Université de Lille 1

PHD THESIS

Revenue Optimization and Demand Response Models Using Bilevel Programming in Smart Grid Systems

Modèles de Gestion du Revenu et de Régulation de la Demande Basés sur la Programmation Mathématique à Deux Niveaux dans un Contexte de Réseaux Intelligents

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Sezin Afşar

Director :	Luce Brotcorne	-	INRIA Lille-Nord Europe
Co-Director :	Gilles Savard	-	École Polytechnique de Montréal

Reviewers :	Bernard Gendron	-	Université de Montréal
	Roberto Wolfler Calvo	-	Université Paris 13
Members :	Dominique Quadri	-	Université Paris Sud XI
	Bernard Fortz	-	Université Libre de Bruxelles
	Miguel F. Anjos	-	École Polytechnique de Montréal
Invited :	Sandrine Charousset	-	EDF

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To my family

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Abstract

This thesis is concerned with revenue optimization of an energy provider. A bilevel programming approach is proposed to model the relationship between the energy provider (leader) and power users (follower). The leader intends to achieve an optimal trade-off between revenue and peak load whereas the follower minimizes total cost of users to achieve system optimality.

A smart grid structure that allows two-way communication is assumed to interconnect users and to schedule their demand regarding the prices. Day-ahead real-time prices are read by each customer's smart meter and the response is coordinated.

In this thesis, we propose several bilinear bilevel programs that are presented and reformulated as single-level mixed integer problems using the KKT conditions of the follower's problem. These MIPs are solved to optimality for randomly generated instances using a commercial software. Different versions of the models are tested and compared.

In order to solve large instances, several heuristics are developed. Two of these methods are shown to be efficient and solve large instances that cannot be solved within a reasonable time interval using exact method. Their outputs are compared to the exact solutions for small instances and their performances are evaluated.

Finally, we address the robust bilevel optimization problem, discuss existing approaches, give illustrative examples, and propose avenues for future research.

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Résumé

Dans cette thèse nous étudions la problématique d'un fournisseur d'électricité qui souhaite à la fois réguler la demande et créer du revenu dans un environnement potentiellement compétitif (PRMDS). Nous proposons des modèles bi-niveaux pour représenter l'interaction hiérarchique entre le fournisseur d'électricité (le meneur) et ses clients (le suiveur). L'objectif du meneur est de maximiser son revenu en décroissant la valeur de pointe de la demande alors que l'objectif du suiveur est de minimiser la somme des coûts des clients.

Nous supposons que les clients résidentiels sont inter-connectés entre eux via un réseau de communication bi-directionnel ce qui permet un pilotage de la demande par rapport aux prix par un agrégateur de réseau intelligent.

Dans cette thèse nous avons proposé plusieurs modèles de programmation mathématique à deux niveaux bilinéaire bilinéaire pour le PRMDS . Ces modèles peuvent être reformulés sous forme de problèmes linéaire avec variables mixte (MIP) en utilisant les conditions de KKT. Ces modèles sont résolus de façon exacte sur des instances de taille moyenne via un logiciel commercial.

Afin de résoudre des instances de plus grande taille, des heuristiques ont été proposées. Deux d'entre elles ont prouvé leur efficacité en terme de qualité de solution obtenue et de temps de calcul.

Finalement nous avons considéré une version robuste du problème de programmation mathématique à deux niveaux. Des propriétés préliminaires ont été prouvées.

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Chapter 1

Introduction

The PhD Thesis deals with *Revenue Optimization and Demand Response Models using bilevel programming in smart grid systems.* The thesis has been carried out at Inria Lille-Nord Europe in connection with Université Lille 1 and collaborating with CIRRELT Research Group in Montréal, Canada. The Canadian collaboration is funded by STEM (deciSion Tools for Energy Management), associated team project of INRIA.

Maintaining supply-demand balance represents a real challenge for energy optimization. When this balance is perturbed, there is higher risk of brownouts and even blackouts. The supply side of the problem is difficult to manage since electricity cannot be stored in bulk amounts with today's technology. Furthermore, the generation capacity of nuclear or fossil-fuel power plants with smaller marginal cost cannot be turned down or up whenever demand fluctuates. Installation of new power plants is a very large investment and usually avoided.

On the other hand, electricity demand increases every day and becomes more and more unpredictable due to technological advancements, customer preferences and population growth. The difference between the peak and minimum load throughout a year is large. Combining the rise and uncertainty

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of demand with seasonal, climatic and daily variations makes it difficult to efficiently manage the whole system.

Instead of tackling the problem from the supply side, demand side management (DSM) aims to influence customers' consumption habits to change the shape of the load curve [Gellings 1985]. DSM programs are used to manage available resources more efficiently rather than employing new ones and they can be grouped as conservation and energy efficiency programs, fuel substitution programs, demand response programs, and residential or commercial load management programs [Gellings 1987].

According to [Torriti 2010, Beaudin 2014] demand response (DR) is a set of decisions and actions that can be taken at the customer side with respect to different payment schemes. As defined in more detail in Chapter 2, there are two main categories in DR, incentive-based and price-based. In the context of this thesis a price-based DR program, namely real-time pricing, is utilized to achieve revenue optimization.

In the context of this work, bilevel programming is utilized to integrate customers' response and choices to the decision making process of the provider and hence facilitate a more efficient use of current capacity without asking the users to modify their consumption habits profoundly. In this framework, smart grid technology provides an automated scheduling mechanism and allows to acquire an interactive power system with two-way communication which helps to smoothly incorporate demand response programs into the relation of the electricity provider and its customers. It also provides an automatic control to customers for their electrical appliances.

The motivation of this thesis is to investigate a hierarchical game that

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is played between an electricity provider (leader) and a smart grid operator (follower). It is assumed that every customer is a residential user who is equipped with a smart meter. In this bilevel setting, the leader tries to achieve a trade-off between revenue maximization and peak load reduction by setting day-ahead prices. The smart grid receives demand and appliance specifications from residential users. Delay tolerance of customers is included in the model by introducing an *inconvenience cost* (similar to the waiting cost in [Mohsenian-Rad 2010a]). Then, the smart grid implements a cost minimizing schedule.

The contributions of this thesis are both on the modelling and the algorithmic side. To the best of our knowledge, we present the first bilevel approach for an electricity provider's revenue optimization problem that involves demand side management in a smart grid context. Bilevel models with preemptive and/or nonpreemptive appliances are defined and studied. Scenarios where the leader faces a competitor firm with fixed prices and monopolistic scenario are considered.

The bilevel models with only preemptive appliances are reformulated as single level mixed integer programs by using the classical approach that integrates KKT conditions of the follower's problem to the leader's problem [Labbé 1998]. A new method is developed to reformulate the models with nonpreemptive appliances as a MIP. Randomly generated instances are solved with different parameter values to better comprehend and depict the underlying structure.

Bilevel programs in general are difficult to solve to optimality and usually fail to produce solutions in moderate CPU time. In that context, efficient

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heuristic methods are developed that are based on the structure of the models.

Numerical results are provided on randomly generated instances. As a general rule, two of the heuristics produce good quality solutions for small and large instances in a short amount of time. Heuristic results are compared to the classical exact method for different parameter values.

Finally, in the last chapter, a novel theoretical concept of robust bilevel programming is introduced and discussed with respect to the existing literature.

Thesis Structure

Chapter 2

This chapter is devoted to the literature review. Demand side management, demand response, smart grid and bilevel programming perspectives are discussed in detail.

Chapter 3

The bilevel models that are developed within the scope of the thesis are introduced. Monopolistic models and their competitive versions are presented alongside the single level mixed integer formulations. Numerical results of the classical exact method are displayed in the last section of the chapter.

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Chapter 4

All heuristic methods that are developed are explained, the two method that are efficient are presented in detail. Inverse optimization, minimum peak and fixed peak subproblems are presented and their roles in heuristic procedures are explained. Comparison between the heuristics and the exact method are provided.

Chapter 5

The robustness approach proposed by Bertsimas and Sim [Bertsimas 2003] is adapted to the our bilevel framework. A list of counter-intuitive examples are given on bilevel toll-setting problem to point out that the solution is far from being trivial.

Chapter 6

The last chapter is dedicated to the analysis of the models, methods and approaches that are developed within the scope of this thesis. Finally, many future research avenues and prospects that are opened up with this work are proposed.

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Chapter 2

Literature Review

Contents

This chapter is devoted to the literature review of four main topics closely related to our work, namely: demand side management, demand response, smart grid technology and bilevel programming.

2.1 Demand Side Management

Electricity demand is increasing and becoming more and more unpredictable. Instabilities trigger many adverse effects for all electricity users. In order to avoid a future imbalance, conservative measures are taken by electricity generators/suppliers. One precaution is to maintain a large capacity. However, both for renewable and non-renewable sources, installation of electricity generation capacities requires a large capital investment. To avoid such complex and long term projects would be beneficial to most firms.

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Demand Side Management (DSM) is a set of measures that aims at a more efficient use of existing resources. DSM methods have started being used since the late 1970s in the United States. Masters [Masters 2004] describes DSM as utility programs that save energy and, in more general terms, alter consumption patterns of customers. It is often employed to control and manipulate the energy consumption at the customer side in order to meet capacity constraints [Masters 2004, Strbac 2008].

DSM methods can be categorized as conservation/energy efficiency, load management, fuel substitution and demand response programs. Energy efficiency methods usually involve replacement of an inefficient equipment with a new one. Load management programs are the methods that modify the load shape and consumption pattern of customers. Fuel substitution methods aim to influence customer's fuel choice. Lastly, demand response programs essentially intend to manipulate consumption by changing prices [Palensky 2011].

[Gellings 1985] states that DSM deals with load shaping objectives that can be categorized as follows:

- Peak clipping: decreasing the peak load, usually through direct load control. The supply capacity or demand load gets reduced.
- Valley filling: increasing the load during off-peak periods. It is especially useful when the average price is higher than the marginal cost.
- Load shifting: transferring load from peak to off-peak periods.
- Strategic conservation: an overall load decrease which might be caused by a reduction in supply or conservation in demand.

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- Strategic load growth: a general increase in energy sales which might be triggered by supply capacity increase or demand growth.
- Flexible load shape objective: providing several different options and services to customers.

In this thesis, in order to have a more efficient utilization of installed generation capacity, achieving load shifting through DSM is targeted.

2.2 Demand Response

In order to achieve load shaping DSM objective, one of the most popular programs is **Demand Response** (DR). DR is defined in the literature [Strbac 2008, Albadi 2008, Palensky 2011] as end-user reaction to changing prices by modifying consumption patterns. DR is related not only to prices, but also to system reliability. In [Albadi 2008], the authors classify DR programs as follows:

- Incentive-based programs (IBP)
  - Classical
    - Direct control
    - Interruptible programs
  - Market-Based
    - Demand bidding
    - Emergency DR
    - Capacity Market

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        * Ancillary services market

- Price-based programs (PBP)
    - Time-of-use (TOU) pricing
    - Critical peak pricing (CPP)
    - Extreme day CPP
    - Real time pricing (RTP)

Classical DR programs are based on payments or discounts to customers. They have two subcategories: direct load control (DLC) and interruptible programs. DLC can be applied on preemptive devices (like heating or air conditioning). These appliances are directly and remotely controlled by a central signaling system and customers can use their devices as they prefer when they are not under direct control. DLC is widely applied in the U.S., although its financial benefits have not been proven yet [Strbac 2008]. Even if DLC is frequently used for industrial customers [Weers 1987, Gomes 2007, Ruiz 2009] combined with special programs, it is unpopular among residential users due to privacy concerns [OpenHAN 2008]. Customers subject to interruptible programs receive an incentive payment or rate discount in advance and then are asked to decrease their load to reach a predefined level. If they fail to do so, they may face penalties.

The second category of IBP, namely market-based programs, rewards customers with money depending on their critical load decrease. There are four subcategories: demand bidding, emergency DR, capacity market and ancillary services market. Demand bidding methods enable customers to bid to reduce

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or give up on their demand in exchange for a price lower than the market price[Strbac 2008, Albadi 2008]. Customers that fail to meet the limit are penalized. Emergency DR programs are used during extraordinary circumstances and customers who reduce their loads receive incentives. Capacity market programs are proposed to customers who are willing to reduce their loads to a predefined amount after receiving a day-ahead notice of load reduction. In ancillary services market, customers are allowed to bid for load decrease in the spot market. When the bid is accepted, customers receive a payment for their commitment and when load reduction is needed, they are paid spot market price.

Price-based programs (PBP) are the third category of IBP and are based on fluctuating prices with respect to time of day, peak hours/days depending on the cost of electricity. The common aim of these programs is to smooth the demand curve through a pricing approach. PBP can be classified as: time-of-use pricing (TOU), critical peak pricing (CPP), extreme day pricing, extreme day CPP and real time pricing. TOU pricing involves two prices (peak and off-peak) that are announced to customers in advance. It is a strong tool for shaping load and convincing customers to shift their demand to off-peak periods [Caves 1984, Çelebi 2012, Yang 2013a]. In CPP, customers are charged pre-specified prices during contingencies for short time and flat or TOU rates at other times. Extreme day pricing is similar to CPP, however the prices are in use during the whole extreme day. Extreme day CPP applies critical peak pricing during the extreme day for 24 hours and a constant rate for the rest of the time. In real-time pricing (RTP) prices fluctuate throughout the day. Customers receive the prices on a day-ahead or hour-ahead basis and adapt

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their demand accordingly. RTP is referred as the most direct and efficient demand response program by many economists [Bloustein 2005, Albadi 2008] and used by many researchers in the literature [Mohsenian-Rad 2010b].

DR programs bring benefits for customers, firms and markets. As mentioned by [Albadi 2008], these benefits are grouped as participant, market-wide, reliability and market performance benefits:

- Participant
  - Incentive payments
  - Bill savings
- Market-Wide
  - Price reduction
  - Capacity increase
  - Avoided/deferred infrastructure costs
- Reliability
  - Reduced outages
  - Customer participation
  - Diversified resources
- Market performance
  - Reduces market power
  - Options to customers
  - Reduces price volatility

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Participants' gains fall into two categories, incentives and bill savings. Customers' bills can be reduced by simply switching demand from peak to off-peak periods which may induce bill savings even though overall consumption is higher. Incentive payments can be allocated to customers as a result of their participation or accomplishment.

Market-wide benefits of DR programs can happen in three ways: price reduction, capacity increase and avoided infrastructure costs [Tan 2007]. More efficient use of installed generation capacity results in reduction of average energy prices. Moreover, when the demand curve is smoother, short-term available energy capacity increases. As a result, distribution and transmission infrastructure enhancements can be avoided which in return helps to reduce expenditures.

Reliability benefits can be categorized as reduced outages, customer participation and diversified resources. Customers who are committed to a DR program help to decrease the risk of electricity service interruption. Meanwhile, their participation increases the system reliability and available resources. Hence the outage risk is diminished.

Market performance benefits can be grouped as reduced market power, reduced price volatility and options for customers. Price elasticity of electricity demand is usually very low, i.e., customers do not react to price fluctuations mostly due to the fact that electricity is perceived as "indispensable and always available" [Kirschen 2003]. DR programs allow people to have more options and to give a proper response to price spikes since small changes in peak demand can result in a large difference in generation cost. As a consequence, the market power of a player with large demand is limited and price volatility

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decreases.

In conclusion, DR programs are applied in order to achieve demand-side management objectives and to increase system efficiency. As previously mentioned, RTP is the most direct and efficient DR program. However, it is not realistic to expect customers to shape their demand according to daily or hourly prices on a regular basis. RTP can be efficiently applied in a smart grid context where customers can stay informed and react accordingly. In this thesis, RTP is mainly used to obtain a smooth supply curve.

2.3 Smart Grid Technology

In the literature, there are several different definitions of smart grid. According to [Dept. of Energy 2009], the smart grid is defined as an electricity distribution system that is programmable and is equipped with two-way communication capability. This definition is assumed in the context of this thesis as well. The smart grid operates more efficiently and more reliably than the traditional system and hence provides a higher quality of service to customers.

This new technology gives rise to many options. For instance, electricity generation can be decentralized and customers can also contribute to the generation in a smart grid system. A house with photo-voltaic panels can provide its own electricity and excess electricity can be sold to the grid. Furthermore, a plug-in hybrid electric vehicles (PHEV) can be charged by solar panels during off-peak hours and then this stored electricity can be sold to the grid during peak hours for a high price or used for other household appliances [Dept. of Energy 2009].

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When the market share of PHEVs becomes more significant, their intensive use may increase the average residential electric load significantly, thus putting the network at risk. In this case, flexibility of the smart grid will gain more importance.

Smart grid optimization has recently received increasing attention in the literature under different assumptions. The approaches vary with respect to the adopted pricing scheme. In [Mohsenian-Rad 2010a], an algorithm is proposed to compute an optimal consumption schedule of customers and to find a trade-off between electricity bill payments and waiting time (in case the usage of an appliance is postponed) with RTP. The authors of [Doostizadeh 2012] propose a day-ahead RTP scheme to maximize energy providers' profits while taking consumer behavior and distribution network data into consideration. In [Wang 2012], a dynamic pricing scenario is analyzed where an electricity provider has the option of selling or buying electricity from users and users decide on their demand with respect to the time-varying prices. A dynamic pricing scheme is proposed in [Caron 2010] to achieve an aggregate load profile and the effect of shared information amount on load profiles is analyzed. In [Costanzo 2012], the authors propose a novel system architecture for autonomous demand side load management which allows to integrate online operation control, optimal scheduling under dynamic pricing techniques. In [Yang 2013b], a game is designed between energy utility companies and users under TOU pricing. It is shown that a Nash equilibrium can be achieved in this setting. An energy provider's cost minimization problem is studied under day-ahead TOU pricing and probabilistic schedule flexibility in [Joe-Wong 2012].

Different modeling approaches are used for DSM in a smart grid system.

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A convex optimization model is proposed in [Samadi 2010] to maximize aggregate utility of customers in a smart grid system which has power supply and communication facilities. In [Ramchurn 2011], the consumption decision of customers is modeled as a mixed integer quadratic program in a smart grid system. The authors divide the household appliances into two subcategories as shiftable static loads and thermal loads and maximize social welfare. Similarly, in [Zhu 2012], the authors propose an integer linear program to achieve a balanced load schedule. They separate the appliances into two categories as time-shiftable and power-shiftable. A network congestion model is proposed in [Ibars 2010b] where customers act selfishly and minimize their individual costs by managing their demand. A similar scenario is discussed in [Mohsenian-Rad 2010b] where an energy consumption scheduling game is played among customers to minimize their individual energy charges.

In addition to the ones that are mentioned above, there are many more articles that approach the energy problem as a Nash game such as [Weber 2002, Overbye 1999, Hobbs 2000, Ibars 2010a]. However, the relation between an electricity provider and its customers better fits the bilevel programming framework due to the hierarchical nature of decision making process. To the best of our knowledge, the relation between the smart grid operator and the electricity provider has not been considered in a bilevel setting so far in the literature.

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2.4 Bilevel Programming

A bilevel program can be defined as a sequential (or static Stackelberg) game that is played between a leader and a follower. The leader makes its decision first while taking the follower's reaction into account. A very explanatory example is given in [Brotcorne 2008b] as follows:

> "In the simplest version of the cake-cutting game, Bob cuts a Sachertorte into two parts, knowing that Alice will select the larger piece. If Bob knows Alice's greedy behavior then, in order to maximize the size of his portion, it is obvious that he should cut the cake into two equal parts. This trivial game, which can be extended to several sequential players, can be modeled as a bilevel program, i.e., an optimization problem where the leader (Bob) integrates within his decision program the mental process of the follower (Alice)."

Bilevel programs were first used in the literature by [Bracken 1973, Bracken 1974, Bracken 1978] to model marketing, production and military applications. However, the term *bilevel* and *multilevel programming* were first mentioned in [Candler 1977] in an agricultural policy development problem.

In general, bilevel programs are intrinsically difficult to solve (NP-hard) [Jeroslow 1985, Hansen 1992]. Besides their nonconvex and combinatorial nature, the feasible region of the leader is generally nonconvex, and can be disconnected or empty [Colson 2005].

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The hierarchical structure of bilevel programs consists of an upper and a lower level problems and it is mathematically expressed as follows:

$$\begin{aligned} \max_{x \in X} \quad & f(x, y) \\ \text{s.t.} \quad & y \in \arg \min_{y' \in Y(x)} g(x, y'). \end{aligned}$$

For a fixed upper level decision x^*, the lower level might have a corresponding non-unique optimal solution $Y^*(x^*)$. In this set of optimal solutions, if the follower chooses to act cooperatively and picks a value for y that maximizes $f(x^*, y)$ or mathematically: $y^*_{OPT} := \arg\max_{y' \in Y^*(x^*)} f(x^*, y')$ then it is called *optimistic* approach and formulated as above. On the other hand, in *pessimistic* bilevel programming, the leader hedges against the worst-case scenario and the pessimistic solution is mathematically expressed as: $y^*_{PES} := \arg\min_{y' \in Y^*(x^*)} f(x^*, y')$.

Pessimistic bilevel programming and its difficulties have been addressed by several researchers in the literature such as [Lucchetti 1987, Loridan 1996, Dempe 2002]. More recently, necessary optimal conditions of pessimistic bilevel programs are analyzed in [Dempe 2014]. A complete discussion and comparison of both approaches can be found in [Loridan 1996]. In the context of this thesis, the optimistic approach is adopted.

An optimization problem that is closely related with bilevel programming is MPEC (mathematical program with equilibrium constraints). MPECs can be defined as bilevel programs where a variational inequality is consisted in the lower level to describe an equilibrium state. MPECs are applied in several fields such as taxation and highway pricing [Labbé 1998, Labbé 2000], traffic

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equilibrium [Marcotte 2007], tax credits for biofuel production [Bard 2000], network design [Marcotte 1986, Marcotte 1992, Gao 2004], electricity planning [Hobbs 1992, Hobbs 2000, Gabriel 2010]. A more extensive study on the theory and application of MPECs and bilevel programs can be found in [Brotcorne 2008b, Colson 2007, Dempe 2003b, Dempe 2003a]. When there are several strategical players at the upper level and each one solves an MPEC, the resulting program is an EPEC (equilibrium program with equilibrium constraints). In [Hu 2007], the authors propose an EPEC to model a restructured electricity market. For a comprehensive study of EPECs, we refer to the PhD thesis of Ehrenmann [Ehrenmann 2004].

Linear bilevel programs (LBP) represent the basic form of bilevel programs with linear objective functions at both levels and linear constraints. LBPs are shown to be NP-complete [Jeroslow 1985, Bard 1991]. An efficient branch-and-bound algorithm was proposed by [Bard 1990] and it is improved in [Hansen 1992]. A genetic algorithm and a KKT approach are developed in [Hejazi 2002] and [Shi 2005], respectively. A mixed-integer version of the problem is studied in [Moore 1990] and an enumeration method is proposed. Later a tabu search method is applied on the MIP version in [Wen 1996].

Bilinear bilevel programs are frequently utilized to model the pricing problem. One of the most known pricing problems is the general taxation problem which is proposed and proven to be strongly NP-hard by [Labbé 1998, Labbé 2000]. Bilevel pricing problems on telecommunication networks have been proposed by [Bouhtou 2007], on transportation networks by [Brotcorne 2001, Brotcorne 2008a] and on freight networks by [Brotcorne 2000]. Algorithms that are specifically based on the structures

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of the problems have been developed in these articles. For the general bilinear bilevel problem, a bundle trust region algorithm is presented in [Dempe 2001].

Revenue management problems which concern high investment and low operation cost sectors with perishable inventories such as airline, railway or energy sector have been modeled as bilevel programs [Talluri 2006, Brotcorne 2008b]. A bilevel formulation that is an extension of the network toll-setting problem addresses several issues of airline industry from a revenue management perspective [Brotcorne 2000, Côté 2003].

Bilevel programs are used to model hierarchical nature of decision making process in the energy field. In [Hobbs 1992], the authors present a nonlinear bilevel program where the leader is a cost-minimizing electric utility and the follower is a power customer. An MPEC approach is proposed later in [Hobbs 2000] to the same problem. In [Hu 2007], an MPEC approach is used to model the problem of each player in a non-cooperative game. A bilevel approach is combined with stochastic pricing for load shifting in [Zugno 2013] and compared to fixed and time-of-use pricing.

In the context of this thesis, we consider a single leader (electricity provider), single follower (smart grid operator) bilevel program. The follower aims to minimize the total cost of customers and hence achieve a system optimum.Therefore it is important to underline the fact that we do not consider an MPEC. Moreover, since there is a single leader in our setting, the problem is also not an EPEC.

The bilevel programming approach gives us the opportunity to integrate DSM into the revenue optimization problem of an energy provider. Moreover, the relationship between an energy provider and a smart grid operator better

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fits the framework of hierarchical decision making.

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Chapter 3

Bilevel Formulation of Revenue Optimization Problem

Contents

In this chapter, we present bilevel programs developed to model the revenue optimization problem of an energy provider. A day-ahead real-time pric-

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ing demand response program [Albadi 2008] is used to achieve load shifting [Gellings 1985] and revenue management [Talluri 2006, Brotcorne 2008b].

The problem we focus on involves two types of decisions of agents acting in a cooperative and sequential way: an energy provider and a smart grid operator. Let us consider a power sharing system among a set of customers where each one of them is equipped with a smart metering device. It is assumed that every customer has a set of electrical residential appliances which are preemptive in §3.1 and §3.1.1, non-preemptive in §3.2 and §3.2.1 and both in §3.3.

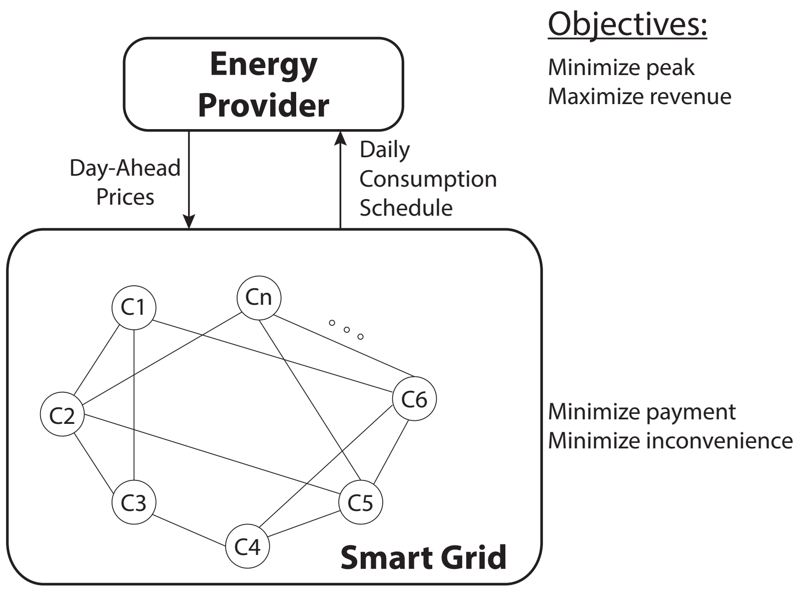

Figure 3.1: Bilevel Structure

It is possible to turn on or off preemptive devices at any time, or to change their level of consumption. Most common examples are air conditioning, radiators, refrigerators, freezers, pool heaters etc. Significant amount of power is consumed by this type of devices, e.g. 45% of household appliances' con-

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sumption comes from preemptive devices in U.S. (Fig. 3.2).

The devices that cannot be stopped, restarted or adjusted such as washing machines, dishwashers, dryers are referred as non-preemptive. In the context of this thesis, their consumption is assumed to be constant from the moment the device starts working until it ends. Different consumption patterns can be implemented as well.

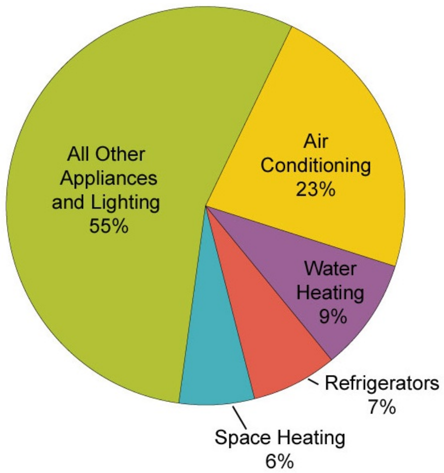

Figure 3.2: Residential Electricity Use in U.S., 2011 [Rep 2012]

In many countries, base load is produced by coal and nuclear power plants whereas peak load is provided by natural gas, hydro or renewable power plants. For this reason, electricity production during peak periods is more costly than off-peak periods. Besides, installed generation capacity has to be larger than peak load in order to assure power supply. Since a reduction in peak load results in a decrease of production and capacity cost, it deserves an extensive analysis. In our models, the leader's objective function is twofold: revenue maximization and peak minimization which allows to assess the trade-off that leader faces.

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Although delays are inevitable for peak maximization objective, some customers might be more sensitive about having to postpone their loads whereas some others are willing to switch to a cheaper time slot, or even the same person might have more restrictions regarding the use of some appliances. In order to capture the price perception difference among customers, an *inconvenience factor* is incorporated in all models that represents delay sensitivity of customers.

3.1 Preemptive Bilevel Model

In this section, we assume that a monopolistic electricity supplier (leader) plays a static Stackelberg game with the smart grid operator in charge of managing the demand of N customers where each customer n owns A_n preemptive appliances.

The smart grid operator is assumed to minimize the sum of the costs of all customers (system optimum) while scheduling the demand. It is formed by interconnected smart meters that consumers own. These devices communicate with each other, receive and deliver data. The grid is connected to a power source and receives the price vector $p^h, h \in H$ from the electricity supplier 24 hours in advance.

The decision variable of the lower level is $x^h_{n,a}$ which represents the consumption amount of appliance a of customer n during time slot h. One time slot h is assumed to be one hour. Since all jobs are preemptive, variables $x^h_{n,a}$ are continuous and for each appliance a of customer n, $\beta^{max}_{n,a}$ that is maximum power limit of the device. For instance, if the device is an air conditioner, it

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consumes most on a hot summer day to cool the room down to 18°C or on a cold winter day to heat the room up to 30°C. Demand $E_{n,a}$ of each customer n for each appliance a is supposed to be known and announced by the customer one day ahead. A time window $T_{n,a}$ is associated with each appliance a of customer n during with the appliance has to be operated. $TW^b_{n,a}$ and $TW^e_{n,a}$ stands for beginning and end of an appliance's time window and it is assumed that the first time slot in a time window is the most desired one. The inconvenience of customer n for usage of appliance a at time h is $C_{n,a}(h)$. It is directly proportional to the demand, length of delay, and an inconvenience coefficient $\lambda_{n,a}$ associated with customer n and appliance a, and inversely proportional to the length of the time window.

$$C_{n,a}(h) := \lambda_{n,a} \times E_{n,a} \times \frac{(h - TW^b_{n,a})}{(TW^e_{n,a} - TW^b_{n,a})}$$

Due to the definition of the time windows, a job with a heavier load is assumed to have a higher inconvenience factor. Besides, a job with a narrow time window has a higher factor than one with a large time window. Customers who define a narrow window are probably willing to pay more rather than postponing their consumption.

All customers receive the same prices at each time slot h. As mentioned earlier, customers are assumed to be residential users. As their consumption habits are similar, it is realistic for the provider to offer the same hourly prices for all customers. Note that if price discrimination were allowed, then the leader's problem would become user-separable and thus much easier to

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solve.

The objective function of the follower is to minimize the total disutility of all customers. It is defined as the sum of electricity payment of all customers as well as the total inconvenience cost. In other words, customers would like to buy electricity at the cheapest possible price without delaying their consumption too much.

The objective function of the leader consists of two terms: revenue and peak cost. The parameter κ defines a trade-off between peak load and revenue. The revenue term of the leader's objective is in fact the billing cost term of the follower's objective. However, the former is maximized whereas the latter is minimized. In other words, the two decision makers have conflicting objectives.

$$\textbf{(PBM)} \quad \max_{p,\Gamma} \quad \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} p^h x^h_{n,a} - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{n\in N}\sum_{a\in A_n} x^h_{n,a} \qquad \forall h \in H \tag{3.1}$$

$$0 \leq p^h \leq p^h_{max} \qquad \forall h \in H \tag{3.2}$$

$$\min_x \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} p^h x^h_{n,a} + \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} C_{n,a}(h) x^h_{n,a}$$

s.t.

$$0 \leq x^h_{n,a} \leq \beta^{max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.3}$$

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$$\sum_{h \in T_{n,a}} x_{n,a}^h \geq E_{n,a} \qquad \forall n \in N, \forall a \in A_n \tag{3.4}$$

Constraint 3.1: guarantees that the peak load Γ is greater than or equal to the load at each hour. (Since the peak cost is minimized in leader's objective, Γ takes the value of maximum load and the time slot that the constraint holds as an equality is named the **peak slot**.)

Constraint 3.2: defines an upper bound on the prices, without which the problem would be unbounded. The definition of p_{max}^h can be dictated by the state to the firm, can be an outcome of market conditions or a reasonable value chosen by the firm in order to decrease the peak.

Constraint 3.3: defines the device's power consumption limit.

Constraint 3.4: ensures demand satisfaction.

Both leader's and follower's objective functions are bilinear. For fixed leader's variables, the follower's objective function is linear and vice versa.

If the leader were to maximize only revenue, all prices would be set to the upper bound, p_{max}, and all jobs would be scheduled to the preferred slot. Then, the revenue would be maximized whereas the inconvenience cost would be minimized. However, the peak would be very high. Therefore, in order to achieve a smoother supply curve and keep a lower capacity, a peak minimization term is needed in the objective function of the leader. Pricing is the only instrument to transfer the upper level's goal to the lower level. Similarly, the revenue (billing cost for lower level) term is needed in the lower level to perceive the intention of the leader. By means of pricing, the leader shows how much he is willing to pay to decrease the peak load.

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3.1.1 Competitive Version

In this section, we assume that the electricity provider (leader) is not in a monopolistic situation anymore but that there exists a competitor provider who declares its prices before the leader. When defining its pricing strategy, the leader has to take the competitor's market strategy into account as well as the behavior of the smart grid. We assume that the competitor's strategy is fixed.

The smart grid operator decides how much power will be supplied by the leader and by the competitor at each time slot. It is important to emphasize that a customer does *not* choose a supplier for all of his demand, but rather the grid chooses the *total amount* of power to be purchased from each supplier. Customers are not aware of the source of energy in this case.

In the competitive pricing setting, the leader has several issues to deal with. In addition to peak minimization and revenue maximization, as well as price ceiling constraints, market shares will only be retained if its prices are competitive. This might significantly restraint the leader's 'degrees of freedom'. However, the leader has a new option with respect to PBM: it does *not* have to cover all of the demand and it may let the competitor take it over. In other words, when deciding not to decrease some prices with respect to the competitor, the leader may loose some revenue. However, his loss is compensated by the reduction in peak cost.

We assume that competitor prices $\bar{p}^h$ are fixed, and actually assume the values $p^h_{\max}$, without loss of generality. If they were higher, then all customers would choose the leader and the situation would be the same as in PBM. If they were lower, then $p^h_{\max}$ would be irrelevant, and the leader's prices would

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be bounded by $\bar{p}^h$. We also assume that the inconvenience factors are identical, whether electricity is supplied by the competitor or the leader. Finally, we make the conservative assumption that, whenever the smart grid buys energy from the competitor, the customers are automatically scheduled to their most preferred time slots.

The continuous variables $\bar{x}^h_{n,a}$ represent the amount of power purchased from the competitor whereas $x^h_{n,a}$ stands for the leader's part. The objective function of the leader stays the same as in model I. The follower minimizes total cost. In addition to the objective function of the follower in model I, the electricity bill and inconvenience cost of the competitor are included.

$$\textbf{(PCBM)} \max_{p,\Gamma} \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} p^h x^h_{n,a} - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{n\in N}\sum_{a\in A_n} x^h_{n,a} \qquad \forall h \in H \tag{3.5}$$

$$0 \leq p^h \leq p^h_{max} \qquad \forall h \in H \tag{3.6}$$

$$\min_{x,\bar{x}} \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} p^h x^h_{n,a} \quad + \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} \bar{p}^h \bar{x}^h_{n,a}$$
$$+ \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} C_{n,a}(h) \quad (x^h_{n,a} + \bar{x}^h_{n,a})$$

s.t.

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$$x^h_{n,a} + \bar{x}^h_{n,a} \le \beta^{max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.7}$$

$$\sum_{h \in T_{n,a}} (x^h_{n,a} + \bar{x}^h_{n,a}) \ge E_{n,a} \qquad \forall n \in N, \forall a \in A_n \tag{3.8}$$

$$x^h_{n,a}, \bar{x}^h_{n,a} \ge 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.9}$$

Constraints 3.5 and 3.6 are the same as in PBM.

Constraint 3.7: limits the power consumption of each device.

Constraint 3.8: guarantees that demand is satisfied for each customer n and appliance a no matter which firm provides it.

3.1.2 Single Level Formulation

PBM is a bilinear bilevel model involving continuous variables. For fixed leader's decision variables, the lower level objective function is linear. As proposed by Labbé et al. [Labbé 1998], PBM and PCBM can be rewritten as a single level mixed integer problem by replacing the lower level program by its primal-dual optimality conditions and then linearizing the bilinear terms. More precisely, let us first consider PBM.

The follower's mathematical program is replaced by a set of constraints that ensures the optimality of the lower level for fixed upper level variables. The dual and primal constraints of the follower define the feasible region of the follower, while complementary slackness constraints ensure optimality.

Let the dual variables corresponding to constraints (3.3) and (3.4) be denoted as $w^h_{n,a}$, $h \in T_{n,a}$ and $v_{n,a}$, respectively. The dual constraint correspond-

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ing to $x^h_{n,a}$ is expressed as:

$$-w^h_{n,a} + v_{n,a} - p^h \leq C_{n,a}(h) \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a}.$$

Complementary slackness between $x^h_{n,a}$ and the dual constraint takes the form:

$$x^h_{n,a}(w^h_{n,a} - v_{n,a} + p^h + C_{n,a}(h)) = 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \quad (3.10)$$

To linearize constraint (3.10), we introduce binary variables $\psi^h_{n,a}$ and since either $x^h_{n,a}$ or $w^h_{n,a} - v_{n,a} + p^h + C_{n,a}(h)$ must be zero, the nonlinear constraint (3.10) can be replaced by linear ones ($\forall n \in N, \forall a \in A_n, \forall h \in T_{n,a}$):

$$\begin{aligned} w^h_{n,a} - v_{n,a} + p^h + C_{n,a}(h) &\leq M_1(1 - \psi^h_{n,a}) \\ x^h_{n,a} &\leq M_1 \psi^h_{n,a} \\ \psi^h_{n,a} &\in \{0, 1\}. \end{aligned}$$

where M_1 is a sufficiently large number.

Similarly, the complementarity constraints between dual variables and primal constraints are linearized yielding two groups of constraints.

Constraint (3.12) and (3.13): follower's primal constraints.

Constraint (3.15) and (3.16): linearized complementary slackness between primal constraint (3.12) and dual variable $w^h_{n,a}$, upon the introduction

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of binary variables $\xi^t_{n,a}$.

Constraint (3.17) and (3.18): linearized complementarity between primal constraint (3.13) and dual variable $v_{n,a}$ upon the introduction of binary variables $\varepsilon_{n,a}$.

Constraint (3.14): dual constraint of the follower' problem which corresponds to $x^h_{n,a}$.

Constraint (3.19) and (3.20): linearized complementary slackness between dual constraint (3.14) and primal variable $x^h_{n,a}$, upon the introduction of binary variables $\psi^t_{n,a}$.

Finally, due to the presence of identical terms (the *billing cost*) in the objective and the constraints, we obtain a linear expression for the leader's objective, and hence the mixed integer program:

$$\max_{\substack{p,\Gamma,x\\ w,v,\psi}} \; -\sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} \beta^{\max}_{n,a} w^h_{n,a} + \sum_{\substack{n\in N\\ a\in A_n}} E_{n,a} v_{n,a} \quad - \sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} C_{n,a}(h) x^h_{n,a} - \kappa\Gamma \tag{3.11}$$

s.t.

$$\Gamma \geq \sum_{\substack{n,a\\ \text{s.t.}\, h\in T_{n,a}}} x^h_{n,a} \qquad \forall h \in H$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H$$

$$0 \leq x^h_{n,a} \leq \beta^{\max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.12}$$

$$\sum_{h\in T_{n,a}} x^h_{n,a} \geq E_{n,a} \qquad \forall n \in N, \forall a \in A_n \tag{3.13}$$

$$-w^h_{n,a} + v_{n,a} - p^h \leq C_{n,a}(h) \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.14}$$

$$-x^h_{n,a} + M_3 \xi^h_{n,a} \leq M_3 - \beta^{\max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.15}$$

$$w^h_{n,a} - M_3 \xi^h_{n,a} \leq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.16}$$

$$\sum_{h\in T_{n,a}} x^h_{n,a} + M_2 \varepsilon_{n,a} \leq M_2 + E_{n,a} \qquad \forall n \in N, \forall a \in A_n \tag{3.17}$$

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$$v_{n,a} - M_2\varepsilon_{n,a} \leq 0 \qquad \forall n \in N, \forall a \in A_n \tag{3.18}$$

$$w^h_{n,a} - v_{n,a} + p^h + M_1\psi^h_{n,a} \leq M_1 - C_{n,a}(h) \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.19}$$

$$x^h_{n,a} - M_1\psi^h_{n,a} \leq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.20}$$

$$\xi^h_{n,a}, \psi^h_{n,a} \in \{0,1\};\ w^h_{n,a} \geq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a}$$

$$\varepsilon_{n,a} \in \{0,1\};\ v_{n,a} \geq 0 \qquad \forall n \in N, \forall a \in A_n.$$

Similar to PBM, PCBM can be expressed as a single level MIP. Following the previous notation, dual variables $w^h_{n,a}$ and $v_{n,a}$ are associated with (3.7) and (3.8), respectively. Next, the primal, dual and complementary slackness constraints of the lower level are appended to the upper level, while the strong duality of the lower level is utilized to linearize the objective of the leader:

$$\max - \sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} \beta^{\max}_{n,a} w^h_{n,a} + \sum_{\substack{n\in N\\ a\in A_n}} E_{n,a}v_{n,a} - \sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} \bar{p}^h \bar{x}^h_{n,a}$$
$$- \sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} C_{n,a}(h)(x^h_{n,a} + \bar{x}^h_{n,a}) - \kappa\Gamma.$$

Due to the additional lower level variables $\bar{x}^h_{n,a}$, additional dual constraints, together with their complementary slackness constraints have to be added. These are linearized as in PBM ($\forall n \in N, \forall a \in A_n, \forall h \in T_{n,a}$):

$$-w^h_{n,a} + v_{n,a} \leq C_{n,a}(h) + \bar{p}^h$$
$$\bar{x}^h_{n,a} \times (w^h_{n,a} - v_{n,a} + \bar{p}^h + C_{n,a}(h)) = 0.$$

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3.1.3 Experimental Results and Interpretation

In this section, we present a set of results obtained from a number of scenarios, and that demonstrate the applicability of the model. The base case (BC) corresponds to setting all prices at $p_{\max}$ and scheduling all appliances to the preferred time slots. Results of BC, PBM and PCBM models are compared in terms of peak cost, peak load, net revenue, billing and inconvenience costs.

The scenarios involve 10 customers, each one controlling three preemptive appliances regulated by the smart grid. The scheduling horizon is composed of 24 time slots of equal duration.

The models are tested with respect to two parameters, peak weight κ and time window width TWW, which is related to customer flexibility. Peak weight reflects the importance to decrease peak load for the leader. A higher weight translates into a higher penalty, hence an effort to smooth out the supply curve. Both models are solved for 5 values of κ: 200, 400, 600, 800 and 1000.

Wider time windows provide the leader with flexibility to induce job shifting through price adjustments, and thus to smooth out the load curve. TWW is tested for 2 different values, 20% and 100%, which specifies that the time window of a job is 20% or 100% larger than the minimum completion time (MCT), ($MCT := \lceil E_{n,a}/\beta_{n,a}^{\max} \rceil$). In other words, MCT denotes the minimum number of time slots required to meet demand $E_{n,a}$ if we could set all devices at their maximum level $\beta_{n,a}^{\max}$.

For each scenario, parameters $\beta_{n,a}^{\max}$ and $E_{n,a}$ are generated for customer n and appliance a. Then, the early time slot of time window for customer n and appliance a, $TW_{n,a}^{b}$ is generated within 0 and $24 - \lceil (1+TWW) \times MCT \rceil$.

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The end of the time window for customer n and appliance a, $TW^e_{n,a}$ takes the value $TW^b_{n,a} + \lceil (1 + TWW) \times MCT \rceil$. For instance, let $\beta^{\max}$ and E be equal to 2 and 8, respectively, for a given customer n, and appliance a. Then, its MCT is 4 hours. If TWW is 1.0, then its time window can start at some time slot in {0,...,16} and must end 8 hours later. If TWW is 0.20, then $TW^b_{n,a}$ belongs to the interval {0,...,19} and $TW^e_{n,a}$ is $TW^b_{n,a} + 5$.

Although all customers are residential users, they may have different levels of sensitivity to delay and hence, they may behave differently. Therefore, a random inconvenience coefficient $\lambda_{n,a}$ is generated for each customer n. When $\lambda_{n,a}$ assumes a low value, customers are less delay-sensitive, which gives the model more flexibility to find a good schedule. Alternatively, when $\lambda_{n,a}$ assumes a large value, certain time slots become too costly and will almost never be selected.

For experimental purposes, 10 instances are randomly generated by assuming a uniform distribution for $E_{n,a}$, $\beta^{\max}_{n,a}$, $TW^b_{n,a}$ and $\lambda_{n,a}$ and tested for each value of κ. In order to test TWW, similar jobs are used with different widths of time windows.

Both models are solved with CPLEX version 12.3 on a computer with 2.66 GHz Intel Xeon CPU and 4 GB RAM, running under the Windows 7 operating system. Whenever an instance could not be solved within the time limit of 4 hours, the best integer solution has been considered.

The first numerical results are displayed in Tables 3.1 and 3.2. They involve 10 random instances of 30 jobs, in both the monopolistic (PBM) and competitive (PCBM) cases. The user costs are split between electricity bill (EB) and inconvenience (EC), both percentages being relative to the base

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Table 3.1: Cost Comparison of PBM and PCBM on 20% TWW instances (BC = 100%)

κ	PBM EB	PBM IC	PBM TC	PCBM EB	PCBM IC	PCBM TC
200	78.02	21.49	99.51	78.31	21.17	99.48
400	77.15	21.59	98.74	78.01	20.13	98.14
600	75.76	21.85	97.61	77.84	18.74	96.58
800	73.52	22.16	95.68	78.07	16.81	94.88
1000	71.50	22.48	93.99	77.63	16.28	93.91

Table 3.2: Cost Comparison of PBM and PCBM on 100% TWW instances (BC = 100%)

κ	PBM EB	PBM IC	PBM TC	PCBM EB	PCBM IC	PCBM TC
200	85.12	14.16	99.28	85.25	14.06	99.30
400	82.87	14.55	97.42	84.05	13.90	97.95
600	80.13	15.29	95.42	84.67	12.69	97.35
800	75.67	16.29	91.96	84.33	11.84	96.17
1000	74.95	16.44	91.39	83.70	11.45	95.15

case (BC). For instance the first line of Table 3.1 indicates that in PBM, out-of-pocket cost is 78.02% and inconvenience cost is 21.49% of the total cost corresponding to BC, the total (TC) being 0.5% less than in BC, for which the billing cost is the highest. PBM and PCBM result in a 2.9% and 3.4% total cost reduction, respectively for 20% TWW instances and a 4.9% and 2.8% total cost reduction for 100% instances. All values are less than 100%, which points out the cost improvement for customers for any peak weight value.

In comparison with the base case, the leader sets lower prices in order to shift some jobs to the off-peak hours, hence the customers' bill is naturally reduced and inconvenience cost is increased. When peak weight κ is large, the leader is willing to give up some revenue in order to achieve a smoother load curve. Hence, it lessens the bill as well. Note that EB is lower in PBM than in PCBM whereas IC is higher. When the leader is a monopoly, he has to

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Table 3.3: Comparison of PBM and PCBM with 20% TWW Instances

	Av Comp Time		Av Gap		# unsolved	
κ	PBM	PCBM	PBM	PCBM	PBM	PCBM
200	1.10	1.20	0.00%	0.00%	0	0
400	3.10	3.50	0.00%	0.00%	0	0
600	6.80	13.10	0.00%	0.00%	0	0
800	8.90	56.60	0.00%	0.00%	0	0
1000	17.90	63.00	0.00%	0.00%	0	0

provide service to all customers. However, in the competitive case, he has the option to give up on some load in order to decrease the peak without lowering prices. According to this reasoning, IC increases as κ increases in both tables for PBM, whereas it decreases for PCBM.

Although the total cost of the follower for 20% TWW instances is lower in the presence of competition, it is not the case for 100% TWW instances, and there lies an interesting fact. For instance, suppose that peak consists of a light-load and a heavy-load job alongside others, and that they are both required to be shifted in order to decrease the peak. Keeping in mind that the heavy-load job has a high inconvenience factor, the leader would have to decrease the price at least by that amount in the monopolistic case. Then, the light-load job would enjoy a price reduction that is larger than its inconvenience, and the total cost would be lower for the light-load job and identical for the heavy one. In contrast, the leader can now give up on the heavy-load job in the competitive case and decrease the price only with respect to the light-load job. Hence, total cost would stay the same for both jobs. This is valid mostly for large time windows, because there are far fewer options for job shifting in 20% instances.

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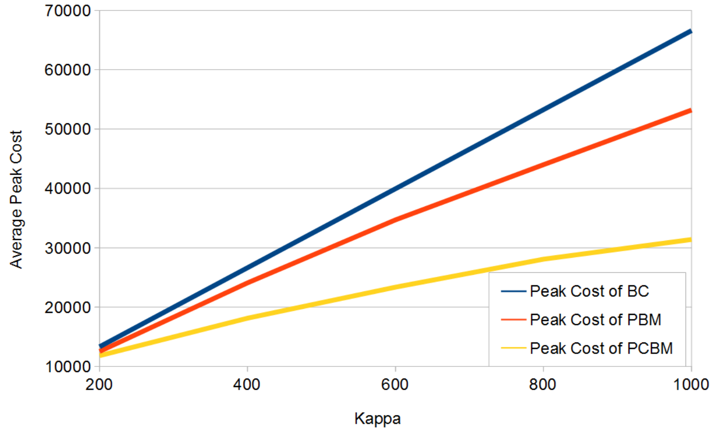
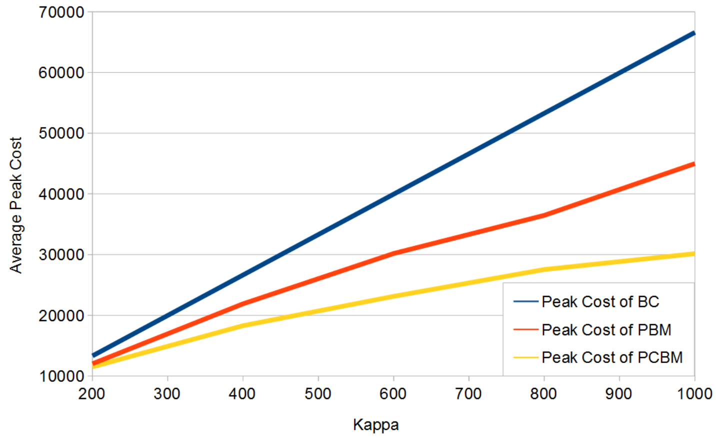
Table 3.4: Comparison of PBM and PCBM with 100% TWW Instances

	Av Comp Time		Av Gap		# unsolved	
κ	PBM	PCBM	PBM	PCBM	PBM	PCBM
200	28.10	321.10	0.00%	0.00%	0	0
400	339.50	592.67	0.00%	0.55%	0	1
600	2040.20	1232.88	0.00%	3.87%	0	2
800	4666.40	2350.67	0.00%	3.73%	0	4
1000	7707.00	2034.14	0.00%	6.04%	0	3



(a) Peak Cost for 20% TWW Instances



(b) Peak Cost for 100% TWW Instances

Figure 3.3: Peak Cost Comparison

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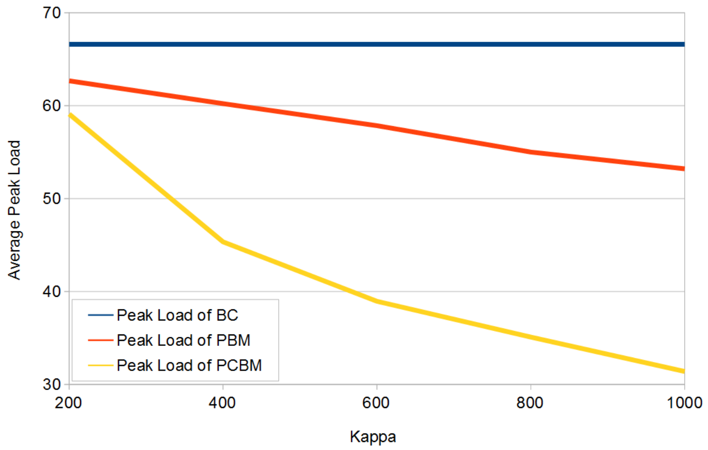
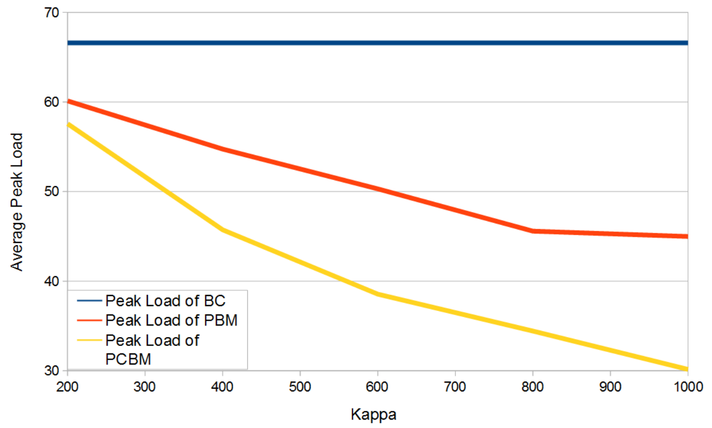


(a) Peak Load for 20% TWW Instances



(b) Peak Load for 100% TWW Instances

Figure 3.4: Peak Load Comparison

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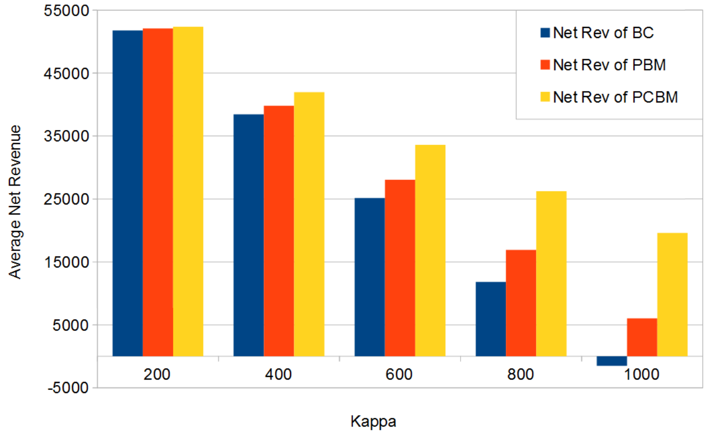
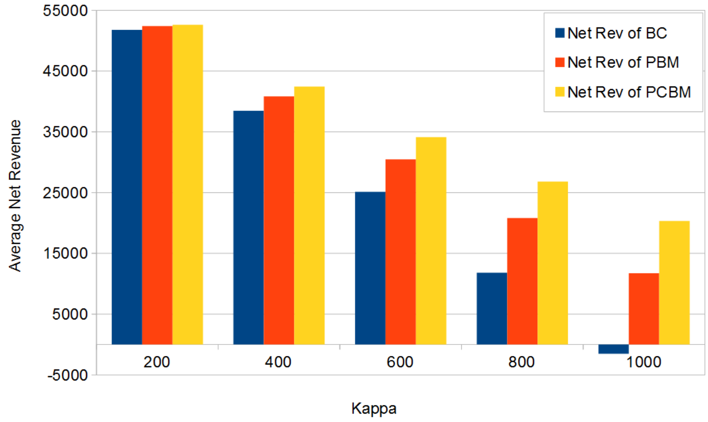


(a) Net Revenue for 20% TWW Instances



(b) Net Revenue for 100% TWW Instances

Figure 3.5: Net Revenue Comparison

In Tables 3.3 and 3.4, the computational results of PBM and PCBM are compared for 20% and 100% TWW instances, respectively. Average computing time (in seconds), the average optimality gap of unsolved instances,

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and the number of unsolved instances are presented. As aforementioned,the running time limit is set to 4 hours (14400 seconds) and the values given in the tables are the average computation time over all instances that could be solved within 4 hours.

In both tables, the average computation time of both models increases together with peak weight κ, since the leader is more willing to modify its prices in order to smooth the load curve, providing extra room for improvement. The average gaps and number of unsolved instances also support this argument. In addition, one can observe that average computation time is larger in Table 3.4. Large time windows induce high running times since there are more options to consider. Another important point is that, on average, PCBM takes longer to solve if we include the unsolved instances. This result can be explained by the increased combinatorics, the leader having the alternative to provide energy for a job or leave it to the competitor. It is in accordance with real life problems, when there is competition, the decision making process of the firms becomes more challenging.

Peak cost, peak load and net revenue (objective function value of leader) comparisons of PBM and PCBM to BC for 20% instances are shown in Figures 3.3(a), 3.4(a) and 3.5(a), respectively. Similar values are shown for 100% instances in Figures 3.3(b), 3.4(b) and 3.5(b). The x-axis consists of the peak weight parameter κ and the y-axis represents the monetary value in Figures 3.3 and 3.5, whereas it represents peak power usage in Figure 3.4. In Figure 3.3, it can be observed that peak costs for PBM and PCBM increases slower than the peak cost of BC, as weight κ increases. Since there is a possibility of *not* satisfying some of the demand for the leader in PCBM, peak load

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and hence peak cost is lower than in PBM, as expected. In Figures 3.3(a) and 3.3(b), it can be observed that peak cost decreases in PBM when time windows are wide, whereas it does not change much in PCBM. In accordance, in Figures 3.4(a) and 3.4(b), it is clear that peak load decreases in PBM. As a result, net revenue in PBM increases considerably when time windows widen.

We now turn our attention the the leader's revenue. Average net revenue of BC is dominated by that of PBM, and the latter is dominated by PCBM. Both bilevel models provide a higher *net revenue* despite the discount on some prices. In view of the peak cost constraint, the leader can adjust its pricing strategy to increase total revenue. Perhaps more surprising, it can benefit from an open market by willingly letting demand flow to the competition, for the sake of meeting the peak constraint. It is important to note that the model behavior is very similar in both the 20% and 100% instances. On average, PBM provides a 13.71% and 24.34% net revenue increase with respect to BC on 20% and 100% TWW instances, respectively. Meanwhile, PCBM provides a 38.31% and 40.31% net revenue increase with respect to BC on 20% and 100% TWW instances, respectively.

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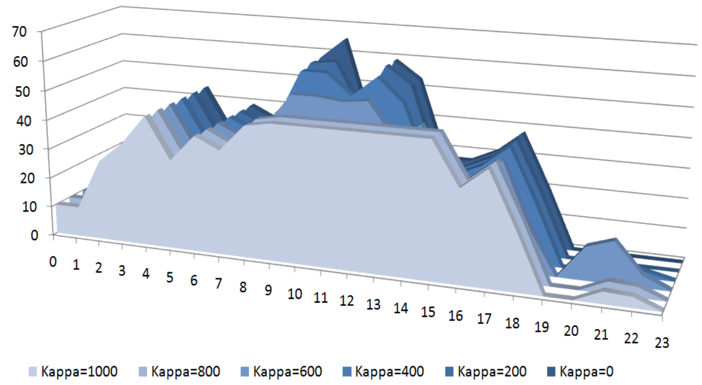
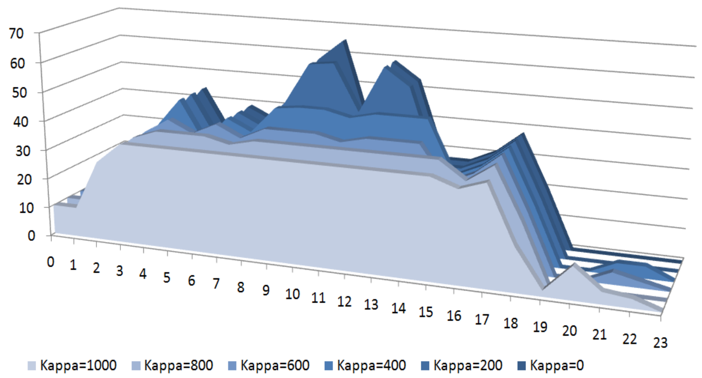


(a) Load Distribution of an Instance (PBM)



(b) Load Distribution of an Instance (PCBM)

Figure 3.6: Load Comparison

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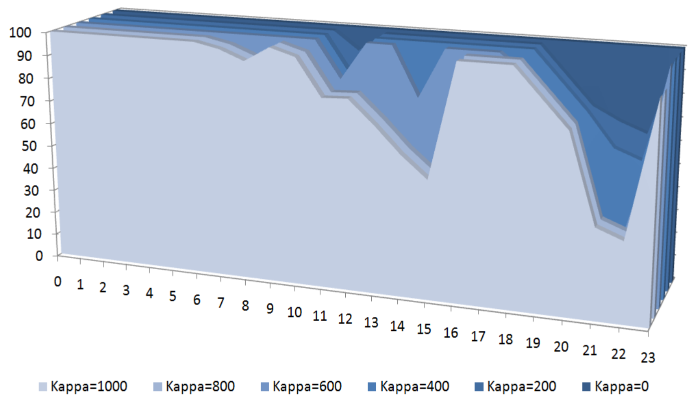
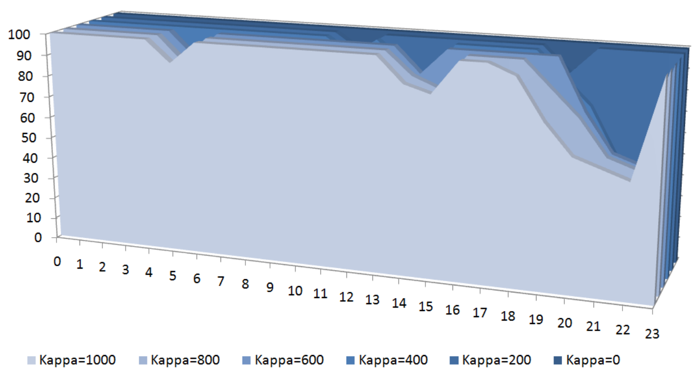


(a) Leader Prices of an Instance (PBM)



(b) Leader Prices of an Instance (PCBM)

Figure 3.7: Price Comparison

Different load shapes are handled differently by the model. If the initial load curve has a single 'high' peak, then it attempts to assign attractive prices around the peak to shift some of the load to later periods. However, when

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there are more than one peak, the shifting issue becomes more complex. Load distributions with respect to different values of κ of an instance under PBM and 100% TWW are shown in Figure 3.6(a). When κ is less than 400, the model ignores the peak at time 12 and focuses on the one at time 10. However, as κ increases, the model tries harder and harder to level the load curve around both peaks. In Figure 3.7(a), where the corresponding price vectors are displayed, low prices illustrate the effort of the leader to shift jobs around. The magnitude of price reduction escalates as κ increases. Besides, it is again clear that prices around the other peak start moving when κ exceeds the value 600. These two graphs provide a better picture of how an energy provider can achieve an optimal trade-off between revenue maximization and peak minimization.

When PCBM is solved on instances of Figures 3.6(a) and 3.7(a), load distribution and prices change as shown in Figures 3.6(b) and 3.7(b), respectively. The leader leaves some load to the competition in return for lower peak value. By applying this strategy, it manages to keep prices higher than in the monopolistic case and achieves a smaller generation capacity. It is further observed that the leader tries to decrease peak to the level of the second highest load value (SHL). In order to achieve this, two strategies are exercised: if the time slot following peak hour has small load, then the leader opts for shifting some load to that time slot. Else, if the difference between peak load and SHL is larger than the difference between SHL and the load at the time slot following the peak, then the residual is left to the competition.

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3.2 Non-preemptive Bilevel Model

In this section, we consider a power sharing system with N customers who own A_n nonpreemptive devices (follower) and a monopolistic power supplier (leader). Similar to PBM, the leader decides on the prices p^h of time slots $h \in H$, then the smart grid schedules the jobs, or in other words, chooses starting time $x^h_{n,a}$ of appliance $a \in A_n$ for customer $n \in N$.

The leader's objective is to find the optimal trade-off between revenue and peak cost. Peak load Γ and peak penalty κ are defined similar to §3.1. The follower's objective is to minimize the sum of billing and inconvenience costs.

When an appliance is started, it has a fixed power consumption per time slot, $k_{n,a}$, during a fixed period, $l_{n,a}$. Besides, every appliance has a time window given by customers which is defined by beginning and end times, $T_{n,a} = [TW^b_{n,a}, TW^e_{n,a}]$, and it cannot be started outside its time window.

Desirability of a time slot decreases linearly within a time window and a job can be started at any slot within the time window in exchange for a penalty cost. We consider the inconvenience factor of a job to be a linear penalty function that is directly proportional to delay length. It is inversely proportional to the width of the desired time window, reflecting the fact that customers that specify narrow time windows are likely to be sensitive to the delay of their tasks by the smart grid. The inconvenience factor of a job, $C_{n,a}(h)$ is computed is slightly different than in the preemptive case, job load and time length are used in place of total demand.

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$$C_{n,a}(h) := \lambda_{n,a} \times k_{n,a} \times l_{n,a} \times \frac{(h - TW^b_{n,a})}{(TW^e_{n,a} - TW^b_{n,a})}$$
$$\forall n \in N, \forall a \in A_n, \forall h \in T_{n,a}.$$

Now we present the bilinear bilevel mathematical model with nonpreemptive appliances and monopolistic pricing:

$$\text{(NBM):} \max_{p,\Gamma} \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} k_{n,a} x^h_{n,a} \sum_{h'=h}^{h+l_{n,a}} p^{h'} \quad - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{n \in N} \sum_{a \in A_n} \sum_{\substack{h'=h-l_{n,a} \\ s.t. h' \in T_{n,a}}}^{h} k_{n,a} x^h_{n,a} \qquad \forall h \in H \tag{3.21}$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H \tag{3.22}$$

$$\min_x \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} k_{n,a} x^h_{n,a} \sum_{h'=h}^{h+l_{n,a}} p^{h'} \quad + \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} C_{n,a}(h) x^h_{n,a}$$

s.t.

$$\sum_{h \in T_{n,a}} x^h_{n,a} = 1 \qquad \forall n \in N, \forall a \in A_n \tag{3.23}$$

$$x^h_{n,a} \in \{0, 1\} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.24}$$

Constraint 3.21: When $x^h_{n,a}$ takes value 1 for a time slot h, it brings a power load of $k_{n,a}$ during $l_{n,a}$ time slots. Peak load Γ is greater than or equal to the value of load during H time slots.

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Constraint 3.23: ensures demand satisfaction.

3.2.1 Non-preemptive Competitive Bilevel Model

Now we turn our attention to a competitive version of non-preemptive model. Similar to §3.1.1, it is assumed that a competitor that offers fixed prices $\bar{p}^h, \forall h \in H$ is interested in providing energy to the clients. In this model, all jobs are non-preemptive. Therefore, whole demand of a job should be ensured by one of the firms. Moreover, jobs are assigned to the most desired time slot when the competitor supplies the energy since its decision process is not a part of this model. The model is as follows where $\bar{x}^h_{n,a}$ represents the competitor supply:

$$\text{(NCBM):} \max_{p,\Gamma} \sum_{n,a,h} k_{n,a} x^h_{n,a} \sum_{h'=h}^{h+l_{n,a}} p^{h'} - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{n,a} \sum_{\substack{h'=h-l_{n,a} \\ s.t. h' \in T_{n,a}}}^{h} k_{n,a} x^h_{n,a} \qquad \forall h \in H \tag{3.25}$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H \tag{3.26}$$

$$\min_{x,\bar{x}} \sum_{n,a,h} k_{n,a} x^h_{n,a} \sum_{h'=h}^{h+l_{n,a}} p^{h'} + \sum_{n,a,h} k_{n,a} \bar{x}^h_{n,a} \sum_{h'=h}^{h+l_{n,a}} \bar{p}^{h'} \tag{3.27}$$

$$+ \sum_{n,a,h} C_{n,a}(h)(x^h_{n,a} + \bar{x}^h_{n,a})$$

s.t.

$$\sum_{h \in T_{n,a}} x^h_{n,a} + \bar{x}^h_{n,a} = 1 \qquad \forall n \in N, \forall a \in A_n \tag{3.28}$$

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$$x^h_{n,a}, \bar{x}^h_{n,a} \in \{0,1\} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.29}$$

The objective function of the leader is identical to the one of NBM since the competitor does not have a direct effect on the revenue or peak load of the leader. However, it plays a major role in the lower level's objective function. It minimizes the total of billing cost paid to the leader, billing cost paid to the competitor and inconvenience cost that is caused by both firms.

Constraint 3.25: defines peak load. When $x^h_{n,a}$ takes value 1 for a time slot h, it brings a power load of $k_{n,a}$ during $l_{n,a}$ time slots. Peak load Γ takes the value of the highest load during H many time slots.

Constraint 3.28: guarantees demand satisfaction of customers, no matter which firm provides it.

Constraint 3.29: Both variables are binary since the devices are non-preemptive and should be started only once.

3.2.2 Single Level Formulation

The bilevel program NBM involves binary variables in the lower level. It belongs to a special class: assignment problem. In our case, every device must be assigned to a time slot within its time window. However, there might be time slots without any devices. This property makes the follower's problem a simpler version of assignment problem. Therefore, we propose a new method to rewrite the bilevel program as a MIP by first relaxing the integrality constraints (3.29) and adding KKT conditions to the upper level [Labbé 1998]. Afterwards, we include the integrality constraints (3.29) on the

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upper level and hence obtain a MIP. We show that although some solutions might be discarded, one optimal integer solution always remains.

There might be several identical lower level solutions corresponding to the same price vector. Since the optimistic approach is adopted, the solution that yields the highest net revenue for the leader is selected. When we solve the aforementioned MIP, the solution that is favored by the leader might be noninteger. For instance, in our problem, a noninteger solution might provide a lower peak load and hence be more preferable for the energy provider. It is illustrated on a small example in Figure 3.8. For the sake of simplicity, there is only one job to be assigned in this example with $k = 10$ and $l = 1$ with time window $[0, 1]$. The price vector of these two hours is $p = (10, 8)$ and the inconvenience factor is $C = (0, 20)$. Peak weight is $\kappa = 5$. It means that if $x = (1, 0)$, the load curve is as in Figure 3.8(a), total cost for the follower is $10 \times 10 + 0 = 100$ and the net revenue of the leader is $10 \times 10 - 5 \times 10 = 50$. If $x = (0, 1)$, the load curve is as in Figure 3.8(b), total cost for the follower is $8 \times 10 + 20 \times 1 = 100$ and net revenue of the leader is $8 \times 10 - 5 \times 10 = 30$. And finally, if $x = (0.5, 0.5)$, the load curve is as in Figure 3.8(c), the total cost for the follower is $5 \times 10 + 5 \times 8 + 20 \times 0.5 = 100$ and the net revenue of the leader is $10 \times 5 + 8 \times 5 - 5 \times 5 = 65$. In all three cases, the total cost is the same for the follower, however the third solution is the best for the leader which is noninteger.

In order to avoid noninteger solutions, another way of reformulating a MIP model is developed. Let us consider an integer program (assignment problem) denoted as (P) and its linear relaxation (R1).

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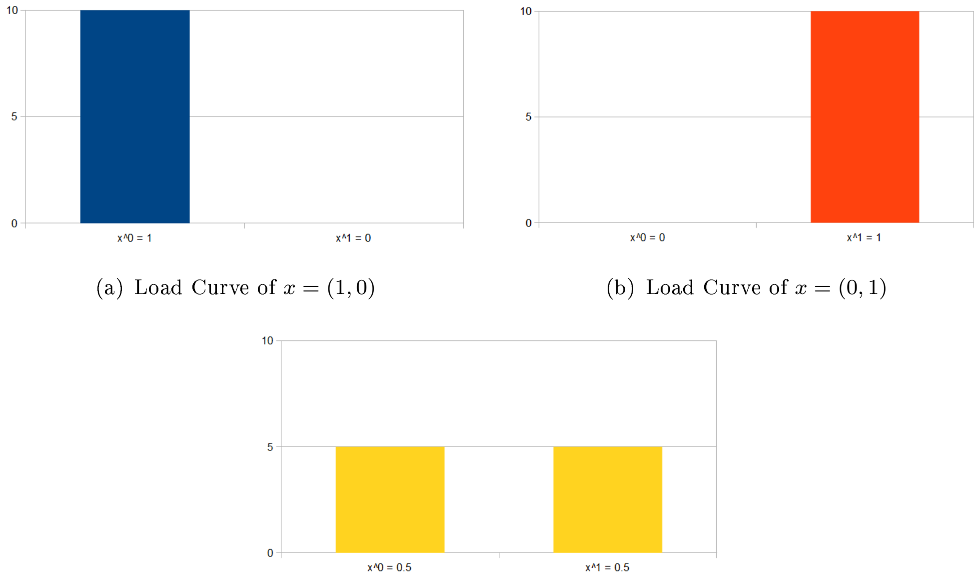

(a) Load Curve of $x = (1, 0)$

(b) Load Curve of $x = (0, 1)$

(c) Load Curve of $x = (0.5, 0.5)$

Figure 3.8: Toy Example

$$(\mathrm{P}) \quad \min \sum_{i,j} c_{i,j} x_{i,j}$$

$$\sum_i x_{i,j} = 1 \qquad \forall j \qquad (3.30)$$

$$x_{i,j} \in \{0, 1\} \qquad \forall i, j \qquad (3.31)$$

$$(\mathrm{R1}) \quad \min \sum_{i,j} c_{i,j} x_{i,j}$$

$$\sum_i x_{i,j} = 1 \qquad \forall j \qquad (3.32)$$

$$x_{i,j} \in [0, 1] \qquad \forall i, j \qquad (3.33)$$

By using (R1), we can write another model, (R2), which consists of the primal and dual constraints along with CSCs. The dual variables corresponding to constraints (3.32) and (3.33) are v_j and $w_{i,j}$, respectively.

$$(\mathrm{R2}) \quad \min \ 0$$

$$\sum_i x_{i,j} = 1 \qquad \forall j \qquad (3.34)$$

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$$x_{i,j} \leq 1 \qquad \forall i,j \qquad (3.35)$$

$$u_j - w_{i,j} \leq c_{i,j} \qquad \forall i,j \qquad (3.36)$$

$$x_{i,j}(c_{i,j} - u_j + w_{i,j}) = 0 \qquad \forall i,j \qquad (3.37)$$

$$w_{i,j}(1 - x_{i,j}) = 0 \qquad \forall i,j \qquad (3.38)$$

$$x_{i,j}, w_{i,j} \geq 0 \qquad \forall i,j \qquad (3.39)$$

Lastly, the nonlinear CSCs (3.37) and (3.38) are linearized by adding the integrality constraint of variable $x_{i,j}$ where M_1 and M_2 are sufficiently large numbers and hence (R3) is built:

$$\text{(R3)} \quad \min \; 0$$

$$\sum_i x_{i,j} = 1 \qquad \forall j \qquad (3.40)$$

$$u_j - w_{i,j} \leq c_{i,j} \qquad \forall i,j \qquad (3.41)$$

$$c_{i,j} - u_j + w_{i,j} \leq M_1(1 - x_{i,j}) \qquad \forall i,j \qquad (3.42)$$

$$w_{i,j} \leq M_2 x_{i,j} \qquad \forall i,j \qquad (3.43)$$

$$x_{i,j} \in \{0,1\} \qquad \forall i,j \qquad (3.44)$$

$$w_{i,j} \geq 0 \qquad \forall i,j \qquad (3.45)$$

Theorem 1. *(R1) has an integer optimal solution $\iff$ it is feasible for (R3).*

Proof. Suppose that (R1) has an integer optimal solution, denoted as x^*_{int}. Then, it means x^*_{int} is feasible with respect to constraint (3.32) and hence, is optimal for (P) as well.

(R2) consists of the optimality conditions of (R1). Therefore, by definition,

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the feasible region of (R2) has only the optimal solution(s) of (R1). Besides, the feasible region of (R3) has only the integer optimal solution(s) of (R1). Hence, x^*_{int} would be feasible for (R2) and (R3).

Assume that (R1) only has a noninteger optimal solution, x^*_{real}. Then, it is suboptimal for (P). The feasible space defined by (R2) consists of only x^*_{real}. Then, x^*_{real} is infeasible for (R3). In fact, the feasible region defined by (R3) is empty, i.e., (R3) is infeasible. □

Based on Theorem 1, we can reformulate NBM as a MIP. If it is feasible, then the only feasible solution is the optimal solution of NBM as well. The MIP formulation:

$$\max_{x,u,w,\Gamma} \sum_{\substack{n\in N\\ a\in A_n}} u_{n,a} - \sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} w^h_{n,a} - \sum_{\substack{n\in N\\ a\in A_n\\ h\in T_{n,a}}} C_{n,a}(h) x^h_{n,a} - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{n\in N} \sum_{a\in A_n} \sum_{\substack{h'=h-l_{n,a}\\ s.t. h'\in T_{n,a}}}^{h} k_{n,a} x^h_{n,a} \qquad \forall h \in H \tag{3.46}$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H \tag{3.47}$$

$$\sum_{h\in T_{n,a}} x^h_{n,a} = 1 \qquad \forall n \in N, \forall a \in A_n \tag{3.48}$$

$$u_{n,a} - w^h_{n,a} \leq k_{n,a} \sum_{h'=h}^{h+l_{n,a}} p^{h'} + C_{n,a}(h) \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{3.49}$$

$$k_{n,a} \sum_{h'=h}^{h+l_{n,a}} p^{h'} + C_{n,a}(h) - u_{n,a} + w^h_{n,a} \leq M_1(1 - x^h_{n,a})$$

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$$\forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \quad (3.50)$$

$$w_{n,a}^h \leq M_2 x_{n,a}^h \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \quad (3.51)$$

$$x_{n,a}^h \in \{0,1\}, w_{n,a}^h \geq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \quad (3.52)$$

Dual variables u and w correspond to primal lower level constraints (3.23) and (3.24), respectively.

Constraint 3.46 and 3.47: upper level constraints of NBM.

Constraint 3.48: lower level primal constraint of NBM.

Constraint 3.49: dual constraint corresponding to lower level variable x of NMP.

Constraint 3.50 and 3.51: linearized complementary slackness constraints of NBM's lower level using the integrality constraint (3.52) of x.

The objective function of NBM contains a bilinear term which means it is linear when one of the variables is fixed. Therefore, using the objective function of lower level's dual problem, the bilinear expression is replaced with a linear equivalent.

3.2.3 Experimental Results and Interpretation

In this section, test results of the non-preemptive bilevel model and its competitive version are presented. All results are obtained using the classical exact method (CEM) which means solving the single level MIP model with a commercial solver. The results of NBM and NCBM are compared to the base case (BC) where BC corresponds to setting all prices to $p_{\max}$ and assigning all appliances to the first preferred time slots.

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The instances that are presented in this section involve 12 customers and each one owns 5 non-preemptive devices that are connected to the smart grid. The scheduling horizon is composed of 24 time slots. The models are tested with peak weight κ and time window width TWW, with values for $\kappa \in \{200, 400, 600, 800, 1000\}$ and for TWW 20% and 100%, same as §3.1.3.

In non-preemptive models, the minimum completion time (MCT) is equal to $l_{n,a}$. Therefore, $TWW = \lceil 1.2 \times l_{n,a} \rceil$ and $TWW = \lceil 2.0 \times l_{n,a} \rceil$ for 20% and 100% TWW instances, respectively. In order to test the models, 10 random instances are generated by using uniform distribution for $k_{n,a}$, $l_{n,a}$, $TW^b_{n,a}$ and $\lambda_{n,a}$.

For each scenario, parameters $k_{n,a}$ and $l_{n,a}$ are generated randomly for customer n and appliance a. Then, the early time slot of time window for customer n and appliance a, $TW^b_{n,a}$ (beginning of time window) is generated within 0 and (24-TWW). The end of time window for customer n and appliance a, $TW^e_{n,a}$ takes the value $TW^b_{n,a} + TWW$.

Like in §3.1.3, a random inconvenience coefficient $\lambda_{n,a}$ is generated for each customer n. The inconvenience penalty function $C_{n,a}(h)$ is directly proportional to $\lambda_{n,a}$ and demand $l_{n,a}$ and duration $k_{n,a}$, and inversely proportional to time window width.

Both models are solved with CPLEX version 12.3 on a computer with 2.66 GHz Intel Xeon CPU and 4 GB RAM, running under the Windows 7 operating system. Whenever an instance could not be solved within the time limit of 4 hours, the best integer solution has been considered.

The cost comparison of NBM and NCBM are presented in Tables 3.5 and 3.6, for 20% and 100% TWW instances, respectively. The tables display the

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Table 3.5: Cost Comparison of NBM and NCBM on 20% TWW instances (BC = 100%)

κ	NBM EB	NBM IC	NBM TC	NCBM EB	NCBM IC	NCBM TC
200	99.43	0.15	99.58	99.43	0.15	99.58
400	99.12	0.21	99.33	98.86	0.30	99.16
600	98.86	0.23	99.10	98.35	0.58	98.94
800	98.69	0.25	98.93	98.53	0.37	98.90
1000	98.69	0.25	98.93	98.49	0.43	98.92

Table 3.6: Cost Comparison of NBM and NCBM on 100% TWW instances (BC = 100%)

κ	NBM EB	NBM IC	NBM TC	NCBM EB	NCBM IC	NCBM TC
200	97.61	1.07	98.67	97.86	2.12	99.98
400	97.09	1.20	98.29	97.35	2.57	99.92
600	96.87	1.25	98.12	97.04	2.71	99.75
800	96.45	1.26	97.71	96.72	2.84	99.56
1000	96.42	1.27	97.69	95.35	4.09	99.44

average of 10 instances with 60 jobs for both models for 5 different peak weight values. The cost of customers are given in terms of electricity bill (EB), inconvenience cost (IC) and total cost (TC). All percentages are given with respect to BC.

It can be observed that all values are less than 100%, which points out the cost improvement for customers for any peak weight value. Besides, all types of cost decrease as κ increases. Cost of 100% TWW instances decreases more than 20% in monopolist case (NBM) which indicates that having wider time windows is beneficial for the customers. In 100% TWW instances, EB values are lower and IC is higher. This shows that the leader has more options to shift some jobs and to achieve a smoother load curve.

Table 3.7 displays the average computation time (secs), average gap (%) and number of unsolved instances within 4 hours for both models. It is important to note that the results are given only for 100% TWW instances since all

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20% TWW instances are solved to optimality under 1 second, all optimality gaps and number of unsolved instances are zero.

Table 3.7: Comparison of NBM and NCBM with 100% TWW Instances

	Av Comp Time		Av Gap		# unsolved	
κ	NBM	NCBM	NBM	NCBM	NBM	NCBM
200	9146	6815	0.46	0.31	4	2
400	7746	9101	0.38	0.33	2	2
600	4590	7305	0.00	0.55	0	2
800	7118	9269	0.21	0.61	1	1
1000	6183	6391	0.00	1.08	0	1

In Table 3.7, it can be observed that the problem does not necessarily get more difficult as κ increases which distinguishes non-preemptive models from the preemptive ones. Also, NCBM does not take particularly longer or shorter time than NBM to solve. However, it is clear that TWW has a direct impact on the solution time and quality.

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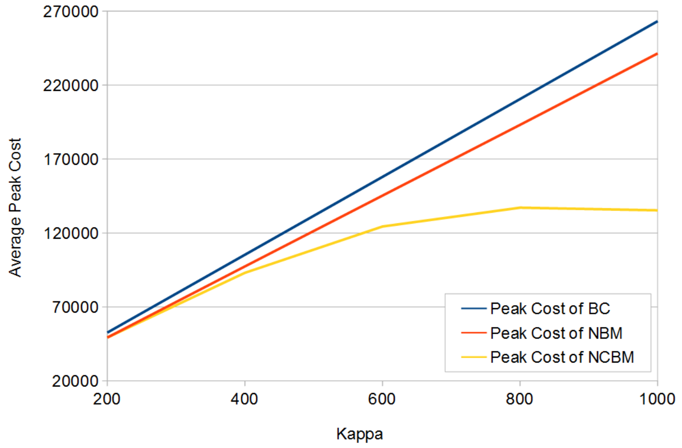
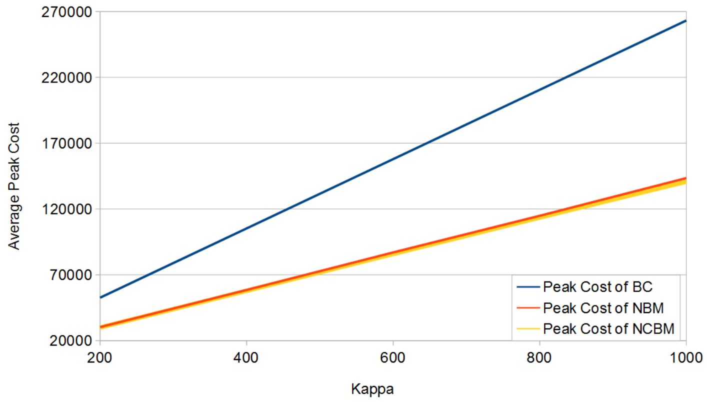


(a) Peak Cost for 20% TWW Instances



(b) Peak Cost for 100% TWW Instances

Figure 3.9: Peak Cost Comparison

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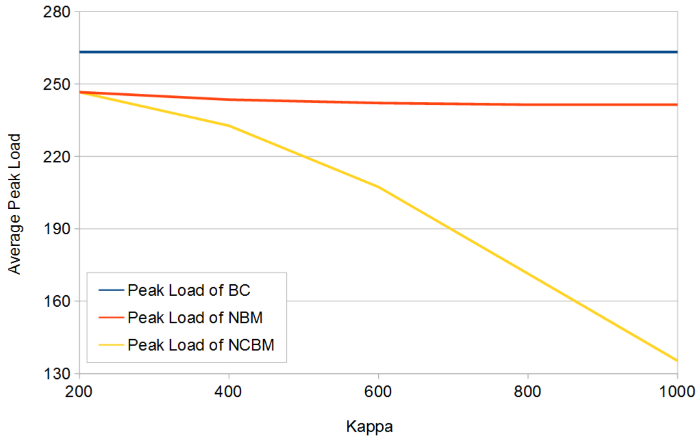
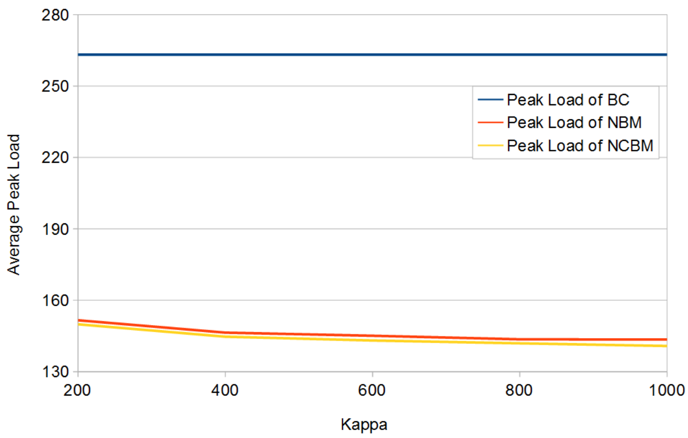


(a) Peak Load for 20% TWW Instances



(b) Peak Load for 100% TWW Instances

Figure 3.10: Peak Load Comparison

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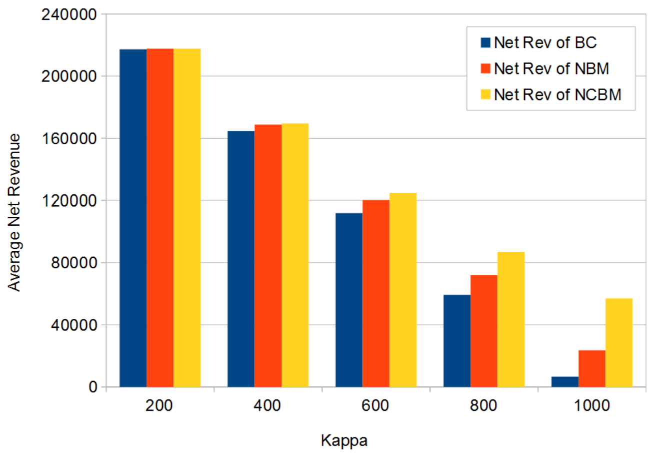
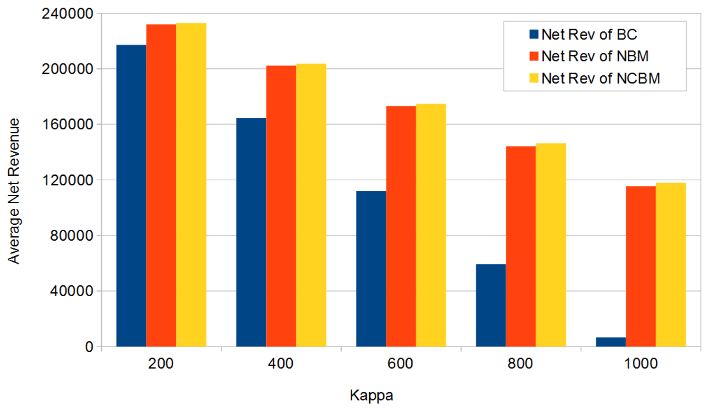


(a) Net Revenue for 20% TWW Instances



(b) Net Revenue for 100% TWW Instances

Figure 3.11: Net Revenue Comparison

Peak cost, peak load and net revenue (objective function value of leader) comparisons of NBM and NCBM to BC for 20% TWW instances are shown in Figures 3.9(a), 3.10(a) and 3.11(a), respectively. For 100% TWW instances,

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the same values can be found in Figures 3.9(b), 3.10(b) and 3.11(b). The x-axis represents peak weight parameter κ and the y-axis represents the monetary value in Figures 3.9 and 3.11, whereas it represents peak power usage in Figure 3.10.

In Figure 3.9(a), peak cost of NCBM increases much slower than NBM and BC. NBM also has a less steep increase than BC which is linear by definition. In the competitive version, the leader can let go of some load to reduce peak if it is more beneficial. Besides, the monopolist solution is always feasible for the competitive model. Therefore, it is clear that the leader exploits this option more and more as peak weight increases. This claim is also supported by Figure 3.10(a). However, in Figure 3.9(b) and Figure 3.10(b), one can observe that peak loads and peak costs of NBM and NCBM are almost the same. In this case, the jobs have wider time windows which gives the leader more options to lower the peak load without passing them on to the competitor. Net revenue values that are depicted in Figure 3.11(a) and 3.11(b) reinforce this conclusion. Wider time windows provide the leader a better chance to compete with its rival. Besides, wider TWW increases the overall net revenue in both models.

On average, NBM provides a 57.64% and 374.53% net revenue increase with respect to BC on 20% and 100% TWW instances, respectively. Meanwhile, NCBM provides a 164.73% and 383.49% net revenue increase with respect to BC on 20% and 100% TWW instances, respectively.

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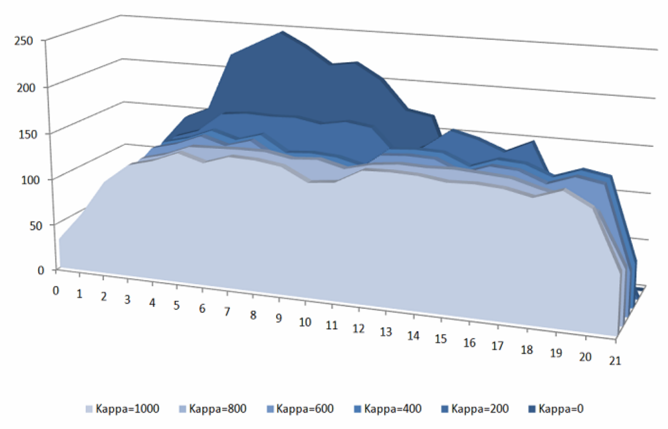
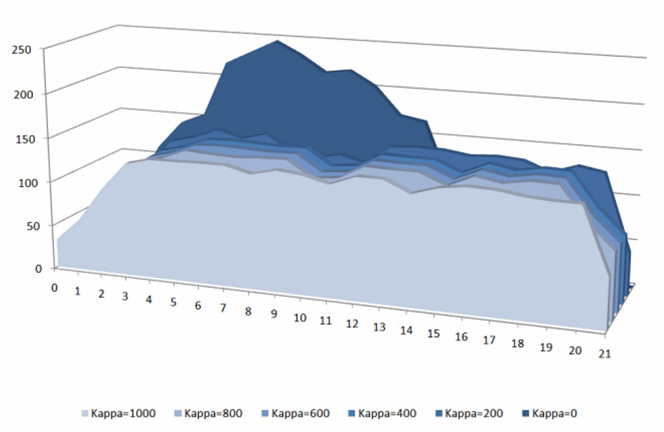


(a) Load Distribution of an Instance (NBM)



(b) Load Distribution of an Instance (NCBM)

Figure 3.12: Load Comparison

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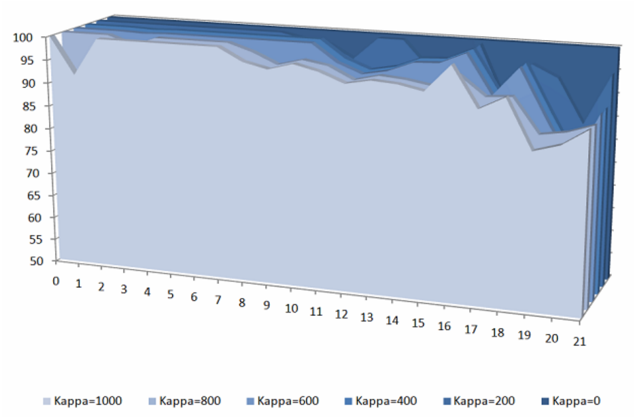
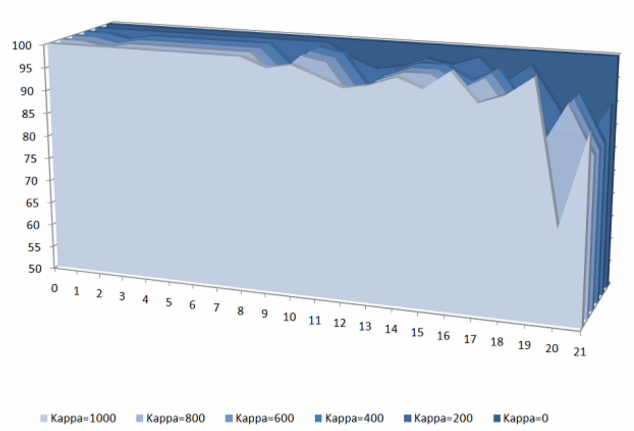


(a) Leader Prices of an Instance (NBM)



(b) Leader Prices of an Instance (NCBM)

Figure 3.13: Price Comparison

Load distributions with respect to different values of κ under NBM and NCBM for a 100% TWW instance are shown in Figure 3.12(a) and Figure 3.12(b), respectively. In Figure 3.12(a), when peak weight is 200, the peak is

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smoothed out and several smaller peaks occur at later time slots. This trend continues as peak weight increases. In Figure 3.12(b), a similar behavior can be observed. However, the load curve is more flat since some load is transferred to the competitor. Prices with respect to different values of κ under NBM and NCBM for a 100% TWW instance are shown in Figure 3.13(a) and Figure 3.13(b), respectively. One can recognize that the load behavior and price changes follow the same pattern. Besides, the prices of NBM change more drastically than NCBM since it is not always profitable to reduce prices to balance the load curve.

3.3 Mixed Bilevel Model

In this section, a mixed model involving both preemptive and non-preemptive appliances is presented. The leader maximizes net revenue by deciding on prices and the follower minimizes total cost by scheduling the use of each appliance.

At the lower level there are N customers, each of them owning A_n^1 preemptive and A_n^2 many non-preemptive devices to use during H time slots. Each appliance has an adequate time window T_{n,a_1} and T_{n,a_2} for type 1 (preemptive) and type 2 (non-preemptive devices respectively, as explained in the previous sections. Non-preemptive devices have fixed power load k_{n,a_2} and operating time l_{n,a_2} whereas preemptive devices have a device power limit $\beta_{n,a_1}^{\max}$ and total power demand E_{n,a_1}.

The objective function of the lower level is minimization of total billing and inconvenience cost. Both terms consist of two parts dedicated to different

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types of appliances. The inconvenience parameter is $C_{n,a_1}(h)$ and $C_{n,a_2}(h)$, the decision variables are x^h_{n,a_1} and y^h_{n,a_2} for for type 1 and 2 appliances, respectively.

The leader's objective function is similar to the previous models. Total revenue term has two parts that come from different types of appliances. Prices p^h are the decision variables of upper level and they are defined for each time period $h \in H$. The other decision variable is peak load Γ which is the highest amount of power consumption throughout the scheduling period.

$$\text{(MBM):}\ \max_{p,\Gamma} \sum_{\substack{n\in N\\ a_2\in A^2_n\\ h\in T_{n,a_2}}} k_{n,a_2} y^h_{n,a_2} \sum_{h'=h}^{h+l_{n,a_2}} p^{h'} + \sum_{\substack{n\in N\\ a_1\in A^1_n\\ h\in T_{n,a_1}}} p^h x^h_{n,a_1} - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{\substack{n\in N\\ a_2\in A^2_n}} \sum_{\substack{h'=h-l_{n,a_2}\\ \text{s.t.} h'\in T_{n,a_2}}}^{h} k_{n,a_2} y^h_{n,a_2} + \sum_{\substack{n\in N\\ a_1\in A^1_n\\ \text{s.t.} h\in T_{n,a_1}}} x^h_{n,a_1} \qquad \forall h\in H \tag{3.53}$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H \tag{3.54}$$

$$\min_x \sum_{\substack{n\in N\\ a_2\in A^2_n\\ h\in T_{n,a_2}}} \left(k_{n,a_2} \sum_{h'=h}^{h+l_{n,a_2}} p^{h'} + C_{n,a_2}(h) \right) y^h_{n,a_2} + \sum_{\substack{n\in N\\ a_1\in A^1_n\\ h\in T_{n,a_1}}} \left(p^h + C_{n,a_1}(h)\right) x^h_{n,a_1}$$

s.t.

$$0 \leq x^h_{n,a_1} \leq \beta^{\max}_{n,a_1} \qquad \forall n\in N, a_1\in A^1_n, h\in T_{n,a_1} \tag{3.55}$$

$$\sum_{h\in T_{n,a_1}} x^h_{n,a_1} \geq E_{n,a_1} \qquad \forall n\in N, a_1\in A^1_n. \tag{3.56}$$

$$\sum_{h\in T_{n,a_2}} y^h_{n,a_2} \geq 1 \qquad \forall n\in N, a_2\in A^2_n \tag{3.57}$$

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$$y^h_{n,a_2} \in \{0,1\} \qquad \forall n \in N, a_2 \in A^2_n, h \in T_{n,a_2} \tag{3.58}$$

Constraint 3.53: sets a lower bound on the peak load. The time slot with the highest power consumption is the binding one, all others become redundant.

Constraint 3.54: represents the price ceiling and stays the same as previous models.

Constraints 3.55 and 3.56: represent the device limit and demand satisfaction, respectively.

Constraints 3.57 and 3.58: represent demand satisfaction and non-preemptive property, respectively.

3.3.1 Single Level Formulation

In this subsection, MBM is reformulated as a single level model. As mentioned before, MBM has both integer and real variables at its lower level. Nevertheless, there is no constraint that involves both variables. The preemptive and nonpreemptive appliances are handled independently. It is important to emphasize the fact that x and y both contribute to the peak, hence they interact with each other at the upper level. However, for a fixed p the lower level problem is separable in terms of variables x and y into an LP and an IP.

The IP part of the lower level is the same as the lower level of NBM. As shown in Chapter 3.2.2, it is possible to reformulate the lower level of NBM equivalently as a MIP using its KKT conditions. Since the lower level of MBM is separable, the same principle is applied here. Hence, the MIP of

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MBM becomes:

$$\max_{x,y,v,w,\Gamma} \sum_{\substack{n\in N\\ a_2\in A^2_n}} v_{2,n,a_2} - \sum_{\substack{n\in N\\ a_2\in A^2_n\\ h\in T_{n,a_2}}} w^h_{2,n,a_2} - \sum_{\substack{n\in N\\ a_1\in A^1_n\\ h\in T_{n,a_1}}} \beta^{\max}_{n,a_1} w^h_{1,n,a_1} + \sum_{\substack{n\in N\\ a_1\in A^1_n}} E_{n,a_1} v_{1,n,a_1}$$

$$- \sum_{\substack{n\in N\\ a_2\in A^2_n\\ h\in T_{n,a_2}}} C_{n,a_2}(h) y^h_{n,a_2} - \sum_{\substack{n\in N\\ a_1\in A^1_n\\ h\in T_{n,a_1}}} C_{n,a_1}(h) x^h_{n,a_1} - \kappa\Gamma$$

s.t.

$$\Gamma \geq \sum_{\substack{n\in N\\ a_2\in A^2_n}} \sum_{\substack{h'=h-l_{n,a_2}\\ \text{s.t.} h'\in T_{n,a_2}}}^{h} k_{n,a_2} y^h_{n,a_2} + \sum_{\substack{n\in N\\ a_1\in A^1_n\\ \text{s.t.} h\in T_{n,a_1}}} x^h_{n,a_1} \qquad \forall h \in H \quad (3.59)$$

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H \quad (3.60)$$

$$0 \leq x^h_{n,a_1} \leq \beta^{\max}_{n,a_1} \qquad \forall n \in N, a_1 \in A^1_n, h \in T_{n,a_1} \quad (3.61)$$

$$\sum_{h\in T_{n,a_1}} x^h_{n,a_1} = E_{n,a_1} \qquad \forall n \in N, a_1 \in A^1_n \quad (3.62)$$

$$\sum_{h\in T_{n,a_2}} y^h_{n,a_2} = 1 \qquad \forall n \in N, a_2 \in A^2_n \quad (3.63)$$

$$- w^h_{1,n,a_1} + v_{1,n,a_1} - p^h \leq C^h_{1,n,a} \qquad \forall n \in N, \forall a_1 \in A^1_n, \forall h \in T_{n,a_1} \quad (3.64)$$

$$- w^h_{2,n,a_2} + v_{2,n,a_2} - k_{n,a_2} \sum_{h'=h}^{h+l_{n,a_2}} p^{h'} \leq C_{n,a_2}(h)$$

$$\forall n \in N, \forall a_2 \in A^2_n, \forall h \in T_{n,a_2} \quad (3.65)$$

$$k_{n,a_2} \sum_{h'=h}^{h+l_{n,a_2}} p^{h'} + C_{n,a_2}(h) - v_{2,n,a_2} + w^h_{2,n,a_2} \leq M_1(1 - y^h_{n,a_2})$$

$$\forall n \in N, \forall a \in A^2_n, \forall h \in T_{n,a_2} \quad (3.66)$$

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$$w^h_{2,n,a_2} \leq M_2 y^h_{n,a_2} \qquad \forall n \in N, \forall a \in A^2_n, \forall h \in T_{n,a_2} \tag{3.67}$$

$$-x^h_{n,a_1} + M_3 \xi^h_{n,a_1} \leq M_3 - \beta^{\max}_{n,a_1} \qquad \forall n \in N, \forall a \in A^1_n, \forall h \in T_{n,a_1} \tag{3.68}$$

$$w^h_{1,n,a_1} - M_3 \xi^h_{n,a_1} \leq 0 \qquad \forall n \in N, \forall a \in A^1_n, \forall h \in T_{n,a_1} \tag{3.69}$$

$$\sum_{h \in T_{n,a_1}} x^h_{n,a_1} + M_2 \varepsilon_{n,a_1} \leq M_4 + E_{n,a_1} \qquad \forall n \in N, \forall a \in A^1_n \tag{3.70}$$

$$v_{1,n,a_1} - M_4 \varepsilon_{n,a_1} \leq 0 \qquad \forall n \in N, \forall a \in A^1_n \tag{3.71}$$

$$w^h_{1,n,a_1} - v_{1,n,a_1} + p^h + M_5 \psi^h_{n,a_1} \leq M_5 - C_{n,a_1}(h) \qquad \forall n \in N, \forall a \in A^1_n, \forall h \in T_{n,a_1} \tag{3.72}$$

$$x^h_{n,a_1} - M_5 \psi^h_{n,a_1} \leq 0 \qquad \forall n \in N, \forall a \in A^1_n, \forall h \in T_{n,a_1} \tag{3.73}$$

$$y^h_{n,a_2} \in \{0,1\} \qquad \forall n \in N, a_2 \in A^2_n, h \in T_{n,a_2} \tag{3.74}$$

$$\xi^h_{n,a_1}, \psi^h_{n,a_1} \in \{0,1\} \qquad \forall n \in N, \forall a \in A^1_n, \forall h \in T_{n,a_1} \tag{3.75}$$

$$\varepsilon_{n,a_1} \in \{0,1\} \qquad \forall n \in N, \forall a \in A^1_n. \tag{3.76}$$

Constraints 3.59 and 3.60: upper level constraints of MBM.

Constraints 3.61, 3.62 and 3.63: lower level primal constraints of MBM.

Constraint 3.64: dual constraint associated with variable $x_{n,a_1,h}$.

Constraint 3.65: dual constraint associated with variable $y_{n,a_2,h}$.

Constraint 3.66: comes from the linearization of the complementary slackness of constraint 3.65 and variable $y_{n,a_2,h}$.

Constraint 3.67: comes from the linearization of the complementary slackness of constraint 3.63 and dual variable w^h_{2,n,a_2}.

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Constraints 3.68 and 3.69: come from the linearization of the complementary slackness of constraint 3.61 and dual variable w^h_{1,n,a_1}.

Constraints 3.70 and 3.71 : come from the linearization of the complementary slackness of constraint 3.62 and dual variable v_{1,n,a_1}.

Constraints 3.72 and 3.73 : come from the linearization of the complementary slackness of dual constraint 3.64 and variable x^h_{n,a_1}.

Constraint 3.74: integrality constraint of $y_{n,a_2,h}$.

3.3.2 Experimental Results and Interpretation

In this section, experimental results of MBM are presented with respect to different parameters. All results are obtained by solving the MIP that is given in Chapter 3.3.1. The results of MBM are compared to the base case (BC) where all prices are equal to $p_{\max}$ and all jobs are scheduled to the most preferred time slots.

The instances that are presented in this chapter consist of 7 customers where each one owns 3 preemptive and 2 non-preemptive appliances. All devices are connected to the smart grid. The scheduling horizon is composed of 24 time slots. The models are tested with peak weight κ and time window width TWW, with values for $\kappa \in \{200, 400, 600, 800, 1000\}$ and for TWW 20% and 100%. TWWs and inconvenience penalty function $C_{n,a}(h)$ of preemptive and non-preemptive appliances are computed as in Chapter 3.1.3 and 3.2.3, respectively.

For experimental purposes, 10 instances are randomly generated as explained in Chapter 3.1.3 and 3.2.3. The instances are solved with CPLEX version 12.3 on a computer with 2.66 GHz Intel Xeon CPU and 4 GB RAM,

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running under the Windows 7 operating system. Whenever an instance could not be solved within the time limit of 4 hours, the best integer solution has been considered.

Table 3.8: Cost Comparison of MBM on 20% TWW instances (BC = 100%)

κ	MBM EB	MBM IC	NBM TC
200	92.45	6.14	98.59
400	91.35	5.81	97.16
600	91.04	5.34	96.38
800	89.91	4.67	94.58
1000	88.80	4.45	93.24

Table 3.9: Cost Comparison of MBM on 100% TWW instances (BC = 100%)

κ	MBM EB	MBM IC	NBM TC
200	94.36	4.82	99.17
400	93.56	5.00	98.56
600	92.47	5.22	97.69
800	92.12	5.21	97.32
1000	91.56	5.26	96.83

The cost values of MBM are presented in Tables 3.8 and 3.9, for 20% and 100% TWW instances, respectively. The tables display the average of 10 instances with 35 jobs for 5 different peak weight values. Customers' cost is given in terms of electricity bill (EB), inconvenience cost (IC) and total cost (TC). All percentages are presented with respect to BC.

Similar to PBM and NBM, all TC values are less than 100% which shows that MBM is beneficial for customers in terms of cost decrease. Moreover, in

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both tables EB and TC values are reduced and IC increases as κ increases since higher peak weight motivates the leader to reduce off-peak prices in order to shift some of the peak load.

Table 3.10: Comparison of MBM with 20% and 100% TWW Instances

	Av Comp Time		Av Gap		# unsolved	
κ	20%	100%	20%	100%	20%	100%
200	1.00	105.60	0.00%	0.00%	0	0
400	1.00	884.40	0.00%	0.00%	0	0
600	1.00	2844.90	0.00%	0.00%	0	0
800	1.00	3487.40	0.00%	0.00%	0	0
1000	1.00	8348.30	0.00%	0.00%	0	0

The 20% TWW and 100% TWW instances are compared in Table 3.10 in terms of average computation time (sec), average gap (%) and number of unsolved instances. It can be observed that 20% TWW instances are considerably easier to solve than 100% TWW ones. All instances are solved within 4 hours time limit, therefore average gap values are zero.

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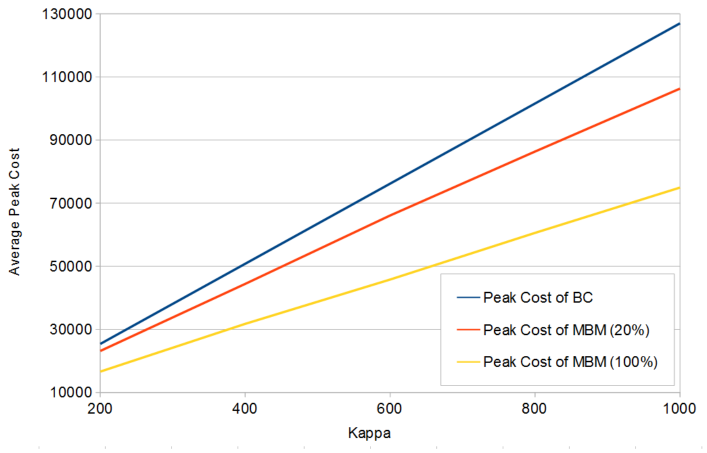

Figure 3.14: Peak Cost Comparison of MBM

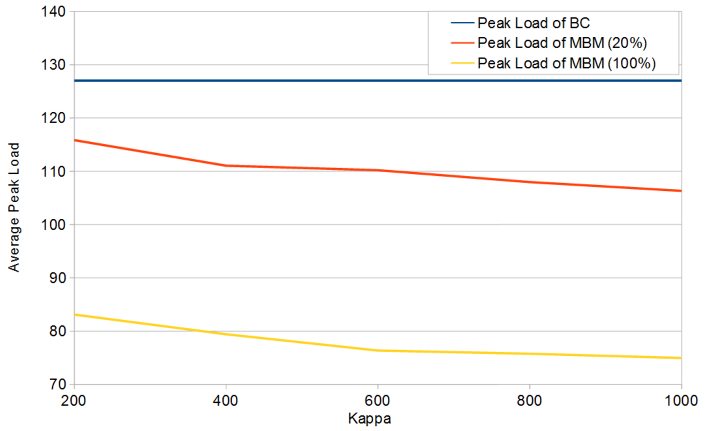

Figure 3.15: Peak Load Comparison of MBM

The 20% TWW and 100% TWW instances are compared in Figure 3.14 and 3.15 in terms of average peak cost and average peak load, respectively. It can be observed that 20% TWW instances tend to have a higher peak load

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than 100% TWW instances and hence, peak cost. When TWW is high, the leader can get a better customer response with respect to changing prices.

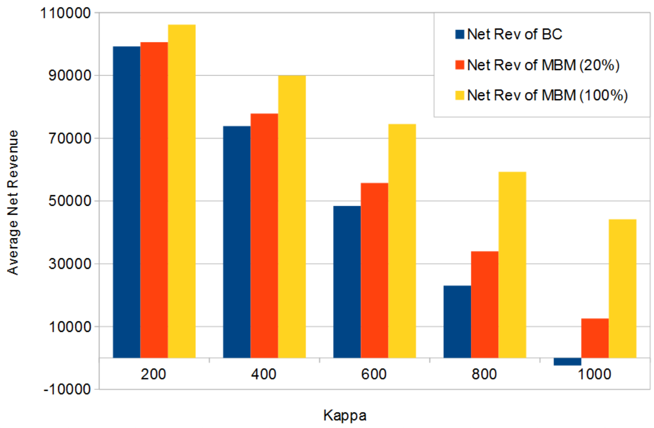

Figure 3.16: Net Revenue Comparison of MBM

The 20% TWW and 100% TWW instances are compared in Figure 3.16 in terms of net revenue. Although both 20% and 100% TWW instances provide a higher net revenue than BC, it is clear that wider time windows are more beneficial for the leader.

3.4 Conclusion

In this chapter, we introduced a bilevel programming approach to the revenue optimization problem where the energy provider and the smart grid operator play a sequential game. Several models are developed regarding different device properties (preemptive and/or non-preemptive) and market conditions (monopolist or competitive).

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It is important to emphasize that we consider a single leader, single follower bilevel program and not a multi-leader one. In the latter case, we would have to solve an equilibrium problem at the upper level, which would sharply increase the complexity of the model definition and resolution.

In the context of our problem, the leader aims to find a trade-off between revenue and peak cost whereas the follower minimizes total cost.

The bilinear bilevel models are reformulated as single level mixed integer problems using both classical and novel methods. These MIPs are solved to optimality with randomly generated instances with respect to time window width TWW and peak penalty. The test results are analyzed.

It is observed that the problem becomes harder to solve and the load curve gets flatter in every case as peak penalty increases. TWW has a similar effect on the problem. It takes more time to solve when time windows are wider and load curve becomes smoother.

When peak penalty and TWW increase, peak load is reduced. Revenue decreases since leader is forced to offer lower prices in order to convince customers to change slots and lower the peak. As the leader offers lower prices, the billing cost of the follower decreases whereas the inconvenience cost increases. It is observed that the total cost of the follower is reduced as a result.

In consequence, it is shown that the bilevel programming approach is beneficial for both supply and demand sides of the problem. It allows a more efficient system utilization due to peak minimization without requiring any further capacity installation. By combining smart grid technology with demand response programs and thus allowing the demand decision to be integrated into the decision making process of the energy provider, our approach

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has very promising results in terms of understanding customer behavior and increasing system efficiency.

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Chapter 4

Heuristic Methods

Contents

In this chapter, heuristic methods are presented to solve the bilevel models defined in Chapter 3. The heuristics, intrinsically based on the structure of the problem, allows to efficiently solve large instances in moderate computation time.

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Models PBM and NBM involve 3 sets of decision variables: prices, peak load and schedule. The main idea behind the heuristic methods is to fix one of these variables and to compute the value of the others. More precisely, Price heuristic defines price vectors, Peak Search and Peak Cuts heuristics fix peak load value, Load Shifting and Divide-and-Stitch heuristics focus on schedules. Once one of these variables is fixed, the corresponding optimal values of other variables are computed by inverse optimization, which allows us to find the optimal value of a parameter (or variable in bilevel context, see §4.1.1) corresponding to a feasible solution.

In order to develop solution methodologies with fast and high quality outputs, several approaches are considered. The objective function value of a feasible solution can be improved in two ways: increasing revenue and/or decreasing peak. In this context, Price heuristic focuses on manipulating prices of a feasible solution to impose a different load distribution with a lower peak. Peak Search heuristic computes feasible schedule-price pairs corresponding to different peak values to find the solution with the highest net revenue. Similarly, Peak Cuts heuristic also fixes a peak value and computes a feasible schedule-pair, however it uses corresponding MIP of the model (See Chapter 3). Load Shifting heuristic focuses on the schedule, it shifts some of the load to different time slots in order to decrease the peak. Divide-and-Stitch heuristic approaches the problem from another angle. It divides the problem into smaller pieces and stitches the solutions back together to solve the problem quicker.

In the following section, the subproblems (inverse optimization, minimum peak subproblem, fixed peak subproblem) that take part in the heuristics are

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presented in detail to provide an insight about different steps of algorithms. Afterwards, all heuristic methods are presented. The results of two efficient methods (Price and Peak Search heuristics) are analysed in §4.7. Other methods are explained in their corresponding sections. §4.8 is the conclusion of the chapter.

4.1 Subproblems for Heuristic Methods

Heuristic approaches are based on dividing the problem into smaller size subproblems by fixing one variable's value and computing others. In this section, we define three subproblems that are the cornerstones of Price and Peak Search heuristics before describing the heuristics in detail.

- *Inverse optimization* consists of finding the leader's optimal prices corresponding to a given feasible lower level solution.
- *Minimum peak subproblem* computes a lower bound on peak load.
- *Fixed peak subproblem* computes an optimal follower's schedule under peak load constraints.

All subproblems are linear and defined separately with respect to PBM and NBM. Inverse optimization is utilized in both heuristics whereas minimum peak and fixed peak subproblems are used in Peak Search heuristic.

4.1.1 Inverse Optimization

Inverse optimization approach has been widely applied on geophysical data [Tarantola 1984, Tarantola 1987]. As defined by [Ahuja 2001], inverse opti-

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mization (IO) “consists of inferring the values of the model parameters, given the values of observable parameters”. In other words, it is used to compute the values of the model parameters corresponding to a feasible solution so that the feasible solution would be optimal with respect to those parameter values.

In bilevel context, it consists in computing the optimal upper level decisions corresponding to any feasible lower level solution [Labbé 1998, Didi-Biha 2006]. It is an extension of inverse optimization where the parameters (prices) are actually decision variables, and where a proximity measure is replaced by the leader’s objective.

The relation between the leader and the follower variables is twofold. For fixed leader prices, it is always possible to find a feasible lower level solution by solving the lower level problem corresponding to these prices. However, as illustrated in Figure 4.1, the converse is not true. Let us consider two preemptive jobs (belonging to two different customers) under a monopolist setting with parameters in Table 4.1.

Parameters	Value
p_{max}	10
T_1	[0, 1]
T_2	[0, 1]
(E_1, E_2)	(10, 20)
(γ_1, γ_2)	(10, 20)
(λ_1, λ_2)	(0.2, 0.2)
(C_1, C_2)	(1, 2)

Table 4.1: Parameters of the example

A corresponding price vector for the schedule in Figure 4.1(a) is (10,10). In general, any price where $p^0 - p^1 \leq 1$ is feasible for the first schedule. In

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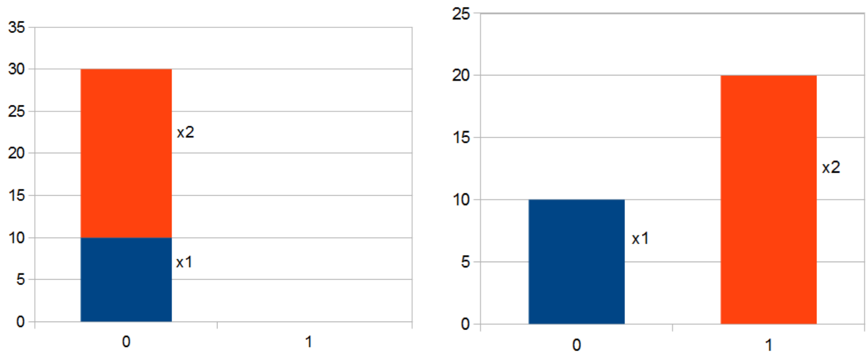
Figure 4.1(b), job 2 is assigned to hour 1 whereas job 1 stays at hour 0. It is not possible to find corresponding prices in this case since the inconvenience cost of job 2 is 2, so it requires a 2-unit reduction in the price to switch to hour 1. Then, $p^0 - p^1 \geq 2$. However, job 1 would be assigned to hour 1 if $p^0 - p^1 \geq 1$. Hence, there is no feasible price corresponding to the second schedule. Even if the example is defined for PBM, such examples can be found for MBM and NBM as well. As illustrated in this example, every price vector has a corresponding feasible schedule whereas every schedule does not have a corresponding feasible price vector in the context of our problem.



(a) Schedule 1 (b) Schedule 2

Figure 4.1: Example

The advantage of IO problem is being a linear program. Therefore it is not time consuming to solve it.

IO formulation for PBM

To define the IO model for PBM, let us assume that $\tilde{x}^h_{n,a}$ are the fixed lower level solutions of the bilevel program. By fixing the variables $x^h_{n,a}$ in the MIP equivalent of PBM (see §3.1.2), we obtain the following model:

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$$\text{IO-PBM: } \max_{p} \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} p^h \tilde{x}^h_{n,a}$$

s.t.

$$0 \leq p^h \leq p^h_{\max} \qquad \forall h \in H$$

$$\tilde{x}^h_{n,a}(w^h_{n,a} - v_{n,a} + p^h + C_{n,a}(h)) = 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{4.1}$$

$$w^h_{n,a}(-\tilde{x}^h_{n,a} + \beta^{\max}_{n,a}) = 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{4.2}$$

$$v_{n,a}(\tilde{x}^h_{n,a} - E_{n,a}) = 0 \qquad \forall n \in N, \forall a \in A_n \tag{4.3}$$

The IO model has a linear objective function that maximizes revenue with respect to a fixed x.

Constraint 4.1: assesses that if appliance a of customer n is used during time period h, then the corresponding dual constraint is satisfied with equality. Otherwise, it stays as an inequality.

Constraint 4.2: assesses that if the usage of appliance a of customer n is equal to the device limit $\beta^{\max}_{n,a}$ during time period h, then the corresponding dual variable takes a nonnegative value. Otherwise, it is zero.

Constraint 4.3: means that if the total consumption for appliance a of customer n is larger than its demand $E_{n,a}$, then the corresponding dual variable is zero. If it is equal, then the dual variable takes nonnegative values.

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IO formulation for NBM

Similarly to PBM, the IO model of NBM is obtained by fixing the lower level variables $\tilde{x}^h_{n,a}$ in the MIP formulation of NBM (see §3.2.2):

$$
\begin{aligned}
\text{IO-NBM:}\ \max_{p} & \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} k_{n,a}\tilde{x}^h_{n,a}\sum_{h'=h}^{h+l_{n,a}} p^{h'} \\
\text{s.t.}\ & 0 \le p^h \le p^h_{\max} \qquad \forall h \in H && (4.4)\\
& \tilde{x}^h_{n,a}\Big(k_{n,a}\sum_{h'=h}^{h+l_{n,a}} p^{h'} + C_{n,a}(h) - u_{n,a} + w^h_{n,a}\Big) = 0 \\
& \qquad\qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} && (4.5)\\
& w^h_{n,a}(1-\tilde{x}^h_{n,a}) = 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} && (4.6)
\end{aligned}
$$

The objective function of the model is linear. Similar to the IO formulation of PBM, complemantarity constraints become linear since the values of $\tilde{x}^h_{n,a}$ are fixed in this formulation.

4.1.2 Minimum Peak Subproblem

The aim of the Minimum Peak subproblem is to compute a lower bound on the peak load. This is not trivial since it is minimized through pricing.

The computed lower bound is not necessarily tight. In fact, the optimal schedule computed by this model might be price infeasible since customers' choice is neglected. In other words the optimal solution of this subproblem would spread the load as much as possible, regardless of inconvenience cost (IC). As previously discussed on a simple example (Figure 4.1), some schedules

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do not have a corresponding price vectors. For instance, there is no price vector that would induce a schedule where a job with higher IC is postponed rather than the one with lower IC. Therefore, a schedule might be price-infeasible when IC is ignored. Nevertheless, solving this subproblem allows us to define an interval for peak values. More details about the purpose of it can be found in §4.3.

Minimum Peak Subproblem for PBM

MinPeak-PBM presented below is an LP with continuous variables $x^h_{n,a}$. Constraints (4.8) and (4.9) are the lower level constraints of PBM. Constraint (4.7) is an upper level constraint of PBM that defines peak. The objective function minimizes peak. It is important to emphasize that the sole purpose of this subproblem is to compute a lower bound on peak, not the optimal value. Therefore, prices and IC are not included.

$$
\begin{aligned}
\text{(MinPeak-PBM)} \quad \min_{\Gamma,x} \quad & \Gamma \\
\text{s.t.} \quad & \Gamma \geq \sum_{n\in N}\sum_{a\in A_n} x^h_{n,a} \quad \forall h \in H && (4.7) \\
& 0 \leq x^h_{n,a} \leq \beta^{max}_{n,a} \quad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} && (4.8) \\
& \sum_{h\in T_{n,a}} x^h_{n,a} \geq E_{n,a} \quad \forall n \in N, \forall a \in A_n && (4.9) \\
& && (4.10)
\end{aligned}
$$

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Minimum Peak Subproblem for NBM

MinPeak-NBM is a linear integer program with binary variables $x^h_{n,a}$. Similar to MinPeak-PBM, the constraints of this subproblem are related to upper and lower level constraints of NBM, and the objective is to minimize peak.

In §3.2.2, it is stated that the lower level of NBM is actually an assignment problem and hence the integrality constraints of $x^h_{n,a}$ can be relaxed. However, it is not the case here due to constraint (4.11). Peak load would be smaller if $x^h_{n,a}$ takes non-integer values. The optimal solution of the relaxed MinPeak-NBM would be lower than the original problem and therefore infeasible.

$$\text{(MinPeak-NBM)} \quad \min_{\Gamma,x} \quad \Gamma$$

$$\text{s.t.} \quad \Gamma \geq \sum_{\substack{n\in N\\ a\in A_n}} \sum_{\substack{h'=h-l_{n,a}\\ s.t. h'\in T_{n,a}}}^{h} k_{n,a} x^h_{n,a} \quad \forall h \in H \tag{4.11}$$

$$\sum_{h\in T_{n,a}} x^h_{n,a} = 1 \quad \forall n \in N, \forall a \in A_n \tag{4.12}$$

$$x^h_{n,a} \in \{0,1\} \quad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{4.13}$$

4.1.3 Fixed Peak Subproblem

Fixed Peak subproblem aims to find a schedule that minimizes the total inconvenience cost (IC) under a fixed load capacity without taking prices into account. The subproblem computes a schedule with a fixed peak load where jobs with smaller IC are postponed to later slots.

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Although prices are not included in the model, it is important to highlight that the solution of this subproblem is price-feasible, i.e. there exists a price vector that would induce this schedule. However, if peak load is fixed to a low value (e.g. to the lower bound computed by Minimum Peak subproblem), jobs might have IC values larger than price ceilings in the resulting schedule. In this case, the leader would be forced to offer negative prices which is not allowed within the scope of this work.

The role of this model and its implications are discussed in more detail in §4.3.

Fixed Peak Subproblem for PBM

The FixedPeak-PBM is an LP with continuous variables $x_{n,a}^h$. Constraints (4.15) and (4.16) are lower level constraints of PBM. Constraint (4.14) assesses that load values of all slots are less than a predefined value, Γ'. Since IC is minimized and x are continuous, peak load is equal to Γ' in the optimal solution.

$$\text{(FixedPeak-PBM):} \quad \min_x \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in T_{n,a}} C_{n,a}(h) x_{n,a}^h$$

$$\text{s.t.} \quad \Gamma' \geq \sum_{n\in N} \sum_{a\in A_n} x_{n,a}^h \qquad \forall h \in H \qquad (4.14)$$

$$0 \leq x_{n,a}^h \leq \beta_{n,a}^{max} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \qquad (4.15)$$

$$\sum_{h\in T_{n,a}} x_{n,a}^h \geq E_{n,a} \qquad \forall n \in N, \forall a \in A_n \qquad (4.16)$$

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Fixed Peak Subproblem for NBM

FixedPeak-NBM is a linear integer program with binary variables $x^h_{n,a}$. Similarly, constraints (4.18) and (4.19) are lower level constraints of NBM.

$$\text{(FixedPeak-NBM):} \ \min_x \sum_{n\in N} \sum_{a\in A_n} \sum_{h\in T_{n,a}} C_{n,a}(h) x^h_{n,a}$$

$$\text{s.t. } \Gamma' \geq \sum_{n\in N} \sum_{a\in A_n} \sum_{\substack{h'=h-l_{n,a} \\ s.t. h'\in T_{n,a}}}^{h} k_{n,a} x^h_{n,a} \geq \Gamma'' \quad \forall h \in H \tag{4.17}$$

$$\sum_{h\in T_{n,a}} x^h_{n,a} = 1 \qquad \forall n \in N, \forall a \in A_n \tag{4.18}$$

$$x^h_{n,a} \in \{0,1\} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a} \tag{4.19}$$

This subproblem is used as a part of Peak Search heuristic that is defined in detail in §4.3. A binary search is performed over peak load values in this method and therefore it is important to obtain a schedule with a certain peak load value. When we provide only an upper bound on peak load, it may not be possible to reach that particular peak value since the x variables are binary. Moreover, there might be multiple optima with different peak values. In order to deal with these issues, a lower limit is also included in constraint (4.17), namely Γ''.

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4.2 Price Heuristic

The main idea of the Price Heuristic (PH) is to generate a sequence of price neighborhoods and to compute corresponding optimal schedules. The target is to find a price-schedule pair with highest possible net revenue. In order to achieve it, the heuristic tries to induce a load alteration via changing prices and hence to decrease peak load.

Price vectors take their values from the interval $[0, p_{\max}]$. There always exists a feasible schedule corresponding to each price vector or in other words, every price vector is schedule-feasible.

At each iteration j, a price vector p_j is fixed and the schedule associated with it x_j is computed. Maximum load and the time slot where it occurs are defined as *peak load* Γ_j and *peak slot* i. The prices of k time slots that follow the peak slot are decreased by a fixed percentage d. Using these updated prices $p_{j(upd)}$, the lower level problem is solved and a new schedule $x_{j(upd)}$ is obtained. Afterwards, the optimal prices $p^*_{j(upd)}$ corresponding to this new schedule is computed by IO (since there can be several price vectors that correspond to the same schedule and we would like to choose the best one for the leader). This process is repeated until there is no improvement in the objective function of the leader.

An iteration of the heuristic is illustrated in Figure 4.2 to give an intuition about the process. Load_0 and Price_0 represent the load and price vector at iteration 0, respectively. Likewise, Load_1 and Price_1 are the load and price vector of the solution after the first iteration. For the sake of simplicity, only one price changes in this example. The peak slot of iteration 0 occurs at time period 12. Therefore, the price of time period 13 is decreased by the

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algorithm in the first iteration and hence a change in load of time periods 11, 12 and 13 is observed.

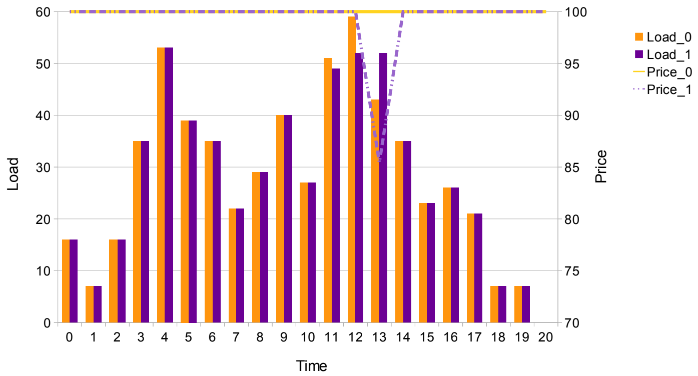

Figure 4.2: Price Heuristic: An Iteration

The steps of the heuristic are explained in more detail as follows:

- 0. Initialization: The lower level problem is solved with randomly generated fixed prices. Then optimal prices corresponding to these schedules are computed. The objective function values of the price-schedule pairs are compared and the best one is selected as the initial solution (p_0, x_0) with objective function value z_0. The incumbent solution is also set as the initial solution $(p_{inc}, x_{inc}) = (p_0, x_0)$ and $z_{inc} = z_0$.

  Iteration $j = 0$

- while **true**

  $j \leftarrow j + 1$

  * 1. Finding Peak: The maximum load Γ_j is determined and named as **peak load**. The associated time slot i is defined as **peak slot**.

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* 2. Price Update: The prices of k time slots that follows peak slot, $(p_j^{i+1}, \ldots, p_j^{i+k})$, are decreased by a certain discount factor d. The updated price vector becomes

$$p_{j(upd)} = (p_j^0, p_j^1, \ldots, p_j^i, (1-d)p_j^{i+1}, (1-d)p_j^{i+2}, \ldots, (1-d)p_j^{i+k}, p_j^{i+k+1} \ldots)$$

.

* 3. Schedule Computation: The lower level problem is solved for fixed prices $p_{j(upd)}$ and a new schedule $x_{j(upd)}$ is computed.

* 4. Inverse Optimization: The optimal prices $x^*_{j(upd)}$ associated with $x_{j(upd)}$ are computed by the inverse optimization.

* 5. Comparison: $z_{j(upd)}$ is the objective function value of the pair $(p^*_{j(upd)}, x_{j(upd)})$
  If $z_{j(upd)} > z_{inc}$, then $(p_{inc}, x_{inc}) = (p^*_{j(upd)}, x_{j(upd)})$ and $z_{inc} = z_{j(upd)}$.
  Else if 0.9 $z_{inc} \leq z_{j(upd)} \leq z_{inc}$, then $p^*_{j(upd)} = p_{j+1}$.
  Else **end loop**.

- 6. MIP Procedure: The algorithm's output (p_{inc}, x_{inc}) is set to the initial solution of the single level MIP formulation (corresponding MIP to each model). Then, MIP is solved with a time limit and an optimistic solution is obtained.

The initial solution of the heuristic is set at Step 0. It starts from a *base case* (BC) that is the solution where all prices are set to the upper bounds and all jobs are scheduled to the beginning of their time windows. The peak slot of BC is computed. Then, price vectors are generated by keeping the

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prices before peak slot (including peak slot itself) the same and randomly generating the rest (using uniform distribution between 0 and $p_{\max}$). The lower level problem is solved with these prices. Afterwards, the optimal prices corresponding to the lower level solution are computed by solving the IO problem. At the end of the process, the solution with the best net revenue (leader's objective) is set to the initial solution.

This random process is applied in order to jump to a different valley than BC since BC is neither a valley nor an optimum and hence hard to improve for the Price Heuristic.

Steps 1-5 are repeated until stopping criterion is fulfilled. Step 6 is executed after the iterative process ends.

In this heuristic, the lower level is solved with fixed prices to compute the corresponding optimal schedule at Step 3. It is important to point out the fact that this schedule is optimal only for the lower level. In the case of multiple optima at the lower level, one of the optima has the highest net revenue for the leader and therefore it is the optimal solution of the optimistic bilevel problem. However, when the lower level is solved with fixed prices separately, the solution does not yield the highest net revenue for the leader. In order to find an optimistic solution, Step 6 is added to the heuristic. At this step, the output of the algorithm (Steps 1-6) is given to the corresponding MIP as an initial solution and MIP is solved by an off-the-shelf solver for a limited time. Thanks to this step, we are able to compute an optimistic solution in the close proximity of the output.

The next example (Figure 4.3) illustrates the effect of Step 7. The output load curve of the algorithm at the end of Step 5 and Step 6 are the blue and

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red curve, respectively. In the context of this example, prices of Step 6 are kept the same as Step 5 for the sake of comparison. It can be observed that the red curve has a lower peak load. It indeed has a 19% higher objective function value than the solution before MIP (Step 5), although they have the same price vector.

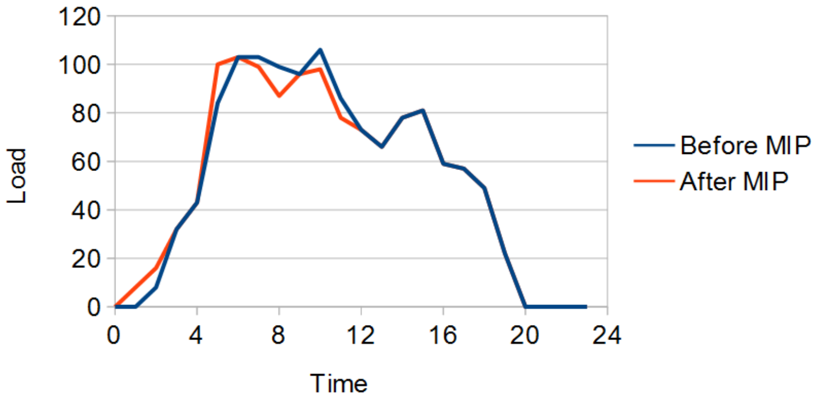

Figure 4.3: Load curves before and after the MIP procedure

4.3 Peak Search Heuristic

The main goal of Peak Search Heuristic (PSH) is to find a price-schedule pair that has a high net revenue by fixing the peak load. In the context of our problem, the leader tries to find an optimal trade-off between revenue and peak. As presented in Chapter 3, it is observed that optimal peak value decreases as peak penalty elevates while the problem becomes increasingly more difficult to solve. Decreasing peak value also results in a lower revenue since the leader offers lower prices to shift jobs. Preliminary analysis shows that net revenue is *roughly* concave with respect to peak. It means that up to a certain level, the leader is willing to give up on some revenue to decrease

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peak. Any peak value lower than this level causes more loss than profit and hence net revenue starts decreasing. This heuristic performs a local search on peak values to find a good balance between revenue and peak.

At the beginning of the algorithm, first an upper and a lower bound for the peak load are computed to define a peak interval (Step 1). Then a **combing** procedure is performed to narrow down this peak interval (Step 2). In combing procedure, the peak interval is divided into equal subintervals. For every fixed peak value, a feasible schedule (Step 2.1) and the corresponding optimal prices (Step 2.2) are computed by IO. Two new peak values with the highest objective value are selected that forms a narrower search interval at Step 3.

At Step 4, a binary search is executed in this new interval to find the *best* one. The Fixed Peak problem is solved to compute a schedule and to obtain the optimal corresponding prices by solving IO problem, then the objective function value of this price-schedule solution pair is calculated. The iterative process ends when the binary search interval is too narrow, i.e., when the difference of upper and lower bounds of peak is too small (Step 5).

When Fixed Peak subproblem is solved, the resulting schedule may not be the one with highest revenue for the leader, i.e. there might be another schedule with the same inconvenience cost and peak load but yielding less revenue. Therefore, similar to PH, a MIP step is applied at the end of PSH. The incumbent solution of the algorithm is set as an initial solution for the corresponding MIP and it is solved by a commercial software to obtain an optimistic solution.

The algorithm consists of the following steps:

- 1. Limiting Peak Load: Upper and lower bounds on the peak load (Γ)

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are computed to form a peak interval. It is clear that BC easily provides an upper bound. The lower bound is obtained by solving the Min Peak model defined in §4.1.2.

- 2. Combing Peak Interval: The combing procedure is performed in the peak interval. The interval is divided into equal subintervals and f many peak values (Γ_i) are computed.
  - 2.1. Schedule with Fixed Peak: For each fixed value Γ_i, the corresponding Fixed Peak model (§4.1.3) is solved. The solution is a schedule with minimal inconvenience cost and a fixed peak load.
  - 2.2. Inverse Optimization: The optimal prices corresponding to the schedule (from Step 2.1) are computed by solving the inverse optimization model (§4.1.1). Note that if a Γ_i is price infeasible, there is no need to continue with smaller peak values (since any value smaller than that one will be price infeasible as well). This procedure is repeated for all Γ_i.
- 3. Narrowing the Interval: Let Γ_a and Γ_b be the two Γ_i values that result in the two highest net revenue at the end of step 2.2 and define a new peak interval.
- 4. Binary Search: A binary search is performed in the new interval. For each peak value, Fixed Peak subproblem is solved to find a feasible schedule and its corresponding prices are computed by IO (as in Steps 2.1 and 2.2). The objective function value of this price-schedule pair is computed. Incumbent solution is updated every time a better solution is found.

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- 5. Stopping Criterion: The algorithm stops when $\Gamma_b - \Gamma_a <= \varepsilon$ for a small enough $\varepsilon > 0$.

- 6. MIP Procedure: The algorithm's output is set to be an initial solution of the single level MIP formulation (corresponding MIP to each model). Then, MIP is solved with a time limit using a commercial solver and an optimistic solution is obtained.

We now illustrate the procedure on small-scale instances. In Figure 4.4, net revenue curves corresponding to 4 different PBM instances after Step 2 are presented to show the impact of combing procedure. These curves show the objective function value of the leader (y-axis) corresponding to different peak values (x-axis). It can be observed that net revenue functions are not quite concave. Moreover, there exist other solutions with much higher net revenue values than BC which is the data point with the highest peak value in every graph. Net revenue is zero at the beginning of the interval at each graph since low peak values lead to price-infeasible schedules and a feasible solution cannot be computed in these cases. These curves also reflect the impact of Step 3. Performing a Binary Search in a more narrow interval gives better results since the net revenue curve tends to fluctuate.

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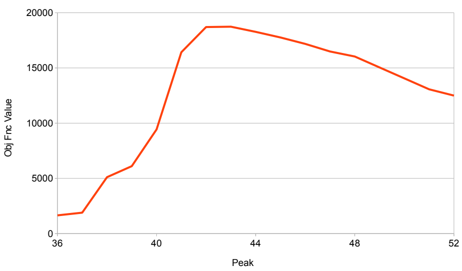
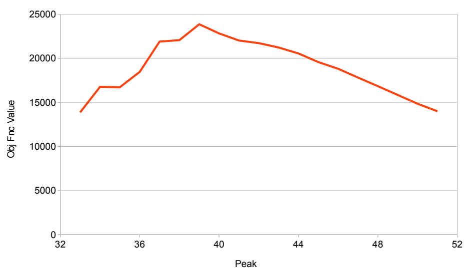
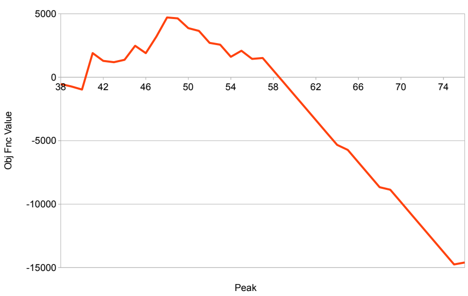


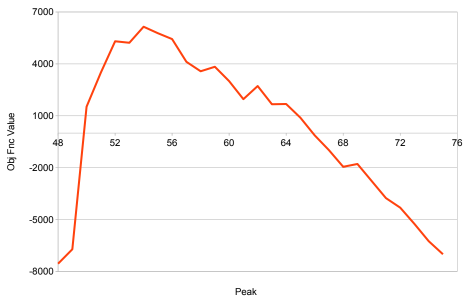
(a) Instance 1



(b) Instance 2



(c) Instance 3



(d) Instance 4

Figure 4.4: Net Revenue Curves vs Peak for Four Instances After Combing

Binary Search is based on the shape of the net revenue curve, therefore picking the correct side at each iteration is important. At each iteration j, two more peak values are evaluated, Γ_j^L and Γ_j^R, that are $c\%$ to the left and to the right of Γ_j, respectively. In other words, at each iteration j, for fixed peak values $(\Gamma_j^L, \Gamma_j, \Gamma_j^R)$, Fixed Peak model is solved; the optimal prices are found by IO, and then net revenue is computed. If Γ_j^L gives the higher net revenue, then the search interval is updated as $[\Gamma_a, \Gamma_j^L]$, else if Γ_j^R gives the higher net revenue, then the search interval is updated as $[\Gamma_j^R, \Gamma_b]$. The incumbent solution is updated accordingly.

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In Figure 4.5, two iterations of Binary Search on the top-left instance (Instance 1) from Figure 4.4 are presented. In this example, initial Γ_j^L and Γ_j^R are obtained at the end of Step 2 are 42 and 43, respectively. The mid point of 42 and 43 (42.5) becomes the incumbent solution since it is better than both of them. Then, 42.5 becomes Γ_j^L and 43 stays as the Γ_j^R. The net revenue value corresponding to their mid point (42.75) is not better than the incumbent hence the incumbent is not updated. This time, 42.75 becomes Γ_j^R and 42.5 stays as the Γ_j^L. The search continues until the stopping criteria is fulfilled.

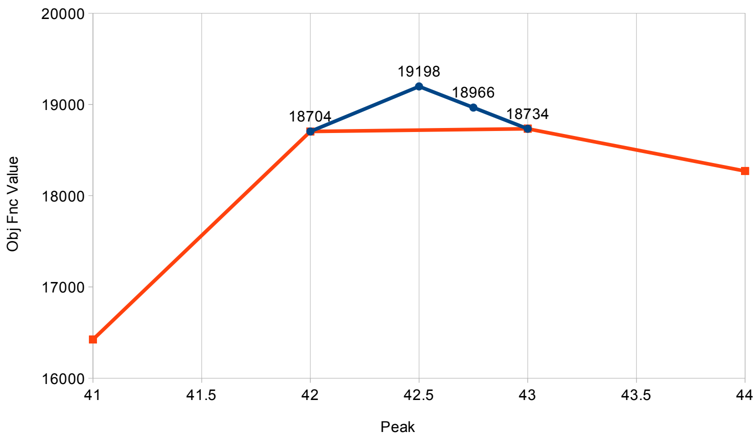

Figure 4.5: Binary Search

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4.4 Load Shifting Heuristic

The main idea of this method is to change the load distribution of a given schedule by shifting some of the jobs that are scheduled in peak slots to later time slots. Shifting load implies decreasing prices and hence revenue losses. Depending on the value of peak penalty, the leader is willing to trade-off revenue loss against lower peak. In this method, it is aimed to find a good trade-off by shifting load and decreasing peak.

For every new schedule, the corresponding optimal prices are computed by solving an IO problem and the net revenue of the solution is calculated.

Given an initial schedule, peak load and peak slot are evaluated. The aim is to reschedule some of the jobs to obtain a new schedule in such a way that peak load is lower and net revenue is higher. At each iteration, the jobs that contribute to the peak are ordered in an ascending inconvenience cost order. The job with the highest inconvenience cost is pushed to the next available slot and a new schedule is obtained. Using IO, corresponding optimal prices and net revenue of the solution are computed. If it is better than the incumbent, then the incumbent solution and hence the peak job list is updated. Otherwise, the next job in the list is chosen and the procedure is repeated. The algorithm stops when it reaches the end of the list.

The main drawback of the method is that the number of jobs contributing to peak might be large. Too many IO problems are required to be solved and the process becomes time consuming. Moreover, since the algorithm operates locally, every iteration is highly dependent on the initial and previous solutions. As previously illustrated in Figure 4.4, objective function is not exactly concave and depending on the instance, there might be several local optima.

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Once the heuristic starts off at a bad solution, it gets stuck there.Hence, the algorithm does not perform as expected.

4.5 Divide-and-Stitch Heuristic

As previously discussed in Chapter 3, computation time increases significantly as instance size increases. The main idea of this method is dividing the problem into smaller ones, and stitching them together in a coherent way in order to reduce computation time.

At each iteration, peak load and peak slot are computed. Corresponding single level MIP consisting of only jobs with positive load around the peak slot and a limited number of adjacent time slots is solved using a commercial solver. Then, partial prices of this reduced problem are combined with the prices of other time slots. The corresponding schedule and net revenue are computed by solving the lower level model. This procedure is repeated until there is no improvement.

The load curve is unidimensional, and hence that it is easy to spot the problematic (congested) time periods. Dividing the problem into smaller subproblems allows us to focus on the most important part of the problem rather than the whole horizon. Unlike other heuristics, values of none of the variables are fixed in this method. Instead, we solve the same model with a limited number of variables.

Several issues have been observed for this method. First, usually many jobs contribute to peak load, since the number of jobs is much larger than the number of time slots. Therefore, dividing the problem does not really

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reduce the complexity. Moreover, when the lower level problem is solved with the combined prices, the resulting schedule may not be well balanced. For instance, assume that peak load occurs at slot 12 and all slots between 12 and 16 are taken into account for the partial problem. Then, the prices $p^{13}, \ldots, p^{16}$ are most probably lower than p^{12} in the partial optimal solution, in order to decrease the peak. However, when these prices are combined with $p^0, \ldots, p^{11}, p^{17}, \ldots, p^{23}$ and the lower level model of the entire problem is solved, the jobs from earlier slots may switch to these lower-price slots. Hence, another peak might be created there (even a higher one) and the algorithm may stop since it cannot improve, much before reaching a good solution. This issue can be eliminated by applying it on instances where peak occurs at the beginning of the horizon. However, complexity issue remains.

4.6 Peak Cuts Heuristic

As mentioned before, there are three variables in our models: prices, peak and schedule. We have shown that when schedule is fixed, the corresponding MIP is an IO problem and the resulting LP can be solved easily. Also, when we fix prices, it is enough to solve the lower level problem and obtain optimal schedule and peak load quickly.

In this heuristic, we try to find an answer to the question whether the optimal solution corresponding to a fixed load can be easily obtained. The procedure is as follows:

- Compute an upper bound Γ_U using BC
- Iteration $j = 0$, $z_{inc} = z_{BC}$

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- while **true**

  $j \leftarrow j + 1$

  * Add the cut $\Gamma = \Gamma_U - j$ to the corresponding MIP and solve.
  * If $z_j \geq z_{inc}$, update z_{inc}.

    Else, **end loop**

The only drawback of this method is the computation time. Even when the peak load is fixed, computation time of MIP increases quickly as j increases and it may even become less efficient than the exact method. Besides, objective function is not concave with respect to peak. Since iterations are time consuming, we cannot keep searching after reaching a local optimum.

This method shows us that unlike p and x, knowing the optimal value of Γ does not simplify the problem.

4.7 Experimental Results

In this section, the experiments of Price and Peak Search heuristics on randomly generated instances of NBM, PBM and MBM models are presented. Comments are given with respect to peak load, net revenue and computation time.

The parameters of the models ($\lambda_{n,a}$, $\gamma_{n,a}$, $E_{n,a}$, $k_{n,a}$, $l_{n,a}$) are generated using uniform distribution over the intervals presented in Table 4.2. $TW^b_{n,a}$ is generated as described in Chapter 3. Price ceiling p^h_{max} has a fixed value for all models and instances.

A random inconvenience coefficient ($\lambda_{n,a}$) is generated for each customer n and it is assumed to be the same for all appliances of the same user.

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Parameters	Intervals
$\lambda_{n,a}$	[1, 5]
$\gamma_{n,a}$	[4, 12]
$E_{n,a}$	[12, 48]
$k_{n,a}$	[8, 15]
$l_{n,a}$	[2, 9]

Table 4.2: Intervals of parameters

The instance sizes differ with models due to their varying difficulty. For NBM, we consider 12 customers, each owning 5 nonpreemptive appliances, for PBM, there are 6 customers owning 5 preemptive devices each and finally, for MBM there are 7 customers each owning 3 preemptive and 2 nonpreemptive appliances. All models are tested with peak penalty parameter κ set to: 200, 400, 600, 800 and 1000.

To assess the quality of heuristic methods, solutions are compared to the Base Case (BC) and the exact solution of the classical exact method (CEM).

All problems are solved with CPLEX version 12.3 on a computer with 2.66 GHz Intel Xeon CPU and 4 GB ram, running on Windows 7 operating system. Time limit is set to 4 hours for classical exact method (CEM). For the instances not solved to optimality within this limit, the heuristic solutions are compared to the best integer solution. At each line, the average results over 10 instances are reported.

The average peak load and net revenue values of instances for PBM are given in Table 4.3. The heuristic solutions are compared to CEM and BC. On average, the heuristics' solutions are 0.31% and 0.05% away from the optimal net revenue for PH and PSH, respectively. The peak load value is 0.22% and 0.07% away from optimal peak for PH and PSH, respectively. These tests show that both peak loads and net revenues tend to decrease as κ increases.

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Table 4.3: Comparison of Heuristic Results of PBM to BC and CEM

	Av Peak Load				Av Net Revenue			
(κ)	BC	CEM	PH	PSH	BC	CEM	PH	PSH
200	65.40	56.07	55.77	56.07	51540.00	52358.11	52328.69	52358.11
400	65.40	50.96	50.96	50.96	38460.00	41612.21	41612.21	41612.21
600	65.40	47.10	47.51	47.10	25380.00	31898.18	31859.72	31898.18
800	65.40	44.96	44.95	45.50	12300.00	22786.64	22786.64	22757.97
1000	65.40	44.63	45.05	44.26	-780.00	13838.26	13647.65	13821.56
Avg(%)			0.22%	0.07%			0.31%	0.05%

Moreover, both heuristics succeed in finding solutions that are close to global optimum, i.e., they do not get trapped in a local optimum.

Table 4.4: Computation Time of CEM and Heuristics for PBM (sec)

(κ)	CEM	PH	PSH
200	31.90	21.70	13.50
400	410.40	102.90	74.70
600	2022.70	141.90	122.10
800	4244.20	154.00	152.20
1000	8717.70	153.60	154.00

The comparison of the computation times of CEM and heuristics for PBM is given in Table 4.4. The exact method's time limit is fixed to 14400 seconds for CEM. The solution time limit of the MIP step at the end of both heuristics is set to 150 seconds. Even if computation times of all three methods increase as κ increases, the heuristics produce considerably good solutions in much shorter time than optimal solution obtained by CEM.

In order to put the performance of the algorithmic part of the heuristics into perspective, optimality gap of CEM after 150 seconds is compared to the net revenue gap values of PH and PSH which is defined as

$$\frac{100 \times (z^* - z_{OM})}{z^*}.$$

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where z^* and z_{OM} are the objective function values of the optimal solution and output of the method (CEM_{150}, PH or PSH), respectively. The comparison of gap for different κ values are given in Figure 4.6. Gap values increase as κ increases for all three methods. Figure 4.6 shows that reaching good quality solutions within such a short time by only solving the MIP is not possible. In other words, the heuristics provide good starting points for the MIP phase.

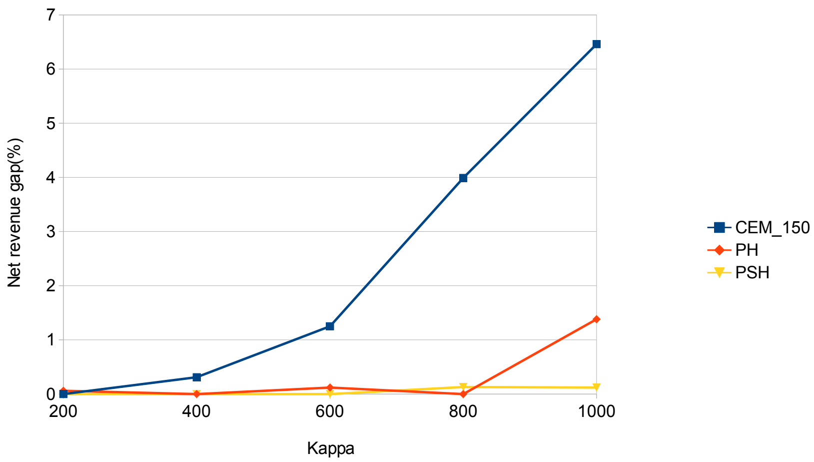

Figure 4.6: PBM: Gap Comparison of CEM_{150}, PH and PSH to OPT(%)

Comparison of the heuristics' net revenue and peak load to BC and CEM for NBM is given in Table 4.5. Both peak load and net revenue decrease as κ increases. However, NBM is not as sensitive to κ as PBM due to nonpreemptive property, i.e., nonpreemptive jobs do not have the same flexibility as preemptive jobs in terms of load distribution. Despite this rigidity, heuristics manage to find good solutions.

Table 4.6 reports computation time of heuristics in comparison to CEM

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Table 4.5: Comparison of Heuristic Results of NBM to BC and CEM

	Av Peak Load				Av Net Revenue			
(κ)	BC	CEM	PH	PSH	BC	CEM	PH	PSH
200	263.20	151.60	158.80	158.00	217180.00	231871.49	231468.69	231458.63
400	263.20	146.40	148.10	148.70	164540.00	202231.53	201831.33	201734.92
600	263.20	145.10	146.60	146.60	111900.00	173139.55	172442.94	171637.63
800	263.20	143.60	144.70	145.50	59260.00	144211.92	143807.34	143721.61
1000	263.20	143.50	144.20	144.20	6620.00	115511.91	115106.32	114526.20
Avg(%)			1.67%	1.75%			0.27%	0.45%

for NBM. There is no direct relationship between computation time and κ for NBM since reshaping a schedule of nonpreemptive devices is a challenging task i.e., even a small change might result in a large perturbation.

Table 4.6: Computation Time of CEM and Heuristics for NBM (sec)

(κ)	CEM	PH	PSH
200	9145.80	153.90	164.90
400	7745.80	153.70	165.20
600	4589.50	153.80	165.40
800	7118.80	153.70	166.50
1000	6182.90	153.70	162.60

The net revenue gap of heuristics in comparison to the solution that CEM finds within 150 seconds for NBM is given in Figure 4.7. PH provides the best solutions for NBM, whereas both heuristics perform much better than CEM. It is considered that PH outperforms PSH since nonpreemptive jobs are not as flexible with respect to peak limit as preemptive ones. Figure 4.7 shows that the heuristics manage to find solutions much closer to optimality than CEM within the same time limit.

The average peak load and net revenue values for MBM are given in Table 4.7. The heuristics' results are compared to CEM and BC in terms of average peak load and average net revenue for different values of κ. On average, the heuristics' net revenue values are 0.13% and 0.20% away from the optimal

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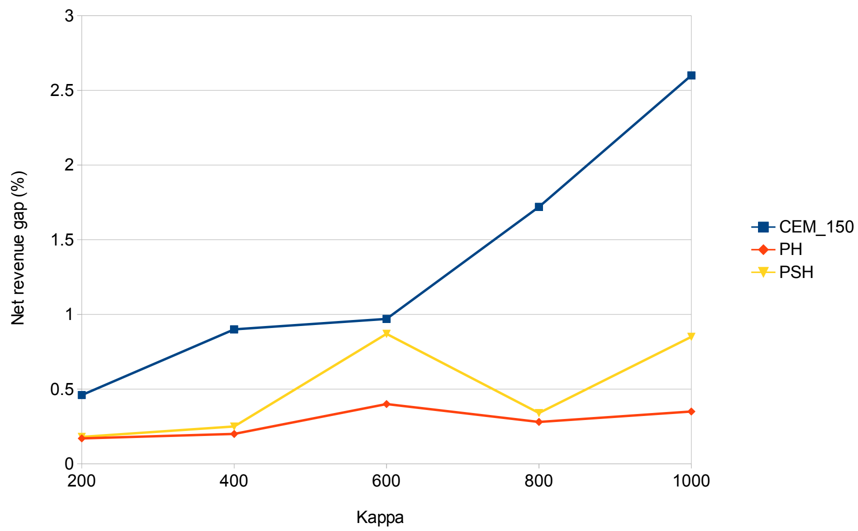

Figure 4.7: NBM: Gap Comparison of CEM_{150}, PH and PSH to OPT(%)

net revenue, peak load value is 0.44% and 0.69% away from optimal peak for PH and PSH, respectively. As previously, both peak load and net revenue decrease as κ increases. Heuristics succeed in finding good solutions even in a problem with both types of appliances. PH and PSH perform better for non-preemptive and preemptive devices, respectively. Therefore, the performance of two heuristics are very similar in the mixed case.

Table 4.7: Comparison of Heuristic Results of MBM to BC and CEM

	Av Peak Load				Av Net Revenue			
(κ)	BC	CEM	PH	PSH	BC	CEM	PH	PSH
200	127.00	83.10	83.30	83.30	99230.00	106161.34	106148.71	106148.71
400	127.00	79.40	79.40	79.90	73830.00	89985.32	89967.83	89894.34
600	127.00	76.35	77.15	77.35	48430.00	74511.72	74403.48	74338.92
800	127.00	75.75	76.70	76.30	23030.00	59262.69	58767.87	59019.60
1000	127.00	74.95	75.35	75.35	-2370.00	44193.59	44050.94	44080.80
Avg(%)			0.44%	0.69%			0.13%	0.20%

In Table 4.8, computation times of heuristics and CEM for MBM are

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Table 4.8: Computation Time of CEM and Heuristics for MBM (sec)

(κ)	CEM	PH	PSH
200	105.60	69.70	82.00
400	884.40	120.50	132.90
600	2844.90	142.20	146.70
800	3487.40	143.40	147.10
1000	8348.30	150.40	148.20

reported. Even if CPU time tends to increase as κ takes higher values for all three methods, the increase is significantly larger for CEM than for the heuristics. As a result, PH and PSH scale better than CEM.

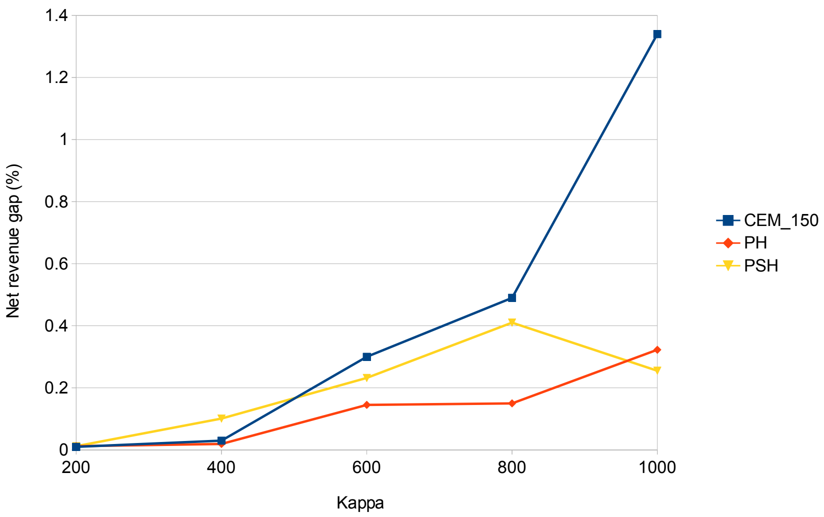

Figure 4.8: MBM: Gap Comparison CEM_{150}, PH and PSH to OPT(%)

Net revenue gap values of heuristics and CEM after 150 seconds with respect to the optimal solution are presented in Figure 4.8. PH provides best solutions for all κ values, and the performance of PSH improves as κ increases.

In order to analyze the scalability of the algorithms, 10 larger instances are generated. For PBM and MBM, these instances involve 20 customers each

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owning 5 appliances, resulting in 100 jobs. For NBM, 10 random instances are generated with 40 customers each owning 5 appliances, resulting in 200 jobs. All instances are solved by CEM and the two heuristics.

Since it is not possible to solve these large instances to optimality within reasonable time, the same time limit as the MIP part of the heuristics (150 seconds) is applied to CEM. As a general rule, CEM cannot even find a feasible solution whereas both heuristics provide solutions for PBM and MBM. Therefore, the heuristic results are compared to BC for these models. It is important to note that both PH and PSH provide a good starting point for the MIP and make it possible to find a solution within 150 seconds. CEM finds feasible solutions within 150 seconds for NBM, however heuristic methods find much better solutions than CEM within the same time interval.

The average net revenue and peak load comparisons of PH and PSH to the BC for PBM model are presented in Figure 4.9 and 4.10, respectively. Heuristic methods allow to increase the net revenue upto 50-60% in comparison to BC. As a general rule, PSH provides considerably higher net revenue and lower peak load than PH for large instances within 150 seconds. Both methods succeed in providing a feasible solution that constitutes a good starting point for the MIP.

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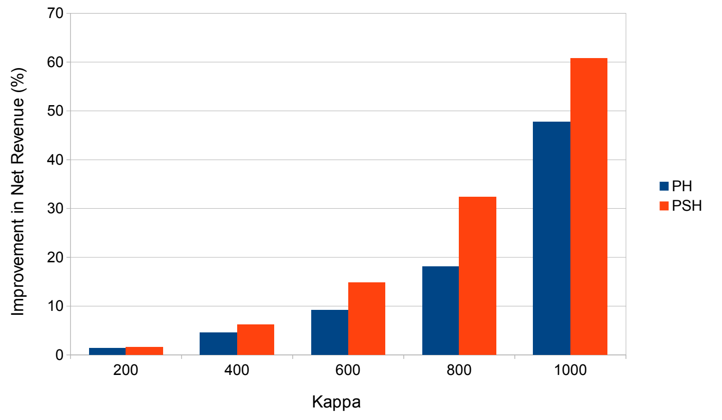

Figure 4.9: PBM: Net Revenue Improvement of PH and PSH wrt BC(%) for 100 jobs

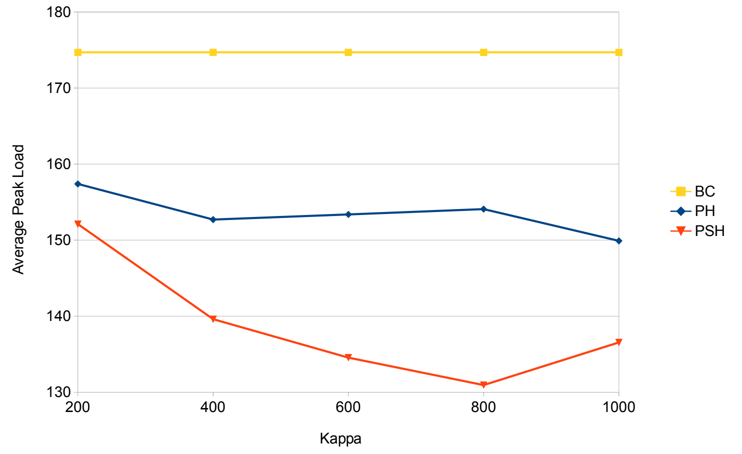

Figure 4.10: PBM: Peak Load of PH and PSH wrt BC for 100 jobs

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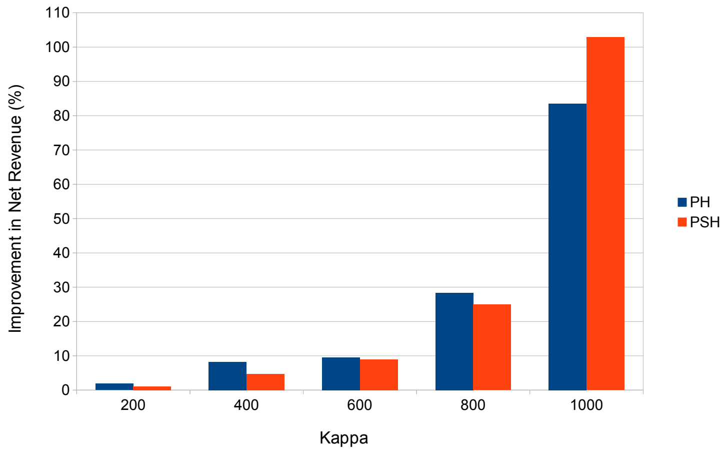

Figure 4.11: NBM: Net Revenue Improvement of PH and PSH wrt CEM_{150}(%) for 200 jobs

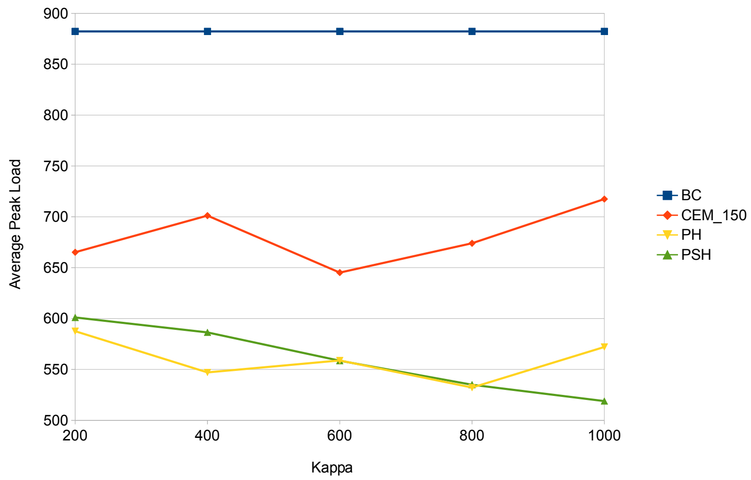

Figure 4.12: NBM: Peak Load of PH and PSH wrt BC and CEM_{150} for 200 jobs

For NBM, a feasible solution can be computed by CEM within 150 seconds.

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Comparison of heuristics with respect to net revenue (%) and peak load with respect to CEM is thus given in Figure 4.11 and 4.12, respectively. It is shown in Figure 4.11 that both PH and PSH provide an improvement of 26.31% and 28.50% on average, respectively. Similarly, average peak loads of PH and PSH are 17.79% and 17.72 % lower that CEM as shown in Figure 4.12, respectively. PH performs better for lower κ values whereas PSH gives better results for the highest value.

The average net revenue and peak load comparisons of PH and PSH to the BC for MBM model are given in Figure 4.13 and 4.14, respectively. Both PH and PSH manage to compute good solutions within 150 seconds, i.e., it is possible to increase the net revenue upto 150% with respect to BC. Besides, their performances are very close to each other for MBM model. PSH performs slightly better for higher κ values, whereas PH is better for lower ones. Again, both PH and PSH are efficient for solving large instances where CEM takes very long time even to reach a feasible solution.

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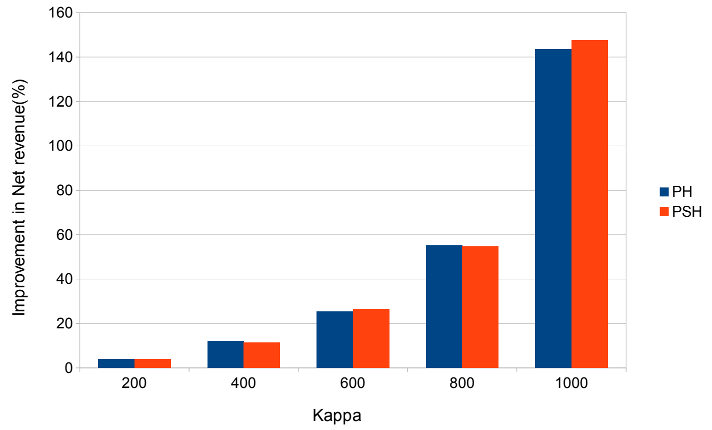

Figure 4.13: MBM: Net Revenue Improvement of PH and PSH wrt BC(%) for 100 jobs

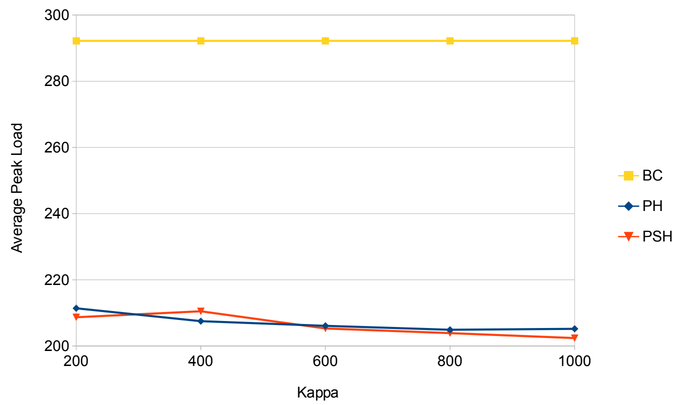

Figure 4.14: MBM: Peak Load of PH and PSH wrt BC for 100 jobs

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4.8 Conclusion

In this chapter, we present Price, Peak Search, Load Shifting, Divide-and-Stitch and Peak Cuts heuristics. All heuristics are intrinsically based on the structure of the bilevel models and the relationship between decision variables. The experimental results of two heuristics that efficiently provide good solutions to PBM, NBM and MBM problems are presented.

As a general rule, Price and Peak Search heuristic solutions have very small optimality gap and it takes both heuristics short time to reach these solutions. In general terms, PH gives better results for nonpreemptive devices whereas PSH works better with preemptive devices.

Both PH and PSH include a MIP procedure as a last step to obtain optimistic solutions. Based on several tests, it is shown that solving MIP alone is not sufficient to compute good solutions, the algorithmic parts of the heuristics are crucial.

In order to measure the scalability, we have also solved large instances with PH and PSH that cannot be solved using classical exact method. It is observed that both heuristics still find good solutions within reasonable time intervals.

Currently, the most time consuming part of PH and PSH is the MIP step. As a future prospect, scalability can be improved by solving the MIP by a more efficient exact algorithm (such as branch-and-bound) rather than directly solving it with an off-the-shelf software. Also, subproblems are quickly solved for small or medium–size instances. However, for much larger instances they would take longer time to solve as well. Efficient exact approaches or heuristics can be used to solve subproblems.

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Adapting the heuristics to different revenue optimization problems and comparing their performances to existing methods can also be an interesting future research subject.

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Chapter 5

Robust Toll Optimization

Contents

5.1 Introduction

Two main approaches are developed in the literature for decision making under uncertainty: stochastic programming and robust optimization. Stochastic programming assumes that data has a known (or estimated) probability distribution. It aims to find optimal policies with by evaluating possible data realizations with different probabilities. However, the parameter may not be stochastic or the probability distribution may not be available. Robust optimization assumes that data is only known to belong to a given uncertainty set

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without any information about the underlying model (set-based representation of uncertainty) [Ben-Tal 1998]. In this approach, the optimal decision is computed by taking all possible scenario realizations into account in a deterministic sense. In the context of this chapter, we focus on robust optimization.

A deterministic linear program (P) can be expressed as $\min\{c^T x \mid Ax \geq b\}$ with input data (A, b, c). Let us consider (P) with uncertain (A, b), i.e., $(A, b) \in \mathscr{U}$ where $\mathscr{U}$ is the set of all possible data realizations of (A, b). The robust counterpart of (P) is $\min\{c^T x \mid Ax \geq b \;\; \forall (A, b) \in \mathscr{U}\}$. The optimal solution of this problem has to be feasible for any realization of (A, b).

Several approaches are developed in the literature for robust optimization. Soyster proposed to use set containment [Soyster 1973] instead of defining feasible region with convex inequalities. Column-wise uncertainty is considered in this case, columns of the constraint matrix belong to a given convex set. Soyster considers the following linear optimization problem:

$$
\begin{aligned}
\max \quad & cx \\
\text{s.t.} \quad & \sum_{j=1}^{n} A_j x_j \leq b \qquad\qquad \forall A_j \in K_j,\ j = 1, \ldots, n \\
& x \geq 0.
\end{aligned}
$$

where A_j are the columns of the constraints matrix and the uncertainty sets K_j are convex. As stated in [Bertsimas 2003], the solution that is feasible for all possible data realizations (uncertain hard constraints) is considered to be over-conservative.

The issue of conservatism is addressed in the literature several

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times. [Ben-Tal 1998], [El Ghaoui 1998], [Ben-Tal 2000], [Ben-Tal 2002], [Ben-Tal 2009] offer uncertain linear problems with ellipsoidal uncertainties. The robust counterparts of these problems are formulated as conic quadratic problems and solved. As mentioned in [Bertsimas 2004], although this reformulation can be useful to approximate more complex uncertainty sets, it requires solving nonlinear convex problems which are computationally even harder than Soyster's formulation [Soyster 1973].

In [Bertsimas 2003] and [Bertsimas 2004], Bertsimas and Sim propose an approach that allows to determine the level of conservatism of the optimization problem while preserving the linear optimization framework of Soyster [Soyster 1973]. As a result, the robust counterpart of discrete optimization problem has the same complexity as the original problem. Bertsimas-Sim (B-S) method remains to be one of the most popular approaches in the area of robust optimization.

In §5.2, we describe robust bilevel programming. Rather than using approaches that lead to solving robust counterparts that are even more complicated than the robust optimization problem, we consider the Bertsimas-Sim approach (§5.3). We apply Berstsimas-Sim approach to a well-studied bilevel problem called toll setting problem in §5.3.1. In the same section, counterintuitive examples with different data sets are presented as well. Then, the Bertsimas-Sim approach is applied on PCBM in two different ways and the implications are discussed in §5.3.2. The chapter is concluded with remarks and future perspectives.

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5.2 Robust Bilevel Programming

A bilevel programming problem in general form is formulated as

$$\begin{aligned} \text{BP:} \quad & \max_{x \in X} \quad f(x,y) \\ & \text{s.t.} \quad y \in \arg\min_{y' \in Y(x)} g(x,y'). \end{aligned}$$

where x and y are upper and lower variables, F and f are upper and lower level objective functions, respectively. Also,X and $Y(x)$ are defined as $X = \{x \mid G(x,y) \leq 0\}$ and $Y(x) = \{y \mid g(x,y) \leq 0\}$, respectively.

In the context of robust optimization, the worst-case solution for the decision maker is computed under data uncertainty. Bilevel programming problems have two decision makers. Therefore it is important to specify whether it is the worst-case for the leader or the follower.

In the robust counterpart of BP, it is assumed that the scalars of the lower level objective f belongs to an uncertainty set $\mathscr{F}$. (Uncertainty may appear at the lower level constraints if g belongs to an uncertainty set $\mathscr{G}$.) If one analyzes the worst-case for the leader, then it should be assumed that a third player (say *nature*) selects these scalars to minimize the value of $F(x,y)$. Else if the worst-case of the follower is considered, then it is assumed that these scalars are chosen from $\mathscr{F}$ to maximize $f(x,y)$. It is important to emphasize that we consider data uncertainty only at the lower level.

In this thesis, we focus our attention to bilinear bilevel programming which is frequently used to model revenue optimization problems. It is formulated

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as:

$$\begin{aligned}
\text{BBP:} \quad & \max_{x,y} && c_1xy + c_2x \\
& \text{s.t.} && A_1x + B_1y = b_1 \\
& && x \geq 0 \\
& \min_{y} && c_3xy + c_4y \\
& \text{s.t.} && A_2x + B_2y = b_2 \\
& && y \geq 0.
\end{aligned}$$

where x and y are upper and lower level variables, respectively. Both objective functions have a bilinear term xy. In this case, it is assumed that the scalars c_3 and c_4 of the lower level objective function belong to an uncertainty set $\mathscr{C}$. The worst-case for the leader of BBP can be reformulated as follows:

$$\begin{aligned}
\text{RBBP:} \quad & \max_{x,y} \min_{(c_3,c_4)\in\mathscr{C}} && c_1xy + c_2x \\
& \text{s.t.} && A_1x + B_1y = b_1 \\
& && x \geq 0 \\
& \min_{y} && c_3xy + c_4y \\
& \text{s.t.} && A_2x + B_2y = b_2 \\
& && y \geq 0.
\end{aligned}$$

It can be observed that nature enters the game as a third player by deciding on the values of c_3 and c_4.

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5.3 The Bertsimas-Sim Approach

First of all, let us explain the Bertsimas-Sim method considering the following linear integer program:

$$\begin{aligned} \min \quad & cx \\ \text{s.t.} \quad & Ax \geq b \\ & x \in \{0, 1\}. \end{aligned}$$

Assume that the objective function coefficients c_j are uncertain and they take values from an interval, $[\bar{c}_j, \bar{c}_j + d_j]$. The B-S approach is based on the assumption that *not all coefficients will be pushed to their upper bounds simultaneously*. The number of coefficients reaching the value $c_j + d_j$ to some constant K are fixed and it is decided which subset of K coefficients would be the most harmful. In other words, they define the maximum restricted damage function as:

$$\max_{\{S|S\subset\{1,\ldots,n\},|S|=K\}} \sum_{j\in S} d_j x_j.$$

where K controls the level of conservatism by taking a value between 1 and n. This function can be equivalently written as:

$$\max_{s} \quad \sum_{j=1}^{n} x_j d_j s_j$$

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$$\text{s.t.} \quad \sum_{j=1}^{n} s_j = K$$
$$s_j \in \{0,1\}, \qquad j = 1,\ldots,n.$$

Integrality constraints of s_j can be linearly relaxed and the model becomes an LP which allows to use the dual model:

$$\min_{y,l} \quad Ky + \sum_{j=1}^{n} l_j$$
$$\text{s.t.} \quad y + l_j \geq x_j d_j \qquad j = 1,\ldots,n.$$

where y and l_j are dual variables. This dual model can be reformulated as a function:

$$\min_{y} \quad Ky + \sum_{j=1}^{n} \max(0, d_j x_j - y).$$

Then, we can reformulate the robust counterpart as:

$$\min_{x \in X, y} \ \bar{c}x + Ky + \sum_{j=1}^{n} \max(0, d_j x_j - y).$$

where $X = \{x \in \{0,1\}^n : Ax \geq b\}$. The max terms can be linearized by introducing nonnegative variables l_j as follows:

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$$\min_{x \in X, y, l \in L} \quad \bar{c}x + Ky + \sum_{j=1}^{n} l_j.$$

where $L = \{l \geq 0 : l_j \geq d_j x_j,\ l_j \geq y\ \forall j\}$. As mentioned before, the complexities of the original problem and the robust counterpart are the same which makes the B-S approach convenient for hard problems.

5.3.1 Bertsimas-Sim Approach on Toll Setting Problem

In order to analyze the application of the Bertsimas-Sim approach on a bilevel program, we now turn our attention to a well-studied problem called toll setting problem. In the context of this chapter, single commodity toll setting problem is studied. Multi-commodity version of the problem can be found in [Brotcorne 2001].

The toll setting problem [Brotcorne 2001, Labbé 1998] consists of setting tolls on a subset of arcs in order to maximize revenue over a network. The set of arcs $\mathcal{A}$ in the network is divided into subset $\mathcal{A}_\infty$ of toll arcs and subset $\mathcal{A}_\in$ of toll-free arcs. The leader aims to maximize revenue by setting prices on the toll arcs of the network while the follower solves a shortest path problem. For each toll arc $a \in \mathcal{A}_1$, there is a minimal unit travel cost c and a toll T. The cost of traveling on a toll-free arc $a \in \mathcal{A}_2$ is defined as d. The amount of flow corresponding to toll and toll-free arcs are x and y, respectively. The bilinear bilevel model is as follows:

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$$\begin{aligned}
\text{TOLL:} \quad & \max_{T} && Tx \\
& \min_{x,y} && (c+T)x + dy \\
& \text{s.t.} && Ax + By = b \\
& && x, y \geq 0.
\end{aligned}$$

where $[A|B]$ is the incidence matrix of the network and b is the demand of the commodity. It is assumed that there does not exist a toll vector that generates revenues and creates negative cost cycles in the network which means that the lower level optimal solution is a shortest path. Moreover, there exists at least one toll-free path between the origin and destination. These assumptions ensure that the problem is bounded.

In the original toll setting problem, the costs of toll-free arcs are fixed. However, they may change due to natural or economical causes, such as an accident or renewal of roads that would create an increase or a decrease of cost, respectively. In the B-S context, this change is referred as the damage that nature/adversary might impose.

It is shown by Bertsimas and Sim [Bertsimas 2003] that robust counterpart of polynomially solvable integer problems are also polynomially solvable. In the PhD thesis of Alessia Violin [Violin 2014], robust counterparts of some (pseudo)-polynomial cases of toll setting problem are studied. B-S approach is applied to the cases of single toll arc multiple commodities, single commodity multiple toll arcs and unit toll. Uncertainty in demand and upper bound on tolls are analysed for all cases. It is shown that robust counterparts of these

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cases remain to be (pseudo)-polynomial. The author also proposes algorithms to find optimal solutions.

In this thesis, we study a more general approach to robust optimization. Let us consider a scenario where the cost of toll-free arcs d are uncertain and belong to a set D. Nature can be also considered as a third player which plays against the leader.

The set D can be defined in several different forms: box constraints (if there are only upper and lower bounds), simplex (if the total cost of arcs stay the same whereas individual costs might change), polyhedron (if there are linear constraints on D) etc. Bertsimas-Sim definition is assumed here, i.e. $d_a \in [\bar{d}_a, \bar{d}_a + e_a]$, $a \in \mathcal{A}_2$ and only K many d_a's would reach $\bar{d}_a + e_a$. Examples of D with different forms and structures are discussed at the end of this subsection.

The toll setting problem with robust toll-free arc costs can be formulated as:

$$
\begin{aligned}
\text{R-TOLL:} \quad & \max_{T} \quad Tx \\
& \min_{d \in D} \quad Tx \\
& \min_{x,y} (c+T)x + dy \\
& \text{s.t.} \quad Ax + By = b && (5.1) \\
& \qquad\; x, y \geq 0. && (5.2)
\end{aligned}
$$

The optimistic version of toll setting problem is considered in TOLL i.e.,

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when there are more than one equal cost paths for the follower, the one with the highest revenue is selected. It is possible to consider R-TOLL in both optimistic and pessimistic ways by using corresponding objective functions:

$$\text{OR-TOLL:} \quad \max_{T,x,y} \quad \min_{d \in D} \quad Tx$$

$$\text{PR-TOLL:} \quad \max_{T} \quad \min_{d \in D,x,y} \quad Tx$$

In both optimistic and pessimistic objective functions, revenue maximization is achieved through pricing. The optimistic case is similar to the original problem, when two paths have equal costs, the better one for the leader is chosen. In the pessimistic case, the follower coordinates with nature. In other words, between two equivalent solutions for the follower, the one that minimizes revenue is selected for the leader.

Lower level problem of R-TOLL can be reformulated using its KKT conditions as previously explained in Chapter 3. The dual variable λ corresponds to Constraint 5.1. Optimistic and pessimistic versions of single level R-TOLL problem:

$$\text{OR-TOLL:} \quad \max_{T,x,y} \quad \min_{d \in D} \quad Tx$$

$$\text{PR-TOLL:} \quad \max_{T} \quad \min_{d \in D,x,y} \quad Tx$$

$$\text{s.t.} \quad Ax + By = b \tag{5.3}$$

$$\lambda A \leq c + T \tag{5.4}$$

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$$x(c + T - \lambda A) = 0 \quad (5.5)$$

$$\lambda B \leq d \quad (5.6)$$

$$y(d - \lambda B) = 0 \quad (5.7)$$

$$x, y \geq 0. \quad (5.8)$$

where Constraints 5.4 and 5.6 are the dual constraints that correspond to variables x and y, respectively. Constraints 5.5 and 5.7 are the complementary slackness constraints.

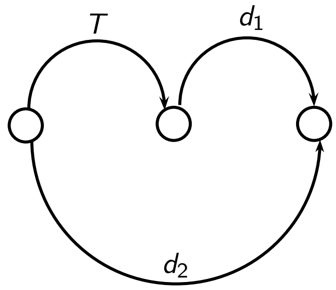

Figure 5.1: Robust Example 1

In Figure 5.1, we consider a simple network containing 3 nodes, 2 toll-free arcs and one toll arc. Let's assume that the costs of toll-free arcs are 3 and 5, respectively ($\bar{d}_1 = 3, \bar{d}_2 = 5$). The cost of one of the arcs may increase by 1 unit ($e_1 = e_2 = 1$, $K = 1$). In the TOLL model, the optimal tariff T would take value 2 and all the flow would be assigned to T and d_1. However, in the robust version, to each toll decision T of the leader, nature responds with a d that minimizes revenue. Since the cost of only one toll-free arc may increase, nature has two possible outcomes: (3,6) or (4,5). The leader has to choose a T value between 0 and 1. If T is larger than 1, revenue would be equal to zero. Moreover, in the pessimistic case, if $T = 1$, all flow will be on d_2 and revenue

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will be zero. Therefore, T should be $1-\varepsilon$, where ε is a small positive number. This illustrates that the pessimistic problem is not well-defined [Dempe 2002].

In a network of toll and toll-free arcs, one may consider that a cost increase in the toll-free arcs would be beneficial for the leader. However, this small example demonstrates that if there is a cost increase on the same path as the toll arc, revenue may decrease and if it happens on a toll-free path, revenue may increase. In other words, depending on where the change occurs, revenue of the leader may be affected positively or negatively.

The Properties of R-TOLL Model

So far, the set D is assumed to be defined as the B-S approach in R-TOLL problem. As mentioned before, it is possible to define it in a more general way. For instance, D can take many forms such as a box, simplex, polyhedron, discrete set etc. R-TOLL problem has a complex structure and we came across several counter-intuitive examples while studying it under different assumptions. They are illustrated on two small examples, Figures 5.2(b) and 5.2(a).

- **The solution is not always extremal of D**

  For instance, a single commodity example is given in Figure 5.2(b). Assume that $D = \{(d_1, d_2)|d_1, d_2 \geq 0, d_1 + d_2 = 6\}$ and $c_1 = c_2 = 3$. The extreme points of the polytope D are (0,6) and (6,0). However, no matter what price vector values are chosen by the leader, the action that is chosen by the nature in this case is (3,3) since it results in zero revenue. Hence, the optimal solution of the problem is not an extremal solution with respect to D.

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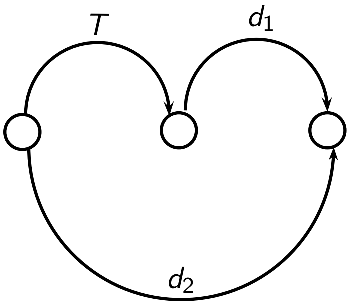
- **One of the toll-free arcs fails**

  Firstly, one may assume that one of the toll-free arcs is doomed to fail in Figure 5.2(a). In this case, if d_1 fails, then the revenue is equal to zero no matter what value is set for T. If d_2 fails, then T can be set to infinity. This small example demonstrates that paths have significance as much as arcs themselves.

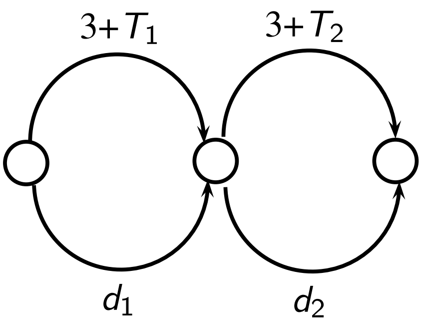
- **D is a discrete and finite set**

  In this scenario, it is assumed that there exists two possible outcomes on Figure 5.2(a), $D = \{(2,5),(6,5)\}$. Although first outcome seems to have smaller values for the toll-free arcs, second outcome results in zero revenue and hence a worse possibility for the leader. Even when D is a discrete and finite set, the solution of robust toll setting problem is not trivial.

The examples illustrate that there is no pure strategy for nature in this three-level game, we cannot simply say that d would take smallest or largest possible values before solving the problem to optimality.



(a) Robust Example 1



(b) Robust Example 2

Figure 5.2: Examples

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It is important to note that if there exists a version of the OR-TOLL problem where the optimal d is always an extreme point of D or D is a finite discrete set, then its objective function can be reformulated as:

$$\max_{T,x,y}\{U \mid U \leq Tx \;\; \forall d \in ext(D)\}$$

where $ext(D)$ is the set of extreme points of D.

5.3.2 Bertsimas-Sim Approach on PCBM

In this section, Bertsimas-Sim method is applied on PCBM, the bilevel model with preemptive devices and a competitor firm (§3.1.1), with robust competition prices. Two versions (price increase and price decrease) are analyzed.

The B-S methodology is based on the observation that a cost increase is harmful for the decision maker. Indeed, when a single level convex minimization problem is considered, it is clear that any perturbation that increases the objective function value is harmful. However, a bilevel program with conflicting objectives is not convex in general and it is not straightforward to decide which player is harmed and which one benefits from the uncertain situation. In other words, a possible cost increase may harm the follower while being profitable for the leader or vice versa.

Let us consider the PCBM setting with robust competition prices. It is assumed that there is a competitor firm with uncertain prices $\bar{p}^h$ together with the energy provider which is also the leader of the bilevel program. In this context, customers choose how much load to buy from each firm while facing

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uncertain prices.

Case 1: Price Increase

In this case, it is assumed that the competitor prices take values independently of each other within the interval $[\Pi^h, \Pi^h + d^h]$ where $d^h \geq 0$. Note that this is a *worst-case analysis* for the follower. If the competitor prices rise, it gives the leader an opportunity to increase its prices as well without losing market share. The maximum restricted damage that is induced by unknown prices on the follower would be:

$$\sum_h d^h (\sum_{n \in N} \sum_{a \in A_n} \bar{x}^h_{n,a})$$

Bertsimas and Sim discuss that not all the prices would reach their upper (lower) bounds at the same time. Therefore, the level of conservatism of the model can be set and assume that only K many of the prices will be at their maximum value. We can choose a K value in $\{1, \ldots, H\}$ and then solve the lower level with the following objective function:

$$\min_x \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} p^h x^h_{n,a} + \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} \Pi^h \bar{x}^h_{n,a} + \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} C_{n,a}(h)(x^h_{n,a} + \bar{x}^h_{n,a})$$
$$+ \max_{\{S | S \subseteq \{1,\ldots,H\}, |S| = K\}} \sum_h d^h (\sum_{n \in N} \sum_{a \in A_n} \bar{x}^h_{n,a})$$

The last term in this objective function is considered as a separate optimization problem which is given as follows:

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$$\max \sum_{h\in H} d^h \sum_{n\in N}\sum_{a\in A_n} \bar{x}^h_{n,a} s^h$$

s.t.

$$\sum_{h\in H} s^h = K$$

$$s^h \in \{0,1\}$$

The relaxed problem always has a binary optimal solution (Theorem 1 of [Bertsimas 2003]). Then, dual of this problem is:

$$\min \qquad Ky + \sum_{h\in H} \lambda^h$$

s.t.

$$\lambda^h \geq d^h \sum_{n\in N}\sum_{a\in A_n} \bar{x}^h_{n,a} - y \qquad \forall h \in H$$

$$\lambda^h \geq 0 \qquad \forall h \in H$$

This model can be reformulated as:

$$\min \qquad Ky + \sum_{h\in H} \max\{d^h \sum_{n\in N}\sum_{a\in A_n} \bar{x}^h_{n,a} - y, 0\}$$

We can replace the last term in the follower's objective with this expression. Second part can be linearized by using an additional continuous variable l^h. Then, the lower level problem becomes:

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$$
\begin{aligned}
\min & \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} p^h x^h_{n,a} + \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} \Pi^h \bar{x}^h_{n,a} \\
& + \sum_{n\in N}\sum_{a\in A_n}\sum_{h\in T_{n,a}} C_{n,a}(h)(x^h_{n,a}+\bar{x}^h_{n,a}) + Ky + \sum_{h\in H} l^h \\
& \text{s.t.} \\
& x^h_{n,a}+\bar{x}^h_{n,a} \leq \beta^{max}_{n,a} && \forall n\in N, \forall a\in A_n, \forall h\in T_{n,a} \\
& \sum_{h\in T_{n,a}} (x^h_{n,a}+\bar{x}^h_{n,a}) \geq E_{n,a} && \forall n\in N, \forall a\in A_n \\
& l^h \geq d^h(\sum_{n\in N}\sum_{a\in A_n} \bar{x}^h_{n,a}) - y && \forall h\in H && (5.9) \\
& l^h \geq 0 && \forall h\in H && (5.10) \\
& x^h_{n,a}, \bar{x}^h_{n,a} \geq 0 && \forall n\in N, \forall a\in A_n, \forall h\in T_{n,a}
\end{aligned}
$$

The term Ky is minimized in the objective function and variable y is free to take negative and positive values. However, it does not cause the problem to be unbounded. The term $\sum_{h\in H} l^h$ is also minimized and constraints (5.9) & (5.10) make sure that y does not go to negative infinity.

In this case, the differences between the lower level problem of PCBM and the model above are $H+1$ many additional continuous variables and two sets of linear constraints. In other words, the complexities of PCBM and its robust version in Case 1 are not different.

It is possible to deduce that in the optimal solution of robust PCBM, K time slots with the highest demand will have the higher competitor prices. This is the worst case for the follower and best case for the leader in our bilevel setting.

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Case 2: Price Decrease

Let us assume that the competitor prices take values within the interval $[\Pi^h - d^h, \Pi^h]$ where $d^h \geq 0$. This is a *best-case analysis* for the follower. If the competitor prices decrease, it jeopardizes the leader's market share unless its prices also reduced. In this case, the leader faces the risk of losing revenue. The maximum restricted gain induced by unknown prices on the follower's objective would be:

$$\sum_h d^h (\sum_{n \in N} \sum_{a \in A_n} \bar{x}^h_{n,a})$$

This term represents a gain for the follower, hence it is subtracted from its total cost:

$$\min_x \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} p^h x^h_{n,a} + \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} \Pi^h \bar{x}^h_{n,a} + \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} C^h_{n,a}(x^h_{n,a} + \bar{x}^h_{n,a})$$
$$- \max_{\{S | S \subseteq \{1,\ldots,H\}, |S|=K\}} \sum_h d^h (\sum_{n \in N} \sum_{a \in A_n} \bar{x}^h_{n,a})$$

Then, the lower level problem can be rewritten as follows:

$$\min \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} p^h x^h_{n,a} + \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} \Pi^h \bar{x}^h_{n,a}$$
$$+ \sum_{n \in N} \sum_{a \in A_n} \sum_{h \in T_{n,a}} C^h_{n,a}(x^h_{n,a} + \bar{x}^h_{n,a}) - \sum_{h \in H} d^h \sum_{n \in N} \sum_{a \in A_n} \bar{x}^h_{n,a} s^h$$

s.t.

$$x^h_{n,a} + \bar{x}^h_{n,a} \leq \beta^{max}_{n,a} \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a}$$

$$\sum_{h \in T_{n,a}} (x^h_{n,a} + \bar{x}^h_{n,a}) \geq E_{n,a} \qquad \forall n \in N, \forall a \in A_n$$

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$$\sum_{h \in H} s^h = K \tag{5.11}$$

$$s^h \in \{0, 1\} \qquad \forall h \in H \tag{5.12}$$

$$x^h_{n,a}, \bar{x}^h_{n,a} \geq 0 \qquad \forall n \in N, \forall a \in A_n, \forall h \in T_{n,a}$$

Constraint (5.11) and (5.12) make sure that only T many s^h get value 1 in order to cause the maximum reduction in the objective for any fixed price decision of the leader. However, in this case, the lower level problem is a MIP with a bilinear objective function which makes it significantly harder than Case 1 and PCBM. The B-S approach uses the dual problem to deal with this complexity issue which is directly applied in Case 1 above. The same method cannot be put to work due to the sign change in the objective function.

5.4 Conclusion

In this chapter, we discuss how to incorporate a robust optimization approach into bilevel programming. It is known that bilevel programs are NP-complete in general form besides being non-convex and non-continuous. In order to not increase the already existent complexity, a method proposed by Bertsimas and Sim is chosen. This method uses interval representation for uncertain parameters to introduce robustness into the problem. It requires to include only continuous variables and linear constraints, and hence formulates a robust counterpart with the same complexity as the original problem.

Bertsimas-Sim approach is based on a clear definition of *damage* caused by an uncertain parameter that takes values from an interval. However, when two decision makers of an optimistic bilevel program have conflicting objectives

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(like toll-setting problem and preemptive competitive bilevel model), it is not always obvious which player would be harmed by a decrease or increase of a coefficient's value. Therefore, although Bertsimas-Sim approach is efficient in terms of complexity, it is not the most suitable method for this class of bilevel programs. Scenario-based robust approach is thus considered to be one of the promising future research directions for this work. In this approach, every scenario corresponds to a realization of uncertain data and a set of (worst-case) scenarios are built. Among the solutions that are feasible for all scenarios, the "best" one is selected as the robust solution. Due to computational difficulty, algorithmic and metaheuristic approaches will likely be considered for solving it.

We believe that robust decision making will be an important concept in bilevel framework in near future and there is a lot of room for novel ideas and methods in this area.

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Chapter 6

Conclusion and Perspectives

Demand for energy grows and becomes more irregular due to several reasons such as technological developments and population growth. Although fluctuating load curves and high peaks cause the energy provider to have a large generation capacity to avoid blackouts, it is an expensive solution. Therefore, a more efficient energy management is required to minimize the peak and to maintain supply-demand balance. Demand side management methods are often utilized to increase efficiency and flatten demand curve.

In this thesis, we propose a bilevel programming approach to revenue optimization and demand side management problem in a smart grid environment. The leader is an energy provider and the follower is customers in this bilevel setting. Instead of handling the issues of the two players separately, customers' concerns are integrated into the decision making process of the provider. To the best of our knowledge, bilevel approach is unprecedented for this problem.

In our setting, the leader aims to find a trade-off between revenue and peak cost whereas the follower minimizes total cost.

Several demand response programs are used in different countries to achieve demand side management (DSM) objectives. We focused our attention to utilizing day-ahead real-time pricing to perform load shifting DSM

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objective in this thesis. Although real-time pricing is theoretically an efficient method for demand management, when the prices change on daily basis, users cannot adapt their behavior quickly. Hence, the method is not quite effective in real life. Therefore, a smart grid structure is proposed such that every user has a smart meter which allows two-way communication and data transfer. Thanks to this automated system, it is possible for the customers to be informed about daily changing prices and behave accordingly.

The leader chooses and communicates the prices to the smart grid. Afterwards, the smart grid determines the schedule of all users' appliances based on their preferences. Demands and time windows of the user are registered by their smart meters. Although time windows are usually larger than the minimum time required to complete the job, it is assumed that users are sensitive to delays. This property is included in the models via an inconvenience cost. The smart grid minimizes both billing and inconvenience costs. In other words, the smart grid decides which jobs can be shifted to other time slots with respect to corresponding inconvenience costs in exchange for a bill discounting.

We developed several bilevel bilinear models for this problem regarding different appliance types (preemptive and/or nonpreemptive) and market conditions (monopoly or competition). In order to reformulate these models as single level mixed integer problems, we used both classical and novel methods. The MIP models are numerically tested on CPLEX with randomly generated instances. The properties of models are analyzed and explained with respect to the sensitivity analysis of peak weight and time window width (TWW). It is shown that the computation time increases as peak weight and TWW

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increases. Furthermore, it is illustrated that bilevel approach provides a solution with lower peak cost and higher net revenue for the leader and lower total cost for the follower in comparison to Base Case. Bilevel approach produces a solution that is better for both players and hence improves system efficiency.

Several heuristic approaches are developed based on the impact of three variables: price, schedule and peak load. Price heuristic computes solutions with high net revenue by changing prices and hence influencing the load curve. Peak Search heuristic performs a search on peak load values. Peak Cuts heuristic solves the MIP with fixed peak load values. Load Shifting heuristic changes the schedule to find a price-schedule pair with high net revenue. Lastly, Divide-and-Stitch heuristic divides the problem into smaller subproblems and focuses on the periods with high peak to decrease computation time.

Two of these approaches, namely Price and Peak Search heuristics, are numerically tested on both small and large instances. It is illustrated that they provide high quality solutions in short time intervals and they can give good solutions for large instances where classical exact method cannot compute a feasible solution within the same time period.

Finally, we presented a brief discussion about integrating robust optimization into bilevel programming framework. Bertsimas-Sim approach is applied on the toll setting problem and on preemptive competitive bilevel model. The difficulties are pointed out and some counter-intuitive examples are displayed.

The importance of energy optimization is rising in recent years. It involves many actors and resources which complicates the problem. We believe that there are many more interesting versions, scenarios and methods that need to be investigated.

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One of such extended scenarios would be data uncertainty. In real life, customer demand might be uncertain due to unexpected events or change of habits. Moreover, the prices of the competitor may be uncertain as previously mentioned in Chapter 5. If the probability distribution function of data is known, then the problem can be modelled using stochastic optimization and evaluated based on different scenarios. On the other hand, if we only have a lower and an upper bound with an unknown distribution, robust optimization would be a better approach. Besides Bertsimas-Sim method, scenario based robust methods can be applied.

Throughout this thesis, demand is assumed to be fixed. However, price elastic demand would be an interesting extension of the problem. Net revenue, peak load and total cost change with respect to different levels of elasticity can be analyzed.

On the supplier level, it can be assumed that the average of all prices is fixed rather than setting a price ceiling. In this case, prices can increase or decrease with respect to demand.

Lastly, a scenario with non-cooperative users can be considered. In this case, the congestion among users would be modelled by using a mathematical model with equilibrium constraints.

In conclusion, revenue optimization and demand side management of energy problems using bilevel programming in smart grid systems are studied in detail and it is believed that the thesis gives rise to many interesting questions for further research.

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