direct consequence of the intricate

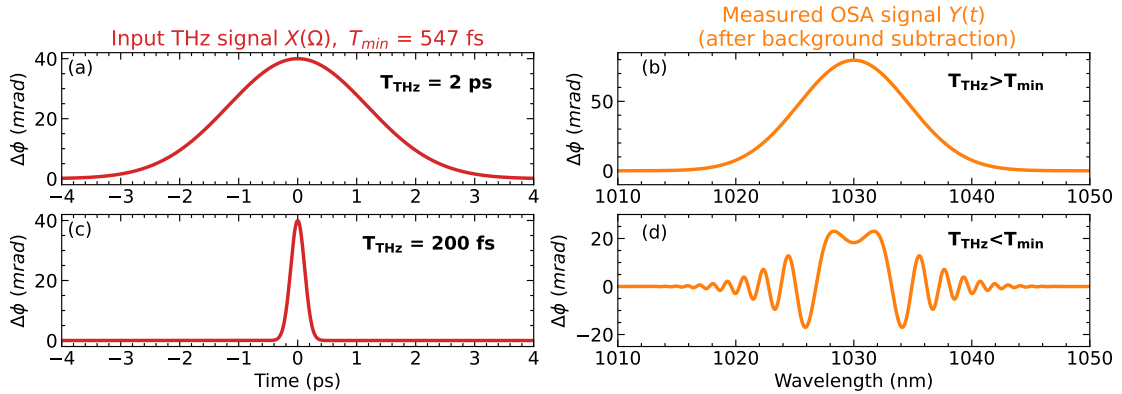


Figure 2.6: Numerical simulation of spectral encoding temporal resolution limitation due to chirped probe pulse. The figure compares the fidelity of THz waveform encoding onto a chirped probe pulse under ideal conditions (i.e., neglecting electro-optic crystal bandwidth effects). (a) Input $T_{THz} = 2$ ps gaussian THz pulse. (c) Input $T_{THz} = 200$ fs gaussian THz pulse. (b) The $T_{THz} = 2$ ps THz pulse, being longer than the minimum temporal resolution $T_{min} = 547$ fs (see equation.2.8), is accurately reproduced in the recorded optical spectrum. (d) The $T_{THz} = 200$ fs THz pulse, falling below $T_{min} = 547$ fs, exhibits distortions in the recorded optical spectrum, highlighting the limitation of spectral encoding for resolving pulse shorter than the theoretical minimum temporal resolution T_{min} .

interplay between the probe pulse's inherent bandwidth and its deliberate temporal stretching.

Sun *et al* [89] first elucidated the temporal resolution limits of spectral encoding. Their seminal analysis demonstrated that a faithful encoded THz waveform is only guaranteed if the desired temporal resolution is larger than a specific minimum value, T_{min} , given by :

$$T_{min} = \sqrt{T_0 T_1} \quad (2.8)$$

Here T_0 represents the Fourier transform limit duration of the unchirped probe pulse, which is the shortest possible duration corresponding to its full spectral bandwidth. In essence, T_0 sets the absolute finest temporal features that the original unchirped probe pulse can inherently resolve. A shorter T_0 implies a broader spectral bandwidth, thus providing the potential for higher temporal resolution.

Conversely, T_1 is the temporal duration of the chirped probe pulse as it interacts with the THz field within the electro-optic crystal. This T_1 effectively defines the "temporal window" over which the THz waveform is mapped onto the probe's spectrum.

Example : Minimum temporal resolution in spectral encoding technique

For instance, in the simulation where we consider a Fourier limit duration of $T_0 = 40$ fs and a temporal window of a duration of $T_1 = 10$ ps, the theoretical minimum resolvable temporal feature would be $T_{min} = 547$ fs. This calculation implies that THz pulses with features shorter than $T_{min} = 547$ fs might suffer from considerable distortion as demonstrated in Fig.2.6.(d).

Typically, the achievable temporal resolution with spectral encoding is on the order of hundreds of femtoseconds to several picoseconds, depending on the system parameters and optimization [11, 90].

2. Transfer function interpretation of temporal resolution limitation

This temporal resolution limitation of the spectral encoding method can be rigorously understood by using the concept of a transfer function. Essentially, the spectral encoding system acts as a filter that determines how faithfully the input THz signal is mapped to the detected optical output.

To formalize this, we can consider the birefringence induced by the THz electric field, $\Delta\phi(t)$, as the input $X(t)$ of our system (e.g., the pulses shown in Figure 2.6.(a) and (c)), and the spectrally encoded optical signal $Y(t)$ (e.g., represented

in Figure 2.6.(b) and (d)). In the Fourier domain, the relationship between this input $\tilde{X}(\Omega)$ and output $\tilde{Y}(\Omega)$ is governed by a transfer function, $H(\Omega)$.

$$\tilde{Y}(\Omega) = H(\Omega)\tilde{X}(\Omega) \quad (2.9)$$

Theoretical derivations, have shown that the transfer function for spectral encoding exhibits a characteristic cosine shape [18, 77, 91]:

$$H(\Omega) = 2m \cos(B\Omega^2 + \phi) \quad (2.10)$$

This specific form of transfer function was also observed in photonic time stretch digitizers [92].

Here, $2m$ is related to the amplitude and phase modulation induced by the instantaneous THz electric field $E_{THz}(t)$. The parameter $B = \frac{1}{2C} = \beta_2 L_1$ is associated with the dispersion of the chirped probe pulse, where C is the chirp rate of the laser and β_2 is the second-order dispersion coefficient of the pre-chirp fiber with length L_1 . The phase term ϕ is dependent on α , the orientation of the THz field with respect to the $[-110]$ direction of the EO crystal, see ref [18].

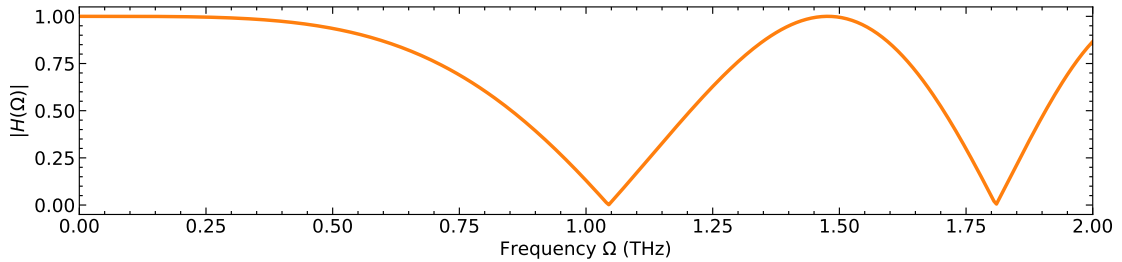


Figure 2.7: Transfer function of a spectral encoding system (plot of the 2.10 equation): The points where $H(\Omega)$ approaches zero indicate frequencies at which the spectral encoding system completely loses spectral information, leading to temporal signal distortion and limiting the achievable temporal resolution. This illustrates the frequency-domain filtering effect.

Parameters: $m = \frac{1}{2}$, $B = 1.44 \text{ ps}^2$ and $\phi = 0$.

The typical shape of this transfer function, clearly showing these information-losing zeros, is displayed in Figure 2.7. The critical feature of this transfer function is its oscillatory nature, which introduces zeros at specific THz frequencies. These zeros represent frequencies at which the system completely loses information, leading to a perfect null in the output spectrum, regardless of the input THz field strength at that frequency. As the input THz pulse, $E_{THz}(t)$, becomes shorter in the time domain, its corresponding spectrum $X(\Omega)$ becomes broader. This broadened spectrum inevitably encompasses these zeros within the system's transfer function, $H(\Omega)$. Consequently, a portion of the THz signal's spectral information is irrevocably lost, resulting in the observed distortions and a reduction in temporal fidelity in the reconstructed waveform.

This direct consequence of the transfer function's frequency-dependent response provides a rigorous explanation for the empirically derived minimum temporal resolution, T_{min} , initially identified by Sun *et al* [89].

Understanding this frequency-domain filtering effect is crucial for optimizing spectral encoding systems and for developing advanced techniques, such as phase diversity [18], to address these inherent limitations.

2.4.2 Diversity Electro-Optic Sampling (DEOS) principle

To overcome the inherent temporal resolution limitation imposed by the $\sqrt{T_0 T_C}$ relationship in standard spectral encoding, a novel approach known as Diversity Electro-Optic Sampling (DEOS) has been developed.

This technique, notably explored by E. Roussel *et al.* [18], provides a pathway to retrieve THz waveforms with fidelities beyond the conventional T_{min} limit.

The core idea behind phase diversity is to go beyond the single measurement by utilizing additional information encoded in the probe pulse.

2.4.2.1 Standard Spectral Decoding: Shared Nulls in the Transfer Function

In a conventional spectral encoding setup, if one were to implement two detection channels by simply splitting the probe beam after the EO crystal and using identical polarization analysis stages, both channels would yield essentially the same signal with opposite sign allowing balanced detection. This is because they would be governed by the same transfer function $|H^{std}(\Omega)|$, and thus, would share the same problematic nulls.

$$\begin{aligned} H_1^{std}(\Omega) &= -H_2^{std}(\Omega) \\ Y_1^{std}(t) &= -Y_2^{std}(t) \end{aligned} \tag{2.11}$$

This scenario is related to the polarization orientation within the EO crystal with respect to the birefringence axis as depicted in Figure 2.9.(a) and the corresponding identical transfer functions in Figure 2.8.(a). Such a configuration provides no new information to mitigate the temporal resolution limitation.

2.4.2.2 Phase Diversity: Interleaved Nulls for Full Bandwidth Retrieval

The innovation of phase diversity lies in purposefully creating multiple outputs, each with a distinct and complementary transfer function. This is achieved by carefully designing the optical setup to capture multiple channels with specific, phase shifts. For instance, this can be realized by employing a different orientation of the EO crystal with respect to the probe beam polarization and the THz field. The phase diversity setup will generate two outputs, $Y_1(t)$ and $Y_2(t)$, from the same input THz field $X(t)$.

In the Fourier domain, these outputs would be related by:

$$\begin{aligned}\widetilde{Y}_1(\Omega) &= H_1(\Omega)\widetilde{X}(\Omega) \\ \widetilde{Y}_2(\Omega) &= H_2(\Omega)\widetilde{X}(\Omega)\end{aligned}\tag{2.12}$$

The critical requirement for phase diversity is that the transfer functions, $H_1(\Omega)$ and $H_2(\Omega)$ must have interleaved nulls, meaning that where $H_1(\Omega)$ has a null $H_2(\Omega)$ must have a non-zero response, and vice-versa.

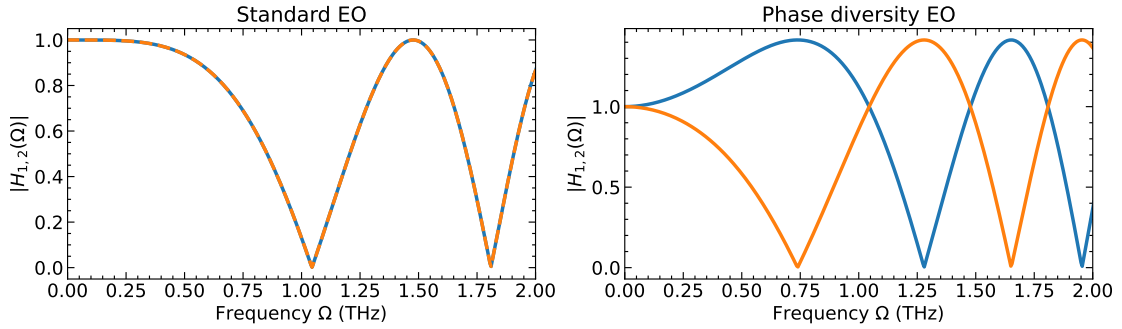


Figure 2.8: **Comparison of transfer functions in standard EOSD vs. DEOS :** (a) In standard EOSD, multiple detection channels would exhibit identical transfer functions $|H_1(\Omega)| = |H_2(\Omega)|$, leading to shared nulls and inherent temporal resolution limitations. (b) In phase diversity, the system is designed to generate two outputs with complementary transfer functions $|H_1(\Omega)|$ and $|H_2(\Omega)|$. The interleaved nulls ensure that when one channel's response is weak or zero at a given frequency, the other channel provides strong signal, thereby overcoming the information loss present in a single-channel system or standard EO configuration.

Parameters Standard EO: $m = \frac{1}{2}$, $B = 1.44 \text{ ps}^2$ and $\phi = 0$.

Parameters Phase Diversity EO: $m = \pm \frac{1}{\sqrt{2}}$, $B = 1.44 \text{ ps}^2$ and $\phi = \pm \frac{\pi}{4}$.

The optimized EO crystal configuration with respect to probe laser and THz polarization is depicted in Figure 2.9.(b). The two transfer function resulting from the optimized phase diversity configuration are presented in Fig. 2.8.(b), showing how the "blind spots" of one channel are covered by the "seeing spots" of the other. This complementarity ensures that, even when one output cannot retrieve the THz pulse due to a null in its transfer function, the other output contains the necessary information, enabling a more complete reconstruction. In essence, $H_1(\Omega)$ and $H_2(\Omega)$ are designed to be complementary, providing a diversity of information that is not available from a single channel, neither in the dual standard EO outputs configuration. This approach necessitates the use of signal processing algorithms to extract the full THz field information from the probe pulse's modified polarization state, ultimately enabling the faithful reconstruction of even ultra-short THz waveforms that would otherwise be distorted or unresolvable by conventional spectral encoding.

A typical setup for implementing phase diversity involves optical polarization components and EO crystal orientation that generate these distinct phase shifts for different detection paths, as shown in Figure 2.10.

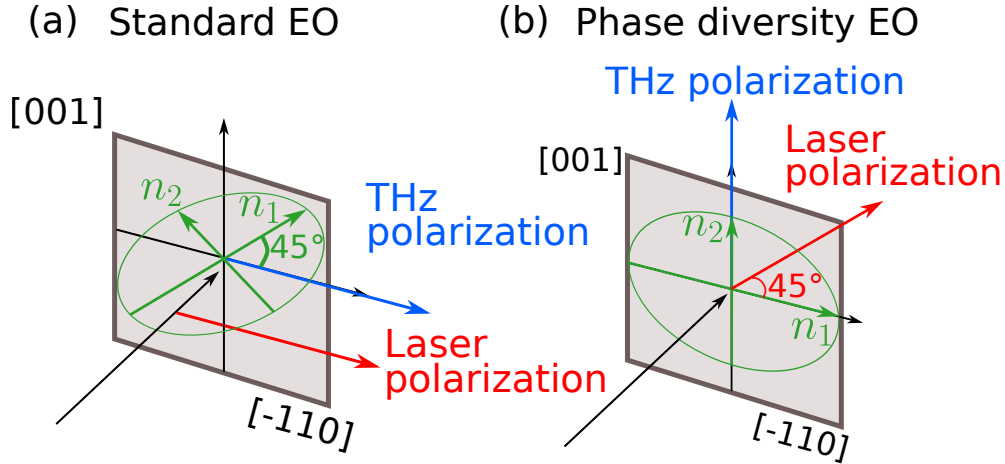


Figure 2.9: **Comparison of standard EOSD and phase diversity EO :** (a) In standard EOSD, the laser and THz polarization are set at 0° with respect to the crystallographic axis $[-110]$. (b) In phase diversity, the laser polarization is set at 45° and THz polarization is set at 90° with respect to the crystallographic axis $[-110]$.

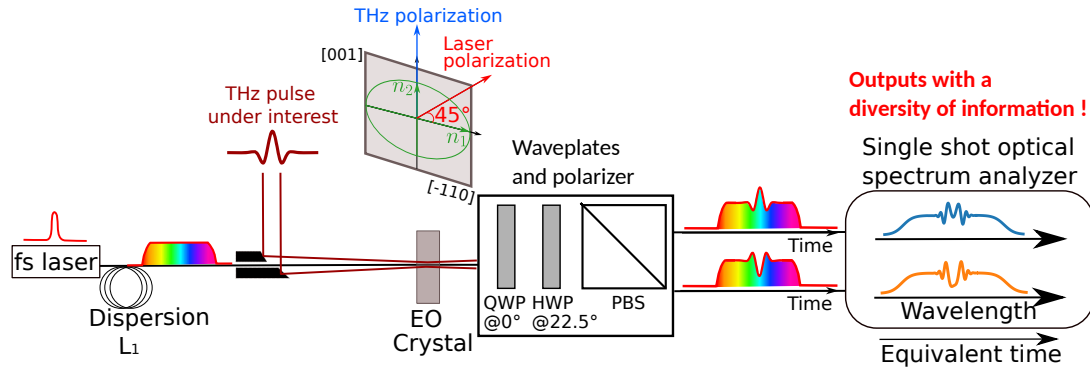


Figure 2.10: **Schematic setup of a phase diversity electro-optic sampling system:** The incoming THz field interacts with a chirped probe pulse in the EO crystal with proper polarization orientations. After the EO crystal, the probe pulse is split into two, each configured to produce a unique and complementary spectral modulation before being sent to ultrafast spectrometers or time-stretch detectors. This allows for the acquisition of multiple output signals with distinct transfer functions, which are then used in a reconstruction algorithm.

2.4.3 Reconstruction using the Maximum Ratio Combining (MRC) algorithm

The successful implementation of phase diversity relies on a reconstruction algorithm capable of combining the information from the multiple, complementary output channels to recover the true THz waveform.

The Maximum Ratio Combining (MRC) algorithm is a powerful technique well-suited for this purpose [93]. It is designed to optimally combine signals from multiple receivers, especially in scenarios where different channels may experience varying degrees of signal degradation or have complementary information, as is the case with the interleaved nulls in phase diversity.

Originating from telecommunication, MRC has been widely used in wireless communication to improve the performance of receivers and mitigate fading effects due to interference [94].

Its application was later extended to photonic systems, notably implemented with photonic time stretch analog-to-digital converters (ADCs) to achieve high-bandwidth RF signal acquisition [92].

1. Principle of the MRC algorithm for DEOS :

The MRC algorithm utilizes both transfer function $H_{1,2}(\Omega)$, to compute and retrieve the true phase retardation spectrum $\Delta\tilde{\phi}^{retr}(\Omega)$.

The combined retrieved spectrum is given by:

$$\Delta\tilde{\phi}^{retr}(\Omega) = \frac{H_1(\Omega)\tilde{Y}_1(\Omega) + H_2(\Omega)\tilde{Y}_2(\Omega)}{H_1^2(\Omega) + H_2^2(\Omega)} \quad (2.13)$$

Here, $\tilde{Y}_1(\Omega)$ and $\tilde{Y}_2(\Omega)$ represent the normalized and subtracted measured output signals from the two channels.

In the special case of phase diversity, $H_1(\Omega)$ and $H_2(\Omega)$ the two transfer function are given by :

$$\begin{aligned} H_1(\Omega) &= \sqrt{2}\cos(B\Omega^2 + \phi) \\ H_2(\Omega) &= -\sqrt{2}\cos(B\Omega^2 - \phi) \end{aligned} \quad (2.14)$$

It is critical to note that the denominator in Equation 2.13 is $H_1^2(\Omega) + H_2^2(\Omega)$. This sum is essential as it ensures that if one transfer function exhibits a null at a specific frequency, the other ideally does not. This arrangement prevents division by zero and facilitates a robust combination of information across the spectrum.

2. Fitting parameters B and ϕ :

In the MRC algorithm a practical difficulty arises from the fact that the parameters B (related to the chirped probe pulse dispersion) and ϕ (related to the polarization orientation within the EO crystal) in Eq.2.14 are generally unknown experimentally. This unknown makes the determination of the transfer functions difficult. To overcome this issue, E.Roussel *et al* developed an algorithm designed to recover these parameters directly from the experimentally measured spectra $\tilde{Y}_1(\Omega)$ and $\tilde{Y}_2(\Omega)$ thus enabling an accurate retrieval of the induced phase shift [18].

The algorithm begins by considering a predefined range of values for B and ϕ . For each combination within this parameter space, a set of hypothetical transfer functions, $H_1^{retr}(\Omega)$ and $H_2^{retr}(\Omega)$ is calculated using Eq. 2.14.

Using these hypothetical transfer functions, a reconstructed THz phase shift spectrum, $\Delta\tilde{\phi}^{retr}(\Omega)$, is then calculated for every single combination of B and ϕ . This calculation is achieved by applying Equation 2.13 to the measured output signals :

$$\Delta\tilde{\phi}^{retr}(\Omega) = \frac{H_1^{retr}(\Omega)\tilde{Y}_1(\Omega) + H_2^{retr}(\Omega)\tilde{Y}_2(\Omega)}{H_1^{retr^2}(\Omega) + H_2^{retr^2}(\Omega)} \quad (2.15)$$

With the hypothetical $\Delta\tilde{\phi}^{retr}(\Omega)$ obtained, the expected spectra of both polarization signals, $\tilde{Y}_1(\Omega)$ and $\tilde{Y}_2(\Omega)$ can be reconstructed using the relationship :

$$\begin{aligned} \tilde{Y}_1^{retr}(\Omega) &\approx H_1^{retr}(\Omega)\Delta\tilde{\phi}^{retr}(\Omega) \\ \tilde{Y}_2^{retr}(\Omega) &\approx H_2^{retr}(\Omega)\Delta\tilde{\phi}^{retr}(\Omega) \end{aligned} \quad (2.16)$$

A classical least-squares fitting approach is employed to find the reconstruction error ε . This error quantifies the discrepancy between the reconstructed signals and the experimentally measured signals :

$$\varepsilon^2 = \int_{-\infty}^{+\infty} (|\tilde{Y}_1(\Omega) - \tilde{Y}_1^{retr}(\Omega)|^2 + |\tilde{Y}_2(\Omega) - \tilde{Y}_2^{retr}(\Omega)|^2) d\Omega \quad (2.17)$$

The algorithm systematically iterates through the parameter space of B and ϕ , identifying the combination that minimizes this error ε . This minimum error corresponds to the optimal values for B and ϕ .

3. Final reconstruction:

Once the optimal values of B and ϕ are determined, the correct transfer functions $H_1(\Omega)$ and $H_2(\Omega)$ can be precisely established. This allows for the final, distortion-free retrieval of the induced phase shift, $\Delta\tilde{\phi}^{retr}(\Omega)$ using Equation 2.13.

Finally, by applying the Inverse Fourier Transform (IFT) to $\Delta\tilde{\phi}^{\text{retr}}(\Omega)$, one obtains the temporal profile of the THz electric field, $\Delta\phi^{\text{retr}}(t)$.

This algorithm is summarized in Figure 2.11, which illustrates the iterative process of parameter estimation and signal reconstruction. Figure 2.12 provides a numerical representation of how the MRC algorithm effectively combines the two complementary channels to achieve a distortion-free recovery of the input $\Delta\phi(t)$.

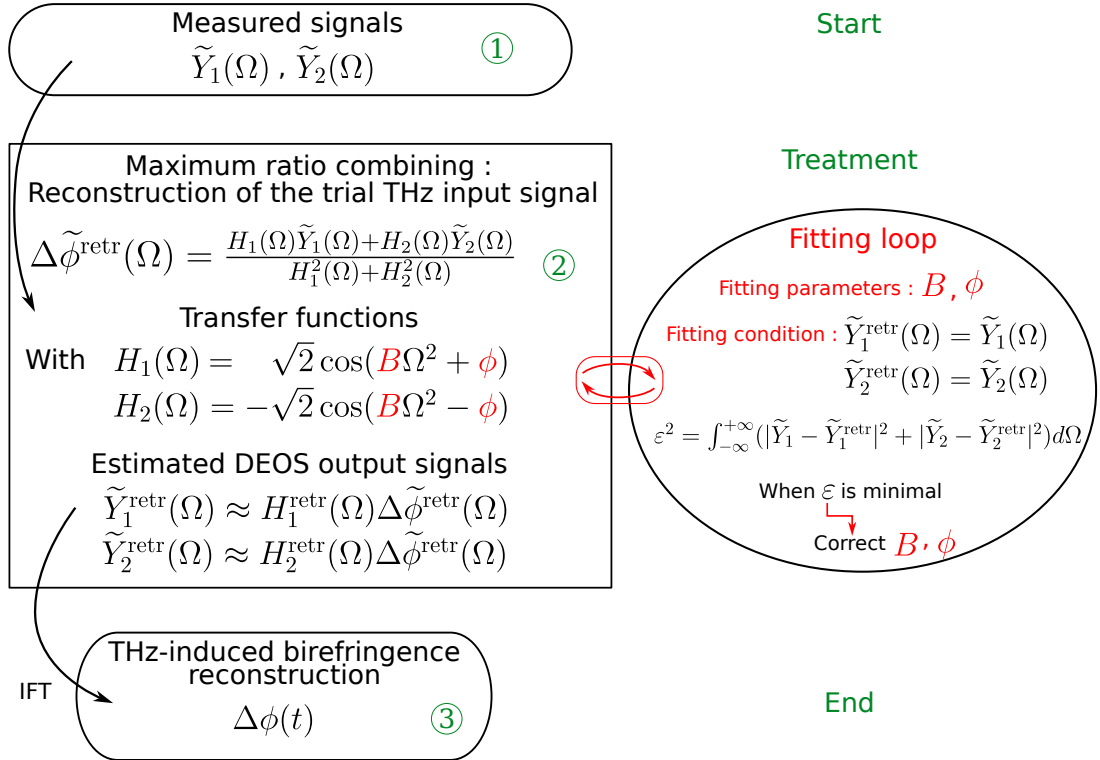


Figure 2.11: **Complete workflow of the MRC algorithm** : The algorithm iteratively determines the optimal B and ϕ values by minimizing the difference between experimentally measured signals and those reconstructed using iterative parameters. Once optimized, the true THz waveform is retrieved.

4. Applications:

By doing so, one can effectively overcome the inherent temporal resolution limitations associated with the cosine-shaped transfer function of single-channel EOSD, even for long chirped pulses or analysis windows.

This phase diversity technique has been demonstrated to be highly efficient for single-shot EOS measurements of THz signals. Its utility has been particularly validated in the context of particle accelerators, notably by measuring the shape of a 200 fs RMS electron bunch at the European X-ray Free Electron Laser (EuXFEL) [18]. More recently, it has also proven effective in measuring coherent transition radiation (CTR) THz spectra in single-shot at the TeraFERMI facility [95]. More detailed and information about phase diversity method and MRC algorithm applied to DEOS can be found in this paper [18].

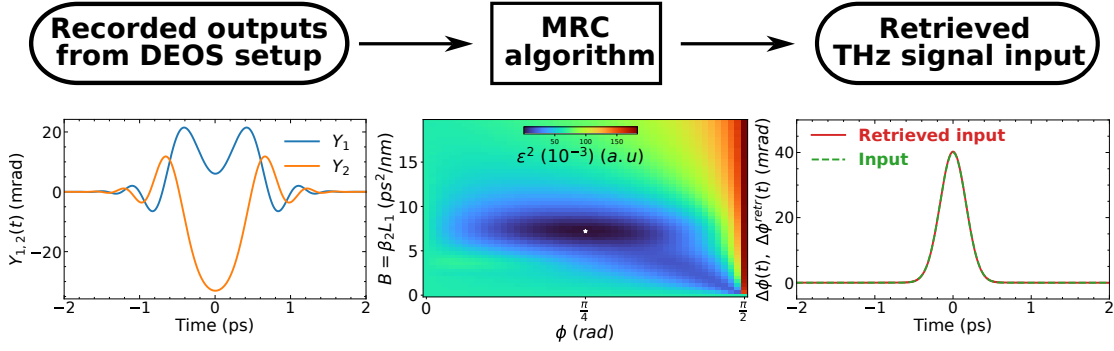


Figure 2.12: **Numerical simulation of THz signal reconstruction using the MRC algorithm with phase diversity:** (a) Example of two normalized and subtracted measured outputs $\tilde{Y}_1(\Omega)$ and $\tilde{Y}_2(\Omega)$, showing the spectral distortions introduced by the individual transfer functions. (b) Colormap showing fitting error ϵ according to different combination of B and ϕ . A minimum is found at optimal B and ϕ parameters. (c) The successfully reconstructed THz spectrum $\Delta\phi^{retr}(\Omega)$ after applying the MRC algorithm, highlighting the recovery of the full spectral information by combining the complementary channels.

5. Conclusion:

By addressing the limitations of the system's transfer function, phase diversity opens up new possibilities for high-resolution, single-shot THz diagnostics, particularly in accelerator facilities where detailed characterization of sub-picosecond phenomena is required.

Nevertheless, the fidelity of the measurement can still be altered by other effects such higher-order dispersion in the dispersive element before the EO crystal.

2.5 Limitations in MRC reconstruction algorithm

The preceding subsection demonstrated a significant enhancement in the temporal resolution and fidelity of single-shot THz measurements, by combining the diversity electro-optic technique with the Maximum Ratio Combining algorithm. Nevertheless, despite these advancements, this section reveals that spectral encoding remains limited by nonlinear dispersion occurring within the chirped probe laser.

1. Issues Arising from Nonlinear Dispersion Relation :

The optimal performance of the MRC algorithm fundamentally relies on a strictly linear time-to-wavelength mapping of the chirped probe pulse and a small THz amplitude signal⁷.

This linear mapping depends on the dominance of the group-velocity dispersion (β_2) over higher-order terms during the initial chirping process. In practical setups, this linearity is compromised when third-order dispersion (β_3) becomes important. This occurs when its characteristic dispersion length ($L'_D = \frac{T_0^3}{|\beta_3|}$) is comparable to or shorter than the second-order dispersion length ($L_D = \frac{T_0^2}{|\beta_2|}$), where T_0 is the probe pulse duration. This condition is concisely expressed as $T_0 \frac{|\beta_2|}{|\beta_3|} \leq 1$. A non-negligible value for this ratio introduces significant nonlinearities into the time-wavelength relationship.

In this case, applying the MRC algorithm under nonlinear time-wavelength conditions leads to a degradation in reconstruction fidelity and a loss of temporal resolution, especially for long THz signals or multi-pulse sequences. Numerical simulations, for instance, clearly illustrate how the MRC algorithm struggles to accurately reconstruct THz waveforms in the presence of non-negligible TOD, as introduced in Fig. 2.13 and further demonstrated in Chapter 3.

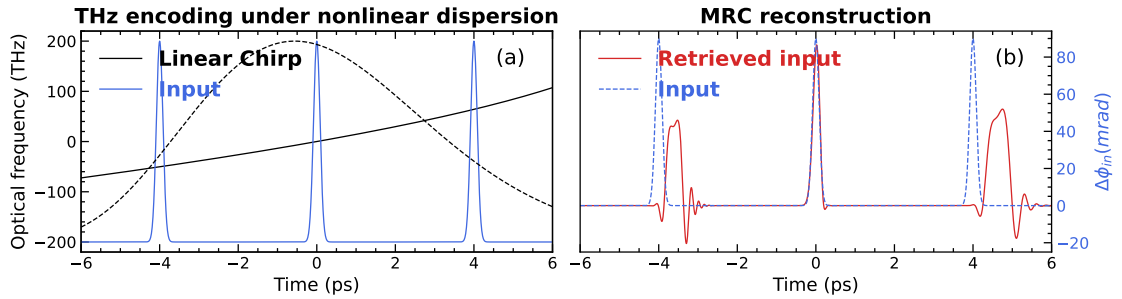


Figure 2.13: **Numerical evidence of MRC algorithm's fidelity degradation under nonlinear dispersion relation :** (a) Three 200 fs THz input pulses, separated by 4 ps, encoded under a nonlinear time-wavelength relationship (black line). The dashed line represent the chirped probe laser temporal profile. (b) Reconstructed signals obtained by the MRC algorithm. Only one peak can be accurately reconstruct.

⁷Which is always the case in our experiment

2. Real-world implications :

The experimental time calibration within the state-of-the-art DEOS setups at DESY (at European XFEL and FLASH), reveals a non-linear time-wavelength relationship due to higher-order dispersion as shown in Fig.2.14.(b).

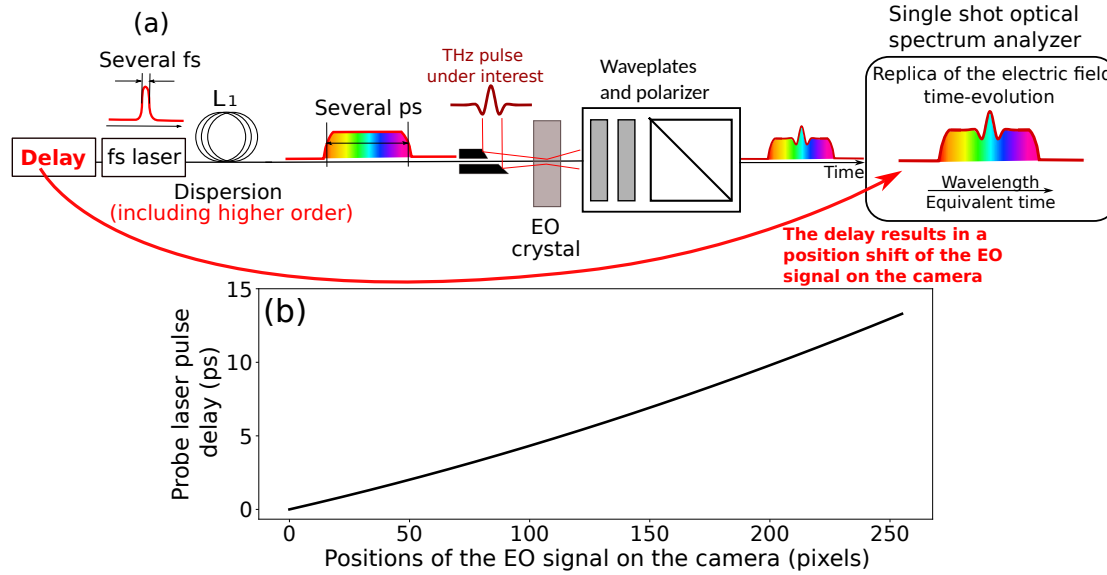


Figure 2.14: **Experimental time-wavelength calibration in a DEOS setup.** (a) Schematic of the electro-optic setup illustrating the method for experimental time-wavelength calibration. A sub-picosecond delay is introduced to the chirped probe laser relative to the THz pulse, causing a corresponding shift of the encoded THz signal (representing the electron bunch) on the measured laser spectrum output. (b) Resulting experimental time calibration curve, plotted against pixels (equivalent to wavelength). This curve highlights a significant non-linear relationship between time and wavelength, which is indicative of higher-order dispersion effects in the system.

Therefore, the application of EO techniques in real-world diagnostics, necessitates a solution for taking into account the third order dispersion effects, to restore the fidelity in our EO measurements. The development of a novel, self-adaptive dispersion algorithm to overcome this limitations is the main objective of Chapter. 3.

Chapter summary

This chapter has provided an overview of electro-optic techniques, indispensable tools for ultrafast THz diagnostics. We began by detailing the principles of conventional EOS, appreciating its high temporal resolution while noting its inherent non single-shot limitation. The discussion then evolved to electro-optic spectral encoding, highlighting its ability to enable single-shot measurements. We explored different readout methods, focusing on ultrafast cameras like KALYPSO and the photonic time stretch technique, both pivotal in pushing spectral encoding capabilities to MHz repetition rates, essential for accelerator-based light sources.

Despite these advancements, we thoroughly examined the intrinsic temporal resolution limitations of spectral encoding technique stemming from the chirped probe pulse and the system's transfer function, alongside challenges posed by non-linear dispersion. To overcome these critical barriers and achieve high-fidelity, sub-picosecond THz waveform and electron bunch shape measurements, the chapter underscored the necessity of a new conceptual approach for signal processing.

Chapters 3 and 4 will detail the development of a novel, self-adaptive dispersion algorithm to overcome MRC limitations under non-linear chirp conditions, and its experimental validation at DESY's EuXFEL and FLASH facilities for high-fidelity THz pulse and electron bunch shape reconstruction.

Part II

High-Fidelity Broadband THz Diagnostics: Diversity Electro-Optic Detection Combined with Self-Adaptive Dispersion (SAD) Algorithm for Routine Accelerator Diagnostics at DESY

Chapter 3

Limitations of the MRC algorithm and development of a new general algorithm for high-fidelity reconstruction using nonlinear chirp probes

In physics, you don't have to go around making trouble; everything is already a mess.

– Richard Feynman

3.1 Introduction

Single-shot electro-optic diagnostics based on spectral encoding using the phase diversity technique represent a promising approach to meet the demanding accelerator diagnostic requirements. The combination of high-speed readouts (ultrafast cameras or Photonic Time Stretch) and this approach enables single-shot THz measurements with hundreds of femtosecond resolution at MHz acquisition rates. However, the reliance on spectral encoding introduces a limitation on the fidelity of the signal reconstruction due to the vulnerability of the chirped probe pulse to non-linear dispersion, in particular the Third-Order Dispersion (TOD).

This non-linear chirp fundamentally breaks the core assumptions of the Maximum Ratio Combining (MRC) signal reconstruction model. The MRC algorithm, which relies on a simple conceptual framework based on a linear-time-invariant system model characterized by a transfer function, is conceptually ill-suited to handle the nonlinear time-to-wavelength mapping introduced by TOD. The resulting distortion on long or multi-pulses THz signals is severe enough to render the reconstructed THz signal unreliable for accelerator diagnostics.

To address this challenge, one of the tasks of this thesis was to rethink the problem from its fundamental principles, moving beyond the limiting conceptual framework of the linear-time-invariant system model. This chapter, therefore, introduces the Self-Adaptive Dispersion (SAD) algorithm, a novel solution that breaks entirely from the linear-time-invariant paradigm. The chapter begins by detailing the underlying limitations of the MRC approach in the presence of TOD in Section. 3.2. This section uses numerical simulations to show the degradation in reconstruction fidelity that occurs when non-linear dispersion is not accounted for. Following this, Section. 3.3 introduces the conceptual and mathematical framework of the SAD algorithm. This new approach exploits a powerful analogy between the phase diversity setup and a Mach-Zehnder Modulator, allowing the full complex amplitude of the chirped probe pulse (amplitude, phase, and dispersion relation) to be retrieved simultaneously with the THz signal. This section concludes by providing numerical validation, demonstrating the SAD algorithm's capacity to accurately reconstruct THz pulses even under nonlinear chirp conditions. Finally, Section. 3.4 provides a table-top experimental proof-of-principle, comparing the SAD reconstruction to a conventional THz scan to demonstrate accurate THz signal retrieval.

Note on computational model parameters used in this chapter

All numerical results and analyses presented in this chapter are derived from simulations conducted under the conditions outlined in Table 3.1. These parameters define the properties of the laser source, the optical stretching element, the incident THz pulse, and the electro-optic detection crystal, forming our computational model.

Laser parameters	
λ_0	1030 nm
$\Delta\lambda$	39 nm FWHM
τ_L	40 fs FWHM
Stretch parameters	
β_2	24 ps ² /km
β_3	0 or 0.1 ps ³ /km
L_1	3 m
THz parameters	
E_{THz}	2 kV m ⁻¹
$\Delta\phi^{phd}$	90 mrad
τ_{THz}^{FEL}	0.2 ps FWHM
EO crystal (GaP)	
n_0	3.1
r_{41}	0.97 pmV ⁻¹
d	5 mm

Table 3.1: Numerical simulation parameters

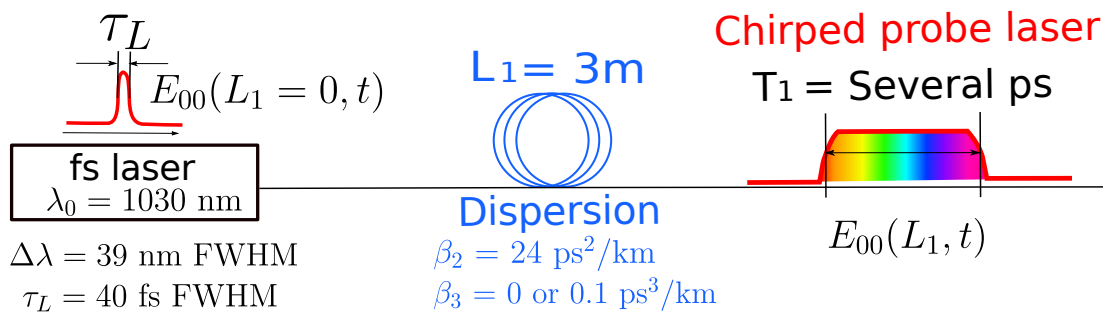


Figure 3.1: Illustration of the Numerical Generation of the Chirped Probe Laser Pulse for Spectral Encoding. A femtosecond (fs) laser pulse, characterized by a center wavelength $\lambda_0 = 1030 \text{ nm}$, and a Fourier-limit pulse duration of $\tau_L = 40 \text{ fs FWHM}$, is stretched by propagating through a dispersive fiber¹ of length $L_1 = 3 \text{ m}$. The fiber imparts² second-order dispersion $\beta_2 = 24 \text{ ps}^2/\text{km}$, and optionally, third-order dispersion β_3 (either 0 or $0.1 \text{ ps}^3/\text{km}$). This process transforms the short input pulse $E_{00}(L_1 = 0, t)$ into a linearly (or nonlinearly) chirped probe pulse $E_{00}(L_1, t)$ with a duration T_1 of several picoseconds, which is then used for spectral encoding.

3.2 Limitations of the MRC algorithm : Mechanisms of failure under nonlinear chirp conditions

This section provides a detailed analysis of the MRC algorithm's behavior under nonlinear chirp conditions, explaining its failure. The failure occurs when the Third-Order Dispersion (TOD) becomes significant, specifically when its characteristic length, L_D' , is comparable to or shorter than the second-order dispersion length L_D [96].

3.2.1 Time invariance breaking by third-order dispersion

3.2.1.1 Numerical evidence of time invariance breaking

The MRC algorithm fundamentally relies on the concept of transfer functions, which are only valid in linear-time invariant (LTI) systems. This implies that the system's impulse response must remain consistent regardless of when the input signal (e.g., an electron bunch or THz signal) is applied. In the spectral decoding method, the input signal is encoded onto a chirped probe laser pulse. To model this chirped probe, we consider the propagation of a femtosecond laser pulse through a dispersive medium, such as an optical fiber¹ of length L_1 . This stretching process is illustrated in Figure. 3.1. The chirped probe laser pulse is defined by its electric field $E_{00}(L_1, t)$ under the slowly varying approximation²:

$$E_{00}(L_1, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{E}_{00}(L_1 = 0, \omega) \exp(i\phi_{\text{disp}}(L_1, (\omega))) d\omega \quad (3.1a)$$

$$\text{With } \phi_{\text{disp}}(L_1, (\omega)) = \frac{\beta_2}{2} \omega^2 L_1 + \frac{\beta_3}{6} \omega^3 L_1 + O(\omega^4 L_1) \quad (3.1b)$$

Here, $\tilde{E}_{00}(L_1 = 0, \omega)$ represents the spectrum of the input femtosecond laser probe, β_2 is the group velocity dispersion (GVD) coefficient, and β_3 is the third-order dispersion (TOD) coefficient. Therefore, the time t is strongly linked with the wavelength due to this dispersion.

To investigate the effects of dispersion, a computational model was employed. When only second-order dispersion ($\beta_3 = 0$) is considered, a linear relationship exists between time and wavelength. In this scenario, encoding three identical short pulses (approximated as Dirac delta functions) at different times onto the linearly chirped probe results in identical impulse responses (see Fig.3.2(a) and (b)).

¹This model remains general, as it can be extended to represent the effect of any dispersive optical element.

²Here, we assume zero nonlinear effects ($\gamma = 0$) in the optical fiber

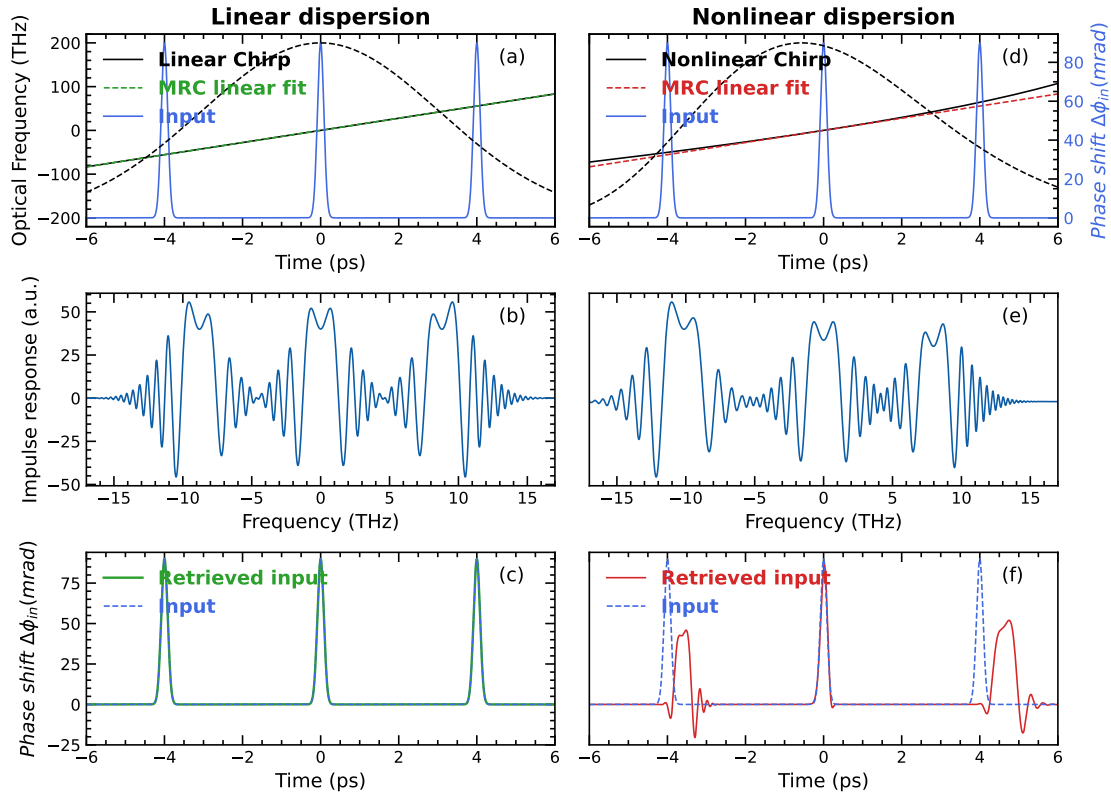


Figure 3.2: Numerical demonstration of time invariance breaking in the EO system and its effect on MRC reconstruction : Panels (a)-(c) Results under pure second-order dispersion ($\beta_3 = 0$). The dashed line in (a) and (d) represents the temporal profile of the chirped probe laser. (a) Three input pulses of 200 fs with 4 ps separation under a linear time-wavelength relationship. (b) Resulting identical impulse responses from the system. (c) Reconstruction of all three input pulses by the MRC algorithm, confirming time-invariance. Panels (d)-(f) Results with third-order dispersion present ($\beta_3 \neq 0$). (d) Three input pulses of 200 fs with 4 ps separation under a nonlinear time-wavelength relationship. (e) Resulting non-identical impulse responses, exhibiting varied amplitude and chirp. (f) Failure of the MRC algorithm to reconstruct the input pulses due to the system's time-invariance breaking, particularly for those off the linear fit (d).

This confirms that the EO system effectively behaves as an LTI system under these conditions, allowing the MRC algorithm to fully reconstruct the input pulses (Fig.3.2(c)).

However, in the general case, the presence of higher-order dispersion in the chirped probe laser introduces a nonlinear relationship between time and wavelength (Fig.3.2(d)). The same computational model, when incorporating third-order dispersion, reveals distinct impulse responses for the three pulses Fig3.2(e). This critical observation highlights the time invariance breaking within the EO system. As a direct consequence, applying the MRC algorithm prevents the complete reconstruction of the three short input pulses (Fig.3.2(f)). Specifically, the MRC algorithm's dispersion fitting parameter B converges to a linear approximation of the nonlinear time-wavelength relationship.

This explains the relatively effective reconstruction of the central pulse, where the linear fit closely aligns with the actual nonlinear dispersion curve, while the other two pulses exhibit unsuccessful reconstruction with an asymmetric broadening, directly attributable to the influence of third-order dispersion.

Therefore, the EO system can no longer be accurately described by a transfer function, leading to the ineffectiveness of the MRC algorithm. To overcome these limitations imposed by higher-order dispersion, two potential solutions were considered:

- **Hardware solution:** This involves designing the chirped probe laser to achieve a chirp as linear as possible, through a complex and time-consuming design using combinations of fibers and gratings. A major drawback lies in the lack of reconfiguration and flexibility: since the hardware compensation is static, the entire design process must be repeated every time the temporal window T_1 is changed. This makes the solution setup-specific and non-portable.
- **Software solution:** This entails developing a new algorithm capable of accounting for higher-order dispersion, making it universally applicable to any EO setup, including for re-analysis of previously acquired data.

Given these considerations, we opted for the software solution. The subsequent section.3.3 will introduce this new algorithm, which is designed to self-adapt to any reasonable laser chirps.

Conceptual Illustration of MRC Temporal Focusing

Note that this box presents a conceptual illustration intended to facilitate understanding of the mechanism of the MRC reconstruction algorithm. It is not a literal physical equivalence.

This conceptual illustration relies on the mathematical time-space analogy between spatial diffraction and temporal dispersion, offering an alternative perspective for understanding the consequences and capabilities of the MRC algorithm. The MRC operation can be conceptually illustrated by focusing a camera lens.

Just as a camera adjusts its focal length f to obtain a sharp image at a specific spatial plane (Figure. 3.3), the MRC algorithm achieves *temporal focusing* by selecting the optimal linear dispersion parameter $B = \beta_2 L_1$ ^a. When the optimal linear dispersion B parameter is fitted, the MRC's inherent linear fitting process precisely matches the underlying nonlinear dispersion at one specific temporal position. This enables a sharp, localized pulse reconstruction at that point. This mechanism is directly similar to

how a camera focuses. It yields a sharp image only at its focal plane, while objects at all other positions appear blurred (as illustrated in Figure. 3.3).



Figure 3.3: **Spatial illustration of the MRC algorithm's operation: focusing a camera lens.** This image illustrates how adjusting a camera's focal length brings objects into sharp focus at a specific spatial plane while blurring others. This process is analogous to the MRC algorithm's optimal reconstruction, where adjusting the dispersion parameter B temporally focuses the probe pulse, leading to sharp reconstruction of the THz signal at a specific temporal position. (cf. Fig.3.2(f))

As an additional illustration of temporal focusing, Figure. 3.4 demonstrates that adjusting the B parameter allows us to deliberately shift the temporal focus to sharply reconstruct a different pulse, akin to re-focusing a camera on a new target.

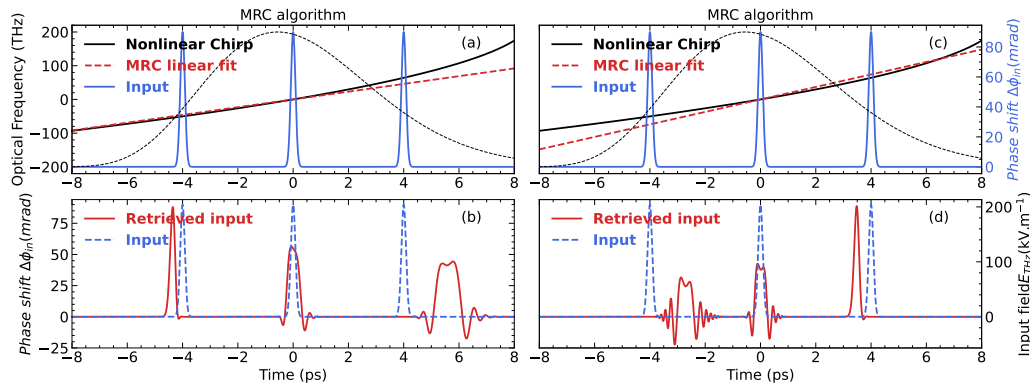


Figure 3.4: **Temporal focusing of individual pulses by B-parameter adjustment :** (a) Three 200 fs input pulses, separated by 4 ps intervals under a nonlinear time-wavelength relation. The MRC linear fit is adjusted for the left pulse. (b) Reconstructed THz pulse, temporally focused on the left pulse. (c) The same input pulses under the same nonlinear time-wavelength relation, but the MRC linear fit is adjusted for the right pulse. (d) Reconstructed THz pulse, temporally focused on the right pulse.

^awhere L_1 represents the propagation distance and β_2 is the group velocity dispersion coefficient of the first fiber used to chirp our probe laser

3.3 A novel Self-Adaptive Dispersion algorithm for Broadband THz Reconstruction

This section introduces a novel algorithm designed to replace the MRC method for THz signal reconstruction, especially under nonlinear dispersion. This innovative approach not only reconstructs the THz signal but also provides a complete characterization of the chirped probe laser, including its amplitude, phase, and dispersion relation (linear or not). The core of this algorithm lies in a fundamental reinterpretation of the spectral encoding electro-optic system, demonstrating its formal and mathematical equivalence to a Mach-Zehnder modulator. Building on this new paradigm, we thoroughly address a specific solution for the phase diversity system. This algorithm's development, inspired by optical backpropagation techniques commonly employed in Mach-Zehnder modulators [88, 97], forms the basis of our new perspective. The complete algorithm is presented in Fig.3.12 and detailed step-by-step within this section. We also provide its computational model, which precisely retrieves the same three THz pulses from Fig.3.2.(d) even in the presence of third-order dispersion.

3.3.1 Reinterpretation of DEOS: The Mach-Zehnder modulator analogy

A direct analogy can be drawn between the phase diversity setup shown in Fig. 3.5(a) and a Mach-Zehnder interferometer in Fig. 3.5(b). The chirped probe laser, initially polarized at 45 degrees, naturally forms the two arms of this conceptual interferometer by splitting into its vertical and horizontal polarization components. The vertical polarization component serves as the modulated arm $E_m(t)$. In the phase diversity configuration, this component is phase-modulated by the THz signal within the electro-optic crystal, $\phi_1(t) = \Delta\phi^{\text{phd}}(t)$. In the interferometer the EO crystal can be interpreted as a phase modulator. In contrast, the horizontal polarization component acts as the reference arm $E_0(t)$, passing through the EO crystal without being phase-modulated by the THz signal $\phi_2(t) = 0$. While quarter-wave plates (QWP) and half-wave plates (HWP) are essential for translating this phase modulation into amplitude modulation, they are not explicitly depicted in the Mach-Zehnder analogy, though they would conceptually reside within each arm. The interference in the phase diversity setup occurs at the polarizing beam splitter (PBS), with its two outputs being the direct result of the interference between the modulated arm $E_m(t)$ and the reference arm $E_0(t)$.

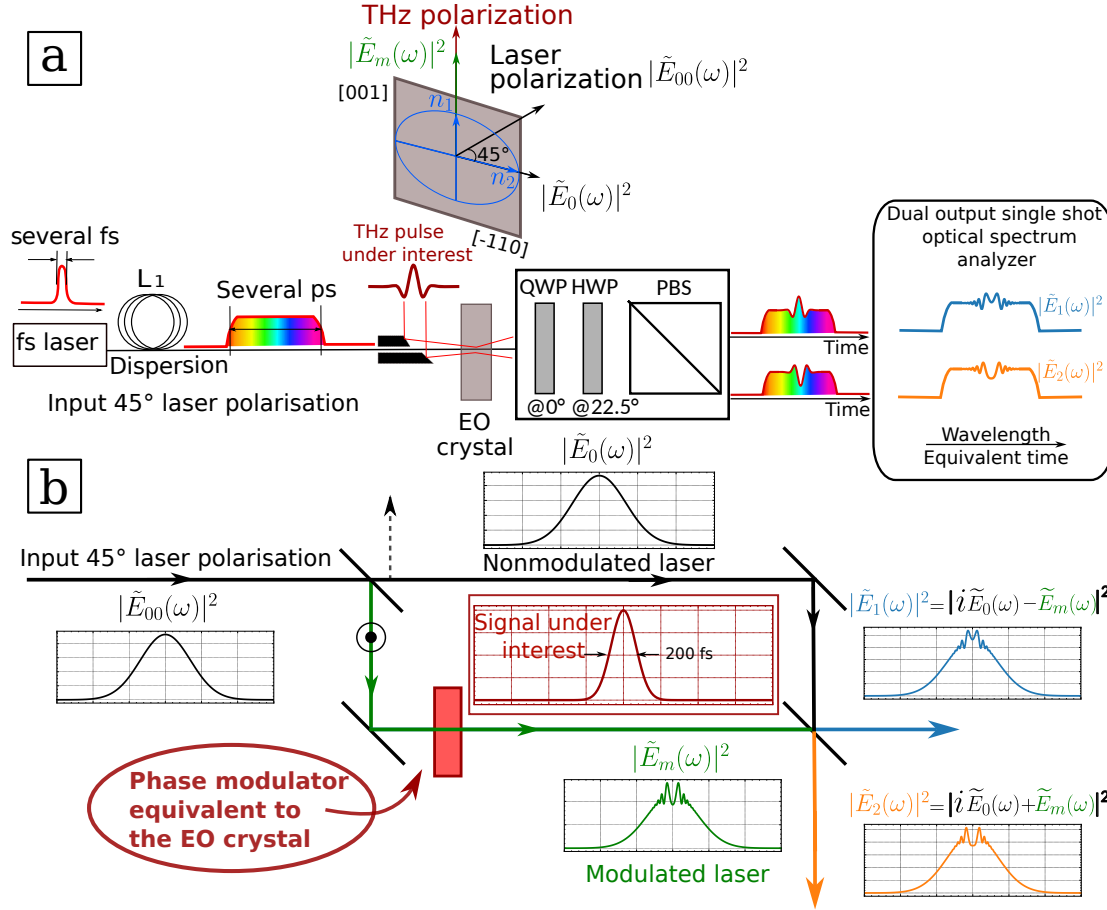


Figure 3.5: Analogy between the phase diversity setup and a Mach-Zehnder interferometer. (a) The phase diversity setup. A chirped probe laser, polarized at 45 degrees, enters the electro-optic (EO) crystal. Inside, its polarization is conceptually decomposed into vertical and horizontal components. The vertical component is phase-modulated by the THz signal, forming the modulated arm $\tilde{E}_m(\omega)$, while the horizontal component remains unmodulated, forming the reference arm $\tilde{E}_0(\omega)$. Quarter-wave plates (QWP) and half-wave plates (HWP) then convert this phase modulation into amplitude modulation. The two outputs from the polarizing beam splitter (PBS) result from the interference between $\tilde{E}_m(\omega)$ and $\tilde{E}_0(\omega)$.

(b) Its conceptual analogue: a Mach-Zehnder modulator. This interferometer incorporates a phase modulator in one arm, directly corresponding to the EO crystal's action on the probe laser's vertical polarization. This results in a modulated arm $\tilde{E}_m(\omega)$ and a reference arm $\tilde{E}_0(\omega)$. While waveplates are integral to the actual phase diversity setup, they are conceptually placed within each arm but omitted from this simplified diagram. The resulting interferometer outputs are formally equivalent to those of the phase diversity setup.

We can now express the phase diversity outputs equation as two interference equations. As established in the appendix of [18], the two outputs of the phase diversity setup, $E_1^{\text{phd}}(t)$ and $E_2^{\text{phd}}(t)$, are given by:

$$E_1^{\text{phd}}(t) = \frac{E_{00}(t)}{2\sqrt{2}}(1+i)(i - e^{i\Delta\phi^{\text{phd}}(t)}) \quad (3.2a)$$

$$E_2^{\text{phd}}(t) = \frac{E_{00}(t)}{2\sqrt{2}}(1+i)(i + e^{i\Delta\phi^{\text{phd}}(t)}) \quad (3.2b)$$

Here, $\Delta\phi^{\text{phd}}(t)$ represents THz-induced birefringence, defined in the phase diversity case as $\Delta\phi^{\text{phd}}(t) = \frac{\pi d}{\lambda_0} n_0^3 r_{41} E_{\text{THz}}$, and $E_{00}(t)$ refers to the chirped probe laser, as detailed in Equation.3.1a.

In these expressions, we recognize the components corresponding to our Mach-Zehnder analogy, the reference arm $E_0(t)$ and the modulated arm $E_m(t)$, defined as:

$$E_0(t) = |E_0(t)|e^{i\phi_0(t)} = \frac{E_{00}(t)}{2\sqrt{2}}(1+i) \quad (3.3a)$$

$$E_m(t) = |E_m(t)|e^{i\phi_m(t)} = \frac{E_{00}(t)}{2\sqrt{2}}(1+i)e^{i\Delta\phi^{\text{phd}}(t)} \quad (3.3b)$$

Where $|E_0(t)|$ and $|E_m(t)|$ are the amplitudes of the reference and modulated arm, respectively, and $\phi_0(t)$ and $\phi_m(t)$ are their corresponding phases.

Accordingly, the two outputs $E_1^{\text{phd}}(t)$ and $E_2^{\text{phd}}(t)$ can be rewritten in terms of $E_0(t)$ and $E_m(t)$, mirroring the form characteristic of a Mach-Zehnder modulator.

$$E_1^{\text{phd}}(t) = iE_0(t) - E_m(t) \quad (3.4a)$$

$$E_2^{\text{phd}}(t) = iE_0(t) + E_m(t) \quad (3.4b)$$

The recorded outputs, typically obtained using a camera combined with a grating (spectrometer) or through a photonic time stretch method (dispersive Fourier transform), are the two optical spectra: $\tilde{I}_{1,2}^{\text{phd}}(\omega) = |\tilde{E}_{1,2}^{\text{phd}}(\omega)|^2$.

Taking the Fourier transform of $E_1^{\text{phd}}(t)$ and $E_2^{\text{phd}}(t)$, yields their spectral representations, $\tilde{E}_1^{\text{phd}}(\omega)$ and $\tilde{E}_2^{\text{phd}}(\omega)$:

$$\tilde{E}_1^{\text{phd}}(\omega) = i\tilde{E}_0(\omega) - \tilde{E}_m(\omega) \quad (3.5a)$$

$$\tilde{E}_2^{\text{phd}}(\omega) = i\tilde{E}_0(\omega) + \tilde{E}_m(\omega) \quad (3.5b)$$

With $\tilde{E}_0(\omega)$ and $\tilde{E}_m(\omega)$ being the fourier transform of $E_0(t)$ and $E_m(t)$.

The optical intensity spectra $I_{1,2}^{\text{phd}}(\omega)$ are obtained by squaring the modulus of the two spectral outputs, which correspond precisely to the classical interference equations³:

$$I_1^{\text{phd}}(\omega) = |\widetilde{E}_1^{\text{phd}}(\omega)|^2 = |\widetilde{E}_0(\omega)|^2 + |\widetilde{E}_m(\omega)|^2 - 2|\widetilde{E}_0(\omega)||\widetilde{E}_m(\omega)|\sin(\Delta\widetilde{\phi}^{\text{phd}}(\omega)) \quad (3.6a)$$

$$I_2^{\text{phd}}(\omega) = |\widetilde{E}_2^{\text{phd}}(\omega)|^2 = |\widetilde{E}_0(\omega)|^2 + |\widetilde{E}_m(\omega)|^2 + 2|\widetilde{E}_0(\omega)||\widetilde{E}_m(\omega)|\sin(\Delta\widetilde{\phi}^{\text{phd}}(\omega)) \quad (3.6b)$$

Here $\Delta\widetilde{\phi}^{\text{phd}}(\omega)$ represents the phase difference between $\widetilde{E}_m(\omega)$ and $\widetilde{E}_0(\omega)$ in the Fourier space.

$$\Delta\widetilde{\phi}^{\text{phd}}(\omega) = \widetilde{\phi}_m(\omega) - \widetilde{\phi}_0(\omega) \quad (3.7)$$

With $\widetilde{\phi}_0(\omega)$ and $\widetilde{\phi}_m(\omega)$ being the phase of $\widetilde{E}_0(\omega)$ and $\widetilde{E}_m(\omega)$ in the Fourier space.

In experimental data analysis, we prefer subtract and normalize $I_{1,2}^{\text{phd}}(\omega)$ by a reference optical intensity spectrum, $I_{1,2}^{\text{ref}}(\omega) = 2|\widetilde{E}_0(\omega)|^2$, obtained without THz signal. The subtraction isolates the contributions of the THz field from any baseline, while normalization emphasizes relative changes induced by the THz field. The subtracted and normalized outputs⁴, $Y_{1,2}^{\text{phd}}(\omega) = Y_{1,2}^{\text{phd}}(t = -\frac{\omega}{C})$, are defined as:

$$Y_{1,2}^{\text{phd}}(\omega) = \frac{I_{1,2}^{\text{phd}}(\omega) - I_{1,2}^{\text{ref}}(\omega)}{I_{1,2}^{\text{ref}}(\omega)} \quad (3.8)$$

These optical intensity spectra $Y_{1,2}^{\text{phd}}(\omega)$ form the starting point of our algorithm. Applying this to Equation 3.6a yields:

$$Y_1^{\text{phd}}(\omega) = \frac{|\widetilde{E}_m(\omega)|^2 - |\widetilde{E}_0(\omega)|^2 - 2|\widetilde{E}_0(\omega)||\widetilde{E}_m(\omega)|\sin(\Delta\widetilde{\phi}^{\text{phd}}(\omega))}{2|\widetilde{E}_0(\omega)|^2} \quad (3.9a)$$

$$Y_2^{\text{phd}}(\omega) = \frac{|\widetilde{E}_m(\omega)|^2 - |\widetilde{E}_0(\omega)|^2 + 2|\widetilde{E}_0(\omega)||\widetilde{E}_m(\omega)|\sin(\Delta\widetilde{\phi}^{\text{phd}}(\omega))}{2|\widetilde{E}_0(\omega)|^2} \quad (3.9b)$$

The spectral outputs $Y_{1,2}^{\text{phd}}(\omega)$, shown in Figure. 3.6, exhibit strong distortions caused by time resolution limitations. The effect of third-order dispersion is manifested by a visible chirp and a time shift in the outer pulses. The asymmetric distortion solely arises from the selected Gaussian shape of the probe laser $\widetilde{E}_{00}(0, \omega)$ and is unrelated to third-order dispersion.

³Note that $\sin(\Delta\widetilde{\phi}^{\text{phd}}(\omega))$ term in the interference equations arises from $\cos(\Delta\widetilde{\phi}^{\text{phd}}(\omega) + \frac{\pi}{2})$. The additional $\frac{\pi}{2}$ is critical for the interference condition and stems from the relative phase between $\widetilde{E}_m(\omega)$ and $\widetilde{E}_0(\omega)$ as expressed in Eq.3.5a.

⁴As shown in the previous chapter.2 $Y_{1,2}^{\text{phd}}(\omega) = Y_{1,2}^{\text{phd}}(t = -\frac{\omega}{C})$

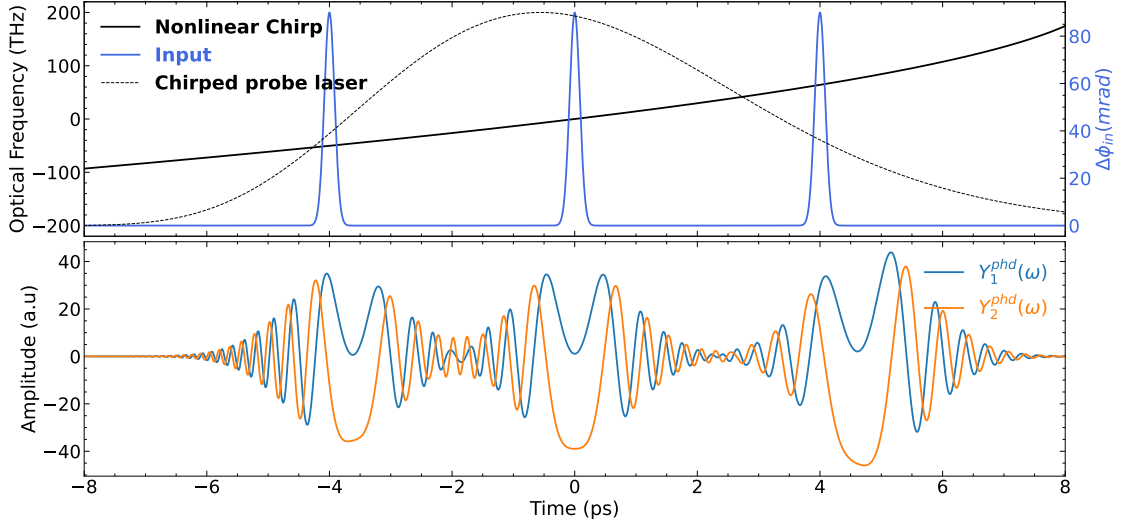


Figure 3.6: **Numerically simulated, subtracted, and normalized optical intensity spectra $Y_{1,2}^{phd}(\omega)$** : Top: Three input pulses of 200 fs with 4 ps separation under a nonlinear time-wavelength relationship. Bottom: The resulting spectral outputs $Y_{1,2}^{phd}(\omega)$, which serve as the initial input for the reconstruction algorithm⁵.

With the phase diversity outputs now understood as equivalent to those of a Mach-Zehnder modulator, we obtain a powerful new conceptual framework for THz pulse retrieval.

3.3.2 Signal reconstruction based on Mach-Zehnder interferometer equations

The recorded optical intensity spectra $Y_{1,2}^{phd}(\omega)$, constitute the initial input data for our algorithm. From these data, the algorithm's core principle is to fully retrieve and reconstruct the complex amplitude (both amplitude and phase) of the modulated arm, $\tilde{E}_m(\omega)$, thereby enabling the ultimate reconstruction of the THz input pulses. This is achieved by employing the classical interference equations and all available information concerning our Mach-Zehnder system. The complete algorithm's scheme is illustrated in Figure 3.12 and detailed step-by-step in the following subsections.

3.3.2.1 Objectives of the algorithm and first step

Our objective is to determine the complex electric field $\tilde{E}_m$:

$$\tilde{E}_m(\omega) = |\tilde{E}_m(\omega)|e^{i\tilde{\phi}_m(\omega)} \quad (3.10a)$$

$$\text{With } \tilde{\phi}_m(\omega) = \Delta\tilde{\phi}^{phd}(\omega) + \tilde{\phi}_0(\omega) = \Delta\tilde{\phi}^{phd} + \phi_{disp}(L_1, \omega) \quad (3.10b)$$

⁵These data exhibit strong distortions due to time resolution limitations. A visible chirp and a time shift on the two outer pulses results from third-order dispersion. Note that the asymmetric distortion is exclusively caused by the Gaussian probe laser shape $\tilde{E}_{00}(0, \omega)$ and is independent of both third-order dispersion and time resolution limitations.

Where L_1 represents the length of the first fiber used to define the temporal recording window.

Using the sum and difference of the two interference equations.3.9a, we can deduce the amplitude of the modulated laser, $|\tilde{E}_m(\omega)|$ and the phase difference $\Delta\tilde{\phi}^{\text{phd}}(\omega)$ between the modulated and unmodulated laser.

$$|\tilde{E}_m(\omega)| = |\tilde{E}_0(\omega)| \sqrt{Y_1^{\text{phd}}(\omega) + Y_2^{\text{phd}}(\omega) + 1} \quad (3.11a)$$

$$\sin(\Delta\tilde{\phi}^{\text{phd}}(\omega)) = \frac{|\tilde{E}_0(\omega)|}{2|\tilde{E}_m(\omega)|} (Y_2^{\text{phd}}(\omega) - Y_1^{\text{phd}}(\omega)) \quad (3.11b)$$

We compute $|\tilde{E}_0(\omega)|$ using a reference measurement in the absence of a THz pulse.

$$|\tilde{E}_0(\omega)| = \sqrt{\frac{I_1^{\text{ref}}(\omega) + I_2^{\text{ref}}(\omega)}{4}} \quad (3.12)$$

The calculated $|\tilde{E}_m(\omega)|$ and $\Delta\tilde{\phi}^{\text{phd}}(\omega)$ using equation.3.11 are displayed in Figure.3.7.

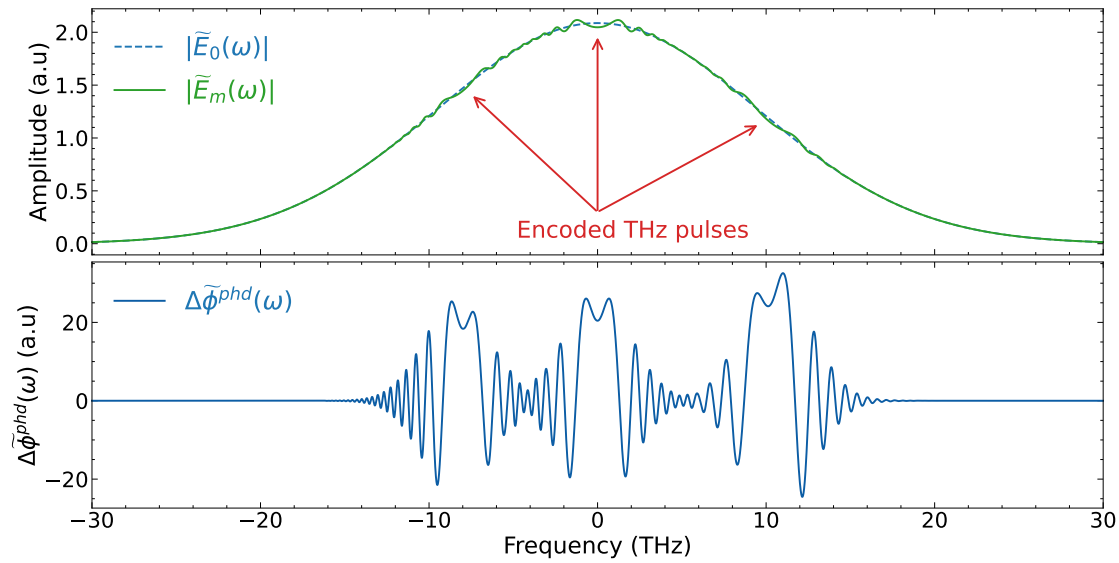


Figure 3.7: First step of the SAD algorithm: Retrieved Spectral Components of the Modulated Laser: Top : Recovered amplitude of the modulated laser $|\tilde{E}_m(\omega)|$, the three distorted pulses are visibly encoded onto the spectrum. Bottom : The phase difference $\Delta\tilde{\phi}^{\text{phd}}(\omega)$ between the modulated and unmodulated laser.

3.3.2.2 Self-adaptive dispersion curve retrieval

To fully reconstruct the complex amplitude of the modulated arm, $\widetilde{E}_m(\omega)$, it is essential to account for the dispersion introduced to achieve the correct temporal window. This requires knowledge of the dispersion parameters⁶. For a fiber⁷, these include the fiber length L_1 , the group-velocity dispersion β_2 and the third order dispersion β_3 in the case of nonlinear chirp⁸. Accurately estimating these dispersion parameters can be challenging.

To address this, we have developed an innovative solution to precisely fit and retrieve all parameters of ϕ_{disp} .

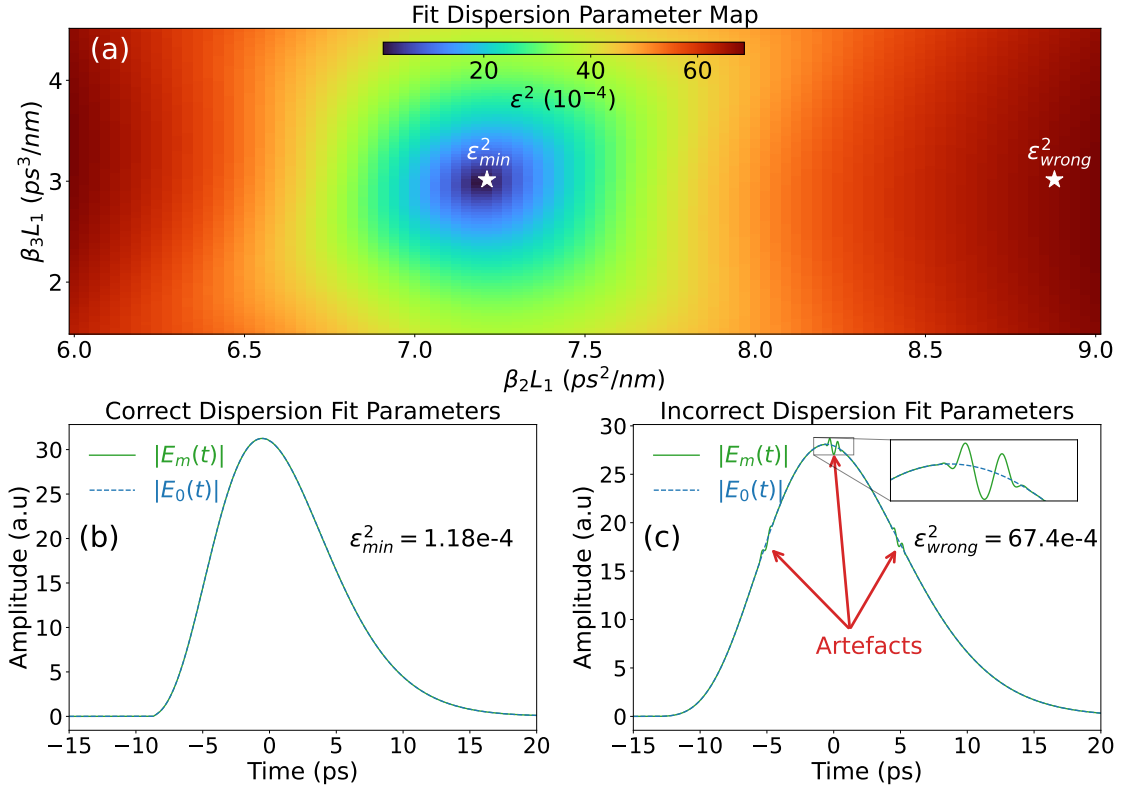


Figure 3.8: **Dispersion parameter fitting process :** (a) Colormap illustrating the error ϵ^2 as a function of various combinations of dispersion parameters $\beta_2 L_1$ versus $\beta_3 L_1$ (b) Reconstructed amplitudes $|E_m(t)|$ and $|E_0(t)|$, when ϵ^2 is minimized, demonstrating that the condition $|E_m(t)| = |E_0(t)|$ is accurately met. (c) Reconstructed amplitudes $|E_m(t)|$ and $|E_0(t)|$, when a high ϵ^2 error occurs, illustrating amplitude differences in the region of the input THz pulses.

This approach relies on a fundamental property of our system. In the time domain, the modulated arm, $E_m(t)$ and the reference arm $E_0(t)$ share the same

⁶Note that this self-adaptive dispersion retrieval step can be bypassed, if the dispersion parameters are known beforehand.

⁷The method described here for fitting dispersion parameters is applicable to any dispersive element

⁸The method can also be extended to fit higher-order dispersion terms, such as β_4, β_5

amplitude, only their phases differ. This means that the modulated arm undergoes a pure phase modulation.

The spectral representations of these arms are given by:

$$\widetilde{E}_0(\omega) = |\widetilde{E}_0(\omega)|e^{i\phi_{\text{disp}}(L_1, \omega)} \quad (3.13a)$$

$$\widetilde{E}_m(\omega) = |\widetilde{E}_m(\omega)|e^{i(\phi_{\text{disp}}(L_1, \omega) + \Delta\phi^{\text{phd}}(\omega))} \quad (3.13b)$$

Based on the time-domain property, when we correctly reconstruct the complex amplitudes of $\widetilde{E}_m(\omega)$ and $\widetilde{E}_0(\omega)$, the condition $|E_m(t)| = |E_0(t)|$ will be satisfied. Therefore, to find the correct dispersion parameters, we minimize the following quantity:

$$\varepsilon^2 = \int_{-\infty}^{+\infty} |E_m(t)|^2 - |E_0(t)|^2 dt \quad (3.14)$$

Figure 3.8 illustrates the entire fitting process, including the error landscape derived from this minimization Fig.3.8(a) and the amplitude reconstruction results under both optimal Fig.3.8(b) and incorrect parameter estimations Fig.3.8(c).

3.3.2.3 Retrieving the THz electric field: The final algorithmic steps

With the dispersion parameters accurately fitted and the complex spectral amplitudes $\widetilde{E}_m(\omega)$ and $\widetilde{E}_0(\omega)$ correctly established, the final step is to reconstruct the THz pulse. We achieve this by performing an inverse Fourier transform on these spectral representations to retrieve their corresponding complex electric fields in the time domain, $E_m(t)$ and $E_0(t)$:

$$E_0(t) = |E_0(t)|e^{i\phi_0(t)} \quad (3.15a)$$

$$E_m(t) = E_0(t)e^{i\Delta\phi^{\text{phd}}(t)} \quad (3.15b)$$

From these time-domain fields, the THz-induced phase difference, $\Delta\phi^{\text{phd}}(t)$, can be directly accessed by taking the argument of the ratio of the modulated and unmodulated arms

$$\Delta\phi^{\text{phd}}(t) = \text{Arg}\left(\frac{E_m(t)}{E_0(t)}\right) \quad (3.16)$$

Finally, the THz electric field, $E_{\text{THz}}(t)$ is straightforwardly obtained by dividing this phase difference by the electro-optic coefficient $\beta^{EO} = \frac{\pi d}{\lambda} n_0^3 r_{41}$:

$$E_{\text{THz}}(t) = \frac{\Delta\phi^{\text{phd}}(t)}{\beta^{EO}} \quad (3.17)$$

Where λ is the laser wavelength, d is the crystal thickness, n_0 is the refractive index of the crystal at the laser wavelength λ , r_{41} is the electro-optic coefficient.

Figure 3.9 provides a visual comparison of the reconstructed THz pulses obtained using the MRC algorithm and this new approach under third-order dispersion, clearly demonstrating its effectiveness.

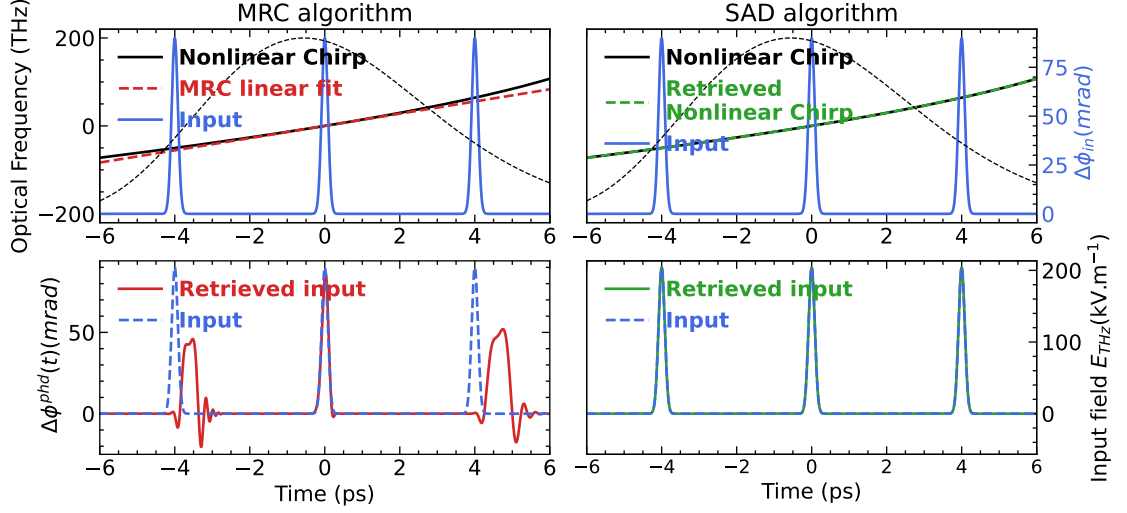


Figure 3.9: Final reconstruction step of the SAD algorithm: Comparison of THz pulse reconstruction methods under third-order dispersion: Top: Three 200 fs input pulses at 4 ps intervals with third-order dispersion. Bottom: Reconstructed THz pulses using the MRC algorithm (left) and the new algorithm (right). The new method demonstrates improved accuracy by fully retrieving the dispersion curve, a significant advantage over the MRC algorithm's linear fit approximation.

This completes the comprehensive framework for our THz pulse reconstruction algorithm, which starts from the recorded optical intensity spectra and culminates in the recovered THz waveform. With the comprehensive framework for our novel THz pulse reconstruction algorithm fully detailed, the next logical step is to demonstrate its practical application and accuracy, which we undertake in the following section through experimental validation.

SAD Algorithm: A Generalized Framework Encompassing MRC Algorithm

The SAD algorithm offers a highly flexible approach by directly fitting the full dispersion phase, ϕ_{disp} . This allows for precise control over the dispersion terms, enabling it to compensate for high-order effects (beyond third-order dispersion) or to be intentionally limited to only the Group Velocity Dispersion (GVD) term.

When the SAD algorithm is restricted solely to GVD compensation (i.e., neglecting third-order dispersion), its behavior become mathematically equivalent to the MRC algorithm. This comparison, demonstrated in Figure . 3.10, shows that the reconstructed THz pulses from both methods are highly similar and share the same characteristic distortion caused by the uncompensated higher-order dispersion.

This agreement serves to validate the new SAD framework by replicating the MRC results under nonlinear dispersion when the SAD is intentionally limited to a linear fit chirp. Furthermore it confirms that the SAD algorithm is a more general and powerful method that encompasses MRC as a specific, limited case.

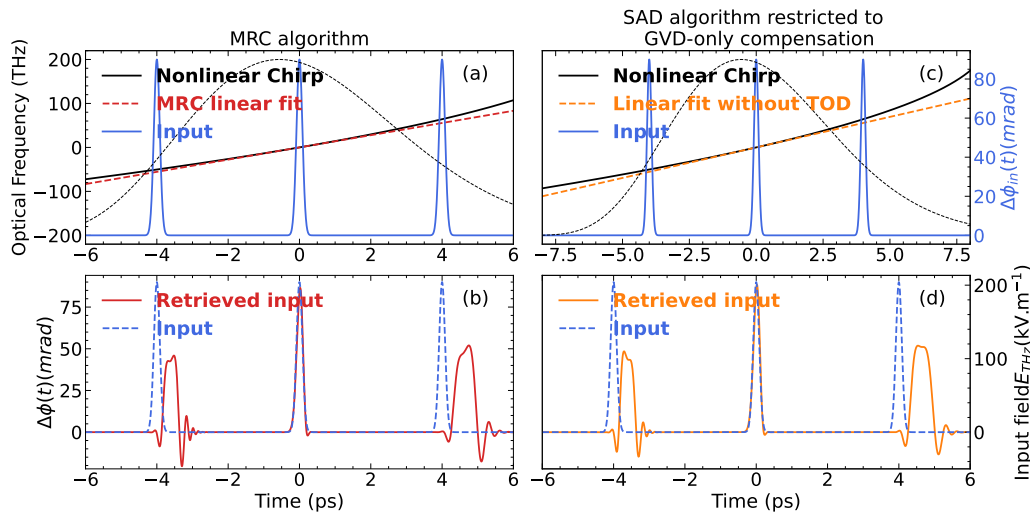


Figure 3.10: **Equivalence of the MRC and the SAD algorithm restricted to GVD-only compensation** : (a) Input pulses and linear fit provided by the MRC algorithm under nonlinear dispersion. (b) THz pulse reconstruction obtained using the MRC algorithm. (c) Input pulses and linear fit (intentionally restricted to GVD-only) provided by the SAD algorithm under nonlinear dispersion. (d) THz pulse reconstruction using the SAD algorithm under the GVD-only restriction. Note that (b) and (d) are highly similar, confirming that SAD algorithm generalizes the MRC algorithm.

Notes on large THz electric field amplitudes

The reconstruction of large THz electric field amplitudes is limited by the reconstruction algorithms.

The MRC algorithm relies on the small-angle approximation (i.e., $\Delta\phi^{phd}(t) \ll 1$), leading to reconstruction distortion when the induced phase shift is large (e.g., 1.5 rad, as shown in Figure. 3.11.(a) under linear dispersion).

The SAD algorithm offers improved performance but is inherently restricted by the domain of the $\sin^{-1}(x)$ function used for phase extraction in equation.3.11, limiting the measurable THz-induced birefringence to a maximum phase shift of $\pm\frac{\pi}{2}$. This means the algorithm becomes ineffective for THz electric fields that induce shifts beyond this range, though it successfully reconstructs large signals below this limit (Figure. 3.11.(b) shows a successful 1.5 rad shift reconstruction under linear dispersion).

A potential solution for reconstructing even higher THz fields involves adding a third output channel to the phase diversity setup. This approach would make the phase diversity system functionally equivalent to a Mach-Zehnder interferometer with three outputs. The three outputs are then used to determine the amplitude, cosine, and sine of the induced phase shift, as demonstrated in Ref. [98, 99] and the Supplemental Material of Ref. [100]. By enabling the use of the tangent function for phase extraction, this modification overcomes the $\sin^{-1}(x)$ limitation ($\pm\frac{\pi}{2}$) inherent to the two-channel using the SAD algorithm. This extends the measurable range to the full interval of $[-\pi, \pi]$, allowing for the reconstruction of higher THz field amplitudes.

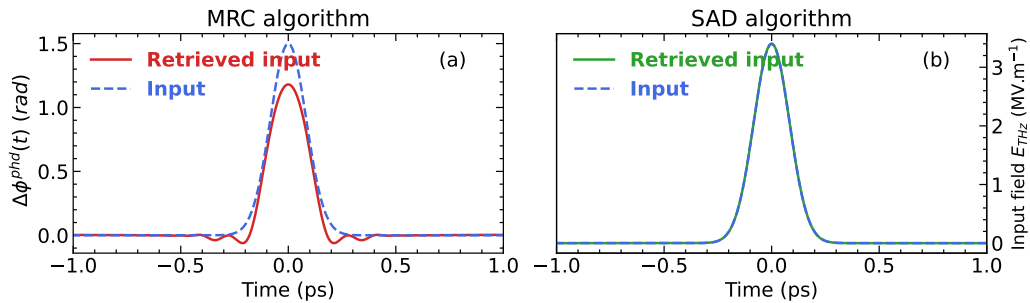


Figure 3.11: **Comparison of the MRC and SAD algorithms for large-amplitude THz reconstruction under linear dispersion:** (a) Reconstructed THz pulse using the MRC algorithm showing distortion when the induced phase shift ($\Delta\phi(t)$) is large (e.g., 1.5 rad), resulting from the breakdown of the small-angle approximation $\cos(x) \approx x$. (b) Reconstructed THz pulse using the SAD algorithm, which successfully reconstructs the large 1.5 rad phase shift as this value remains within the inherent $\pm\frac{\pi}{2}$ limit imposed by the $\sin^{-1}(x)$ function used for the phase extraction.

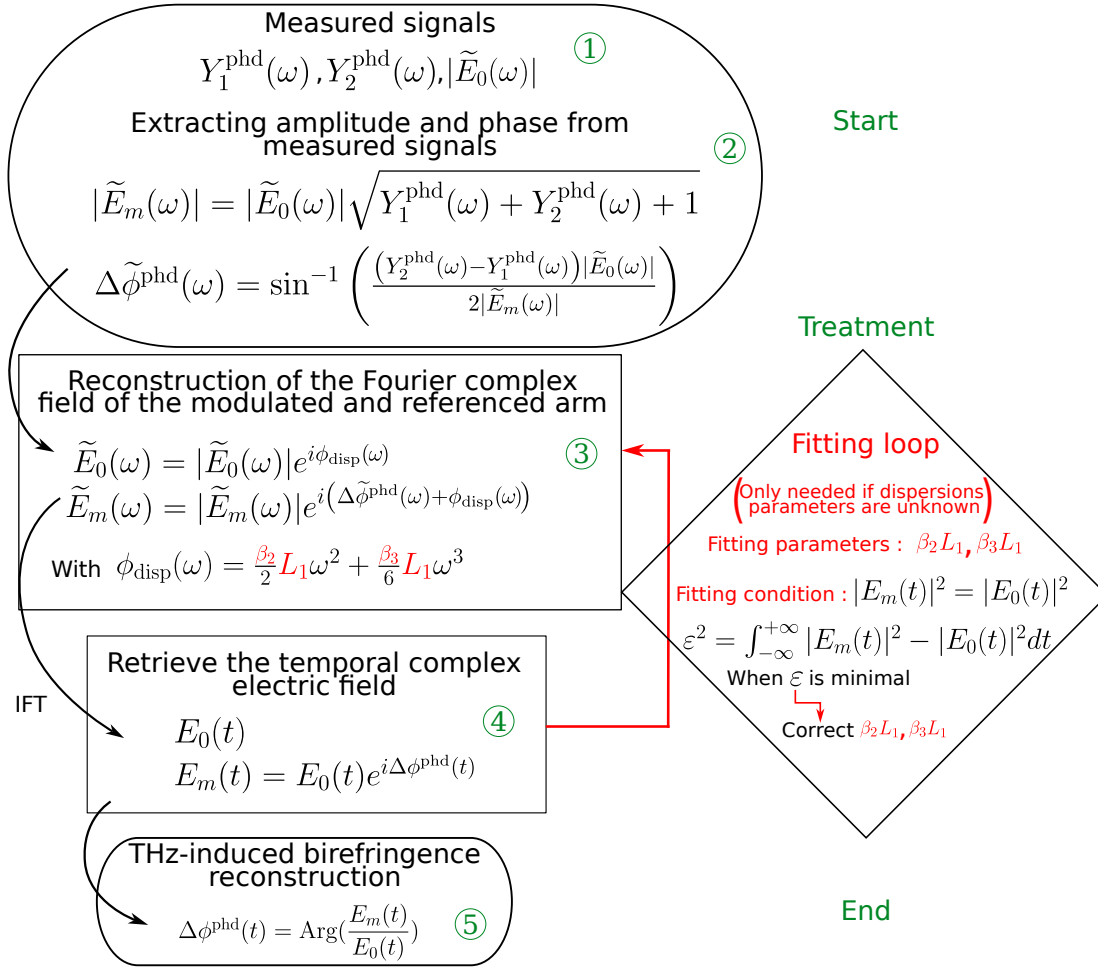


Figure 3.12: Complete workflow of the novel THz pulse reconstruction algorithm. This detailed flowchart outlines every mathematical step of the algorithm, from the initial recorded optical intensity spectra $\tilde{Y}_{1,2}^{\text{phd}}(\omega)$ derived from the Mach-Zehnder modulator analogy (Equations 3.10-3.12), to the final reconstruction of the THz electric field $E_{\text{THz}}(t)$. Key stages include the calculation of the modulated arm's amplitude $|\tilde{E}_m(\omega)|$ and phase difference $\Delta\tilde{\phi}^{\text{phd}}(\omega)$ (Equation 3.13), followed by a dispersion parameter fitting process using the minimization of ε^2 (Equation 3.16) to accurately account for linear and nonlinear chirp. The algorithm culminates in the inverse Fourier transform to obtain time-domain fields $E_m(t)$ and $E_0(t)$, from which $\Delta\phi^{\text{phd}}(t)$ and ultimately $E_{\text{THz}}(t)$ are derived (Equations 3.17-3.19). This comprehensive approach allows for robust THz signal recovery even in the presence of nonlinear dispersion.

3.4 Experimental validation: A proof of principle using laser-generated THz pulses

We apply this new algorithm to a reference dataset, originally acquired prior to this thesis and published in the phase diversity EO paper [18]. This dataset is particularly valuable for a proof of principle because its single-shot reconstructions can be directly compared against well-established electro-optic scanned data, providing a reliable, known THz pulse shape. While this experimental data does not exhibit strong third-order dispersion and contains only a single THz pulse, which allows the MRC algorithm to perform adequately and focus clearly, it nonetheless serves as a critical benchmark for confirming our new approach's fundamental accuracy and practical performance on real-world measurements.

3.4.1 Experimental apparatus for THz characterization : DEOS single-shot vs scanned EOS

The experimental data analyzed by the new algorithm were acquired using the setup shown in Fig.6.3. The setup was designed for operational flexibility, allowing the grating stretcher to be bypassed or operated without altering overall alignment. This versatility enables the analysis of laser-based terahertz pulses⁹ using two distinct methods: single-shot diversity electro-optic detection with chirped pulses, providing a 20 ps temporal window, or traditional electro-optic scanning by delaying femtosecond laser pulses relative to the THz signal. This dual capability was essential for validating our single-shot method, as it enabled direct comparison of reconstructed DEOS results from our new algorithm against corresponding scanned EO signals.

3.4.2 Applying the novel SAD algorithm: Experimental tabletop validation results

The validation begins by outlining the reconstruction procedure, starting with the raw DEOS outputs required as inputs by the algorithm. This is followed by the intermediate needed calculation to estimate the dispersion parameters using the self-adaptive fitting method, and finally, a direct comparison of the reconstructed signal against the known scanned THz reference data.

⁹These pulses are produced via optical rectification of 800 nm millijoule-range laser pulses in a Zinc Telluride (ZnTe) crystal.

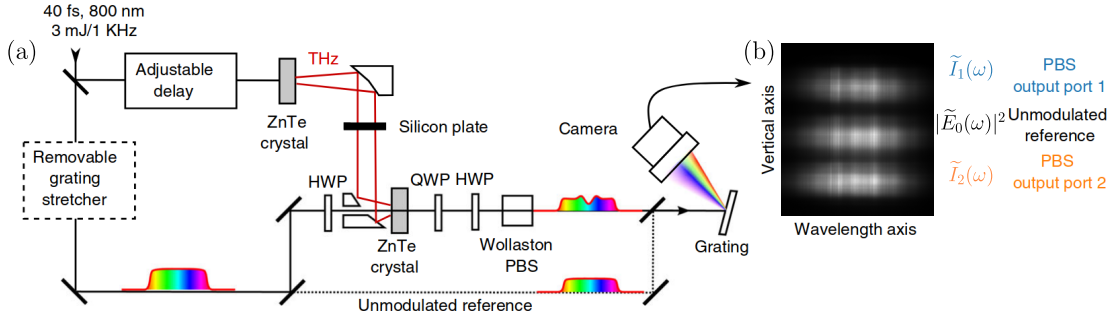


Figure 3.13: DEOS experimental apparatus for laser-based THz pulse recording : (a) This figure illustrates the experimental setup used for comparing traditional electro-optic scanning with single-shot Diversity Electro-Optic Sampling (DEOS) for recording laser-based THz pulses. Key components include : ZnTe: 1 mm-thick, 110-cut Zinc Telluride crystal, HWP: Half-wave plates, QWP: Quarter-wave plate, PBS: Wollaston polarizing cube beam-splitter. (b) Representative raw camera image showing the recorded single-shot spectra: the two polarization outputs with THz pulses $\tilde{I}_{1,2}^{\text{phd}}(\omega)$ and without THz pulses $\tilde{I}_{1,2}^{\text{ref}}(\omega)$, along with the unmodulated laser spectrum $|E_0(\omega)|^2$. Figure adapted from [18].

3.4.2.1 Raw experimental data: processed DEOS outputs for algorithm input

The subtracted and normalized EO signal ¹⁰ $\tilde{Y}_{1,2}^{\text{phd}}(\omega) = Y_{1,2}^{\text{phd}}(t = -\frac{\omega}{C})$ derived from the recorded optical intensity spectra $\tilde{I}_{1,2}^{\text{phd}}(\omega)$ and $\tilde{I}_{1,2}^{\text{ref}}(\omega)$ as shown in Fig.6.3.(b)), are presented in Fig.3.14. As expected from the phase diversity configuration, the two outputs exhibit distinct spectral shapes. Their characteristics, when compared with scanned and reconstructed data, notably highlight the time resolution limitations inherent to classical spectral decoding methods. Furthermore, the reflection of the THz pulse within the electro-optic crystal is clearly visible in these outputs.

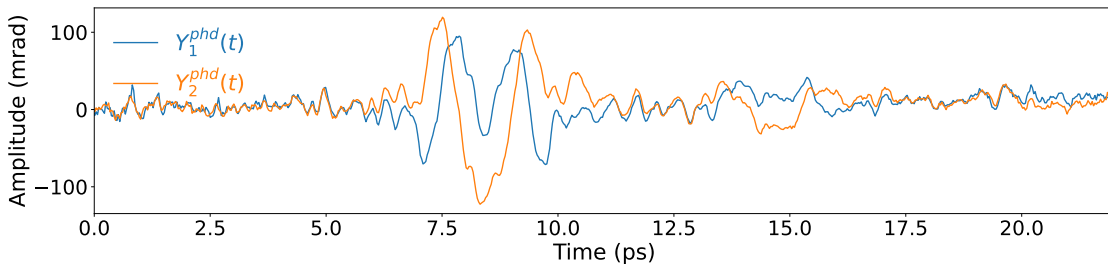


Figure 3.14: Subtracted and normalized experimental outputs $\tilde{Y}_{1,2}^{\text{phd}}(\omega)$: These figures display the raw experimental data, processed by subtraction and normalization, which serve as the direct input for our new algorithm. Distinct shapes are observed for the two outputs, reflecting the phase diversity configuration. The notable distortions compared to the scan data in Fig.3.17 correspond to the time resolution limitations discussed in Section.2.4

¹⁰As shown in the previous chapter.2 $\tilde{Y}_{1,2}^{\text{phd}}(\omega) = \tilde{Y}_{1,2}^{\text{phd}}(t = -\frac{\omega}{C})$

3.4.2.2 First step of the SAD algorithm: extracted modulated amplitude and phase difference

Applying Equation 3.11 to the processed EO signals $\tilde{Y}_{1,2}^{\text{phd}}(\omega)$, we retrieve the modulated amplitude $|E_m(\omega)|$ and the phase difference $\Delta\tilde{\phi}^{\text{phd}}(\omega)$. As shown in Fig.3.15.(a), the difference between the retrieved modulated amplitude $|E_m(\omega)|$ and the reference unmodulated amplitude $|E_0(\omega)|$, clearly exhibit the encoded THz pulse and its reflection within the EO crystal onto the modulated amplitude.

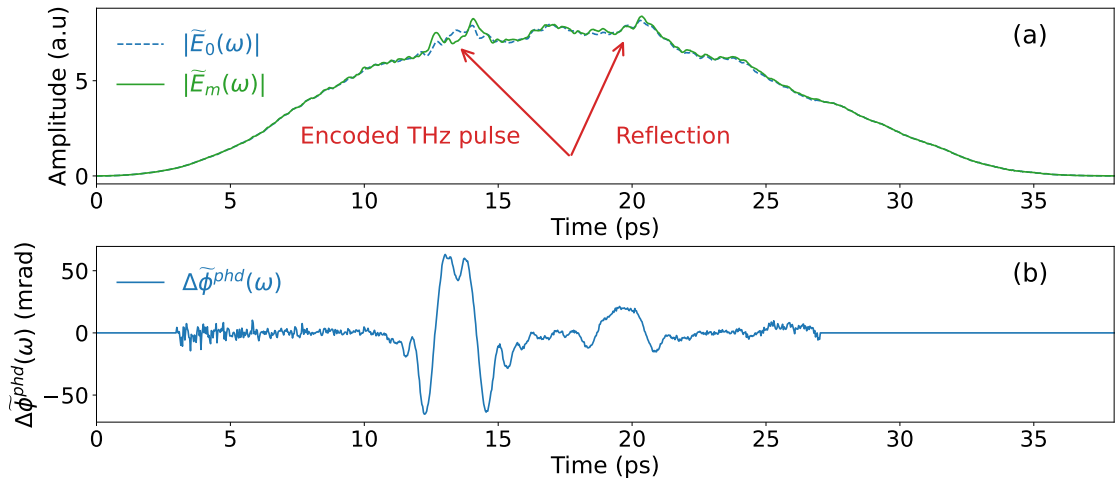


Figure 3.15: **First step of the SAD algorithm : Experimentally retrieved spectral components of the modulated laser:** (a) Recovered amplitude of the modulated laser $|E_m(\omega)|$, showing the encoded THz pulse and its reflection onto the spectrum. (b) The phase difference $\Delta\tilde{\phi}^{\text{phd}}(\omega)$ between the modulated and unmodulated laser. Note that the ω axis has been expressed in time unit for clarity ($\omega = -Ct$). The final reconstruction step is presented in Figure 3.17.

3.4.2.3 Experimental dispersion parameter estimation: application of the self-adaptive fitting method

Figure.3.16.(a) presents the colormap of the minimized quantity ε^2 , clearly indicating a well-defined minimum. This minimum corresponds to the optimal fitting condition where the temporal amplitudes of the modulated $|E_m(t)|$ and unmodulated $|E_0(\omega)|$ arms are made as identical as possible, as illustrated in Fig.3.16.(b). Figure 3.16.(c) displays the retrieved dispersion curve, characterized by its key parameters $\beta_2 L_1$ and $\beta_3 L_1$. The dispersion relation is found to be predominantly linear, with the third-order dispersion being notably weak relative to the group velocity dispersion.

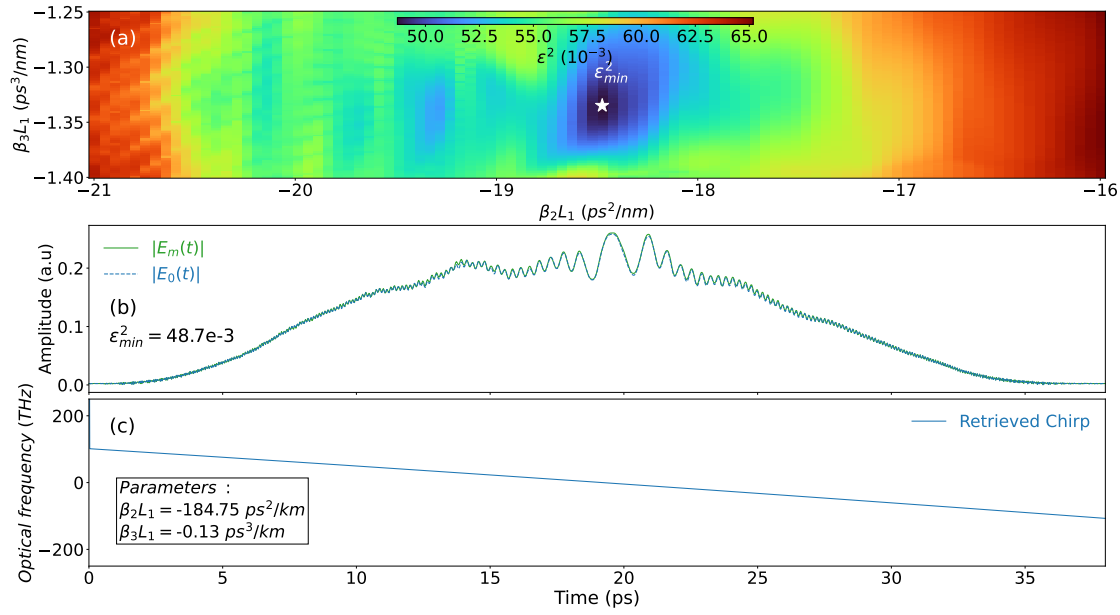


Figure 3.16: **Experimental dispersion parameter fitting process** : (a) Colormap illustrating the error ϵ^2 as a function of the dispersion parameters $\beta_2 L_1$ and $\beta_3 L_1$, showing a distinct minimum. (b) Reconstructed temporal amplitudes $|E_m(t)|$ and $|E_0(t)|$ at the minimized ϵ^2 , demonstrating their near-perfect identity. (c) The experimentally retrieved chirp, including the fitted parameters, $\beta_2 L_1$ and $\beta_3 L_1$, indicating a predominantly linear dispersion with weak third-order contribution relative to the group velocity dispersion.

3.4.2.4 Validation through direct comparison with scanned THz data

Figure 3.17(a) presents a comparison between the reconstructed THz pulse, from our new algorithm's DEOS data, and the reference THz pulse obtained via traditional electro-optic scanning. Their shapes are found to be in excellent agreement. Remarkably, the reconstruction accurately reproduces subtle features, including the small oscillations observed between 10 and 20 ps. These oscillations are attributed to water vapor absorption in air and reflections within the silicon plate at 15.5 ps, further validating the algorithm's fidelity.

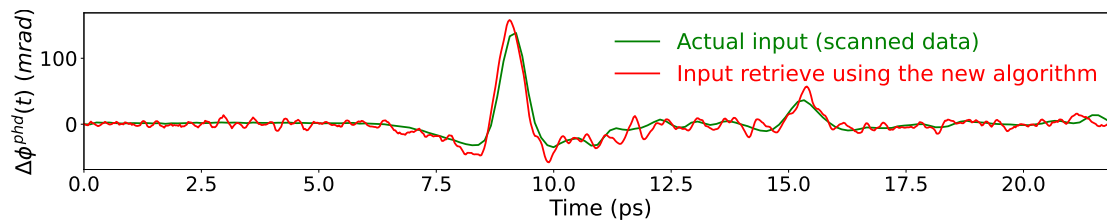


Figure 3.17: **Final reconstruction step of the SAD algorithm: Comparison of reconstructed THz pulse (DEOS) with reference scanned data** : (a) Overlap of the THz pulse reconstructed by the new algorithm from single-shot DEOS data (red line) and the reference THz pulse obtained via traditional electro-optic scanning (green line). The excellent agreement confirms the new algorithm's accuracy. Fine details, such as oscillations from water vapor absorption and silicon plate reflections, are accurately reproduced.

These results conclusively confirm the effectiveness of this new reconstruction algorithm, demonstrating its ability to accurately retrieve both the THz pulse waveform and the dispersion parameters from the experimental data.

Chapter summary

This chapter examined the fundamental limitations of the MRC algorithm when applied to phase diversity electro-optic measurements, particularly in the presence of third-order dispersion. Through numerical simulations, we unequivocally demonstrated how TOD breaks the time-invariance of the EO system, rendering the MRC algorithm's underlying linear time-wavelength assumption invalid and leading to inaccurate THz pulse reconstructions. This failure underscored the pressing need for an advanced reconstruction method, based on a new conceptual framework, to accurately handle higher-order dispersion.

To address this challenge, we developed and introduced a novel, self-adaptive dispersion algorithm for THz signal reconstruction. This algorithm builds upon a powerful conceptual and mathematical analogy between the phase diversity setup and a Mach-Zehnder modulator, enabling the retrieval of both the THz signal and the full complex amplitude (amplitude, phase, and dispersion relation) of the chirped probe laser. The detailed mathematical framework and computational model of this new approach were presented, demonstrating its theoretical capacity to accurately reconstruct THz pulses even in the presence of complex, non-linear dispersion. Furthermore, unlike the MRC algorithm, the SAD algorithm enables the reconstruction of higher THz fields (up to an equivalent phase shift of $\pm\frac{\pi}{2}$), expanding the range and fidelity of measurable signals.

The algorithm's effectiveness was experimentally validated using previously acquired data, demonstrating its performance for accurate THz signal retrieval against known reference pulses.

To conclude, by moving beyond linear assumptions, this method offers a more precise characterization tool for accelerator facilities and THz sources. The technique's performance and inherent adaptability enable its practical implementation in routine accelerator diagnostics, making it ready for future challenges.

Therefore, Chapter 4 shifts the focus to the practical application of this method. We will discuss its integral role in routine single-shot EO accelerator diagnostics at DESY, specifically for the EuXFEL and FLASH facilities, demonstrating its real-world impact.

Single-Shot DEOS diagnostics with the SAD algorithm: Results from EuXFEL and FLASH

*To see something new, one must
build something new.*

– Georg Christoph Lichtenberg
(Motto of Bjørn Wiik)

4.1 Introduction

Chapter 3 established the theoretical framework and initial experimental validation of the Self-Adaptive Dispersion (SAD) algorithm, a novel reconstruction algorithm developed to restore high-fidelity signal measurements in Diversity Electro-Optic Sampling (DEOS) by taking into account the nonlinear dispersion of the chirped probe pulse.

This chapter now transitions the SAD algorithm from a mathematical framework and proof of principle to a robust, practical application, directly combining it with the DEOS experimental setups at EuXFEL and FLASH. This practical integration necessitated key hardware component upgrades on both duplicated DEOS setups. These modifications were made to enable single-shot dual-output acquisition (i.e., recording both DEOS outputs simultaneously), thereby fulfilling the requirements for single-shot electron bunch characterization at MHz repetition rates. Therefore, the primary objective of this chapter is to prove that this upgraded DEOS system, in combination with the SAD algorithm, is capable of enabling the high-fidelity, single-shot, MHz-rate diagnostics essential

for routine FEL operation at DESY.

To structure this proof, the chapter first introduces the key operational characteristics of the EuXFEL and FLASH facilities and highlights the relevance of longitudinal electron bunch diagnostics, including electro-optic methods, for their operation. The DEOS experimental apparatus is then detailed in Section. 4.2. These duplicated setups, implemented with minor differences across the EuXFEL and FLASH accelerator environments, also demonstrate the robustness and adaptability of the SAD algorithm.

The core results are then presented in two phases, beginning with the DEOS-SAD system's performance at EuXFEL. The section. 4.3 first provides preliminary results, obtained prior to the dual-output simultaneous acquisition upgrade, which demonstrate the minimum temporal resolution achievable during variable electron bunch compression. Building upon this, we then detail the first single-shot electron bunch measurements using the new dual-output acquisition upgrade, alongside statistics on arrival time and bunch length. This section concludes with a performance comparison between the SAD algorithm and the conventional MRC algorithm. Finally, we present the results from the DEOS-SAD system at FLASH in Section. 4.4, where measurements under various compression and charge settings confirm the system's operational versatility and suitability for routine diagnostics across varied accelerator settings.

4.1.1 The EuXFEL facility

The European X-ray Free-Electron Laser (EuXFEL) in Hamburg, Germany, is a 3.4 km underground accelerator, shown in Figure 4.1, that generates extremely intense, femtosecond X-ray pulses for groundbreaking research across diverse scientific fields.

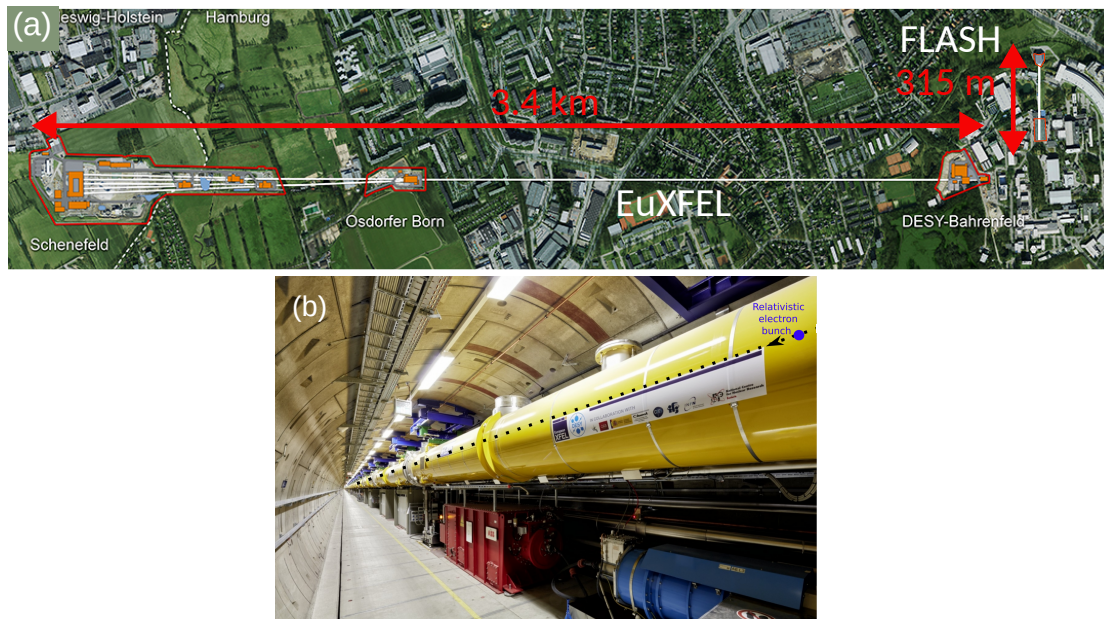


Figure 4.1: Overview of the DESY Campus and EuXFEL/FLASH Accelerators
 : (a) An aerial view of the DESY campus in Hamburg, Germany, highlighting the EuXFEL and FLASH facilities (marked by the red lines). EuXFEL injector is on the right, leading to the beamlines on the left. Credit: European XFEL / Luftaufnahmen: FHH, Landesbetrieb Geoinf. und Vermessung. (b) A view inside the EuXFEL underground tunnel, showing the superconducting linac (yellow) through which relativistic electron bunches travel. Credit: DESY / Dirk nölle

4.1.1.1 Key characteristic of EuXFEL

EuXFEL stands out for its superconducting linear particle accelerator (linac), a distinctive feature that enables it to accelerate long electron bunch trains at a high repetition rate [4]. Table 4.1 summarizes the main operational parameters of the EuXFEL accelerator.

1. Electron bunch length and compression

The EuXFEL accelerator supports electron bunch charges ranging from 20 pC to 1,000 pC [32]. After compression, the resulting bunch lengths can vary from 3 fs to 150 fs (FWHM) [101].

To achieve the desired short bunch lengths, EuXFEL employs a three-stage bunch compression system, which divides the linac into three distinct sections (L1, L2 and L3). See Figure 4.2(a). Each magnetic chicane within this system is

widely tunable, providing flexibility in compression scenarios. This tunability is crucial for balancing parameters like peak current and arrival time stability with overall accelerator performance.

2. Electron bunch distribution and beamlines

A defining characteristic of EuXFEL is its ability to accelerate up to 2700 electron bunches within a single 600- μ s-long RF pulse, achieving a maximum intra-train repetition rate of 4.5 MHz. With the RF pulse repeating at 10 Hz, this facility can generate up to 27 000 X-ray pulses per second, corresponding to each electron bunch.

The overview of this electron bunch structure is provided in Figure 4.2(b).

These electron bunches are precisely distributed among three separate undulator beamlines (SASE1, SASE2, SASE3), where they produce tunable, femtosecond X-ray pulses spanning wavelengths from 0.05 nm to 4.7 nm (25 keV to 260 eV). A sophisticated distribution system, which uses multiple fast, high-precision kickers, can individually guide electron bunches to any beamline. This facilitates the simultaneous operation of multiple experiments [102]. Furthermore, an upstream kicker system can deflect single bunches into a dump beamline, offering complete flexibility in tailoring bunch patterns for each electron beamline while maintaining constant beam loading in the linac [103].

Figure 4.2(c) illustrates the three beamlines and their respective applications.

Parameter	Design	Unit
Energy range	8–17.5	GeV
Bunch charge	20–1,000	pC
RF pulse repetition rate	10	Hz
RF pulse length	600	μ s
Intra-train repetition rates	4.5	MHz
No. of bunches within the RF pulse	1–2,700	
Bunch length (FWHM)	3–150	fs

Table 4.1: Key performance parameters of the European XFEL accelerator [4]

4.1.1.2 EuXFEL’s EO diagnostic station

The electro-optic detection system, central to this thesis, is strategically installed within the EuXFEL accelerator tunnel. It is positioned downstream of the second bunch compression chicane, where the electron beam energy is approximately 700 MeV. Its precise location is indicated in Figure 4.2(a). At this diagnostic station, the EO system is designed to provide a time resolution that matches the expected RMS bunch lengths, typically ranging from 200 fs to 300 fs for

bunch charges between 100 pC and 1 nC [104]. This placement and optimized time resolution are essential for accurately characterizing the electron bunches after a significant portion of their compression, providing essential data for understanding and optimizing the FEL's performance.

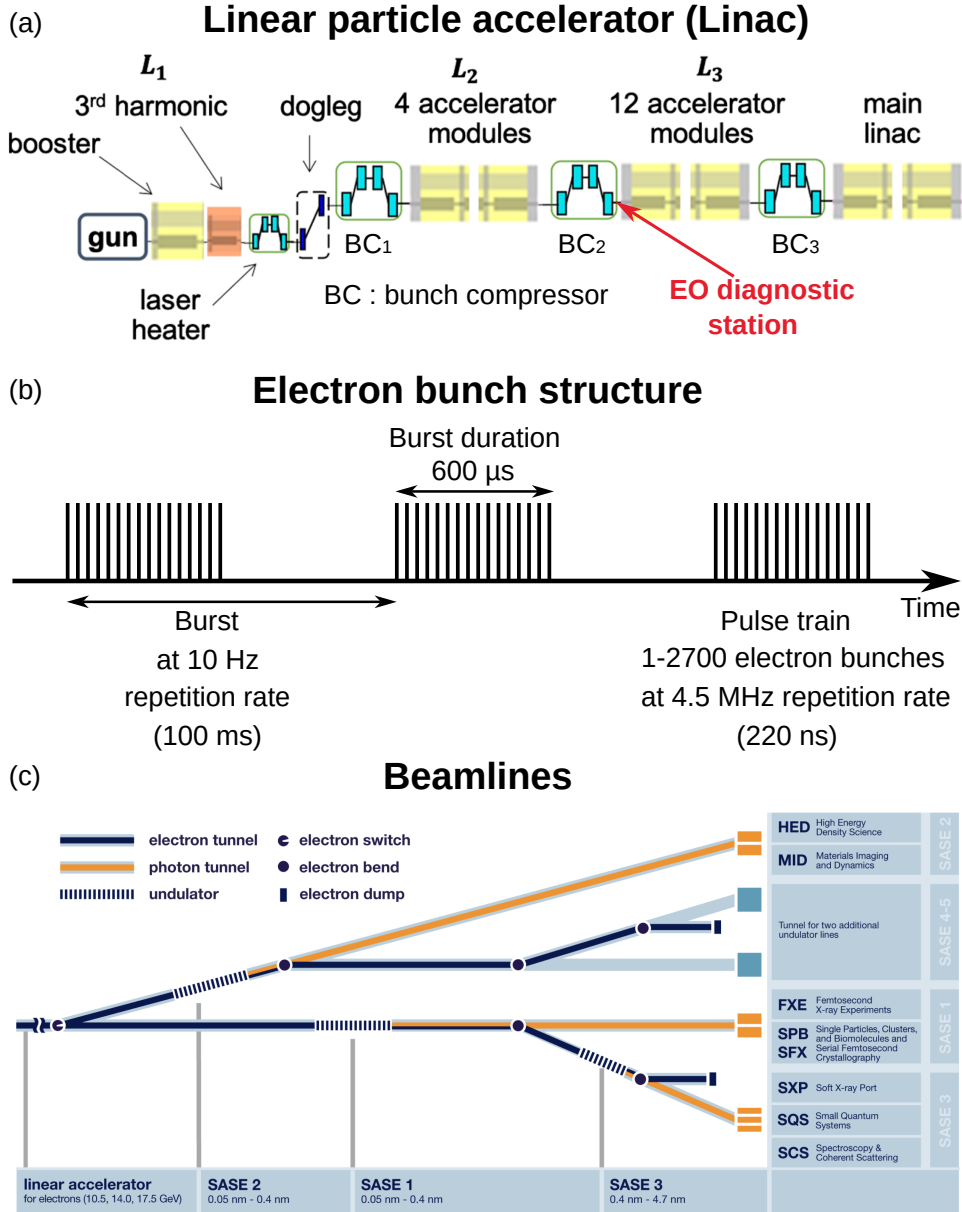


Figure 4.2: EuXFEL accelerator layout, electron bunch structure, and beamline distribution. (a) Schematic representation of the EuXFEL linear accelerator, employing a three-stage bunch compression system. The EO diagnostic station is located downstream the second bunch compressor. (b) Illustration of the EuXFEL's bunch structure, showing multiple electron bunches at 4.5 MHz within a single 600- μ s RF pulse train, with a burst repetition rate of 10 Hz. (c) Schematic overview of the three distinct undulator beamlines at EuXFEL (SASE1, SASE2, SASE3) and their applications, demonstrating how electron bunches are distributed to enable simultaneous experiments.

4.1.2 The FLASH facility

The Free-electron LASer in Hamburg (FLASH) at DESY, is renowned as the world's pioneering free-electron laser operating in the XUV and soft X-ray range [105]. Operational for the photon science user community since 2005, FLASH produces extremely intense, ultrashort X-ray laser flashes, enabling the direct observation of atomic and molecular dynamics on femtosecond timescales. The facility achieves laser amplification and saturation through the Self-Amplified Spontaneous Emission process (a detailed explanation of SASE FEL can be found in Section 1.4.2). A bird's eye view of the DESY campus, including the FLASH facility, is presented in Figure 4.1(a).

4.1.2.1 Key characteristic of FLASH

FLASH is a superconducting linear accelerator facility that currently operates two parallel SASE beamlines: FLASH1 and FLASH2, as depicted in Figure 4.3(a). Table 4.2 summarizes the key operational parameters for the FLASH accelerator.

Parameter	Design	Unit
Energy range	0.35–1.35	GeV
Bunch charge	0.01–1.2	nC
RF pulse repetition rate	10	Hz
RF pulse length	500	μ s
Intra-train repetition rates	0.04-1	MHz
No. of bunches within the RF pulse (shared between FL1 & FL2)	1-500	
Bunch length	20	fs

Table 4.2: Key performance parameters of the FLASH accelerator [106]

1. Electron bunch distribution and beamlines

At FLASH, electron bunches are generated by illuminating a Cs₂Te photocathode with ultraviolet laser pulses, producing initial bunches with a charge of 0.2 to 1 nC, a peak current of 50–100 A, and an RMS length of approximately 4 ps. These bunches are produced in pulse trains (or bursts) that typically have a repetition rate of 10 Hz. Each train can consist of 1–500 bunches with a spacing of 1 μ s, resulting in a 1 MHz intra-train repetition rate [106]. The intra-train repetition rate can be reduced to selected sub-harmonics of 1 MHz (e.g., 500 kHz, 100 kHz, among others) to meet experimental needs. The overall bunch structure is illustrated in Figure 4.3(b).

A total of 5000 bunches per second are available and shared between FLASH1

and FLASH2, enabling the two beamlines to provide FEL radiation for simultaneous experiments at the full 10 Hz repetition rate [107]. FLASH1 produces FEL radiation spanning the XUV range from 4.2 nm to 52 nm (295 to 24 eV). Additionally, FLASH1 provides secondary THz radiation covering a range from 1 μm to above 300 μm [108]. Meanwhile FLASH2 covers 3.3 nm to 90 nm (375 to 14 eV) [109]. A third beamline, FLASHForward, is dedicated to pioneering beam-driven plasma-wakefield experiments [39]. More detailed information on FLASH beamlines can be found in reference [106].

2. Electron bunch length and compression

Intense relativistic electron bunches with durations of 100 femtoseconds or less are essential for SASE-based FELs like FLASH.

Achieving these ultrashort durations is realized using a two-step longitudinal bunch compression process at FLASH. This process divides the linac into two parts, as depicted in Figure 4.3(a), enabling up to two distinct bunch compression stages within a single pulse train.

4.1.2.2 FLASH's EO diagnostic station

The electro-optic detection system at FLASH, a copy of the EuXFEL apparatus, is located upstream of the second bunch compressor [110]. Its position is illustrated in Figure 4.3(a).

4.1.2.3 Upgrade : FLASH 2020+

The FLASH facility continues to evolve with significant upgrades. Two separate shutdowns were scheduled, the first for the linac accelerator (including the EO setup), and the second for the beamlines. In the context of this thesis, the electro-optic measurements recorded at FLASH were taken in May 2024, just one month before the start of the beamline shutdown [111].

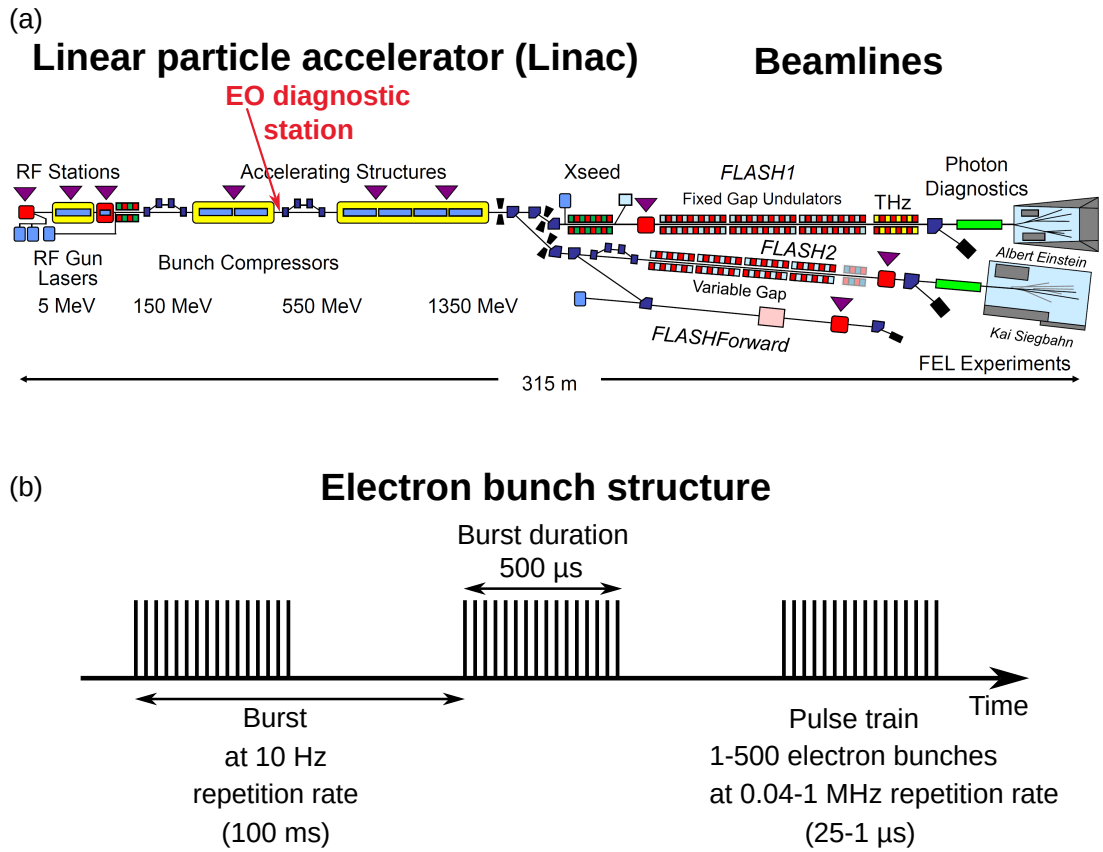


Figure 4.3: **FLASH accelerator layout, electron bunch structure, and beamline distribution.** (a) Schematic representation of the complete FLASH accelerator, including the linear accelerator section and beamlines (FLASH1, FLASH2). The EO diagnostic station is located upstream of the second bunch compressor. (b) Illustration of the FLASH's bunch structure, showing 1 to 500 electron bunches delivered at an intra-train repetition rate of 0.04 to 1 MHz within a single 500 μ s RF pulse train, with a burst repetition rate of 10 Hz.

4.1.3 Role of longitudinal electron bunch diagnostics at DESY

The emitted photon pulses depend directly on the properties of the electron bunches. Hence the diagnostics of their longitudinal shape has been the subject of an intense research in the recent years [110]. Information regarding longitudinal electron bunch properties is essential for optimizing and for providing feedback on the performance of EuXFEL and FLASH. Key parameters such as the electron bunch's temporal shape, its bunch length, and the arrival time of each electron bunch within the pulse train directly influence the quality and timing of the generated X-ray pulses. The electro-optic diagnostic system is invaluable in this regard, providing single-shot, non-destructive measurements of the bunch's arrival time and profile with high temporal resolution. These data are intended for correcting long-term drifts to stabilize the bunch compression and arrival time. Furthermore, the EO diagnostics are not currently used for rapid, intra-train feedback due to firmware limitations and the data's insufficient resolution compared to measurement jitter. The fast feedback currently relies on the Bunch Arrival time Monitor (BAM) for arrival time stabilization.

4.2 Experimental apparatus presentation

The DEOS setup described in this section corresponds to the two experimental apparatus replica used at EuXFEL and FLASH. A more detailed overview of the setup, especially regarding the laser probe and synchronization, can be found in Ref [11]. This section described the implementation of the diversity EO configuration and the recent upgrades made during this thesis to those systems.

4.2.1 Femtosecond probe laser

The probe laser is a custom-built Yb-fiber laser system, optimized for generating short pulses with a broad spectral distribution essential for spectral decoding techniques. Non-linear effects within the system broaden the spectrum, providing a bandwidth up to 80 nm centered at 1050 nm. The laser operates at a repetition rate ranging from 0.1-2 MHz and delivers an output pulse energy from 5-100 nJ. This laser system is located in a rack outside the accelerator tunnel, mitigating radiation exposure from EuXFEL and FLASH. Key parameters are summarized in Table 4.3.

4.2.2 Chirped probe laser

Laser pulses, with adjustable energies from 5 nJ to 100 nJ, are transported into the accelerator tunnel via a $L_1 = 15\text{ m}$ polarization-maintaining fiber. Immediately after the fiber coupler, a grating compressor adjusts the pulse length. This

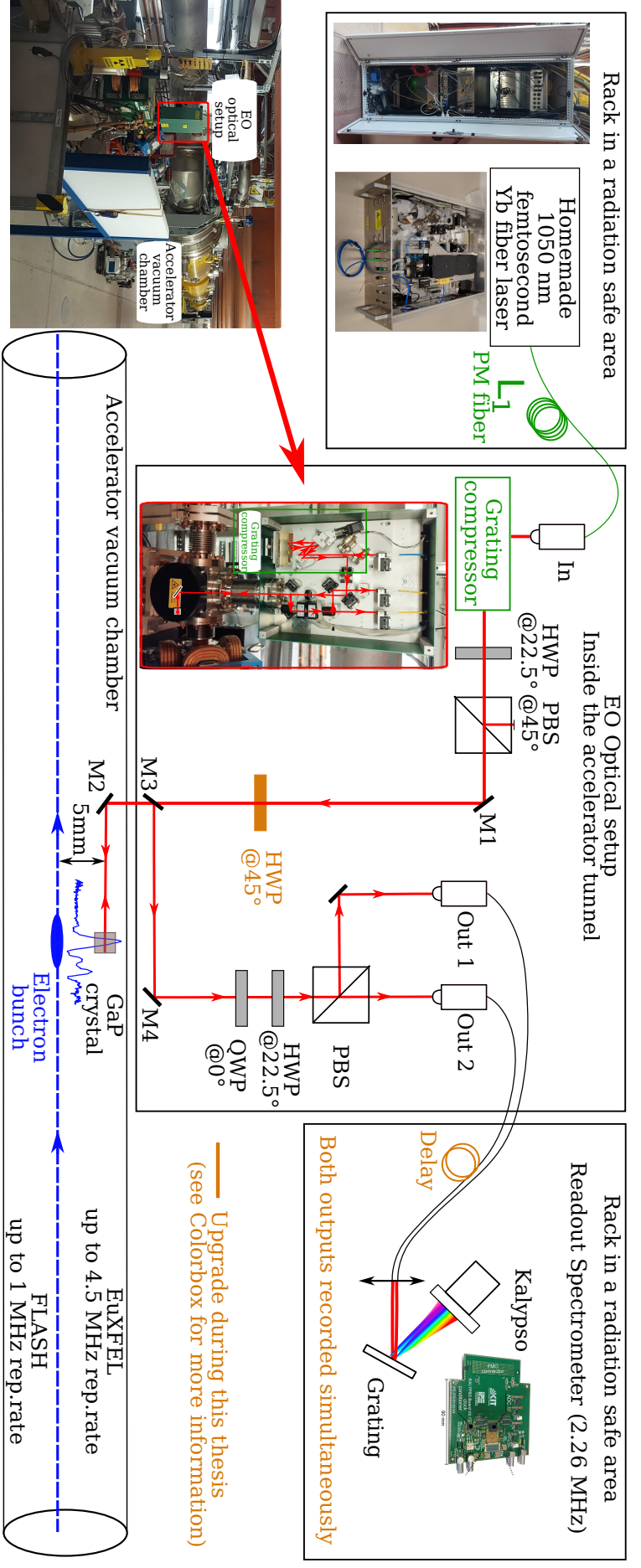


Figure 4.4: **Overview of the Single-Shot Electro-Optic Diagnostic Setup at DESY (EuXFEL and FLASH):** Detailed schematic of the Diversity Electro-Optic Sampling system utilized for high-repetition-rate longitudinal electron bunch diagnostics. The setup is divided into two main areas. Inside the accelerator tunnel, where the electron bunch interacts with the chirped probe laser inside the electro-optic crystal (GaP), converting the THz field into a spectral phase modulation. The EO system's probe laser source and readout components are housed in a dedicated rack. For FLASH, this rack is in a radiation-safe area, while for EuXFEL, it is situated inside the accelerator tunnel. This rack contains the Ytterbium (Yb) femtosecond fiber laser (the probe source) and the KALYPSO high-speed readout spectrometer. The spectrometer records the two orthogonal polarization components (Out 1 and Out 2) simultaneously at 2.26 MHz to enable high-fidelity reconstruction of the electron bunch profile. This 2.26 MHz readout rate allows for recording every bunch at FLASH with a repetition rate of 1 MHz. However, when EuXFEL operates at its standard 2.26 MHz intra-train repetition rate, the system can only record every second bunch. EuXFEL is also capable of reaching an intra-train repetition rate of up to 4.5 MHz. Elements highlighted in orange represent the upgrades performed during this thesis. These modifications involved the implementation of a HWP rotated at 45° to compensate the metallic mirror induced phase-shift only before the EO crystal, and the addition of an extra fiber delay to enable the simultaneous recording of both DEOS outputs.

HWP: Half-wave plate, QWP: Quarter-wave plate, M: Mirror.

Laser parameters	
λ_0	1030-1050 nm
$\Delta\lambda$	40-80 nm
Pulse length (Fourier limit)	50 fs (FWHM)
Pulse energy	5-100 nJ
Average power	5-100 mW
Repetition rate	0.1-2 MHz

Table 4.3: Parameters of the Yb-fiber laser system [11]

compressor, is composed by two 1200 lines/mm gratings, one grating is motorized, allowing pulse length adjustment from full compression of less than 100 fs to more than 5 ps (FWHM), crucial for controlling the temporal measurement window.

4.2.3 EO head

Directly after the grating compressor, the chirped probe laser's polarization is set at 45° using a half-wave plate (HWP) and a polarizing beam splitter (PBS) to ensure a clean polarization state. However, the use of metallic mirror to guide the beam introduces a parasitic phase shift to the 45° laser polarization, which mandates compensation (see Colorbox. 4.2.3).

The laser is then directed to a mirror and a Gallium Phosphide (GaP) crystal positioned within the electron beamline on a motorized vacuum feed-through. The laser pulses first counter-propagate with the electron bunches, entering the anti-reflective coated rear side of the GaP crystal, and then reflect from its highly reflective front side. The reflected laser pulses then co-propagate with the electron bunch's Coulomb field, inducing polarization modulation within the crystal. After reflection back through the vacuum window, the laser pulses pass through a QWP and HWP, set at 0° and 22.5° respectively, to satisfy the phase diversity condition. Finally, a PBS splits the modulated signal into orthogonal components, which are then coupled into optical fibers for readout (denoted as Out1 and Out2 in Fig. 4.4).

Recent upgrades : Metallic mirror induced phase-shift compensation

The phase diversity configuration requires the probe laser polarization to be set at 45° . However, the necessity of guiding the beam to the in-vacuum GaP electro-optic crystal mandates the use of at least one metallic mirror, which inherently introduces a parasitic wavelength-depend phase shift between the horizontal and vertical polarization components. In the context of electro-optic measurements, which rely on detecting small changes in ellipticity induced by the Coulomb field of the electron bunch, this mirror-induced shift constitutes a significant source of errors. Quantitatively, the phase shift from a single metallic mirror is on the order of 13 mrad over the probe wavelength. A robust solution has been identified to passively compensate at each probe wavelength for this parasitic effect by incorporating an achromatic half-wave plate oriented at 45° and strategically utilizing mirror symmetries within the optical path, as detailed in Fig.4.5.

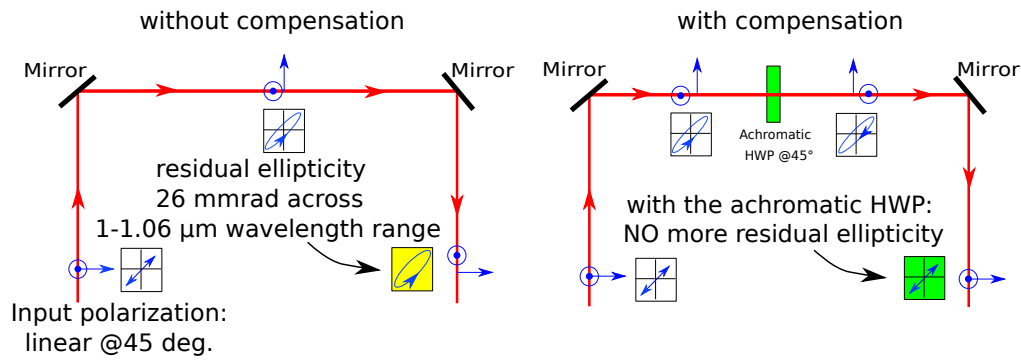


Figure 4.5: Compensation of metallic mirror induced wavelength-dependent phase-shift. The uncompensated path using two identical metallic mirrors introduces a total parasitic ellipticity of approximately 26 mrad across the laser probe wavelength range, which constitutes unacceptable error for high-fidelity EO measurements. The compensated path uses an achromatic Half-Wave Plate strategically placed between the two identical metallic mirrors. This HWP rotates the polarization base by 90° , causing an opposing phase shift introduced by the second mirror. This phase shift exactly cancels out that of the first mirror, resulting in zero residual ellipticity for the initial linear polarization state at 45° .

Presently, only a partial compensation is implemented, the HWP (colored orange in Fig.4.4) compensates the phase shift only before the EO crystal. A future upgrade is planned during the next EuXFEL shutdown to achieve full compensation both before and after the crystal, utilizing an alternative periscope configuration equivalent to the HWP/symmetry method.

4.2.4 Readouts : KALYPSO spectrometer

The two polarization components are routed via optical fibers to a custom-made grating spectrometer, designed for measuring the spectral distribution of the chirped laser pulses. This spectrometer utilizes a KALYPSO linear array detector [60], specifically developed at Karlsruher Institut für Technologie (KIT) for EOSD at facilities like EuXFEL and Karlsruhe Research Accelerator (KARA) .

It enables continuous data readout at frame rates up to 2.7 MHz, composed of 256 pixels with a spectral resolution of approximately 0.2 nm/pixel. An upgrade was implemented during this thesis period to enable the simultaneous capture of both phase diversity outputs, allowing the first single-shot measurement of a single electron bunch (see Colorbox.4.2.4 and section.4.3.2). More information can be found in Ref [11].

Recent upgrades : Dual-Channel KALYPSO Spectrometer Readout for Phase Diversity

Our custom-made spectrometer, utilizing the KALYPSO linear array detector, was initially designed to record only one output, as standard spectral decoding techniques typically use a single output. However, adopting the phase diversity configuration requires measuring two outputs simultaneously to perform effective single-shot measurements.

Previously, we were forced to measure the outputs sequentially by manually rotating the final HWP. This meant the two single-shot measurements were not captured from the same electron bunch, but from two different bunches.

An upgrade was implemented during this thesis period to overcome this limitation. We now are able to record both single-shot outputs simultaneously from a single electron bunch. This is achieved by splitting the signal into two channels, delaying one channel, and then sending both to the KALYPSO camera. The trade-off for this simultaneous capability is a reduction in the achievable effective repetition rate by a factor of two. For example, if the Free-Electron Laser (FEL) operates at 1.13 MHz, the KALYPSO camera runs at 2.26 MHz to capture the two time-separated outputs from a single bunch. If the FEL operates at 2.26 MHz, we are only able to record every second electron bunch.

4.3 DEOS diagnostic of bunch shape at EuXFEL

This section presents the results from the DEOS experiments conducted at EuXFEL, starting with preliminary non-simultaneous measurements of a special configuration of an electron bunch train comprising three distinct bunch compression. These initial measurements motivated a critical system upgrade to implement a spectrometer enabling simultaneous measurement of the two DEOS outputs. With this capability in place, we present the first single-shot, MHz-rate characterization of a full bunch train, providing statistics on bunch length and arrival time jitter, and conclude with a rigorous comparison of the SAD and MRC reconstruction algorithms, highlighting the performance of the new Self-Adaptive Dispersion algorithm in accurately characterizing ultrashort bunches under non-linear dispersion.

4.3.1 Preliminary results with non-simultaneous DEOS system motivating a system upgrade

The measurements presented here were conducted in February 2024 at EuXFEL. These data were acquired prior to the KALYPSO spectrometer readout upgrade that enabled simultaneous recording of both phase diversity outputs. Thus, the two phase diversity outputs shown in Figures 4.6(a), (b), and (c) were recorded sequentially by manually adjusting the final half-wave plate. Therefore, these outputs do not correspond to the same electron bunch.

During these experiments, electron bunches were generated with a charge of 255 pC at a repetition rate of 2.26 MHz. These bunches formed 1261 bunches trains, of which only the even-numbered bunches were recorded (see colorbox4.2.4).

Figure 4.6(d) displays a single-shot measurement of an electron bunch train that was configured to exhibit three distinct bunch compression settings within a single train. This capability is necessary for accommodating user requirements, as different X-ray light properties are often needed for three simultaneous experiments on separate beamlines. A clear shift in arrival time, bunch length and varying signal amplitudes, indicative of the different electron bunch compressor settings, are discernible along the train.

To highlight these distinct compression states, the temporal profiles of three representative bunches from the train, bunch 300, bunch 700, and bunch 1240 are presented in Figure 4.6(e), (f), and (g), respectively. The measured root mean square (RMS) temporal bunch lengths for these bunches confirm the successful implementation of varying compression, as evidenced by their different duration. For bunch 700, a particularly short RMS bunch length of less than 200 fs was

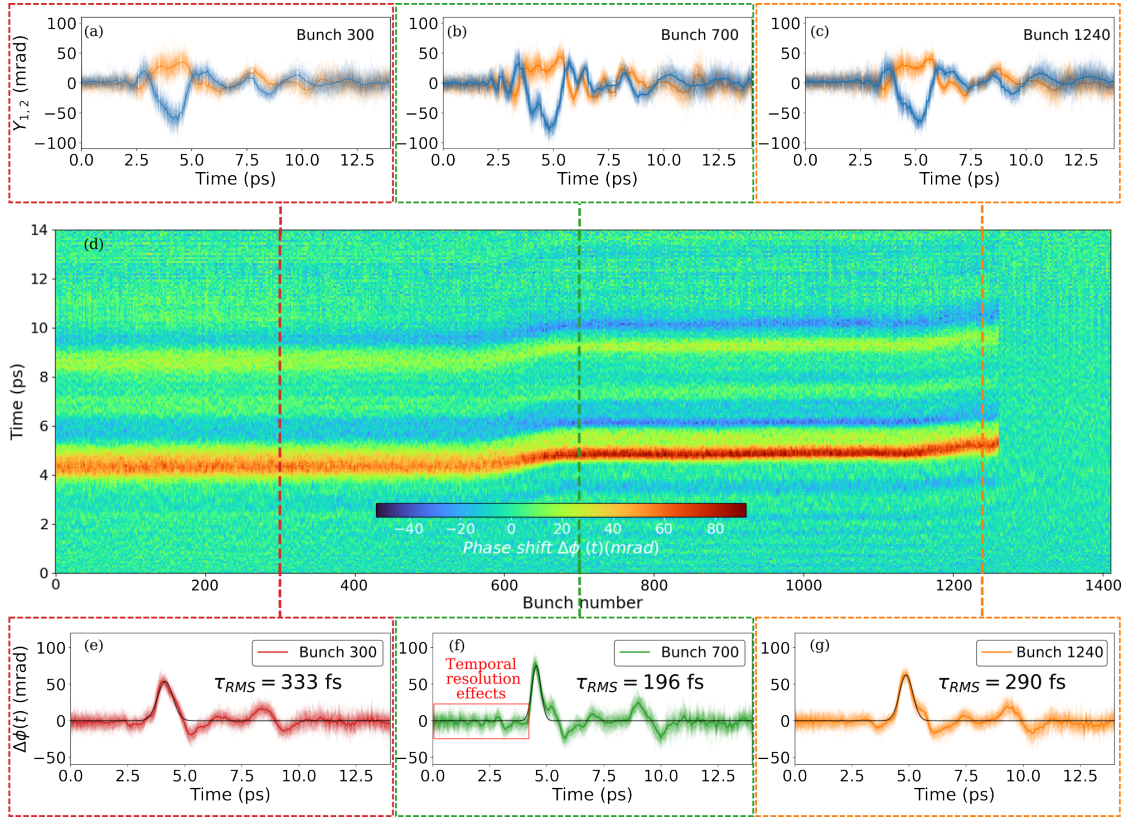


Figure 4.6: **EuXFEL bunch train characterization with varying electron bunch compression settings.** (a)-(c) Recorded of the two outputs of individual electron bunches (300, 700, 1240, respectively) required for phase diversity detection. (d) Single-shot measurement of an electron bunch train at EuXFEL, demonstrating three distinct bunch compression configurations. (e)-(g) Reconstructed temporal profiles of individual electron bunches (300, 700, and 1240, respectively) from the displayed train, highlighting their resulting different bunch lengths.

measured. However, as the current electro-optic system was primarily designed for RMS electron bunch lengths greater than 200 fs, a temporal resolution limit becomes apparent in Figure 4.6(f). This limitation manifests as unphysical oscillations preceding the electron bunch, in a region where no signal is expected.

This temporal resolution limit is attributed to the phase matching condition between the chirped probe laser pulse and the Coulomb electric field of the electron bunch within the GaP EO crystal, which is directly dependent on its thickness. The GaP crystal currently employed has a thickness of 2 mm. Figure 4.7(a) shows the EO response function $G(f, d)$, and Figure 4.7(b) displays the effective response $G(f, d) \times r_{41}$, representing the crystal's bandwidth for various thicknesses, including 2 mm, 1 mm, and 500 μm . The first zero of the 2 mm GaP crystal's effective response appears at 4.35 THz, which corresponds to a temporal feature of approximately 230 fs. The measured 196 fs bunch length for bunch 700 overcome this limits and falls very close to another zero. This proximity affects measurement accuracy, leading to the observed oscillation that

defines the temporal resolution limit.

To overcome this limitation and enable more accurate characterization of ultrashort bunches, a future upgrade is planned to replace the current 2 mm GaP crystal with a thinner 500 μm GaP crystal. This modification is expected to increase the crystal's effective bandwidth up to 6 THz, thereby improving the temporal resolution to approximately 166 fs.¹

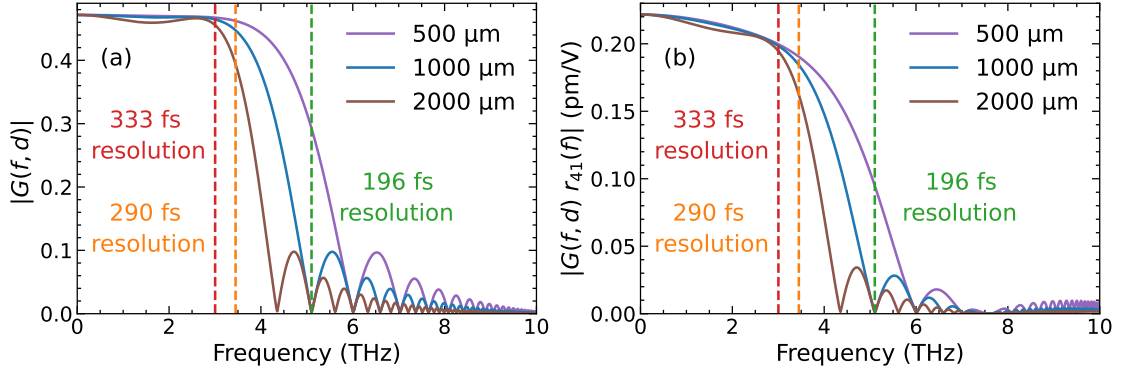


Figure 4.7: **Electro-optic bandwidth dependence on GaP crystal thickness.** (a) Electro-optic response function $G(f, d)$ as a function of frequency f and crystal thickness d . (b) Effective electro-optic response $G(f, d) \times r_{41}$, illustrating the bandwidth of the GaP crystal for various thicknesses: 2 mm (current EO crystal), 1 mm, and 500 μm (planned Eo crystal upgrade).

¹The phonon GaP electro-optic bandwidth is higher, around 8 THz.

4.3.2 Results from the upgraded simultaneous DEOS system: First DEOS single-shot bunch shape characterization

Building directly upon the successful preliminary results obtained using the DEOS setup combined with the SAD algorithm, detailed in the preceding sections. This motivate the deployment of the simultaneous readout upgrade to enable real-time, single-shot diagnostic of the electron bunch train at EuXFEL.

Recent upgrades : Implementation and Deployment Strategies of the Hardware Upgrades

The dual-channel spectrometer, which enables simultaneous measurement of both phase diversity outputs, was first successfully commissioned and tested at the FLASH facility (Section. 4.4). This initial deployment benefited from the easier electronics access at FLASH during its operation compared to the radiation restrictions at EuXFEL. Following this successful validation, the system was installed at EuXFEL a few months later, enabling the data acquisition presented in this section.

The recorded data corresponds to a typical machine configuration, where an electron bunch train consisting of 980 bunches with a nominal bunch charge of 255 pC was accelerated, operating at an intra-train repetition rate of 2.26 MHz. Due to the required readout speed for the simultaneous recording of both phase diversity outputs, necessary for the reconstruction of a single electron bunch, only odd bunches were recorded (see Colorbox 4.2.4).

4.3.2.1 Information extract from bunch profile DEOS measurement : Statistic of bunch length and arrival time jitter

The core capability of the single-shot EO monitor is the retrieval of the electron bunch's longitudinal profile. Figure 4.8 summarizes the key information extracted from the EO measurements across the entire bunch train.

1. Single-shot profile reconstruction Figure 4.8.(a-b):

A typical electron bunch profile reconstruction is demonstrated for electron bunch number 100 using the SAD algorithm. Figure 4.8(a) displays the two raw recorded outputs from the DEOS setup, which serve as the input for the reconstruction. The resulting longitudinal electron bunch profile is shown in Figure 4.8(b). For this specific bunch, the profile was successfully reconstructed, and the root-mean-square (RMS) bunch length was measured to be 545 fs.

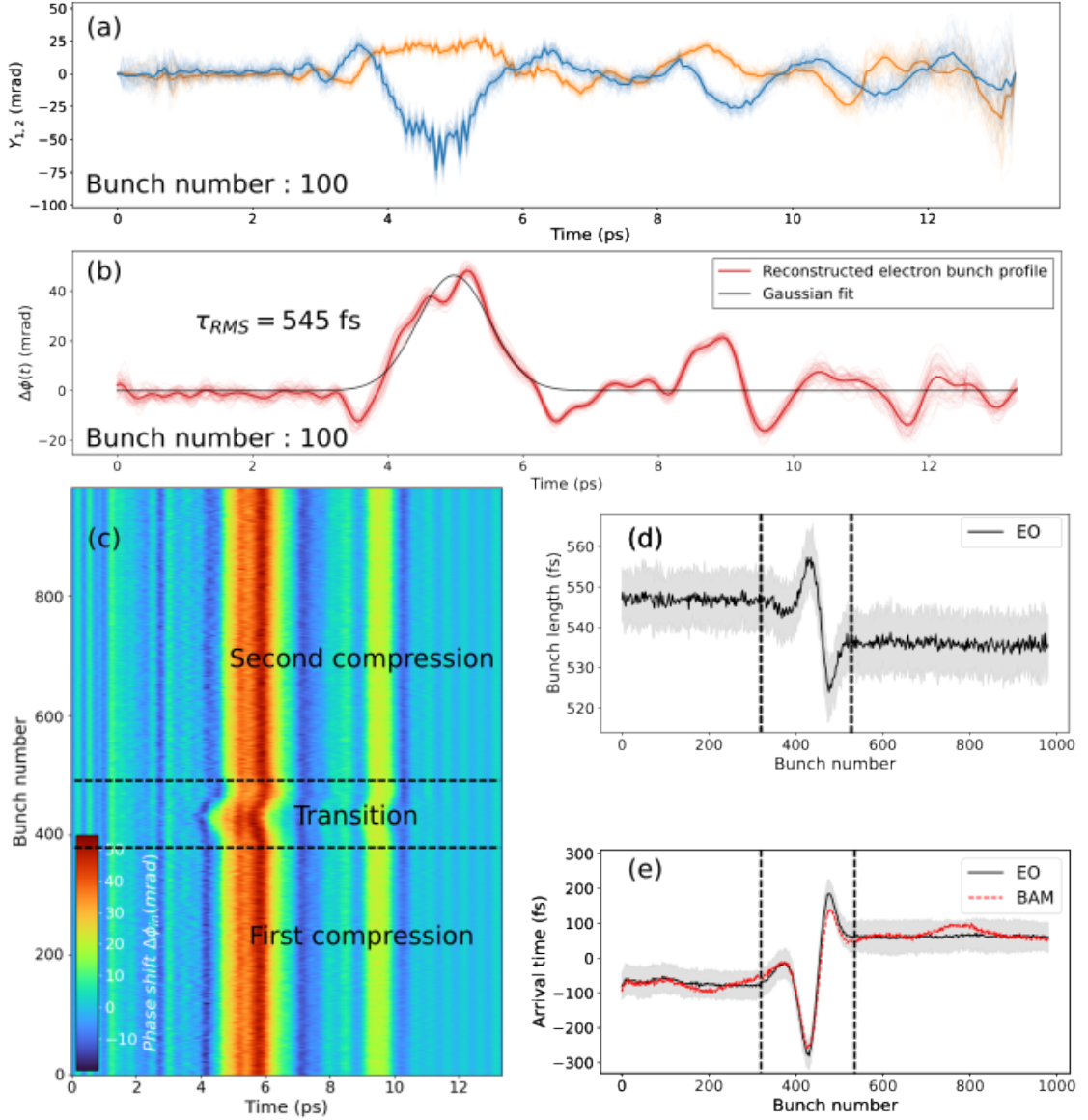


Figure 4.8: **Summary of extracted information from single-shot EO measurements on a 980-bunch train at EuXFEL.** (a) Two recorded outputs from DEOS setup for bunch 100. (b) Reconstructed longitudinal profile for bunch 100 using the SAD algorithm, fitted with Gaussian indicating an RMS bunch length of 545 fs. (c) Full 980-bunch train, highlighting the two distinct compression stages. (d) Bunch-by-bunch RMS bunch length extracted from EO measurements. The grey area represents the shot-to-shot fluctuation over 50 consecutively recorded bunch trains. The black curve is the average RMS length over those 50 shots. (e) Comparison of bunch arrival time between the EO measurement and the independent Beam Arrival Monitor (BAM) signal (red dashed curve). For the EO data, the grey area represents the shot-to-shot fluctuation over 50 recorded trains, and the black curve is the average arrival time over those 50 shots.

2. Full electron bunch train overview Figure 4.8.(c):

Figure 4.8(c) presents the full train data across the 980 bunches, of which only the even-numbered bunches were recorded (see colorbox4.2.4). This visualization clearly delineates three distinct zones. The initial first compression, followed by a transition region, and finally the second compression.

3. Bunch length statistics Figure 4.8.(d):

The successful single-shot profile reconstruction enables the extraction of statistical properties over 50 consecutive shots of the entire bunch train, as presented in Figure 4.8(d) and (e). Figure 4.8(d) plots the fitted RMS bunch length for all 980 bunches. Analysis of this data shows a clear bunch length around 545fs for the first compression area and a slightly shorter length of 535 for the second compression area. A noticeable perturbation is observed around bunch 320 to 520, corresponding to the transition region in Figure 4.8(c), where the bunch length momentarily increases and then stabilizes.

4. Arrival time jitter statistic Figure 4.8.(e):

Figure 4.8(e) tracks the bunch-by-bunch arrival time jitter. This temporal center is determined by fitting the reconstructed bunch profile. The arrival time measurement extracted from the EO measurement is compared with the signal from the independent Beam Arrival Monitor (BAM) [112, 113], which is situated next to the EO setup and serves as an external reference. The excellent agreement between the EO and BAM systems, particularly their concurrent capture of the sharp transient event between bunches 320 and 520, confirms the EO monitor's high fidelity and its capability to capture fast, sub-picosecond longitudinal dynamics within the bunch train. The main limitation of the arrival time accuracy of the EO system come from the stability of the laser synchronization, which is about 40 fs RMS [11].

4.3.2.2 Comparison between MRC vs SAD algorithm

Another key objective of this part is to compare the performance of different profile reconstruction algorithms under the influence of nonlinear dispersion, as evidenced by the measured time-calibration data from EuXFEL (Figure 2.14b). This comparison illustrates the longitudinal electron bunch profile reconstructed using different algorithms from the same EO data trace shown in Figure 4.9.(a). Specifically, Figures 4.9.(b),(c) and (d) show the longitudinal electron bunch profiles reconstructed using the MRC algorithm, the SAD algorithm intentionally limited to GVD-only compensation (i.e., neglecting the third order dispersion), and the full SAD algorithm accounting for third-order dispersion, respectively.

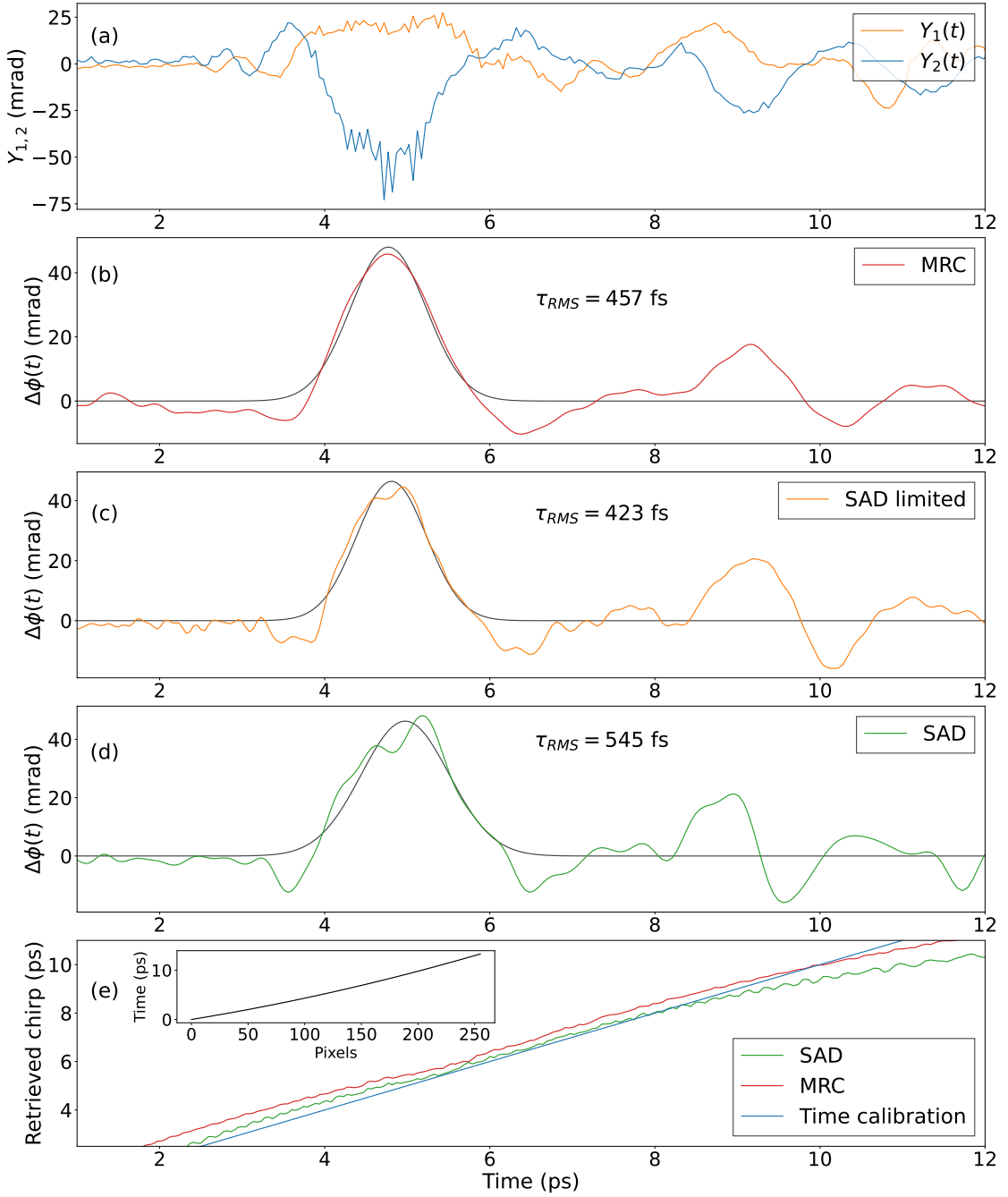


Figure 4.9: **Comparison of electron bunch profile reconstruction using MRC, GVD-only restricted SAD, and full SAD algorithms on identical single-shot EO data.** (a) The two simultaneously recorded outputs from DEOS setup for bunch 100. (b) Reconstructed longitudinal electron bunch profile using the Maximum Ratio Combining (MRC) algorithm. (c) Reconstructed longitudinal electron bunch profile using the Self-Adaptive Dispersion (SAD) algorithm restricted to Group Velocity Dispersion (GVD) compensation only. (d) Reconstructed longitudinal electron bunch profile using the full SAD algorithm with third-order dispersion compensation (e) Comparison of the retrieved chirp from the MRC and full SAD algorithms against the measured time-calibration reference. For visualization, all data are plotted against the measured time calibration, which therefore appears as a perfectly linear line serving as the accuracy benchmark. The non-linearity of the measured time calibration is visualized in the inset and in Figure 2.14.

1. Profile fidelity and wakefield difference.

The three algorithms reconstruct a similar main electron bunch profile. However, a significant difference is observed in the wakefield structure following the main peak. The MRC and restricted SAD algorithms result in a shorter main peak and a longer wakefield compared to the full SAD algorithm, which successfully incorporates third-order dispersion. This discrepancy is interpreted as a manifestation of the temporal focusing effect on the main signal, as previously discussed in Colorbox 3.2.1.1. This effect arises from the inherent limitation of MRC and GVD-only restricted SAD approaches to perform only a linear fit to the chirped probe pulse's dispersion. This restricted linear fitting forces an optimal compromise that temporally smears the reconstructed profile, reducing the duration of the main peak while artificially lengthening the wakefield.

2. Algorithms chirp retrieval vs measured time-calibration.

Further evidence supporting the fidelity of the full SAD algorithm is seen in the comparison of the retrieved instantaneous chirp against the measured time calibration (Figure 4.9.(e)). This time calibration was measured by introducing a sub-picosecond delay to the chirped probe laser relative to the electron bunch. This delay causes a corresponding and measurable shift of the encoded signal on the laser spectrum output, as explained previously in Figure.2.14.

The non-linearity of this time calibration is illustrated in the inset of panel (e). For visualization clarity, all retrieved chirps (from MRC and SAD) are plotted against the measured time calibration. This methodology results in the measured time calibration appearing as a perfectly linear reference, providing an immediate visual benchmark for algorithm accuracy. The chirp retrieved by the SAD algorithm remains closer to the measured calibration than the chirp retrieved by the MRC algorithm. This demonstrates SAD's superior ability to accurately map the nonlinear dispersion relation, thereby enabling high-fidelity reconstruction.

3. Discussion on the artifacts visible in the SAD algorithm's results

The artifacts visible in the SAD algorithm's results (e.g., negative values before the main peak) are attributable to a misorientation of the waveplates in the experimental setup, as further detailed in the following colorbox 4.4.1. While an improvement to the MRC algorithm has already been implemented to handle the effects of waveplate misorientation by treating the associated phase term ϕ as a fit parameter, as described in Section 2.4.3. It is highly probable that the absence of this numerical compensation in the restricted SAD algorithm is responsible for the subtle differences in reconstructed bunch length duration

observed between the MRC and restricted SAD algorithms, as well as the artifacts observed in the full SAD algorithm. A future improvement in the SAD algorithm's mathematical framework should be realized to explicitly model and account for the effects of waveplate misorientation, thereby achieving even greater robustness against non-ideal experimental alignments.

4.4 DEOS diagnostic of bunch shape at FLASH

This section presents the DEOS measurement results obtained at FLASH, these data were acquired just a few days before the shutdown for the facility's upgrade FLASH 2020+. These measurements were made using the implementation of the simultaneous readout upgrade to the KALYPSO spectrometer. KALYPSO was operating at 2 MHz to record both phase diversity outputs from the same electron bunch delivered by FLASH at 1 MHz. This capability allows for single-shot characterization of both phase diversity outputs from the same electron bunch.

4.4.1 Single-shot characterization of electron bunch train containing two different bunch charge and compression

Figure 4.10(c) displays a single-shot recording of an electron bunch train at FLASH, demonstrating two distinct electron bunch compression configurations. This configuration supports the facility's multi-user operation, allowing different bunch properties to be optimized for parallel experiments. Within this train, a single electron bunch with a charge of 250 pC and an RMS length of 900 fs, dedicated to the FLASH1 beamline, while a sequence of one hundred electron bunches with a charge of 125 pC, and an RMS length of 830 fs is directed towards the FLASH2 beamline.

Figure 4.10(a) displays the two simultaneously recorded phase diversity outputs for the single electron bunch directed to FLASH1. While Fig 4.10(b) presents the corresponding outputs for a representative electron bunch directed to the FLASH2 beamline. Distortions resulting from the time-resolution limit intrinsic to the spectral decoding method (as detailed in Section 2.4.1), are evident in these raw outputs. These distortions become particularly pronounced as the electron bunch duration shortens, a condition noticeable in Figure 4.10(b).

The reconstructed temporal profiles for two representative electron bunches, each corresponding to a different compression setting within the train, are presented in Fig 4.10(d) for the FLASH1-destined single bunch, and Fig 4.10(e) for a representative electron bunch destined for FLASH2. These reconstructions

reveal the differing bunch lengths and peak amplitudes resulting from distinct compression settings and different bunch charges. Note that the reconstructed profiles exhibit unphysical artifacts (e.g., negative values) due to waveplate misorientation, which is discussed in detail in Colorbox4.4.1.

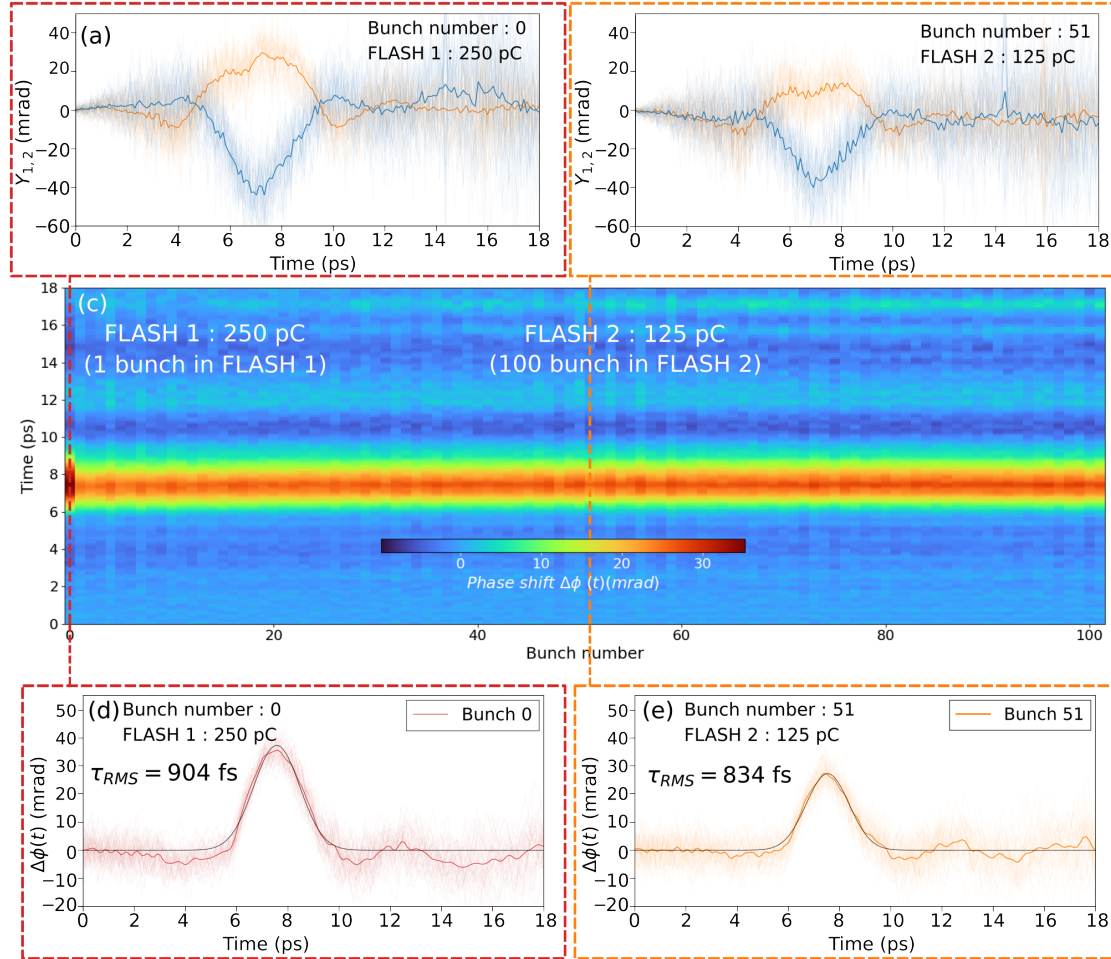


Figure 4.10: **FLASH Electron bunch train characterization** : (a)-(b) Simultaneously recorded phase diversity outputs for a single electron bunch. (c) Single-shot measurement of an electron bunch train at FLASH, highlighting two different electron bunch compression settings for FLASH1 and FLASH2 beamlines. (d) Reconstructed temporal profile of a 250 pC electron bunch and an 900 fs RMS electron bunch for the FLASH1 beamline. (e) Reconstructed temporal profile of a 125 pC electron bunch and an 830 fs RMS electron bunch for the FLASH2 beamline.

Note on artifacts from waveplate misorientation

Artifacts, typically appearing both before and after the main peak, are observed in the experimentally SAD-reconstructed electron bunch profiles (e.g., Figures 4.9.(d), and Figures 4.10(d) and (e)). While negative values appearing after the peak could potentially be attributed to physical phenomena like wakefields, the presence of negative values before the peak are unphysical, as electrons carry a negative charge and cannot precede the main bunch with a positive charge. These artifacts directly indicate an incorrect orientation of the optical waveplates within the DEOS setup. The challenge in setting the waveplates arises from their initial orientation, typically determined by minimizing the transmission power to identify the zero points, is corrupted by ellipticity. This ellipticity is induced by the metallic mirrors positioned downstream of the GaP crystal. Thus, the QWP is miscalibrated, resulting in an incorrect final orientation for the phase diversity technique. This wavelength-dependent phase shift induced by the mirrors requires future compensation (as detailed in the *Recent Upgrades* 4.2.3). To confirm this hypothesis, a simulation of an electron bunch reconstruction was performed with deliberately misoriented HWP and QWP. The results, depicted in Figure 4.11, show artifacts in the reconstructed pulse, specifically negative values in the front and tail that are strikingly similar to those observed in the reconstructed experimental data. This confirms that misoriented waveplates is the source of the observed artifacts.

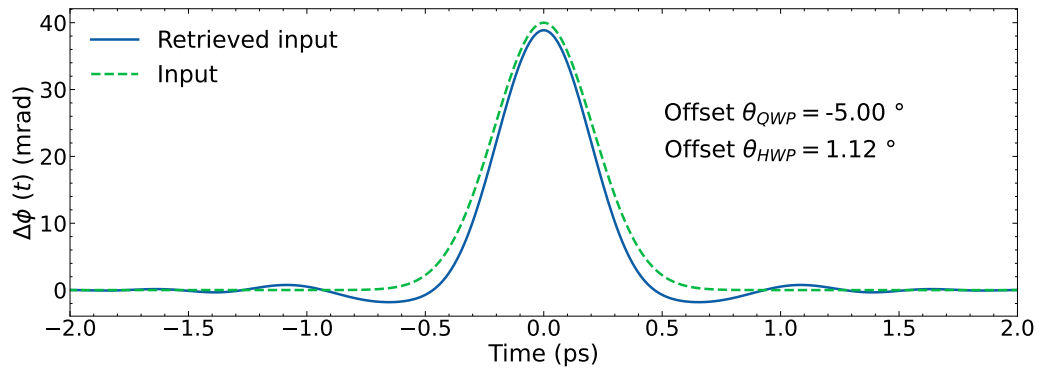


Figure 4.11: Simulation of electron bunch reconstruction with misoriented waveplates : The reconstructed temporal profile exhibits artifacts when the HWP and QWP are misoriented by a respective angular offset of θ_{QWP} and θ_{HWP} , resulting in profile distortion compared to the original input pulse (dashed line). This simulation confirms the source of artifacts observed in the reconstructed experimental data.

This numerical uncompensated waveplate misalignment is a key difference between the algorithms. The MRC approach has been numerically improved to handle this error by treating the associated phase term ϕ as a fit parameter, as described in Section 2.4.3. In contrast, the current SAD algorithm do not yet incorporate this numerical compensation, leading directly to the observed artifacts.

4.5 Chapter summary and Perspectives

Chapter summary : Now capable of routine diagnostics with a resolution of a few hundred femtoseconds at MHz repetition rates with high fidelity

This chapter detailed the complete DEOS diagnostic setup and its recent upgrades with the implementation of the novel Self-Adaptive Dispersion algorithm within the distinct environments of two major accelerator facilities (EuXFEL and FLASH).

The measurements presented in this chapter confirm that the developed DEOS system, combined with the SAD algorithm, is now fully operational for routine accelerator diagnostics.

The DEOS system is now able to achieve longitudinal electron bunch characterization with few hundreds femtoseconds temporal resolution with high fidelity at the MHz repetition rates, including the measurement of arrival time and bunch length stability within the electron bunch train. Furthermore, the comparison of the measured time calibration and the algorithm retrieval dispersion confirms that the SAD algorithm offers a superior approach to dispersion management compared to the MRC algorithm, ensuring higher accuracy even under realistic non-linear chirp conditions.

Perspectives

Despite the results obtained in this chapter, the experimental campaigns highlighted several key areas for future optimization and research.

A primary area of focus involves algorithmic refinement for enhanced robustness. Particularly, the artifacts observed in the SAD reconstructed profiles, attributed to waveplate misorientation (detailed in Colorbox 4.4.1), clearly indicate the necessity of improving the SAD mathematical framework to explicitly model the associated phase term ϕ . Implementing this model, similar to the current MRC approach, will achieve greater robustness against non-ideal experimental waveplates orientation, making the improved SAD model applicable for both high fidelity and high robustness. In addition, this algorithmic improvement must be implemented in a real-time reconstruction process, with a user-friendly interface. This ensures that accelerator specialists, even those unfamiliar with electro-optic diagnostics, can easily access and monitor the reconstructed longitudinal profile of the electron bunch.

Beyond the algorithm, several hardware modifications are required to push the performance limits of the system. To further increase the temporal resolution, switching from a 2 mm thick GaP EO crystal to a thinner 500 μm GaP crystal will enable bunch measurement from 200 fs down to 160 fs RMS. This gain in temporal resolution, however, comes at the cost of a fourfold decrease in the measurable signal amplitude.

Finally, a key optical setup correction involves addressing the ellipticity induced by the metallic mirrors downstream of the GaP crystal with periscopes to eliminate the induced ellipticity (see Colorbox 4.2.3). This upgrade ensures correct waveplate orientation by removing the ellipticity effect, thereby eliminating the fundamental cause of the resulting artifacts.

The deployment of the SAD algorithm and DEOS system's upgrades concludes the second part of this thesis, which focused on high-fidelity, broadband THz diagnostics. The subsequent Part III will pivot toward characterizing narrowband THz fields by introducing a novel electro-optic detection method that combines all-optical heterodyning with the Photonic Time-Stretch technique.

Part III

From Theoretical Design to Experimental Applications: Realization of a Novel Narrowband THz EO-Based Diagnostic System at FELBE

A novel single-shot electro-optic method using all-optical heterodyne with photonic time stretch for narrowband THz diagnostics

Before anything else, preparation is the key to success.

– Alexander Graham Bell

5.1 Introduction

The capability to record the complete electric field of multicycle THz pulses in a single shot is essential for accelerator-based THz sources. This capability is required both for monitoring the FEL dynamics on a pulse-to-pulse basis and for providing the information required by pump-probe users, such as the Carrier-Envelope Phase (CEP).

Conventional single-shot electro-optic diagnostics are limited by the readout bandwidth necessary to sample the signal. To faithfully capture a multicycle THz signal, the measurement system must satisfy the Nyquist criterion by sampling a minimum of two points per THz cycle. For techniques based on spectral encoding, this constraint manifests as limitations in the number of pixels and the spatial resolution of the spectrometer. In contrast, the Photonic Time-Stretch (PTS) technique offers a method for single-shot measurement by lowering the required electronic bandwidth. However, as derived in Colorbox. 5.1, the required oscilloscope bandwidth is determined solely by the total number of cycles of the

multicycle pulse, and is independent of its original THz frequency. Consequently, capturing pulses with many cycles still necessitates the use of high-bandwidth oscilloscopes (e.g., > 10 GHz). Nevertheless, using high-bandwidth oscilloscopes to capture a narrowband multicycle signal remains an inefficient use of bandwidth resources. Since the temporal signal's envelope occupies a narrow spectral width in the Fourier domain, a substantial portion of the high-frequency bandwidth records negligible information (i.e., zeros), as schematically illustrated in Figure 5.1. This bandwidth waste is a major impediment for achieving real-time single-shot diagnostics, which are typically restricted to a few GHz bandwidth by commercially available data acquisition (DAQ) systems.

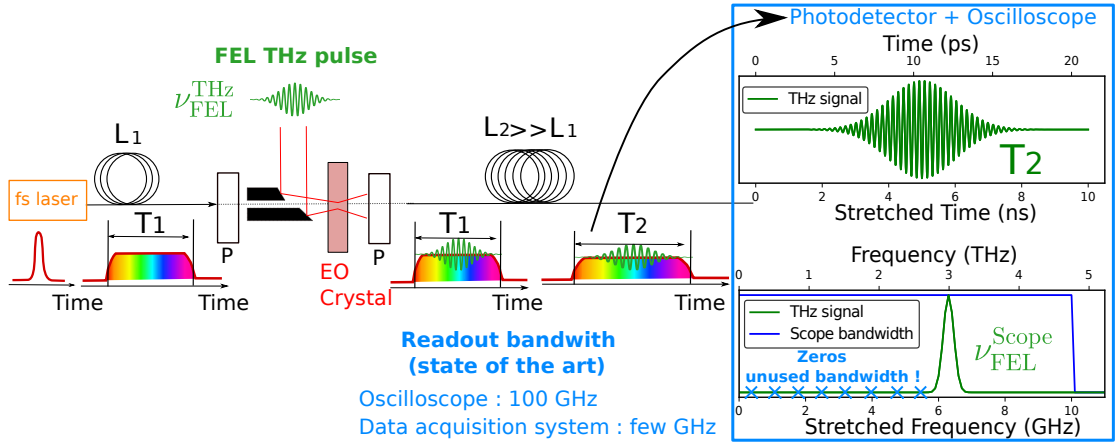


Figure 5.1: Photonic Time-Stretch Bandwidth Requirement for Narrowband THz Signals: The figure illustrates the photonic time-stretch transformation of the THz signal from its original duration $\approx T_1$ to the stretched duration $\approx T_2$, which slow down the THz frequency $\nu_{\text{FEL}}^{\text{THz}}$ to the detected lower frequency $\nu_{\text{FEL}}^{\text{Scope}}$. For a narrowband multicycle pulse, the spectrum signal occupies only a small spectral window within the oscilloscope's maximum bandwidth. This leaves an amount of unused bandwidth, highlighting the inefficiency of standard PTS for real-time diagnostics.

To overcome this electronic readout bottleneck, this chapter introduces a novel solution: the Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS) method. This new EO technique integrates an all-optical heterodyning process with the high-repetition-rate single-shot capability provided by the combination of chirped probe laser pulses and the Photonic Time-Stretch technique.

The conceptual groundwork for the HEARTS methods is provide in Section. 5.2. This involves detailing the conceptual idea of heterodyning and exploring various conceptual approaches (i.e, mixing the signals before or after PTS, electronically or optically). This exploration defines the most effective strategy for achieving the required frequency downshift of the THz pulse to an intermediate frequency that fits within the desired (typically few GHz) readout bandwidth.

The complete theoretical framework for a compact, all-optical implementation of the HEARTS method is presented in Section. 5.3. This innovative approach circumvents the technological constraints of complex electronic local oscillators, by showing how a simple birefringent crystal (YVO4) can optically generate the required THz local oscillator. The theoretical analysis includes important key design considerations, such as the optimal THz orientation within the EO crystal, which directly impact the detected down-converted signal's quality. This analysis demonstrates that by employing a specific phase diversity electro-optic configuration, the method ensures a flat and non-deforming transfer function, thereby guaranteeing that the detected, down-converted intermediate frequency signal is a faithful replica of the original high-frequency THz pulse.

Photonic Time-Stretch Bandwidth Requirement for Narrowband THz Signals

The Photonic Time-Stretch (PTS) technique slows down the original THz frequency $\nu_{\text{FEL}}^{\text{THz}}$ to a detected frequency $\nu_{\text{FEL}}^{\text{Scope}}$ by the time-stretch factor M , as display in Figure.5.1:

$$\nu_{\text{FEL}}^{\text{Scope}} = \frac{\nu_{\text{FEL}}^{\text{THz}}}{M} \quad \text{with } M \approx \frac{T_2}{T_1}$$

Where T_1 is the duration of the chirped probe laser (adjusted to match the duration of the THz signal) and T_2 is the duration of the stretched chirped probe laser (which is limited by the laser probe's repetition period, T_{rep}).

For a multicycle pulse with n_{FEL} cycles within the duration T_1 , the original THz frequency is $\nu_{\text{FEL}}^{\text{THz}} = \frac{n_{\text{FEL}}}{T_1}$. By substituting this expression into the PTS scaling equation, we find that the detected frequency is:

$$\nu_{\text{FEL}}^{\text{Scope}} = \frac{1}{M} \left(\frac{n_{\text{FEL}}}{T_1} \right) = \frac{T_1}{T_2} \left(\frac{n_{\text{FEL}}}{T_1} \right) = \frac{n_{\text{FEL}}}{T_2}$$

Therefore the required oscilloscope bandwidth is independent of the THz frequency $\nu_{\text{FEL}}^{\text{THz}}$ and depends solely on the number of cycles n_{FEL} to be recorded and the maximum stretch duration T_2 .

Numerical Example: For a laser probe repetition rate of $T_{\text{rep}} = 78$ MHz (i.e., $T_2 \approx 12.8$ ns), the resulting detected frequencies are:

- $n_{\text{FEL}} = 40$ cycles: detected at $\nu_{\text{FEL}}^{\text{Scope}} = 3.12$ GHz
- $n_{\text{FEL}} = 80$ cycles: detected at $\nu_{\text{FEL}}^{\text{Scope}} = 6.25$ GHz
- $n_{\text{FEL}} = 160$ cycles: detected at $\nu_{\text{FEL}}^{\text{Scope}} = 12.50$ GHz

The subsequent Chapter. 6 transitions this conceptual and theoretical ground-work directly into an experimental proof-of-principle demonstration at the FELBE facility, where the HEARTS technique is successfully employed to diagnose the CEP of the FEL.

5.2 Concept of a new EO detection method for narrowband THz diagnostics

This section introduces the concept of the Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS) method, which addresses the electronic readout bottleneck by employing frequency down-conversion combined with photonic time-stretch. The conceptual idea of heterodyning is first detailed before exploring the various technical options for generating the local oscillator and performing the mixing process before or after PTS.

5.2.1 Conceptual idea of heterodyning

The down-conversion of the multicycle FEL signal is achieved using the heterodyne detection principle. This technique operates by combining (i.e mixing) the signal wave at the FEL frequency ν_{FEL} with an externally generated Local Oscillator (LO), at a controlled frequency ν_{LO} . Mathematically, when two waves are multiplied (or beat) together, they generate new frequency components according to the trigonometric product-to-sum identity:

$$\cos(2\pi\nu_{LO}t)\cos(2\pi\nu_{FEL}t) = \frac{1}{2}\cos(2\pi(\nu_{LO} + \nu_{FEL})t) + \frac{1}{2}\cos(2\pi(\nu_{LO} - \nu_{FEL})t) \quad (5.1)$$

By applying the heterodyne technique, two copies of the signal are produced at intermediate frequencies (IF), corresponding to both the sum and difference of ν_{FEL} and ν_{LO} , as defined by Equation.5.1. This process yields a lower-frequency copy, the difference frequency $\nu_{IF}^- = |\nu_{LO} - \nu_{FEL}|$, which is the desired FEL signal wave copy, now shifted within the desired electronic readout bandwidth, as display in Fig. 5.2.

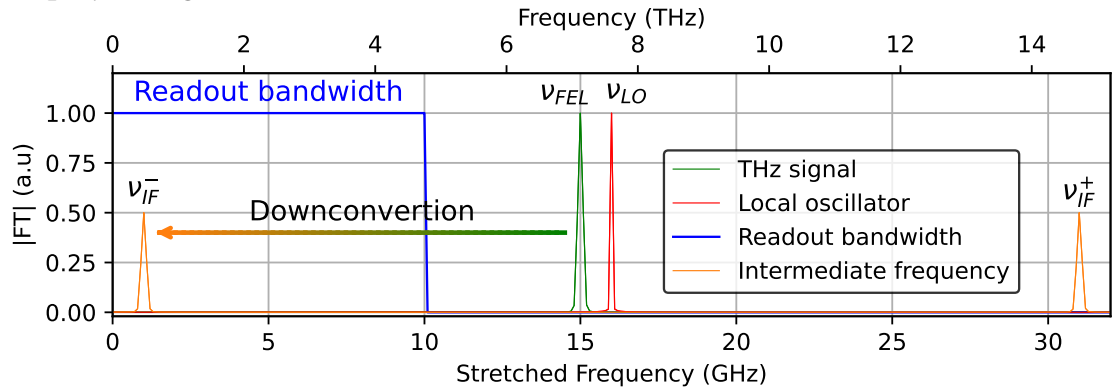


Figure 5.2: **Principle of heterodyning.** A THz signal ν_{FEL} beats with a local oscillator ν_{LO} , generating sum ν_{IF}^+ and difference ν_{IF}^- frequencies. The copy of the THz at the intermediate frequency ν_{IF}^- falls within the oscilloscope bandwidth used for detection

5.2.2 Technical options for heterodyning

Building upon the conceptual idea of using heterodyning to downconvert the THz signal to a lower frequency, this subsection explores the various technical options for its implementation combined with the photonic time-stretch technique. Several approaches can be considered for the implementation depending on different parameters. Hence, the local oscillator and the heterodyne mixing process can be realized either electrically or optically. Moreover, the heterodyne mixing itself can be performed either directly in the THz domain, before the time-stretch, or in the GHz range, after the time stretch. Three approaches are briefly explored and each of them comes with their own set of advantages and constraints.

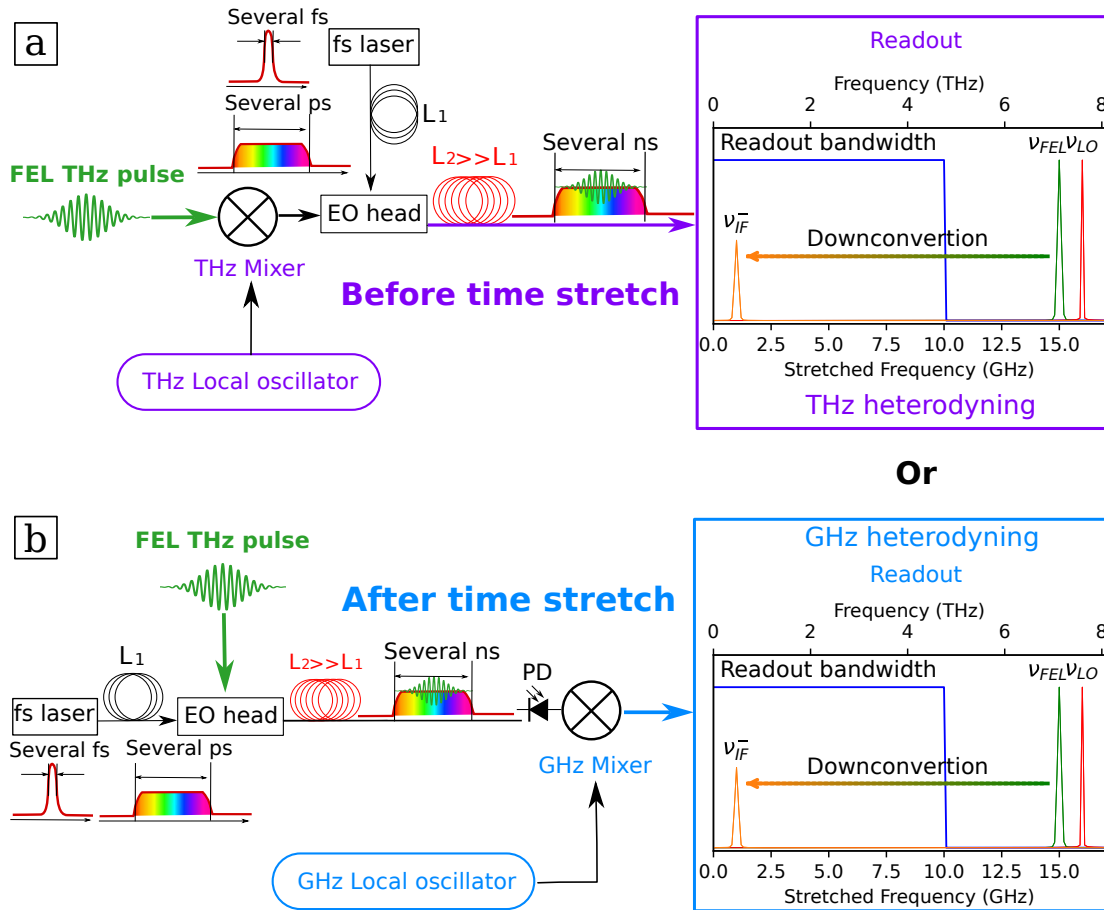


Figure 5.3: **Comparison of two possible heterodyne detection approaches.** (a) Mixing before the photonic time stretch using a THz local oscillator and a THz mixer. (b) Mixing after the time stretch with a GHz local oscillator and a GHz mixer. PD : photodiode

5.2.2.1 Option 1: Electronics THz Local oscillator and mixers before the photonic time stretch

One way to generate the local oscillator is through purely electronic means, directly in the THz domain. This approach typically relies on frequency multiplication from microwave sources, where a low-frequency signal (GHz) is progressively doubled or multiplied to reach the desired THz range. State-of-the-art electronic THz sources, such as those based on Schottky diode multipliers or photoconductive antennas driven by ultra-fast electronics, can provide stable and tunable signals up to a few THz [114–117].

However, electronic generation faces several challenges, including limited output power at higher frequencies and phase noise accumulation throughout the multiplication stages. These constraints can impact the efficiency of the heterodyne process, particularly for high-precision diagnostics requiring a clean and stable reference.

To implement heterodyne detection, the generated THz local oscillator must be mixed with the incoming THz signal. This is typically achieved using Schottky diode mixers [118, 119] or superconducting hot-electron bolometers (HEBs) [120, 121], which enable frequency down-conversion with high sensitivity. However, the efficiency of these mixers decreases at higher frequencies, and their noise characteristics impose further constraints on the achievable signal quality [122, 123].

More details about THz receiver can be found in those paper [124, 125].

5.2.2.2 Option 2: Electronics GHz local oscillator and mixers after the photonic time-stretch

Instead of generating a THz local oscillator, an alternative approach is to perform heterodyne mixing after the PTS. In this configuration, the local oscillator operates in the GHz range, where stable and high-quality sources are readily available [126].

Mixing at GHz frequencies offers practical advantages, as high-performance electronic mixers and amplifiers are widely available in this range [127].

However, this approach introduce certain drawbacks. The electronic mixers often have bandwidth limitations, which may restrict the range of detectable frequencies. Furthermore the frequency conversion process introduces additional active components, thereby increasing the system complexity compared the subsequent all-optical option.

5.2.2.3 Option 3: Heterodyning by all-optical means

Our approach, offers a fundamentally different and more elegant solution to the limitations encountered by electronic THz and GHz heterodyning. Instead of relying on complex electronic components, we leverage a purely optical technique to generate the local oscillator and perform the mixing.

A stable, tunable THz local oscillator ν_{LO} is generated not electronically, but using a Lyot filter resulting with a modulation of the chirped probe laser spectrum.

The heterodyne mixing is performed optically by placing the birefringent Lyot crystal either before or after the electro-optic crystal. This optical THz mixing down-converts the high-frequency THz signal ν_{FEL} to a lower intermediate frequency in the THz range. Finally, this intermediate THz frequency is stretched down to the GHz range using photonic time stretch, making the signal recordable with cost-effective, low-bandwidth electronic readout systems.

Brief Summary: Heterodyne Possibilities

In summary, while the THz heterodyning approach would be a dream solution, it remains difficult to implement and is limited to only a few THz, far from the 30 THz we need. The GHz approach offers a viable solution but complicates the setup compared to the all-optical option, which enables the heterodyne process using optical techniques with a simple passive birefringent crystal, thereby bypassing the challenges of electronics. The details of this all-optical heterodyning approach are described in the following section.

5.3 Theoretical Framework and Design of the All-Optical Heterodyne EO System

Building upon the concepts introduced in the previous section, this section transitions to the theoretical framework and detail the design of the all-optical HEARTS method.

This framework begins with the optical generation of the THz local oscillator onto the chirped probe laser via a Lyot filter, detailed in subsection. 5.3.1. It then proceeds to a didactic explanation of the optical heterodyning and photonic time stretch process in subsection. 5.3.2. Finally, the last subsection. 5.3.3 provides an analytical derivation of the downconverted output, along with a numerical computational model of the entire all-optical HEARTS method. This includes a study on the THz orientation relative to the electro-optic crystal and probe laser polarization, showing how it affects the system's overall transfer function, and therefore the final downconverted output.

The primary goal of this section is to verify the feasibility and expected performance of the HEARTS concept, thereby ensuring all technical prerequisites are met before the practical experimental implementation in Chapter.6. An illustration of the complete HEARTS method via all-optical means is provided in Fig.5.5.

5.3.1 Optical THz Local Oscillator Generation via a Lyot Filter

The Lyot filter's operational principle for generating the THz local oscillator is detailed, beginning with its function as an optical modulator described through an interferometric analogy, followed by the derivation of the modulation frequency's relation according to the material properties.

5.3.1.1 Lyot Filter as an Optical Modulator: An Interferometric Analogy

To overcome the challenges associated with electronic local oscillators, an optical approach to generate this reference signal onto the chirped probe laser is explored. A modulated pulse can be generated using an interferometer, such as a Mach-Zehnder interferometer, where a delay is introduced in one of the arms (Figure 5.4.(b)). Alternatively, an elegant and equivalent way to achieve this is by using a Lyot filter [128], shown in Figure 5.4.(a)

In this setup, a birefringent material is inserted at a 45-degree angle relative to the vertical laser polarization, decomposing the light into ordinary and extraordinary polarization components. Due to the material's birefringence, defined as $\Delta n(\lambda) = n_o(\lambda) - n_e(\lambda)$, where $n_o(\lambda)$ and $n_e(\lambda)$ are the ordinary and extraordinary refractive indices, respectively, these two components propagate at different phase velocities. This differential propagation over the material thick-

ness e , introduces a wavelength-dependent phase delay between the ordinary and extraordinary pulses $\Delta\phi_{\text{Lyot}}(\lambda)$:

$$\Delta\phi_{\text{Lyot}}(\lambda) = \frac{2\pi}{\lambda} \Delta ne = \frac{2\pi}{\lambda} (n_o(\lambda) - n_e(\lambda)) e \quad (5.2)$$

By introducing this controlled delay between the two orthogonal polarization components, the Lyot filter replicates the physical path-difference of the Mach-Zehnder interferometer.

The beam then passed through a vertical polarizer (i.e, the analyzer), where the phase delay is converted into an intensity modulation $I(\lambda)$ described by:

$$\begin{aligned} I(\lambda) &= I_0(\lambda) \cos^2 \left(\frac{\Delta\phi_{\text{Lyot}}(\lambda)}{2} \right) \\ I(\lambda) &= \frac{I_0(\lambda)}{2} \left(1 + \cos(\Delta\phi_{\text{Lyot}}(\lambda)) \right) \end{aligned} \quad (5.3)$$

Where $I_0(\lambda)$ is the intensity of the input chirped probe laser spectrum incident on the Lyot filter. The Lyot filter provides a powerful tool for generating stable, THz-frequency optical modulation onto the chirped probe laser using a purely passive birefringent element.

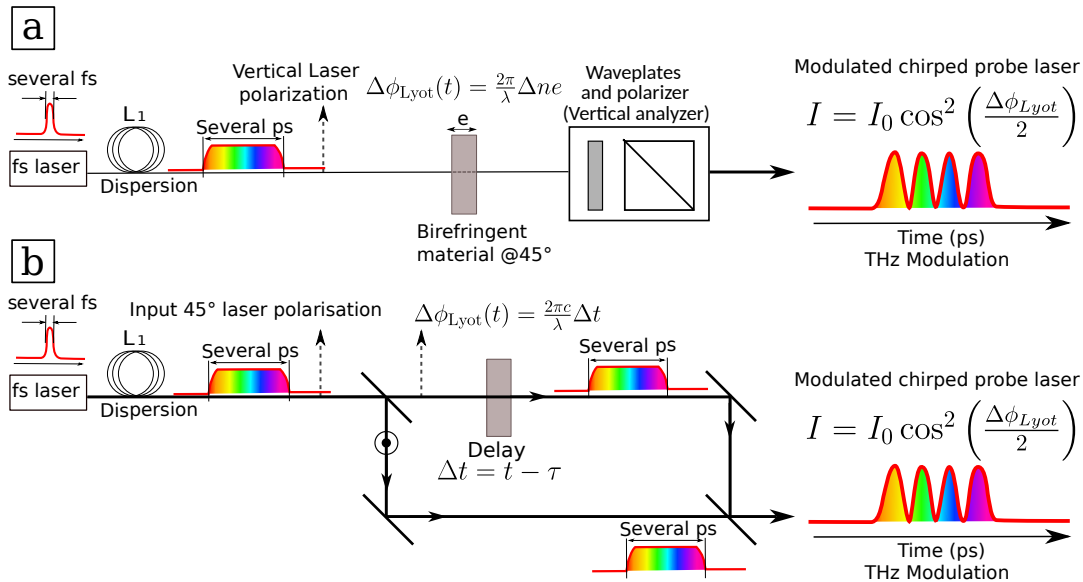


Figure 5.4: Analogy between a Lyot filter and a Mach-Zehnder interferometer. In both cases, a controlled delay is introduced between two optical paths, resulting in an intensity modulation. The Lyot filter achieves this by exploiting birefringence, while the Mach-Zehnder interferometer uses a physical path difference. This modulation initially occurs at THz frequencies but is later stretched into the GHz range through photonic time stretch, making it compatible with standard electronic detection (See Fig.5.5).

5.3.1.2 Modulation Frequency of the Lyot Filter

The equation for the modulation frequency ν_{LO}^{THz} of the optical local oscillator is found by mapping the Lyot filter's wavelength-dependent phase delay, $\Delta\phi_{Lyot}(\lambda)$, into an equivalent time-dependent phase delay $\Delta\phi_{Lyot}(t)$. This mapping utilizes the probe laser's chirp rate C via the equivalent time-to-frequency relation $t = -\frac{\omega}{C}$ [18].

The derivation of this modulation frequency ν_{LO}^{THz} is detailed in Appendix. A. The resulting equivalent time-dependent phase delay $\Delta\phi_{Lyot}(t)$, the optical frequency modulation ν_{LO}^{THz} and the corresponding output intensity $I(t)$, are given by:

$$\Delta\phi_{Lyot}(t) = 2\pi\nu_{LO}^{THz} t \quad (5.4)$$

$$\nu_{LO}^{THz} = \frac{\Delta ne}{\lambda_0^2 |D_1| L_1} \quad (5.5)$$

$$I(t) = \frac{I_0(t)}{2} \left(1 + \cos(2\pi\nu_{LO}^{THz} t) \right) \quad (5.6)$$

Where $I_0(t)$ is the intensity of the input chirped probe laser spectrum $I_0(\lambda)$ expressed in the equivalent time domain via the time-to-frequency relation $t = -\frac{\omega}{C}$. This result demonstrates that the modulation frequency ν_{LO}^{THz} of the optical local oscillator is governed by the birefringent material's properties $(\Delta n, e)$, the laser's central wavelength (λ_0) , and the dispersion characteristics $(|D_1|, L_1)$ of the temporal window, making it possible to tune ν_{LO}^{THz} by adjusting the thickness of the birefringent material or by modifying the temporal window.

5.3.2 Optical THz Mixing Combined with Photonic Time Stretch

With the tunable optical THz local oscillator ν_{LO}^{THz} now defined, the next step is to efficiently mix it with the incoming FEL THz signal ν_{FEL}^{THz} . For didactic purposes, this subsection first derives the intensity expression that proves the heterodyne process is at work, establishing the lower intermediate frequency in the THz domain (or hundreds of GHz), $\nu_{IF}^{THz} = |\nu_{FEL}^{THz} - \nu_{LO}^{THz}|$. This signal is then temporally stretched using photonic time stretch, which yields the final measurable frequency in the few GHz range, ν_{IF}^{GHz} , thereby further reducing the electronic readout bandwidth. The full, rigorous calculation starting from the electric field, along with the corresponding numerical computational model, will then be presented in the following subsection.5.3.3.

5.3.2.1 Optical THz mixing

For effective mixing, the birefringent material must be placed either directly before or after the EO crystal¹ and aligned along the same axis orientation as the EO crystal. Both the EO crystal and the birefringent material are set at 45 degrees relative to the laser polarization and analyzer, as displayed in Fig. 5.5).

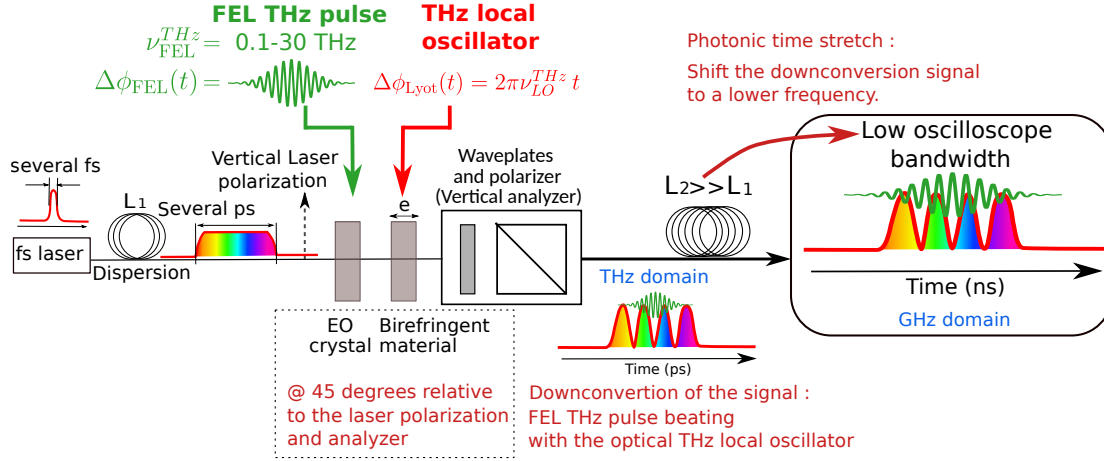


Figure 5.5: **Illustration of the All-Optical HEARTS method.** This figure illustrates the two-stage frequency reduction performed by the all-optical HEARTS method. First, the heterodyne detection principle is used to mix the incoming FEL THz signal $\nu_{\text{FEL}}^{\text{THz}}$ with the THz optical local oscillator $\nu_{\text{LO}}^{\text{THz}}$. For effective mixing, the EO crystal and the birefringent material must be aligned at 45 degrees relative to the probe laser's polarization and the analyzer. The resulting downconverted signal is then temporally stretched using the photonic time-stretch technique, which yields the final measurable frequency $\nu_{\text{IF}}^{\text{GHz}}$ and enables the use of a low oscilloscope readout bandwidth.

The electric field vector after the heterodyne mixing step, $\mathbf{E}_{\text{Het}}(t)$, is given by the product of the following Jones matrices:

$$\mathbf{E}_{\text{Het}}(t) = \mathbf{M}_{\text{HWP}} \left(\frac{\pi}{8} \right) \mathbf{M}_{\text{Lyot}} \left(\Delta\phi_{\text{Lyot}}(t) \right) \mathbf{M}_{\text{EO}} \left(\Delta\phi_{\text{FEL}}(\alpha, t) \right) \mathbf{E}_{\text{in}}^{45}(t) \quad (5.7)$$

The THz-induced phase modulation in the EO crystal is $\Delta\phi_{\text{FEL}}(\alpha, t) = \phi_1(\alpha, t) - \phi_2(\alpha, t) = P(\alpha)E_{\text{FEL}}(t)$, where $\phi_1(\alpha, t)$ and $\phi_2(\alpha, t)$ are the phase retardations along the crystal's two birefringent axes. The coefficient $P(\alpha)$ is the Pockels effect coefficient, which depends on the orientation angle α of the FEL THz electric field $E_{\text{FEL}}(t)$ with respect to the $[-110]$ direction². The optimal orientation of the THz polarization relative to the EO crystal and probe laser is discussed in the following subsection 5.3.3.

The Jones matrices for the half-wave plate rotated by 22.5° ($\mathbf{M}_{\text{HWP}}$), the Lyot

¹The Jones matrices of the EO crystal and the birefringent material are commutative

² $P(\alpha) = \frac{\pi d}{\lambda_0} n_0^3 r_{41} \sqrt{1 + 3 \cos^2(\alpha)}$ is defined in Chapter 2, Equation 2.1

filter's birefringent material (M_{Lyot}), the EO crystal (M_{EO})³, and the 45° polarization chirped probe laser ($E_{\text{in}}^{45}(t)$)⁴ are explicitly defined as:

$$\begin{aligned} \mathbf{M}_{\text{HWP}}\left(\frac{\pi}{8}\right) &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & \mathbf{M}_{\text{Lyot}}(\Delta\phi_{\text{Lyot}}(t)) &= \begin{pmatrix} e^{i\frac{\Delta\phi_{\text{Lyot}}(t)}{2}} & 0 \\ 0 & e^{-i\frac{\Delta\phi_{\text{Lyot}}(t)}{2}} \end{pmatrix} \\ \mathbf{M}_{\text{EO}}(\Delta\phi_{\text{FEL}}(\alpha, t)) &= \begin{pmatrix} e^{i\frac{\Delta\phi_{\text{FEL}}(\alpha, t)}{2}} & 0 \\ 0 & e^{-i\frac{\Delta\phi_{\text{FEL}}(\alpha, t)}{2}} \end{pmatrix} & \mathbf{E}_{\text{in}}^{45}(t) &= \frac{E_0(t)}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned} \quad (5.8)$$

The resulting output electric fields, $E_1(t)$ and $E_2(t)$ are expressed as:

$$\mathbf{E}_1(t) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{E}_{\text{Het}}(t) \quad \mathbf{E}_2(t) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{E}_{\text{Het}}(t) \quad (5.9)$$

The intensities $I_{1,2}(t) = |E_{1,2}(t)|^2$ resulting from this configuration are given by:

$$I_{1,2}(t) = \frac{I_0(t)}{2} \left(1 \pm \cos(\Delta\phi_{\text{Lyot}}(t) + \Delta\phi_{\text{FEL}}(\alpha, t)) \right) \quad (5.10)$$

The differential detection signal, $\Delta I_{\text{Het}}(t) = I_1(t) - I_2(t)$, is given by:

$$\Delta I_{\text{Het}}(t) = I_0(t) \cos(\Delta\phi_{\text{Lyot}}(t) + \Delta\phi_{\text{FEL}}(\alpha, t)) \quad (5.11)$$

This expression is expanded using the angle addition formula for cosine:

$$\Delta I_{\text{Het}}(t) = I_0(t) \left[\cos(\Delta\phi_{\text{Lyot}}(t)) \cos(\Delta\phi_{\text{FEL}}(\alpha, t)) - \sin(\Delta\phi_{\text{Lyot}}(t)) \sin(\Delta\phi_{\text{FEL}}(\alpha, t)) \right] \quad (5.12)$$

By performing a first-order Taylor series expansion for the small FEL THz-induced phase $\Delta\phi_{\text{FEL}}(\alpha, t) \ll 1$, the resulting intensity is simplified:

$$\Delta I_{\text{Het}} = I_0(t) \left[\cos(\Delta\phi_{\text{Lyot}}(t)) - \sin(\Delta\phi_{\text{Lyot}}(t)) \Delta\phi_{\text{FEL}}(\alpha, t) \right] \quad (5.13)$$

Assuming a FEL narrowband THz pulse $E_{\text{FEL}}(t) = A(t) \sin(\omega_{\text{FEL}} t)$ with a Gaussian envelope $A(t)$, thus $\Delta\phi_{\text{FEL}}(\alpha, t) = P(\alpha) A(t) \sin(\omega_{\text{FEL}} t)$, and substituting $\Delta\phi_{\text{Lyot}}(t) = 2\pi \nu_{\text{LO}}^{\text{THz}} t = \omega_{\text{Lyot}} t$:

$$\Delta I_{\text{Het}} = I_0(t) \left[\cos(\omega_{\text{Lyot}} t) - P(\alpha) A(t) \sin(\omega_{\text{Lyot}} t) \sin(\omega_{\text{FEL}} t) \right] \quad (5.14)$$

³The global phase factor is omitted here as it cancels out when calculating the detected intensities $I_{1,2}(t)$

⁴With $E_0(t)$ being the chirped probe pulse and expressed in Eq.5.19

The resulting differential intensity is therefore the product of two sines⁵, one corresponding to the FEL THz pulse and the other to the local oscillator generated by the Lyot filter. This represents the heterodyne process at work :

$$\Delta I_{\text{Het}} = I_0(t) \left[\cos(\omega_{\text{Lyot}} t) - \frac{P(\alpha)A(t)}{2} \left[\cos((\omega_{\text{Lyot}} - \omega_{\text{FEL}})t) - \cos((\omega_{\text{Lyot}} + \omega_{\text{FEL}})t) \right] \right] \quad (5.15)$$

This last equation contains the two intermediate frequency $\nu_{IF}^{\pm} = |\nu_{\text{FEL}}^{\text{THz}} \pm \nu_{\text{LO}}^{\text{THz}}|$, and confirms that the all-optical heterodyne process successfully downconverts the high-frequency $\nu_{\text{FEL}}^{\text{THz}}$ FEL signal to an intermediate lower frequency $\nu_{IF}^{-} = \nu_{IF}^{\text{THz}}$, defined as:

$$\begin{aligned} \nu_{IF}^{\text{THz}} &= |\nu_{\text{FEL}}^{\text{THz}} - \nu_{\text{LO}}^{\text{THz}}| \\ \nu_{IF}^{\text{THz}} &= \left| \nu_{\text{FEL}}^{\text{THz}} - \frac{\Delta n e}{\lambda_0^2 |D_1| L_1} \right| \end{aligned} \quad (5.16)$$

5.3.2.2 Temporal Stretching of the Downconverted Frequency using Photonic Time Stretch

The ultimate purpose of the HEARTS method is to select an appropriate frequency of the downconverted signal ν_{IF}^{THz} (typically, few GHz), ensuring compatibility with electronic detection while preserving the information carried by the original signal. Using photonic time stretch adds an additional tuning parameter that helps further reduce the intermediate frequency ν_{IF}^{THz} , making it more accessible for electronic detection. By applying additional dispersion after the heterodyning stage, as shown in Fig.5.5, we obtain a new expression for the observed GHz-range intermediate frequency ν_{IF}^{GHz} :

$$\nu_{IF}^{\text{GHz}} = \frac{(\nu_{\text{FEL}}^{\text{THz}} - \nu_{\text{LO}}^{\text{THz}})}{M} \quad \text{with} \quad M = 1 + \frac{D_2 L_2}{D_1 L_1} \quad (5.17)$$

$$\nu_{IF}^{\text{GHz}} \approx (\nu_{\text{FEL}}^{\text{THz}} D_1 L_1 - \frac{\Delta n e}{\lambda^2}) \frac{1}{D_2 L_2} \quad \text{with} \quad L_2 \gg L_1 \quad (5.18)$$

where M is the stretch factor, D_2 and L_2 are the dispersion and the length of the stretch fiber, respectively. To minimize the intermediate frequency ν_{IF}^{GHz} , several

⁵While the local oscillator term $\cos(\omega_{\text{Lyot}} t)$ remains present, its high frequency ν_{LO} will exceed the bandwidth of the readout electronics and will be filtered out during acquisition.

parameters need to be optimized. One important factor is the birefringent crystal used in the Lyot filter. The birefringence itself, Δn , is an intrinsic property of the material and cannot be modified. Therefore, the only way to adjust the local oscillator frequency ν_{LO}^{THz} is by varying the thickness of the birefringent crystal. Additionally, the amount of dispersion in the first fiber, characterized by $D_1 L_1$, should be minimized and adjusted to match the duration of the FEL signal T_{FEL} . The temporal window in this case is constrained by $T_1 = \Delta \lambda D_1 L_1 \approx T_{FEL}$. Meanwhile, the dispersion in the stretch fiber, represented by $D_2 L_2$, should be maximized to further reduce the intermediate frequency but is limited by the repetition rate of the probe laser ν_{rep}^{probe} , i.e., $T_2 = \Delta \lambda D_2 L_2 \approx \frac{1}{\nu_{rep}^{probe}}$.

Brief Summary of the All-Optical HEARTS Method

In summary, the FEL THz monochromatic pulse is first downconverted using a THz optical local oscillator generated by a Lyot filter in an optical heterodyning process. This process is implemented by setting up the EO and Lyot crystals adjacent to each other and oriented at 45° relative to the input laser polarization and analyzer. Then the lower intermediate frequency in the THz domain (or hundreds of GHz) $\nu_{IF}^- = \nu_{IF}^{THz}$ is stretched down to a few GHz (ν_{IF}^{GHz}) using photonic time stretch, making it possible to record it with a low bandwidth photodetector and oscilloscope.

5.3.3 Orientation of the THz Polarization Relative to the EO crystal and Probe Laser

As mentioned in the previous subsection, the relative orientation of the THz polarization with respect to both the electro-optic crystal and the probe laser polarization, plays a crucial role in the optical heterodyne mixing process. Two configurations are possible: the phase diversity and the standard EO configurations. Each of these configurations leads to different outputs, which can have deformation due to the transfer functions of the system.

To fully understand the effect of the THz orientation on the mixing process, its implications are analyzed through both analytical calculations and numerical simulations. This subsection first examines the phase diversity configuration, then the standard electro-optic configuration, and concludes by comparing the results of both to identify the most effective approach for optimal performance.

5.3.3.1 Phase Diversity EO Configuration : Signal Conservation Due to the System Transfer Function

In the phase diversity electro-optic detection scheme, the THz polarization is set perpendicular to the $[-110]$ axis of the EO crystal, and the laser polarization is aligned at 45° relative to the $[-110]$ axis, as display in Figure.5.6.(a). Both the EO and birefringent crystals share the same axis orientation. The analyzer is set at 45° relative to the $[-110]$ axis using a half-wave plate turned at 22.5° and a polarizing beam splitter (see Fig.5.6.(b)).

This setup in Phase Diversity (PD)⁶ configuration can be viewed as an optical equivalence of a Mach-Zehnder interferometer, where one arm of the interferometer, $E_{LO}^{PD}(t - \tau)$, introduces a temporal delay $\Delta t = t - \tau$ through the Lyot filter, while the other arm, $E_m^{PD}(t)$, experiences a phase modulation induced by the FEL THz signal $\phi_{FEL}(t)$, an illustration is given in Fig.5.6.(c) [18].

⁶Note that the following superscripts PD stand for the phase diversity configuration throughout this section.

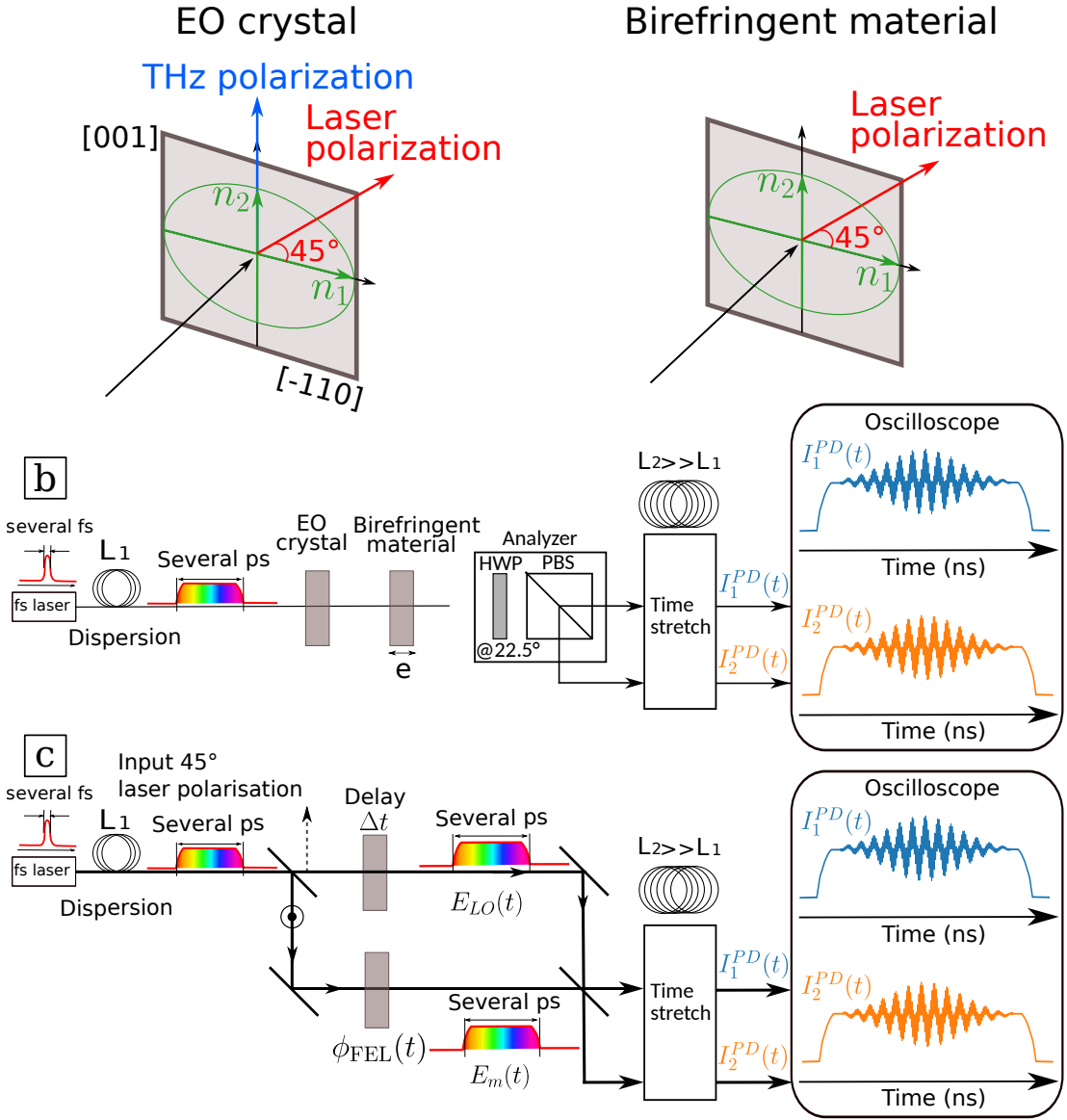
a HEARTS with the phase diversity electro-optic configuration

Figure 5.6: Principle of the all-optical HEARTS method in phase diversity electro-optic configuration (a) Experimental arrangement showing the relative orientation of the THz polarization and laser polarization with respect to the EO crystal and birefringent material axes. (b) Illustration of the all-optical HEARTS method using the phase diversity configuration. The laser polarization and the analyzer are set at 45° relative to the [-110] axis of the EO crystal. The EO and Lyot crystals share the same axis orientation and are display in (a). The analyzer is composed of an Half-Wave-Plates (HWP) oriented at 22.5° and a Polarizing Beam Splitter (PBS). The two resulting outputs $I_1(t)$ and $I_2(t)$ are temporally stretched using photonic time-stretch technique and sent to the low bandwidth oscilloscope readout. (c) Optical analog of a Mach-Zehnder interferometer, where the 45° input laser polarization is split into two arms: one arm experiences a time delay, while the other is phase-modulated by the THz signal. The two outputs are then temporally stretched using photonic time-stretch and are visualized on an oscilloscope, showing the intensities $I_1(t)$ and $I_2(t)$ over time.

1. Theoretical Study of the All-Optical HEARTS Method in Phase Diversity Configuration Based on Mach-Zehnder Interferometer Equivalent :

To quantify how the relative orientation of the THz polarization influences the optical heterodyne mixing process and signal deformation, we derived the analytical transfer function for the all-optical HEARTS method in the phase diversity configuration, modeled as the Mach-Zehnder Interferometer equivalence. We begin with the chirped probe laser pulse that encodes both the local oscillator and the FEL THz signal. The electric field of this chirped probe pulse is given by:

$$E_0(t) = A(t)e^{-\frac{i}{2}Ct^2} \quad (5.19)$$

where $C = \frac{\partial\omega}{\partial t}$ is the chirp rate of the laser and $A(t)$ is the envelope. The two arms of the Mach-Zehnder interferometer are described as follows:

$$E_{LO}^{PD}(t-\tau) = \frac{E_0(t-\tau)}{2} = \frac{A(t-\tau)}{2}e^{-\frac{i}{2}C(t-\tau)^2} \quad (5.20)$$

$$E_m^{PD}(t) = \frac{E_0(t)}{2}e^{i\phi_{FEL}(t)} \approx \frac{A(t)}{2}e^{-\frac{i}{2}Ct^2}(1 + i\phi_{FEL}(t)) \quad (5.21)$$

where $\phi_{FEL}(t) = m\cos(\omega_{FEL}t + \psi)$ represents the phase modulation introduced by the FEL THz signal, with m being small enough for the approximation $e^{i\phi_{FEL}(t)} \approx 1 + i\phi_{FEL}(t)$ to hold.

The interference between the two arms results in the following expressions for the two temporal electric field outputs $E_1^{PD}(t)$ and $E_2^{PD}(t)$:

$$E_1^{PD}(t) = E_{LO}^{PD}(t-\tau) + E_m^{PD}(t) = \frac{1}{2}(E_0(t-\tau) + E_0(t)(1 + i\phi_{FEL}(t))) \quad (5.22)$$

$$E_2^{PD}(t) = E_{LO}^{PD}(t-\tau) - E_m^{PD}(t) = \frac{1}{2}(E_0(t-\tau) - E_0(t)(1 + i\phi_{FEL}(t))) \quad (5.23)$$

To analyze the spectral components of the temporal interference signals $E_1^{PD}(t)$ and $E_2^{PD}(t)$, we simplify the calculations by assuming a constant envelope $A(t) = A$. Thus the Fourier transform $\widetilde{E}_0(\omega)$ of the chirped probe laser can be expressed as:

$$\widetilde{E}_0(\omega) = A\sqrt{\frac{\pi}{C}}(1-i)e^{i\frac{\omega^2}{2C}} \quad (5.24)$$

The Fourier transform of $E_1^{PD}(t)$ and $E_2^{PD}(t)$, yields the following:

$$\widetilde{E}_1^{PD}(\omega) = \frac{\widetilde{E}_0(\omega)}{2} \left(e^{-i\omega\tau} + 1 + im\cos\left(-\frac{\omega\omega_{FEL}}{C} + \psi\right) e^{i\frac{\omega_{FEL}^2}{2C}} \right) \quad (5.25)$$

$$\widetilde{E}_2^{PD}(\omega) = \frac{\widetilde{E}_0(\omega)}{2} \left(e^{-i\omega\tau} - 1 - im\cos\left(-\frac{\omega\omega_{FEL}}{C} + \psi\right) e^{i\frac{\omega_{FEL}^2}{2C}} \right) \quad (5.26)$$

The intensities corresponding to these outputs of the Mach-Zehnder are related to the optical frequency ω and the time t by the relation $t = \frac{-\omega}{C}$. This allows us to express the recorded intensities $I_{1,2}^{PD}(t)$ with THz and $I_{1,2}^{ref}(t)$ without THz⁷, in the following form:

$$I_{1,2}^{PD}(t) = \left| \widetilde{E}_{1,2}^{PD}(\omega = -Ct) \right|^2 \quad (5.27)$$

$$I_{1,2}^{ref}(t) = \frac{|\widetilde{E}_0(\omega = -Ct)|^2}{2} = \frac{\pi A^2}{C} \quad (5.28)$$

The two signals of interest $Y_{1,2}^{PD}(t)$, which isolates and normalizes the FEL THz modulation, are defined as the difference between the intensity $I_{1,2}^{PD}(t)$ with the THz signal and without $I_{1,2}^{ref}(t)$, divided by the reference intensity :

$$Y_{1,2}^{PD}(t) = \frac{I_{1,2}^{PD}(t) - I_{1,2}^{ref}(t)}{I_{1,2}^{ref}(t)} \quad (5.29)$$

This subtraction isolates the contributions of the THz field and local oscillator from any baseline or unmodulated signal, and the normalization helps focus on the relative changes induced by the THz field.

A Taylor expansion of $Y_{1,2}^{PD}(t)$ around $m = 0$ is performed up to the first order to isolate the linear terms from the harmonics.

$$Y_{1,2}^{PD}(t) \approx Y_{1,2}^{PD(0)}(t) + Y_{1,2}^{PD(1)}(t) + O(m^2) \quad (5.30)$$

Applying a Fourier transform to this expansion yields:

$$\widetilde{Y}_{1,2}^{PD}(\omega) \approx \widetilde{Y}_{1,2}^{PD(0)}(\omega) + \widetilde{Y}_{1,2}^{PD(1)}(\omega) + O(m^2) \quad (5.31)$$

The zero-order term, $\widetilde{Y}_{1,2}^{PD(0)}(\omega)$, represents the local oscillator contribution and is given by:

$$\widetilde{Y}_{1,2}^{PD(0)}(\omega) = \pi [\pm \delta(\omega - C\tau) \pm \delta(\omega + C\tau)] \quad (5.32)$$

By substituting $C\tau = 2\pi\nu_{LO}^{THz} = \omega_{LO}$, this expression can be rewritten as:

$$\widetilde{Y}_{1,2}^{PD(0)}(\omega) = \pi [\pm \delta(\omega - \omega_{LO}) \pm \delta(\omega + \omega_{LO})] \quad (5.33)$$

⁷The reference intensity measured without the THz signal contains the local oscillator, but this high-frequency component is removed by the low-bandwidth readout. Thus the measured reference intensity corresponds solely to the envelope of the chirped probe laser spectrum.

The first-order term $\widetilde{Y}_{1,2}^{PD(1)}(\omega)$, describes the heterodyne mixing of the THz signal and is expressed as:

$$\widetilde{Y}_{1,2}^{PD(1)}(\omega) = \frac{\pi m}{2} \left[\begin{array}{l} \mp i \delta(\omega + \omega_{LO} - \omega_{FEL}) e^{-i\left(\frac{\omega_{FEL}^2}{2C} + \psi\right)} \\ \pm i \delta(\omega - \omega_{LO} + \omega_{FEL}) e^{i\left(\frac{\omega_{FEL}^2}{2C} + \psi\right)} \end{array} \right\} \begin{array}{l} \text{Lower Sideband} \\ \omega_{IF}^- = |\omega_{FEL} - \omega_{LO}| \end{array}$$

$$\left[\begin{array}{l} -2\delta(\omega - \omega_{FEL}) \sin\left(\frac{\omega_{FEL}^2}{2C}\right) e^{-i\psi} \\ -2\delta(\omega + \omega_{FEL}) \sin\left(\frac{\omega_{FEL}^2}{2C}\right) e^{i\psi} \end{array} \right\} \begin{array}{l} \text{FEL THz Signal} \\ \omega_{FEL} \end{array}$$

$$\left[\begin{array}{l} \mp i \delta(\omega + \omega_{LO} + \omega_{FEL}) e^{i\left(-\frac{\omega_{FEL}^2}{2C} + \psi\right)} \\ \pm i \delta(\omega - \omega_{LO} - \omega_{FEL}) e^{-i\left(-\frac{\omega_{FEL}^2}{2C} + \psi\right)} \end{array} \right\} \begin{array}{l} \text{Upper Sideband} \\ \omega_{IF}^+ = |\omega_{FEL} + \omega_{LO}| \end{array}$$
(5.34)

This result confirms that an encoded FEL THz signal copy appears at both the sum and difference frequencies of the THz signal and the local oscillator⁸.

The two final observed signals on a low-bandwidth oscilloscope $\widetilde{Y}_{1,2}^{(-)PD}(\omega) = \widetilde{Y}_{1,2}^{Scope PD}(\omega)$, representing the downconverted THz signal $\omega_{IF}^- = |\omega_{FEL} - \omega_{LO}| = \pm\omega_{FEL} \mp \omega_{LO}$, are expressed as:

$$\widetilde{Y}_{1,2}^{Scope PD}(\omega) = \frac{\pi m}{2} \left[\begin{array}{l} \mp i \delta(\omega + \omega_{LO} - \omega_{FEL}) e^{-i\left(\frac{\omega_{FEL}^2}{2C} + \psi\right)} \\ \pm i \delta(\omega - \omega_{LO} + \omega_{FEL}) e^{i\left(\frac{\omega_{FEL}^2}{2C} + \psi\right)} \end{array} \right]$$

(5.35)

These two final signals represent clean replicas of the original FEL THz field, confirming that the demodulation process preserves its integrity. This preservation indicates that the system's transfer function does not introduce any distortion, as evidenced by the presence of only phase terms in the resulting Equation.5.35⁹. To quantify this behavior and mathematically validate the absence of distortion, we determine the transfer functions of the downconverted THz signal $\mathcal{H}_1^{PD}(\omega_{IF}^-)$ and $\mathcal{H}_2^{PD}(\omega_{IF}^-)$. These transfer functions link the input signal $X_{in}^{scope}(t) = m \cos(|\omega_{FEL} - \omega_{LO}| t + \psi)$ (i.e, the temporal downconverted copy of

⁸Note that the presence of sinusoidal terms in the two FEL THz Signal components indicates that distortion from the system's transfer function affects the THz peak.

⁹Additionally, the two final observed signals are identical in content but possess opposite signs, enabling balanced detection. The balanced detection configuration results in an improvement in the signal-to-noise ratio.

the FEL THz signal) to the recorded output signals $Y_{1,2}^{Scope\ PD}(t)$ in the Fourier domain:

$$\widetilde{Y}_{1,2}^{Scope\ PD}(\omega_{IF}^-) = \mathcal{H}_{1,2}^{PD}(\omega_{IF}^-) \widetilde{X}_{in}^{scope}(\omega_{IF}^-) \quad (5.36)$$

The Fourier transform $\widetilde{X}_{in}^{scope}(\omega_{IF}^-)$ of the input signal $X_{in}^{scope}(t)$ is expressed as:

$$\widetilde{X}_{in}^{scope}(\omega_{IF}^-) = \frac{\pi m}{2} \left[\delta(\omega + \omega_{LO} - \omega_{FEL}) e^{-i\psi} + \delta(\omega - \omega_{LO} + \omega_{FEL}) e^{i\psi} \right] \quad (5.37)$$

Thus, the transfer function is given by:

$$\mathcal{H}_{1,2}^{PD}(\omega_{IF}^-) = \pm i e^{i\left(\frac{\omega_{FEL}^2}{2C}\right)} \quad (5.38)$$

$$|\mathcal{H}_{1,2}^{PD}(\omega_{IF}^-)| = 1 \quad (5.39)$$

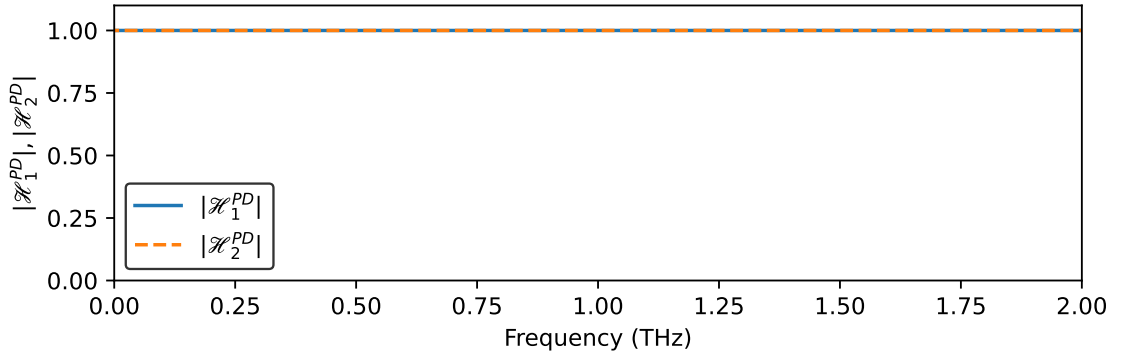


Figure 5.7: **Magnitude of the transfer functions $\mathcal{H}_1^{PD}$ and $\mathcal{H}_2^{PD}$ as a function of frequency.** The plot illustrate that both transfer functions remain constant at 1 across the frequency range, confirming that the system does not introduce amplitude distortion.

This result confirms that the magnitude of the transfer function is unitary, as demonstrated by Equation.5.39 and illustrated by Figure.5.7, indicating that no distortion is introduced to the signal's amplitude, ensuring a faithful representation of the downconverted THz signal.

This analysis demonstrates that the all-optical HEARTS method in phase diversity configuration enables the retrieval of the FEL THz signal with high fidelity. The system's transfer function preserves both the amplitude and phase information, ensuring an accurate and undistorted measurement of the downconverted THz field. In addition, the opposite-sign outputs allow for the use of a balanced detection scheme, further improving the signal-to-noise ratio. These characteristics validate the effectiveness of the measurement system in providing direct measurements of the downconverted FEL THz electric field.

2. Computational Model of the Phase Diversity Configuration Based on Jones Matrix Formalism:

To validate the previously derived equations for the all-optical HEARTS method in the phase diversity configuration, we conduct a computational model based on the Jones matrix formalism. The resulting Jones matrix $\mathbf{M}_{Het}^{PD}(t)$, is expressed as¹⁰:

$$\mathbf{M}_{Het}^{PD}(t) = \mathbf{M}_{HWP}\left(\frac{\pi}{8}\right)\mathbf{M}_{Lyot}\left(\Delta\phi_{Lyot}(t)\right)\mathbf{M}_{EO}\left(\Delta\phi_{FEL}(\alpha = \frac{\pi}{2}, t)\right) \quad (5.40)$$

The expressions for the Jones matrices were detailed previously in Subsection 5.3.2, Equation 5.8. The only element that differs is the electro-optic crystal matrix, $\mathbf{M}_{EO}\left(\Delta\phi_{FEL}(\alpha = \frac{\pi}{2}, t)\right)$. In the phase diversity configuration, the THz-induced phase shift is set such that $\Delta\phi_{FEL}(\alpha = \frac{\pi}{2}, t) = \phi_1(\alpha = \frac{\pi}{2}, t)$ and $\phi_2(\alpha = \frac{\pi}{2}, t) = 0$. This results in the following Jones matrix [18]:

$$\mathbf{M}_{EO}\left(\Delta\phi_{FEL}(\alpha = \frac{\pi}{2}, t)\right) = \begin{pmatrix} e^{i\phi_1(\alpha=\frac{\pi}{2},t)} & 0 \\ 0 & 1 \end{pmatrix} \quad (5.41)$$

The temporal output electric fields, $E_1^{PD}(t)$ and $E_2^{PD}(t)$, are computed as follow:

$$\mathbf{E}_1^{PD}(t) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{M}_{Het}^{PD}(t) \mathbf{E}_{in}^{45}(t) \quad \mathbf{E}_2^{PD}(t) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{M}_{Het}^{PD}(t) \mathbf{E}_{in}^{45}(t) \quad (5.42)$$

The detected intensities are given by $I_{1,2}^{PD}(t) = \left| \widetilde{E_{1,2}^{PD}}(\omega = -Ct) \right|^2$.

To ensure accurate simulation results, the following parameters were carefully chosen to reflect the physical properties of the system components and the experimental conditions. The full set of parameters used in the computational model is summarized in the Table. 5.1.

Based on the parameters in Table. 5.1 and Equation. 5.5, the local oscillator frequency is expected to be $\nu_{LO} = 16.56$ THz. This frequency is essential for downconverting the $\nu_{FEL} = 15$ THz FEL pulse to an intermediate frequency of $\nu_{IF}^{THz} = 1.56$ THz, which, after time stretch, reaches $\nu_{IF}^{GHz} = 3.28$ GHz according to Equation 5.18, placing it within a measurable range.

¹⁰For reference, the orientation of each component in the phase diversity configuration is illustrated in Figure 5.6.

Parameters for the Computational Model based on Jones Matrix Formalism	
Laser parameters	
λ_0	1550 nm
$\Delta\lambda$	65 nm
Stretch parameters	
D_1, D_2	17 ps/km/nm
L_1	3 m
L_2	1.424 km
M	475
FEL THz parameters	
E_{THz}^{FEL}	8 kV m^{-1}
τ_{THz}^{FEL}	10 ps FWHM
ν_{THz}^{FEL}	15 THz
EO crystal (GaAs)	
n_0	3.662
r_{41}	1.5 pmV $^{-1}$
d	100 μm
Birefringent Lyot crystal	
Δn	0.203
e	10 mm
ν_{LO}^{THz}	16.56 THz

Table 5.1: Computational model parameters

Figure 5.8 presents the temporal profiles of the two output intensities, $I_{1,2}^{ref}(t)$ and the corresponding subtracted signals, $Y_{1,2}^{ref}(t)$ in the absence of a THz signal. Panel (a) shows the resulting $I_{1,2}^{ref}(t)$ using a high-bandwidth readout, where the modulation resulting solely from the Lyot filter is clearly visible. Conversely, for the low-bandwidth readout shown in Panel (b), this modulation is filtered out, leaving only the pulse envelope. Panel (c) displays the subtracted signals, $Y_{1,2}^{ref}(t)$ (without THz), for the high-bandwidth readout case¹¹, which correspond to the zero-order term $Y_{1,2}^{PD(0)}(t)$ in equation.5.30.

Finally, the spectrum $\tilde{Y}_{1,2}^{PD(0)}(\omega)$ (Eq.5.33) is displayed in Panel (d), confirming the agreement between the computational model and the analytical formula of $\nu_{LO}^{THz} = 16.56$ THz from Eq. 5.5.

Figure 5.9 displays the spectral output components, $\tilde{Y}_1^{PD}(\omega)$ and $\tilde{Y}_2^{PD}(\omega)$, in the presence of a THz signal. Panel (a) shows the full spectra of $\tilde{Y}_1^{PD}(\omega)$ and $\tilde{Y}_2^{PD}(\omega)$, where the high-intensity local oscillator peaks are the most prominent features. Panel (b) provides a vertical zoom to reveal the heterodyne mixing peaks while saturating the LO signal. The observed behavior is consistent with Eq. 5.34. The insets highlight that while the direct FEL THz peak is altered by distortions from the sinusoidal terms in Eq. 5.34, the two heterodyne sidebands remain unaffected. Furthermore, the down-converted signal appears at $\nu_{IF}^- = |\nu_{FEL} - \nu_{LO}| = |15 - 16.56| = 1.56$ THz, successfully shifting the information into the low-bandwidth readout range as predicted.

¹¹For a low-bandwidth readout, a zero signal is expected

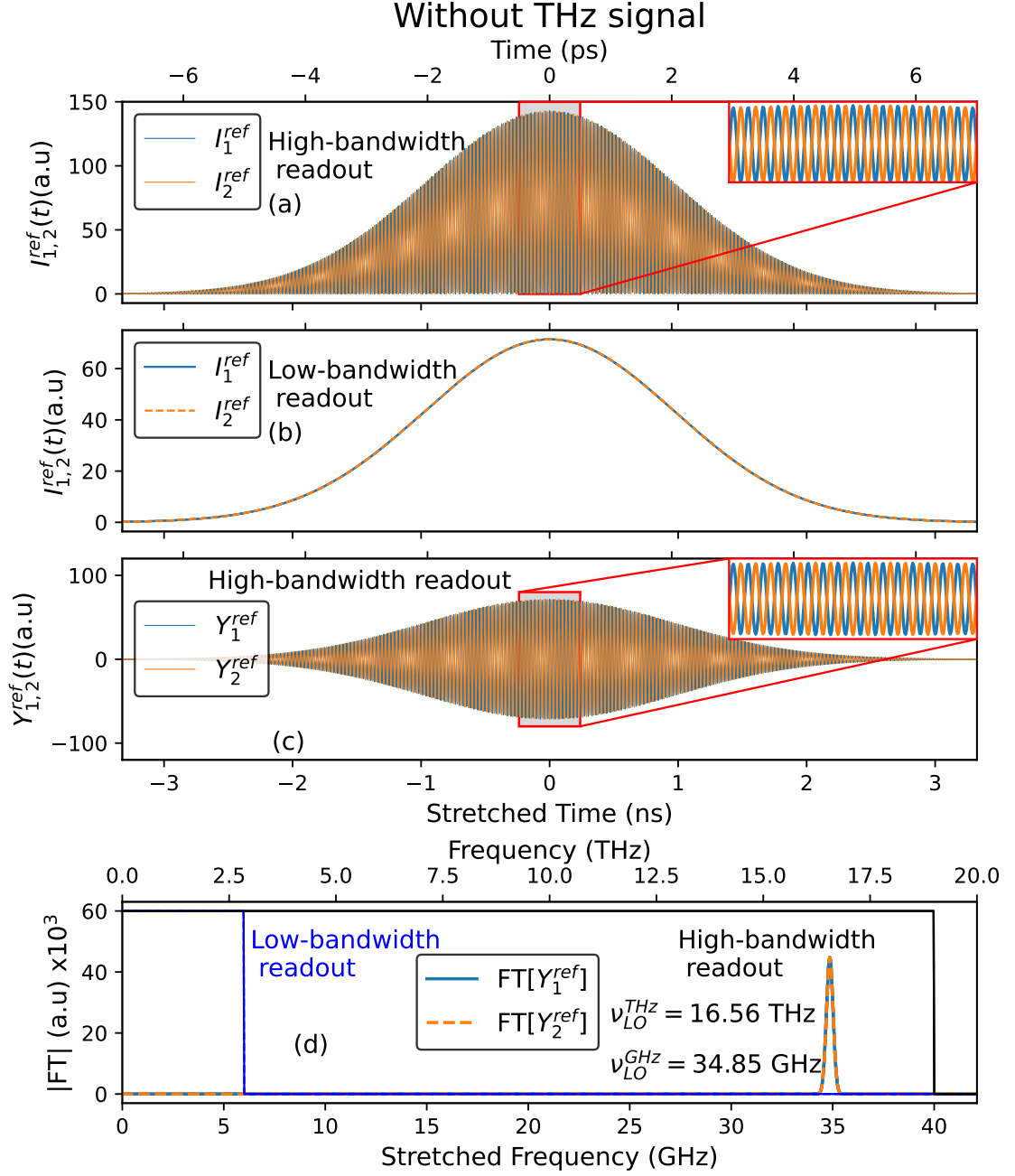


Figure 5.8: **Temporal and spectral profiles of the reference signals $Y_{1,2}^{ref}(t) = Y_{1,2}^{PD(0)}(t)$, in absence of the THz signal, under different readout bandwidths.** (a) Temporal intensity of the two references outputs without THz using a high-bandwidth readout. (b) Temporal intensity of the two references outputs without THz using a low-bandwidth readout. (c) The two temporal signal $Y_{1,2}^{ref}(t) = Y_{1,2}^{(0)}(t)$ defined in equation.5.33 without THz using high-bandwidth readout. (d) Frequency spectrum of (c) exhibiting solely the local oscillator at $\nu_{LO}^{THz} = 16.56$ THz. This plot also illustrates the high- and low-bandwidth readout responses, demonstrating that the LO peak is filtered out when using a low-bandwidth detection system.

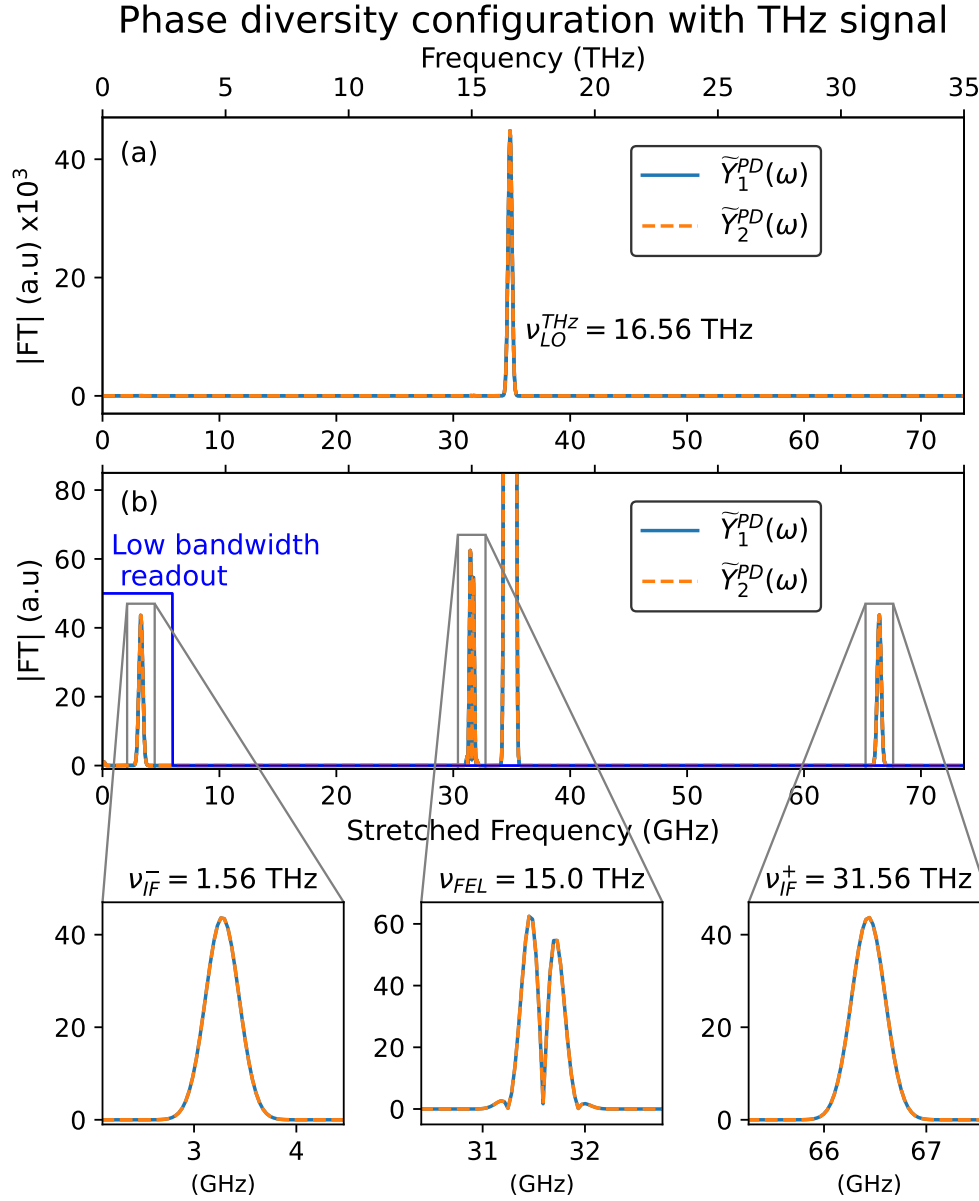


Figure 5.9: **Spectral output components $\tilde{Y}_1^{PD}(\omega)$ and $\tilde{Y}_2^{PD}(\omega)$ in presence of a THz signal.** (a) Fourier transform of $Y_1^{PD}(t)$ and $Y_2^{PD}(t)$ in the presence of a THz signal using a high-bandwidth readout. The local oscillator peak dominates the spectral amplitude. (b) Vertical zoom of (a) revealing the heterodyne mixing terms. As predicted by Eq. 5.34, the direct FEL THz peak at 15 THz exhibits distortions from the sinusoidal transfer function, while the heterodyne sidebands remain unaffected. The mixing of the FEL signal (15 THz) and the LO (16.56 THz) results in down-converted and up-converted signals at 1.56 THz and 31.56 THz, respectively. The low-bandwidth readout response is plotted to demonstrate the isolation of the down-converted signal through spectral filtering.

The temporal profile of the down-converted signal is now examined under low-bandwidth readout conditions. In contrast to the results shown in Figure 5.13, Figure 5.10(b) incorporates a Gaussian envelope to provide a more realistic model of the THz pulse:

$$\phi_{\text{FEL}}(t) = m e^{-(t/\tau_{\text{THz}}^{\text{FEL}})^2} \cos(\omega_{\text{FEL}} t + \psi) \quad (5.43)$$

Where $\tau_{\text{THz}}^{\text{FEL}}$ represents the duration of the THz envelope.

The resulting down-converted signal exhibits a temporal shift originating from the phase of the transfer function (Eq. 5.38). Furthermore, the simulation confirms that the two outputs are identical but with opposite signs. This behavior is consistent with the analytical predictions of Eq. 5.35 and effectively enables balanced detection.

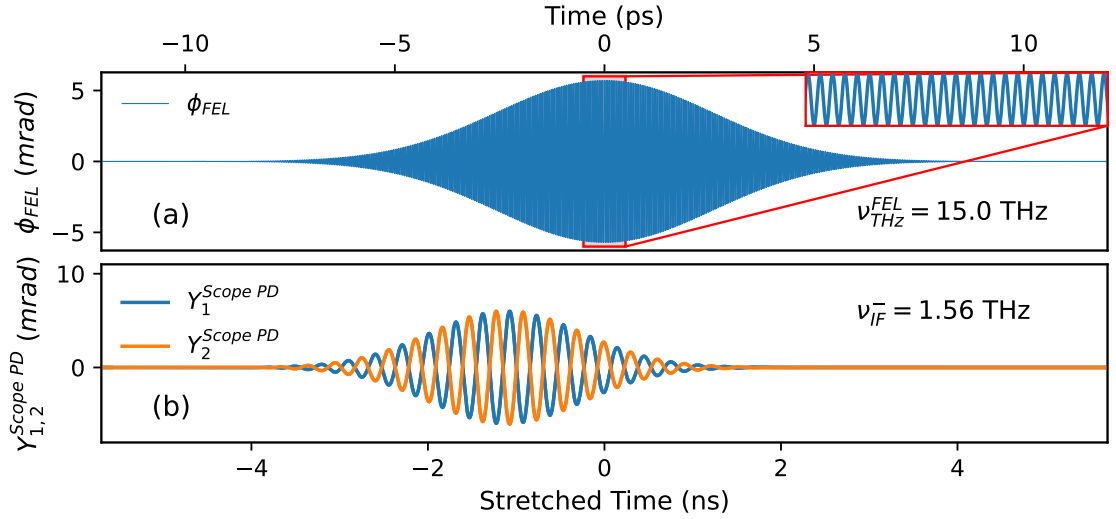


Figure 5.10: **Numerical comparison between the input signal and the downconverted heterodyne outputs in the phase diversity configuration.** (a) Multicycle input signal at 15 THz. (b) 1.56 THz downconverted heterodyne outputs in the phase diversity configuration. A time shift is observed due to the phase in the frequency domain. Additionally, both output components are identical with opposite signs, as predicted by Equation 5.34. Note that in this simulation, a Gaussian envelope is added to the THz signal, in contrast to the analytical derivation where no envelope was assumed.

The computational model results validate the derived equations for the all-optical HEARTS method in phase diversity configuration. Key features such as the preservation of the downconverted signal's profile (i.e., no distortion due to the transfer function), and balanced detection with opposite-sign outputs, are confirmed. Additionally, the computed local oscillator ν_{LO} and the intermediate frequencies ν_{IF}^- align with the analytical expressions. Overall, both analytical and numerical analyses underscores the effectiveness of the phase diversity configuration.

To achieve better signal-to-noise ratio, one could consider using the standard EO configuration, where the THz-induced phase shift is twice as strong as in the phase diversity configuration¹². Nevertheless, analytical and numerical studies of the transfer functions for the all-optical HEARTS method in a standard EO configuration are required to assess its viability. This will be explored in the following pages.

5.3.3.2 Standard EO Configuration: Signal Deformation Due to the System Transfer Function

In the Standard (Std)¹³ EO configuration, the THz polarization and the input laser polarization are both set parallel to the $[-110]$ axis of the EO crystal, as shown in Figure.5.11.(a). Both the EO crystal and the birefringent Lyot crystal share the same axis and are oriented at 45° relative to the input laser polarization and the final horizontal analyzer (see Fig.5.11.(b)).

This setup can be modeled as a Mach-Zehnder interferometer, where the two optical paths experience different modulations. The first arm, $E_{LO}^{Std}(t - \tau)$, undergoes a temporal delay imposed by the Lyot filter and a phase modulation of $\phi_2(\alpha = 0, t) = -\phi_{FEL}(t)$ induced by the THz field. Meanwhile, the second arm, $E_m^{Std}(t)$, is only modulated by the THz signal with a phase shift $\phi_1(\alpha = 0, t) = \phi_{FEL}(t)$, an illustration is given in Fig.5.11.(c) [18].

¹²The THz-induced phase shift in the standard configuration is $\Delta\phi^{std}(t) = \frac{2\pi d}{\lambda_0} n_0^3 r_{41} E_{FEL}(t)$, while in the phase diversity configuration is $\Delta\phi^{PD}(t) = \frac{\pi d}{\lambda_0} n_0^3 r_{41} E_{FEL}(t)$

¹³Note that the following superscripts Std stand for the standard configuration throughout this section.

a HEARTS with the standard electro-optic configuration

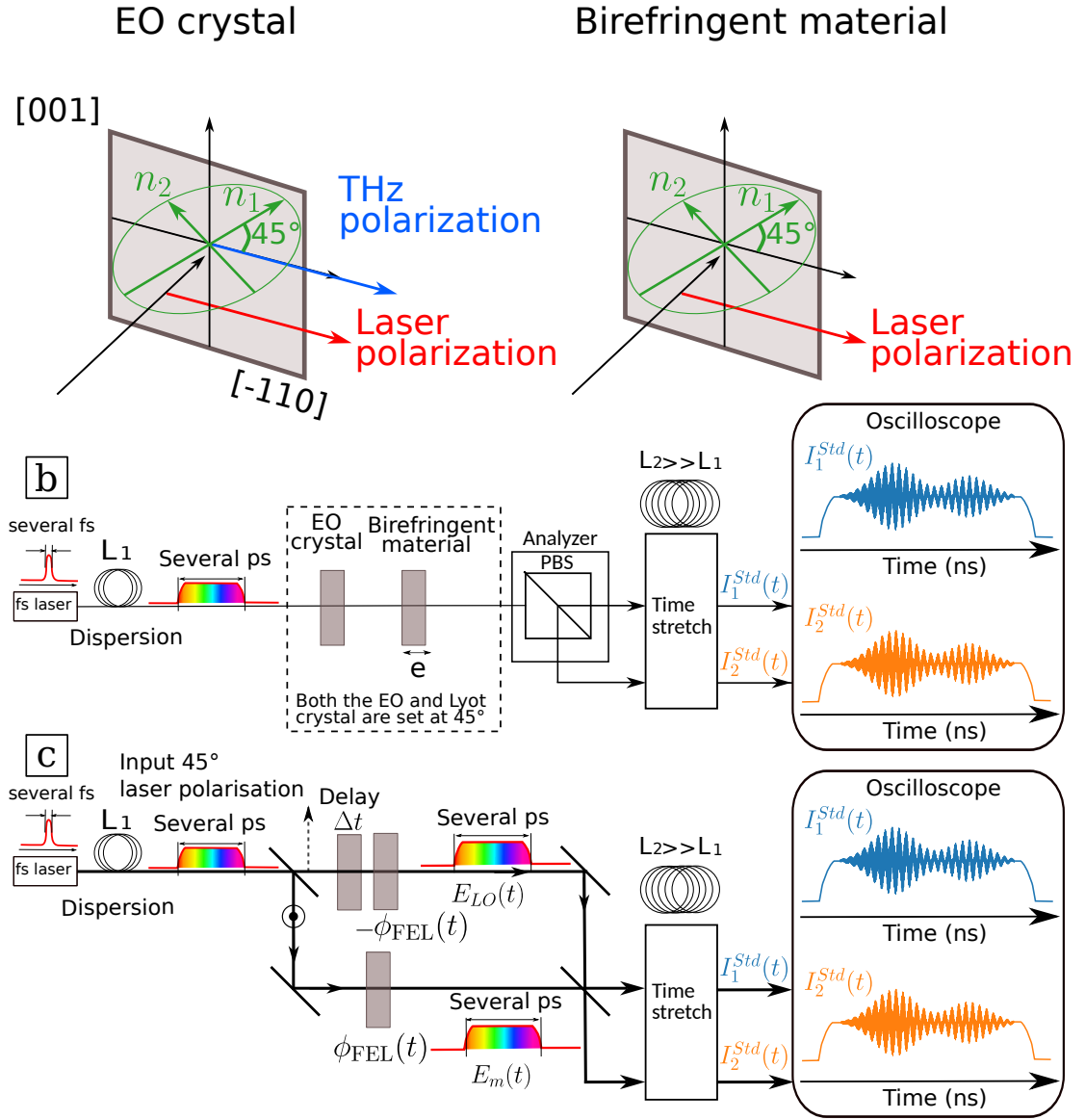


Figure 5.11: Principle of the all-optical HEARTS method in the standard electro-optic configuration (a) Experimental arrangement showing the relative orientation of the THz polarization and laser polarization with respect to the EO crystal and birefringent material axes. (b) Illustration of the all-optical HEARTS method using the standard EO configuration. The laser polarization and the analyzer are set parallel to the [-110] axis of the EO crystal. The EO and Lyot crystals share the same axis orientation and are displayed in (a). The analyzer is a Polarizing Beam Splitter (PBS). The two resulting outputs $I_1^{Std}(t)$ and $I_2^{Std}(t)$ are temporally stretched using photonic time-stretch technique and sent to the low bandwidth oscilloscope readout. (c) Optical analog of a Mach-Zehnder interferometer, where the 45° input laser polarization is split into two arms: one arm experiences a time delay and a phase modulation by the THz field, while the other is only phase-modulated by the THz signal. The two outputs are then temporally stretched using photonic time-stretch and are visualized on an oscilloscope, showing the intensities $I_1^{Std}(t)$ and $I_2^{Std}(t)$ over time.

1. Theoretical Study of the All-Optical HEARTS Method in Standard EO Configuration Based on Mach-Zehnder Interferometer Equivalence :

Following a similar approach as in the phase diversity configuration subsection. 5.3.3.1, we derive the expressions for the local oscillator $E_{LO}^{Std}(t - \tau)$ and modulated $E_m^{Std}(t)$ electric fields:

$$E_{LO}^{Std}(t - \tau) = \frac{E_0(t - \tau)}{2} e^{-i\phi_{FEL}(t)} \approx \frac{A(t - \tau)}{2} e^{-\frac{i}{2}C(t-\tau)^2} (1 - i\phi_{FEL}(t)) \quad (5.44)$$

$$E_m^{Std}(t) = \frac{E_0(t)}{2} e^{i\phi_{FEL}(t)} \approx \frac{A(t)}{2} e^{-\frac{i}{2}Ct^2} (1 + i\phi_{FEL}(t)) \quad (5.45)$$

To focus on the behavior of the heterodyne peaks, we apply the same derivation as in the phase diversity case to the standard configuration fields, $E_{LO}^{Std}(t - \tau)$ and $E_m^{Std}(t)$. The resulting first-order Fourier transforms of the normalized output signals¹⁴, $\tilde{Y}_{1,2}^{Std(1)}(\omega)$, are given by:

$$\tilde{Y}_{1,2}^{Std(1)}(\omega) = \frac{\pi m}{2} \left[\begin{array}{l} \mp 2i \cos\left(\frac{\omega_{FEL}^2}{2C} + \psi\right) \delta(\omega + \omega_{LO} - \omega_{FEL}) e^{-i(\tau\omega_{FEL})} \\ \pm 2i \cos\left(\frac{\omega_{FEL}^2}{2C} + \psi\right) \delta(\omega - \omega_{LO} + \omega_{FEL}) e^{-i(\tau\omega_{FEL})} \end{array} \right\} \left. \begin{array}{l} \text{Lower Sideband} \\ \omega_{IF}^- = |\omega_{FEL} - \omega_{LO}| \end{array} \right\} \\ \left[\begin{array}{l} -4i \sin\left(\frac{\omega_{FEL}^2}{2C}\right) \sin\left(\frac{\omega_{FEL}\tau}{2}\right) \delta(\omega - \omega_{FEL}) e^{-i(\psi + \frac{\omega_{FEL}\tau}{2})} \\ +4i \sin\left(\frac{\omega_{FEL}^2}{2C}\right) \sin\left(\frac{\omega_{FEL}\tau}{2}\right) \delta(\omega + \omega_{FEL}) e^{-i(\psi - \frac{\omega_{FEL}\tau}{2})} \end{array} \right\} \left. \begin{array}{l} \text{FEL THz Signal} \\ \omega_{FEL} \end{array} \right\} \\ \left[\begin{array}{l} \mp 2i \cos\left(\frac{\omega_{FEL}^2}{2C} + \psi\right) \delta(\omega + \omega_{LO} + \omega_{FEL}) e^{i(\tau\omega_{FEL})} \\ \pm 2i \cos\left(\frac{\omega_{FEL}^2}{2C} + \psi\right) \delta(\omega - \omega_{LO} - \omega_{FEL}) e^{-i(\tau\omega_{FEL})} \end{array} \right\} \left. \begin{array}{l} \text{Upper Sideband} \\ \omega_{IF}^+ = |\omega_{FEL} + \omega_{LO}| \end{array} \right\} \quad (5.46)$$

Compared to the phase diversity configuration, an important distinction is that the sinusoidal terms affect not only the THz peak but also the up and downconverted components.

The downconverted signal $\tilde{Y}_{1,2}^{(-)Std}(\omega) = \tilde{Y}_{1,2}^{Scope Std}(\omega)$, extracted from the full expression, are given by:

$$\tilde{Y}_{1,2}^{Scope Std}(\omega) = \frac{\pi m}{2} \left[\begin{array}{l} \mp 2i \cos\left(\frac{\omega_{FEL}^2}{2C} + \psi\right) \delta(\omega + \omega_{LO} - \omega_{FEL}) e^{-i(\tau\omega_{FEL})} \\ \pm 2i \cos\left(\frac{\omega_{FEL}^2}{2C} + \psi\right) \delta(\omega - \omega_{LO} + \omega_{FEL}) e^{-i(\tau\omega_{FEL})} \end{array} \right] \quad (5.47)$$

Due to the presence of cosine terms, neither output provides a direct, undistorted copy of the original THz field.

¹⁴As the zero-order components are identical to those in the phase diversity case, corresponding to the LO peaks described by Eq.5.33

To further analyze this behavior, we derive the transfer functions for the downconverted THz signal $\mathcal{H}_{1,2}^{Std}(\omega_{IF}^-)$ associated with the standard EO configuration:

$$\mathcal{H}_{1,2}^{Std}(\omega_{IF}^-) = \pm 2i \cos\left(\frac{\omega_{FEL}^2}{2C}\right) e^{-i\tau\omega_{FEL}} \quad (5.48)$$

$$|\mathcal{H}_{1,2}^{Std}(\omega_{IF}^-)| = 2 \left| \cos\left(\frac{\omega_{FEL}^2}{2C}\right) \right| \quad (5.49)$$

The magnitude of this transfer function, shaped by a cosine function, confirms that the system introduces amplitude distortions affecting both outputs. Moreover, since both transfer functions are identical, performing deconvolution using the two signals is ineffective. The zeros of their transfer functions coincide, leading to a loss of information in both arms simultaneously, ultimately preventing the reconstruction or retrieval of the original THz field.

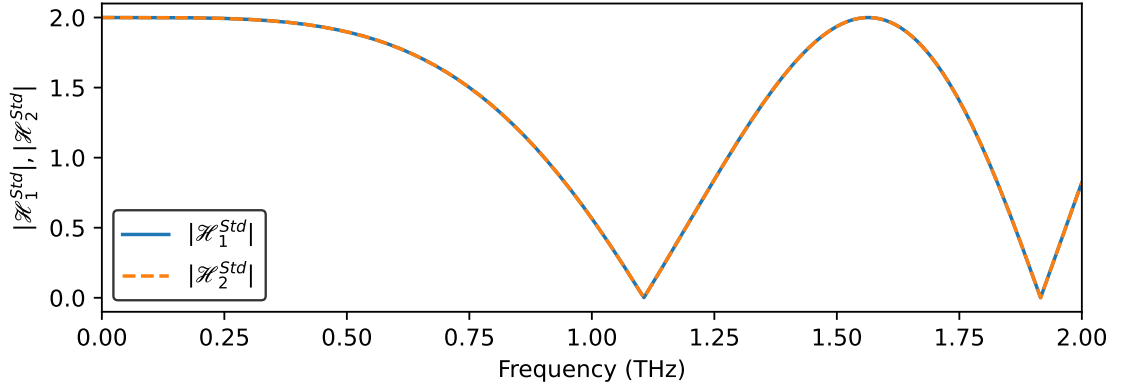


Figure 5.12: **Magnitude of the transfer function for the standard EO configuration.** The cosine dependence leads to amplitude distortion in both output signals, preventing accurate retrieval of the original THz field.

This analysis demonstrates that all-optical HEARTS method in standard EO configuration, does not allow direct access to the original THz field, even via algorithmic post-processing techniques.

2. Computational Model of the Standard EO Configuration Based on Jones Matrix Formalism: :

To validate the analytical results obtained for the standard EO configuration, we perform a computational model based on the Jones matrix formalism, following the same approach as in the phase diversity configuration. In this case, the resulting Jones matrix is given by:

$$\mathbf{M}_{Het}^{Std}(t) = \mathbf{M}_{Lyot}\left(\frac{\pi}{4}, \Delta\phi_{Lyot}\right) \mathbf{M}_{EO}\left(\frac{\pi}{4}, \Delta\phi_{FEL}(t)\right) \quad (5.50)$$

The Jones matrix of the input laser polarization is:

$$\mathbf{E}_{in}^0(t) = E_0(t) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5.51)$$

Using this formalism, the two temporal output electric fields, $E_1^{Std}(t)$ and $E_2^{Std}(t)$ are computed as follow:

$$\mathbf{E}_1^{Std}(t) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{M}_{Het}^{Std}(t) \mathbf{E}_{in}^0(t) \quad \mathbf{E}_2^{Std}(t) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{M}_{Het}^{Std}(t) \mathbf{E}_{in}^0(t) \quad (5.52)$$

To ensure a consistent comparison, the computational parameters used here, in the standard configuration, are identical to those employed in the phase diversity configuration. The resulting spectral components, $\tilde{Y}_1^{Std}(\omega)$ and $\tilde{Y}_2^{Std}(\omega)$, in presence of a THz signal are presented in Figure 5.13.

Panel (a) shows the full spectra of $\tilde{Y}_1^{Std}(\omega)$ and $\tilde{Y}_2^{Std}(\omega)$, where the high-intensity LO peaks are the most prominent features. Panel (b) displays a vertical zoom of (a), revealing the central THz peak along with the up- and down-converted sidebands. All of them exhibit distortions introduced by the standard configuration transfer function, $\mathcal{H}_{1,2}^{Std}(\omega)$, in the standard EO configuration (eq. 5.46).

To further examine the impact of these distortions, we perform the same simulation as in Figure. 5.13 while incorporating a Gaussian temporal envelope into the THz signal. The results are illustrated in Figure 5.14, where the temporal input THz field (a) is compared with the down-converted output signal (b). This comparison highlights how the Gaussian-enveloped signal is reshaped by the transfer function of the standard EO configuration.

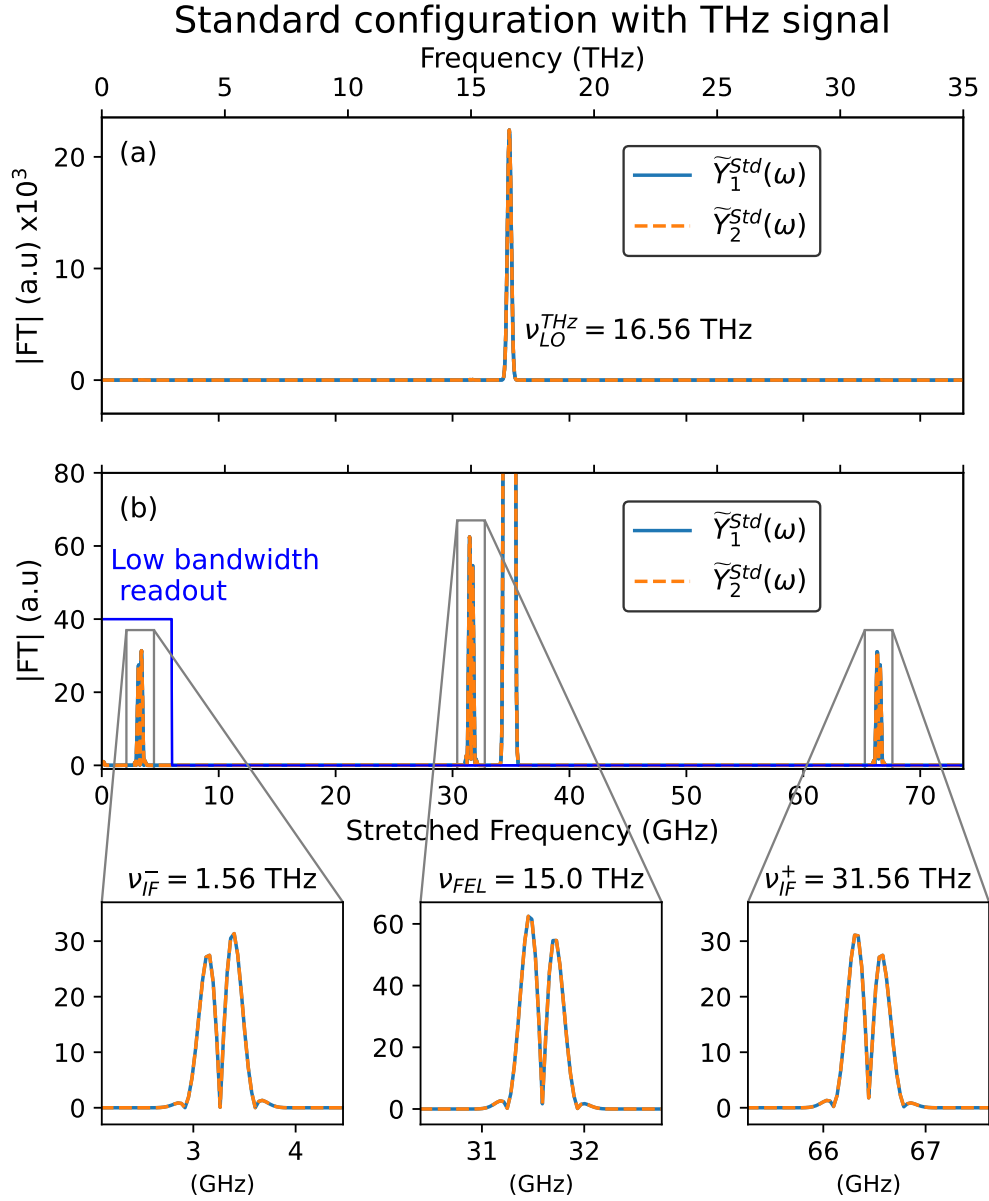


Figure 5.13: **Spectral output components $\tilde{Y}_1^{Std}(\omega)$ and $\tilde{Y}_2^{Std}(\omega)$ in presence of a THz signal.** (a) Fourier transform of $Y_1^{Std}(t)$ and $Y_2^{Std}(t)$ in the presence of a THz signal using a high-bandwidth readout. The local oscillator peak dominates the spectral amplitude. (b) Vertical zoom of (a) revealing the heterodyne mixing terms. As predicted by Eq. 5.46, the direct FEL THz peak and the sidebands exhibits distortions introduced by the cosinusoidal and sinusoidal transfer function. The mixing of the FEL signal (15 THz) and the LO (16.56 THz) results in down-converted and up-converted signals at 1.56 THz and 31.56 THz, respectively. The low-bandwidth readout response is plotted to demonstrate the isolation of the down-converted signal through spectral filtering.

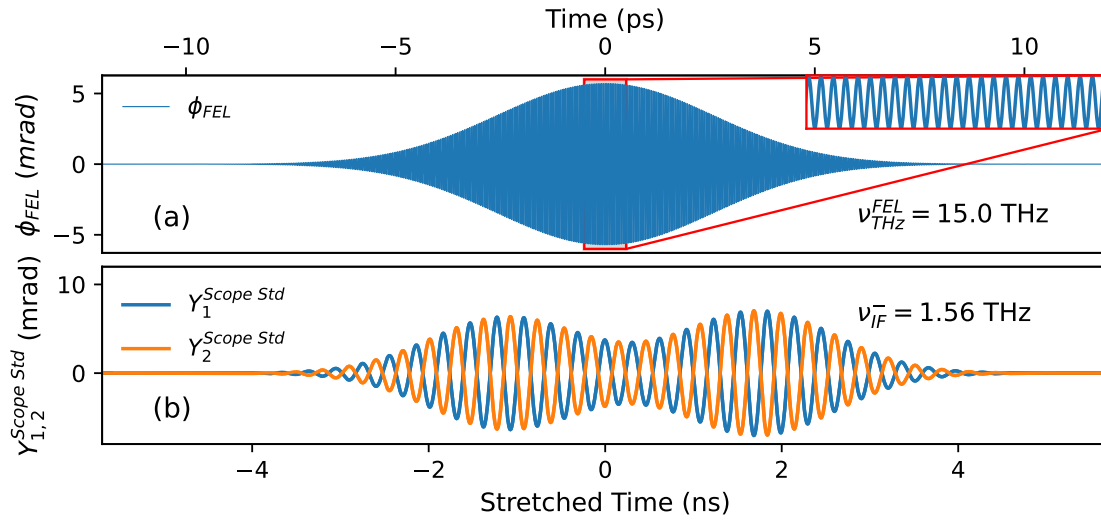


Figure 5.14: **Numerical comparison between the input signal and the down-converted heterodyne outputs in the standard EO configuration.** (a) Multicycle input signal at 15 THz. (b) 1.56 THz downconverted heterodyne outputs in the standard configuration. A noticeable distortion arises due to multiple peaks in the Fourier space, caused by zeros in the system's transfer function. Moreover, as predicted in Equation. 5.46, both output components exhibit the same shape but with opposite signs.

The simulation results for the standard EO configuration align with the analytical predictions derived in the previous subsection. In particular, the simulated spectral components, $\tilde{Y}_1^{Std}$ and $\tilde{Y}_2^{Std}$, exhibit the central THz peak along with the up- and down-converted components, with distortions arising from the system's transfer function. The comparison between the input THz field and the down-converted output clearly demonstrates a distortion, which arises due to multiple peaks in the Fourier space caused by the presence of zeros in the transfer function¹⁵.

Overall, both analytical and numerical analyses highlight the limitations of the standard EO configuration in accurately reconstructing the original THz field.

¹⁵Additionally, as predicted by Equation 5.46, both output components show the same shape but with opposite signs.

Conclusion on the choice of the all-optical HEARTS configuration

In comparing the phase diversity and standard EO configurations, several key differences and implications for THz signal retrieval emerge.

The phase diversity configuration is advantageous in that it does not introduce any distortion on the down-converted signal. This enables the direct observation of the THz signal, eliminating the need for reconstruction algorithms. Furthermore, the output signals are identical in shape but opposite in sign, which allows for effective balanced detection schemes, improving signal-to-noise ratio.

In contrast, the standard EO configuration, while offering an improvement in signal-to-noise ratio by a factor of 2, is intrinsically limited. It introduces distortions into the down-converted signal via the system's transfer function, preventing direct access to an undistorted version of the THz field. Furthermore, reconstruction algorithms are ineffective because both transfer functions are identical and share zeros at the same frequencies. This results in an irrecoverable loss of information, rendering the recovery of the original signal impossible through deconvolution.

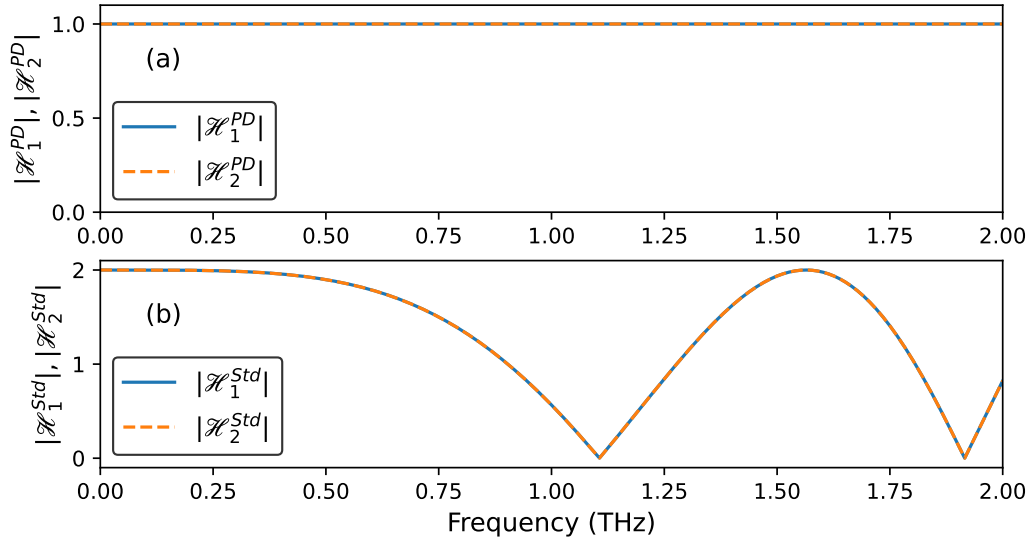


Figure 5.15: Comparisons of the transfer functions between the all-optical HEARTS phase diversity and standard EO configurations:

- (a) Transfer function of the down-converted signal for the phase diversity configuration. The absence of zeros ensures a direct representation of the THz field, eliminating the need for reconstruction algorithms.
- (b) Transfer function of the down-converted signal for the standard EO configuration. The presence of zeros introduces distortions in the THz signal, and their occurrence at the same positions makes reconstruction algorithms ineffective.

Thus, despite the potential benefits in terms of SNR, the standard EO configuration proves unusable for accurate THz field measurement. Therefore, the phase diversity configuration is the essential choice for the all-optical HEARTS detection scheme, as it uniquely provides a direct, undistorted copy of the THz signal at the lower intermediate frequency $\nu_{\text{IF}}^{\text{GHz}}$.

5.4 Chapter summary

Chapter summary

This chapter established the theoretical foundation for the novel Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS) method.

We first demonstrate the limitation of conventional single-shot diagnostics, where in the case of photonic time stretch, the required high-bandwidth oscilloscope leads to inefficient resource use (bandwidth waste) for narrow-band multicycle THz pulses, thereby impeding the realization of real-time diagnostics systems.

To overcome this, we presented the HEARTS method, which integrates all-optical heterodyning with the photonic time-stretch technique. The theoretical framework detailed how the use of simple birefringent crystals (YVO4) allows for the optical generation of a necessary tunable THz local oscillator via a Lyot filter.

Most importantly, the theoretical and numerical analysis demonstrated that the HEARTS method must employ the phase diversity electro-optic configuration to ensure a flat and non-distorting transfer function. This finding guarantees that the detected, down-converted intermediate frequency signal is a faithful and undistorted replica of the original high-frequency THz pulse, all within an affordable readout bandwidth.

This theoretical analysis validated the HEARTS design required for the experimental proof-of-principle demonstration at FELBE in Chapter. 6.

Experimental demonstration and implementation of the HEARTS detection technique at FELBE

*In the field of observation, chance
favors only the prepared mind.*

– Louis Pasteur, 1854, Lille
university

6.1 Introduction

Chapter 5 established the theoretical foundation for the novel Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS) technique. This single-shot electro-optic strategy was designed for narrowband THz sources to resolve the inefficient use of high-bandwidth electronic readouts and enable future real-time diagnostics. By integrating all-optical heterodyning, Photonic Time-Stretch, and a phase diversity configuration, HEARTS guarantees the acquisition of a faithful, down-converted signal within an affordable readout bandwidth.

This chapter now transitions from the theoretical design to the practical, experimental realization of the HEARTS technique. To demonstrate both its experimental proof-of-principle and its immediate applicability to accelerator diagnostics. A dedicated measurement campaign was conducted at the oscillator Free-Electron Laser in Dresden (FELBE). FELBE generates tunable, narrowband THz radiation (1.2–60 THz) at a 13 MHz repetition rate. However, the source is affected by Carrier-Envelope Phase (CEP) drift, a parameter that needs to be

known or stabilized for THz pump-probe experiments. Therefore, the ultimate goal of this campaign was to implement the HEARTS technique within an experimental setup, enabling Carrier-Envelope Phase (CEP) diagnostics for the FELBE source using a low readout bandwidth.

The chapter begins by detailing the characteristics of the FELBE facility and outlining the motivations for CEP diagnostic. Following this, the Section 6.2 provides a detailed description of the experimental setup using a 1550 nm chirped probe laser. This setup allows easy switching between the DEOS (without heterodyne) combined with photonic time-stretch and the HEARTS configurations. The experimental system preparation is then addressed, detailing the experiment system noise floor (Section. 6.3) and the alignment steps, including the THz properties, transport, the spatial overlap and timing between the chirped probe laser and the THz field (Section. 6.4).

Before proceeding to the HEARTS demonstration, Section 6.5 details a preliminary experiment using a high-bandwidth acquisition system in a DEOS configuration. This preparatory step was necessary to confirm the observability of the FEL THz signal, as there was initial uncertainty regarding the performance of the GaAs EO crystal at the 3.44 THz frequency¹.

The core results are presented in Section 6.6, detailing the experimental demonstration of the all-optical HEARTS in its phase diversity configuration. This section includes the first recorded EO down-converted signal achieved with a low-bandwidth acquisition system (a 2.5 GHz photodetector and a 4 GHz oscilloscope).

The final section of this chapter, Subsection 6.6.3, is dedicated to the study of the CEP drift of the FELBE source. This CEP diagnostic was realized using EO measurements recorded in the HEARTS configuration with a low-bandwidth readout, thus directly proving its utility for accelerator diagnostics.

¹This was the first time we used an experimental setup at 1550 nm, which required the use of a GaAs EO crystal, to record a narrowband THz signal at this frequency.

6.1.1 The ELBE facility

The Electron Linac for beams with high **B**rilliance and low **E**mittance (ELBE), is a user-oriented research facility located at Forschungszentrum Dresden-Rossendorf in Germany, providing powerful radiation sources since 2002. The central component of ELBE is a superconducting linear accelerator that is able to produce ultrashort electron bunches with an energy of 40 MeV.

The ELBE facility offers flexible operational modes to satisfy various experimental requirements. These modes are produced at the repetition rate up to 26 MHz, in continuous or burst mode. The accelerated electron beams are directed to multiple experimental halls enabling the creation of multiple secondary radiation and particle beams. These include 2 free-electron lasers (FELBE), Bremsstrahlung, positron beams (pELBE), electron and neutron beams (nELBE), hard X-rays, and superradiant THz radiation, all of which enable a wide range of scientific investigations.

In this thesis, the measurement campaign is performed at FELBE the oscillator FEL.

6.1.1.1 The Free Electron Lasers of the ELBE facility (FELBE)

The Free-Electron Lasers at Dresden (FELBE) are narrowband THz radiation sources operating as an FEL oscillator.

For a detailed explanation of the oscillator-type FEL operating principle, the reader is referred to Section 1.4.3.

1. Key characteristic of FELBE and undulator choice

FELBE is equipped with two FEL systems utilizing different undulators, the U37-FEL and the U100-FEL, with undulator periods of 37 mm and 100 mm, respectively. These systems produce intense, coherent electromagnetic radiation tunable across the mid- to far-infrared range. The U37-FEL covers the range from 5–40 μm , while the U100-FEL is designed to cover the range from 18–250 μm . A summary of the key performance parameters for the emitted radiation is provided in Table 6.1, and further details about the two undulators are listed in Table 6.2.

Parameters	Design	Unit
Energy range	5–40	MeV
Pulse length	1–25	ps
Pulse energy	0.1–2	μJ
Peak power	up to 1	MW
Average power	up to 20	W

Table 6.1: Key characteristics of FELBE

In order to test the novel all-optical HEARTS method, the primary objective was to measure FEL pulses in the low-frequency, long-wavelength regime. Therefore, the experiment was performed using the U100-FEL, as its operational range of 1.2–16 THz (18–250 μm) is ideally suited to generate this required low-frequency THz radiation. The U100 undulator is composed of 38 periods. This number of periods determines the order of magnitude of the number of THz cycles.

Undulator characteristics	U37	U100
Undulator period λ_u	37 mm	100 mm
Number of period N_u	54	38
Undulator parameter K_u	0.4–1.35	0.5–2.8
Wavelength range	5–40 μm	18–250 μm
Frequency range	60–7 THz	16–1.2 THz

Table 6.2: Parameters of the undulators of FELBE

2. The FELBE time structure

FELBE can be configured to operate in three distinct modes, in continuous wave at a 13 MHz repetition rate, in macropulses, or single-pulse switched mode. For the experiment described in this chapter, we selected the macropulse mode operating at a 13 MHz micropulse repetition rate, as illustrated in Figure 6.1. This operational mode was chosen for two primary reasons. First, it allowed us to maintain high peak power levels, while limiting thermal effect (damage) due to the average power². Second, the macropulse structure was essential to enable a detailed observation of the FEL’s turn-on and turn-off dynamics during the course of the macropulse. The temporal structure of the macropulses is defined

²In practice, the average power was around 20 mW

by an approximate duration of 0.5 ms and a repetition period of 40 ms, with the 13 MHz micropulses being emitted only within this 0.5 ms window.

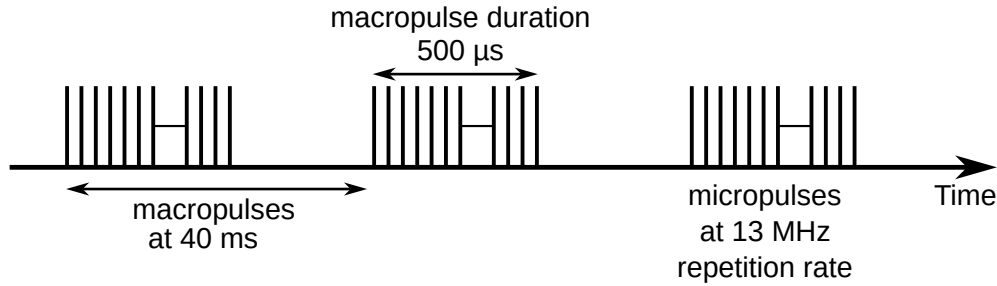


Figure 6.1: **Temporal structure of THz emission** : The figure shows 13 MHz micropulses contained within 0.5 ms macropulses, with a 40 ms repetition rate.

6.1.2 Motivations for Carrier-Envelope Phase (CEP) Diagnostics

In an oscillator FEL, the synchronization between the arrival time of the electron bunch and the round-trip time of the existing FEL pulse within the resonant cavity is required. This synchronization is achieved and controlled by a parameter known as cavity detuning, which is adjusted by changing the position of one of the cavity mirrors. This detuning directly influences the efficiency and the Carrier-Envelope Phase (CEP) of the FEL.

1. Principle and Consequences of Carrier-Envelope Phase (CEP) Drift

The CEP is defined as the phase of the carrier wave relative to the peak of its pulse envelope. The phenomenon of CEP drift is a consequence of the velocity mismatch between the group velocity V_g of the pulse envelope and the phase velocity V_ϕ of the carrier wave as they propagate through the oscillator cavity. As the THz pulse travels through the resonant cavity, this velocity mismatch causes the carrier wave to slip relative to its envelope on each round trip. This phase slippage accumulates over time, leading to a continuous drift of the CEP. Management of the CEP drift for the THz pump-probe experiments is required either by suppressing it, or by recording the CEP (in real-time) during the measurements. A clear representation of this effect is provided in Figure 6.2.

2. Review of Single-Shot CEP Diagnostics at FELBE

Previous single-shot measurement have been performed to characterize the carrier-envelope phase drift of the THz radiation sources at FELBE.

One such notable study, by I. Ilyakov *et al.* [26], focused on field-resolved THz time-domain spectroscopy using the pulsed radiation from FELBE. While this work successfully diagnosed the CEP drift by using a *measure-and-sort* technique,

it did not provide access to the full temporal profile of the THz pulses in a single shot.

Another notable study of the CEP drift and THz characterization at FELBE has been realized by C. Hanoun *et al.* [129]. Their measurements at FELBE, using a 1030 nm phase diversity EO setup successfully provided access to the full temporal profile, including both the carrier and the envelope, of the 4 THz monochromatic pulses in a single-shot. This achievement enabled the first-ever observation and characterization of the FEL's dynamics and CEP diagnostic in a single-shot.

3. Advancing Single-Shot CEP Diagnostics with the HEARTS Technique

The work presented in this chapter aims to further advance these single-shot CEP diagnostic capabilities by reducing the readout electronic bandwidth requirements. This bandwidth reduction is achieved by employing the novel all-optical HEARTS technique, which combined single-shot EO techniques with a heterodyning mixing process and photonic time-stretch techniques (as detailed in Chapter. 5). The results presented herein provide the foundation for future real-time, single-shot characterization of narrowband THz signals. Furthermore, this work serves as a proof-of-principle demonstration for capturing multiple cycles of a narrowband THz signal without the need for high-bandwidth oscilloscope (e.g >10 GHz).

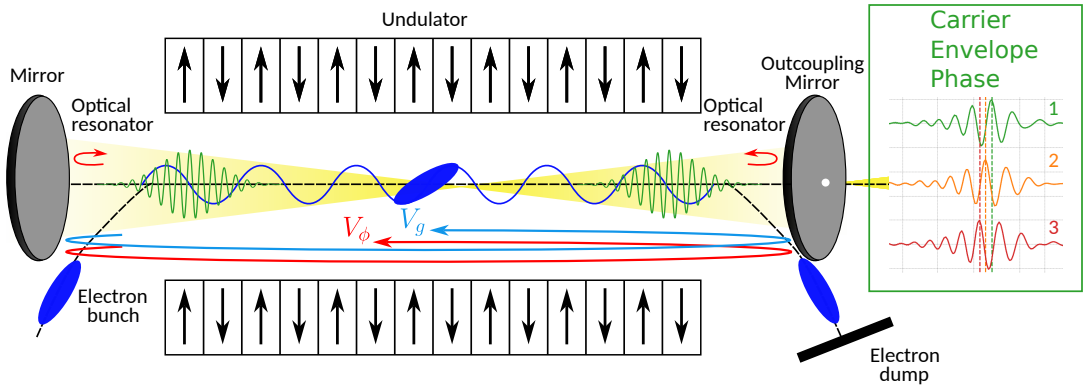


Figure 6.2: Principle of carrier-envelope phase (CEP) drift in an oscillator FEL : This figure illustrates the concept of carrier-envelope phase drift from one optical pulse to the next (detailed within the green rectangle). The CEP drift is caused by a velocity mismatch between the group velocity V_g of the pulse envelope and the phase velocity V_ϕ of the carrier wave in the oscillator cavity.

6.2 Experimental apparatus presentation

This section provides a description of the single-shot all-optical HEARTS experimental apparatus, detailing its implementation using the phase diversity configuration³.

Notes on chirped probe wavelength choice : 1030 nm vs 1550 nm

The choice of laser wavelength was an important factor in the experimental design, with a decision to be made between 1030 nm and 1550 nm.

The 1030 nm wavelength was considered a reliable and proven option. It had been successfully used in previous experiments at FELBE by C. Hanoun *et al*, and its compatibility with GaP crystals as established.

In contrast, the 1550 nm wavelength, a standard for telecommunications, offered significant advantages. The telecommunications market has led to the development of a wide range of high-quality optical components, that could be used to optimize the experimental setup. Furthermore, 1550 nm pulses can be more easily stretched due to lower power losses within optical fiber compare to 1030 nm [84], leading to the use of lower bandwidth readouts. Nevertheless, the choice of a GaAs electro-optic crystal introduced a major uncertainty, as its performance and response had not been thoroughly tested at the THz frequencies of interest for this experiment.

Given these considerations, we decided to choose the following experimental strategy.

Plan A, considered the most promising due to its long-term potential, was to use the 1550 nm setup.

Plan B involved a fallback to the well-proven 1030 nm configuration, to be implemented only if the initial tests with 1550 nm were unsuccessful. Fortunately, the success of the 1550 nm implementation meant we did not have to resort to Plan B.

1. Femtosecond probe laser :

The probe laser is a femtosecond erbium laser (Toptica) centered at 1550 nm. The output pulses are compressed, close to their Fourier limit. It exhibits a 62 nm bandwidth, corresponding to 7.8 THz, as depicted in Fig. 6.4, and delivers an output power of 250 mW in free space. The laser operates at a repetition rate of 78 MHz and is synchronized to the FEL using a laser timing synchronization system, achieving a phase jitter of ≤ 500 fs.

³The importance of the phase diversity configuration to ensure a flat and non-deforming transfer function is detailed in Section 5.3.

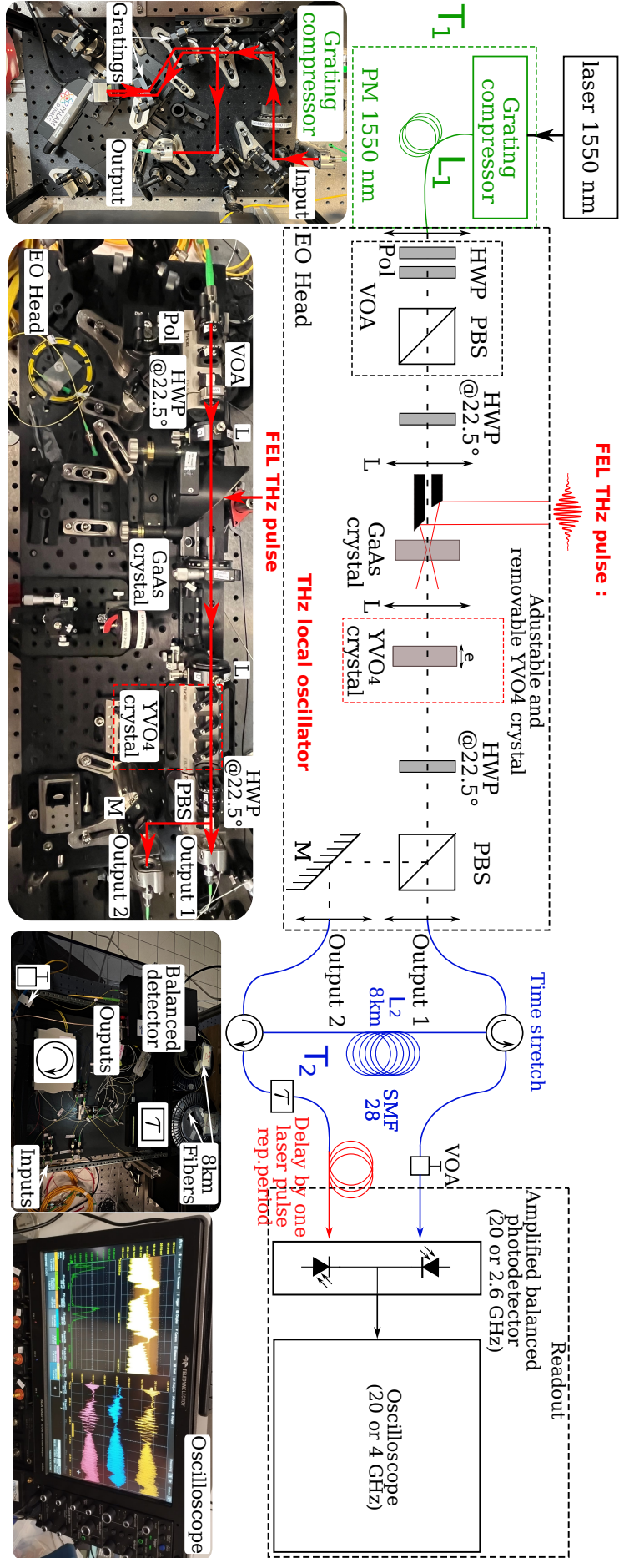


Figure 6.3: **Single-shot electro-optic detection setup for narrowband THz FEL signals using the all-optical HEARTS technique in phase diversity configuration.** A chirped 1550 nm laser pulse and the THz FEL signal are co-focused onto a Gallium Arsenide (GaAs) electro-optic (EO) crystal, inducing a phase modulation. The modulated laser pulse is heterodyned using a local oscillator generated by a Lyot filter with a YVO₄ birefringent crystal. The resulting modulated and heterodyned laser pulse is separated into orthogonal polarization states by a polarizing beam splitter (PBS), each directed via optical circulators into an 8 km single-mode fiber (SMF-28) for temporal stretching. One polarization arm is further delayed by a repetition period. Balanced detection is then performed on both arms, and the individual polarization components are recorded using an oscilloscope. Pol: Polarizer, HWP: Half-wave plate, VOA: Variable optical attenuator, M: Mirror, L: Lens.

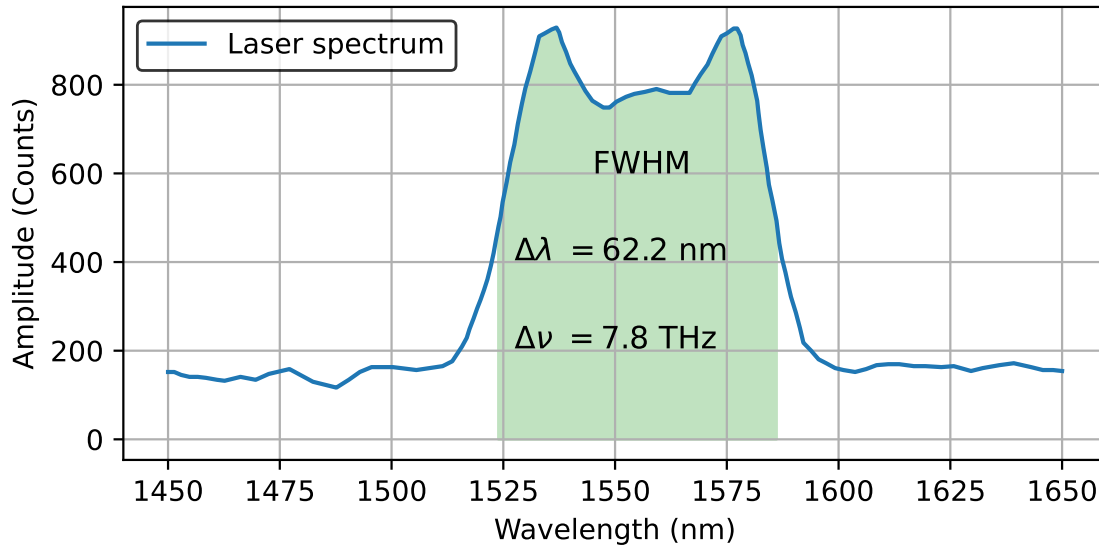


Figure 6.4: Probe laser spectrum measured with an optical spectrum analyzer, showing a bandwidth of 62.2 nm (7.8 THz).

2. Chirped probe laser :

The femtosecond probe laser pulses are chirped using a homemade grating compressor consisting of a pair of gratings with 600 lines/mm as shown in Fig.6.3. The grating compressor exhibits a transmission efficiency of 65 %.

It is capable of stretching the probe laser pulses to a duration of up to 20 ps, a PM1550 fiber can be add to increase further the recording analysis window T_1 . The use of a grating compressor offers two primary advantages.

Firstly, it prevents nonlinear effects that could distort the laser spectrum if short, high-power pulses were coupled into optical fibers.

Secondly, it enables a continuous adjustment of the temporal window, T_1 , to match the THz signal, an important aspect in this EO heterodyne experiment as defined by Eq.5.18.

3. EO head :

The electro-optic head was constructed by mounting optical components on a 20 cm x 60 cm optical breadboard, as depicted in Fig. 6.3. Initial beam collimation was achieved using a fiber collimator (Thorlabs TC06 APC-1550), with a 6.18 mm focal length. A linear polarizer was then employed to ensure a well-defined linear polarization state. To control the beam intensity and prevent detector saturation, a free-space variable optical attenuator (VOA) was implemented. This VOA consisted of an achromatic half-wave plate (HWP) (Thorlabs FBR-AH3), in conjunction with a 5 mm polarizing beam splitter (PBS) cube (Thorlabs PBS054). Following the VOA, a second HWP, carefully aligned at 22.5° , was used to orient the laser polarization at a 45° angle relative to the $[-110]$ axis of the GaAs crystal, thus implementing a phase diversity configuration. Subsequently, a plano-convex lens (Thorlabs LA1207-C N-BK7 C-coated) with a 10 cm focal

length was incorporated to focus the laser beam onto the EO crystal.

The horizontally polarized THz beam was directed to the EO board using a 2-inch diameter off-axis parabolic mirror with a 2-inch focal length. This parabolic mirror focused the THz beam onto the crystal while allowing the laser beam to pass through a central aperture to reach the EO crystal. Three 110-cut Gallium Arsenide (GaAs) crystals with AR coating at 1550 nm, from Del Mar Photonics, tested during the experimental run: a $10 \times 10 \times 3$ mm, a 0.5 mm thick crystal, and a 0.1 mm thick crystal. A second lens (Thorlabs LA1207-C), with a 100 mm focal length, was implemented, followed by a dedicated space for the insertion of a Lyot filter. This Lyot filter, made of YVO4 birefringent crystals with an "adjustable" thickness (achieved by adding or removing YVO4 crystals), is the key component used to switch the EO system into the Heterodyne EO (HEARTS) configuration (when inserted) or the non-heterodyne Phase Diversity configuration (when removed). Then a HWP, rotated to 22.5° , was used to ensure the phase diversity configuration. Finally, a 5 mm PBS was utilized to decompose the modulated signal's polarization state, generating two orthogonal polarization components that were subsequently directed through collimators.

4. Photonic Time stretch :

The fourth part covers the photonic time-stretch technique, which involves slowing down the modulated signal by introducing dispersion using optical fibers. Following the collimators, all subsequent components are interconnected via single-mode (SM) fiber. The dual optical outputs of the EO head are directed through two identical optical circulators from ANYLINK, each with losses below 0.88 dB. These circulators redirect the modulated laser pulses, representing orthogonal polarizations, into a single 8 km FS SMF28 fiber. This fiber, characterized by a dispersion of 18 ps/(nm·km) and an attenuation of 0.16 dB/km, induces a temporal stretch of up to 8 ns, as illustrated in Fig. 6.6a⁴.

To ensure identical temporal stretching and delay for both polarization states, a symmetrical design is employed. However, minor variations in fiber lengths can lead to small temporal delays between the polarizations. These fiber length mismatch are compensated using an AFR variable optical delay line, providing a 0-500 ps adjustment range, within one optical arm. Additionally, a variable optical attenuator is incorporated into the second arm to balance the laser power between the two polarization states, ensuring equal power levels.

Finally, to enable the independent recording of each polarization state, a fiber with a length equivalent to the temporal period of a 78 MHz laser pulse is intro-

⁴Note that an experimental observation of the Dispersive Fourier Transform can be found between the similarity of the laser spectrum, measured with an OSA in Fig. 6.4, and the temporally stretched waveform recorded by an oscilloscope in Fig. 6.6a.

duced into one optical path. This temporal separation allows for distinct data acquisition of each polarization. A clear visualization of this configuration is provided in Figure 6.5.

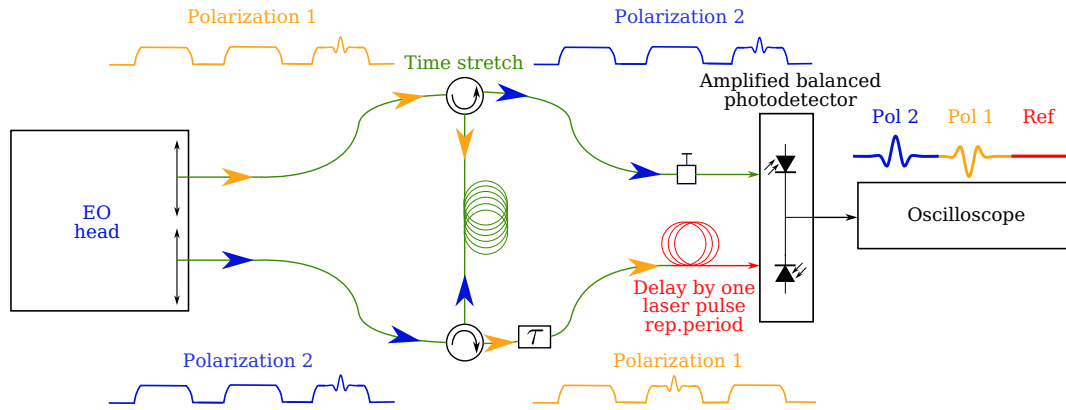


Figure 6.5: **Illustration of the time-stretch configuration in phase diversity technique** showing a fiber-induced delay (laser repetition period) for polarization state separation. Analog balanced detection is then employed to reduce common mode laser contributions.

5. Readouts :

High-bandwidth readout configuration enabling preliminary tests:

For this experiment, initially, a high-bandwidth readout system was utilized to establish a baseline for signal acquisition. This system comprised a Discovery Semiconductors DSC-R412 balanced photoreceiver (20 GHz bandwidth, variable gain ranging from 30 to 2800 V/W, 40 pW/ $\sqrt{\text{Hz}}$ optical noise equivalent power) and a Teledyne Lecroy SDA820Zi-B20 WaveMaster 8 Zi-B oscilloscope (20 GHz bandwidth, 80 GS/s). This configuration allowed for the initial detection of signals through time-stretch methods, providing a familiar starting point before progressing to more complex heterodyne experiments. Furthermore, this setup facilitated a proof-of-principle experiment, enabling the observation of the FEL, the local oscillator, and the resulting downconverted frequency.

Low-bandwidth readout configuration demonstrating bandwidth reduction capabilities of the HEARTS technique:

To demonstrate the bandwidth reduction capabilities of our novel HEARTS technique, a second readout configuration was implemented. This setup employed a InGaAs balanced amplified photodetector (Thorlabs PDB780/AC) (1 MHz to 2.5 GHz bandwidth, 2125 V/W @1310 nm RF output conversion gain, 15 pW/ $\sqrt{\text{Hz}}$ noise equivalent power) and a 4 GHz Rohde & Schwarz oscilloscope. By employing this lower-bandwidth configuration, we aimed to selectively observe and record the downconverted intermediate frequency, thus validating the HEARTS method's potential for reducing the required detector and oscilloscope

bandwidth.

In both experimental setups, the oscilloscopes were dual-triggered, synchronized to both the macropulse signal and the micropulse repetition rate of 13 MHz.

6.3 Experiment System Noise Floor Characterization

This section details the noise floor characterization of the experimental system using the high-bandwidth readouts configuration (20 GHz) in subsection.6.3.1 followed by a comparative analysis of noise floor between balanced and independent measurements in Subsection 6.3.2.

Noise floor definition :

The noise floor defines the system's limit of detection, representing the minimum detectable signal that can be reliably distinguished from background noise. With no THz modulation applied to the laser pulse, in an ideal scenario, the balanced detection signal would be null. However, in practice, residual signals arising from optical component imperfections are observed, as depicted in Fig. 6.6.(b). The noise floor is quantified as a noise equivalent phase shift (radians) or electric field strength (V/m), derived from the ratio of the residual noise to the input laser intensity profile. Notably, the system's sensitivity demonstrates a direct dependence on the input laser power, increasing the power results in an enhanced sensitivity.

6.3.1 Noise floor measurement of the experimental setup using the high-bandwidth readouts (20 GHz)

We characterized the system's noise floor using the high-bandwidth (20 GHz) readout configuration and the 8 km time-stretch fiber, under conditions relevant to subsequent measurements. The measured noise floor was approximately 4.2 mrad⁵ over an 8 ns time window, as illustrated in Fig. 6.6.(c). As a reference, we also evaluated the intrinsic noise floor of the readout system, comprising the balanced detector and oscilloscope, in the absence of both the THz and probe laser signals. This baseline noise floor was determined to be approximately 0.5 mrad (Fig. 6.6.(c)).

⁵Average value over the 8 ns recording window

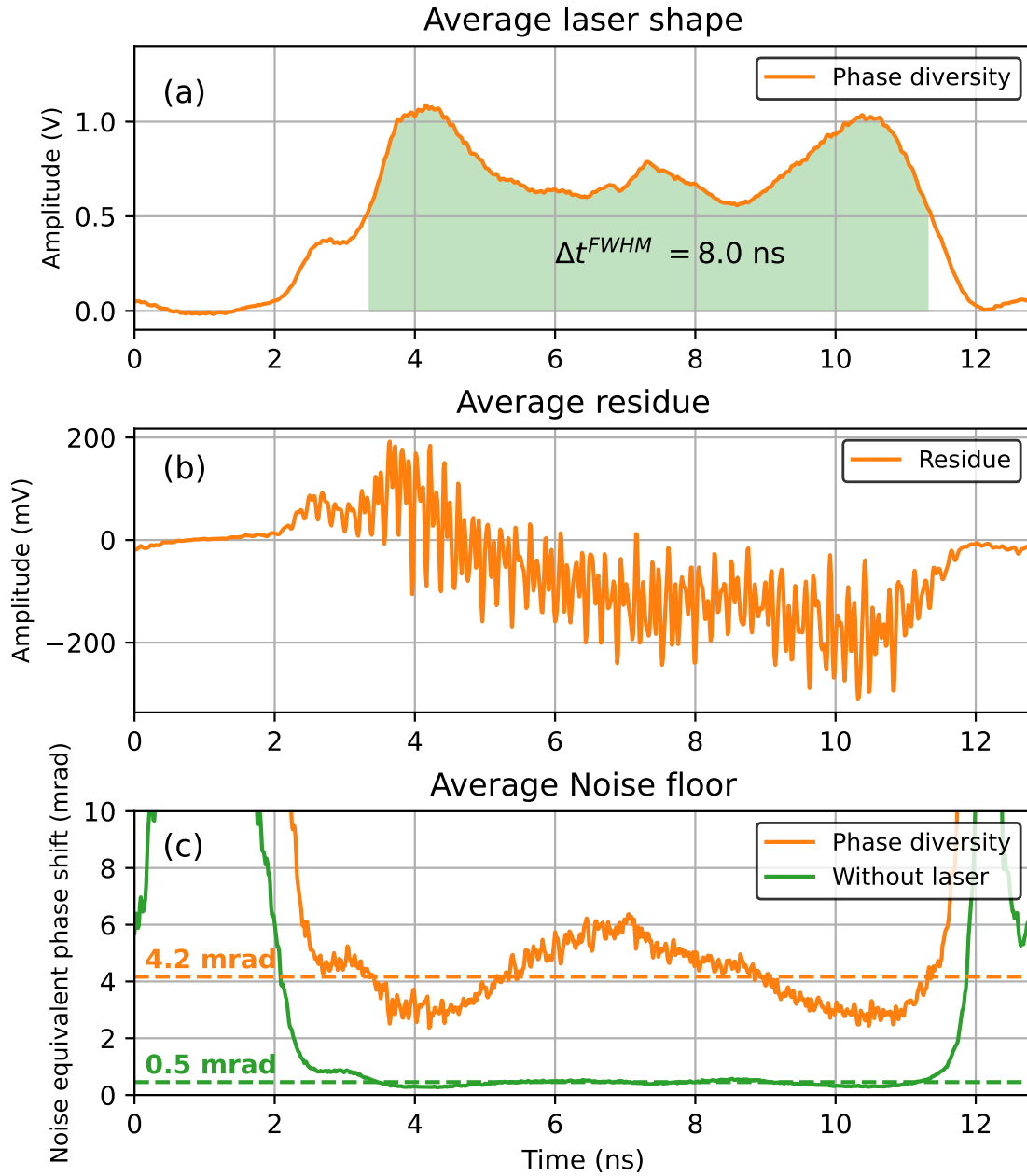


Figure 6.6: Characterization of system noise floor (a) Measured laser pulse shape at a single input of the balanced detector. (b) Residual signal resulting from the balanced detector's output difference, demonstrating optical component imperfections. (c) System sensitivity (orange) measured at approximately 4.2 mrad, and combined noise contribution of the 20 GHz balanced detector and oscilloscope (green) measured at approximately 0.5 mrad, both in the absence of a THz signal.

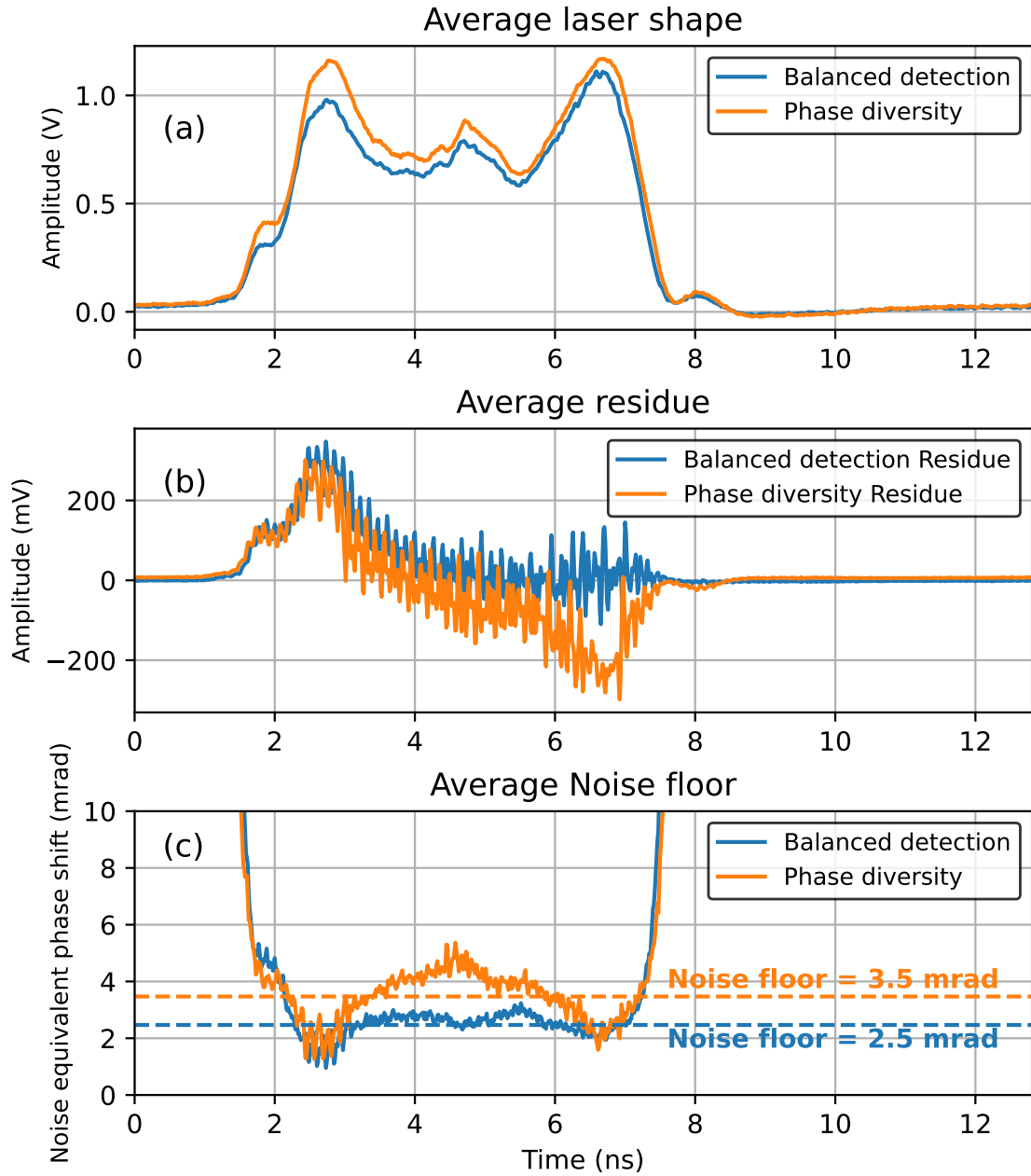


Figure 6.7: **Noise floor comparison between balanced detection (subtracting identical laser pulses) and phase diversity configuration (neighboring pulse subtraction)** (a) Measured laser pulse shape at a single input of the balanced detector. (b) Residual signal resulting from the balanced detector's output difference, demonstrating optical component imperfections, blue: balanced detection configuration, orange: phase diversity configuration. (c) Comparison of the system's sensitivity using balanced detection (subtracting the identical pulses) in blue versus phase diversity (subtracting neighboring laser pulses) in orange.

6.3.2 Noise floor comparison: Independent versus balanced measurements

Unlike balanced detection, which uses identical pulses, phase diversity measurements generate two simultaneous outputs by subtracting neighboring laser pulses, leading to unbalanced signals (see Fig. 6.5). Consequently, the shot-to-shot fluctuations present in each output are not perfectly canceled, leading to an elevated noise floor.⁶ A comparative analysis of the noise floor in balanced detection (subtracting identical laser pulses) and phase diversity measurements (subtracting neighboring laser pulses) is presented in Fig. 6.7. This comparison demonstrates the reduction in noise floor achieved by subtracting identical shot-to-shot fluctuations. As detailed in Chapter 5, we have shown that for the all-optical HEARTS technique in a phase diversity configuration, the two outputs provide equivalent information with opposite signs. This property enables balanced detection, thereby enhancing the signal-to-noise ratio. A compelling experimental evidence of the signal-to-noise ratio difference between independent and balanced signals using EO heterodyne data can be found in Appendix C.

6.4 THz properties, transport and alignment

This section details the specific THz beam properties and the procedures used to achieve optimal spatial overlap and timing synchronization with the 1550 nm probe laser at the electro-optic crystal.

6.4.1 THz frequency choice

For this experiment, a fixed THz wavelength of 87.1 μm , corresponding to a central frequency of 3.44 THz, was selected. The THz signal was generated in macropulse mode with a 13 MHz repetition rate.

The selection of the 3.44 THz frequency was driven by multiple factors, including the optimization of the FEL efficiency and the avoidance of water absorption lines. A key consideration was the trade-off between FEL output power and the required EO crystal thickness. Higher frequencies FEL operation generally produces increased power but necessitate thinner EO crystals to satisfy phase-matching conditions.

⁶These measurements were conducted at the beginning of the experimental shift, utilizing a 5 km time-stretch fiber.

6.4.2 THz beam properties

To ensure optimal electro-optic interaction, the THz beam profile at the EO crystal position was characterized using a pyroelectric camera (Pyrocam).

The measured beam diameter was $D_{4\sigma_x} = 490 \mu\text{m}$ and a $D_{4\sigma_y} = 500 \mu\text{m}$ corresponding in a beam waist of $w_{0x} = 245 \mu\text{m}$ and $w_{0y} = 250 \mu\text{m}$ ⁷.

6.4.3 Alignment and focus of the THz and laser probe on the EO crystal

Initial transverse alignment of the terahertz beam with the 1550 nm laser was achieved using a visible 532 nm alignment laser. This alignment laser from the FELBE beamline was previously aligned to the THz beam path. Fine transverse alignment was then performed by replacing the GaAs EO crystal with a $400 \mu\text{m}$ pinhole and superimposing the 1550 nm and 532 nm laser beams. This ensured a spatial overlap of the beams at the interaction point as illustrated in Fig. 6.8.

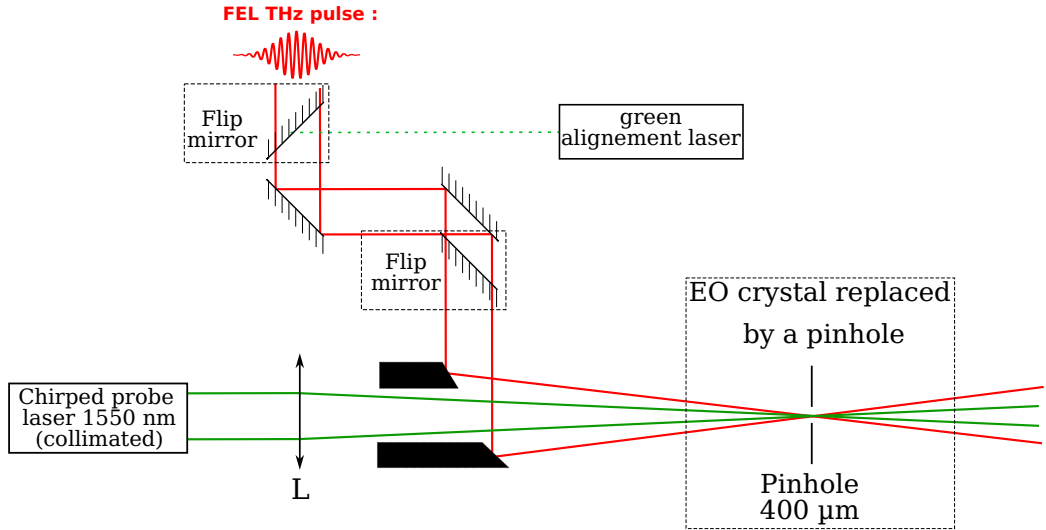


Figure 6.8: **Illustration of the precise spatial alignment of the terahertz and 1550 nm laser beams at the interaction point.** Alignment is achieved by replacing the electro-optic crystal with a $400 \mu\text{m}$ pinhole.

⁷ $w_0 = D_{4\sigma}/2$

6.4.4 Temporal synchronization of THz and 1550 nm laser

The temporal delay between the THz and 1550 nm laser pulses was controlled to ensure their optimal overlap at the electro-optic crystal.

To achieve optimal temporal synchronization (see Figure.6.9), the temporal delay between the THz and 1550 nm laser pulses was monitored using a fast THz detector (a field-effect transistor (FET)) and a 5 GHz InGaAs high-speed photodetector (Thorlabs DET08CFC).

The relative timing was adjusted by a variable delay to the radio frequency (RF) reference signal supplied to the laser's repetition rate synchronization system. This delay was implemented using a mechanical RF delay line (ARRA 2448), which was used to scan over 1 ns with 7 ps step, ensuring the entire THz signal was accurately centered within the 1550 nm laser pulse's temporal window.

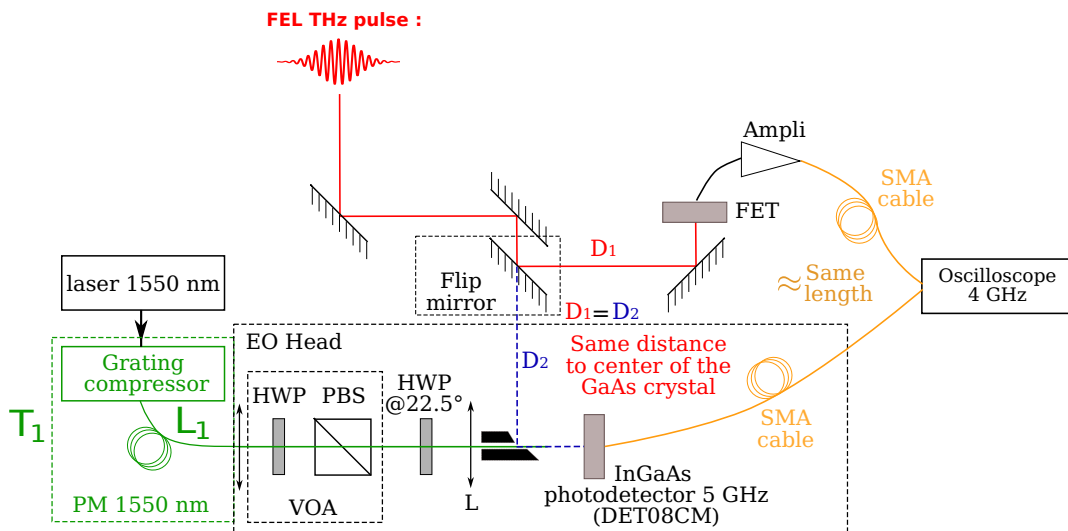


Figure 6.9: Temporal synchronization setup. The 1550 nm laser pulses were detected with a 5 GHz InGaAs high-speed photodetector (Thorlabs DET08CFC), and the THz pulses were detected using a field-effect transistor (FET) THz detector. To ensure accurate timing, the respective distances between the FET detector and the original EO crystal position (now occupied by the InGaAs photodetector), as well as the SMA cable lengths. Both signals were then displayed on a 4 GHz oscilloscope. Temporal adjustments were made using an RF delay line (ARRA 2448).

6.5 Preliminary experiment :

Diversity electro-optic detection measurement using photonic time stretch

This section details the preliminary THz measurements using the diversity electro-optic sampling (without heterodyne) combined with the photonic time stretch technique and the high-bandwidth readouts configuration (20 GHz).

Contextually, similar EO measurements employing the time-stretch method had previously been conducted at 1 μm [129] (e.g., by C. Hannoun et al.). However, transposing this detection scheme to a 1550 nm probe wavelength introduces several unknowns, particularly regarding phonon absorption and the bandwidth characteristics of the Gallium Arsenide (GaAs) crystal at the frequency of 3.44 THz.

To ensure the observability of the FEL THz signal, this first investigation using the DEOS setup was an essential step before proceeding to the more complex HEARTS technique with our 1550 nm system.

Several GaAs crystal thicknesses (including 500 μm and 100 μm) were tested. Both thicknesses were found to be suitable for THz signal detection. Therefore, this section presents results obtained using the 100 μm thick GaAs EO crystal.

6.5.1 Recording window and theoretical stretch factor M

1. Recording window :

We opted for a recording window T_1 of 40 ps⁸, achieved through the grating compressor (equivalent to 18 meters of PM1550 fiber at its maximum setting) and an additional 17 meters of PM1550 fiber, resulting in a total effective fiber length $L_1 = 35$ m. This temporal window has been chosen to allow the full THz pulse to be recorded.

2. Theoretical stretch factor M :

With the total effective fiber length $L_1 = 35$ m, along with an SMF28 stretch fiber of $L_2 = 8$ km, the theoretical stretch factor M is calculated to be 228. Consequently, the stretched FEL frequency observed on the oscilloscope is expected to be approximately 15 GHz. To detect this frequency within the phase diversity configuration, we employed a 20 GHz balanced photodetector connected to a 20 GHz oscilloscope for signal acquisition.

⁸Previous EO time-stretch measurements at 1 μm required at least twice the expected FEL pulse duration [129].

6.5.2 Phase diversity dual-output

Figures 6.10(a) and (b) present the single-shot measurements of the two orthogonal output polarization states, Y_1 and Y_2 for a single emitted monochromatic 3.44 THz pulse. The spectra demonstrate a narrowband response centered at a peak frequency of 3.44 THz, corresponding to a 15.13 GHz peak frequency on the oscilloscope. Based on these measurements, the experimental stretch factor M is determined to be 227, which is consistent with our theoretical prediction of 228.

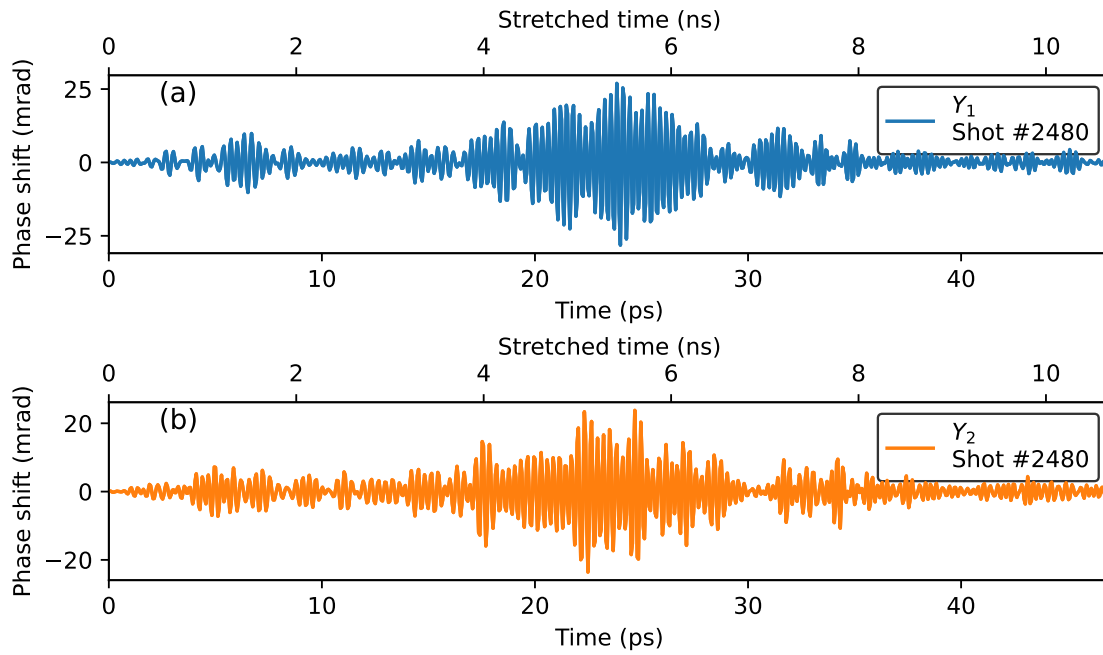


Figure 6.10: **Single shot temporal profiles of two measured polarizations** corresponding to the phase shift induced by the FEL THz signal of one typical single shot signal, here the 2480 shot of Fig.6.12.

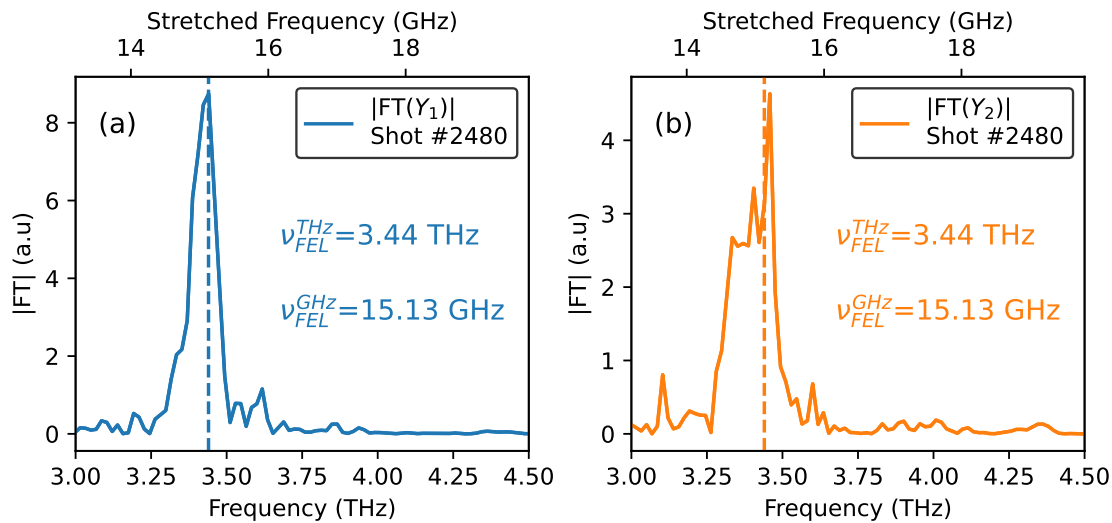


Figure 6.11: **Single shot spectra of the two temporal profile** represented in Figure.6.10.

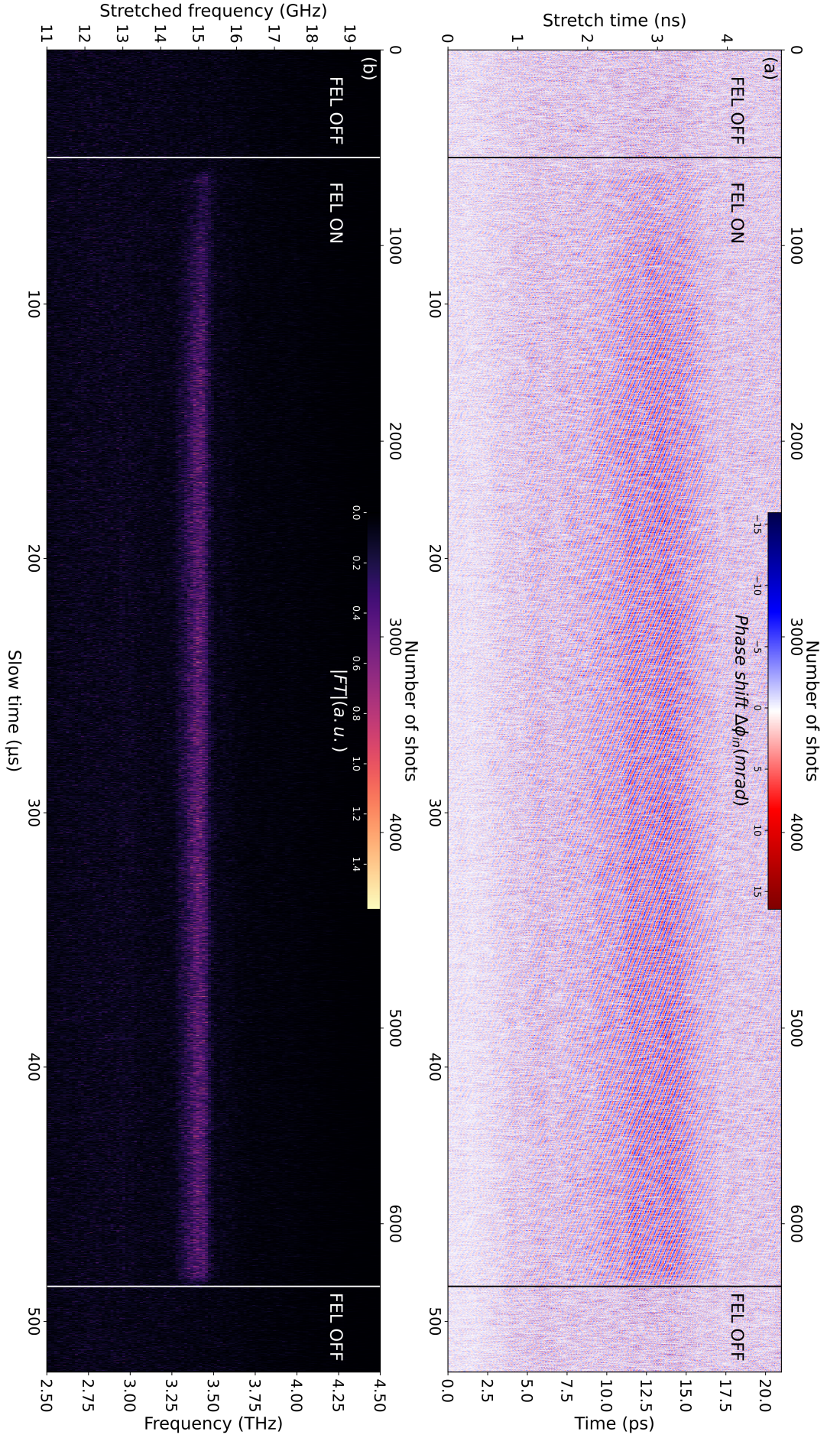


Figure 6.12: **Complete FEL macropulse temporal profile and spectrum of Y_1** : (a) Temporal evolution of the macropulse exhibiting both the turn-on and turn-off. The carrier-envelope phase drift is visible within this evolution. The actual values are on the order of ± 25 mrad, as shown in Fig.6.10. (b) Spectrum of the macropulse, indicating a stable output at 3.44 THz.

6.5.3 FEL macropulse in phase diversity configuration

The complete FEL macropulse, captured in a single acquisition, is shown in Figure.6.12. This figure provides the macropulse's temporal evolution, including its turn-on and turn-off characteristics. The temporal profile in Figure 6.12.(a) reveals the carrier-envelope phase drift, while Figure.6.12.(b) shows a stable frequency of 3.44 THz throughout the macropulse.

Notes on experimental data analysis: Noise reduction and temporal recalibration

The balanced detector signal is composed of waveform affected by the electro-optic modulation (at 13 MHz) and waveform that are not affected. The first step of the analysis consist in performing a subtraction between these waveforms, in order to reduce the low-frequency laser common noise. Following this step, the signals are normalized by the average laser shape without THz to yield the dual-outputs equivalent phase-shifts, Y_1 and Y_2 , expressed in radiant.

A detailed explanation of the data treatment is provided in the Appendix.C.

The data analysis also involved resampling the recorded waveforms based on a time calibration to compensate for the higher-order dispersion introduced by the 8 km stretch fiber.

A detailed explanation of the temporal correction algorithm is provided in Appendix B.

Noise reduction and temporal calibration were applied systematically to all data presented in this chapter, including to the HEARTS data in Section.6.6.

This preliminary experiment using the DEOS system (without heterodyne) combined with photonic time-stretch confirmed the observability of the THz signal, employing a 1550 nm probe laser and a 100 μm thick GaAs crystal.

6.6 Experimental Demonstration of the All-optical Heterodyne Electro-optic Analysis and Reconstruction of THz Signals (HEARTS) Technique

Building upon the preliminary experiment, this section details the implementation and results of the all-optical Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS) detection technique obtained in November 2024.

The implementation began with the generation of the THz optical local oscillator (LO), accomplished via a Lyot filter composed of birefringent YVO_4 crystals. By beating the LO with the FEL signal, we observed the first downshifted FEL signal. This measurement demonstrated the system's primary advantage by enabling distortion-free downconverted signal acquisition using a low-bandwidth readout system (a 2.5 GHz photodetector and a 4 GHz oscilloscope).

Finally, to highlight the direct application of the HEARTS method, the section concludes with a study investigating the relationship between cavity detuning and the carrier-envelope phase drift. This demonstrates the capability to diagnose and monitor the CEP stability using the HEARTS method combined with a low-bandwidth readout.

6.6.1 Generation of the THz optical local oscillator via a Lyot Filter

The THz optical local oscillator is generated using a Lyot filter, as detailed in Chapter 5. To experimentally implement the Lyot filter for LO generation, it is essential to select an appropriate birefringent material. Several birefringent materials are available, each offering a distinct degree of birefringence Δn , as summarized in Table 6.3.

Birefringent material	Δn
PM fiber	0.0004
Quartz	0.008353
CaCO_3	0.1572
YVO_4	0.203

Table 6.3: Birefringence of different materials

One potential solution for generating a high-frequency THz local oscillator involves the use of a polarization-maintaining (PM) fiber. Despite its weak birefringence ($\Delta n = 4.10^{-4}$ at 1550 nm), a significant phase delay between the orthogonal polarizations can be achieved over several meters, potentially leading to multi-THz oscillation. However, employing a PM fiber would undesirably alter both the recording window duration and the temporal delay between the THz and probe pulses.

Another solution involves using a birefringent crystal. For this experiment, we selected YVO4 due to its highest birefringence ($\Delta n = 0.203$) among the available options, which also included Quartz and CaCO3. We selected and purchased several YVO4 crystals, each with a length of 9.6 mm. These crystals can be placed in series to further increase the total length and, consequently, enhance the phase delay between the horizontal and vertical polarization components of the laser probe.

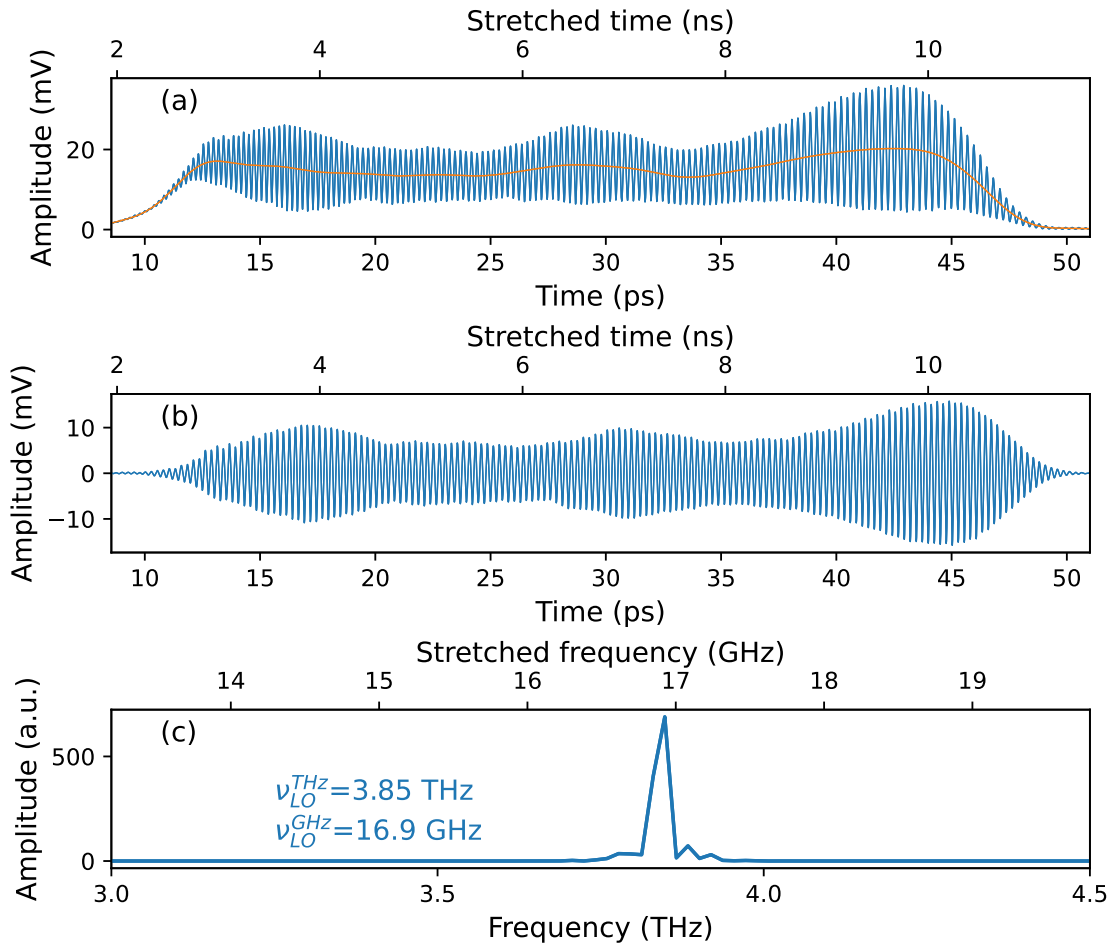


Figure 6.13: THz optical local oscillator generated by the Lyot filter. (a) Laser pulse shape without (orange) and with (blue) modulation. (b) Isolated THz modulation (i.e, the difference between the laser pulse shape with and without modulation). (c) Fourier transform of the THz modulation revealing a 3.85 THz modulation (16.9 GHz after photonic time stretch).

The YVO4 crystals were positioned immediately after the second lens following the EO crystal, as depicted in Fig.6.3. Their birefringent axes were aligned with those of the EO crystal. Furthermore, these crystals were easily removable, allowing for a straightforward switch between DEOS and the all-optical HEARTS measurement configurations.

We opted for three YVO4 birefringent crystals. This configuration was chosen to generate a 3.85 THz modulation, corresponding to 16.9 GHz as shown in Fig.6.13, which represents a downconverted frequency of 1.7 GHz matching perfectly the photodetector bandwidth (2.6 GHz).

6.6.2 Experimental results from the all-optical HEARTS setup in phase diversity configuration

This subsection details the experimental results obtained using the all-optical HEARTS setup in phase diversity configuration.

By mixing the FEL signal at 3.44 THz (15.13 GHz) with the optical local oscillator at 3.85 THz (16.9 GHz), a downshifted replica of the FEL spectrum is obtained at an intermediate frequency of 0.41 THz (1.76 GHz), enabling detection with the lower bandwidth readouts configuration (2.5 GHz photodetector and a 4 GHz oscilloscope).

A 500 μm thick GaAs crystal was selected as the EO crystal for this experiment, offering a suitable balance between SNR and temporal resolution.

6.6.2.1 Single-Shot Measurements of the Downshifted FEL Signal using a 2.5 GHz Photodetector and a 4 GHz Oscilloscope

1. Measurements of the Two Downshifted FEL Signal Outputs:

Figure.6.14.(a) presents a single-shot measurement of the two orthogonal polarization components of the downshifted FEL signal, denoted as Y_1^{IF} and Y_2^{IF} .

As predicted by the theoretical framework in Chapter.5 in Subsection. 5.3.3.1, these two outputs exhibit similar temporal profiles free from transfer function distortions. Furthermore, the two outputs have opposite signs, enabling balanced detection that will effectively cancel common-mode noise⁹.

The corresponding spectra of these single-shot temporal profiles, are shown in Figure.6.14.(b), each present a single peak at 0.39 THz (1.72 GHz).

The absence of spectral zeros further confirms the distortion-free nature of the HEARTS method in phase diversity configuration.

⁹Data obtained with balanced detection on the same laser pulse, including the difference signal $Y^{BD} = Y_1^{IF} - Y_2^{IF}$, are presented in Appendix.C.

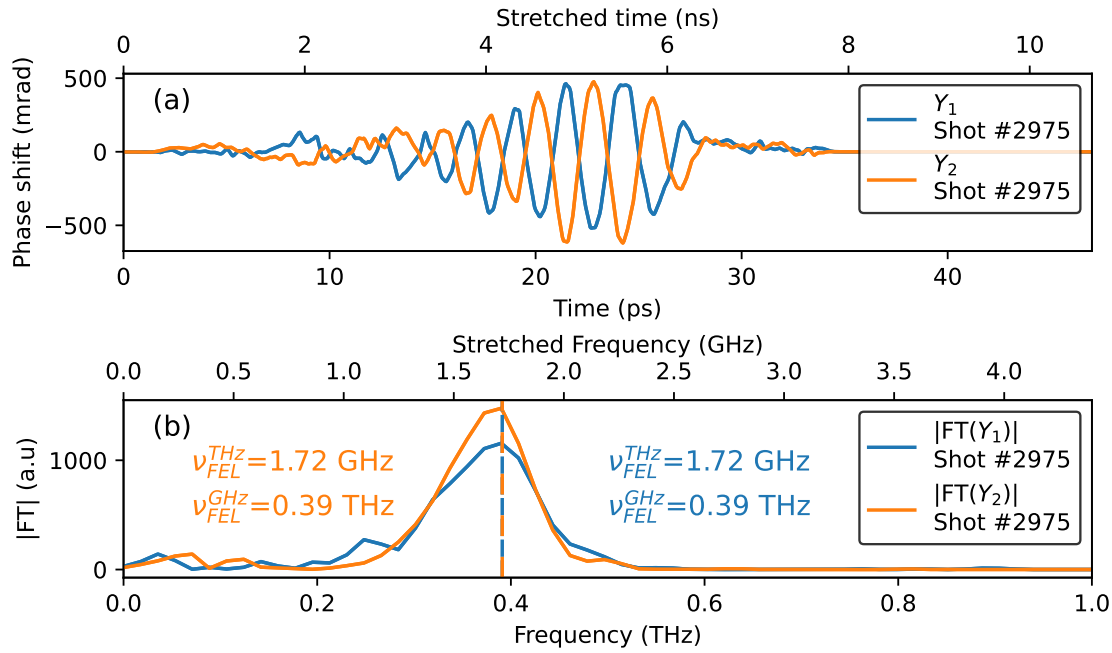


Figure 6.14: Single-Shot Down-Conversion and Distortion-Free Acquisition of THz Signal using the HEARTS Method in Phase Diversity Configuration. (a) Single-shot temporal profiles of the two downshifted orthogonal polarization components Y_1^{IF} and Y_2^{IF} resulting from the heterodyne mixing of the FEL signal (15.13 GHz) and the optical local oscillator (16.9 GHz), enabling their acquisition with a 2.5 GHz balanced photodetector and a 4 GHz oscilloscope. (b) Corresponding spectra of the single-shot temporal profiles, exhibiting an intermediate frequency peak at 1.72 GHz (equivalent to 0.39 THz) without zeros confirming the distortion-free nature of the heterodyne phase diversity configuration.

2. Downconverted FEL macropulse characterization :

The complete temporal and spectral evolution of the downconverted FEL macropulse, acquired using our low-bandwidth detection system, is presented in Figure.6.15.(a) and (b). The temporal profile of the macropulse in Fig.6.15.(a) clearly reveals the CEP drift. The spectral profile in Fig.6.15.(b) demonstrates a stable central frequency at 1.72 GHz throughout the macropulse.

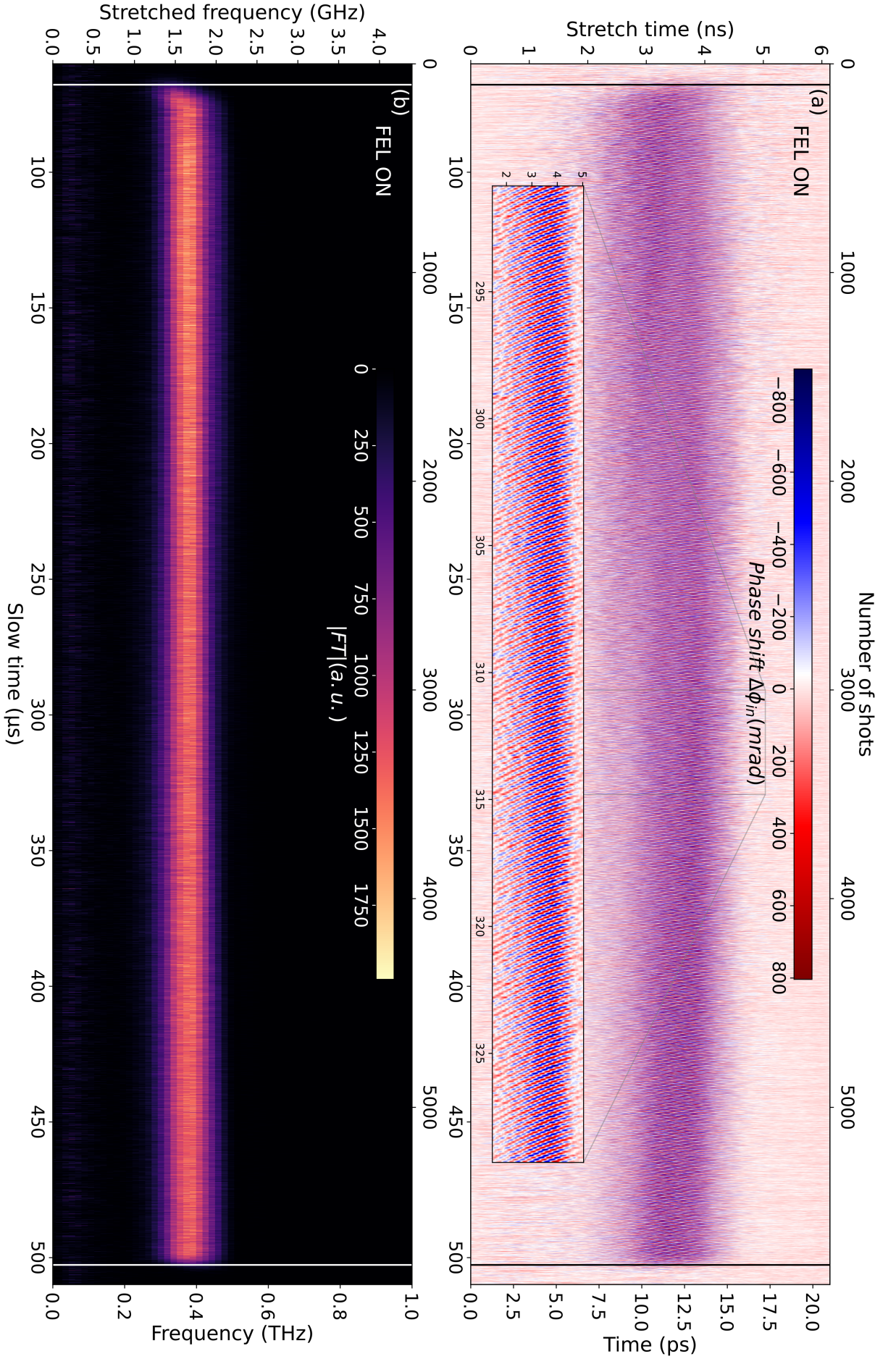


Figure 6.15: **Characterization of the complete downconverted FEL macropulse.** (a) Temporal evolution Y_1^{IF} of illustrating the macropulse turn-on and turn-off, with the carrier-envelope phase drift clearly visible. (b) Spectrum of the macropulse, showing a stable intermediate frequency output at 0.39 THz resulting from the beating of the 3.44 THz FEL and the 3.85 THz local oscillator.

Brief summary: HEARTS Heterodyne Down-Conversion Demonstration

The first experimental demonstration of the all-optical HEARTS technique proved the heterodyne down-conversion of a multicycle THz signal to a lower intermediate frequency, that is temporally stretch down to few GHz. As illustrated below, the initial FEL THz signal (a) at $\nu_{\text{FEL}} = 3.44$ THz was mixed with an optically generated Local Oscillator (b) at $\nu_{\text{LO}} = 3.85$ THz. This process resulted in an intermediate frequency signal (c) at $\nu_{\text{IF}} = 0.39$ THz, thus validating the heterodyne principle $\nu_{\text{IF}} = |\nu_{\text{LO}} - \nu_{\text{FEL}}|$. This down-conversion reduced the required electronic readout bandwidth from 15.13 GHz to 1.72 GHz^a after PTS, confirming HEARTS as an efficient, low-bandwidth, single-shot diagnostic for narrowband THz sources.

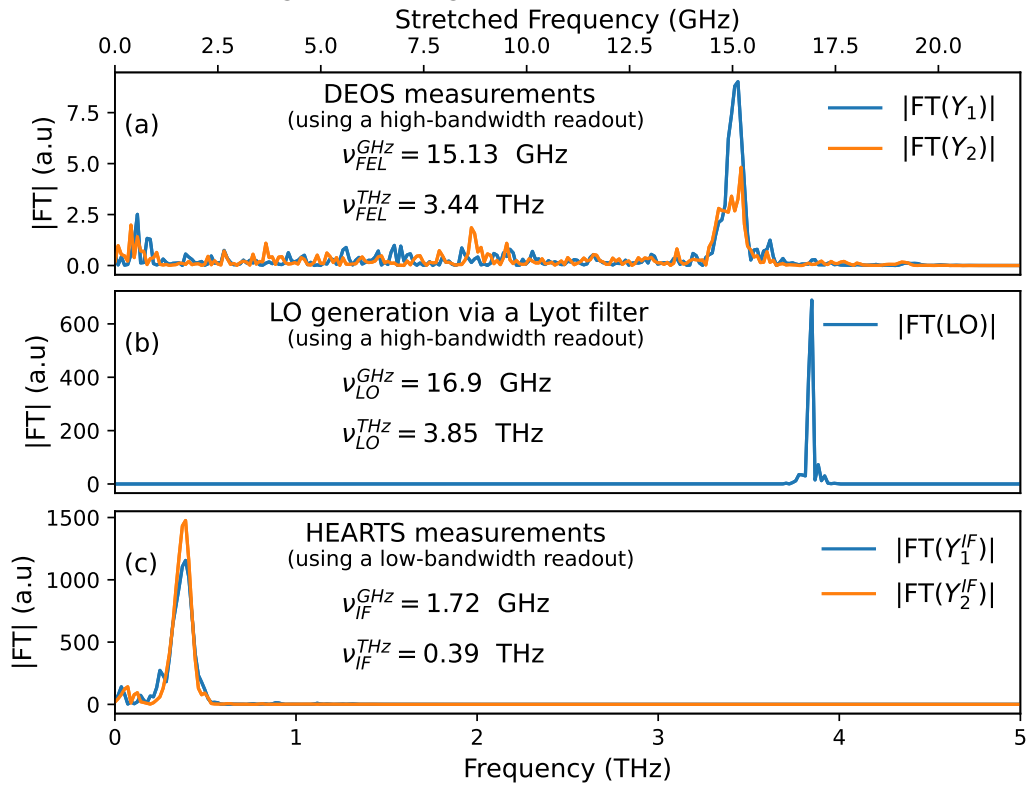


Figure 6.16: Spectra demonstrating the heterodyne down-conversion principle of HEARTS. (a) The initial FEL THz signal measured using the DEOS with Photonic time-stretch configuration, showing the FEL spectral components at a stretched frequency $\nu_{\text{FEL}}^{\text{GHz}} = 15.13$ GHz ($\nu_{\text{FEL}}^{\text{THz}} = 3.44$ THz). (b) The spectrum of the Local Oscillator (LO) generated via the Lyot filter, peaked at $\nu_{\text{LO}}^{\text{GHz}} = 16.9$ GHz ($\nu_{\text{LO}}^{\text{THz}} = 3.85$ THz). (c) The resulting Intermediate Frequency (IF) signal spectrum after down-conversion, showing a frequency shift to a lower frequency $\nu_{\text{IF}}^{\text{GHz}} = 1.72$ GHz ($\nu_{\text{IF}}^{\text{THz}} = 0.39$ THz), successfully demonstrating the heterodyne principle. Note that this down-converted signal have no distortion resulting from the system's transfer function compared to (a).

^aThis result in an hetero-stretch factor $M_{\text{HS}} = \frac{3.44 \text{ THz}}{1.72 \text{ GHz}} = 2000$

6.6.3 Measurements of the CEP drift

To investigate the terahertz emission characteristics as a function of cavity detuning, we conducted a study by varying the position of one of the two cavity mirrors. This section will demonstrate that the HEARTS detection setup in a phase diversity configuration provides CEP drift and temporal profile measurements using a low-bandwidth readouts. The mirror position was scanned from 8.234 mm to 8.316 mm. The measured THz power, acquired using a THz powermeter, is presented in Figure 6.17 as a function of the mirror position. This detuning curve reveals a clear dependence of the FEL emitted power on the cavity length, with an optimal efficiency observed on the left side of the curve.

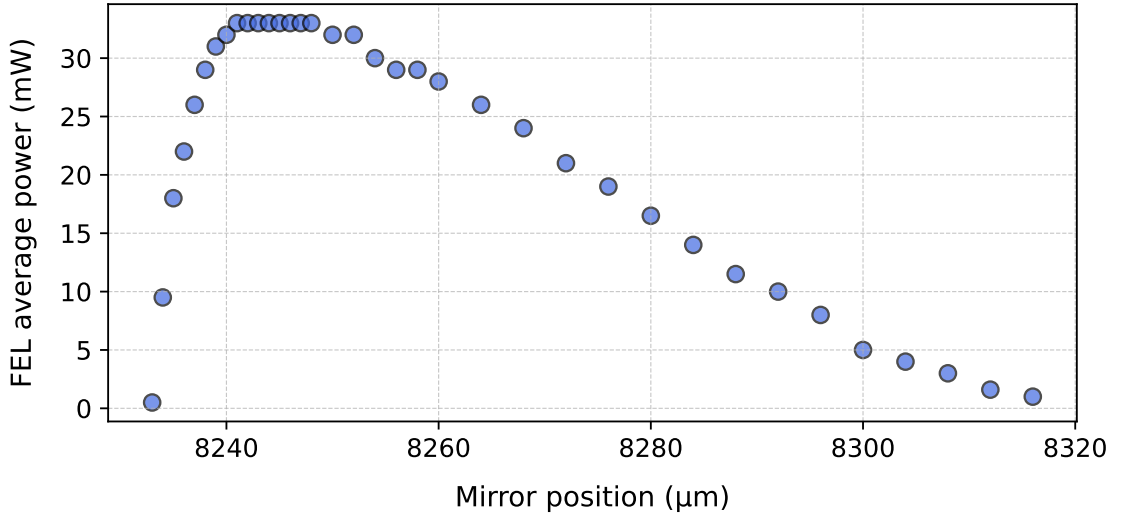


Figure 6.17: **FEL emitted power as a function of the cavity mirror position** : The detuning curve shows the dependence of the THz power on the cavity length.

To analyze the CEP drift, we performed a two-dimensional Fourier transform $\tilde{E}_{EO}(\nu_{drift}, \nu_{GHz})$ of the single-shot EO measured signals $E_{EO}(n, t)$ over n round-trips, according to the following relationship:

$$E_{EO}(n, t) \Rightarrow \tilde{E}_{EO}(\nu_{drift}, \nu_{GHz}) \quad (6.1)$$

Here, ν_{drift} represents the frequency associated with $n\tau_R$, (the time over n round-trips) and corresponds to the THz repetition rate. This frequency quantifies the speed at which the FEL pulses drift from one pulse to the next, effectively describing the CEP drift speed. Meanwhile, ν_{GHz} denotes the frequency relative to the stretch time t .

Increasing the mirror position resulted in a periodic behavior of the CEP drift speed. This periodicity, with a period of $43.6 \mu\text{m}$, corresponds to half the wavelength of the 3.44 THz FEL frequency, as illustrated in Figure 6.18.(a). This plots shows that the CEP frequency drift cross zero points $\nu_{drift} = 0$. This observation confirms the potential for stabilizing the FEL emission, for instance, by implementing a feedback loop system.

To further highlight this periodic behavior, we present several EO HEARTS measurements over a $20 \mu\text{s}$ duration that exhibit different CEP drift according to different mirror's position. The data displayed in Figures 6.18.(b) and (d) were acquired at mirror positions of 8.252 mm and 8.296 mm , which represent a separation of $44 \mu\text{m}$, approximately half the wavelength period. As expected, these two datasets exhibit similar temporal profiles. The corresponding two-dimensional Fourier transforms, shown in Figures 6.18.(c) and (e), reveal an equivalent CEP drift.¹⁰

The three green points highlighted in Figure 6.18.(a) indicate the passage through a zero drift condition. The mirror positions of $8.256 \mu\text{m}$, $8.260 \mu\text{m}$, and $8.264 \mu\text{m}$ correspond to positive, near-zero, and negative drift, respectively. This progression is clearly demonstrated by the temporal profiles in Figures 6.19.(a), (c), and (e) and is further confirmed by the Fourier transforms in Figures 6.19.(b), (d), and (f), which show the spectral drift evolving through a zero-drift point.

¹⁰The decrease in THz power was observed at the 8.296 mm mirror position, leading to a reduced phase shift $\Delta\phi_{in}$ relative to the 8.252 mm measurement, which is consistent with the measured average FEL power shown in Figure 6.17.

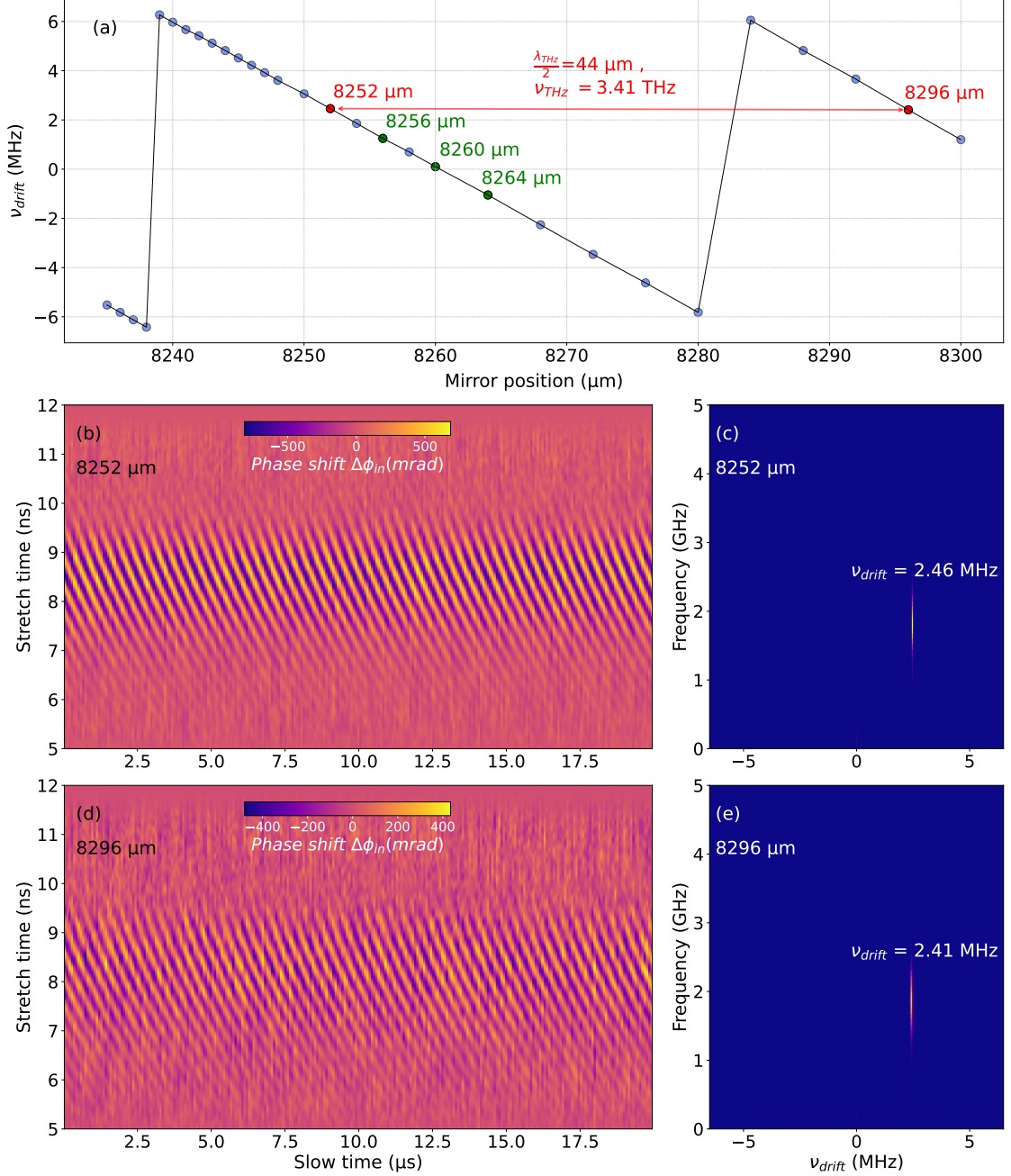


Figure 6.18: **CEP drift as a function of the cavity mirror position and its measurement using EO HEARTS detection** : (a) Periodic variation of the CEP drift frequency with mirror position, the period being half the 3.44 THz FEL wavelength. (b) and (d) show temporal EO signals at two mirror positions separated by approximately half a wavelength. (c) and (e) are the 2D Fourier transforms of (b) and (d), indicating a consistent CEP drift frequency.

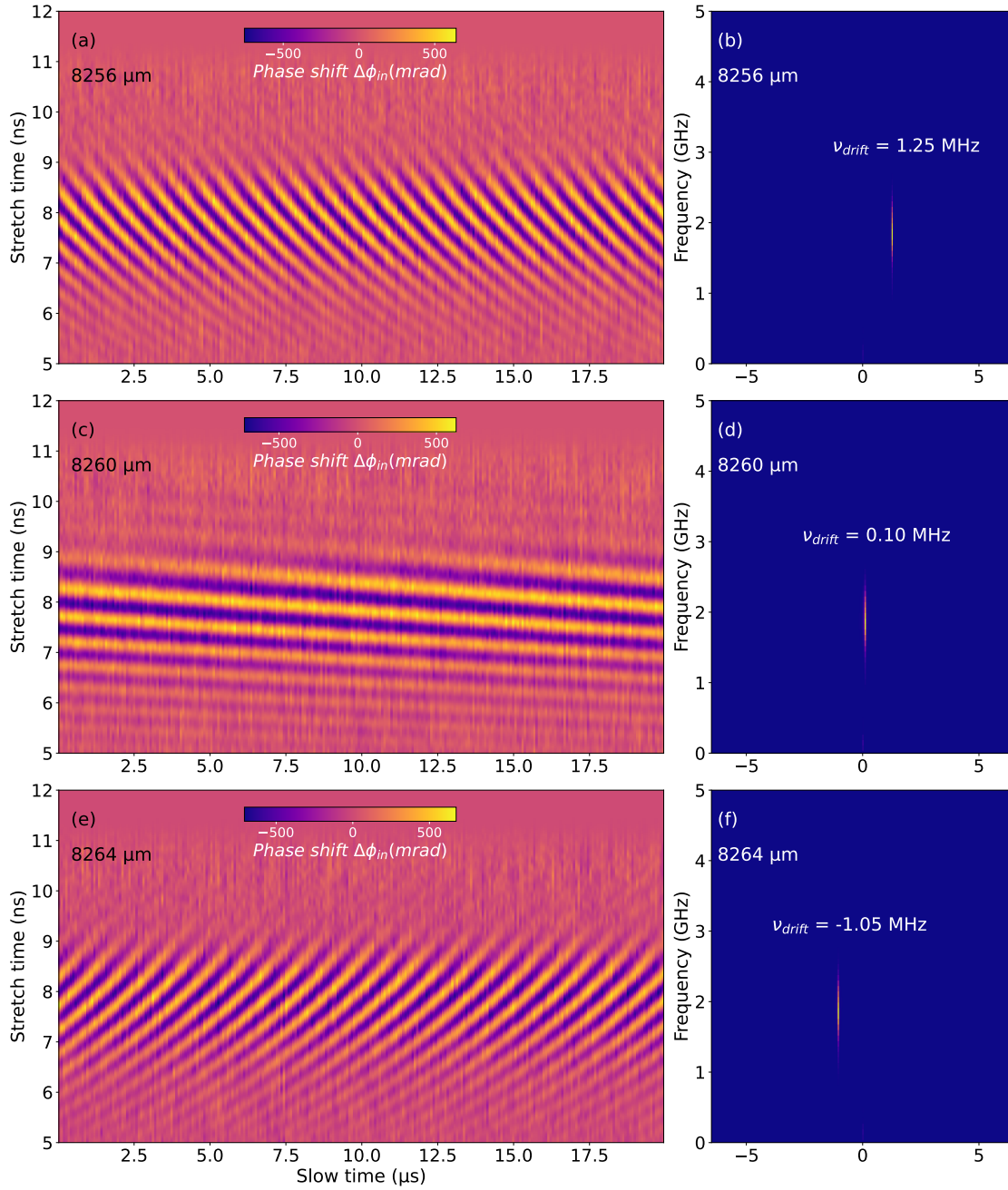


Figure 6.19: **Visualization of the CEP drift passing through zero** : (a), (c), and (e) present the temporal profiles of the drift for three specific mirror positions: 8.256 μm (positive drift), 8.260 μm (near-zero drift), and 8.264 μm (negative drift), indicated by green points in Figure 6.18.(a). The corresponding 2D Fourier transforms in (b), (d), and (f) confirm this observation by exhibiting the transition through zero CEP drift.

6.7 Chapter summary and Perspectives

Chapter summary: Experimental Demonstration of the HEARTS technique at FELBE

This chapter details the experimental demonstration and implementation of the novel Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS) technique at the Free-Electron Laser in Dresden (FELBE). The experimental campaign at FELBE focused on characterizing the narrowband THz radiation at 3.44 THz, employing an experimental setup with a 1550 nm chirped probe laser and a GaAs EO crystal.

A preliminary measurement utilizing the DEOS setup with photonic time-stretch and a high-bandwidth readout, first confirmed the observability of the THz signal at this frequency.

The HEARTS technique was then demonstrated in its phase diversity configuration. This implementation used a Lyot filter with YVO_4 crystals to optically generate a local oscillator at 3.85 THz, successfully down-converting the FEL signal to an intermediate frequency of 0.39 THz. This down-converted signal was then recorded using a low-bandwidth acquisition system (2.5 GHz photodetector and 4 GHz oscilloscope), where it appeared at 1.72 GHz after the photonic time-stretch. This result validates the core principle of HEARTS, which is to capture multicycle narrowband THz pulses without requiring high-bandwidth electronics, while confirming that no distortions are introduced by the system's transfer function.

The down-converted signal enabled the observation of the FEL macropulse's full temporal profile and dynamics, including the Carrier-Envelope Phase (CEP) drift. A CEP study, made as a function of the cavity mirror position, successfully identified the optimal position for zero CEP drift. This finding lays the foundation for future real-time CEP stabilization systems.

In conclusion, this work fully validates the 1550 nm all-optical HEARTS system as a single-shot technique capable of simultaneously characterizing multicycle THz pulses and providing the CEP diagnostics necessary for sorting data from THz pump-probe experiments at FELBE.

Immediate and Future Perspectives:

An immediate perspective is the realization of a future real-time diagnostic combined with a feedback loop to stabilize the Carrier-Envelope Phase drift of the FELBE source. This can be achieved by utilizing the HEARTS setup with a low-bandwidth data acquisition system to continuously monitor the CEP and dynamically adjust the FEL cavity mirror position.

A future perspective addresses the recent developments in THz accelerator-based sources are pushing the field toward the generation of high-frequency narrowband THz radiation [19–25] (up to 30 THz, with 60 THz already achieved at FELBE). Therefore, future THz diagnostics must focus on enhancing the operational frequency range by addressing existing limitations.

Firstly, to overcome the constraint imposed by the probe laser's insufficient bandwidth, spectral broadening techniques [56, 57, 130, 131] must be implemented to enable the capture of higher-frequency THz components. Secondly, pushing detection towards higher THz frequencies requires mitigating the limitations of GaAs phonon absorption and intrinsic bandwidth. This necessitates investigating alternative electro-optic materials, such as organic crystals [132], or utilizing the Air-Biased-Coherent-Detection (ABCD) technique in free space [133].

General Conclusion

*No phenomenon is a real
phenomenon until it is an observed
phenomenon.*

– John Archibald Wheeler

This thesis addressed current challenges in single-shot electro-optic diagnostics for both broadband and narrowband terahertz electric fields, providing new tools for the characterization and optimization of accelerator-based light sources.

For broadband THz signal measurement in Part. II, a novel reconstruction algorithm for the Diversity Electro-Optic Sampling (DEOS) technique called the Self-Adaptive Dispersion (SAD) algorithm was developed. This SAD algorithm overcome the THz reconstruction fidelity issues caused by nonlinear dispersion within the chirped probe laser pulse (Chapter. 3). This algorithm, which moves beyond the limiting linear model, accurately retrieves both the THz signal and the full dispersion properties of the chirped probe laser. Its practical utility was demonstrated through integration with the DEOS system at the EuXFEL and FLASH facilities, enabling high-fidelity, single-shot, MHz-rate electron bunch diagnostics (Chapter. 4).

To tackle the separate challenge of characterizing narrowband, multicycle THz pulses, the Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS) method was introduced in Part. III. This novel approach integrates all-optical heterodyning with the Photonic Time-Stretch technique to resolve the inefficient use of high-bandwidth electronic readouts (Chapter. 5). The HEARTS method down-converted the high-frequency THz signal to a lower intermediate frequency, demonstrating the ability to record the complete field with an affordable, low-bandwidth acquisition system. This capability was successfully applied to perform single-shot Carrier-Envelope Phase (CEP) diagnostics on the FELBE source, confirming its immediate value for accelerator diagnostics and laying the groundwork for real-time feedback control of the CEP (Chapter. 6).

General Perspectives and Future Work

*The future cannot be predicted, but
futures can be invented.*

– Dennis Gabor

The ensuing perspectives detail the immediate and long-term future work required to fully optimize the developed methods in thesis.

To improve the broadband characterization system at DESY, the SAD reconstruction algorithm should be implemented in real-time, complete with a user-friendly interface within the DESY's control room. This will ensure that accelerator specialists, even those unfamiliar with EO diagnostics, can easily access and view the reconstructed longitudinal profile of the electron bunch. Furthermore at EuXFEL and FLASH, the temporal resolution could be significantly enhanced by replacing the current electro-optic crystal with a thinner, 500 μm GaP crystal, enabling measurements of electron bunches as short as 160 fs RMS. Addressing artifacts by correcting the ellipticity induced by metallic mirrors, through the use of periscopes, is also a necessary hardware upgrade.

For narrowband diagnostics, an immediate perspective is the realization of a real-time diagnostic to provide the CEP values necessary for sorting data from THz pump-probe experiments at FELBE. Ultimately, this system could be combined with a feedback loop to dynamically adjust the FEL cavity mirrors and stabilize the Carrier-Envelope Phase drift.

Motivated by the requirements of several THz facilities, including STERN, PITZ and FELBE (along with its future upgrade, DALI), a common perspectives and challenge of both broadband and narrowband EO diagnostics is extending the operational range toward 30 THz [19–25]. Overcoming the constraint of the probe laser's insufficient bandwidth necessitates implementing spectral broadening techniques [56, 57, 130, 131] to ensure the capture of higher-frequency THz components. Finally, the intrinsic material limitations of GaAs phonon absorption must be mitigated by investigating alternative electro-optic crystals like organic materials [132] or by utilizing the free-space Air-Biased-Coherent-Detection (ABCD) technique [133].

Derivation of the frequency modulation of a Lyot filter

This appendix provides the complete derivation for the time-domain modulation frequency ν_{LO}^{THz} imprinted onto a chirped probe laser via a Lyot filter. This frequency acts as the local oscillator for the all-optical HEARTS¹ detection principle discussed in Chapter.5. The derivation links the Lyot filter's spectral response $\Delta\phi_{Lyot}(\lambda)$ to the equivalent time-domain modulation $\Delta\phi_{Lyot}(t = \frac{-\omega}{C})$ via the properties of the initial dispersive element (i.e, pair of grating, fibers).

A.1 Lyot Filter Response in the Optical Frequency Domain

The Lyot filter imposes a wavelength-dependent phase delay $\Delta\phi_{Lyot}(\lambda)$ between the ordinary and extraordinary polarization components of the chirped probe pulse. This phase delay and the resulting intensity modulation $I(\lambda)$ are initially expressed in the wavelength domain λ :

$$\begin{aligned}\Delta\phi_{Lyot}(\lambda) &= \frac{2\pi}{\lambda} \Delta ne \\ I(\lambda) &= \frac{I_0(\lambda)}{2} \left(1 + \cos(\Delta\phi_{Lyot}(\lambda)) \right)\end{aligned}\tag{A.1}$$

Where $\Delta n = n_o(\lambda) - n_e(\lambda)$ and e are the birefringence and the thickness of the crystal, respectively.

To transition toward the time domain, this expression is first converted to the optical frequency domain ν , using the relationship $\lambda = c/\nu$, where c is the speed of light:

$$\begin{aligned}\Delta\phi_{Lyot}(\nu) &= 2\pi \frac{\Delta ne}{c} \nu \\ I(\nu) &= \frac{I_0(\nu)}{2} \left(1 + \cos(\Delta\phi_{Lyot}(\nu)) \right)\end{aligned}\tag{A.2}$$

Note that the term $c/(\Delta ne)$ represents the Free Spectral Range (FSR) of the filter in the optical frequency domain.

¹Heterodyne Electro-Optic Analysis and Reconstruction of THz Signals (HEARTS)

A.2 Relating Optical Frequency to an Equivalent Time via Dispersion

In a spectral encoding system, the probe laser is chirped using the dispersion of a fiber with a length L_1 . This causes the instantaneous optical frequency ν to be related to the equivalent time t within the pulse. The relationship between the equivalent time t and the optical frequency ν is defined by the chirp parameter C in the appendix of Ref. [18].

$$t = -2\pi \frac{\nu}{C} \quad \text{where} \quad C = \frac{1}{\beta_2 L_1} \quad (\text{A.3})$$

Where β_2 represents the Group Velocity Dispersion (GVD) parameter and L_1 the fiber length. The GVD parameter β_2 is commonly defined in terms of the fiber dispersion coefficient D_1 (in ps/(nm · km)) at the central wavelength λ_0 :

$$\beta_2 = \frac{-D_1 \lambda_0^2}{2\pi c} \quad (\text{A.4})$$

Substituting this expression for β_2 into the time-frequency relationship yields the required mapping:

$$\nu = \frac{c}{D_1 \lambda_0^2 L_1} t \quad (\text{A.5})$$

A.3 Equivalent Time-Domain Modulation Frequency (ν_{LO}^{THz})

The equivalent time-domain phase modulation, $\Delta\phi_{Lyot}(t)$, is found by substituting the expression for ν back into the frequency-dependent phase delay from Equation.A.2:

$$\Delta\phi_{Lyot}(t) = 2\pi \frac{\Delta n e}{-D_1 \lambda_0^2 L_1} t = 2\pi \nu_{LO}^{THz} t \quad (\text{A.6})$$

$$I(t) = \frac{I_0(t)}{2} \left(1 + \cos(\Delta\phi_{Lyot}(t)) \right)$$

This expression clearly shows the phase $\Delta\phi_{Lyot}(t)$ is an equivalent time-domain modulation $\Delta\phi_{Lyot}(t) = 2\pi \nu_{LO}^{THz} t$. The modulation frequency imprinted onto the chirped probe laser pulse is²:

$$\nu_{LO}^{THz} = \frac{\Delta n e}{|D_1| \lambda_0^2 L_1} \quad (\text{A.7})$$

This expression confirms that the frequency ν_{LO}^{THz} is entirely determined by the birefringent material properties ($\Delta n, e$), the dispersion parameters ($|D_1|$ and L_1) and the central wavelength of the probe laser (λ_0).

²By definition, the local oscillator frequency ν_{LO}^{THz} represents a physical magnitude and must be a non-negative, thus, the absolute value of the dispersion coefficient, $|D_1|$, is required.

Peak-based temporal correction algorithm

Third-order dispersion is always neglected... well, it should not be...

– Serge Bielawski

This appendix details a method developed for the temporal recalibration of signals exhibiting a chirp encountered at FELBE, as discussed in Chapter 6. This chirp primarily arises from higher-order dispersion within the stretch fiber. This effect is evident in the local oscillator signal generated by a Lyot filter incorporating three 9.6 mm YVO4 crystals, shown in Fig.B.1.(a). Ideally, the Lyot filter should produce a pure cosine wave with a narrowband spectrum. However, the higher-order dispersion in the stretch fiber introduces a chirp, resulting in the spectral broadening observed in Fig.B.1.(b).

To compensate for this temporal chirp and enable accurate signal analysis, we have implemented a peak-based temporal correction algorithm. This method involves identifying key temporal features, specifically the peaks, within the chirped signal and establishing a correspondence between their measured temporal positions to the expected positions of the peaks in a theoretical, unchirped signal (i.e., a signal devoid of higher-order dispersion). This correspondence is achieved through interpolation, allowing for the resampling of the chirped signal onto a corrected time axis, resulting on a corrected unchirped signal. The corrected time axis is use to resample any other measured signal, include phase diversity and heterodyne FEL signal.

To illustrate and validate the effectiveness of this peak-based temporal correction algorithm, the following section details its application to a computational model, followed by its use for the temporal recalibration of the local oscillator signal presented in Fig.B.1.

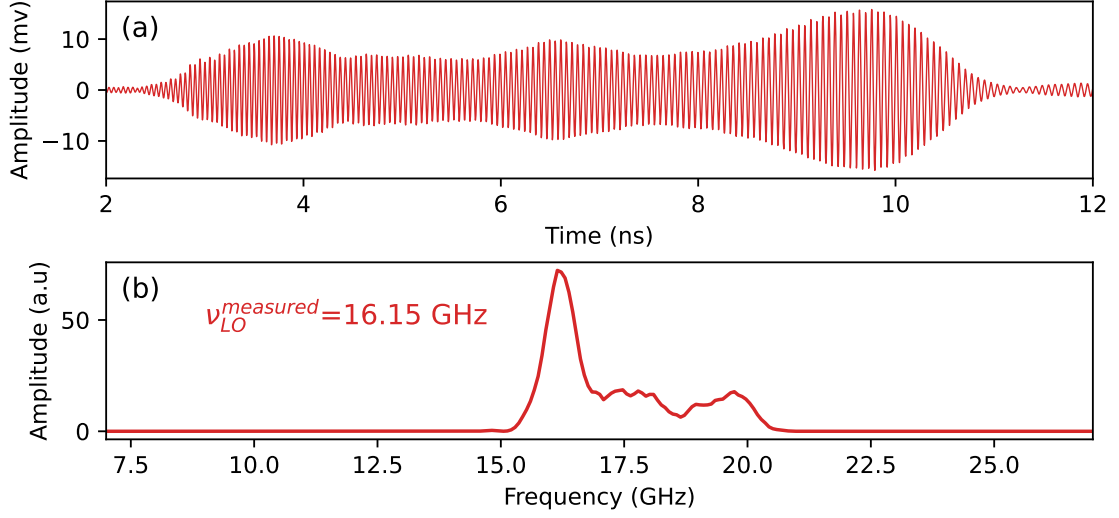


Figure B.1: **Demonstration of higher-order dispersion-induced chirp on the YVO4 optical local oscillator signal.** (a) Measured temporal profile of the local oscillator signal generated by a Lyot filter with three 9.6 mm YVO4 crystals, exhibiting a chirp due to higher-order dispersion in the stretch fiber. (b) Corresponding broadened spectrum of the chirped local oscillator signal.

B.1 Computational model validation

1. Generation of simulated signals :

This section details the application of the peak-based temporal correction algorithm to numerically-generated data, allowing for a controlled validation of its effectiveness. We begin by generating two signals, a pure cosine wave and a chirped cosine wave, designed to mimic the distortions observed in the experimental data due to higher-order dispersion. The unchirped cosine signal is defined as:

$$S_{\text{unchirped}} = A \cos(2\pi\nu t + \phi_0) \quad (\text{B.1})$$

Where $A = 1$ is the amplitude, $\nu = 10$ GHz is the frequency of the cosine wave, and $\phi_0 = 0$ is the initial phase.

To simulate the effect of higher-order dispersion, we introduce a temporal chirp to the cosine signal. In this computational model, we employ a linear chirp, where the instantaneous frequency changes linearly with time. The chirped signal is defined as:

$$S_{\text{chirped}} = A \cos(2\pi(\nu t + \alpha t^2) + \phi_0) \quad (\text{B.2})$$

Where $\alpha = 1$ is the chirp rate. The specific value of α will determine the extent of the spectral broadening observed in the frequency domain. This broadened spectrum, resulting from the temporal chirp, is analogous to the experimental observations attributed to higher-order dispersion in the stretch fiber, as shown in Fig.B.1.(b). The time and frequency domain representations of both the pure and chirped cosine signals are compared in Fig.B.3.(a) and Fig.B.3.(b), respectively.

2. Time correspondence of chirped and unchirped peaks :

The first step of the correction algorithm involves identifying the temporal positions of the peaks in the chirped signal, denoted as t_i^{chirped} , where $i = 0, 1, 2, \dots, N - 1$ and N is the total number of detected peaks. These peaks are detected using a peak-finding algorithm.

Next, we estimate the theoretical time instances of the peaks for an unchirped cosine with the frequency ν , here it is 10 GHz. Assuming the first detected peak of the chirped signal corresponds to the first peak of the unchirped signal, the peak times of a cosine without chirp is:

$$t_i^{\text{unchirped}} = t_0^{\text{chirped}} + i \cdot T \quad (\text{B.3})$$

Where $T = \frac{1}{\nu}$ is the period of the unchirped cosine wave. The positions of the detected peaks in the chirped signal and their corresponding estimated positions without chirp are visualized in Fig.B.2.(a).

To establish a continuous correspondence between the chirped time base t_i^{chirped} and the unchirped time base $t_i^{\text{unchirped}}$, we employ a one-dimensional interpolation function, $t_i^{\text{unchirped}} = f_{\text{interp}}(t_i^{\text{chirped}})$. In our implementation, a cubic interpolation is used due to its ability to accurately capture the relationship between the two time bases. This interpolation function is then evaluated at each time point of the original chirped signal t to yield the corrected time axis $t_{\text{corrected}}$ shown in Fig.B.4.(b):

$$t_{\text{corrected}} = f_{\text{interp}}(t) \quad (\text{B.4})$$

3. Signal resampling :

Finally, to obtain the temporally corrected signal, the original chirped signal $S_{\text{chirped}}(t)$ is resampled onto the newly computed corrected time axis, $t_{\text{corrected}}$. This is achieved by creating another interpolation function, $S_{\text{chirped}} = g_{\text{interp}}(t_{\text{corrected}})$ that maps the corrected time axis to the amplitude of the original chirped signal at the corresponding original time points t :

$$S_{\text{corrected}} = g_{\text{interp}}(t) \quad (\text{B.5})$$

Evaluating this function at the original time points effectively provides the temporally recalibrated signal. The effectiveness of this correction is assessed by comparing the original chirped signal with the corrected signal in both the time domain (Fig.B.3.(a)) and the frequency domain (Fig.B.3.(b)). In the spectral domain, the recalibration using our peak-based algorithm demonstrates a significant recovery of a narrowband spectrum, indicating a successful compensation for the temporal chirp.

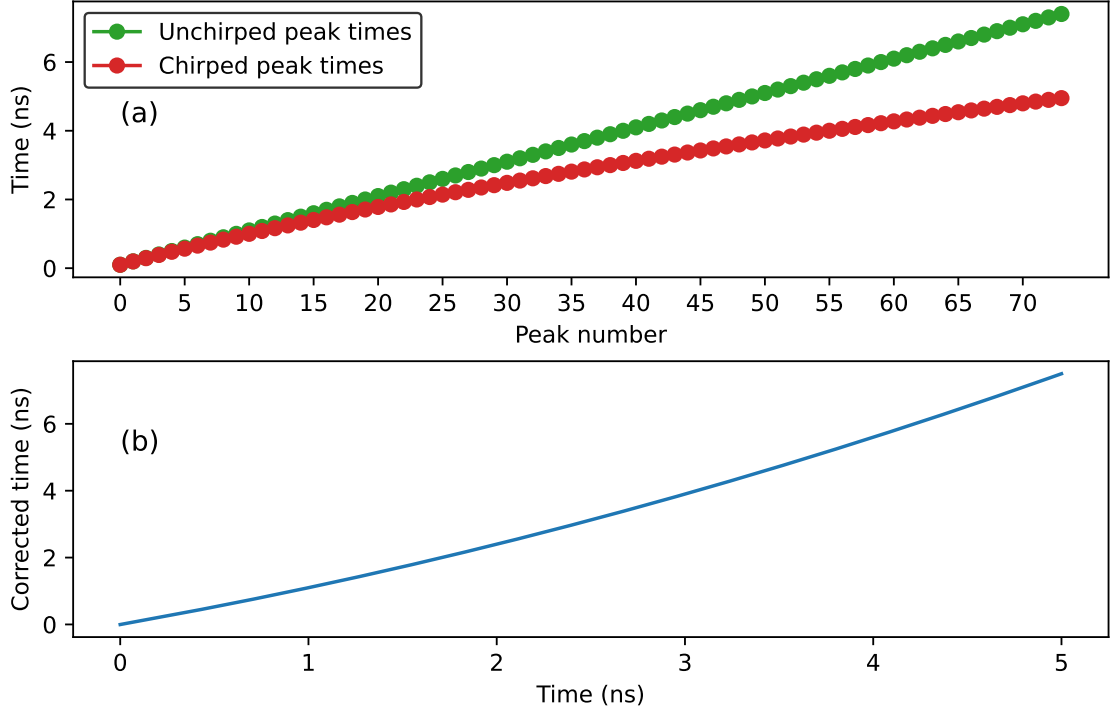


Figure B.2: **Input for the peak-based temporal correction algorithm.** (a) Temporal locations of the peaks identified in the simulated chirped signal (red circles) are plotted alongside the corresponding estimated peak locations for an unchirped signal (green circles). (b) The resulting corrected time axis $t_{\text{corrected}}$ obtained through cubic interpolation of the correspondence between the chirped and unchirped peak times.

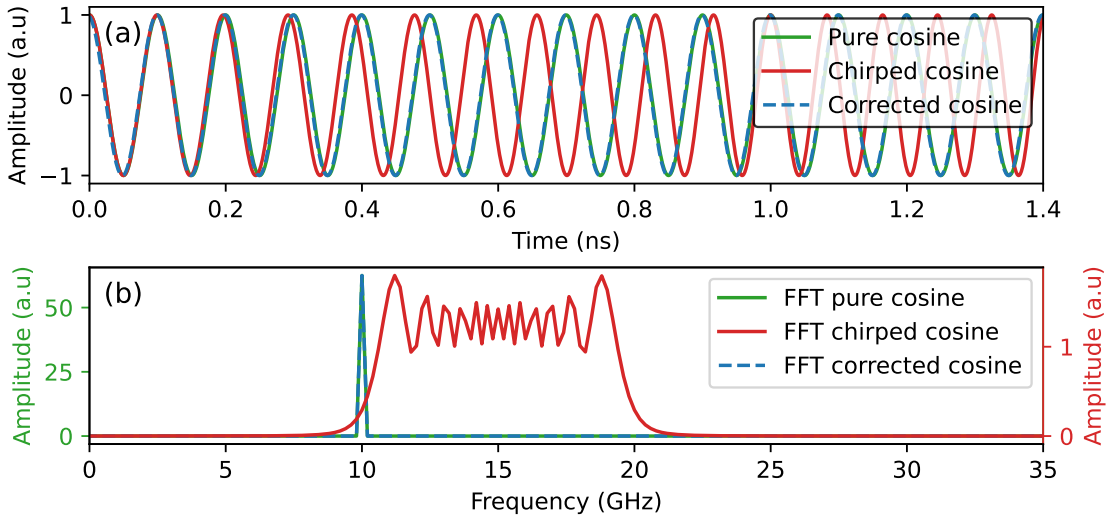


Figure B.3: **Effectiveness of peak-based temporal correction.** (a) Time-domain comparison of the original chirped (red), unchirped (green), and temporally corrected (blue) signals. (b) Frequency spectra showing the broadened spectrum of the chirped signal (red) and the recovered narrowband spectrum of the corrected signal (blue), effectively matching the spectrum of the original unchirped signal (green).

B.2 Experimental validation with local oscillator signal at FELBE

4. Local oscillator generation and theoretical frequency :

This section details the application of the peak-based temporal correction algorithm to an experimentally obtained local oscillator signal generated at FELBE. This local oscillator, produced using a Lyot filter with three 9.6 mm YVO4 crystals (total length 28.8 mm). The theoretical modulation frequency of the Lyot filter is given by the equation:

$$\nu_{LO}^{GHz} = \frac{\Delta n e}{\lambda^2 |D_2| L_2} \quad (B.6)$$

Where $\Delta n = 0.203$ is the birefringence of YVO4 crystal at 1550nm, $e = 28.8\text{mm}$ is the length of the YVO4 crystal, $\lambda = 1550\text{nm}$ is the central wavelength of the chirped probe laser, $D_2 = 18 \text{ ps/nm.km}$ and $L_2 = 8\text{km}$ are the dispersion the length of the SMF28 stretch fiber.

Using these parameters, the theoretical modulation frequency ν is calculated to be 16.9 GHz. However, as observed in the experimental data, higher-order dispersion in the stretch fiber introduces a temporal chirp, leading to a broadening of the expected narrowband spectrum as shown in FigB.5.(b).

5. Peak detection and time correspondence :

Figure B.4.(a) illustrates the temporal positions of the peaks detected in the measured local oscillator signal t_i^{chirped} and the corresponding estimated peak positions for a pure cosine wave at 16.9 GHz without the influence of higher-order dispersion $t_i^{\text{unchirped}}$. As evident in the figure, a slight deviation or curvature in the measured peak times compared to the theoretical linear progression reveals the impact of the TOD.

The temporal correction establishes a continuous correspondence between the measured peak times t_i^{chirped} and the estimate peak times $t_i^{\text{unchirped}}$ using a one-dimensional cubic interpolation. This interpolation is linearly extrapolated beyond the range of detected peaks to cover the entire probe laser duration ¹, yielding the corrected time axis $t_{\text{corrected}}$, as shown in Fig.B.4.(b).

It is this corrected time axis that will be used to resample any phase diversity of the heterodyne FEL signal.

¹Note that the FEL signal is encoded only in the interpolated region corresponding to the full width half maximum duration of the chirped probe laser

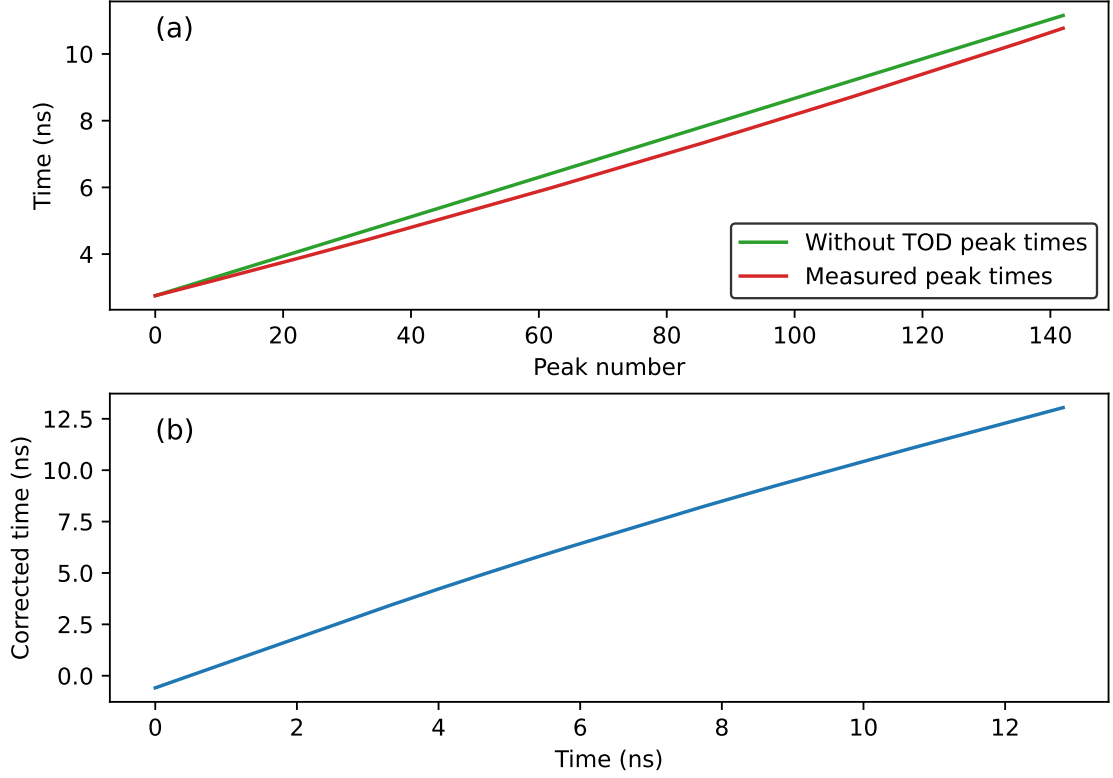


Figure B.4: (a) Temporal positions of the peaks detected in the measured local oscillator signal (red) and the estimated peak positions for a pure 16.9 GHz cosine wave (green), highlighting the deviation due to higher-order dispersion. (b) The corrected time axis $t_{\text{corrected}}$ obtained through cubic interpolation of the correspondence between the measured and estimated peak times, linearly extrapolated.

6. Local oscillator resampling :

The measured local oscillator (LO) signal S_{measured}^{LO} is resampled onto this corrected time axis using the interpolation function $S_{\text{measured}}^{LO} = g_{\text{interp}}(t_{\text{corrected}})$, yielding the temporally recalibrated LO signal $S_{\text{corrected}}^{LO}$. Figure B.5 compares the measured local oscillator signal and the corrected signal. The spectrum of the corrected signal exhibits a narrowband peak at 16.9 GHz, indicating a reduction in the spectral broadening caused by higher-order dispersion.

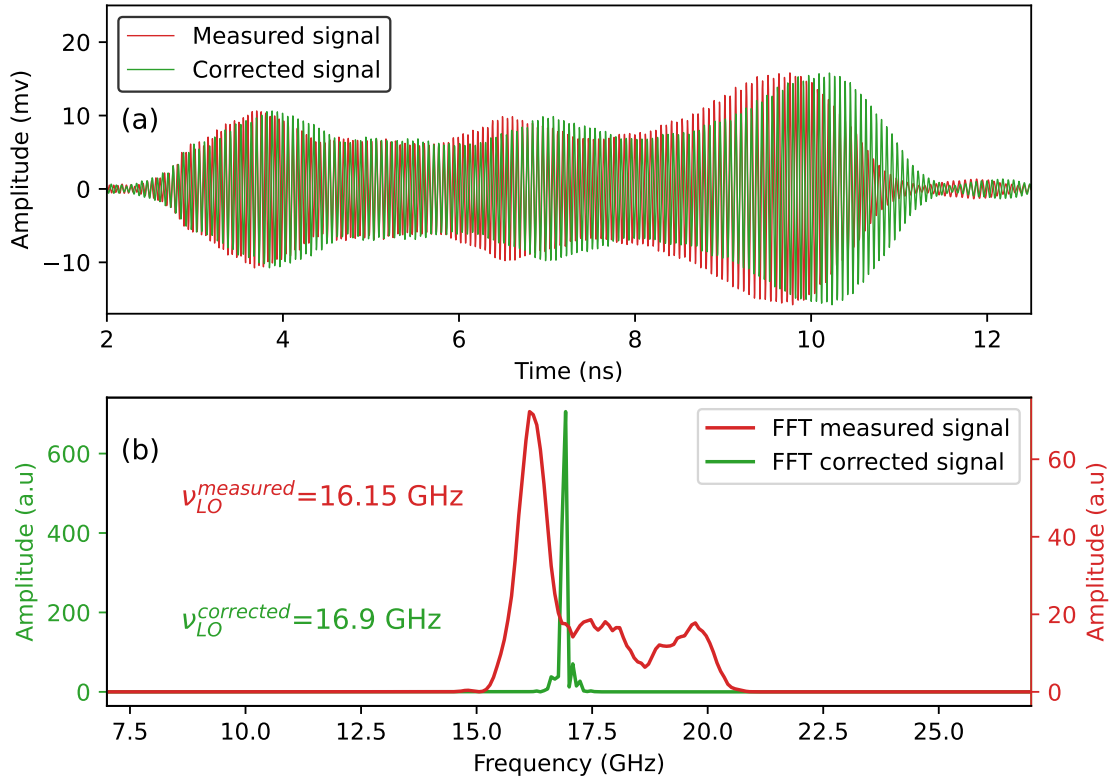


Figure B.5: Peak-based temporal correction on the local oscillator signal : (a) Time-domain comparison of the original measured local oscillator signal (red) and the resampled, temporally corrected signal (green). (b) Frequency domain representation showing the broadened spectrum of the measured LO signal (red) and the narrowed spectrum of the corrected LO signal (green), with a peak at 16.9 GHz.

7. Comparison with Optical Spectrum Analyzer (OSA) :

During preliminary testing at the PhLAM laboratory (prior to the FELBE measurements), another modulation generated via a Lyot filter employing the same YVO4 birefringent crystals was also analyzed by directly measuring the resulting optical spectrum using an Optical Spectrum Analyzer (OSA), as shown in Figure. B.6(a). This approach allowed us to observe the spectral modulation without any distortion related to the effects of β_3 (third-order dispersion), which are typically introduced when converting the spectrum to the time domain. The resulting Fourier spectrum obtained from this direct spectral measurement exhibits a peak very similar to the recalibrated peak observed in the time-domain measurements, confirming the presence of a narrow, well-defined spectral peak introduced by the Lyot filter configuration, as display in Figure. B.6(b).

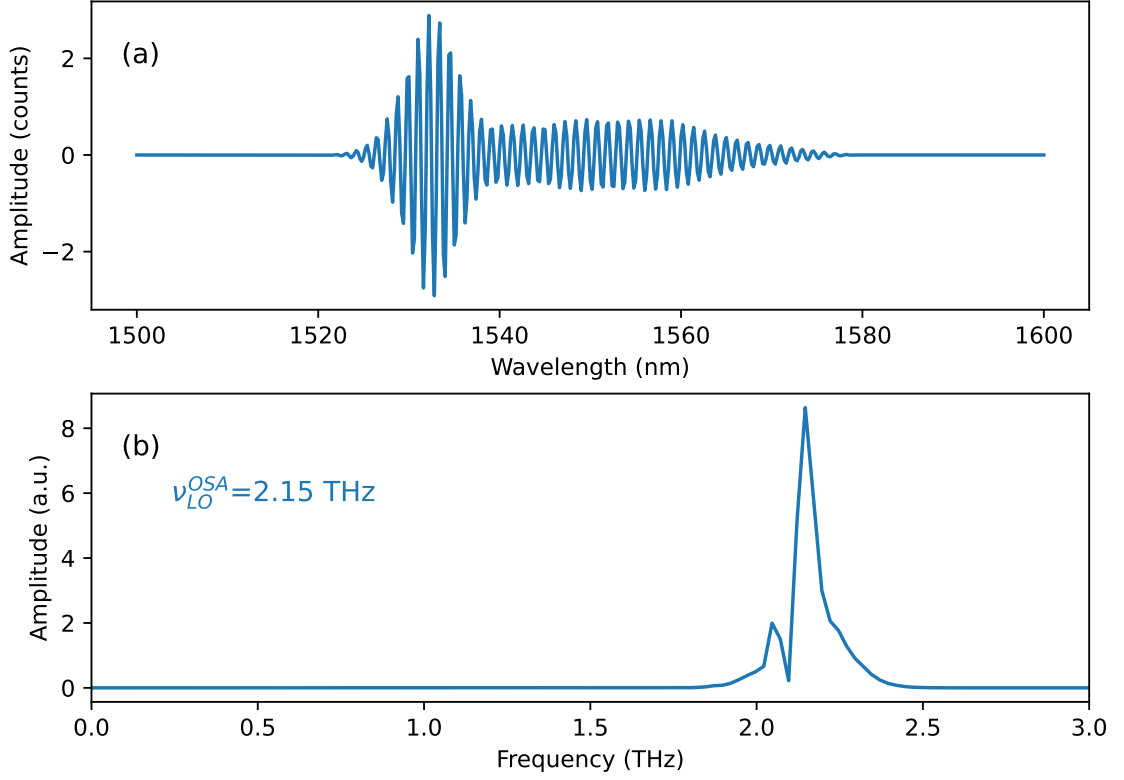


Figure B.6: **Optical spectrum of the modulation produced by the Lyot filter, measured directly using an Optical Spectrum Analyzer (OSA).** The use of the OSA allows for the direct observation of the spectral modulation, avoiding distortions associated with β_3 (third-order dispersion) that affect time-domain measurements. The analysis of this spectrum's Fourier transform reveals a narrow, distinct peak, similar to the recalibrated peak found in the time-domain data, as display in Figure.B.4.(b).

8. Note on low bandwidth readouts :

Furthermore, it is worth noting that alternative materials or fiber types could be employed to generate the time calibration signal. For instance, polarization-maintaining fiber (PM1550) or materials with smaller birefringence, such as Quartz or CaCO_3 , could be used to produce a smaller frequency modulation, which might be advantageous for satisfying the bandwidth limitations of the 2.6 GHz balanced photodetector used in our experiment in the spirit of reducing the readout's bandwidth.

Appendix C

Data analysis of the all-optical HEARTS method in phase diversity configuration: Comparison of independent and balanced measurements

I am pleased to see that it is working as expected; it is such a rare thing!

– Christophe Szwaj

This appendix provides a comparative analysis of the two distinct data acquisition strategies available for electro-optic heterodyne detection in a phase diversity configuration. The comparison is between recording the two orthogonal polarization states independently versus combining them via a balanced photodetector based on the fact that they are identical but opposite in sign.

For these characterization measurements, a 500 μm thick GaAs crystal was used. The signal was acquired using the 2.6 GHz balanced detector, but its output was recorded on the 20 GHz Teledyne Lecroy oscilloscope only due to the user-friendly interface compared to the alternative 4 GHz Rohde & Schwarz unit from the 90's.

The two following paragraph detail and compare these two acquisition options, independent versus balanced detection, with respect to the resultant signal-to-noise ratio. A fair comparison is ensured as all measurements were performed under identical THz and probe laser power levels.

C.1 Independent polarization measurement

In the phase diversity configuration, the two orthogonal polarization states of the probe laser are measured independently. To achieve this separation, as illustrated in Figure C.1(b) and detailed in Figure 6.5, one of the polarization arms incorporates an additional fiber length. This extra fiber introduces a temporal delay equivalent to one probe laser repetition period.

The recorded signals in this configuration, denoted as S_1^{PD} and S_2^{PD} represent the difference between a THz-encoded probe laser pulse S_1, S_2 and a temporally delayed reference probe laser pulse (S_0^b preceding S_1 and S_0^a following S_2)

The S_1^{PD} and S_2^{PD} can be expressed as :

$$\begin{aligned} S_1^{PD} &= S_1 - S_0^b \\ S_2^{PD} &= S_0^a - S_2 \end{aligned} \quad (C.1)$$

However, this method of independent measurement inherently prevents the cancellation of shot-to-shot laser fluctuations, which can impact the signal quality.

To mitigate the impact of background noise, a correction is applied to the recorded signals by subtracting background signals s_{01} and s_{02} . These background signals are calculated by averaging of two residual signals acquired without THz interaction: one recorded before $s_{01,02}^b$ and one after $s_{01,02}^a$ the arrival of the THz-encoded probe, as depicted in Figure C.1(b).

$$\begin{aligned} s_{01} &= \frac{1}{2} (s_{01}^b + s_{01}^a) \\ s_{02} &= \frac{1}{2} (s_{02}^b + s_{02}^a) \end{aligned} \quad (C.2)$$

Finally, the background-subtracted EO signals are normalized by the average temporal profile of the reference probe laser S_0^{Ref} , which is recorded separately as shown in Figure C.1(a). This normalization step includes a gain factor $G = \frac{G_{EO}}{G_{Ref}}$, which accounts for the difference in gain applied during the EO signal and reference measurements to prevent photodetector saturation during the latter (when only one detection arm is used). The normalized and background-subtracted EO signals $Y_{1,2}^{PD}$ are given by:

$$\begin{aligned} Y_{1n}^{PD} &= \frac{(S_{1n}^{PD} - s_{01n})}{S_0^{Ref}} G \\ Y_{2n}^{PD} &= \frac{(S_{2n}^{PD} - s_{02n})}{S_0^{Ref}} G \end{aligned} \quad (C.3)$$

The analysis of these resulting EO signals reveals a phase shift in the order of 500 mrad as shown in Figure. C.1.(c) and two peak at frequency of 0.39 THz in the spectrum in Figure. C.1.(d), note that both peaks are not identical mainly due to the different shot-to-shot laser fluctuations.

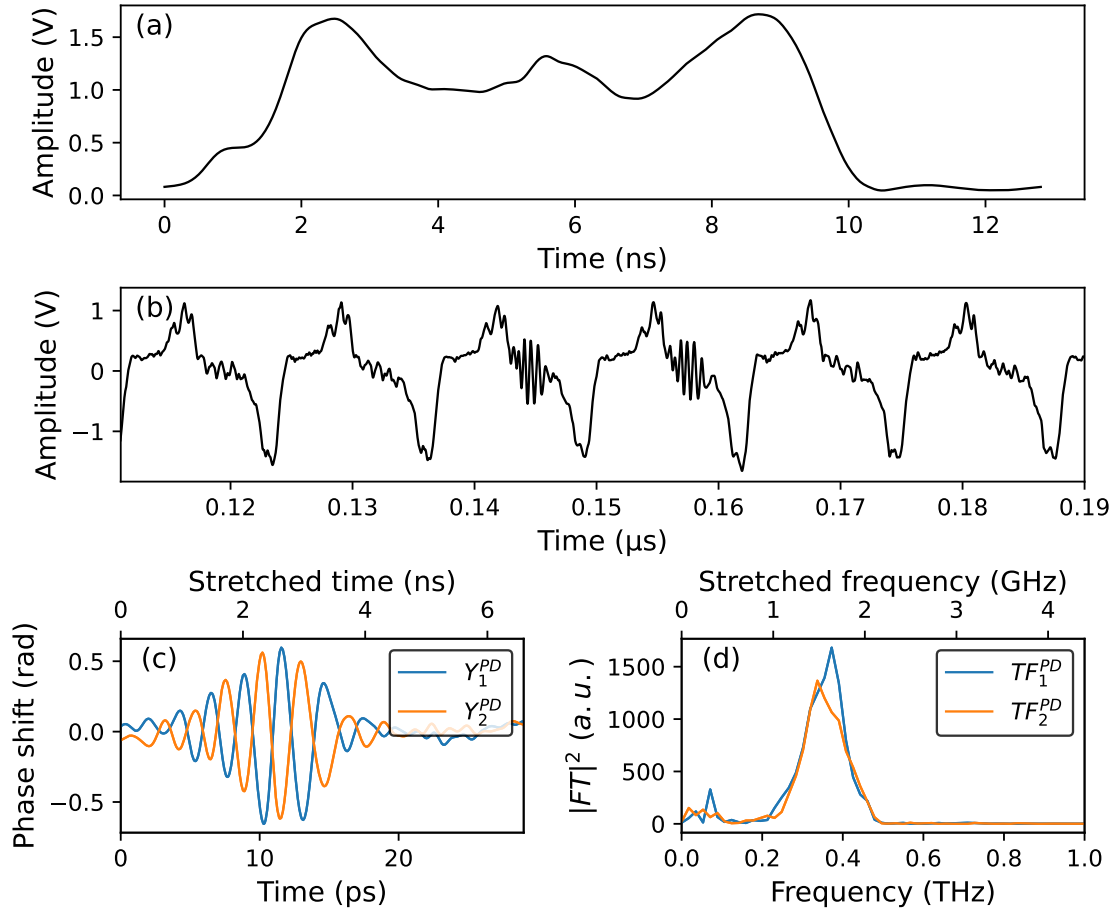


Figure C.1: **Independent polarization measurement in phase diversity configuration.** (a) Average temporal profile of the reference probe laser used for normalization. (b) Raw data displaying the two orthogonal polarization states on distinct temporal backgrounds due to the optical delay. The background signals used for background correction are also indicated. (c) Resulting single shot EO signals showing an approximate 500 mrad phase shift. (d) Frequency spectrum of the EO signals, exhibiting two peaks at 0.39 THz with slightly different amplitudes due to residual shot-to-shot laser fluctuations.

C.2 Balanced detection measurement

Given that the two output signals in the previous configuration were identical but with opposite signs, the extra fiber, previously used for the independent recording, was removed. This allowed for the subtraction of the same laser pulse on both arms of the polarizing beam splitter at the input of the balanced detector, as illustrated in Figure C.2.(b). This crucial change effectively suppresses common laser shot-to-shot fluctuations, a significant source of noise. The electro-optic phase shift Y_n^{BD} for each shot n recorded at the output of the balanced detector is determined after background suppression s_{0n}^{BD} and normalization by the average reference laser profile S_0^{Ref} , which is displayed in Fig.C.2.(a). This calculation is performed according to the following equation:

$$Y_n^{BD} = \frac{(S_n^{BD} - s_{0n}^{BD})}{2S_0^{Ref}} G \quad (C.4)$$

The resulting EO signal, obtained through this balanced detection scheme, exhibits a strong phase shift, on the order of 1 radian, as shown in Figure C.2(c). Furthermore, its frequency spectrum (Figure C.2(d)) displays a prominent and clean gaussian peak at 0.39 THz. Importantly, by employing this analog balanced detection technique, the signal-to-noise ratio (SNR) is improved by a factor of 2 compared to one arm of the previous configuration where the polarization states were recorded independently. This enhancement underscores the effectiveness of subtracting the common-mode laser noise. It is worth noting that while a numerical balanced detection could be performed on the independent data to enhance the SNR, the fundamental limitation remains that the common-mode laser noise cannot be perfectly cancelled due to the subtraction of signals from distinct laser pulses.

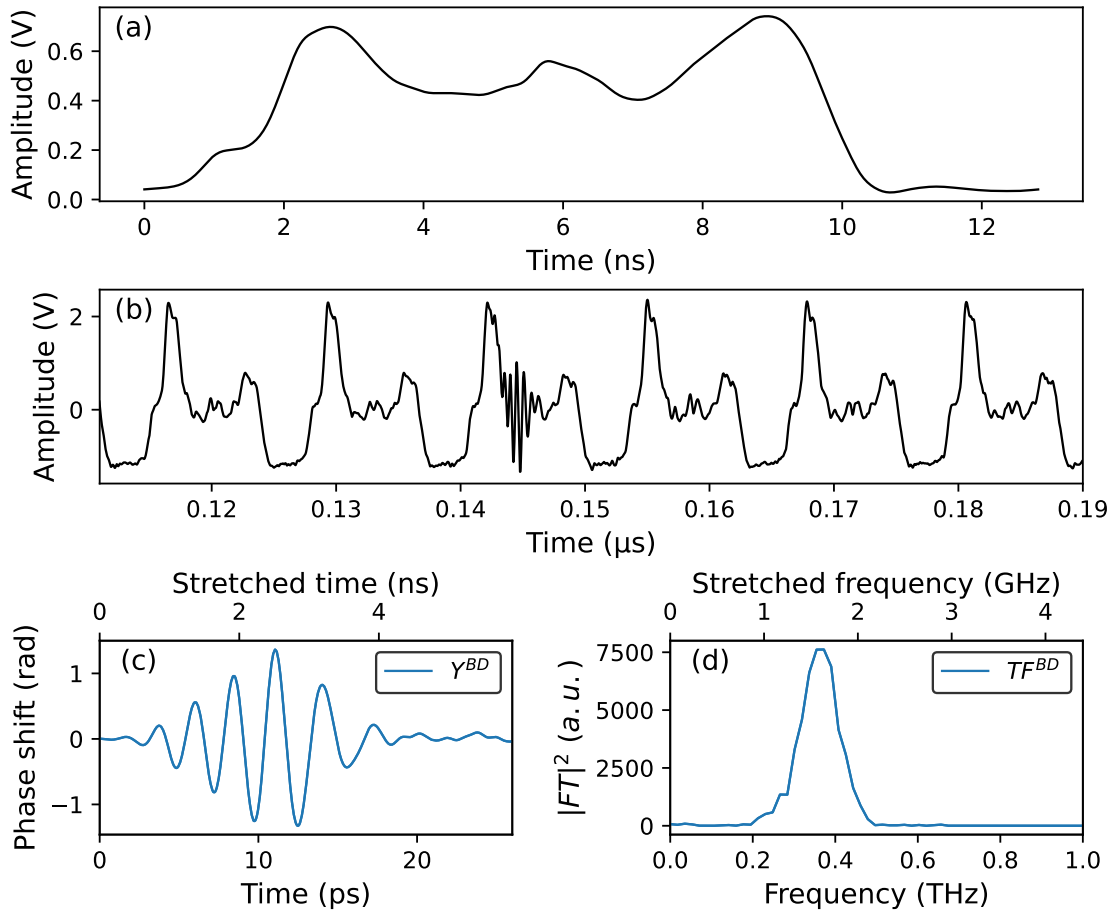


Figure C.2: Common-mode laser noise suppression via balanced detection. (a) Average temporal profile of the reference probe laser used for normalization. (b) Raw data showing the subtraction of the same laser pulses on both arms of the detector. (c) Resulting electro-optic signal showing a strong phase shift of approximately 1 radian. (d) Frequency spectrum of the EO signal, exhibiting a prominent and clean gaussian peak at 0.39 THz, highlighting the enhanced spectral purity due to noise reduction.

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HIGH-REPETITION-RATE ELECTRO-OPTIC DIAGNOSTICS FOR ACCELERATOR-BASED LIGHT SOURCES : BROADBAND AND NARROWBAND THz CHARACTERIZATION

Abstract

Accelerator-based light sources are indispensable research tools, delivering intense, short light pulses ranging from the infrared to X-ray spectral regions. The routine operation and investigation of these sources relies on the real-time characterization of relativistic electron bunches and the resulting photon beams. To address this, the objective of this thesis is the development of detection setups that deliver sub-picosecond temporal resolution at MHz repetition rates, capable of characterizing in single-shot both broadband and narrowband THz signals.

For broadband THz characterization, although Diversity Electro-Optic Sampling (DEOS) combined with the use of a THz reconstruction algorithm, has improved temporal resolution, existing reconstruction algorithms based on linear dispersion models remain unreliable due to non-linear dispersion in chirped probe pulses. This work introduces a novel Self-Adaptive Dispersion (SAD) algorithm for DEOS. This generalized framework transcends previous linear-time-invariant assumptions, providing a high-fidelity reconstruction solution for any arbitrary dispersion profile while simultaneously characterizing the complex amplitude (amplitude, phase, and dispersion) of the chirped probe laser. The DEOS-SAD system was successfully implemented at the European XFEL and FLASH facilities enabling routine, single-shot MHz-rate longitudinal electron bunch characterization.

For narrowband THz characterization, capturing complete multicycle waveforms in high-repetition-rate environments typically requires prohibitively high electronic bandwidths. This thesis proposes and demonstrates a novel electro-optic approach combining all-optical heterodyning with the Photonic Time-Stretch technique. By down-converting THz signals to an intermediate frequency and applying temporal stretching, this method records complete multicycle electric fields while preserving signal integrity. A proof-of-principle demonstration was conducted at the FELBE facility, where a 3.44 THz signal was successfully down-converted to 1.72 GHz, directly enabling the single-shot diagnosis of carrier-envelope phase (CEP) drift using lower electronic readout bandwidths.

By integrating these innovations, this thesis advances the state of the art in THz diagnostics, providing new possibilities for the real-time monitoring of high-repetition-rate accelerator-based light sources.

Keywords: accelerator, photonic, ultrafast measurement, free electron laser

DIAGNOSTICS ÉLECTRO-OPTIQUES À HAUT TAUX DE RÉPÉTITION POUR LES SOURCES DE LUMIÈRE BASÉES SUR DES ACCÉLÉRATEURS : CARACTÉRISATION THz À LARGE BANDE ET À BANDE ÉTROITE

Résumé

Les sources de lumière basées sur des accélérateurs sont des outils de recherche indispensables, délivrant des impulsions lumineuses intenses et brèves allant de l'infrarouge aux rayons X. L'opération courante et l'étude de ces sources reposent sur la caractérisation en temps réel des paquets d'électrons relativistes et du faisceau de photons qui en résulte. Pour répondre à ce besoin, l'objectif de cette thèse est le développement de dispositifs de détection offrant une résolution temporelle sub-picoseconde à des taux de répétition de l'ordre du MHz, capables de caractériser en monocoup des signaux térahertz (THz) à la fois à large bande et à bande étroite.

Pour la caractérisation THz à large bande, bien que l'échantillonnage électro-optique de diversité (DEOS) combiné à un algorithme de reconstruction ait amélioré la résolution temporelle, les algorithmes existants basés sur des modèles de dispersion linéaire restent peu fiables en raison de la dispersion non linéaire des impulsions de sonde étirées. Ce travail introduit un nouvel algorithme de dispersion auto-adaptative (SAD) pour le DEOS. Ce cadre généralisé dépasse les hypothèses précédentes d'invariance temporelle linéaire, fournissant une solution de reconstruction fidèle pour n'importe quel profil de dispersion arbitraire, tout en caractérisant simultanément l'amplitude complexe (amplitude, phase et dispersion) du laser de sonde. Le système DEOS-SAD a été implémenté avec succès sur les installations European XFEL et FLASH, permettant une caractérisation monocoup longitudinale des paquets d'électrons à des taux MHz.

Concernant la caractérisation THz à bande étroite, la capture de formes d'onde multicycles complètes dans des environnements à haut taux de répétition nécessite généralement des bandes passantes électroniques prohibitives. Cette thèse propose et démontre une nouvelle approche électro-optique combinant l'hétérodynage tout-optique avec la technique d'étirement temporel photonique (Photonic Time-Stretch). En convertissant les signaux THz vers une fréquence intermédiaire et en appliquant un étirement temporel, cette méthode enregistre des champs électriques multicycles complets tout en préservant l'intégrité du signal. Une démonstration de principe a été réalisée à l'installation FELBE, où un signal de 3,44 THz a été converti avec succès en un signal de 1,72 GHz, permettant directement le diagnostic monocoup de la dérive de la phase enveloppe-porteuse (CEP) avec une bande passante électronique réduite.

En intégrant ces innovations, cette thèse fait progresser l'état de l'art des diagnostics THz, offrant de nouvelles possibilités pour la surveillance en temps réel des sources de lumière de nouvelle génération.

Mots clés : accélérateur, photonique, mesure ultra-rapide, laser à électrons libres
